

Technological progress and diffusion in Brazilian manufacturing*

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Summary: 1. Introduction; 2. Methodology; 3. Data base; 4. Empirical results; 5. Final remarks.

This paper uses frontier production functions to estimate total factor productivity (TFP) growth in 80 sectors of Brazilian manufacturing in 1970-80 and to decompose it into technological progress and changes in technical efficiency. For most sectors, the former was found to be the main source of TFP change: while TFP grew 2.6 percent annually, best practice TFP advanced 3.3 percent and efficiency declined 0.7 percent. These results suggest a slow rate of technological diffusion. Estimates for sector TFP growth were found to be rather similar to those obtained elsewhere using index numbers. The results cast doubt on the usefulness of estimating frontier production functions through elaborate and time consuming procedures such as maximum likelihood, whose advantage relies on taking into account a presumed asymmetry of the distribution of firms' technical efficiency.

Este artigo utiliza fronteiras de produção para estimar o crescimento da produtividade total dos fatores (PTF) em 80 setores da indústria de transformação brasileira em 1970-80 e para decompô-la em progresso tecnológico e mudanças na eficiência técnica. Para a maioria dos setores, o primeiro se revelou a principal fonte da mudança na PTF: enquanto a PTF cresceu 2,6% ao ano, a PTF das firmas mais eficientes avançou 3,3% e a eficiência técnica declinou 0,7%. Esses resultados sugerem um ritmo lento de difusão tecnológica. Estimativas para o crescimento da PTF no nível de setor mostraram-se bastante semelhantes às obtidas em outro estudo usando números-índices. Os resultados colocam em questão a utilidade de se estimar fronteiras de produção através de procedimentos complexos e caros como a estimativa por máxima verossimilhança, cuja vantagem se baseia em levar em conta uma suposta assimetria na distribuição da eficiência técnica das firmas.

1. Introduction

Productivity change has been a leading source of output growth in nowadays developed countries. It is not surprising, therefore, that so much effort has been spent trying to determine the sources of total factor productivity (TFP) growth, a variable that Moses Abramovitz once described as the "measure of our ignorance" with respect to what drives economic growth. A step in this direction consists in decomposing the growth of TFP into changes in technical efficiency and advances in the technological frontier.

TFP change measures growth of output that is not caused by a larger consumption of inputs. Best-practice TFP change reflects technological progress; that is, shifts in the frontier production function not explained by changes in the consumption of inputs. The difference between the two is given by the change in the technical efficiency with which known technology is used. Distinguishing between these two components not only helps to understand the growth process but also has important policy implications. Potential productivity gains

* This paper has benefited from comments by Albert Fishlow, Bronwyn Hall, Sherman Robinson and two anonymous referees and from computer assistance by Marcia Pimentel Pinto.

** From Ipea/Dipes and FEA/UFRJ.

arising from technological diffusion may surpass — and cost less than — those stemming from technological progress. Nishimizu and Page (1982), for instance, show that Yugoslavia's stationary productivity level in 1965-78 resulted from a combination of technological progress and technical efficiency decline.

The objective of this paper is to estimate TFP change in 1970-80 for 80 sectors of Brazilian manufacturing and to decompose it into technological progress and diffusion (defined here as an improvement in technical efficiency). To that end, frontier production functions are estimated for each sector in 1970 and 1980 and then combined to yield the desired results.

TFP growth in Brazilian manufacturing has been a recurrent topic of research. Several studies have examined different time intervals in the 1960-85 period according to distinct sector aggregations (e.g., Bonelli, 1975 and 1992; Braga and Rossi, 1988; Braga and Hickmann, 1988; Espinal, 1991; and Pinheiro, 1989). Most authors have resorted, however, to index number decompositions and some used value added rather than production functions. The most notable exception is the study by Braga and Rossi (1988), who have used a translog cost function. In the same fashion, frontier production functions have been an object of several empirical studies in Brazil (Rossi, 1984; Braga and Rossi, 1986; Alves, 1987; and Gomes, 1992).

To the best of my knowledge, nonetheless, this is the first study to combine these two topics to both estimate and decompose productivity growth in Brazilian manufacturing. It also contrasts with most work done in the field in that it considers a rather thin sector aggregation and allows for the substitution in production among energy, material inputs, labor and capital.

The methodology used here follows, to some extent, the one adopted by Nishimizu and Page (1982), Handoussa, Nishimizu and Page (1986) and Noh (1987). Differently from these previous studies, however, the frontiers are estimated in this paper using the more efficient maximum likelihood procedure. In addition, I estimate deterministic, stochastic and corrected least squares frontiers, whereas the analysis in these three studies was restricted to deterministic frontiers.

The plan of the paper is as follows. Next section presents the methodology used in the analysis. In section 3 the data used in the frontier estimation is briefly described. The results obtained are discussed in section 4. A final section summarizes the paper's main conclusions.

2. Methodology

The production possibility set of an establishment i at time t is defined by:

$$G[y(i, t), x(i, t); i, t] \leq 0 \quad (1)$$

where y and x are, respectively, the vector of outputs and inputs. The arguments i and t appear in G to indicate the levels of marginal factor products of establishment i at time t . If y is separable from x , and there is an appropriate index of output ($Y(i, t)$), then the set of feasible output and input vectors can be equivalently represented by

$$Y(i, t) \leq F[x(i, t); i, t] \quad (2)$$

where $F[\cdot; i, t]$ describes the frontier production function for establishment i at period t . Technical efficiency for i at time t is defined as

$$e(i, t) = Y(i, t)/F[\mathbf{x}(i, t); i, t], \quad (0 \leq e \leq 1) \quad (3)$$

Taking first differences in the logs, and after some algebra, one has a (parametric) measure of TFP change given by ¹

$$\begin{aligned} \text{TFPC}_p &= \ln[Y(i, t)/Y(i, t-1)] - \\ &- \{\ln F[\mathbf{x}(i, t); i, t-1] - \ln F[\mathbf{x}(i, t-1); i, t-1]\} = \\ &= \ln F[\mathbf{x}(i, t); i, t] - \ln F[\mathbf{x}(i, t); i, t-1] + \ln e(i, t) - \ln e(i, t-1) \end{aligned} \quad (4)$$

Expression (4) shows that TFP change can be decomposed into two elements. First, a gain in productivity arising from shifts in the frontier not accounted for by a larger consumption of inputs ($\ln F[\mathbf{x}(i, t); i, t] - \ln F[\mathbf{x}(i, t); i, t-1]$). This term will be called technological progress or best-practice TFP change. Second, a change in TFP arising from variations in technical efficiency ($\ln e(i, t) - \ln e(i, t-1)$). This term will be named efficiency change and will be associated with technological diffusion.

When $u_i = -\ln e(i, t-1)$ is assumed to be non-negative, $F(\mathbf{x})$ is said to be a deterministic frontier. If $\ln F(\mathbf{x})$ is linear in the parameters, four different methods can be used to estimate it: linear programming (LP), quadratic programming (QP), corrected ordinary least squares (COLS) and maximum likelihood (MLE).

LP has been used by Nishimizu and Page (1982), Handoussa, Nishimizu and Page (1986) and Noh (1987) in their analyses of the Yugoslavian, Egyptian and Korean, and Japanese manufacturing, respectively. This method presents two noteworthy problems. First, as no assumption is made with respect to the distribution of the inefficiency residuals, it is not possible to derive the statistical properties of the estimators.² Second, there will be only as many observations on the frontier as there are parameters to be estimated. With QP, although this second problem does not arise, one still gets estimators that have unknown statistical properties.^{3,4} The COLS procedure consists in estimating an average production function by OLS and correcting the value of the intercept to obtain a frontier production function. COLS estimators for the slope coefficients are best linear unbiased and consistent.

¹ Following Nishimizu and Page (1982), I call the estimates of TFP change obtained using the production function "parametric estimates", whereas the values obtained using index numbers (that require assuming price-taking and profit-maximizing behavior by firms) will be termed "nonparametric". Note also that there is an index number problem, as TFPC_p could be alternatively decomposed as

$$\begin{aligned} \text{TFPC}_p &= \ln[Y(i, t)/Y(i, t-1)] - \ln F[\mathbf{x}(i, t); i, t] - \ln F[\mathbf{x}(i, t-1); i, t] = \\ &= \ln F[\mathbf{x}(i, t-1); i, t] - \ln F[\mathbf{x}(i, t-1); i, t-1] + \\ &+ \ln e(i, t) - \ln e(i, t-1). \end{aligned}$$

² Schmidt (1976) has shown that if the u_i 's are independently and identically distributed (i.i.d.) according to an exponential distribution, then LP yields maximum likelihood estimators (MLE). This, however, is of little consequence since in this case MLE do not necessarily have their usual desirable asymptotic properties.

³ If the u_i 's are i. i. d. and their common distribution is half-normal, then QP yields MLE (Schmidt, 1976); but, as with LP, these MLE may lack their usual properties.

⁴ The solutions to the LP and QP problems are, respectively, the least absolute deviation and the least squares estimators of \mathbf{a} . It is interesting to note that while the latter is more popular in the average production function problem, the former is preferred in the estimation of frontier production functions.

To obtain a consistent estimator of the intercept one most often exploits the fact that the errors are unilaterally distributed to shift the production function upward, such that all observations but one lie below the frontier.

MLE are efficient, in the sense that they take into account the skewness of the assumed distribution of the inefficiency residual, have known statistical properties and can have their significance tested, at least asymptotically. MLE present, however, two important drawbacks. First, they are sensitive to the choice of distribution for u_i . Since there is no *a priori* argument on behalf of any particular distribution, this can be regarded as an important disadvantage of the method. Second, in the deterministic frontier set-up, all observations (Y_i) must lie on or below the frontier ($F(\mathbf{x}_i)$) and, as a consequence, the condition that the support does not depend on the parameters to be estimated is not satisfied. It follows that in this case one cannot guarantee that the MLE will have their ordinary properties. Greene (1980a) showed, however, "that the usual desirable properties of maximum likelihood estimators still hold if the density of u satisfies the following conditions: (a) the density of u is zero at $u = 0$; (b) the derivative of the density of u with respect to its parameters approaches zero as u approaches zero". He also noted that the gamma density satisfies both conditions.⁵

The deterministic frontier problem is then defined as finding a solution to

$$\begin{aligned} \text{Max}_{\mathbf{a}, \tau, P} \quad L^* &= T (P \ln \tau - \ln \Gamma(P)) + (P-1) \sum_i^T \ln(\varepsilon_i) - \tau \sum_i^T \varepsilon_i \\ \text{s. t.} \quad \varepsilon_i &= \ln(F(\mathbf{a}, \mathbf{x}_i)) - \ln(Y_i) \geq 0 \\ \tau &> 0, \text{ and} \quad P > 2 \end{aligned} \quad (5)$$

where \mathbf{a} is the vector of parameters of the frontier production function $F(\mathbf{a}, \mathbf{x}_i)$, and T is the number of observations in the sample.

Deterministic frontiers present two important problems arising from the requirement that all firms lie on or below the frontier. First, the estimators become very sensitive to measurement errors and, in particular, to the presence of outliers. Second, there is an implicit assumption that all inefficiency is endogenous; that is, there is no room for exogenous shocks or inefficiencies arising from factors outside the control of the establishment. A way of overcoming these problems is through the use of the stochastic frontier model, that can be represented by

$$\ln Y_i = \ln F(\mathbf{a}, \mathbf{x}_i) + (u_i + v_i) \quad (6)$$

In (6), the stochastic frontier is given by $F(\mathbf{x}_i) \cdot \exp(v_i)$, where v_i has a symmetric distribution and reflects measurement errors and exogenous shocks, while u_i has a one-sided distribution and accounts for the establishment's technical inefficiency. Assuming u_i is distributed according to a half-normal and v_i to a normal distribution,⁶ the stochastic frontier estimation problem is given by

$$\text{Max}_{\mathbf{a}, \theta, \sigma^2} \quad L^* = T/2 \ln(2/\pi) - T \ln(\sigma)$$

⁵ Neither the exponential nor the half-normal densities meet the above conditions, and this makes the estimates obtained by LP and QP less attractive.

⁶ Since now the value of Y is no longer required to be lower than $F(\mathbf{x}_i)$, the regularity conditions are satisfied and the MLE have their desirable asymptotic properties.

$$+ \sum_i^T \ln[1 - \Phi(\varepsilon_i \theta / \sigma)] - (1/2\sigma^2) \sum_i^T \varepsilon_i^2$$

$$\text{s. t. } \varepsilon_i = u_i + v_i = \ln(F(\mathbf{a}, \mathbf{x}_i)) - \ln(Y_i)$$

$$u_i \geq 0, \quad -\infty \leq v_i \leq \infty$$

$$\sigma^2 = \sigma_u^2 + \sigma_v^2 \quad \text{and} \quad \theta = \sigma_u / \sigma_v \quad (7)$$

where $\Phi(\cdot)$ is the cumulative distribution of the standard normal.

To test the robustness of some of the results with respect to frontier specification and estimation procedure, I also estimated a corrected least squares (CLS) frontier by maximum likelihood, assuming that the error term is normally distributed. The problem in this case can be defined as finding a solution to

$$\text{Max}_{\mathbf{a}, \sigma} L^* = T/2 \ln(2\pi) - T \ln(\sigma^2) - (1/2\sigma^2) \sum_i^T \varepsilon_i^2 \quad (8)$$

$$\text{s. t. } \varepsilon_i = \ln(F(\mathbf{a}, \mathbf{x}_i)) - \ln(Y_i)$$

$$-\infty \leq \varepsilon_i \leq \infty$$

In all cases the frontiers are represented by a translog production function,

$$\begin{aligned} \ln(F(\mathbf{a}, \mathbf{x}_i)) &= \mathbf{a}_0 + \mathbf{a}'_i \mathbf{Z} + \mathbf{Z}' \mathbf{A} \mathbf{Z} = \\ &= \mathbf{a}_0 + \sum_j^4 \mathbf{a}_j \ln(\mathbf{x}_j) + 1/2 \sum_k^4 \sum_j^4 \mathbf{a}_{jk} \ln(\mathbf{x}_j) \ln(\mathbf{x}_k) \end{aligned} \quad (9)$$

Although flexible functional forms such as the translog may capture more closely the actual technology being used, it is not unusual in empirical studies to obtain coefficients that do not represent a true production function. In particular, it is not uncommon to reach a specification that is not concave and sometimes not even monotonic. To avoid this, the following restrictions were imposed

$$\sum_j^4 \mathbf{a}_j = 1 \quad (10)$$

$$\sum_j^4 \mathbf{a}_{jk} = 0, \quad k = 1, 4 \quad (11)$$

$$\mathbf{a}_j \geq 0, \quad j = 1, 4 \quad (12)$$

$$\mathbf{a}_{jj} \leq 0, \quad j = 1, 4 \quad (13)$$

Restrictions (10) and (11) are necessary and sufficient to impose constant returns to scale on the translog, while (12) is necessary and sufficient to guarantee monotonicity at the

approximation point. Assuming that the three hold, then (13) is both necessary and sufficient to guarantee global concavity.⁷ It is worth pointing out, however, that attaining global concavity is not costless. In particular, as shown by Diewert and Wales (1987), there is a loss of flexibility and the risk of overestimating the elasticities of substitution between the inputs.

To solve the maximization problems described by (6) to (8), subject to (9) to (13), I used the optimization software GQOPT, developed by Richard Quandt and Stephen Goldfeld; in particular, its subroutine GRADX. The variance-covariance matrix of the asymptotic distribution was obtained from the inverse information matrix. Accuracy was fixed at 1^{-10} . The program converges in case one of the following is less than accuracy: (a) the attempted change in the value of each of the parameters, (b) the norm of the gradient, and (c) the relative improvement in the function value at any step.

3. Data base

The data used in the analysis consists of all establishments with more than five employees, that had been active the entire year of the 1970 and 1980 industrial censuses and for which all elements of the input vector were strictly positive.⁸ To allow for enough degrees of freedom, only sectors with more than 20 establishments were considered. To control expenses with computer facilities, systematic samples were taken for all sectors with a large number of observations, setting an upper bound of around one thousand establishments in each sector.⁹

Four different inputs — capital, labor, material inputs and energy — were considered. Output was defined as the current value of the goods and services produced by the establishment. The flow of capital services was assumed to be proportional to the stock of machinery, equipment and installations, which was used to measure the capital input. The labor input measure tried to take into account the influence of different skill compositions and was defined as $L_i^{k,t} = \sum_h^H w_h^{k,t} L_{h,i}^{k,t}$, where $w_h^{k,t}$ is the average wage paid to workers of skill h , in sector k and year t , and $L_{h,i}^{k,t}$ is the corresponding figure for the number of employees in establishment i . The material input and energy variables were measured by the value of goods and services consumed in production and the establishment's expenses with electric energy and fuels, respectively.

⁷ A proof of necessity and sufficiency of these conditions was first advanced by Jorgenson and Fraumeni (1981). It relies on the one-to-one correspondence between the elements of the matrix of (constant) share elasticities A (see expression 9) and those of its Cholesky decomposition. For global concavity it is necessary and sufficient that the diagonal elements of the Cholesky factorization of A be non-positive, what in this case is equivalent to conditions (13). See Jorgenson (1986, 1859/60) for a more detailed proof and Lau (1978) for a discussion on imposing and testing monotonicity and concavity using Cholesky decompositions. Note the particular role played by the constant returns to scale assumption.

⁸ The unit of observation in the census is the establishment, defined as "the part of the organization that is in charge of the industrial activity and has installations and means to produce industrial goods".

⁹ Table 4.B.1 of Pinheiro (1989) lists the number of establishments in each sector before and after sampling. The last column of that table shows the number of establishments in the working sample with more than 50 employees and after eliminating outliers.

4. Empirical results

The estimates obtained for the frontiers are not reported here due to space limitations, but can be found in Pinheiro (1989).¹⁰ Two observations about them are noteworthy. First, the hypothesis of a Cobb-Douglas specification for the frontier production function is rejected for almost all sectors, as most translogs present statistically significant second-order parameters. In many cases, though, the coefficients a_j ($j=1, 4$) are all not significantly different from zero and the concavity restrictions (13) are binding. This result reflects the difficulty to estimate the slope of isoquants when moving away from the approximation point, not an uncommon problem in the estimation of flexible functional forms.

Second, the estimates for both P and the intercept (a_0) in the deterministic frontier are, for most sectors, very large, reflecting the lack of asymmetry of the residual distribution: as P goes to infinite, the gamma distribution becomes symmetric.¹¹ The estimates obtained for the inefficiency term using the stochastic frontier also present this lack of skewness. In this case, θ (equal to σ_u/σ_v) was of the order of magnitude of 10^{-4} for all sectors. From the analysis of Jondrow et al. (1982) it follows that $M(u|e)$ — that can be interpreted as the MLE of $u|e$, the conditional inefficiency error — is of the order of magnitude of 10^{-8} . A consequence of this symmetry is that the asymptotic efficiency gain of MLE diminishes considerably.

It is interesting to observe, in this regard, that similar results have been reported elsewhere. Aigner, Lovell and Schmidt (1977) were not able to identify the technical inefficiency term in their analysis of the American metals industry; neither were Lee and Tyler (1978) for two of the five sectors of the Colombian manufacturing industry they examined, nor Braga and Rossi (1986) for two-thirds of the 136 Brazilian manufacturing sectors they analyzed. A relatively symmetric distribution is also reported by Alves (1987). Probably, equivalent (unreported) results were also obtained in studies using LP, QP or COLS.

The symmetric distribution of the inefficiency term suggests that best-practice technology is used by few establishments, with most firms adopting average efficiency production techniques, and a few companies being relatively inefficient. This scenario reflects a slow diffusion of best-practice techniques, as discussed by Salter (1966). Causes for that would stem from differences in firms' sizes to training of managers, capital sunkness, indivisibilities, ownership and expectations about future demand. In a developing inward-oriented economy as Brazil in the 70's, few firms in each sector are sufficiently large to profitably exploit more up-to-date, usually imported, technologies. For most firms, their small scale would not warrant the investment in acquiring, adapting or developing modern technologies. Another possibility is the presence of measurement errors that overwhelm the inefficiency term. The data covers a large number of different firms all over Brazil, and it would not be surprising if the variables used are not perfectly measured, especially the capital input.

A case could be made that the asymmetric distribution would describe the situation in mature economies and sectors with well-established technologies. The symmetric profile, on the other hand, would apply to sectors or economies undergoing rapid change. This picture contrasts with the scenario underlying the utilization of skewed distributions, which

¹⁰ Another set of results from the frontiers' estimation concerns the technical efficiency of individual establishments. Pinheiro (1990) examines these results and correlates technical efficiency with a set of variables reflecting firms' characteristics such as size and share of exports in output.

¹¹ The skewness of the gamma distribution is equal to $2/\sqrt{P}$.

presupposes a significant concentration of establishments close to the frontier. This implicitly assumes that technological progress is exogenous to firms and that they have equal access to best-practice technology, so that sectors are characterized by rapid technological diffusion. If this scenario is not consistent with reality, there is little justification for the use of frontier rather than average production functions.

Irrespective of the actual cause, however, it seems that the heterogeneity of the working sample may be an important factor contributing to the symmetrical distribution of ε . To assess the relevance of this hypothesis, the frontiers were re-estimated using a reduced sample that includes only establishments with more than 50 employees. After estimating the frontier with this reduced sample, 5 percent of the establishments in each extreme of the distribution were deleted to eliminate outliers and the deterministic frontier was re-estimated. With this procedure skewness increased for most sectors, although not for all of them.

After estimating the frontier production functions one should be able to measure the technical efficiency of establishments and sectors. However, one of the consequences of the symmetry of the distributions is that in general the technical efficiency estimates lack economic sense. For the stochastic frontier, they are equal to one for all sectors. For the deterministic frontier, on the other hand, the opposite result was found. As the estimated error distribution becomes symmetric, its range of variation tends to $(-\infty, +\infty)$. Since the observed residual is required to be positive, it is necessary that the probability of ε_i being negative be very low, especially if the number of observations is large. As a consequence, the estimators of P and a_0 become highly positively correlated, a result which is actually observed empirically. A high value for the intercept implies that the technical efficiency levels for all establishments are close to zero.

To give economic meaning to the technical efficiency measures the intercepts were changed, shifting the frontiers. The resulting distributions of sector technical efficiencies are reported in table 1 (estimates for efficiency levels in each sector are presented in table 2).¹² Column (1) presents my preferred measure — to be used in the remainder of the paper — that is obtained using the deterministic frontier and the complete sample, and letting 5 percent of the establishments be above the frontier. In this preferred case, technical efficiency levels lie around 60 percent in 1970 and 55 percent in 1980 for most sectors. In 1970 technical efficiency goes from a minimum of 31 percent to a maximum of 86 percent. These limits are 32 percent and 78 percent, respectively, in 1980. Columns (2) and (3) summarize the results obtained for 1970 using alternative assumptions with respect to the distribution of the error term. Two facts should be noted. First, the technical efficiency levels obtained with the stochastic and the deterministic frontiers are very alike for most sectors. Second, when the distribution of the residuals is assumed to be normal (the corrected least squares procedure), technical efficiency is found to be significantly smaller for all sectors. This highlights the influence of the assumed distribution on the efficiency estimates.

Columns (4) and (5) assess the impact of letting different proportions of the sample lie beyond the frontier. As expected, while technical efficiency increases with the percentage of extra-efficient establishments, the cross-sector variance is significantly reduced. Two other points seem worth calling attention to. First, in sectors with few establishments, there is very little or no difference between letting 0 percent or 1 percent of the sample be beyond

¹² Sector technical efficiency is measured as the technical efficiency of the average establishment, defined as a hypothetical firm that consumes sector average amounts of each of the inputs and produces sector average quantities of output.

the frontier. Second, for sectors with asymmetric error distributions, technical efficiency does not change much when allowing 5 percent of the firms to be super-efficient.

Table 1
Distribution of average sector efficiency

Interval	1970						1980			
	(1)	(2)	(3)	(4)	(5)	(6)	(1)	(4)	(5)	(6)
0.0 - 0.1			0.09	0.10				0.20		
0.1 - 0.2			0.23	0.19	0.04			0.18	0.04	
0.2 - 0.3			0.38	0.17	0.13			0.34	0.24	
0.3 - 0.4	0.03	0.04	0.24	0.23	0.21		0.06	0.14	0.38	0.01
0.4 - 0.5	0.08	0.09	0.04	0.17	0.36	0.03	0.19	0.08	0.25	0.05
0.5 - 0.6	0.28	0.28		0.07	0.19	0.08	0.45	0.02	0.05	0.23
0.6 - 0.7	0.46	0.46	0.01	0.05	0.05	0.31	0.28	0.04	0.04	0.49
0.7 - 0.8	0.11	0.10		0.01	0.01	0.49	0.02			0.20
0.8 - 0.9	0.03	0.03				0.09				0.02
0.9 - 1.0										
Total	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Minimum	0.31	0.31	0.00	0.03	0.12	0.48	0.32	0.02	0.13	0.40
Maximum	0.86	0.87	0.67	0.71	0.71	0.86	0.78	0.68	0.68	0.85
Average	0.61	0.61	0.24	0.32	0.42	0.70	0.55	0.24	0.36	0.64
Weighted average	0.58	0.60	0.26	0.20	0.37	0.71	0.56	0.18	0.34	0.64

(1), (2) and (3) are, respectively, the deterministic, stochastic and CLS frontiers, with 5 percent of the establishments above frontier. (4) is the deterministic frontier with 0 percent of the establishments above frontier. (5) is the deterministic frontier with 1 percent of the establishments above frontier. (6) is the deterministic frontier with 5 percent of the establishments above frontier, using the reduced sample (>50 employees).

At last, using the reduced sample (>50 employees) and letting 5 percent of the observations be above the frontier, one obtains average sector technical efficiency levels of 70 percent in 1970 and 64 percent in 1980 (column 6). It is also found that the variance of the technical efficiency term is considerably smaller for the reduced sample than for the original one (>5 employees). This is consistent with the assumption of higher homogeneity among large firms and with the idea that, on average, these firms tend to be closer to the frontier.

Two conclusions may be drawn from this sensitivity analysis. First, not too much stress should be put on sector efficiency levels *per se*. Second, irrespective of how efficiency is measured, it has gone down in the 70's. On average, this annual decline equaled 0.7 percent for the complete sample and reached 0.9 percent for the sample of large firms. For individual sectors the yearly change in efficiency ranged from -4.2 percent to 1.9 percent for the complete sample and from -4.9 percent to 1.7 percent for the reduced sample. These results are similar to the ones reported by Nishimizu and Page (1982) for Yugoslavia, by Handoussa, Nishimizu and Page (1986) for Egypt, and by Noh (1987) for Japan and Korea. In all cases there was a general decline in technical efficiency. Note, though, that sector technical efficiency reflects firms' position with respect to the most efficient concerns, and not absolute productivity levels — that is, as some firms adopt new technologies and others do not,

variance increases and so does inefficiency. This does not imply that firms start to misuse the technology they control. It is also important to note that firms may rationally decide not to adopt best-practice technologies for several reasons, such as sunkness of assets and the existence of quasi-fixed inputs or non-convexities.

Table 2
Average technical efficiency and decomposition of total factor
productivity change (% per year)

Sector	Effic. level		TFPC _P	Techn. progress	Effic. change	TFPC _{np} - TFPC _P	TFPC _{np}
	1970	1980					
<i>Construction</i>	57.7	51.0	2.87	4.15	-1.23	-1.11	1.75
Stones for construction	48.6	43.8	6.40	7.24	-1.03	-1.32	5.08
Lime	43.4	32.2	4.97	7.89	-2.99	-8.17	-3.20
Ceramics	44.8	53.0	1.59	-0.10	1.69	-0.23	1.35
Cement	59.8	39.3	4.13	8.35	-4.21	-0.23	3.90
Cement products & artifacts	62.4	53.6	0.11	1.65	-1.53	-0.28	-0.17
Glass	60.2	48.1	5.58	7.70	-2.24	1.39	6.97
Processed non-metallic minerals	37.1	36.6	13.39	12.72	-0.15	-4.99	8.40
Misc. prod. of non-metallic miner.	68.0	48.2	0.83	4.33	-3.44	-1.22	-0.39
Wood	58.1	51.1	0.93	2.22	-1.29	-2.22	-1.29
Wood furniture	62.9	64.0	3.40	3.16	0.18	-1.61	1.79
Metallic furniture	68.7	59.2	3.98	5.40	-1.48	0.56	4.55
Upholstery	72.5	66.0	4.77	5.61	-0.95	0.52	5.29
<i>Metal-mechanic complex</i>	62.1	55.2	1.77	2.99	-1.19	1.12	2.89
Iron and steel	63.0	53.5	2.31	3.92	-1.62	-0.77	1.55
Nonferrous metals	59.2	54.6	1.09	1.90	-0.81	2.07	3.17
Metallic structures		55.8					2.17
Iron and steel artif.	64.4	59.2	0.31	1.15	-0.83	0.02	0.33
Metal stamping	56.5		0.68			0.35	1.03
Metal tanks & recipients	69.2	60.5	2.92	4.24	-1.34	0.25	3.18
Cutlery/hand tools/hardware & guns	71.2	59.6	1.01	2.79	-1.78	0.93	1.94
Special ind. machin.	60.8	48.6	2.22	4.45	-2.23	-0.03	2.19
Ind. mach. for hidr. & termic inst.	67.4	56.7	3.04	4.74	-1.73	-1.03	2.01
General industrial machinery		48.1					1.56
Machin. & equip. for agriculture	54.7	56.6	1.75	1.40	0.33	1.01	2.76
Home & office machin. & equip.	55.3	55.9	2.63	2.50	0.10	2.68	5.31
Clocks, whatches & chronometers		44.0					6.10
Tractors & earth-moving machines	55.2	54.8	-2.25	-2.19	-0.08	8.32	6.07
Equip. for electrical energy	61.9	59.0	2.28	2.73	-0.48	0.79	3.08
Electrical material ^a	61.6	56.8	6.22	6.85	-0.81	0.76	6.98
Lamps		69.8					5.50
Electr. material for vehicles ^b	61.1	63.8	3.89	3.38	0.44	4.35	8.23
Electrical appliances	66.9	58.3	2.86	4.21	-1.38	2.97	5.83
Electronic material	55.3	51.1	1.01	1.78	-0.77	2.72	3.73
Communication equipment	55.0	58.5	-2.24	-2.89	0.62	1.54	-0.70
Naval industry	44.6		6.03			3.19	9.22
Railway stock		47.1					4.70
Autom. vehicles & parts	65.6	56.9	0.93	2.36	-1.42	2.10	3.03
Bicycles	82.6	67.5	0.99	3.03	-2.03	0.15	1.14
Aircrafts		43.7					14.50
Other vehicles	66.6	53.3	2.20	4.42	-2.22	2.20	4.40

(continues)

(continuation)

Sector	Effic. level		TFPC _p	Techn. progress	Effic. change	TFPC _{np} - TFPC _p	TFPC _{np}
	1970	1980					
<i>Paper and printing</i>	61.0	63.5	4.59	4.18	0.39	0.55	5.14
Paper	64.7		0.36			2.58	2.94
Paper products	58.8	62.2	5.86	5.15	0.55	0.29	6.15
Newsp., books, manuals & period.	57.1	56.4	6.10	6.05	-0.13	-0.51	5.59
Teaching/ind./comm. printed matter	64.7	78.4	7.33	5.17	1.92	-0.39	6.94
Other print. & publish. services	54.9	45.0	2.82	4.81	-2.01	-0.94	1.87
<i>Chemical complex</i>	56.6	56.3	3.61	3.67	-0.06	-0.65	2.96
Rubber	74.6	59.2	3.69	5.96	-2.31	-0.54	3.15
Chemical elem. & composit.	30.9	33.0	3.00	2.32	0.64	-3.23	-0.23
Oil-refining & petrochem.	51.4	58.0	4.35	3.05	1.22	-1.58	2.77
Artific. threads & resins		56.8					1.59
Raw vegetable oils	57.2	50.3	0.75	2.06	-1.30	0.56	1.31
Aromatic concentrates	72.6	58.8	-2.29	-0.18	-2.11	5.72	3.43
House clean. prod. & pesticides	60.4	61.7	8.10	7.57	0.22	-1.34	6.75
Pigments, paints & solvents	63.3	56.1	1.02	2.23	-1.21	3.24	4.26
Fertilizers	65.2	58.8	0.81	1.83	-1.02	1.83	2.64
Other chemical products		54.6					3.00
Pharmaceuticals	53.2		3.11			3.50	6.61
Perfumery	56.4	54.2	0.68	1.07	-0.40	1.65	2.33
Soaps		59.4					2.29
Candles	57.6	64.4	6.64	5.33	1.10	0.81	7.45
Plastic sheets	64.6	54.1	3.14	4.88	-1.77	0.12	3.26
Plastics for industrial use	64.2	55.5	4.33	5.71	-1.46	-0.29	4.04
Plastics for domestic use		74.3					-0.07
<i>Textile and footwear complex</i>	57.8	54.2	2.38	3.05	-0.65	-0.49	1.89
Leather and hides	66.2	60.7	-2.04	-1.20	-0.86	0.50	-1.55
Spinning and weaving	52.1	47.2	1.46	2.45	-1.00	-0.46	1.00
Other textile artifacts	55.5	49.8	7.03	7.89	-1.09	-0.52	6.52
Special textile products	68.0	51.9	2.83	5.53	-2.70	1.97	4.80
Clothing	64.9	62.4	7.12	7.27	-0.39	-1.40	5.72
Hats	58.0	54.9	4.58	5.03	-0.55	-0.46	4.12
Footwear	74.5	63.3	0.48	2.12	-1.63	0.07	0.55
Other clothing products	72.0	56.7	-1.00	1.42	-2.40	-0.31	-1.30
<i>Agri-industrial complex</i>	60.5	55.3	2.03	2.95	-0.89	-0.48	1.55
Agribusiness	63.2	59.2	2.84	3.46	-0.65	0.05	2.89
Canned & preserved fruits & veget.	61.5	62.4	1.00	0.85	0.15	-1.39	-0.39
Meat products & animal fats	54.8	50.2	2.28	3.14	-0.88	-1.02	1.26
Fish & other sea products	63.7	42.8	-0.43	3.64	-3.99	0.91	0.48
Candies & chocolates	73.9	61.2	-1.70	0.20	-1.89	-0.21	-1.90
Bakery products	67.6	56.0	-3.24	-1.39	-1.89	0.60	-2.65
Pasta & cookies	76.4	60.3	-0.14	2.24	-2.36	0.80	0.66
Veget. oil & fats & misc. food prod.	48.7	50.1	2.67	2.35	0.28	-0.44	2.23
Alcoholic beverages	64.7	61.7	3.85	4.26	-0.48	-2.85	0.99
Nonalcoholic beverages	54.8	56.2	7.40	6.88	0.26	-2.25	5.15
Tobacco	71.6	65.9	-0.11	0.73	-0.83	2.03	1.91

Note: TFPC_p and TFPC_{np} are, respectively, the rates of total factor productivity change estimated using frontier production functions and index numbers. See text for description of the other variables. Efficiency levels were estimated using the deterministic frontier, the complete sample of firms and allowing 5 percent of the establishments to be above the frontier.

^a Exclusive vehicles.

^b Excluding train engines.

Sector efficiency levels were correlated with a set of variables to search for stylized facts about the most or least efficient sectors. These variables were selected on an *ad hoc* basis from the data available and there are no maintained hypotheses regarding the sign or size of the correlation coefficients. The results were not very revealing, but tended to loosely confirm that low or decreasing technical efficiency is associated with structural change.¹³ For both 1970 and 1980 a negative and significant association was obtained between technical efficiency and different measures of capital intensity. On the other hand, positive and significant correlations resulted for the rates of return on the stock of total capital and on the stock of machinery in 1980. For male participation in the labor force, the degree of capital utilization, and the share of skilled workers in total labor force in production correlations were negative. They were significant in 1970 for the first two and in 1980 for the latter.¹⁴ In addition, it was found that sectors in which technical efficiency increased were the ones that in 1970 had low ratios of investment to capital stock and large shares of skilled workers in production. Average technical efficiency increased with the rate of return to capital and profitability, and decreased with capital intensity, male participation in the labor force and the share of imports in the consumption of material inputs.

In table 2, the values of $TFPC_p$ and the two terms of the decomposition yielded by (4) for 1970-80 — using the deterministic frontier, the complete sample and allowing 5 percent of the establishments to be above the frontier — are reported for each sector. The two additional columns in table 2 give the nonparametric or index number measure of TFP growth in 1970-80 ($TFPC_{np}$) and the difference between this measure and the one obtained using the frontiers.

The two measures of TFP change may differ due to: (a) the use of cost shares to approximate output elasticities with respect to the inputs; and (b) the fact that the elasticities for “interior” establishments, for which the shares provide estimates, may differ from those for establishments at the frontier, used in the parametric estimation. To see this, recall the expression for the translog index of (nonparametric) TFP change from $t-1$ to t :

$$TFPC_{np} = \ln [Y(k, t)/Y(k, t-1)] - \sum_j^4 v_j [\ln (x_j(k, t)/x_j(k, t-1))] \quad (14)$$

where,

$$v_j = [v_j(k, t) + v_j(k, t-1)]/2, \text{ and}$$

$v_j(k, t)$ is the cost share of input x_j in the output of sector k in period t , so that

¹³ Correlation coefficients are reported in Pinheiro (1989).

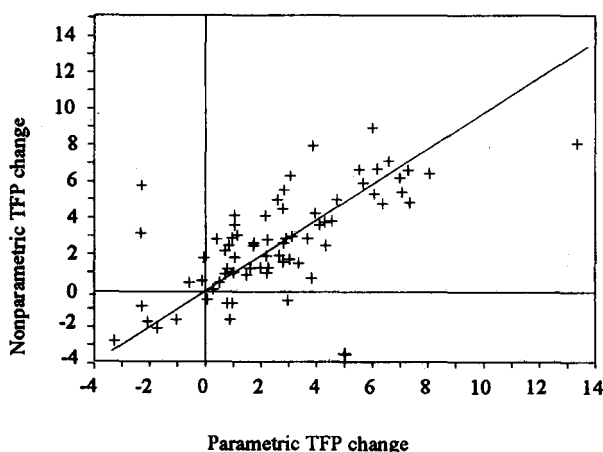
¹⁴ In addition, negative correlations were observed in both years between technical efficiency and the shares of direct exports in output and of imports in material inputs, the average establishment size, and the share of less-than-five-year-old establishments in production. None of these correlations was, however, significant. For the investment-capital ratio, the composition of the stock of capital and the degree of industrial concentration the correlations were positive and non significant for 1970 and 1980. Finally, for the share of imports in investment in machinery, for the ratio of royalties to profits and for profitability the correlations were non significant and of different signs in the two years.

$$\begin{aligned} \text{TFPC}_{np} - \text{TFPC}_p = & \ln F[\mathbf{x}(k, t); k, t-1] - \ln F[\mathbf{x}(k, t-1); k, t-1] - \\ & - \sum_j^4 v_j [\ln x_j(k, t) - \ln x_j(k, t-1)] \end{aligned} \quad (15)$$

On average, both measures of TFP growth yielded yearly averages of 2.6 percent in 1970-80. The rates of TFPC_p and TFPC_{np} were also very close for most sectors, although for extreme observations significant discrepancies arose (figure 1). These differences are, however, of the same order of magnitude of the ones reported for Yugoslavia (Nishimizu and Page, 1982) and for Korea and Japan (Noh, 1987). These results suggest that the strong assumptions necessary for using index numbers to compute TFP growth may not seriously compromise the estimates.

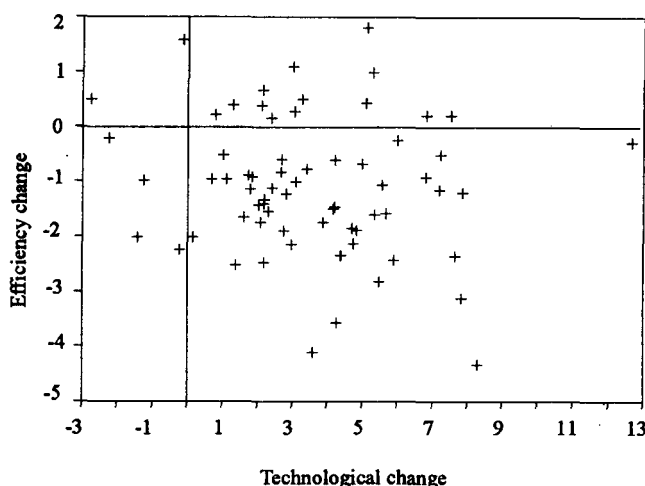
Results for TFP growth decomposition revealed the leading role played by technological improvement. While best practice TFP advanced annually 3.3 percent, technical efficiency declined 0.7 percent. This reinforces the idea that when structural change is significant, TFP change is caused mainly by the adoption of new technologies (see figure 2), with technological diffusion taking place at a slower pace, increasing cross-firm heterogeneity and average sector inefficiency.

Figure 1
Parametric vs. nonparametric TFP change
(% p. a.)



Source: Table 2.

Figure 2
Efficiency vs. technological change
(% p. a.)



Source: Table 2.

Technological progress was found to have taken place at significant rates in the six complexes examined here.¹⁵ It was more important in the paper and printing (4.2 percent per year), construction (4.2 percent per year) and chemical (3.7 percent per year) complexes, but was likewise remarkable in the textile and footwear (3.1 percent per year), metal-mechanic (3.0 percent per year) and agri-industrial (3.0 percent per year) complexes.

The cement and glass sectors were responsible for the significant productivity growth and technological progress in the construction complex during the 70's. Both sectors expanded their output substantially during this period and invested in reducing the consumption of energy after the oil shock. The two are highly concentrated, the second with a significant share of output in the hands of transnational corporations, in contrast with the ceramics and wood sectors, which acted to slow down the complex's performance.

The iron and steel and the automobile sectors answer for 45 percent of the metal-mechanic complex's output, the most important of the six considered here. The high growth rates of average and best practice productivity in iron and steel were probably due to the large investments done by the public enterprises that used to answer for a major share of the sector's output. The automobile sector experienced a more modest advance in productivity and in the technological frontier. The naval industry presented the highest rate of productivity growth among the transport equipment sectors. Mechanic and electrical equipment also showed advances with the exceptions of tractors and earth-moving machinery in the first

¹⁵ "A segment of the economy forms an industrial complex when it comprises a set of industries that are technologically interdependent, that are regulated by a common set of rules or that produce goods that are highly complementary in consumption." (Araujo et al., 1990: 9).

and of communication equipment in the second. Electrical equipment was the sector with most significant technological diffusion in the complex.

Paper and printing experienced the highest rates of productivity growth and technological progress and diffusion. Only one sector — other printing and publishing services — showed a significant reduction in technical efficiency. Such good performance may be explained by the fact that in the 70's these sectors were largely stimulated by BNDES, the Brazilian National Development Bank, receiving 14 percent of the total loans provided for basic input sectors in 1974-79, and also being favored by substantial fiscal incentives for forestry. Technological progress resulted in part from the indigenous development of short fiber pulp.

The good performance of the chemical complex was pulled by four of its most important sectors: rubber, chemical elements and composites, oil refining and petrochemicals, and pharmaceuticals, responsible for 58 percent of the complex's output. The negligible loss of efficiency in the complex resulted from the very positive performance of oil refining and petrochemicals sector, that answers for about a third of the complex's output. Possibly, the role played by Petrobrás in this sector helps to explain why it was so successful in diffusing technological progress.

TFP in the textile and footwear complex grew 2.4 percent yearly in the 70's. Technological progress was fast, but efficiency declined 1 percent annually, revealing a slow rate of technological diffusion. Three sectors responded for almost 90 percent of the complex's output: spinning and weaving (58 percent), clothing (20 percent) and footwear (11 percent). While the first and the last presented low rates of productivity growth and below average rates of technological progress, the clothing sector showed rapid change in average and best practice TFP.

Finally, in the agri-industrial complex the three main sectors — agribusiness, meat and vegetable food products, with 25 percent, 26 percent and 16 percent of output, respectively — showed a similar performance in 1970-80, with annual rates of TFP growth and of technological progress in the (2.3 percent, 2.8 percent) and (2.4 percent, 3.5 percent) intervals, respectively. The best results were accomplished by the beverage sector.

5. Final remarks

Two sets of conclusions may be drawn from the analysis undertaken in this paper. The first has to do with the methodology used in the estimation and decomposition of TFP growth in 1970-80. The second concerns the empirical results proper.

Results from the frontier estimation were not as anticipated. Technical efficiency distribution was found to be symmetric for most sectors, contrary to prior expectations, with two consequences. First, the gain in estimation efficiency I had expected to achieve using the more expensive maximum likelihood procedure was not accomplished. Second, estimates for the deterministic and the stochastic frontiers turned out to be much alike, precluding a more thorough comparison of the two methodologies.

The symmetry of the distribution of technical efficiency suggests a slow rate of technological diffusion, which may result from the fact that in a developing inward-oriented country as Brazil in the 70's few firms in each sector are sufficiently large to profitably exploit more up-to-date technologies. For most firms, their small scale does not justify investing in the acquisition, adoption or development of modern technologies. This picture contrasts

with the scenario inherent in the utilization of skewed distributions, which assumes a concentration of establishments close to the frontier. If this scenario does not conform with reality, there is little justification for the use of frontiers rather than average production functions in the estimation of technical efficiency. This may be important to consider given the time and resources usually consumed in frontier estimation.

The analysis of sector technical efficiency levels showed that not too much emphasis should be placed upon measures of efficiency *per se*, as they are quite sensitive to the number of establishments allowed to be above the frontier and to the distribution assumed for the inefficiency term. A correlation analysis with a set of variables reflecting sector characteristics revealed little, but suggested that low or decreasing efficiency levels tend to be associated with high investment rates and capital intensity.

Using frontier production functions it was found that TFP had increased 2.6 percent annually in the 70's, exactly the same rate estimated elsewhere using index numbers. Not very significant differences were observed between the two measures of TFP change for most sectors, although in a few cases significant discrepancies arose. The likeness of the two sets of estimates of TFP growth suggests that the strong assumptions that one needs to make when using index numbers to compute TFP growth may not be so binding and the estimates obtained in this fashion not so biased as some fear.

The decomposition of TFP change revealed the leading role played by technological progress in Brazilian manufacturing during the 70's. While the technological frontier advanced about 3.3 percent annually, technical efficiency fell for most sectors, averaging a yearly decline of 0.7 percent in 1970-80. These empirical findings seem to support the conclusion that during periods of rapid structural change, as the 70's in Brazil, the main source of TFP growth is rapid technological progress. It is important to stress, however, that the argument refers to relative, rather than absolute technical efficiency. In this way, reductions in average technical efficiency open new opportunities for policies targeted at the diffusion and adoption of best-practice techniques among a large number of industrial establishments.

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