Equity Valuation with Fuzzy Multicriteria Decision Analysis
(Análise Multicritério Nebulosa de Empresas)

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Hugo Ghiaroni de Albuquerque e Silva**

Abstract
We consider the problem of equity valuation. The use of fuzzy multicriteria decision analysis is proposed to solve the problem. The resulting methodology allows the use of the multiples most often calculated by equity analysts from audited balance sheets, with the addition of qualitative criteria, such as corporate governance, sustainability indicators and credit ratings, as well as risk measures (liquidity and market) based on trading prices and volumes. Also, the proposal facilitates incorporating uncertainty into the problem with the use of fuzzy mathematics. The resulting methodology proved to be robust and offered detailed information about expected performance under adverse scenarios, enhancing the decision-making process faced by equity analysts and portfolio managers. Numerical examples obtained with data from the Brazilian stock market are exhibited for illustrative purposes.

Keywords: Equity Valuation, Fuzzy Mathematics, Multicriteria Decision Analysis, Uncertainty.

JEL Codes: C44, C63, G11, M41

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1. Introduction

Equity valuation is the single most used methodology by equity portfolio managers when selecting assets for investment (Fabozzi & Markowitz, 2011). The traditional equity valuation allows analysts to compare different companies for possible investment based on multiples (i.e., quantitative indicators) obtained, most often, from audited balance sheets and economic projections (Stowe et al., 2014).

Given the growing sophistication of financial markets, the traditional equity valuation must be enhanced by the introduction of concepts such as corporate governance, risks (liquidity, market and credit) and sustainability. Therefore, equity valuation turns out to be a financial problem characterized by the existence of many quantitative and qualitative indicators that need to be computed and compared simultaneously.

The Multicriteria Decision Analysis (MCDA) offers methodologies to reach compromise solutions when several criteria are used to compare several alternatives (Belton, Stewart, 2002; Ehgott et al., 2010; Ishizaka, Nemery, 2013; Wallenius et al., 2008). MCDA is suited for equity valuation because it facilitates identifying the most promising investments when quantitative and qualitative indicators are adopted to compare companies.
Applications of MCDA to areas such as accounting and finance are well documented in the academic literature (Bana e Costa, Soares, 2010; Doumpos, Zopounidis, 2010; Doumpos, Zopounidis, 2011; Doumpos et al., 2016; Gomes et al., 2016; Hallerbach, Spronk, 2002; Lisboa, Duarte, 2013; Steuer, Na, 2003; Steuer et al, 2007; Xidonias et al, 2011; Xidonias et al, 2012; Yu et al, 2014; Zavadskas, Turskis, 2011; Zhang et al., 2014; Zopounidis, Doumpos, 1998; Zopounidis et al., 2015), although concentrated in credit rating, portfolio management and project analysis. In the case of equity valuation, the single attempt to address the problem so far has been documented in Duarte (2018).

Equity analysts cannot rely solely on historical data. They must forecast the multiples of the companies under analysis to obtain realistic results (Damodaran, 2012, Maginn et al., 2012). This step leads to the introduction of uncertainties into the problem. As an example, let us consider that the rating issued by Moody’s (Moody’s, 2015) for a company has been published as “Baa”. It is reasonable to assume that one year from now the rating will remain at this level, but it may well happen that Moody’s revises it upwards (for example, to “A”) or downwards (for example, to “Ba”), meaning that uncertainty is naturally present in this credit quality indicator. It becomes necessary to modify MCDA to incorporate uncertainties when applied to the equity valuation problem. In this work we rely on fuzzy mathematics (Bellman, Zadeh, 1970; Kahraman, 2008; Kaufmann, Gupta, 1985; Klir, Yuan, 1995; Zimmermann, 1991). Although there are several multicriteria methods that can be adapted to incorporate fuzziness, in this work, for illustrative purposes, we have chosen the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS), originally proposed in Hwang and Yoon (1981).

Our objective in this work is to propose and illustrate the use of a fuzzy MCDA to the equity valuation of Brazilian companies listed in the BM&FBOVESPA. We incorporate uncertainty in the valuation problem with the use of different membership functions for the fuzzy numbers, with varying levels of uncertainty, including asymmetries, all studied in a series of sensitivity analysis performed under different economic scenarios. We also extend the traditional equity valuation by the introduction of qualitative indicators in the analysis (related to corporate governance, sustainability and credit risk) and stock market data (related to volatility and trading volume), mixing them with the traditional multiples derived from balance sheets (usually related to profitability, liquidity and costs) which are frequently used by financial analysts, according to Stowe et al.
Also, we extend the proposal for equity valuation presented in Duarte (2018) in two ways: (a) by allowing risk measures and qualitative indicators to be introduced, given that only multiples were used in that previous work, and (b) by proposing a more general methodology that can be used when comparing companies from different economic sectors, contrary to what was presented in that article, which remained limited to a single economic sector (i.e., financial institutions). We also differ from Duarte (2018) in the fuzzy MCDA used to obtain the numerical examples: we rely on the Fuzzy TOPSIS method, instead of the Fuzzy TODIM method, with the former demanding a simpler computational implementation.

In terms of organization, in the second section we detail the data and the transformations that need to be imposed to the traditional equity valuation problem to incorporate fuzziness and multicriteria analysis. The third section displays the numerical results, including all studies with their sensitivity analyses. The last section brings in our final comments and future directions for the use of fuzzy mathematics and MCDA in equity valuation. Finally, in the appendix, we detail the multicriteria method used in the numerical analyses for those who may want to replicate our results.

2. The Methodology: Fuzziness, Multicriteria Analysis and the Data

The traditional equity valuation can be extended to incorporate fuzziness and multicriteria analysis the following way:

i. Select all the stocks of interest for possible investment. In the numerical examples presented later we concentrate on the nine largest market capitalizations in BM&FBovespa at the end of 2014: Ambev SA (ABEV3), Banco do Brasil SA (BBAS3), Banco Bradesco SA (BBDC4), BRF SA (BRFS3), Cielo SA (CIEL3), Itaú-Unibanco Holding SA (ITUB4), Companhia Brasileira de Distribuição (PCAR4), Petróleo Brasileiro SA (PETR4) e Vale SA (VALE5). It is important to mention that our proposal can handle any number of stocks but, given space limitation in this article, we focus only on these nine for illustrative purposes.

ii. Identify all the criteria (multiples, stock market data and qualitative indicators) to be used to compare the previously selected stocks. In the numerical examples, we adopt ten criteria for the sake of numerical illustration: five of the most traditional multiples used by stock analysts (Stowe et al., 2014), three risk indicators (market, liquidity and credit) and two related to the
organizational structure of the company (corporate governance and sustainability). Table 1 summarizes the ten criteria. It is opportune to mention that our proposal can handle any number of criteria but, given space limitation, we focus only on the ten shown in Table 1, for illustrative purposes.

iii. Establish the relative importance of the criteria previously defined. This is a critical step when using any MCDA, requiring attention from the decision maker (Figueira et al., 2005). In the numerical examples, we shall experiment with different levels of relative importance, making use of sensitivity analysis to that end.

iv. Collect and/or compute all the multiples and qualitative indicators for each criteria and company of interest. In this work, we have chosen to use only historical data, all obtained from commercial data feeders (Economatica and Bloomberg) or directly from audited balance sheets provided by the investors’ relations area of each company. The values for the criteria summarized in Table 1 are exhibited in Table 2.

v. Generate scenarios for the possible evolution of each criterion in the future. For example, it is important to understand how a projected scenario can alter the scores (and by consequence, the orderings) of the companies under analysis. We illustrate this step with the use of sensitivity analysis, which is crucial when trying to understand how small changes in the inputs can alter the results, as thoroughly illustrated by the numerical examples presented ahead.

vi. Apply a multicriteria method combined with fuzzy mathematics to the data to obtain the scores (and by consequence, the orderings). In this article we alter the original TOPSIS method to incorporate fuzzy numbers, finally resulting in a method we refer to simply as Fuzzy TOPSIS from now on. This fuzzy and multicriteria method allows the decision maker to experiment with a myriad of ways to handle uncertainty in the equity valuation problem, as illustrated later. The Fuzzy TOPSIS used in the numerical examples is detailed in the appendix, in order to keep the focus of this article on equity valuation (and not on the mathematical approach adopted in the numerical examples).
### Table 1. Criteria Used in the Numerical Examples

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Earnings Per Share (EPS) is a profitability multiple calculated as the ratio between the net income, after subtraction of dividends paid for preferred stocks, and the number of common stocks outstanding. It summarizes the company’s profit allocated to each common stock. The values in Table 2 are the medians obtained from quarterly data collected between the second quarter of 2010 and the first quarter of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Return on Equity (ROE) is a profitability multiple calculated as the ratio between the net income and shareholders’ equity. It summarizes how much money the company earns for each unit invested by shareholders. The values in Table 2 are the medians obtained from the quarterly data collected between the second quarter of 2010 and the first quarter of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Price-to-Book Ratio (P/B) allows the analyst to compare the stock’s market value to the book value at the end of every quarter. It is used by financial analysts to understand if the stock should be considered undervalued or not. The values in Table 2 are the medians obtained from quarterly data starting in the second quarter of 2010 until the first quarter of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Price-Earnings Ratio (P/E) is used to value companies by comparing the market price of each share to the per-share earnings of the company in the last quarter. It is one of the most used multiples by financial analysts, according to Clayman et al (2012). The values in Table 2 are the medians obtained for the last day in each quarter starting in the second quarter of 2010 until the first quarter of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Quick Ratio (QR) is a short-term liquidity multiple computed as the ratio between the summation of cash equivalents, marketable securities and accounts receivable, divided by the current liabilities. The values in Table 2 are the medians obtained from quarterly data collected between the second quarter of 2010 and the first quarter of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>A strong corporate governance structure is important to guarantee long-term positive results for stockholders. The BM&amp;FBovespa attributes four levels of corporate governance for all companies listed in the exchange: from the “Novo Mercado”, the best possible classification, to the “Tradicional”, the worst possible classification, passing through the segments “Nível 2” and “Nível 1”. The classification in Table 2 was valid at the end of the first quarter of 2015. This is a qualitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Índice de Sustentabilidade Empresarial (ISE), first released in 2005 by the BM&amp;FBovespa (BM&amp;FBOVESPA, 2015), covers the listed companies that “excelled in their compromise to reach a sustainable development … with a strategic alignment with the demands from the local society”. The data in Table 2 is related to those companies that composed the ISE (“Inside”) or not (“Outside”) at the end of the first quarter of 2015. This is a qualitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The annualized Volatility is the most used market risk measure according to Defusco et al (2007). There are several possibilities to estimate this risk measure, but we chose to compute it using maximum likelihood estimation under the normality assumption (Brooks, Persand, 2003) based on daily returns from the beginning of the second semester of 2010 until de end of the first semester of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Rating is a qualitative assessment indicator of the credit quality of a company published by rating agencies. In this work, we rely on the long-term scale published by the agency Standard &amp; Poor’s for debts issued by Brazilian companies in R$ (STANDARD &amp; POOR’S, 2014). The ratings in Table 2 were valid at the end of the first quarter of 2015. This is a qualitative criterion.</td>
<td></td>
</tr>
<tr>
<td>The Average Volume (AVol) is a stock liquidity measure computed as the amount negotiated daily of each asset in an exchange. The values in Table 2 were obtained as the median of the amount negotiated daily at the BM&amp;FBovespa between the second quarter of 2010 and the first quarter of 2015. This is a quantitative criterion.</td>
<td></td>
</tr>
<tr>
<td>Company</td>
<td>EPS</td>
</tr>
<tr>
<td>---------</td>
<td>-----</td>
</tr>
<tr>
<td>PETR4</td>
<td>0.42</td>
</tr>
<tr>
<td>PBR</td>
<td>3.34%</td>
</tr>
<tr>
<td>BBVA</td>
<td>1.29</td>
</tr>
<tr>
<td>BFA</td>
<td>1.36</td>
</tr>
<tr>
<td>BBDC</td>
<td>3.40</td>
</tr>
<tr>
<td>BNAS</td>
<td>4.36</td>
</tr>
<tr>
<td>AVA</td>
<td>9.00%</td>
</tr>
</tbody>
</table>

Table 2: Values for Each Criterion
Table 3. Conversion Scale for the Criterion Corporate Governance

<table>
<thead>
<tr>
<th>Segment</th>
<th>Numerical Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tradicional</td>
<td>1</td>
</tr>
<tr>
<td>Nível 1</td>
<td>3</td>
</tr>
<tr>
<td>Nível 2</td>
<td>5</td>
</tr>
<tr>
<td>Novo Mercado</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 4. Conversion scale for the Criterion Sustainability

<table>
<thead>
<tr>
<th>ISE</th>
<th>Numerical Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside</td>
<td>1</td>
</tr>
<tr>
<td>Inside</td>
<td>3</td>
</tr>
</tbody>
</table>

3. Numerical Analyses

We present in this section the numerical results (including sensitivity analyses) obtained when the methodology presented above was applied to the nine stocks and the data displayed in Table 2.

The initial step requires defining conversion scales for the three qualitative criteria presented in Table 1. The initial conversion scales adopted are as exhibited in Table 3, Table 4 and Table 5. With respect to the conversion scale for credit ratings, we handle intermediary positions by adding or subtracting 0.5 – for example, given that the conversion value for the rating br.A is set at 7, the ratings br.A+ e br.A− receive values 7.5 and 6.5, respectively. Table 6 summarizes the data after the application of the three conversion scales to Table 2. We anticipate that sensitivity analyses will be performed for each conversion scale later, to help clarify how sensitive the defuzzified scores of the nine companies are to changes in these parameters.
Table 5. Conversion Scale for the Credit Ratings

<table>
<thead>
<tr>
<th>Rating</th>
<th>Numerical Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>br.C</td>
<td>1</td>
</tr>
<tr>
<td>br.CC</td>
<td>2</td>
</tr>
<tr>
<td>br.CCC</td>
<td>3</td>
</tr>
<tr>
<td>br.B</td>
<td>4</td>
</tr>
<tr>
<td>br.BB</td>
<td>5</td>
</tr>
<tr>
<td>br.BBB</td>
<td>6</td>
</tr>
<tr>
<td>br.A</td>
<td>7</td>
</tr>
<tr>
<td>br.AA</td>
<td>8</td>
</tr>
<tr>
<td>br.AAA</td>
<td>9</td>
</tr>
</tbody>
</table>

We denote the fuzzy triangular components of the weight vector as

\[
\left( (w^a_1;w^b_1;w^c_1); ...; (w^a_n;w^b_n;w^c_n) \right) \tag{1}
\]

with \(0 \leq w^a_j \leq w^b_j \leq w^c_j\) \(\forall j = 1, ..., n\). Their membership functions can be generically written as

\[
\Gamma_{w^i_j}(y) = \begin{cases} 
\frac{y-w^a_j}{w^b_j-w^a_j} & w^a_j \leq y < w^b_j \\
\frac{w^b_j-y}{w^c_j-w^b_j} & w^b_j \leq y \leq w^c_j \\
0 & \text{otherwise}
\end{cases} \forall i = 1, ..., m \text{ and } j = 1, ..., n \tag{2}
\]

We introduce the parameter \(\alpha\) to facilitate handling fuzziness in the weight vector, writing

\[
w^a_j = (1 - \alpha) \times w^b_j \quad \forall j = 1, ..., 10 \tag{3}
\]

and

\[
w^c_j = (1 + \alpha) \times w^b_j \quad \forall j = 1, ..., 10 \tag{4}
\]
Table 6: Decision Matrix

<table>
<thead>
<tr>
<th>Company</th>
<th>EPS</th>
<th>ROE</th>
<th>PB</th>
<th>PL</th>
<th>QR</th>
<th>CGI</th>
<th>CCA</th>
<th>GOV</th>
<th>Valuation</th>
<th>Sustainability</th>
<th>Valence</th>
<th>Range</th>
<th>A/A</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALFA</td>
<td>9.0</td>
<td>3.0</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.5</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>PETRA</td>
<td>9.0</td>
<td>4.4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.8</td>
<td>2.0</td>
<td>4.8</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>POURA</td>
<td>9.0</td>
<td>3.0</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>IIIA</td>
<td>2.0</td>
<td>0.9</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>CGI</td>
<td>1.2</td>
<td>0.7</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>BEES</td>
<td>1.3</td>
<td>0.6</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>BDO</td>
<td>0.3</td>
<td>0.3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>BASS</td>
<td>0.3</td>
<td>0.3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
<tr>
<td>BVEA</td>
<td>0.3</td>
<td>0.3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3.0</td>
<td>4.4</td>
<td>2.0</td>
<td>4.4</td>
<td>5.4</td>
<td>5.4</td>
<td>5.4</td>
<td>3.0</td>
<td>4.4</td>
</tr>
</tbody>
</table>

The table above presents the decision matrix for various companies, including EPS, ROE, PB, PL, QR, CGI, CCA, GOV, Valuation, Sustainability, Valence, Range, A/A, and Avg.
For example, if \( \alpha = 20\% \), we say that the uncertainty level is set at 20\% for each component of the weight vector, resulting in the triangular fuzzy number \((80\% \times w_j^0; w_j^0; 120\% \times w_j^0)\) for \( j = 1, \ldots, 10 \).

We denote the generic element of the fuzzy decision matrix as

\[
M_{ij} = (a_{ij}; b_{ij}; c_{ij}) \quad \forall \ i = 1, \ldots, m \ and \ j = 1, \ldots, n
\]

and assume that

\[
a_{ij} \leq b_{ij} \leq c_{ij} \quad \forall \ i = 1, \ldots, m \ and \ j = 1, \ldots, n
\]

with membership function written as

\[
\Gamma_{M_{ij}}(y) = \begin{cases} 
\frac{y-a_{ij}}{b_{ij}-a_{ij}} & a_{ij} \leq y < b_{ij} \\
\frac{c_{ij}-y}{c_{ij}-b_{ij}} & b_{ij} \leq y \leq c_{ij} \\
0 & \text{otherwise}
\end{cases} \quad \forall \ i = 1, \ldots, m \ and \ j = 1, \ldots, n
\]

We introduce the parameter \( \beta \) to facilitate handling fuzziness in the elements of the fuzzy decision matrix, denoting

\[
a_{ij} = (1 - \beta) \times b_{ij} \quad \forall \ i = 1, \ldots, 9, j = 1, \ldots, 10
\]

and

\[
c_{ij} = (1 + \beta) \times b_{ij} \quad \forall \ i = 1, \ldots, 9, j = 1, \ldots, 10
\]

For example, if we set \( \beta = 10\% \), we say that the uncertainty level is set at 10\% for each element in the decision matrix, resulting in the triangular fuzzy number \((90\% \times b_{ij}; b_{ij}; 110\% \times b_{ij})\) for \( i = 1, \ldots, 9, j = 1, \ldots, 10 \).

We present next three numerical examples:

i. The first example considers only the five multiples (EPS, ROE, P/B, P/L and QR).

ii. The second example expands the set of criteria used by considering the five multiples already adopted in the first example, plus the qualitative criteria corporate governance and sustainability.
iii. The third example considers all ten criteria simultaneously, adding the three related to market, credit and liquidity risks to the seven already adopted in the second example.

The analysis of these three examples will allow us to understand how the inclusion of criteria tend to alter the defuzzified scores and the relative positions of the nine companies considered.

3.1 First Numerical Example

We consider the following parameters in the first numerical example:

i. Only the five multiples are used: EPS, ROE, P/B, P/L and QR.

ii. We assume that the five criteria are equally important (i.e. \( w_1^b = w_2^b = \ldots = w_5^b = \frac{1}{5} \)), with uncertainty at \( \alpha = 20\% \), resulting in a fuzzy weight vector with generic element given by \((1 - 20\%) \times \frac{1}{5}; \frac{1}{5}; (1 + 20\%) \times \frac{1}{5}\) = \((\frac{4}{25}; \frac{5}{25}; \frac{6}{25})\).

iii. The uncertainties in the components of the fuzzy decision matrix are all set equal to 20\% (i.e., \( \beta = 20\% \)). For example, if we consider the criterion EPS related to ABEV3, we observe in Table 6 a value equal to 0.63, leading to a triangular fuzzy number equal to \((1 - 20\%) \times 0.63; 0.63; (1 + 20\%) \times 0.63\) = (0.51; 0.63; 0.76). The fuzzy values for the criterion EPS are summarized in Table 7 (compare to the column EPS in Table 6). The complete triangular fuzzy decision matrix is exhibited in Table 8.
The second column in Table 9 exhibits the defuzzified scores obtained after applying the Fuzzy TOPSIS to the data in Table 8. For example, BBAS3 and ABEV3 are the companies with the highest and lowest defuzzified scores, implying the best and the worst expected performance, respectively. We can also observe that the nine defuzzified scores are concentrated in the interval between 43% and 60%, indicating that there is not much separation between the expected performance of the nine companies selected.
<table>
<thead>
<tr>
<th>Company</th>
<th>EPS</th>
<th>ROE</th>
<th>PL</th>
<th>PD</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAEG3</td>
<td>0.94</td>
<td>0.76</td>
<td>0.96</td>
<td>0.78</td>
</tr>
<tr>
<td>BHCD4</td>
<td>1.34</td>
<td>1.82</td>
<td>2.18</td>
<td>2.57</td>
</tr>
<tr>
<td>BHSSA</td>
<td>1.45</td>
<td>1.98</td>
<td>2.40</td>
<td>2.74</td>
</tr>
<tr>
<td>IPHAN</td>
<td>0.75</td>
<td>1.30</td>
<td>1.56</td>
<td>1.95</td>
</tr>
<tr>
<td>JASUL</td>
<td>0.95</td>
<td>1.15</td>
<td>1.63</td>
<td>1.95</td>
</tr>
<tr>
<td>JETRZ</td>
<td>1.00</td>
<td>1.10</td>
<td>1.33</td>
<td>1.50</td>
</tr>
<tr>
<td>CARA4</td>
<td>1.01</td>
<td>1.29</td>
<td>1.58</td>
<td>1.86</td>
</tr>
<tr>
<td>TILTA</td>
<td>0.75</td>
<td>1.02</td>
<td>1.25</td>
<td>1.46</td>
</tr>
<tr>
<td>PETR4</td>
<td>0.34</td>
<td>0.42</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>CPET4</td>
<td>0.34</td>
<td>0.42</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>VALES</td>
<td>1.14</td>
<td>1.84</td>
<td>1.69</td>
<td>1.69</td>
</tr>
<tr>
<td>PETR4</td>
<td>0.34</td>
<td>0.42</td>
<td>0.50</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 8: Fuzzy Decision Matrix: First Example
Table 9. Defuzzified Scores and Orderings for Nine Stocks Using Fuzzy TOPSIS

<table>
<thead>
<tr>
<th>Company</th>
<th>First Example: Five Multiples</th>
<th>Second Example: Five Multiples plus Corp. Gov. and Sustainability</th>
<th>Third Example: All ten criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABEV3</td>
<td>43.37% (9ª)</td>
<td>40.15% (9ª)</td>
<td>39.07% (9ª)</td>
</tr>
<tr>
<td>BBAS3</td>
<td>59.49% (1ª)</td>
<td>60.90% (1ª)</td>
<td>57.31% (1ª)</td>
</tr>
<tr>
<td>BBDC4</td>
<td>54.28% (3ª)</td>
<td>53.61% (3ª)</td>
<td>52.16% (3ª)</td>
</tr>
<tr>
<td>BRFS3</td>
<td>44.12% (8ª)</td>
<td>47.92% (6ª)</td>
<td>45.94% (8ª)</td>
</tr>
<tr>
<td>CIEL3</td>
<td>50.77% (5ª)</td>
<td>53.06% (4ª)</td>
<td>50.84% (4ª)</td>
</tr>
<tr>
<td>ITUB4</td>
<td>55.67% (2ª)</td>
<td>54.82% (2ª)</td>
<td>54.63% (2ª)</td>
</tr>
<tr>
<td>PCAR4</td>
<td>51.97% (4ª)</td>
<td>49.85% (5ª)</td>
<td>47.17% (7ª)</td>
</tr>
<tr>
<td>PETR4</td>
<td>49.43% (6ª)</td>
<td>46.69% (7ª)</td>
<td>49.24% (5ª)</td>
</tr>
<tr>
<td>VALE5</td>
<td>45.06% (7ª)</td>
<td>45.36% (8ª)</td>
<td>48.08% (6ª)</td>
</tr>
</tbody>
</table>

An interesting question to ask at this point is: How are the defuzzified scores (and the relative positions) of the nine companies altered if the two qualitative criteria related to corporate governance and sustainability are included in the analysis?

3.2 Second Numerical Example

In this second numerical example we maintain the uncertainties in the fuzzy weight vector and decision matrix at 20% (i.e., \( \alpha = \beta = 20\% \)). Also, the relative importance of all seven criteria (five multiples, corporate governance and sustainability) are considered equal (i.e. \( w_1^b = w_2^b = \cdots = w_7^b = \frac{1}{7} \)), resulting in a fuzzy weight vector with equal components given by \( \left( \left( 1 - 20\% \right) \times \frac{1}{7} ; \frac{1}{7} ; \left( 1 + 20\% \right) \times \frac{1}{7} \right) = \left( \frac{4}{35} ; \frac{5}{35} ; \frac{6}{35} \right) \).

The fuzzy decision matrix has now seven columns: the first five as in Table 7, and the last two corresponding to the qualitative criteria corporate governance and sustainability, represented as triangular fuzzy numbers.

The final defuzzified scores obtained in this second example are as given in the third column of Table 9. For example, we can observe a decrease in the scores of ABEV3 e PETR4 (compare the second and third
columns in Table 9), given that both companies were classified by the BM&FBovessa in the least demanding corporate governance segment (Tradicional) at the end of the first quarter of 2015. On the other hand, the scores of BRFS3 and CIEL3 increase, since they are both classified in the most demanding corporate governance segment (Novo Mercado).

Overall, a comparison between the second and third columns on Table 9 allows us to observe that the proposed methodology responds correctly to the introduction of the two qualitative criteria by increasing the defuzzified scores of companies classified in more demanding corporate governance segments at the end of the first quarter of 2015, as well as those that belonged to the index ISE.

3.3 Third Numerical Example

The third numerical example extends the two previous by adopting all ten criteria described in Table 3, all considered as equally important (i.e. $w_1^b = w_2^b = \cdots = w_{10}^b = \frac{1}{10}$). In this case the generic element of the fuzzy weight vector results to be $\left(\left(1 - 20\%\right) \times \frac{1}{10}; \frac{1}{10}; \left(1 + 20\%\right) \times \frac{1}{10}\right) = \left(\frac{4}{50}; \frac{5}{50}; \frac{6}{50}\right)$. The fuzzy decision matrix is obtained by considering a 20% level of uncertainty (i.e., $\beta = 20\%$) applied to all the data in Table 3.4.

The scores and orderings obtained are exhibited in the last column of Table 9. We can observe that the defuzzified scores of companies with more volatile stock prices and lower credit ratings tend to decrease (such as BBAS3, although it remained as the largest one). On the other hand, companies with the most liquid stocks have their defuzzified scores increased (for example, PETR4 and VALE5), leading to an upgrade on their relative positions, as expected.

Overall, we can observe in Table 9 that:

i. Although the defuzzified scores fluctuated with the inclusion of more criteria, BBAS3, ITUB4 and BBDC4 remained as the three most promising alternatives. Since they all belong to the same economic sector (i.e., financial institutions), this should be sufficient to illustrate that this sector was the most rewarding in the Brazilian economy during the period analysed.

ii. ABEV3 displayed the worst expected performance for the data considered in all three numerical examples.
iii. The other five companies (BRFS3, CIEL3, PCAR4, PETR4 and VALE5) remained in intermediary positions, with close-by scores.

At this point, we must remember that whenever using MCDA, it is important to fully understand how changes in the input parameters impact the scores (and the orderings) obtained, with special attention towards the conversion scales adopted. In the remaining of this section, we present four sensitive analyses that address these concerns. We depart from the third example just presented in all sensitivity analyses displayed below.

3.4 First Sensitive Analysis

The first sensitivity analysis considers changes only in the conversion scale of the qualitative criteria sustainability, with all other inputs maintained as used to obtain the third numerical example above (i.e., the defuzzified scores in the fourth column of Table 9).

If we consider Table 4, we can observe that the relative importance attributed to the companies composing the ISE is three times that of not being part of the index composition. In this first set of analysis, we change the relative importance attributed to the companies composing the ISE to test three other levels of relative importance: 5, 7 and 9. Figure 1 resumes the results obtained.

It can be observed a clear tendency of decreasing defuzzified scores for the three companies that do not compose the ISE (ABEV3, PCAR4 e PETR4), as it could be anticipated. For example, when PETR4 and VALE5 are compared, the second shows a tendency to outperform the first as the importance of belonging to the ISE is increased, this being true also in the case of PCAR4 and BRFS3.
3.5 Second Sensitivity Analysis

The second sensitivity analysis is applied to changes only in the conversion scale of the qualitative score corporate governance. This is achieved by maintaining the scores for the segments Tradicional, Nível 1 and Nível 2 as in Table 3, but increasing the weight for the segment Novo Mercado from 7 to 9, to 11 and, finally, to 13. In other words, in this sensitivity analysis we aim at investigating how the increase in the relative importance of the most demanding segment of corporate governance according to BM&FBovespa (i.e., Novo Mercado) impacts the scores and orderings displayed in the fourth column of Table 9. Figure 2 resumes our observations.

It can be observed that the defuzzified scores of the three companies that belonged to the segment Novo Mercado (BBAS3, CIEL3 e BRFS3) at the end of the first quarter of 2015 tend to increase. For example, a comparison between the orderings of PCAR4 and BRFS3 shows that the second company surpasses (higher score) the first with the increase of importance attributed to the segment Novo Mercado, the same being true for the pair BBDC4 and CIEL3.

Figure 1. Sensitivity Analysis Applied to Sustainability
3.6 Third Sensitivity Analysis

The third sensitivity analysis considers changes in the level of uncertainty in the fuzzy weight vector. In this sensitivity analysis, we consider that the conversion scales are exactly as exhibited in Table 3, Table 4 and Table 5, and that the level on uncertainty in the elements of the fuzzy decision matrix is kept fixed at 20% (i.e., $\beta = 20\%$). On the other hand, the level of uncertainty in the fuzzy weight vector varies substantially, with $\alpha \in \{0\%, 5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$. Figure 3 summarizes the results obtained.

We can observe in Figure 3 that the defuzzified scores tend to converge, with their values varying slowly with the changes in the level of uncertainty. Also, let us observe that the orderings are not subjected to changes. In other words, increasing symmetric uncertainties in the fuzzy weight vector tend not to produce relevant changes in the final orderings, with defuzzified scores converging slowly.
3.7 Fourth Sensitivity Analysis

In this last sensitivity analysis, the conversion scales are as in the preceding case, we keep the level of uncertainty in the fuzzy weight vector fixed at 20% (i.e., $\alpha = 20\%$), but now we vary the level of uncertainty in the components of the fuzzy decision matrix, experimenting with $\beta \in \{0\%, 5\%, 10\%, 15\%, 20\%, 25\%, 30\%, 35\%, 40\%, 45\%, 50\%\}$. Figure 4 summarizes the trends observed.

We can observe that the defuzzified scores tend to converge smoothly, with no changes in the final orderings. We can observe basically the same behavior as in the case of symmetric modifications in the level of uncertainty of the fuzzy weight vector, causing Figure 3 and Figure 4 to display basically the same pattern.
4. Conclusion

We have proposed and illustrated the combined use of fuzzy mathematics and multicriteria methods to address the problem of equity valuation. Our proposal allows the use of both quantitative and qualitative criteria, combining the traditional multiples used by most financial analysts (profitability, costs etc.), with indicators related to concepts such as sustainability, corporate governance and risk (market, liquidity and credit). In addition, our proposal facilitates modeling uncertainties in the problem with the use of fuzzy mathematics, enriching the information made available for the financial analyst.

Several numerical examples were presented. The nine largest market capitalizations negotiated in the Brazilian stock market at the beginning of 2015 were adopted for the sake of illustration. The data set used was obtained from audited balance sheets (published by the investors’ relations area of each company) and commercial databases, covering twenty consecutive quarters. Sensitivity analyses were also performed to help understand how changes in the inputs altered the final defuzzified scores (and orderings) obtained for the stocks considered. The numerical examples were obtained with the use of the TOPSIS method modified to incorporate fuzziness. This fuzzy method was detailed in the appendix for those who may want to replicate our results.

Our conclusion was that the use of fuzzy multicriteria methods for equity valuation have led to a sound decision-making process, conferring
more credibility to the final recommendations of the financial analyst, as properly illustrated in the third section. Incorporating qualitative information related to risks, sustainability and corporate governance is a necessary step nowadays, forcing the extension of the traditional equity valuation, based solely on multiples.

In terms of future developments, there are three immediate extensions possible. The first extension is to try other fuzzy numbers (i.e., other membership functions), besides the triangular numbers reported in this article. As we have already mentioned, we did experiment with trapezoidal fuzzy numbers, which results (orderings) did not differ significantly from those obtained with triangular fuzzy numbers. A second possibility is to consider the introduction of more criteria. We have not tried other criteria than those ten shown in this work. We do understand that properly selecting criteria is an important point for a financial analyst when fine-tuning the methodology for his daily practice. A third possibility is to apply other multicriteria methods, in addition to TOPSIS. We mention that we have tried a limited number of runs with fuzzy variants of another multicriteria method (TODIM), with only minor changes (orderings) when compared to those reported in this work.

References


Appendix: The Fuzzy TOPSIS

For the sake of completeness, we have chosen to detail in this appendix the version to the Fuzzy TOPSIS used in the numerical examples given before. The original TOPSIS method can be found in Hwang e Yoon (1981).

Adapting the original TOPSIS method to incorporate fuzzy mathematics can be achieved in several ways, as illustrated by Ashzafzadeh et al (2012), Chen (2000), Dymova et al (2013) and Marbini and Saati (2009). We have followed the versions already proposed in several steps, except for three specific points: (a) we have changed the normalization process (see (A7) below) and (b) the distance computations from every alternative and the ideal solutions (see (A12) and (A13) below), as well as (c) we allow uncertainties in the relative importance of all criteria. The reason why we have chosen to differ from the other versions is simple: our proposal captures the original TOPSIS as a very particular case if fuzziness is removed from the problem, as mentioned below.

The Fuzzy TOPSIS detailed below is presented with the use of triangular fuzzy numbers. We take this opportunity to mention that we did experiment with other fuzzy numbers, with no substantial changes when the results were compared to those exhibited in the third section, this being the reason why we have chosen to display only the results obtained with triangular fuzzy numbers in this article.

We denote the fuzzy weight vector for the criteria as
\[
\left( (w_1^a, w_1^b, w_1^c); \ldots; (w_n^a, w_n^b, w_n^c) \right) \quad (A1)
\]
and assume that \(0 \leq w_j^a \leq w_j^b \leq w_j^c \quad \forall \ j = 1, \ldots, n\).

We denote the generic element of the fuzzy decision matrix as
\[
M_{ij} = (a_{ij}; b_{ij}; c_{ij}) \quad \forall \ i = 1, \ldots, m \ and \ j = 1, \ldots, n \quad (A2)
\]
and assume that \(a_{ij} \leq b_{ij} \leq c_{ij} \quad \forall \ i = 1, \ldots, m \ and \ j = 1, \ldots, n\). The fuzzy decision matrix can be written as
\[
\begin{pmatrix}
(a_{11}; b_{11}; c_{11}) & (a_{12}; b_{12}; c_{12}) & \cdots & (a_{1n}; b_{1n}; c_{1n}) \\
(a_{21}; b_{21}; c_{21}) & (a_{22}; b_{22}; c_{22}) & \cdots & (a_{2n}; b_{2n}; c_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
(a_{m1}; b_{m1}; c_{m1}) & (a_{m2}; b_{m2}; c_{m2}) & \cdots & (a_{mn}; b_{mn}; c_{mn})
\end{pmatrix} \quad (A3)
\]
which normalized element becomes

$$M_{ij} = (a_{ij}; b_{ij}; c_{ij}) = \left( \frac{w_i^x a_{ij}}{\sum_{m=1}^{M} w_{m}^x a_{mj}}, \frac{w_i^y b_{ij}}{\sum_{m=1}^{M} w_{m}^y b_{mj}}, \frac{w_i^z c_{ij}}{\sum_{m=1}^{M} w_{m}^z c_{mj}} \right) \forall i = 1, \ldots, m \text{ and } j = 1, \ldots, n \quad (A4)$$

We denote the sets of criteria classified as benefits and costs as $C^+$ and $C^-$, respectively. The fuzzy positive ideal solution is obtained from

$$\begin{align*}
(a^*_j; \beta^*_j; \gamma^*_j) &= \left\{ (\max\{a_{ij}; \ldots; a_{im}\}; \max\{b_{ij}; \ldots; b_{im}\}; \max\{c_{ij}; \ldots; c_{im}\}) \right\} \forall j \in C^+ \quad (A5) \\
(b^*_j; \gamma^*_j) &= \left\{ (\min\{a_{ij}; \ldots; a_{im}\}; \min\{b_{ij}; \ldots; b_{im}\}; \min\{c_{ij}; \ldots; c_{im}\}) \right\} \forall j \in C^- 
\end{align*}$$

while the fuzzy negative ideal solution is given by

$$\begin{align*}
(a^{-}_j; \beta^-_j; \gamma^-_j) &= \left\{ (\max\{a_{ij}; \ldots; a_{im}\}; \max\{b_{ij}; \ldots; b_{im}\}; \max\{c_{ij}; \ldots; c_{im}\}) \right\} \forall j \in C^- \quad (A6) \\
(b^-_j; \gamma^-_j) &= \left\{ (\min\{a_{ij}; \ldots; a_{im}\}; \min\{b_{ij}; \ldots; b_{im}\}; \min\{c_{ij}; \ldots; c_{im}\}) \right\} \forall j \in C^+ 
\end{align*}$$

The fuzzy difference between each element of the fuzzy decision matrix and the fuzzy ideal positive solution is obtained as

$$P^+_i = (a^+_{ij}; b^+_{ij}; c^+_{ij}) = (a^*_{ij} - \gamma^*_j; b^*_{ij} - \beta^*_j; c^*_{ij} - \alpha^*_j) \forall i = 1, \ldots, m \text{ e } j = 1, \ldots, n \quad (A7)$$

Analogously, the difference between each element of the fuzzy decision matrix and the fuzzy negative ideal solution is calculated from

$$P^-_i = (a^-_{ij}; b^-_{ij}; c^-_{ij}) = (a^-_{ij} - \gamma^-_j; b^-_{ij} - \beta^-_j; c^-_{ij} - \alpha^-_j) \forall i = 1, \ldots, m \text{ e } j = 1, \ldots, n \quad (A8)$$

The distance from each alternative to the fuzzy ideal positive solution can be computed as

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left( y^+ \cdot \left( x^+_{ij} - \gamma^+_j \right)^2 + y^- \cdot \left( x^-_{ij} - \gamma^-_j \right)^2 \right) + \sum_{k=1}^{m} \left( \left( y^+ \cdot x^+_{ik} - \gamma^+_k \right)^2 + y^- \cdot \left( x^-_{ik} - \gamma^-_k \right)^2 \right)} \forall i = 1, \ldots, m \quad (A9)$$

while the equivalent in the case of the fuzzy ideal negative solution is
Finally, the relative proximity is obtained from

\[ D_i \Rightarrow = \frac{\int_{\alpha_1}^{c_1} ... \int_{\alpha_n}^{c_n} (y_1 \times \Gamma_{n_1}(y_1) + ... + y_n \times \Gamma_{n_n}(y_n)) dy_1 ... dy_n}{\int_{\alpha_1}^{c_1} ... \int_{\alpha_n}^{c_n} (\Gamma_{n_1}(y_1) \times ... \times \Gamma_{n_n}(y_n)) dy_1 ... dy_n} = \cdots =
\]

\[ = \frac{1}{6} \times \sum_{i=1}^{n} \left( (a_{i1})^2 + (b_{i1})^2 + (c_{i1})^2 + a_{i1} \times b_{i1} + a_{i1} \times c_{i1} + b_{i1} \times c_{i1} \right) \quad \forall \ i = 1, ..., m \]

\[(A10)\]

Finally, the relative proximity is obtained from

\[ v_i = \frac{D_i^-}{D_i^- + D_i^+} \quad \forall \ i = 1, ..., m \]

\[(A11)\]

where the values \( v_1, ..., v_m \) vary between 0% and 100%. It is possible to interpret these values as the “scores” or “grades” obtained for the investment alternatives: the most promising investment alternatives will present scores/grades close to 100% (i.e., \( \max\{v_1; \ldots; v_m\} \)), while the not so promising alternatives will have scores/grades close to 0% (i.e., \( \min\{v_1; \ldots; v_m\} \)).

Since the work of Belton and Gear (1983), showing the existence of rank reversal in the Analytical Hierarchy Process (Saaty, 1980), the phenomenon has demanded some attention in the MCDA literature. Several methods can be influenced by this phenomenon, in addition to the Analytic Hierarchy Process, such as the ELECTRE, the PROMETHEE and the TOPSIS, as illustrated in García-Cascales and Lamata (2012), and Wang and Luo (2009). We point out that this phenomenon was not observed in the numerical examples presented in this article. We also recall that avoiding rank reversal is not considered crucial by many MCDA users, as properly argued in Saaty and Vargas (1984) and Tversky (1969), among others.

Finally, we must mention one very interesting feature of the Fuzzy TOPSIS presented: the original TOPSIS can be captured as a particular case if we remove uncertainty by setting \( w_j^a = w_j^b = w_j^c \) and \( a_{ij} = b_{ij} = c_{ij} \forall \ i = 1, ..., m \ e \ j = 1, ..., n \), as it can be easily verified.