Empirical analyses of the effectiveness of interventions in the foreign exchange market in Brazil

Fernando Nascimento Oliveira[†]

Abstract This paper discusses the effectiveness in Brazil of the traditional instrument of exchange rate intervention (spot interventions) as well as an instrument based on exchange rate derivatives (foreign exchange swaps). We show that these instruments are capable of affecting the conditional mean of the process of the nominal exchange rate throughout our sample period, from January 2006 to April 2016. Our results are robust to different techniques of estimation (GMM in continuous time and in discrete time), specifications and sample periods.

Keywords: Central Bank; Intervention in the foreign exchange market; Spot interventions; Foreign exchange derivatives

JEL Code: E58, F31, E52.

1. Introduction

Our objective, in this paper, is to analyze the effectiveness of the instruments that the Central Bank of Brazil (hereafter CBB) adopted to intervene in the foreign exchange market from January 2006 to April 2016. The instruments of intervention we consider are spot market operations and foreign exchange swap operations.

A first step to analyze the effectiveness of the central bank instruments is to measure the effect of different instruments in the dynamics of the foreign exchange rate. To do this, we specify a stochastic process of the foreign exchange rate, introduce the interventions of the central bank and estimate the process with the interventions.

We follow a literature of interventions of central banks in the foreign exchange market (Sarno and Taylor; 2001) and another literature that models the dynamics of foreign exchange rates using continuous time models (Erdemlioglu et al.; 2015). This latter literature describes these models as diffusion or jump-diffusion processes.

In this paper, we model the dynamics of the foreign exchange rate as jump-diffusion processes, considering spot and foreign exchange swap interventions of the CBB as jump processes. In this strategy, the instruments of

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[†]Central Bank of Brazil and IBMEC/RJ, Brazil: fernando.nascimento@bcb.gov.br

intervention are effective if they affect the conditional mean of the process of the foreign exchange rate. In doing so, we follow [Sarno and Taylor \(2001\)](#) and [Edison \(1993\)](#).¹

To estimate the parameters of the dynamics of the foreign exchange rate, we use the work of [Hansen and Scheinkman \(1995\)](#). The authors obtain moment conditions that allow the use of the Generalized Method of Moments (GMM). To test the robustness of the results we use different specifications, sample periods and a more common technique of estimation that transforms the continuous process into a discrete process ([Chan et al.; 1992](#)).

Our results show that the instruments used by the CBB were effective throughout our sample period. The results are robust to different specifications, sample periods and to the techniques of estimation mentioned above. The results are in line with those of other papers that analyze the effectiveness of foreign exchange interventions in Brazil on the level of the foreign exchange rate, such as [Meurer et al. \(2010\)](#), [Chamon et al. \(2017\)](#) and [Kohlscheen and Andrade \(2014\)](#).

The remainder of the paper is organized as follows. Section 2 briefly discusses the literature on interventions in foreign exchange markets by central banks. Section 3 describes the policy of interventions of the CBB in our sample period and the data that we use. Section 4 discusses the models that we estimate. Section 5 presents the results of the empirical analyses and Section 6 concludes.

2. Review of the literature on foreign exchange rate interventions

There are two ways by which central banks can intervene in the foreign exchange market: nonsterilized interventions, those that alter the domestic monetary base; and sterilized interventions, which do not alter the monetary base.

According to [Sarno and Taylor \(2001\)](#), foreign exchange intervention carried out by a monetary authority can be effective in influencing the exchange rate if it affects its conditional mean or the conditional volatility of the foreign exchange rate. Sarno and Taylor suggest that the effectiveness of intervention depends on the degree of industrialization of the economy, the dependence on the international capital market and the degree of substitution between financial assets with the appreciation or depreciation of the currency.

Central banks may announce transactions in the foreign exchange market, known as the coordination channel ([Bryant; 1995](#)). They can signal the goals

¹These authors also consider interventions effective if they impact the conditional volatility of the foreign exchange rate.

they want to achieve, to influence the formation of expectations (signaling channel) (Mussa; 1981). Finally, they may also vary the amount of financial assets indexed to the existing dollar in the market and, thus, change the relative value of international assets vis-à-vis domestic assets (portfolio channel) (Dooley and Isard; 1983).

Dominguez (1998) finds that intervention policies always influence the conditional variance of the exchange rate; effects on volatility depend on the type of intervention. Estimated regressions showed that hidden interventions increase the volatility of the currency, while those made public during the analysis period lead to a reduction in volatility.

Domac and Mendoza (2004) study whether the measures carried out by the Central Bank of Mexico and the Central Bank of Turkey impact the conditional volatility of the exchange rate, since the adoption of floating exchange rate regimes in these countries. They used E-GARCH model and their empirical findings are that the volume and frequency of interventions decrease exchange rate volatility in these countries. These results corroborate the theory that currency intervention does not aim to defend a particular exchange rate, but it can be an important ally to contain adverse effects of temporary shocks in the exchange rate on inflation and financial stability.

Empirical studies on the effectiveness of interventions in emerging economies may differ from the results of studies on the most developed economies. This difference is due to the complexity of exchange rate regimes, the sophistication of the market and the regulatory systems of foreign transactions. In Brazil, for example, intervention tools are quite atypical, not just the traditional instruments of intervention such as the purchase and sale of foreign currency in the spot market or changes in interest rates. The CBB also influences the exchange rate through foreign exchange derivatives such as foreign exchange swaps.

The empirical literature on the effectiveness of interventions of the CBB on the foreign exchange market is still rather sparse. Papers that are more in line with ours include those of Meurer et al. (2010), Chamon et al. (2017) and Kohlscheen and Andrade (2014).

Meurer et al. (2010) show that the interventions affect foreign exchange dynamics, however they are more effective in periods of lower volatility of the foreign exchange rate.

Chamon et al. (2017) estimate the effects of announcements of swap programs by the CBB from August 22, 2013 to December 19, 2013. Their conclusion is that the initial announcement generated a persistent foreign exchange appreciation in the first weeks. However, daily interventions that occurred in the rest of the period did not create any additional impact on the

foreign exchange rate.

[Kohlscheen and Andrade \(2014\)](#) use high-frequency data to study the effects of currency swap auctions by the CBB on the spot exchange rate. The authors find that official currency swap auctions affect the level of the exchange rate, even though they do not directly alter the supply of foreign currency in the market. The maximum impact occurs 60 to 70 minutes after the initial official announcement of an auction, and again typically shortly after the results of the auctions become public.

Other papers analyze different effects of intervention of the CBB on the foreign exchange market, such as those on the conditional volatility of the exchange rate and on the level and volatility of futures of foreign exchange rate. In the case of the former, [Oliveira and Plaga \(2011\)](#) use daily data of nominal exchange rate and the interventions for the period from January 1999 to September 2006. In all sample periods studied, including those with currency crises, some instrument of intervention affects the conditional volatility of the nominal exchange rate.

Regarding effects on future contracts of the foreign exchange rate, [Janot and Macedo \(2016\)](#) estimate the effects of CBB interventions on the return and volatility of the BRL/USD future exchange rate using intraday data from October 2011 to March 2015. Their results show that interventions not anticipated by the market affect the level of exchange rate persistently. They also show that the effects vary with the size of the intervention, and that higher volumes of intervention affect the exchange rate more strongly than minor volumes of interventions.

[Nakashima \(2012\)](#) shows the existence of abnormal returns in future contracts of foreign exchange rate near the foreign exchange auctions of the CBB. On the other hand, [Nogueira \(2014\)](#) finds that interventions of the CBB had an impact of the foreign exchange future only when they happened in an unexpected way.

3. Data

To investigate the effectiveness of the CBB's interventions in the foreign exchange market we use daily data. Our sample runs from January 02, 2006 to April 20, 2016. The whole sample period contains 2581 observations. Figure 1 shows how the foreign exchange rate evolved in this period.

The first type of intervention that we consider is spot interventions. We use the official daily spot interventions of the Central Bank of Brazil. We observe 1,209 interventions of this type. Of these, 1,160 are interventions in which the CBB supplied dollars to the market and 49 are interventions

Figure 1
Foreign exchange rate (R\$/US\$)

Figure 2
Spot interventions (millions \$)

where the CBB bought dollars from the market. Figure 2 shows the date and intensity of the interventions in dollars.²

²The data on spot interventions is confidential.

Figure 3
Foreign exchange swap auctions: CBB in the sell side (millions of US\$)

The second type of intervention we consider is foreign exchange swaps. In swap operations, the CBB exchanges the first difference of interest rate in a certain period with the depreciation (or appreciation) of the foreign exchange rate in the same period. In these operations, the CBB can be either short or long in the depreciation of foreign exchange rate.

We use the net daily supply of swap auctions as the intensity of the intervention. Our information comes from the Open Market Department of the CBB. In our sample period, there are 853 interventions with this instrument. Of these, 721 are ones in which the CBB is short on the depreciation of foreign exchange rate and 132 are those in which the CBB is long on the depreciation of foreign exchange rate. Figure 3 displays the interventions with this instrument.

4. The dynamics of the foreign exchange rate

The main objective of this paper is to study the effectiveness of distinct instruments of intervention in the foreign exchange market. To do this, we need a model that is sufficiently flexible to describe the dynamics of the nominal exchange rate with and without the intervention of the central bank.

Although the literature on foreign exchange rates has not converged to a standard model for its dynamics, a class of models based on continuous time has gained relevance in recent years. Among several advantages, with these models one can obtain closed expressions for the moments of the process that

allow for statistical tests related to the impact of the interventions.

As Erdemlioglu et al. (2015) stress, decomposing foreign exchange into jump-diffusion processes or simply into diffusion processes is important because these two components imply different modeling and hedging strategies. The authors investigate which of these types of continuous-time models best describe intraday exchange rate fluctuations. In particular, they differentiate between processes that have finite jumps, that is, a finite number of jumps in a time interval, from those models with an infinite number of jumps in a time interval.

We model the dynamics of the depreciation of the nominal exchange rate dX_t by the following process belonging to the class of diffusion processes with finite jumps:

$$dX_t = (\mu X_t + K_i) dt + \sigma dW_t + \sum_f I_t^f dJ_t^f \quad (1)$$

where i represents SELIC-FedFunds; I_t^f is the intensity of spot and swap interventions of CBB and J_t^f represents the Poisson process corresponding to spot and swap interventions.

Intuitively, the multiplicative term dt is the predetermined trend of the process. This trend can be decomposed into two components, the first of which reflects the impact of the depreciation of the nominal exchange rate in time t .

The second term of the drift reflects the interest rate differential between Brazil and the rest of the world, measured by the difference between effective SELIC and effective FedFunds (both measured annually). Economic theory addresses the relationship between interest rate and exchange rate mainly through the Uncovered Interest Parity (UIP). It postulates that similar assets in different currencies should have the same rate of return, taking into account perfect capital mobility. Therefore, the interest rate differential between two currencies is understood as reflecting depreciation expectations of the high-yield currencies against the low-yield currencies. Thus, we expect the sign of the coefficient of this interest rate differential to be negative. Figure 4 displays the SELIC rate, FedFunds rate and their difference in our sample period.

Two types of shocks disturb the predetermined trend. The first one, σdW_t , is not predictable. The second refers to the discontinuous interventions of CBB in the foreign exchange market. These discontinuous interventions are modeled as Poisson processes. The Poisson processes are such that there is a probability $\lambda_f dt$, (f equal to the spot, or swap) of a discontinuous intervention occurring, and once it occurs its intensity is known to be I_t^f .

Figure 4
SELIC, FedFunds, SELIC-FedFunds

From Equation (1), we have the following conditional expectation in t of the depreciation of the nominal exchange rate:

$$\mathbf{E}_t[dX_t] = (\mu X_t + K i_t) dt + \lambda^{\text{spot}} I^{\text{spot}} + \lambda^{\text{swap}} I^{\text{swap}}. \quad (2)$$

Equation (2) shows that the expected depreciation of the nominal exchange rate is a function of the foreign exchange rate, the difference between SELIC and FedFunds, the probability of interventions in the spot market, the intensity of spot market interventions, the probability of interventions in the foreign exchange market, and the intensity of swap interventions.

If the probability associated with one of the non-continuous instruments is statistically significant, then we can affirm that this instrument is capable of affecting the conditional expectation of the nominal exchange rate. For example, if in t the depreciation is 0.2%, and the central bank supplies dollars in a value corresponding to R\$10 million, and the probability of the foreign exchange intervention is 10^{-6} , then the supply of foreign exchange by the central bank implies an increase in the appreciation (or a decrease of expected depreciation) of 2.5%.

An effective policy, in the above sense, does not only affect the conditional expectation of the nominal exchange rate in the next period. Realizing that this expectation is affected by interventions, the market tries to infer the frequency and magnitudes of interventions. So, as indicated by [Krugman \(1991\)](#), [Cadenillas and Zapatero \(2000\)](#), and [Jorion \(1988\)](#), the policy of interventions affects all dynamics of the exchange rate.

We assume that the policy of interventions in the foreign exchange market by the CBB is known by market participants. Therefore, we consider them exogenous relative to the foreign exchange rate dynamics. We think that despite being a strong hypothesis, it is reasonable, taking into account the adoption of Inflation Targeting (hereafter IT) by the CBB in 1999.

Several central banks worldwide have adopted IT as their monetary policy in recent years. In IT, central banks that respond endogenously to movements in the exchange rate take the risk of transforming it into a nominal anchor for monetary policy that takes precedence over IT.

One possible way to avoid this problem for central banks that adopt IT is to have transparent mechanisms which can ensure that policies to influence the exchange rate are aimed only at smoothing the impact of temporary shocks. In other words, in an IT regime, interventions of a central bank in the foreign exchange markets should be communicated in advance to market participants in a clear and transparent way. That is, they should be known and should not reflect any intention of central banks to achieve a certain optimum level of the foreign exchange rate.

We think CBB has been following the strategies of interventions in the foreign exchange market of central banks that adopt IT described above.³ This is the main justification of our hypothesis that the market knows the decision process of the CBB that generates the Poisson process of interventions in the foreign exchange market. Thus, the process of interventions is a predetermined variable of the model and our main interest is to know if the depreciation of the nominal exchange rate is affected by a given (known) intervention.

This hypothesis of predetermining the policy of interventions allows us to ignore problems of endogeneity of the estimation of the parameters λ_f . An estimate statistically significantly equal to zero – our null hypothesis – must be understood as evidence that the policy of interventions does not affect the conditional expected depreciation of the nominal exchange rate instead of the more natural interpretation that the probability of intervention of the central bank is zero.

³All governors of the CBB since the adoption of IT have made clear in their official communications that the CBB does not have a target for the level of the foreign exchange rate. Anecdotal evidence also shows that the CBB has been quite transparent and predictable in its objectives and in the amount of interventions in the foreign exchange market since the implementation of IT in Brazil in 1999.

5. Empirical analyses

5.1 Hansen and Scheinkman (1995) method

Estimating a stochastic process in continuous time, as in Equation (1), usually involves some discretization method, such as the Euler approximation. However, the resulting discrete-time specification is merely an approximation of the continuous-time process. Lo (1988) shows that the maximum likelihood estimator of the discrete-time process is not consistent, in general.

Hansen and Scheinkman (1995) propose a method to estimate continuous-time processes, which does not resort to discretization by exploiting the properties of the infinitesimal generator of moments in stationary Ito processes. Intuitively, the infinitesimal generator of a continuous-time stochastic process indicates the slope of the stochastic process at a certain point. Or rather, the generator reflects the trend (or dragging) of the process at a certain point. Or rather, the generator is the trend (or dragging) of the process at a certain point. For example, in the process of diffusion, $dX_t = \mu_t dt + \sigma_t dW_t$, the infinitesimal generator of the original process, $A(X_t)$, is μ_t .

In the same manner, the infinitesimal generator applied to a function of the process provides the trend of the function of this process at that point. By means of an appropriate choice of these functions, known as test functions, Hansen and Scheinkman (1995) obtain the moment conditions that allow the application of the Generalized Method of Moments (GMM) to estimate the parameters of the original process.

Hansen and Scheinkman (1995) choose only stationary test functions and prove that the infinitesimal generator of these functions satisfies the first class of the moments below. The intuition for that class is that the average of the slopes of a stationary process should be zero. More formally,

$$\mathbf{E}[A\phi(X_t)] = 0, \quad (3)$$

where $\phi(X_t)$ is a test function and $A\phi(X_t)$ is the infinitesimal generator of the test function.

A second class of moment conditions from Hansen and Scheinkman (1995) arises from a property of the infinitesimal generator. The property is that the infinitesimal generator commutes with the conditional expected operator one period ahead. More precisely, Hansen and Scheinkman prove that

$$\mathbf{E}_t[A\phi(X_{t+1})] = A\mathbf{E}_t[\phi(X_{t+1})]. \quad (4)$$

To implement conditions (3) and (4), Hansen and Scheinkman (1995) write the moment conditions in the form of their equivalent samples and find the

Table 1
Estimation of foreign exchange rate with Hansen and Scheinkman (1995): Absolute value of swap auctions and interest rate differential

The horizon of estimation extends from January 2006 to April 2016. The data are daily. The moment conditions were obtained taking in consideration the two classes of moment conditions (C1) e (C2), described in the text, using X and X^2 as test functions. We fix $\sigma = 1$ for identification purposes. Presented below the statistics, in parentheses, are the p -values.

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
μ	0.000735 (0.00)	0.001264 (0.00)	0.000219 (0.00)
K	−0.000045 (0.00)	−0.000122 (0.00)	-5.87×10^{-7} (0.72)
λ_{spot}	−4.52 (0.00)	−2.75 (0.00)	−3.80 (0.00)
λ_{absswap}	−2.93 (0.00)	−1.06 (0.00)	−3.10 (0.00)
J statistic	0.40	0.73	0.90
obs.	2518	647	1871

estimators using GMM. For models similar to ours (Equation (1)), the authors suggest as test functions X and X^2 , which generate four moment conditions to estimate all the models. We fix $\sigma = 1$ to identify the model and work with these four conditions. Appendix 1 discusses in detail the technique used in Hansen and Scheinkman (1995) and Appendix 2 derives the infinitesimal generators and the moment conditions for the estimated models in this paper.

5.2 Estimation of the dynamics of the foreign exchange rate by Hansen and Scheinkman (1995)

5.2.1 Main empirical analyses

In this subsection, we estimate the dynamics of the depreciation of the nominal exchange rate (1), using the Hansen and Scheinkman (1995) method. We estimate using the whole sample period; the period before the subprime crisis, from February 1, 2006 to September 15, 2008; and after the subprime crisis from September 16, 2008 to April 28, 2016.⁴

Initially, we use absolute values of swap operations of CBB. We present the results of this estimation in Table 1. We also estimate the model, considering swap interventions in which the CBB is short or long in the foreign exchange rate. The results of this estimation are reported in Table 2.

⁴The subprime crisis affected Brazil mainly after the demise of Lehman Bros, in September 2008. Therefore, September 2008 is a more natural breakpoint.

Table 2
Estimation of foreign exchange rate with Hansen and Scheinkman
(1995): Discriminating between long and short position in foreign
exchange swaps and interest rate differentials

The horizon of estimation extends from January 2006 to April 2016. The data are daily. The moment conditions were obtained taking in consideration the two classes of moment conditions (C1) e (C2), described in the text, using X and X^2 as test functions. We fix $\sigma = 1$ for identification purposes. Presented below the statistics, in parentheses, are the p -values.

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
μ	5.96×10^{-8} (0.50)	6.13 (0.00)	7.54×10^{-6} (0.00)
K	-7.71×10^{-9} (0.81)	-9.91×10^{-6} (0.00)	-1.21×10^{-6} (0.00)
λ_{spot}	4.95×10^{-4} (0.07)	5.46×10^{-1} (0.00)	-8.35×10^{-2} (0.00)
$\lambda_{\text{swap_short}}$	-4.36×10^{-11} (0.00)	-5.23×10^{-12} (0.00)	-4.80×10^{-2} (0.00)
$\lambda_{\text{swap_long}}$	-2.93×10^{-9} (0.00)	2.32×10^{-12} (0.00)	-6.59×10^{-6} (0.00)
J statistic	0.41	0.03	0.094
obs.	2518	647	1871

Tables 1 and 2 show the effectiveness of the instruments used by the CBB independent of the period we choose to estimate. They confirm the relevance of the interventions in the spot market of foreign currencies and currency swaps to affect the conditional expectation of the depreciation of the nominal exchange rate. At least, one of the instruments is effective, in the periods considered, as can be noted by the p -values of the λ probabilities associated with the Poisson jumps of the intervention and of the interest rate. The J -statistics do not reject the specifications of the models in any of the periods that we consider.

The coefficient of the nominal exchange rate, μ , is positive and statistically significant in all estimations. The coefficient of the interest rate differential is negative and statistically significant in all estimations.

5.3 Robustness analyses

To investigate the robustness of our results, we consider alternative models for the dynamics of the depreciation of the nominal exchange rate, as well as Monte Carlo experiments, and using a more common GMM technique based on the discretization of the process.

Table 3
Estimation of foreign exchange rate with Hansen and Scheinkman
(1995): Absolute value of swap auctions and SELIC

The horizon of estimation extends from January 2006 to April 2016. The data are daily. The moment conditions were obtained taking in consideration the two classes of moment conditions (C1) e (C2), described in the text, using X and X^2 as test functions. We fix $\sigma = 1$ for identification purposes. Presented below the statistics, in parentheses, are the p -values.

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
μ	0.00061 (0.00)	0.0011 (0.00)	0.00056 (0.00)
K	−0.0000011 (0.20)	−0.000051 (0.00)	−0.000066 (0.00)
λ_{spot}	−4.67 (0.00)	−2.48 (0.00)	−2.36 (0.00)
λ_{swap}	−3.15 (0.00)	−1.14 (0.00)	−6.87 (0.00)
J statistic	0.38	0.76	0.78
obs.	2518	647	1871

5.3.1 Alternative specifications of the depreciation of the exchange rate

Tables 3 and 4 present the results of the estimation of the basic model of the dynamics of the foreign exchange rate, Equation (1), substituting the interest rate differential by the SELIC rate.

The SELIC rate should not be considered an instrument of intervention in the foreign exchange market. The reason is that the Central Bank of Brazil, in our sample period, was on an IT regime, as we mentioned above, and had as its main objective when using the interest rate keeping inflation on target and not intervening in the foreign exchange market. Of course, as is well known in the literature (Sarno and Taylor; 2001), interest rate movements are correlated to the dynamics of foreign exchange, and that is the reason we include it on the drift part of our continuous model in our robustness exercises.

Tables 3 and 4 again show the effectiveness of the instruments used by the CBB, independent of the period chosen, the coefficient of the foreign exchange rate is positive and statistically significant, and the coefficient of the SELIC rate is negative and statistically significant in all estimations.

We also estimate a diffusion model with only the drift term of Equation (1). The results of the estimation are presented in Table 5 for the different sample periods. They show once more that the estimated coefficient of the foreign exchange rate is positive and statistically significant, while the estimated coefficient of the interest rate differential is negative and statistically significant in all sample periods.

Table 4
Estimation of foreign exchange rate with Hansen and Scheinkman (1995): Discriminating between long and short position in foreign exchange swaps and SELIC

The horizon of estimation extends from January 2006 to April 2016. The data are daily. We estimate parameter $\sigma = 1$. The moment conditions were obtained taking in consideration two classes of moment conditions (C1) e (C2), described in the text, using X and X^2 as test functions. Presented below the statistics, in parentheses, are the p -values.

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
μ	2.50×10^{-6} (0.41)	0.0028 (0.00)	-0.097 (0.00)
K	-6.35×10^{-7} (0.40)	-0.00033 (0.00)	-0.039 (0.00)
λ_{spot}	1.82×10^{-8} (0.36)	-2.14 (0.00)	-4.61×10^{-1} (0.00)
$\lambda_{\text{swap_short}}$	-3.10×10^{-12} (0.00)	-8.65×10^{-8} (0.04)	-2.55×10^{-12} (0.17)
$\lambda_{\text{swap_long}}$	-8.53 (0.00)	-4.92×10^{-10} (0.00)	1.44×10^{-12} (0.62)
J statistic	0.011	0.36	0.64
obs.	2518	647	1871

Table 5
Estimation of a diffusion process of the foreign exchange rate with Hansen and Scheinkman (1995)

The horizon of estimation extends from January 2006 to April 2016. The data are daily. We estimate parameter $\sigma = 1$. The moment conditions were obtained with two classes of moment conditions (C1) e (C2), described in the text, using X and X^2 as test functions. Presented below the statistics, in parentheses, are the p -values.

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
μ	0.073 (0.00)	0.0990 (0.00)	0.093 (0.00)
K	-0.034 (0.00)	-0.0036 (0.00)	0.038 (0.00)
J statistic	1.00	1.02	0.99
obs.	2518	647	1871

5.3.2 Testing for endogeneity of the intervention instruments

In another attempt to test the robustness of our main results, we verify if the hypothesis of exogeneity of our intervention instruments impact our main results. We do this in three different ways. In the first two, we do two different Monte Carlo experiments. In the third, we estimate Equation (1) from August 22, 2013 until the end of the sample period. In this period, the CBB announced its interventions officially and prospectively.

In the first Monte Carlo experiment, we generate 2000 draws of the series of spot and swap interventions, not taking into account the possibility of endogeneity of interventions with these instruments. For each draw, we estimate Equation (1) using Hansen and Scheinkman (1995). Table 6, Panel A, presents the average coefficients of all estimations (p-values inside the parentheses). The results are very similar in qualitative terms to the previous ones, and once again show the effectiveness of at least one form of intervention in each of our sample periods.

In the second Monte Carlo experiment, we assume that the central bank intervenes in the spot and swap markets conditional on its expectations of depreciation of the nominal exchange rate. We consider perfect foresight and that the central bank is leaning against the wind, that is if the depreciation is higher than the average of the depreciation in the whole period plus one standard deviation of this series, the central bank is on the long side of swap and spot operations. On the contrary, if the depreciation is less than the average of the depreciation of the nominal foreign exchange rate minus one standard deviation of this series, the CBB is on the short side of swap and spot operations.

Table 6, Panel B, presents the average coefficients of all estimations (p-values under parentheses). Once again, the results are very similar in qualitative terms to the previous ones and reflect the effectiveness of at least one form of intervention in our sample periods.

In a third attempt to test the exogeneity hypothesis, we estimate Equation (1) in the period that we know for sure that the CBB was announcing its interventions. The results presented in Table 6, Panel C, exhibit the effectiveness of swap interventions in our sample periods.

5.3.3 Test of robustness using the method of differentiation

In a final attempt to test the robustness of our results, we estimate the process of the depreciation of the foreign exchange rate, using the most common GMM technique based on the discretization of the process.⁵

⁵See Chan et al. (1992) for some examples of the use of this technique.

Table 6
Estimation by Hansen and Scheinkman (1995): Monte Carlo estimations

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
Panel A: Monte Carlo with exogenous interventions			
μ	0.23 (0.00)	−0.00325 (0.179)	0.000883 (0.84)
K	-7.50×10^{-5} (0.00)	0.00192 (0.09)	−0.00062 (0.179)
λ_{spot}	2.98×10^{-6} (0.00)	−9.30 (0.00)	−8.86 (0.00)
λ_{swap}	-9.67×10^{-2} (0.00)	−1.40 (0.00)	−1.60 (0.00)
Panel B: Monte Carlo with endogenous interventions			
μ	0.067 (0.32)	−0.0880 (0.00)	−0.090 (0.24)
K	−0.029 (0.073)	−0.0053 (0.00)	−0.037 (0.04)
λ_{spot}	−1.400 (0.00)	−0.2700 (0.0)	−0.120 (0.0)
λ_{swap}	-9.70×10^{-2} (0.00)	−0.0760 (0.00)	−0.021 (0.00)
Panel C: Estimation of jump-diffusion of foreign exchange rate with Hansen and Scheinkman (1995) from August 22, 2013 until the end of the sample period			
μ	0.00038 (0.32)		
K	−0.00021 (0.02)		
λ_{swap}	−3.02000 (0.00)		
R2	0.03		
obs.	657		

The stationary process in continuous time is

$$dX_t = (\mu X_t + K i_t) dt + \sigma dW_t + \sum_f I_t^f dJ_t^f. \quad (5)$$

We discretize this process according to the Euler approximation, subdividing the time interval in 252 parts ($h=1/252$), as follows:⁶

$$X_{t+1} - X_t = (\mu X_t + K i_t) h + \sum_f I_t^f dJ_t^f + \varepsilon_{t+1} \quad (6)$$

where $J_{t+1}^f - J_t^f = 1$ with probability λ_f and equals zero with probability $1 - \lambda_f$, and where f = spot or foreign exchange swaps. In order to discretize

⁶As shown by Lo (1988), this differentiation may lead to inconsistent estimators.

Table 7
Estimation using the method of differentiation: OLS estimation

The horizon of estimation extends from January 2006 to April 2016. The data are daily. We estimate the model discretized by OLS. Presented below the statistics, in parentheses, are the p -values.

	whole sample 02/01/2006–04/28/2016	before subprime 02/01/2006–09/15/2008	after subprime 09/16/2008–04/28/2016
μ	0.000859 (0.58)	0.004800 (0.09)	0.0002744 (0.83)
K	–0.000045 (0.85)	–0.000543 (0.17)	0.0002100 (0.49)
λ_{spot}	–0.000010 (0.00)	–1.110000 (0.00)	–1.0200000 (0.01)
λ_{absswap}	0.251000 (0.83)	0.710000 (0.57)	0.2300000 (0.88)
J statistic	0.04	0.03	0.0078
obs.	2518	647	1871

the process (6), we must impose that

$$\mathbf{E}[\varepsilon_{t+1}] = 0 \quad \text{and} \quad \text{Var}(\varepsilon_{t+1}) = \sigma^2.$$

In Table 7, we present the results of the estimation with this technique for the two specifications of Section 4.1. The results are clearly in line with those we obtain using Hansen and Scheinkman (1995). They confirm the relevance of the interventions with the spot market operations and currency swap operations in all our sample periods.

6. Conclusion

This paper studies the effectiveness of the interventions of the CBB in the foreign exchange market from January 2006 to April 2016. We estimate several models in continuous time of the foreign exchange rate, using two distinct econometric techniques: GMM according to Hansen and Scheinkman (1995), and GMM based on the differentiation process.

We show that the instruments of intervention were effective in the sample periods we study, in the sense that they affected the conditional mean of the process of the foreign exchange rate.

One relevant question not discussed in this paper is the analysis of the efficiency of these interventions, that is, the cost-benefit relationship of these interventions. Discussions surrounding this question involve a definition of a loss function of the central bank in the foreign exchange market and an optimal choice of instruments of intervention in order to minimize the loss func-

tion. We think that our paper can help future authors to respond appropriately to issues related to cost-benefit analyses of foreign exchange interventions.

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A. Continuous time GMM estimation – Hansen and Scheinkman (1995)

A.1 Introduction

The Hansen and Scheinkman (1995) technique is different from the usual practice of GMM because of the following reasons: the asymptotic identification is not obvious; the identification in small sample, uniqueness and the existence of an estimator is not an autonomous problem with respect to the asymptotic identification; the lack of asymptotic identification is such that implies lack of small sample identification.

The Hansen and Scheinkman (1995) paper can be divided in two parts. In the first part, the authors' intent is to characterize the infinitesimal generator and to find the moment conditions related to this generator. In the second part, the authors analyze asymptotic proprieties of the infinitesimal generator and find moment conditions that assure that the law of large numbers and the central limit theorem apply to processes whose samples are discrete and obtained in regular intervals. In this appendix, we will discuss questions related to the first part of the paper.

Hansen and Scheinkman (1995) is used for the estimation of strictly stationary continuous time processes, observed in regular discrete time intervals whose frequencies are normalized to one.

A.2 Concept of an infinitesimal generator

A.2.1 Definition of $L^2(Q)$

Let Q be the true distribution of probability of the stationary stochastic process X_t defined over $\mathbf{R}^n$: then $L^2(Q)$ denotes the set of Q -square integrable functions from $\mathbf{R}^n$ to $\mathbf{R}$.

A.2.2 Definition of a semigroup T_t

$T_t, t \geq 0$ is a semigroup over $L^2(Q)$ if and only if:

- (i). T_t is a linear operator on $L^2(Q), t \geq 0$;
- (ii). $T_{t+s} = T_t T_s, t, s \geq 0$.

Given a semigroup T_t over $L^2(Q)$, we can define:

A.2.3 Definition of a set of test functions ϕ

$$D = \left\{ \phi \in L^2(Q) : \lim_{t \rightarrow 0} \frac{T_t \phi - \phi}{t} \text{ exists} \right\}.$$

A.2.4 Definition of an infinitesimal generator $A\phi$

$$A\phi = \lim_{t \rightarrow 0} \frac{T_t \phi - \phi}{t}, \quad \forall \phi \in D. \quad (7)$$

The infinitesimal generator defined in (7) describes locally the stationary stochastic process in continuous time. The motivation for the moment conditions based on the local behavior of the continuous process X_t , characterized by the infinitesimal generator is derived from the fact that this behavior is sometimes specified in the first place. Normally, we start with the stochastic differential that must be satisfied for X_t . There is a known correspondence between the coefficients of the equation and the infinitesimal generator.

The main advantage to characterize the moment conditions in terms of infinitesimal generators is the fact that it is not necessary for computing the sample analog that transition functions of the process be found. This would require that partial differential equations be solved or conditional density functions of the process be built.

A.3 Infinitesimal generator characteristics and other definitions

A.3.1 Continuity

We will consider strictly stationary processes with marginal distribution q and semigroups or correspondent transition functions⁷ $\{X_t : t \geq 0\}$:

A.3.2 Definition of T_t and T_t^*

$$\begin{aligned} T_t &= \mathbf{E}[\phi(X_{s+t} | X_s = y)] \\ T_t^* &= \mathbf{E}[\phi(X_s | X_{s+t} = y)] \end{aligned}$$

$A\phi$ is the true infinitesimal generator of the process X_t , using the semigroup T_t above.

A.4 Moment Conditions

The moment conditions suggested by Hansen and Scheinkman (1995) for the true candidates of the true infinitesimal generator of the continuous stochastic process are:

$$\mathbf{E}^Q [\hat{A}\phi(X_t)] = 0, \quad \forall \phi \in \hat{D} \quad (C1)$$

⁷In our paper, this process is the depreciation of the nominal exchange rate.

$$\mathbf{E}^Q [\hat{A}\phi(X_{t+1})\phi^*(X_t) - \phi(X_{t+1})\hat{A}^*\phi^*(X_t)] = 0, \quad \forall \phi \in \hat{D}, \phi^* \in \hat{D}^* \quad (C2)$$

$\hat{A}$ is the candidate generator, $\hat{A}^*$ is the adjoint operator of $\hat{A}$, $\hat{D}$ is a candidate set of test functions, $\hat{D}^*$ is the domain of $\hat{A}^*$, and Q is the true distribution of the process.⁸

Condition (C1) above explores the informational content of the generator candidate. The second condition eliminates the candidate whose marginal distributions are incorrect. The formal demonstrations for the two conditions can be found on the paper.

The inspiration or intuition for the first condition comes from the fact that X_t is a stationary process. In particular, it must be true that:

$$\mathbf{E}[\phi(X_t)] = \mathbf{E}[\mathbf{E}[\phi(X_t) | X_0]] = \mathbf{E}[T_t\phi(X_0)] \quad (8)$$

$$\frac{d}{dt}\mathbf{E}[\phi(X_t)] = 0 = \frac{d}{dt}\mathbf{E}[T - t\phi(X_0)] = \mathbf{E}[A\phi(X_0)] = \mathbf{E}[A\phi(X_t)] \quad (9)$$

A.5 Identification

The moment conditions are in the ideal format to be replicated by their sample counterparts. However, to understand the identification problem it is convenient to obtain a set of equivalent conditions. We show that:

Proposition A.1 X_t satisfies (C1) $\iff \hat{Q} = Q$, where $\hat{Q}$ is a candidate stationary distribution.

Proof ($\Leftarrow$) Let us suppose that Q is a stationary distribution and $\hat{Q} = Q$. Then, $\forall \phi \in L^2(Q)$,

$$\mathbf{E}[T_t\phi(X_0)] = \mathbf{E}[\mathbf{E}[\phi(X_t) | X_0]] = \mathbf{E}[\phi(X_t)] = \mathbf{E}[\phi(X_0)] \quad (10)$$

$$\mathbf{E}[\hat{A}\phi(X_t)] = \mathbf{E}[\hat{A}\phi(X_t) | X_0] = \mathbf{E}[\hat{A}\phi(X_0)] = \lim_t \frac{\mathbf{E}[T_t\phi(X_0)] - \mathbf{E}[\phi(X_0)]}{t} = 0 \quad (11)$$

($\Rightarrow$) To show the converse, consider the process $\{X_t, t \geq 0\}$ initialized with a distribution $\hat{Q}$ and having transition functions $\hat{T}$ and $\hat{T}^*$. We need to show that if $\hat{A}$ satisfies (C1) then it has a stationary distribution Q .

$$\int \hat{T}_s\phi \, ds \in \hat{D} \quad (12)$$

⁸ $L^2(Q)$ is a Hilbert space. $\hat{A}: L^2(Q) \rightarrow L^2(Q)$ is a bounded linear operator. Let ϕ be a fixed point of $L^2(Q)$. It can be shown because of the Riesz representation theorem (Naylor and Sell; 2000) that there is only one ϕ^* such that: $\langle \hat{A}x, \phi \rangle = \langle x, \phi^* \rangle$, where $\langle \cdot, \cdot \rangle$ is the inner product defined on $L^2(Q)$ and $\phi^* = \hat{A}^*\phi$. $\hat{A}^*$ is the adjoint operator and is unique.

$$\hat{A} \int \hat{T}_s \phi \, ds = \hat{T} \phi - \phi \quad (13)$$

Integrating the right side of the above equation with respect to Q we have

$$\int \hat{T} \phi - \phi \, dQ = \int \hat{A} \left[\int \hat{T}_s \phi - \phi \right] dQ = \mathbf{E}^Q \left[\hat{A} \int \hat{T}_s \phi \, ds \right] = 0 \quad (14)$$

In particular the above equation is valid for $\phi = 1_B$, B Borelian in $\mathbf{R}^n$. Opening the first left term we have: $\hat{Q}(B) - Q(B) = 0$, Q is the unconditional distribution of X_t . Q.E.D.

Proposition A.2 X_t satisfies (C2) $\iff \hat{A}T_1\phi(X_t) = T_1\hat{A}\phi(X_t)$, or $\hat{A}$ and T_1 commute.

Proof ($\Rightarrow$) Let us suppose that $\hat{A}$ satisfies (C1), hence there exists a stationary distribution Q and T_t , $t \geq 0$, is a semigroup in $L^2(Q)$. $\forall \phi \in \hat{D}, \phi^* \in \hat{D}^*$:

$$\mathbf{E} [\hat{A}\phi(X_{t+1})\phi^*(X_t)] = \mathbf{E} [\mathbf{E} [\hat{A}\phi(X_{t+1}) | X_t] \phi^*] = \mathbf{E} [T_1\hat{A}\phi(X_t)\phi^*] = \langle T_1\hat{A}\phi, \phi^* \rangle \quad (15)$$

$$\mathbf{E} [\phi(X_{t+1})\hat{A}^*\phi^*(X_t)] = [\mathbf{E} [\phi(X_{t+1}) | X_t] \hat{A}^*\phi^*] = [T_1\phi(X_t)\hat{A}^*\phi^*] = \langle T_1\phi, \hat{A}^*\phi^* \rangle \quad (16)$$

where $\langle \cdot, \cdot \rangle$ is the scalar product. Therefore (C2) can be written as:

$$\langle T_1\hat{A}\phi, \phi^* \rangle = \langle T_1\phi, \hat{A}^*\phi^* \rangle, \quad \forall \phi \in \hat{D}, \phi^* \in \hat{D}^*. \quad (17)$$

Therefore,

$$\langle \hat{A}T_1\hat{A}\phi, \phi^* \rangle = \langle T_1, \phi, \phi^* \rangle, \quad \text{or} \quad \hat{A}T_1\hat{A}\phi = T_1\phi. \quad (18)$$

This equality is only valid if $\forall \phi \iff \hat{A}$ and $\hat{T}_1$ commute.

($\Leftarrow$) $\hat{A}$ and T_1 commute in D then:

$$\langle T_1\hat{A}\phi, \phi^* \rangle = \langle \hat{A}T_1\hat{A}\phi, \phi^* \rangle = \langle T_1\phi, \hat{A}^*\phi^* \rangle, \quad \forall \phi \in \hat{D}, \phi^* \in \hat{D}^*. \quad (19)$$

Q.E.D.

Among the candidates with the right distribution, condition P2 eliminates the one that does not commute with the true generator T_1 . The true infinitesimal generator commutes with T_1 . Therefore, the candidate of infinitesimal generator must also commute. The two conditions must be used together for estimation and identification.

The identification may not be complete. In particular, KA commutes with T_1 . If there are multiple candidates with the same true marginal distribution and with infinitesimal generators that are multiples of one another then they must be equivalent.

A.5.1 Definition of reversible processes

Continuous reversible processes are those in which $A = A^*$, that is the infinitesimal generator is equal to its adjoint.

Hansen and Scheinkman (1995) show that for reversible processes there does not exist the phenomenon of aliasing. This phenomenon corresponds to the fact that different continuous stochastic processes seem identical when estimated by discrete samples. Diffusion scalar processes are reversible. However, processes that are a combination of diffusion and jump processes are not reversible (Duffie and Glynn; 2004).

A.6 Infinitesimal generator of jump-diffusion processes

We use as candidates for the infinitesimal generators of a jump diffusion process the sum of the candidates of infinitesimal generators of a diffusion process and the candidates of infinitesimal generators of the jump processes (Duffie and Glynn; 2004).

Proposition A.3 *The candidate infinitesimal generator above satisfies the moment conditions specified by Hansen and Scheinkman (1995).*

Proof (C1) Let

$$A\phi = A'\phi + A''\phi \quad (20)$$

where $A'\phi$ is the infinitesimal generator of the diffusion process and $A''\phi$ is the infinitesimal generator of the jump process.

We will show that the infinitesimal generator satisfies conditions A.1 and A.2. For A.1, $E^Q[A'] = 0$; $E^Q[A''] = 0$, thus $E^Q[A' + A''] = 0$. We must show that $AT_1 = T_1A$. $A = A' + A''$, therefore $AT_1 = A'T_1 + A''T_1 = T_1A' + T_1A'' = T_1[A' + A'']$.

The first equality comes from the linearity of the infinitesimal generator and the second equality derives from the fact that A' and A'' are the infinitesimal generators of the diffusion and jump processes respectively. The third equality derives from the linearity of T_1 . The extension to processes with more than one jump process is trivial. Q.E.D.

We model the jump process as Merton (Jorion; 1988): there is a probability that a jump occurs, and if it occurs its intensity is a random variable whose probability density function is lognormal.

A.6.1 Infinitesimal generators of continuous, ergodic diffusion processes

$$dX_t = \mu(X_t)dt + \sigma(X_t)dW_t, \mu(X_t), \sigma(X_t), dW_t \in \mathbf{R}.$$

$$A(\phi(X_t)) = \mu\phi'(X_t) + \frac{1}{2}\sigma^2\phi''(X_t) \quad (21)$$

A.6.2 Infinitesimal generators of jump processes with probability density functions of a Merton type

$$A\phi(X_t) = \lambda \frac{1}{v\sqrt{2\pi}} \int_S (\phi(X+Y) - \phi(X)) e^{-\frac{(y-\mu)^2}{2v^2}} dy \quad (22)$$

A.7 Ways of using Hansen and Scheinkman (1995)

We start with a parametric family of candidates, supposedly including the true candidate. We choose test functions and estimate the parameters of interest completely identifies by its sample counterparts. We find moment conditions to estimate the parameters of interest in all models presented in section 3.

A.8 The Choice of Test Functions.

[Hansen and Scheinkman \(1995\)](#) suggest a simple polynomial for the scalar diffusion with polynomials drifts. [Duffie and Glynn \(2004\)](#) also suggest simple polynomials for *jump*-diffusion processes in which the Laplace transform of the jump probability is known. This is the case of the processes we are modeling. Considering these facts we chose as test functions the simple polynomials X and X^2 .

The intuition for the choice of such functions is related to the fact that the drift of the process is a simple polynomial of the first degree. As the infinitesimal generator describes the process locally, infinitesimal generators built from these polynomials seem to be more adequate.

In appendix B, we show the infinitesimal generators for each test function as well as the respective moment conditions. To find the moment conditions we use the sample counterparts of propositions A.1 and A.2 above.

B. Infinitesimal generators of diffusion processes

B.1 Infinitesimal generator of diffusion processes

Test function X_t : $A\phi(X_t) = \mu X_t + K i_t$

Test function X_t^2 : $A\phi(X_t) = 2\mu X_t^2 + 2K X_t i_t + \sigma^2$

B.2 Infinitesimal generator of jump processes

f = spot or swap.

Test function X_t : $A\phi = \lambda_f \frac{1}{v_f \sqrt{2\phi}} \int_S ((X+Y) - Y) e^{(y-v_f)/(2v_f^2)} dy = \lambda_f \mathbf{E} \left[I_t^f \right].$

Test function X_t^2 : $A\phi = \lambda_f \frac{1}{v_f \sqrt{2\phi}} \int_S \left((X+Y)^2 - Y^2 \right) e^{(y-v_f)/(2v_f^2)} dy = 2\lambda_f \mathbf{E} \left[I_t^f \right] + \lambda_f \mathbf{E} \left[(I_t^f)^2 \right]$

B.3 Class of moment conditions (C1): $\mathbf{E}[A\phi] = 0$

Test function X_t : $\mathbf{E} \left[\mu X_t + K i_t + \lambda_{\text{spot}} \text{spot}_t + \lambda_{\text{swap}} \lambda_{\text{swap}} \right] = 0$;

Test function X_t^2 : $\mathbf{E} \left[2\mu X_t^2 + 2K X_t i_t + \sigma^2 + \sum_f 2\lambda_f \mathbf{E} \left[I_t^f \right] + \lambda_f \mathbf{E} \left[(I_t^f)^2 \right] \right] = 0$.

B.4 Moment conditions of class (C2) : $\mathbf{AT} = \mathbf{TA}$

Test function X_t :

$$\mu X_{t+1} + K i_{t+1} + \sum_f \lambda_f I_{t+1}^f = \mathbf{E}_t \left[\mu X_{t+1} + K i_{t+1} + \sum_f \lambda_f I_{t+1}^f \right];$$

Test function X_t^2 :

$$\begin{aligned} \mathbf{E}_t \left[2\mu X_{t+1}^2 + 2K X_{t+1} i_{t+1} + \sigma^2 + \sum_f 2\lambda_f \mathbf{E} \left[I_{t+1}^f \right] + \lambda_f \mathbf{E} \left[I_{t+1}^{f^2} \right] \right] \\ = 2\mu X_{t+1}^2 + 2K X_{t+1} i_{t+1} + \sigma^2 + \sum_f 2\lambda_f \mathbf{E} \left[I_{t+1}^f \right] + \lambda_f \mathbf{E} \left[I_{t+1}^{f^2} \right] \end{aligned}$$