

# Money and growth: from a quasi-neoclassical standpoint

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**Summary:** 1. Introduction; 2. Money in the utility function; 3. Money in the production function; 4. Cash in advance models; 5. Inflation and growth; 6. Price flexibility and money; 7. Endogenous growth; 8. Money, innovation and endogenous growth; 9. Conclusions.

## 1. Introduction

The relationship between money and growth has been much debated in the literature. The favorable effect of inflation on the long-run level of capital and consumption, obtained by Tobin's growth model, was never satisfactory. According to Tobin (1965), money and capital are substitutes and inflation would reduce the rate of return of money and so stimulate the increase of capital per capita. As a result, the economy would reach a steady state path with higher per capita consumption.

Tobin's work was very important and sets the guides of the discussion. Money is treated as an alternate asset to capital within the framework of Solow's growth model. The real money holdings are endogenous and money becomes neutral, but the rate of growth of money affects the rate of return of money through the inflation rate. Inflation induces agents to change their portfolio composition away from money and in favor of capital. The economy shifts to a new steady state with higher capital intensity. As a result, money is not superneutral and inflation increases per capita income. The intuition behind it is somehow related to Keynes. The lower bound of the rate of interest that would halt investment could be overcome by inflation.

In a way Tobin's results replicate the Phillips curve in the long-run setting. However, the results were controversial and this may explain the growing literature on the subject. From a neoclassical standpoint the system should have real anchors particularly in the long run. From an empirical point of view it becomes increasingly clear that there is an inverse association between inflation and growth.<sup>1</sup>

Sidrauski (1967) provided the first serious answer to Tobin, reestablishing the superneutrality of money. Dornbusch & Frenkel (1973) introduced money in the production function and showed that Sidrauski's conclusion was not unambiguous. Lucas (1980) and Stockman (1981) brought money in the expenditure function along Clower's suggestions and the results point to "anti-Tobin" effects. Furthermore, rigidity of prices may change consider-

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<sup>1</sup> Empirical evidence can be found in Cardoso (1989), De Gregorio (1993) and Fischer (1993).

ably the dynamics of these models. Finally, the resurgence of growth theory, in the form of endogenous growth, allows one to reexamine the relationship of money and growth taking into account its effect on the savings, investment and technology parameters.

Two kinds of studies have been done: one emphasizes money as a transaction device and the other as a financial device. Under the former, inflation clearly inhibits growth. By the latter, it is shown that the development of the financial sector has an important effect on growth.

From another point of view, money should be considered an essential feature of growth. The idea can be traced back to John Stuart Mill but it is due mainly to Schumpeter. The point is that credit facilitates growth. The opportunities of moving to other "combinations" of resources and factors of production cannot be reached in the absence of credit. We try to convey the idea that credit is an important component of financial intermediation and has a direct relationship to monetary policy. In that sense we can interpret those models mentioned above as saying that availability of credit affects growth. In this spirit we present an endogenous growth model along AK lines, in which availability of credit appears as an externality.

To be sure, the role of money recognized in this review is limited by the assumptions embedded in the framework. In the long run, within a steady state model, there is not much play for uncertainty. In that sense, the role of money is limited to a transaction device. But even so, it may be questioned that money is not necessary.<sup>2</sup>

## 2. Money In the utility function

Sidrauski (1967) is the first to introduce money within a dynamic optimizing framework. He follows Ramsey with infinite horizon. Consider  $n$  as the rate of growth of the population. At any time  $s$  families maximize their utility:

$$W_s = \int_s^{\infty} u(c_t, m_t) e^{-\rho(t-s)} dt \quad \text{where} \quad u_{c,c} u_{m,m} > 0 \quad (1)$$

$$u_{cc}, u_{mm} < 0$$

with the following budget constraint:<sup>3</sup>

$$c + \dot{k} + nk + \dot{m} + \pi m + nm = w + rk + x \quad (2)$$

where small caps stand for per capita values,  $c$  consumption,  $m$  real money holdings,  $w$  wage,  $r$  rate of interest,  $k$  capital,  $x$  government transfers, and  $\pi$  inflation.

<sup>2</sup> See Hicks (1982) and Hahn (1969) on this regard. In Hicks: "Money as medium of indirect exchange plays no part in the Lausanne equilibrium... the tacit assumption of perfect foresight deprive the numeraire of any monetary function", p. 33. In Hahn: "... Patinkin model always contains a non monetary solution. Moreover it is not at once clear how we could establish that it also contains a solution with  $P^* > 0$ " (p. 194).

<sup>3</sup> This is known as the dynamic budget constraint and it is consistent with the intertemporal budget constraint if NPG is attended. See Blanchard & Fischer (1989).

The per capita household wealth is by definition:

$$a = k + m \quad (3)$$

Taking derivatives and substituting in the above expression, it follows:

$$\dot{a} = [(r - n)a + w + x] - [c + (\pi + r)m] \quad (4)$$

the interpretation of the last part is "full consumption", including the consumption of services of money.

We assume that the NPG (non-Ponzi game condition) is satisfied.<sup>4</sup>

The Hamiltonian corresponding to this dynamic optimization problem is as follows:

$$H(.) = [u(c, m) + \lambda[(r - n)a + w + x - c - (\pi + r)m]e^{-\rho t}] \quad (5)$$

The first order conditions for the maximum are:

$$\begin{aligned} u_c(c, m) &= \lambda \\ u_m(c, m) &= \lambda(\pi + r) \\ \dot{\lambda} &= \lambda(\rho + n - r) \end{aligned} \quad (6)$$

We assume the terminal condition is satisfied:

$$\lim a_t \lambda_t e^{-\rho t} = 0 \quad (7)$$

$$\text{The first two conditions (6) imply: } u_m = u_c(\pi + r) \quad \text{or} \quad \frac{u_m}{u_c} = \pi + r \quad (8)$$

To close the model we assume efficient (cost minimizing) production:

$$\begin{aligned} r &= f_k(k) \\ w &= f(k) - kf_k(k) \end{aligned} \quad (9)$$

Government transfers are considered to be lump sum, and in the absence of taxation correspond to seigniorage from money issue:

$$x = \sigma m \quad (10)$$

$$\text{In steady state we assume that: } \dot{a} = \dot{m} = \dot{\lambda} = 0 \quad (11)$$

$$\text{The condition: } \dot{m} = 0 \Rightarrow \sigma m = \pi m + nm \Rightarrow \pi = \sigma - n \quad (12)$$

Combining the last condition (6) with (9), and considering steady state, it follows:

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<sup>4</sup> The non-Ponzi game condition guarantees that the indebtedness of the families will not grow more than the interest rate, otherwise the intertemporal budget would become unbounded.

$$f_k(k^*) = \rho + n \quad (13)$$

the asterisk stands for steady state equilibrium values. This is called the *modified golden rule condition*.

Using the budget restriction (2), and the competitive profit conditions (9), and considering steady state we have:

$$c^* = f(k^*) - nk^* \quad (14)$$

Using the conditions (8), (9) and (13), steady state level of real money balances are determined as:

$$\begin{aligned} u_m &= u_c(\pi + r) \\ u_m &= u_c(\pi + f_k(k^*)) \\ u_m &= u_c(\pi + n + \rho) \\ u_m(c^*, m^*) &= (\rho + \sigma)u_c(c^*, m^*) \end{aligned} \quad (15)$$

Equations (13), (14) and (15) characterize the steady state solution of the system, that can be represented as follows:

$$\begin{array}{ccccccc} \text{Eq. 13} & & \text{Eq. 14} & & \text{Eq. 15} & & \\ (\rho, n) & \longrightarrow & k^* & \longrightarrow & c^* & \longrightarrow & m^* \end{array}$$

Clearly, neither the nominal money nor the rate of growth of money matters for the determination of  $c^*$  and  $k^*$ . Money is said to be neutral and superneutral. However, by (15), it is possible to show that the optimal quantity of money holdings depend on the rate of growth of money.

The way money has been introduced in the economy does not satisfy Clower's critique: money is indistinguishable from any other commodity and so it cannot matter (see Clower, 1969). The development of the literature, that follows Sidrauski seminal contribution, can be seen as a response to Clower's criticism. Most of the research can be summarized as experiments into how changes in the specifications of money affect its neutrality. Dornbusch & Frenkel (1973), Brock (1974), Lucas (1980), Stockman (1981) and Fischer (1979) are among the most notorious contributions. Recently Wang & Yip (1992) presented a unified framework to compare the different experiments, and we will make extensive use of it. Our basic contribution in these sections is to examine more fully the effects of money in the production function as well as in the accumulation equation.

### 3. Money In the production function

Sidrauski's model analyzes the influence of money in the economy through the utility of the representative agent. The nature of money is considered to be the same as any other

commodity. Let us examine what happens when money does not enter the utility function but instead affects the production function.

It has been argued that money reduces the transaction costs of the exchanges related to the production process. The representation of the transaction services provided by money holdings could be made directly in the production function or indirectly through a *shopping cost* function in the manner of Dornbusch & Frenkel (1973). With money in the production function the problem can be presented as follows:

$$\text{Max } W_s = \int_s^{\infty} u(c_t) e^{[-\rho(t-s)]} dt \quad (16)$$

supposing all the product belongs to the representative agent the budget restriction can be expressed as:

$$\dot{k} + \dot{m} = f(k, m) - nk - c - (\pi + n)m + x \quad (17)$$

The difference between this model and Sidrauski's is that in this case money enters the production function instead of the utility function. Using the first order conditions of the maximum principle we get:

$$\begin{aligned} u_c &= \lambda_t \\ \dot{\lambda} &= -\lambda [f_k(k, m) - n - \rho] \\ \dot{\lambda} &= -\lambda [f_m(k, m) - (\pi + n) - \rho] \end{aligned} \quad (18)$$

In steady state we consider  $\dot{\lambda} = \dot{k} = \dot{m} = 0$

Consequently the system above can be rewritten as (omitting the asterisk):

$$u_c = \lambda \quad (19)$$

$$\begin{aligned} f_k(k, m) &= n + \rho \\ f_m(k, m) &= \sigma + \rho \end{aligned} \quad (20)$$

Given the budget constraint and the steady state we have the following expression for per capita consumption:

$$c = f(k, m) - nk \quad (21)$$

Equations (20) determine the values of  $k$  and  $m$ . The comparative static exercise presents the following results:

$$\frac{dm}{d\sigma} = \frac{f_{kk}}{E}$$

$$\frac{dk}{d\sigma} = \frac{-f_{km}}{E} \quad (22)$$

$$E = f_{kk}f_{mm} - f_{mk}f_{km}$$

Sidrauski's superneutrality of money is not confirmed, and the rate of inflation has real effects. However, the signs of the effects are ambiguous. If the production function is concave in  $k$  and  $m$  then  $E > 0$  and  $f_{km} > 0$ . So  $\frac{dk}{d\sigma} < 0$ . This assumption, however, cannot be made *a priori* as is demonstrated by Fischer (1974).<sup>5</sup>

Similar results were obtained by Dornbusch & Frenkel (1973). They introduce a *shopping cost* function  $v(m)$  that represents the fraction of real resources that are necessary for transactions. The assumption is that money holdings facilitate transactions so  $v_m < 0$ . Money enters the economy through the budget constraint as follows:

$$\dot{k} + \dot{m} = f(k) \cdot [1 - v(m)] - nk - c - (\pi + n)m + x \quad (23)$$

This model is equivalent to the earlier one with the following transformation:

$$f(k, m) = f(k) \cdot (1 - v(m)) \quad (24)$$

The introduction of money directly in the production function or indirectly through the shopping cost function in the budget constraint breaks the superneutrality of money. However, the sign of the effects of inflation on the steady state values is ambiguous. It can be said that "anti-Tobin" results are more likely to be obtained the greater is the complementarity between money holdings and capital.

#### 4. Cash in advance models

The cash in advance model developed by Lucas (1980) and by Stockman (1981) can be traced back to Clower (1969). Clower points out that the specificity of money *vis-à-vis* other commodities is its capacity to be generally accepted as a medium of exchange. As the aphorism goes: "money buy goods, goods buy money but goods do not buy goods". Money should be brought into the system through the budget constraint. To the usual income constraint we should add a liquidity constraint. That constitutes basically the Cash in Advance Model (CIA).

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<sup>5</sup> It is important to see Wang & Yip (1992) in this regard.

According to cash in advance models, the way to introduce money in the economy is to assume that money balances are necessary prior to any transaction.

The model can be presented as follows:

$$\text{Max } W_s = \int_s^{\infty} u(c_t) e^{-\rho(t-s)} dt \quad (25)$$

subject to the budget constraint

$$\dot{k} + \dot{m} = f(k) - nk - c - (\pi + n)m + x \quad (26)$$

and the restriction of cash in advance or liquidity constraint:

$$g(m) \geq \Gamma \dot{k} + \Phi c \quad (27)$$

where  $g_m(m) \geq 1$

$$0 \leq \Gamma \leq 1$$

$$0 \leq \Phi \leq 1$$

assuming equality in the constraints become:

$$\begin{aligned} \dot{m} &= f(k) - nk - c \left( 1 - \frac{\Phi}{\Gamma} \right) - (\pi + n)m - \frac{g(m)}{\Gamma} + x \\ \dot{k} &= \frac{g(m) - \Phi c}{\Gamma} \end{aligned} \quad (28)$$

And the Hamiltonian is:

$$H(.) = \left[ u(c) + \lambda \left( f(k) - nk - c \left( 1 - \frac{\Phi}{\Gamma} \right) - (\pi + n)m + x - \frac{g(m)}{\Gamma} \right) + \phi \left( \frac{g(m) - \Phi c}{\Gamma} \right) \right] e^{-\rho t} \quad (29)$$

The first order conditions are:

$$\begin{aligned} \dot{\lambda} &= \lambda \left( \rho + \pi + n + \frac{g_m}{\Gamma} \right) - \frac{\phi g_m}{\Gamma} \\ \phi &= \lambda(n - f_k) + \phi \rho \end{aligned} \quad (30)$$

Considering the steady state conditions:  $\dot{\lambda} = \dot{\phi} = 0$  we obtain:

$$f_k(k) = \rho + n + \frac{\rho}{g_m}(\sigma + \rho)\Gamma \quad (31)$$

It is clear from the modified golden rule that changes in the rate of inflation affect negatively the capital labor ratio:

$$f_{kk} dk = \Gamma \frac{\rho}{g_m} d\sigma \quad (32)$$

We assume  $g_{mm} = 0$  and  $f_{kk} < 0$ .

As a result the superneutrality disappears, and the higher the rate of growth of money the lower will be the value of  $k$ . The result is the opposite of Tobin's one.

It is noteworthy that the cash in advance to consumption expenditures do not affect the neutrality. This becomes clear if  $\Gamma = 0$ . Observe that the modified golden rule (31) becomes insensitive to inflation. If that is the case, let us assume CIA is not necessary for consumption expenditures and consider  $\Phi = 0$ . The modified golden rule is the same as before and it points to "anti-Tobin" results. *Note however that in steady state this result does not hold* because money balances will be redundant as long as  $\dot{k} = 0$ . The steady state conditions lead to the redundancy of money balances, as a pay in advance device, and its co-state variable will go to zero. If money is necessary only for investment expenditures, it is meaningless in the steady state, and we are back to a model where money does not enter the economy. If we were to assume that the money held in advance should not be redundant, equilibrium would imply hyperinflation. The only equilibrium possible would be the one in which  $m$  becomes zero.

Consider now a variant of the foregoing models in which money balances appear in the production function as well as in the accumulation function. The assumption, as before, is that money affects the productivity of capital, reducing transaction costs. The first order results are a slight modification of the earlier ones (see 30), and, in steady state, can be presented as follows:

$$\begin{aligned} \dot{\lambda} &= \lambda \left( \rho + \pi + n + \frac{g_m}{\Gamma} - f_m(k, m) \right) - \frac{\phi g_m}{\Gamma} \\ \dot{\phi} &= \lambda (n - f_k(k, m)) + \phi \rho \end{aligned} \quad (33)$$

Combining those we reach the following:

$$f_k(k) = \rho + n + \frac{\rho}{g_m} (\sigma + \rho - f_m(k, m)) \Gamma \quad (34)$$

However, the implication of the condition (34) is not clear. Both  $k$  and  $m$  are affected by the rate of growth of money. In steady state, however, we know that  $\Phi c = g(m)$  since  $\dot{k} = 0$ . Consider  $\Phi = 1$  to simplify. In that case:  $dc = g_m dm$ . Differentiating the steady state budget constraint we get the following:

$$dc = (f_k - n)dk + (f_m - \sigma)dm$$

Using the relation between  $c$  and  $m$  we obtain:



$$\frac{dn}{dk} = \frac{f_k - n}{g_m - f_m + \sigma} = \omega \quad \text{where } \omega > 0 \quad (35)$$

This expression shows that the steady state values of  $m$  and  $k$  move together. In that case it is clear that money growth is negatively related to  $k$  and  $m$ :

$$f_{kk} dk = \frac{\rho}{g_m} (d\sigma - f_{mm} dm) \Gamma = \frac{\rho}{g_m} (d\sigma - f_{mm}(k, m) \omega dk) \Gamma \quad (36)$$

Stockman (1981) had shown that CIA models, when the liquidity constraint applies to consumption and capital expenditures, lead to “anti-Tobin” results. The inclusion of money in the production function and in the expenditure function confirms the conclusion that money is not superneutral and that its growth reduces per capita capital, consumption and money holdings. The basic reason for these “anti-Tobin” results lies in the complementarity of money and capital.<sup>6</sup>

## 5. Inflation and growth

Some of the results obtained above are due to the fact that the economies converge to steady state where the equilibrium growth is zero. The growing literature on endogenous growth models (Romer, 1994) makes it possible to analyze the effects of technology, investment and savings along the growth path.

Money may affect growth through its effect on the return of money. In a model like the one developed by Stockman it would be easy to show that the cash in advance is a transaction cost to investment much like a cost of adjustment. As a result, an increase of the rate of growth of money implies a shift of the inflation rate and so an increase in the transaction cost of investment. A model developed in a similar vein is due to De Gregorio (1993). In these models, inflation increases the transaction cost of investment and reduces the growth rate. We shall come back to this point in section 7 below.

## 6. Price flexibility and money

The main characteristic of these dynamic models is that real balances are endogenous. This means that money is neutral. The discussion is limited to the question of superneutrality or to the relation between inflation and the growth path. The assumption of price flexibility and full employment is fundamental to these results.

It becomes clear that the complementary role played by money, that produces the “anti-Tobin” results, will enhance capital accumulation and growth, if the economy is below full capacity utilization or the prices are fixed.

The relaxation of price flexibility may bring serious problems mainly in the cash in advance framework.

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<sup>6</sup> This point is very well made by Orphanides & Solow (1990).

As it is well known the equilibrium path of the dynamic models presented above is a saddle equilibrium, that is: one of the  $n$  eigenvalues of the  $n$ -dimensional Jacobian from the linearized system is negative while the others are positive. This instability of the system allows one to define the value of the variables uniquely since it is usual to exclude all the other paths that do not lead to equilibrium. What had been considered a fatal flaw became a great benefit, according to Sargent & Wallace (1973). If there is to be a unique path to the steady state it is necessary that the model exhibits a saddlepoint instability. The problem with this approach is that it excludes all the problems related with dynamic stability. This is known as the Hahn problem.<sup>7</sup> We will argue that the economy can be thrown out of the stable arm.

The cash in advance models developed above show that money is not superneutral. Nonetheless money continues to be neutral. The neutrality of money comes from the fact that the optimal quantity of real money holdings is endogenous to the model. Once price stickiness is introduced, however, neutrality disappears in the short run. Furthermore, the economy may become highly unstable. The problem comes from the fact that *if prices are given it is unlikely that both the liquidity constraint and the budget constraint will be satisfied.*

Consider the same economy we had before with the difference that in the short run prices are fixed.<sup>8</sup> The economy is on the equilibrium path and the monetary authorities decide to reduce the rate of growth of the money supply. The reduction of the expected money supply embodied in the prices makes the cash in advance constraint binding and end up reducing the capital accumulation of the current period. To gain simplicity let us assume the cash in advance is necessary only for investment. The reduction in the money supply should constrain the investment, in a way that  $\dot{k} < \tilde{k}$ . (The tilde over a variable means its equilibrium value, not liquidity constrained.) The rate of return on money is higher than that on capital and the latter has to adjust, that is  $f_k \geq f_{\tilde{k}}$ , the consumers would increase the rate of consumption accordingly. As a result, the shock could displace the representative agent from the stable arm path to an unstable path. If prices change in order to absorb the shock, the liquidity constraint will remain binding. The reason is that now the rate of growth of capital will be higher because the economy is farther from equilibrium than before.<sup>9</sup> Unless prices changes accordingly to the required amount the economy is doomed to remain in the liquidity constraint path.

The proposition that follows is that within a cash in advance model, in an optimizing framework, the equilibrium may be very tenuous. Any disturbance will throw the economy out of the Ramsey equilibrium to the liquidity constrained equilibrium. It could come from this that the best policy in this framework would be one in which the money supply would be completely endogenous. In this case, the stochastic elements from the supply side<sup>10</sup> will be reduced. What the model suggests is that stochastic elements in the supply of money de-

<sup>7</sup> See Hahn problem in Hahn (1987).

<sup>8</sup> L. Svensson (1986) developed a model along these lines. A slightly different model in the context of open economies appears in Obstfeld & Rogoff (1994). This problem is formally identical to an unexpected shock with flexible prices. An unexpected reduction in the money supply should constrain the investment.

<sup>9</sup> Barro & Sala-i-Martin (1994) present a rigorous demonstration of the convergence property of Ramsey type economies.

<sup>10</sup> Black (1972) reached a similar conclusion.

stabilize the economy if it is liquidity constrained. The same results occurs with the more comprehensive notion of CIA in which liquidity constraint applies also to consumption expenditures.

### *Finance and convergence*

One of the key elements in this economy is the fact that money is introduced through a cash in advance mechanism. This makes money complementary to capital and consumption. There are some similarities between money as cash in advance and money as finance.<sup>11</sup> Both represent command over resources prior to the transaction, but finance introduces credit. In a way the models so far hide an important contribution of money. Although money as finance is very close to cash in advance models, finance introduces some important attributes like:

- (a) anticipation of purchasing power,
- (b) creation of inside money-credit money.

It is notorious that this important characteristic of money has been, so often, forgotten. Money as finance is a fundamental characteristic of the market system and it pertains to a particular kind of money: the credit money. Credit money is "inside money", and is different from "outside money" on two accounts: it is "real" money and it does not compete with capital. By "real money" we mean that credit money appears in the system backed by real collateral. Furthermore, since inside money does not represent wealth for the agents, it does not compete with capital, and it should not appear in the budget constraint.<sup>12</sup>

The model generates a similar optimization problem to that which we had before; with the differences that money does not appear in the budget constraint function, and the cash in advance constraint is real. We assume that credit is necessary only for investment. The model can be presented as follows:

$$\text{Max } W_s = \int_s^{\infty} u(c_t) e^{-\theta(t-s)} dt \quad (37)$$

subject to the budget constraint

$$\dot{k} = f(k) - nk - c$$

<sup>11</sup> It is noteworthy the similarity of the cash in advance approach and the Keynesian model. In the latter, capital accumulation depends on finance. The investment function could be defined as  $\dot{k}(m) = g(m)$ . There is no reason to assume that prices are on the right level. The finance constraint becomes binding. Another version of the Keynesian approach eliminates the budget constraint and considers only the liquidity constraint. In that case animal spirits would be the driving force of the economy.

<sup>12</sup> Gale (1983) introduces inside money in a Diamond type overlapping generation model and shows an anti-Tobin result. He criticizes Tobin for not recognizing inside money. "Specifically, it ignores the distinction between inside and outside money and this has serious consequences for the analysis of monetary policy. In the model studied in the preceding section, all money is outside money... In reality, a large part of the money supply consists of inside money, that is, bank deposits generated by the process of borrowing and lending within the private sector. ... Unlike outside money, holding inside money is not, from the macroeconomic point of view, an alternative to holding capital goods" (pp. 110-1).

and the restriction of cash in advance to capital

$$g(\bar{m}) \geq \dot{k}$$

where  $g(\bar{m})$  represents the maximum credit provided by the system. Until this limit is reached the supply of credit is endogenous. In this case the constraint will be:

$$\min \{f(k) - nk - c, g(\bar{m})\} \quad (38)$$

If the credit constraint is binding, the rate of growth of the economy will be defined by  $g(\bar{m})$ . In that case the rate of growth of the economy would be defined by the rate of growth of capital and it would be inferior to the optimal rate of growth. That is:  $\dot{k} = g(\bar{m}) < \tilde{k}$ , where the variable with the tilde is the optimal one, not liquidity constrained. *The crucial point behind this simple expression is that the economy is driven by the availability of finance credit.*<sup>13</sup> It is important to note the assumptions that are being made. One is that the debt is paid at the same rate that investment is being made. In other words, there is no debt outstanding. Money is created to finance the purchases of capital goods, and as soon as the expenditures are made the debt is paid out and the money disappears. This supposition allows us to represent the model in the same vein as the CIA model. Once we drop this assumption it is necessary to change the function  $g(\cdot)$  and introduce a demand for money holdings. Another assumption is that the supply of credit from the banks is closely related to the money supply. No currency is held by agents, only deposits. In other words, all the money is inside money. Finally, the analysis is based on the existence of an upward limit of real credit. This seems to be a controversial assumption, nonetheless it is being done in different frameworks.<sup>14</sup>

Credit creates inside money, tied to real goods. The demand for loans depends on the real rate of return and in a sense is neutral to inflation. The question is on the supply of funds. It is true that the liquidity of the system is greater depending on the price level. However, the capacity of lending of the system depends on the liquidity but even more so on institutional factors as well. Hence it is reasonable to assume that there is an upper real limit to the supply of credit.

*The important point however is that real money is exogenous to the system. If that is the case the price level is not determined in the model, but is exogenous too.*

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<sup>13</sup> An interesting development was made by Foley (1982, 1986). He shows that in steady state and within a Marxian framework credit is necessary for the economy to be able to grow. The assumption is that there is a lag in the production process between the initial moment of production and the moment of sale of the production. Credit is necessary to advance the payment of the labor force as well as the inputs during the production process. Income has to be anticipated by credit before it is received by the capitalists through the market, otherwise the system will come to a halt by the lack of demand. In this sense Foley demonstrates that there is a relationship between capital accumulation and the amount of credit consistent with the model presented above.

<sup>14</sup> One example is the real bills doctrine. Another case can be mentioned on the discussion between finance and growth that comes in Shaw and McKinnon type models. See Fry (1988) and King (1993).

One important outcome of this discussion has to do with the convergence hypothesis. As it is well known, the transient dynamics, of Solow-Swan type models or the Ramsey type models, is that the farther the economy is from the steady state the greater is its optimal rate of growth. What that implies is that the needs of finance are greater the poorer is the economy. It is reasonable to suppose that the availability of credit is directly related to the wealth of the economy. If that is true the likelihood that the poorer economies are constrained by finance is very high.<sup>15</sup>

The discussion so far has emphasized the finance aspect of inside money very much along the lines raised by Keynes' finance motive. The concept of finance used by Keynes differs from that used by the mainstream, which we wish to emphasize. Credit money is money that has no real funding behind it. In other words, credit provides finance of a short run nature and is not backed by savings. Financial intermediation provided by the banks has this character. This allows savings to be separated from investment.<sup>16</sup>

The model developed here echoes the literature that came with Shaw (1973), McKinnon (1973) and their followers. The basic difference is that these models do not contemplate Ramsey type economies. Their main conclusion is that credit liberalization increases the availability of finance, shifting the liquidity constraint.<sup>17</sup>

## 7. Endogenous growth

As is known, "AK type models" permit growth to persist, without technological progress, by the assumption of constant returns to capital. The idea is simple, the accumulation of capital does not lead to diminishing returns and so to reduction of the rate of return. In this case the accumulation is kept positive and constant. The constant returns of capital can reflect learning-by-doing and spillovers of knowledge.

To be able to analyze the influence of money in the context of endogenous growth theory it is worth separating household behavior from firm behavior.<sup>18</sup>

Household behavior is the same as before:

$$W_s = \int_s^{\infty} u(c) e^{-\rho[t-s]t} dt \quad (39)$$

Population is constant and we assume a utility function given by:

$$u(c) = (c^{1-\theta} - 1) / (1-\theta) \quad (40)$$

which incorporates a constant elasticity of marginal utility,  $-\theta$ .

<sup>15</sup> For a discussion on the convergence hypothesis see Barro (1990).

<sup>16</sup> A very complete exposition of credit money can be found in Wray (1990). See also Davidson (1990a and 1990b). A recent account of the post-Keynesian view on money that incorporates empirical studies can be seen in Dymski & Pollin (1994).

<sup>17</sup> Another difference is that their models contemplate long-run debt.

<sup>18</sup> We follow Barro & Sala-i-Martin (1992).

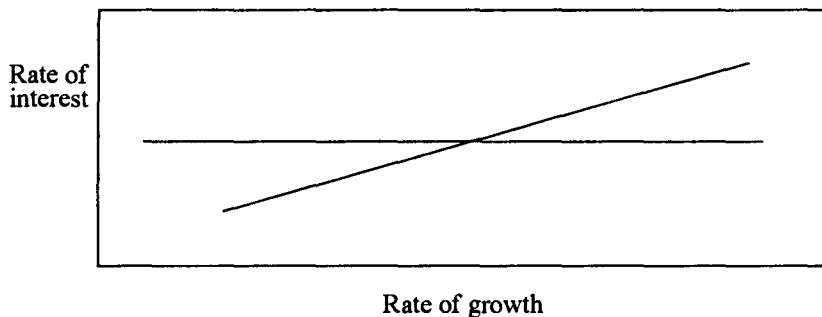
Households' budget constraint determines the dynamics of the assets:

$$\dot{a} = ra - c \quad (41)$$

The first order condition for maximization of utility under the dynamic constraint requires the growth rate of per capita consumption to be:

$$\gamma_c = \frac{\dot{c}}{c} = \frac{1}{\theta}(r - \rho) \quad (42)$$

This establishes a relationship between the growth rate and the interest rate that depends on the preferences of the households, expressed by the parameters  $\rho$  and  $\theta$ . This relationship, the *preference locus*, can be represented in the rate of interest and growth space as a straight line with positive slope.



From the side of production we assume that firms maximize their present cash flow value subject to an investment function. The difference from the previous models is that the production function exhibits constant returns in capital. The rationale behind this assumption has to do with the comprehensive definition of capital that includes human capital and spill-over effects. This production function is called an AK type function. The representative firm is supposed to:

$$\text{Max} \int_s^{\infty} (Ak - \eta i) e^{-r[t-s]} dt \quad \text{subject to} \quad \dot{k} = i \quad (43)$$

where  $\eta$  stands for the price of capital goods in terms of consumption goods. The first order conditions:

$$\begin{aligned} q &= \eta \\ \dot{q} - rq &= -A \end{aligned} \quad (44)$$

In steady state the equilibrium condition becomes:

$$\frac{A}{\eta} = r \quad (45)$$

This relationship can be viewed as the *production locus* and in the diagram above is represented by the horizontal line. The equilibrium relation for the rate of growth of per capita consumption and capital becomes the following:

$$\gamma_c = \frac{1}{\theta} \left[ \frac{A}{\eta} - \rho \right] = \gamma_k \quad (46)$$

There are no transitional dynamics and the economy is always in a position of constant steady state growth. This equation is fundamental to show the variables involved in the determination of the rate of growth:  $A$ , base line technology;  $\eta$ , the cost of capital;  $\rho$ , the rate of intertemporal preference and  $\theta$ , elasticity of substitution. The last two parameters reflect the savings behavior of the representative agent. In this model the rate of inflation reduces the real rate of return of capital and brings down investment, as De Gregorio (1993) shows.

The model introduces money through a transaction costs function. In a sense, it is a development of the shopping cost and the cash in advance model.<sup>19</sup> The demand side of the model is basically the same as that presented earlier; the difference is in the production side.

Now assume that there are transaction costs in investment expenditure. Firms require money to buy capital goods and the more money the better. Suppose the cost of capital,  $\eta$ , is affected by the shopping cost function  $s(m/i)$ . Investing  $i$  units of new capital goods is  $\eta_i = i(1 + s(m/i))$  where  $s(\cdot)$  is decreasing and convex. Differently from the previous model, the cash holdings of firms are subject to the inflation tax. The firms maximize the present discounted value of cash flows, net of the inflation tax.

The first order conditions, from the production side, become:

$$\begin{aligned} q &= 1 + s\left(\frac{m}{i}\right) - \frac{m}{i} s'\left(\frac{m}{i}\right) \\ -s'\left(\frac{m}{i}\right) &= \pi + r \\ \dot{q} - rq &= -A \end{aligned} \quad (47)$$

The first two equations reveal the relationship among the shadow value of investment, the rate of inflation, and the rate of return. The last equation shows, in steady state, the inverse relation between the shadow price of the investment and the rate of return.

Using these relationships, it is possible to define implicit functions between  $q$  and  $\pi$ , and between  $r$  and  $\pi$ . It can be shown that they are increasing and decreasing, respectively,

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<sup>19</sup> These results follow from CIA Stockman model: "As long as capital cannot be costlessly obtained by barter, however, higher inflation reduces the steady-state capital stock. This conclusion does not seem to require a rigid cash-in-advance constraint ... Any transactions model of money demand, in which money must be used for purchases and there are costs of exchanging money for other assets, would seem to imply this same result" (Stockman, 1981, p. 393).

that an increase of inflation increases the cost of capital, and reduces the rate of return of the capital, and so the rate of investment.

$$q = q^+(\pi) \quad r = r^-(\pi)$$

In the diagram this would correspond to a downward shift of the horizontal line. The usual cash in advance model is a particular case of this model in which  $s(\cdot)$  is zero. In this case, the cost of the investment is:

$$q = 1 + r + \pi \quad (48)$$

As before, the steady state rates of growth of per capita consumption, and capital are given by:

$$\gamma_c = \frac{\dot{c}}{c} = \frac{1}{\theta} (r(\pi) - \rho) = \gamma_k \quad (49)$$

It is possible to add in this framework another effect of inflation that was not emphasized by De Gregorio. The point is that the  $s(\cdot)$  function is itself dependent on inflation. Inflation raises uncertainty in several ways. It is a well known fact that inflation is associated with high price variance. The chances of capital gains and losses are higher in this context. It is reasonable to assume that the demand for cash balances goes up during inflationary periods to take advantage of opportunities. In that case  $s(\pi, m/i)$ , with  $s_\pi > 0$ . If we introduce a positive effect of inflation on the demand for cash balances, due to this uncertainty, the negative effect of inflation on growth will be strengthened.

Besides the negative effect of inflation on growth that comes (directly and indirectly) from the inflation tax it is possible to analyze the relationship of inflation and growth independently of cash balances. The basic assumption that is necessary to make is that risk associated with investment increases with inflation, and that the investors are risk averse. For the same rate of interest, the higher the inflation rate, the higher risk premium the investors will require. That will correspond to a reduction in the rate of return of the investment.<sup>20</sup>

## 8. Money, innovation and endogenous growth

The importance of finance to growth has been raised by several authors and we should mention Goldsmith (1969), Shaw (1973) and McKinnon (1973).<sup>21</sup> According to these authors quantity and quality of services provided by financial institutions could partly explain why countries have grown at different rates.

Solow and Swan type models, however, were not very useful for this literature since they assume savings are exogenous. With the development of endogenous growth models,

<sup>20</sup> We acknowledge Geoff Wyatt for calling our attention to these points.

<sup>21</sup> It is interesting to note that most of these models used Leontieff type functions that are the predecessors of the AK type models.



several studies have appeared reemphasizing the old message that financial intermediation does matter. Finance is not neutral as money used to be. The literature follows basically three directions:

Finance affects growth through its effects on: (a) savings; (b) baseline technology; (c) technological progress.

Most of the traditional literature is associated with development. How to mobilize savings to investment was the main question. The repression of the financial sector through taxation, rate of interest subsidy and credit control was responsible for the lack of finance of the less developed countries. The services of the financial sector affect the composition of savings reducing self-finance and the needs of liquidity for random events. These points are integrated in an interesting model of intertemporal optimization by Bencivenga & Smith (1991). Basically they show that financial intermediation increases the average rate of return on productive assets. In a dynamic optimizing framework, with AK type function, it results that financial development is beneficial to growth.

An imperfect way of dealing with the influence of credit on technology is to analyze it in the context of steady state AK type models. This is obviously a serious simplification of the way Schumpeter viewed the process of growth and innovation. There are two alternatives to show the influence of credit and financial intermediation that in fact turn out to be the same. One way is to point out the effect of financial intermediation on the baseline technology. Another alternative is to assume that credit availability has externalities.

Roubini & Sala-i-Martin (1992) present an endogenous growth model in which financial development affects the rate of growth of the economy through the baseline technology. The effect of financial intermediation is on the value of  $A$ . The level of financial development increases microeconomic efficiency of the macroeconomy through a better matching between saving and investment and more efficient allocation of investment. Raising the value of  $A$ , the productivity of capital, financial intermediation increases the steady state rate of endogenous growth of the economy. It corresponds to a shift up of the production locus in the diagram above.

In the same vein as Roubini and Sala-i-Martin, we propose bank credit externalities. Let us assume that availability of credit allows firms to have access to new combinations and processes that compensate for the diminishing returns to capital.<sup>22</sup>

$$Y_i = A_i L_i^{1-\alpha} K_i^\alpha \left[ \frac{M}{P} \right]^{1-\alpha} \quad \text{for } i = 1 \text{ to } n \quad (50)$$

$\frac{M}{P}$  has the meaning of the overall credit availability of the economy and in that sense it includes only inside money. The idea is that it should be considered as an externality for the individual firms. This good that we could call as the level of development of the financial sector affects the whole macroeconomic efficiency as in the previous model. The value of profit can be expressed as:

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<sup>22</sup> The model developed here is similar to the one presented by Barro & Sala-i-Martin (1994). The difference is that the latter considered government expenditure in infrastructure instead of credit.

$$\Pi_i = L_i \left[ A_i k_i^\alpha m^{1-\alpha} - w - r k_i \right] \quad (51)$$

where  $k_i = \frac{K_i}{L_i}$  and  $m = \frac{M}{P}$

Making  $k_i = k$  and  $A_i = A$  we have:

Maximum profit implies:

$$r = \alpha A k^{-(1-\alpha)} m^{1-\alpha} \quad (52)$$

It is easy to see that as long as  $k$  and  $m$  grow at the same rate,  $r$  stays constant. The representative household maximizes utility:

$$W = \int_0^\infty e^{-\rho t} \left[ \frac{c^{(1-\theta)} - 1}{(1-\theta)} \right] dt \quad (53)$$

subject to:

$$\dot{k} = A k^\alpha m^{1-\alpha} - c \quad (54)$$

we assume that the labor force is constant.

The first order conditions and profit maximization imply:

$$\frac{\dot{c}}{c} = \frac{1}{\theta} (r - \rho) = \frac{1}{\theta} (\alpha A k^{-(1-\alpha)} m^{1-\alpha} - \rho) \quad (55)$$

Let us assume that there is a constant relation between real credit and capital that can be expressed as follows:

$$m = z^{\frac{1}{1-\alpha}} k \quad (56)$$

This relationship measures the degree of development of the financial sector.

Using this relation in the expression that represents the growth of consumption, it follows that:

$$\frac{\dot{c}}{c} = \frac{1}{\theta} (\alpha A z - \rho) \quad (57)$$

In equilibrium the steady state rate of growth of consumption that will be the same as the rate of growth of capital depends on the elasticity of intertemporal substitution, on the technological parameters  $\alpha$  and  $A$  and on the relative availability of credit, or degree of development of the financial sector,  $z$ . The limitations of this model is the *ad hoc* character of the parameter  $z$ .

Turning to the effect of financial deepening on technical progress, King & Levine (1993) emphasize the role of finance in fostering technological innovation *à la* Schumpeter. The role of financial institutions would be twofold: evaluate prospective entrepreneurs and fund the most promising ones.

In this case financial intermediation affects the rate of return of the innovation activity. The variable that is used to deal with the cost of intermediation is a tax on the financial sector. It is shown that a tax cut in the financial sector, that is, a reduction in the financial repression, induces a positive shift in the rate of growth of the economy. It is interesting that the rate of interest is not used since it is real and endogenously determined. It would be equally possible to increase the effect of the financial sector if we considered a credit constraint. It is noteworthy that King and Levine present Schumpeterian innovation as a smooth process.<sup>23</sup> That enables them to set the economy in the dynamic optimizing framework, ending up with a steady state growth with full employment. It should be mentioned that theirs is one of the few works on endogenous growth that emphasizes the role of the finance in Schumpeter's analysis.<sup>24</sup>

The analysis of King and Levine, and, for that matter, most of the recent work on endogenous innovation miss one of the crucial points of Schumpeter's model, namely the discontinuity of innovation. The discontinuous process of innovation characterizes the process of cyclical growth of the economy and is the basis for the fundamental role of credit. For Schumpeter the new set of opportunities could only be put forward by anticipation of purchasing power. If in steady state net savings are zero, how can the economy finance the development and the adoption of new technologies, that take time and experimentation, before coming to the full use in the production process? Credit plays a crucial role in Schumpeter's analysis, and more so in the supposition of aggregate discontinuity of innovations. Credit is fundamental for understanding the changing path of the economy. This approach can be followed within this framework, as a transient dynamics to a sequence of different paths, according to the sequence of innovations.

## 9. Conclusions

The role of money in neoclassical growth models depends largely on the hypothesis embodied in the models. If money is seen as producing utility like any other commodity, it is neutral and superneutral. If money is understood as an asset, competitive to capital, inflation will stimulate accumulation, and the economy will end up with more capital per head. This is Tobin's famous result. On the other hand, if money is viewed as highly complementary to capital, either because it is necessary for transactions or because it economizes transaction costs, inflation will bring down accumulation, and the economy will be in a steady state with lower consumption and capital per capita. Inflation has a real effect in the economy. There is a Phillips curve after all. Extending the analysis to the endogenous growth we see further effects of inflation on growth. Inflation increases the transaction costs associated with investment and reduces growth.

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<sup>23</sup> Most of the literature on endogenous technological innovation assumes that the discontinuities happen in the micro level but cancel out at the macro level. Investment in R&D increases the probability of successful innovation. Innovation is seen as more efficient production processes or change in the quality of intermediate goods. A constant flow of innovation guarantees a steady state growth of per capita product, with full employment. See Grossman & Helpman (1991, 1994), and Solow (1994).

<sup>24</sup> See King & Levine (1993).

Price rigidity in the context of cash in advance models leads to frequent changes of equilibrium paths. Liquidity-constrained paths may exaggerate instability from stochastic shocks in the supply of money.

We argue further, that money has an important function as finance. As such, it can be a bridge between the CIA type model and Keynesian models. Furthermore, we show that finance has some real limits. In that sense, the availability of finance may constrain growth. Secondly, it is possible to argue that finance has an important bearing in the fostering and adoption of innovation in a Schumpeterian framework. In that sense finance is an important determinant of innovation and growth.

All these findings are restricted to the notion of money as a transaction device. The framework of dynamic optimizing, with rational expectations, is hardly adequate to deal with uncertainty. That however will be treated in another paper.

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