The formation of inflation expectations in Brazil: a study of the futures market for the price level*

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Summary: 1. Introduction; 2. An asset pricing model for the price level futures market; 3. Tests for the martingale hypothesis and for an unit root; 4. Term structure of the price level futures contracts; 5. Conclusion.

This paper examines the formation of inflations in Brazil by exploring the data from the futures market for the price level. These data are uniquely suited for this purpose, providing a basis for more precisely defined statistical tests than the existing ones in the literature. In the first part, an asset pricing model *à la* Lucas shows that even under risk neutrality these futures prices do not follow a martingale. The econometric results indicate that those futures prices have positive bias *vis-à-vis* the expected price level, and that risk aversion is a better characterization of this Brazilian market. Since those futures prices were used by the Central Bank to infer agent's expected inflation, this result means that an excessively high real return might have been paid on government bonds. In the second part, contracts with subsequent maturity dates are used to infer expected inflation for subsequent months. How these expectations jointly react to shocks to the economy provides a basis for tests of the stochastic structure of expected inflation. The tests show that expected inflation has a unit root for the "well-behaved" period of Finance Minister Bresser Pereira, who did not fight backward inflation indexation of prices. When a new economic policy was put in place, trying (unsuccessfully) to dismantle the several indexation mechanism, this unit root is statistically rejected. These findings have important implications for the pricing of fixed income securities in a highly inflationary economy.

Este trabalho examina a formação de expectativas inflacionárias no Brasil, valendo-se de dados do mercado de futuros para avaliar o nível de preços. Tais dados servem apenas para tal finalidade, fornecendo uma base para testes estatísticos mais precisos que os existentes na literatura. Na primeira parte, o modelo *à la* Lucas de apreçamento de ativos mostra que mesmo havendo risco de neutralidade, esses preços futuros não seguem um modelo progressivo. Segundo os resultados econométricos, esses preços futuros apresentam tendenciosidade positiva com relação ao nível de preços esperado, e a aversão ao risco caracteriza melhor esse mercado brasileiro. Como o Banco Central utilizava esses preços para inferir a inflação esperada pelos agentes econômicos, a constatação é de que podiam estar sendo pagos rendimentos excessivamente elevados pelos títulos governamentais. Na segunda parte, foram utilizados contratos com prazos de vencimento subseqüentes, para inferir a inflação esperada nos meses subseqüentes. O modo como essas expectativas, tomadas em conjunto, reagem a choques na economia dá a base para que se teste a estrutura estocástica da inflação esperada. Os testes mostram que a inflação esperada foi a raiz unitária para o período "bem-sucedido" de Bresser Pereira no Ministério da Fazenda, quando não foi combatida a indexação de preços pela inflação passada. Ao serem adotadas novas políticas econômicas que tentaram (sem sucesso) desmontar os vários mecanismos de indexação, essa raiz unitária foi estatisticamente rejeitada. Tais constatações têm implicações importantes no apreçamento de títulos de renda fixa numa economia com inflação alta.

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1. Introduction

The formation of inflation expectations is a central question in both macroeconomics and finance. In several important rational expectations macroeconomic models, the existence of unbiased inflation expectations is a requisite to obtain the neutrality of monetary policy (Lucas, 1973, 1975; Barro, 1980). An important question in finance is whether markets successfully aggregate private information. The main goal of this paper is to address the question of how inflation expectations are formed by looking at a dataset uniquely suited to this purpose. The existence in Brazil of a futures market for the price level allows a direct analysis into the process of how inflation expectations are formed, because in this market the futures price is essentially determined by inflation expectations. Other determinants of assets' prices, e.g., expected real interest rates, are far less important in the determination of this futures price. Being less affected by this unavoidable noise, these Brazilian data shall provide a basis for more precisely defined statistical tests than the existing ones, which rely on the returns of nominal (Hamilton, 1985; Huizinga & Mishkin, 1984) and indexed securities (Huberman & Schwert, 1985; Woodward, 1990). The tests based on this futures market data are also superior to those based on actual inflation data or surveys of inflation expectations. Actual inflation data contain forecast errors, which decrease the power of tests regarding expected inflation. Also, survey data are flawed by the fact that, unlike futures prices data, agents do not commit resources to back their forecasts.

A futures market for the price level has existed in Brazil since 1987. This market trades futures contracts which are valued through an official index (*OTN: Obrigações do Tesouro Nacional*; and later *BTN: Bônus do Tesouro Nacional*). Before the current administration took office, this index was updated according to the evolution of the consumer price index (IPC). The IPC, usually announced on the last day of the month, is an average of the four weekly price levels between the 16th of the previous month and the 15th of the current month. Agents trade on the future value of this index. Much before this futures market began operating, there were markets in Brazil for securities indexed to inflation, as well as huge daily overnight market with repurchasement agreements.

This paper uses the Brazilian dataset to undertake several tests. Section 2 develops a simple asset pricing model in the tradition of Lucas (1978). Since the sequence of futures prices may be seen as a sequence of forecasts at different dates of the same event, it is tempting to conclude that the futures prices follow a martingale stochastic process,[1] i.e., that the futures price at t is the expected value at t of the futures price at $t+1$. The model of section 2 shows that, even if risk neutrality is assumed, the daily sequence of futures prices (settlement prices) does not necessarily follow a martingale. This result comes from the unpredictability of the purchasing power of money, as shown by Frenkel & Razin (1980) when studying the efficiency of forward exchange rates markets. The main implication of the model is that, under risk neutrality, the futures price systematically underestimates the expected price level. This downward bias decreases gradually as the contract's maturity approaches, and the uncertainty about the future price level declines. This, in turn, implies that unit roots tests should not reject the existence of a unit root in the futures price series.

Section 3 presents an econometric investigation of the existence of a unit root in the futures price level data. Results indicate that the futures price does not have a unit root, and

[1] The martingale model is originally due to Samuelson (1965).

therefore does not follow a martingale. These results imply that risk aversion may play an important role in the determination of the futures price.

Section 4 uses the *term structure* of the futures contracts for the price level to infer the stochastic structure of the *expected* inflation process. On any given day, several futures contracts maturing at the end of subsequent months are traded. These contracts pay off according to the price level at the end of the previous month. If P_t is the price level at t, and π_{t+1} the inflation rate between t and t+1, then $P_{t+1} = (1 + \pi_{t+1}) P_t$. Hence, if an informational shock to the economy raises π^e_{t+1} by 1 percent, P^e_{t+1} will also rise by 1 percent. What happens to the subsequent expected prices, P^e_{t+2}, P^e_{t+3}, ..., will depend on the stochastic process followed by expected inflation. For example, if expected inflation follows a martingale difference, that is, if price increases today do not imply further price increases or decreases in the future, an increase of 1 percent in P^e_{t+1} is associated with a 1 percent increase in all expected future price levels, P^e_{t+2}, P^e_{t+3}, and so on. At the other extreme, if expected inflation follows a martingale, that is, a price increase today implies a similar price increase in the future, an increase of 1 percent in P^e_{t+1} is associated with a 2 percent increase in P^e_{t+2}, 3 percent in P^e_{t+3}, and so on. The existence of overlapping futures contracts on the price level of subsequent monthly maturities makes it possible to perform precisely defined tests about the stochastic process of expected inflation, because, as already mentioned, these data are less susceptible to the extraneous noise that inevitably affects financial assets'prices.

Finally, section 5 summarizes the main findings of the paper and suggests some points for further research.

2. An asset pricing model for the price level futures market

This section presents an asset pricing model following Lucas (1978). Within the Lucas framework, the main assumption of this model is risk neutrality. This assumption is believed to be a reasonable one for the market analyzed here, given that only financial institutions trade in the OTN/BTN futures market. These institutions usually have very good technical staff and operate in this market either to speculate or to hedge their operations with indexed and non-indexed securities. Many of these institutions collect price data themselves in order to outguess the Brazilian agency which computes the IPC.[2] Given the risky nature of these institutions' activities, doing away with risk aversion may not be a bad approximation. The assumption of risk neutrality generates a testable implication which is tested in section 3.

The first simplifying assumption is to neglect the differences between futures and forward prices. A futures contract differs from a forward contract maturing on the same date because the former is marked to market, i.e., payments between the parties, called resettlements,[3] are made daily depending on the variations in the futures prices (for a full characterization of the relation between futures and forward prices, see Cox, Ingersoll & Ross, 1981). Empirical evidence suggests that these two speculative prices are not very different (Cornell & Reinganum, 1981; Hodrick & Srivastava, 1987).

[2] The IPC (Índice de Preços ao Consumidor - Consumer Price Index) is computed by the Fundação Instituto Brasileiro de Geografia e Estatística (IBGE). The IPC's monthly variation (inflation rate) reflects the consumption basket price change of families with incomes from 1 to 8 times the minimum wage (approximately from US$60 to US$480 a month). The data are collected, approximately, between the 16th of the previous month and the 15th of the current month. The IPC is announced on the last days of the month. The price data is collected in nine Brazilian state capitals and in Brasília (Banco Central do Brasil, 1990).

[3] "The process of collecting and paying variation margin is called resettlement" (Duffie, 1989, p. 12).

A standard asset pricing model in which a representative agent intertemporally maximizes utility, gives the following pricing equation:

$$0=\frac{P_t}{U'(C_t)}E_t\left(\beta^{T-t}\,U'(C_T)\frac{(S_T-F_{t,T})}{P_T}\right)^4 \tag{1}$$

where

$U'(.)$ = marginal utility;
C_t = consumption at date t;
S_T = spot price at the maturity date T;
$F_{t,T}$ = futures price at date t of a contract maturing at date T;
P_T = price level at date T.[5]

The intuition behind equation (1) is the following. A futures contract has zero marginal cost — left-hand side of equation (1). The expected marginal benefit of its payoff at the maturity date ($S_T - F_{t,T}$) is given by the right-hand side of equation (1). The other terms on the right-hand side of equation (1) may be interpreted as a present value operator that converts dollar payoffs at date T into dollar values at date t (Hodrick & Srivastava, 1987). From equation (1), noting that the spot price at the maturity date, S_T, is the price level announced at that date, P_T, the futures price at t of the price level at T is given by equation (2).

When risk neutrality is assumed, the risk premium represented by the conditional covariance term vanishes, and equation (2) is turned into equation (2a).

$$F_{t,T}=\frac{1}{E_t(1/P_T)+\dfrac{\mathrm{Cov}_t(U'(C_T),1/P_T)}{E_t\left(U'(C_T)\right)}} \tag{2}$$

By Jensen's inequality, the second term on the right-hand side of equation (2a) is non-positive ($E_t(1/P_T) \geq [1/E_t(P_T)]$). Therefore, one implication of this model is that the futures price tends to underestimate the expected value of the price level.[6] The downward bias is given

$$F_{t,T}=\frac{1}{E_t(1/P_T)}=E_t(P_T)+\frac{1-E_t(P_T)E_t(1/P_T)}{E_t(1/P_T)}=E_t(P_T)+\frac{\mathrm{Cov}_t(P_T,1/P_T)}{E_t(1/P_T)} \tag{2a}$$

[4] For the derivation of this equation, see Appendix.

[5] The relevant variable is the price level at day T, which is different from the price index announced by the government at T. P_T is a non-linear transformation of the daily price levels. Assuming inflation can be linearly approximated within its measurement period, P_{T+30} is a good approximation for the price level at T. The effects derived below still go through if this approximation is used.

[6] The intuition behind this result is better understood by means of an example. Suppose the distribution of the price level at date T is binomial: P_T may equal either .5 or 1.5 with probability 1/2. The expected price level is therefore 1. The value at t (P_t = 1) for a risk-neutral investor (with B=1) of a US\$1.00 payment at T is US\$1.33. In other words, the implicit deflator used was 0.75, which underestimates the expected price level of 1. This example shows that, in the calculation of the implicit deflator, the low price levels in the distribution carry more weight than the higher ones, because of the non-linear way prices enter in this calculation. To be sure, in the example above, the gain for the investor of moving from the expected P_T of 1 to the low P_T of .5 is much greater than the loss of moving from the P_T of 1 to the high P_T of 1.5. Another way to appreciate this non-linearity is to remember that prices may not fall below 0, but are unbound upward.

by the second term on the right-hand side of equation (2a). If this downward bias is not constant over time, daily futures prices will not follow a martingale process even under risk neutrality.

In equation (2), in which derivation risk neutrality is *not* assumed, the downward bias may be offset by a negative conditional covariance. Intuitively, under risk neutrality, the downward bias is created by the fact that movements from the expected price level to lower price levels generate a much greater gain than the loss generated by moving from the expected price level to higher price levels. However, if greater marginal utility is obtained at high price levels than at low price levels, this downward bias can be offset or transformed in an upward bias. For this to be true, inflation must be a risk; that is, the endowment process must be such that the representative agent has a greater marginal utility of consumption when inflation is high.

I now focus on how this downward bias evolves over time. Expanding $(1/P_T)$ in a Taylor series around $E_t(P_T)$, taking expected values, and neglecting the Lagrange form of the remainder after the second moment, equation (3) is obtained.

$$E_t(1/P_T) \cong \frac{1}{E_t(P_T)}\left(1+\frac{\mathrm{Var}_t(P_T)}{\left(E_t(P_T)\right)^2}\right) = \frac{1}{E_t(P_T)}\left(\frac{E_t(P_T{}^2)}{\left(E_t(P_T)\right)^2}\right) \tag{3}$$

Simulations showed that for the time horizons involved in this study (at most two months) this is an acceptable approximation. Substituting equation (3) into equation (2a) gives equation (4).

$$F_{t,T} = E_t(P_T)\left(1-\frac{\mathrm{Var}_t(P_T)}{\left(E_t(P_T)\right)^2+\mathrm{Var}_t(P_T)}\right) = E_t(P_T)\left(1-\frac{\mathrm{Var}_t(P_T)}{E_t(P_T{}^2)}\right) \tag{4}$$

It is not straightforward to test this model because one cannot observe the daily stochastic process of prices, upon which the conditional expectation and variance of P_T depend upon. The only observable variable is P_T itself, which has a monthly frequency. Other variables with frequencies higher than monthly are available to the econometrician and were probably used by agents to forecast P_T. These observable variables could be used in a Kalman filter framework to uncover $E_t(P_T)$ and $\mathrm{Var}_t(P_T)$.

An alternative route to test the model is provided by one testable implication of equations (2a) or (4). This implication is better understood by examining the middle expression in equation (4). The expression for the downward bias depends on the conditional expectation and variance of P_T. A mean preserving spread increases the downward bias. As maturity approaches and more information about inflation is revealed, $\mathrm{Var}_t(P_T)$ declines and the futures price converges from below to $E_t(P_T)$. Therefore, in a large sample, a regression of the futures price on itself lagged once must generate a coefficient greater than 1. This is because of the upward convergence of $F_{t,T}$ to $E_t(P_T)$, which equals P_T at the maturity date $t = T$. Formally, in regression (5) below, for each contract, α_t converges to 1 from below.

$$F_{t+1,T} = \alpha_t F_{t,T} + \varepsilon_t \tag{5}$$

Therefore, $E\left(\frac{\sum F_{t+1\,T} F_{t\,T}}{\sum F_{t\,T}^{2}}\right) > 1$. As the number of contracts grows large, the time invariant estimate of α_t should be greater than 1. A natural way to test this hypothesis is to use unit root tests, because they are one-sided tests. If the null hypothesis of a unit root is rejected, so is the Lucas model with risk neutrality.

The next section tests whether or not the series of daily futures prices has a unit root. If it does indeed have a unit root, one would be interested in testing whether the futures price is a martingale, which could be the case if the downward bias in equations (2a) or (4) is negligible or approximately constant over time. The martingale tests are also presented in the next section.

3. Tests for the martingale hypothesis and for a unit root

This section presents several econometric tests for the existence of a unit root in the futures price series and for the hypothesis that the daily futures price is a martingale. The large sample justification of the tests can be found in Hodrick & Srivastava (1987). The martingale null hypothesis is:

$$E_t\ (F_{t+1,T}) = F_{t,T} \tag{6}$$

The dataset for this test is composed of 21 contracts. To make clear how the dataset was constructed, only the first 15 futures contracts for the OTN are used in the charts. This is because after this 15-month-period a monetary reform (Plano Verão) took place, and indexation was outlawed. With the failure of the Plano Verão, indexation resumed and so did the trade in the futures market for the official index, then renamed BTN. Six more contracts of this later period are included in the dataset. After March, 1991, the trade in this market was significantly disrupted by another stabilization plan (the first Collor plan).

Figure 1 shows a typical contract in this futures market. The x-axis represents the number of calendar days before the maturity date, in this case, November 1, 1988. The value in Cz$ of the ex-post OTN was normalized to equal 100, so that the over-or underestimation of the true price level given by the futures prices (the line in figure 1) can be readily assessed. The bars represent the volume in Cz$ of open contracts at each given date. This is a measure of how much risk is changing hands through this market. The peak in the value of open contracts is around US$75 million. This contract was traded for the first time on June 15, 1988. However, trade was very thin until approximately three months before maturity. As inflation escalated, trade became increasingly thin for all open contracts except the *nearby contract*.[7]

To guard against the criticism that the futures price data early in the contract are not representative of the market, I decided to use only the last 40 observations of each contract (covering approximately two calendar months). Figure 2 shows the observations that are used in the estimation. Note that for each date there are usually two observations from two different

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[7] The nearby contract is "... the contract with the earliest delivery date, [and is] often the most actively traded..." (Duffie, 1989, p. 23).

Figure 1
OTN futures market — last 48 calendar days of the November, 1988 contract

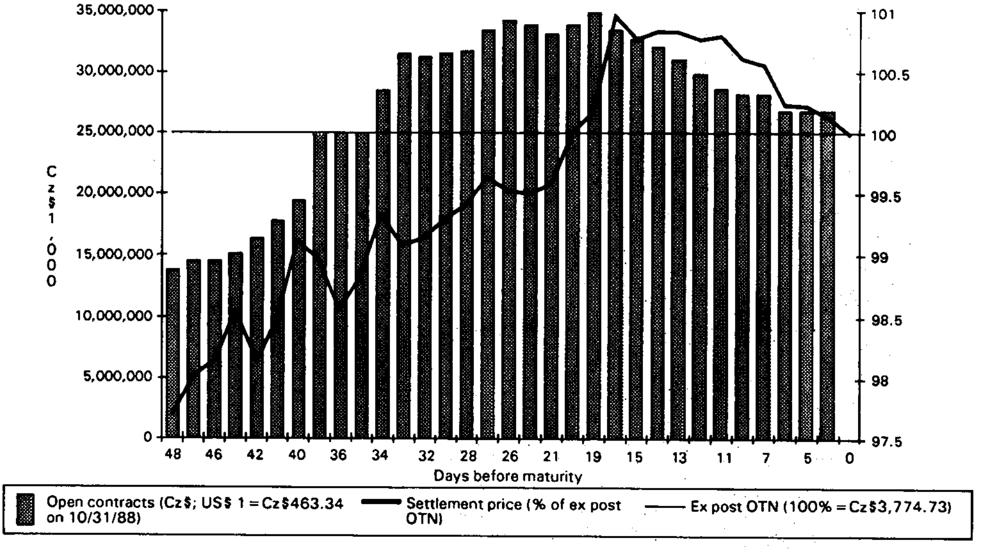
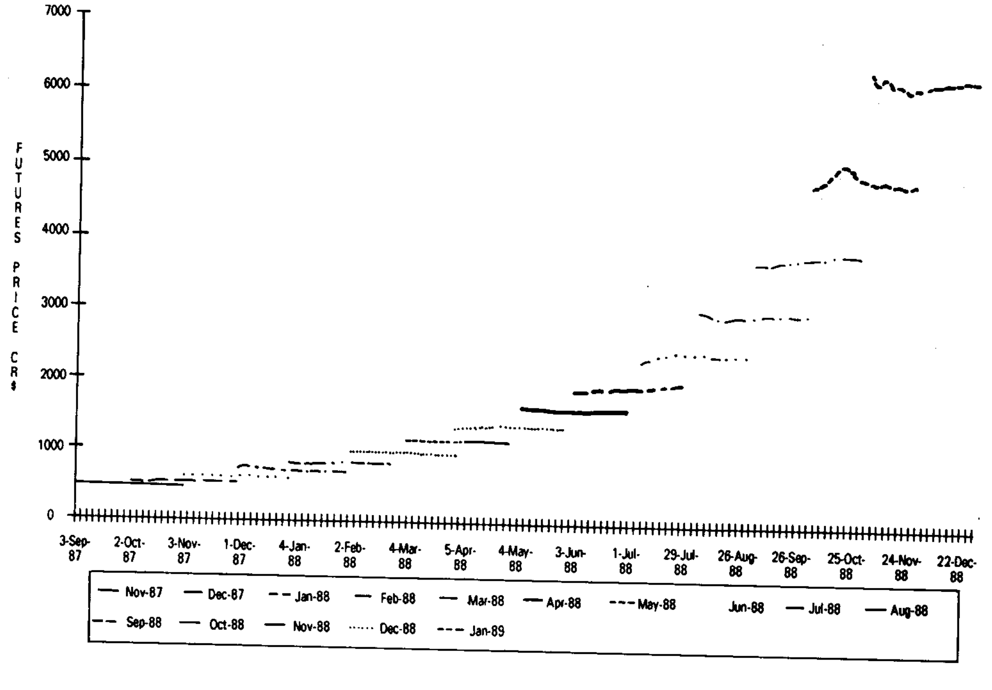


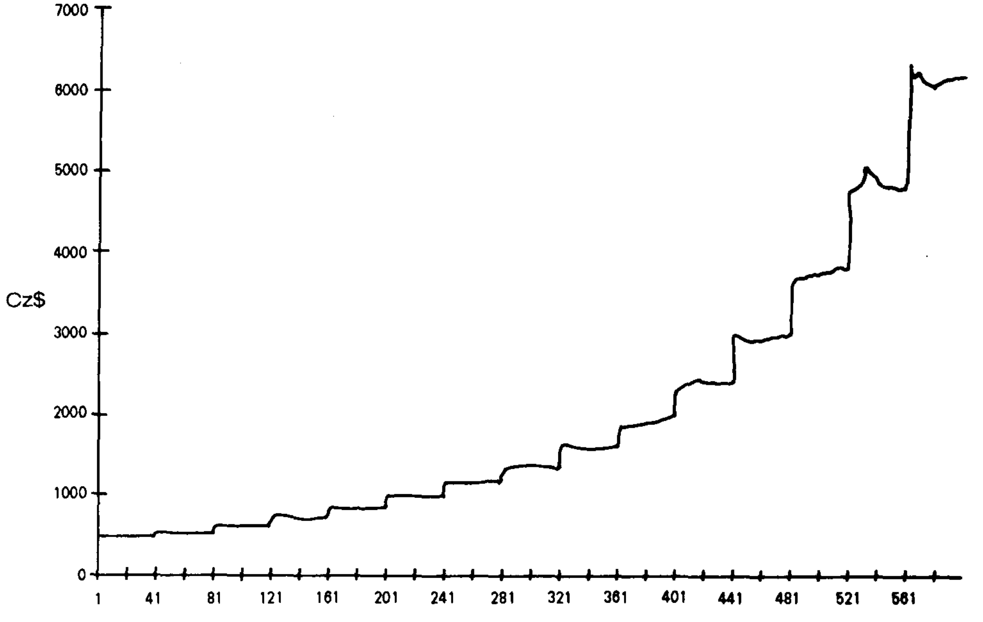
Figure 2
Futures price of the price level in Brazil (OTN)



contracts. Each segment is a sequence of 40 "forecasts" of the true price level. To obtain the price level curve in figure 2, one has merely to link the end points of each segment.

The dataset used in the empirical work is created by appending the farthest observation to maturity (the 40th observation) of a contract to the observation on the maturity day of the previous contract. Figure 3 shows the resulting picture. Note that the x-axis no longer contains dates; the data used in the tests are no longer time-series data.

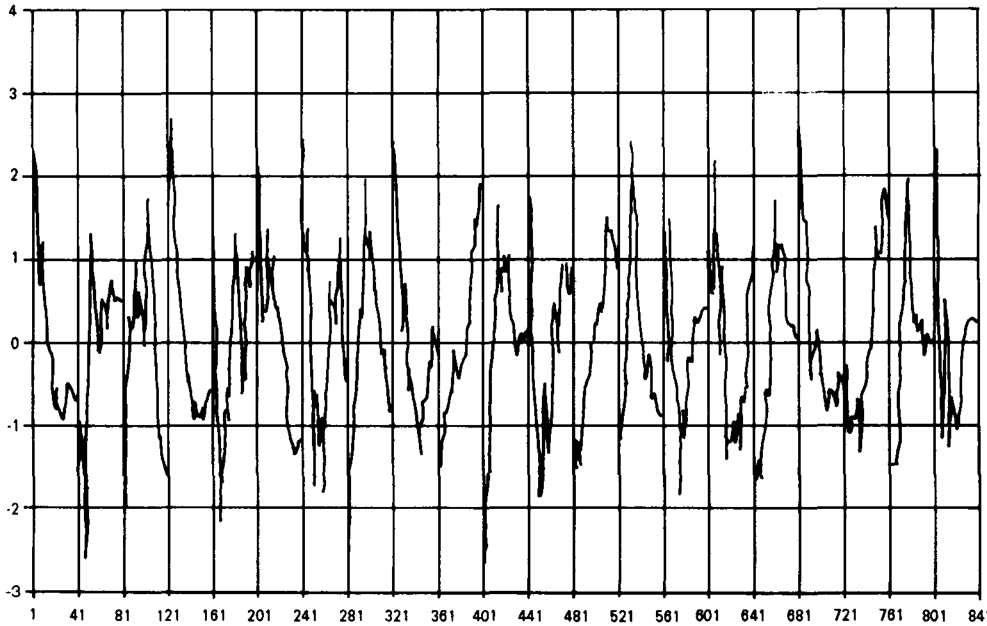
Figure 3
OTN futures market — settlement prices



One cannot undertake tests with the data shown in figure 3, because the futures prices of a given contract are not comparable to the futures prices of a different contract. In order to render the data homogeneous, alternative methods were used. The first one was to subtract from each daily settlement price the observed mean for that contract. The result was then divided either by the mean of that contract or by its standard deviation. These latter normalizations were conceived to ensure the homoscedasticity of the data. Figure 4 shows the resulting series when the standard deviation is used as the normalization factor. In this chart the contracts for the BTN from October 89 to March 90 are also included. After every 40 observations in figure 4 there is a change of contract (at observations 41, 81, 121 and so on). Therefore, the "jumps" on the series when the contracts are changing do not represent ordinary price movements in the series. There are 840 observations (21 contracts and 40 observations per contract).

The other way to ensure homogeneity was to first-difference the data. Note that this will generate 39 observations per contract, or 39 x 21 = 819 total observations. The first-differenced data were then divided either by the mean per contract or the standard deviation per contract to induce homoscedasticity.

Figure 4
Normalized data



In both cases, when the normalization factor used was the standard deviation, a more homoscedastic series resulted. The normalization by the mean tended to excessively damp the fluctuations of the latter contracts in figure 3. Therefore, results will only be reported for the former normalization. Nevertheless, the use of the series normalized by the mean did not change the tests'results.

The notation used here is the following:[8]

y_{t+1} = ($F_{t+1,T}$ - mean per contract) / standard deviation per contract;

x_t = ($F_{t,T}$ - mean per contract) / standard deviation per contract;

Δy_{t+1} = ($F_{t+1,T}$ - $F_{t,T}$) / standard deviation per contract;

Δx_t = ($F_{t,T}$ - $F_{t-1,T}$) / standard deviation per contract.

Assuming rational expectations implies that:

$$y_{t+1} = E_t(y_{t+1}) + \varepsilon_t \tag{7}$$

[8] The use of x_t and y_{t+1} is intended to emphasize the correct alignment of the data used in the tests. x_t cannot be obtained simply by lagging y_{t+1}, because of the existence of different contracts. If plain lagging were done in the tests explained below, one would be regressing the farthest observation to maturity (the 40th observation) of a contract on the observation for the maturity date of the previous contract, which would be uninterpretable.

A test of the martingale hypothesis can be constructed by using the orthogonality conditions between the forecast error and any information in the econometrician's information set. We define the function $h(y_{t+1}, x_t, \delta_0) = y_{t+1} - \alpha - \beta x_t$, where δ_0 is the true parameter vector. The null hypothesis is $\alpha = 0$ and $\beta = 1$. Tests were performed using Hansen's (1982) Generalized Method of Moments (GMM). As mentioned in section 2, the conditional variance of the series is believed to decrease as the maturity date approaches. A nice feature of the GMM is that no additional auxiliary assumption of conditional homoscedasticity is necessary when constructing the covariance matrix of the parameters' estimates (Hodrick & Srivastava, 1987, p.8). In the linear setting used here, GMM reduces to OLS with White's (1980) correction for heteroscedasticity. The large sample theory invoked to perform the estimation and testing with the peculiar structure of the futures contracts is fully spelled out in Hodrick & Srivastava (1987, p. 11). There, it is recognized that the asymptotics of the testing rely on the number of contracts growing large.[9]

The regressions of the one day ahead futures price on the previous day's futures price (with the mean per contract subtracted and the result divided by the standard deviation per contract) rejected the martingale hypothesis (see table 1). The χ^2 test for $\beta = 1$ was significant well below the 1 percent level.

Table 1

Test for $\alpha = 0$ and $\beta = 1$ in the regression $y_{t+1} = \alpha + \beta x_t$

Dependent variable	4	YSTDEV	
From 87: 9:2 UNTIL	90:11:20		
Total observations	840	Skipped/missing	0
Usable observations	840	Degrees of freedom	838
R** 2	.85863241	RBAR**2	.85846372
SSR	115.78005	S.E.E.	.37170197
Durbin-Watson	1.86076168		
Q (84) =	91.2151	Significance level	.27673265

Nº	Label	Var	Lag	Coefficient	Stand. error	T-statistic	Signif. L
1	Constant	0	0	-0.5533361E-15	0.1280966E-01	-0.4319679E-13	1.000000
2	XSTDEV	3	0	.9266242	0.1643097E-01	56.39499	.0000000

χ^2 test for $\beta = 1$			
Chi-square (1) =	19.94249	Significance level	.0000000

The evidence from the regressions of Δy_{t+1} on Δx_t was not conclusive (see table 2). On the one hand, the overall regression was not significant at the 10 percent level. On the other hand, the t statistic for Δx_t was significant at the 10 percent level. Therefore, this test was inconclusive as to whether Δy_{t+1} is a martingale difference. Even if the t statistic for Δx_t were *not* significant, it could be argued that other variables known at t could affect Δy_{t+1}, i.e., that this test has low power against other possible interesting alternatives.

[9] "Because the asymptotic distribution [of the covariance matrix] depends on the number of contracts growing large, the degrees of freedom in the analysis using future data are inherently less than the number of observations" (Hodrick and Srivastava, 1987, p.8).

One could argue that the martingale test performed here assumed stationarity to ensure the consistency of the standard errors, and therefore is inconsistent if the null hypothesis is true. To guard against this criticism, I ran the following forms of Dickey-Fuller tests for the whole sample (840 observations): without a constant, with a constant, with a constant and a linear time trend, with a constant and a quadratic time trend, and with a constant, a linear time trend and a quadratic time trend. Given the structure of the data used here, the time trend is 1 for the 40 observations relative to the first contract, 2 for the 40 observations relative to the second contract, and so forth. Table 3 summarizes these results.

Table 2

Test for $\alpha = 0$ and $\beta = 0$ in the regression $\Delta y_{t+1} = \alpha + \beta \Delta x_t$

Dependent variable	8	DIFYSTDV	
From 87:9:23 Until 90:10:23			
Total observations	819	Skipped / missing	0
Usable observations	819	Degrees of freedom	817
R**2	.00968469	RBAR**2	.00847255
SSR	127.32298	S.E.E.	.39476839
Durbin-Watson	1.98394832		
Q(84) -	81.8880	Significance level	.54488798

No.	Label	Var	Lag	Coefficient	Stand. error	T-statistic	Signif. L
1	Constant	0	0	-0.6980620E-02	0.1379068E-01	-.5061840	.6127275
2	DIFXSTDV	7	0	.1016154	0.5327785E-01	1.907273	0.5648526

Null hypothesis
The following coefficients are zero
Series Constant (0)
Series DIFXSTDV (7)
Chi-square (2) - 4.073354 Significance level .1304615

All tests rejected the existence of a unit root. This rejection implies two things. First, these futures prices cannot be characterized as a martingale. Second, the theoretical model of section 2 assuming risk neutrality is not a good characterization of these futures prices either.

Table 3

Dickey-Fuller tests

x_t	Constant	T	T^2	R^2	D. W.	Q(84)	S. E. E.
-0.12				0.09	1.77	132.4	0.37
(-9.24)							
-0.12	-0.004			0.09	1.77	132.4	0.37
(-9.23)	(-0.28)						
-0.12	-0.008	0.43E-3		0.09	1.77	132.4	0.37
(-9.22)	(-0.31)	(0.21)					
-0.12	-0.003		-.53E-5	0.09	1.77	132.1	0.37
(-9.23)	(-0.14)		(-0.06)				
-0.12	-0.044	0.01	-.43E-3	0.10	1.78	130.4	0.37
(-9.22)	(-1,05)	(1.11)	(-1.09)				

Dependent variable: Δx_{t+1}; 819 usable observations. DF statistics are the t statistics for x_t (in bold face); t statistics in parentheses. The trend variable (T) is 1 for the first contract (first 40 observations), 2 for the second (observations 41 to 80), and so on.

This latter point is discussed at the end of this section. Before that, however, a few econometric points must be discussed.

Two econometric problems are present in the former Dickey-Fuller tests. The first one, already mentioned, is the existence of conditional heteroscedasticity in the data. As each contract gets closer to maturity, agents gather more information about inflation, and the variance of the futures price declines. The risk neutral model of section 2 has this feature. If risk aversion is assumed, instead, the risk premium will also decline as maturity approaches.

The other problem is a non-standard form of autocorrelation that arises in these data even under the null hypothesis that the futures price is a martingale. Figures 2 and 3 can be used to illustrate this autocorrelation problem. Figure 2 shows that for most dates the dataset is composed of the futures prices of two contracts with maturity in subsequent months, i.e., the two contracts with the earliest maturity dates. These futures prices are basically forecasts of the price levels for that and the following month. Since price levels have a unit root by definition, any shock that affects $F_{t,T}$ (the "forecast" of the nex month's price level) will also affect $F_{t,T+30}$ (the "forecast" of the price level in two months). This effect is present in the data even if the martingale hypothesis holds, because, as figure 3 makes clear, the data used here are not strictly time-series data, but an "unfolding" of the times-series data displayed in figure 2. This kind of autocorrelation is non-standard exactly because of this "unfolding" process. For any contract except the first one, the first 20 observations (closest to the maturity date) correlate with the last 20 observations (farthest from the maturity date) of the subsequent contract. Each of these *first 20 observations* correlates with the observation "leaded" 20 periods, as figure 3 makes clear. However, for any contract except the last one, the *last 20 observations* (farthest from the maturity date) correlate with the first 20 observations (closest to the maturity date) of the previous contract. Each of these last 20 observations correlates with the observation "lagged" 20 periods, as figure 3 makes clear. This "asymmetry" in the autocorrelation originates from the "unfolding" process described earlier.

To deal with the heteroscedasticity problem, table 4 reports the results when White's (1980) correction for heteroscedasticity was applied. The solution adopted for the non-

Table 4
Dickey-Fuller tests

x_t	Constant	T	T^2	R^2	D.W.	Q (60)	S. E. E.
-0.055				0.03	1.65	59.9	0.24
(-3.38)							
-0.055	0.013			0.03	1.66	60.6	0.24
(-3.41)	(1.14)						
-0.58	-0.21	0.003		0.04	1.66	61.8	0.24
(-3.52)	(-.82)	(1.71)					
-0.057		.16E-1		0.03	1.66	61.4	0.24
(-3.51)		(2.00)					
-0.057	-.39E-2		-.11E-3	0.03	1.66	61.0	0.24
(-3.48)	(-0.21)		(1.41)				
-0.060	-0.054	0.012	-.40E-3	0.04	1.66	63.7	0.24
(-3.57)	(-1.53)	(1.69)	(-1.33)				

Dependent variable: Δx_{t+1}; 420 usable observations.
The t-statistics are computed using White's (1980) standard errors corrected for heteroscedasticity.
DF statistics are the t statistics for x_t (in bold face); t statistics in parentheses.
The trend variable (T) is 1 for the first contract (first 20 observations), 2 for the second (observations 21 to 40), and so on.

standard autocorrelation problem was to consider only 20 observations per contract. Once the contracts no longer overlap, the "unfolding" process no longer takes place, and the non-standard autocorrelation no longer arises. The resulting dataset is the one shown in figure 2 once all the observations in the subsequent contract overlapping with the previous one have been eliminated. To account for other forms of autocorrelation that could be present, table 5 reports the results when White's (1986) autocorrelation-heteroscedasticity consistent standard errors are used. Despite the reduction on the DF statistics relative to those shown in table 3, one can still reject the existence of a unit root in the series.

Table 5
Dickey-Fuller tests

x_t	Constant	T	T^2	R^2	D.W.	Q(60)	S. E. E.
-0.055				0.03	1.65	59.9	0.24
(-2.80)							
-0.055	0.013			0.03	1.66	60.6	0.24
(-2.79)	(0.80)						
-.058	-0.21	0.003		0.04	1.66	61.8	0.24
(-3.04)	(-.64)	(1.28)					
-0.057		.16E-1		0.03	1.66	61.4	0.24
(-3.89)		(1.37)					
-0.057	-.39E-2		-.11E-3	0.03	1.66	61.0	0.24
(-2.98)	(-0.16)		(1.02)				
-0.060	-0.054	0.012	-.40E-3	0.04	1.66	63.7	0.24
(-3.03)	(-1.16)	(1.19)	(-0.91)				

Dependent variable: Δx_{t+1} ; 420 usable observations.
The t-statistics are computed using White's (1986) heteroscedasticity-autocorrelation consistent standard errors.
DF statistics are the t statistics for x_t (in bold face); t statistics in parentheses.
The trend variable (T) is 1 for the first contract (first 20 observations), 2 for the second (observations 21 to 40), and so on.

Augmented Dickey-Fuller tests were also undertaken. These results are not reported here. Again, although the ADF statistics are no longer near the values of the DF statistics of table 3, one can still reject the existence of a unit root in the series.

The rejection of the existence of a unit root in futures prices also characterizes the rejection of the risk neutrality hypothesis within the Lucas (1978) framework. In section 2, it was shown that an asset pricing model *à la* Lucas (1978) with risk neutrality implies that, in a large sample, a regression of the futures price on itself lagged once must generate a coefficient greater than 1. The coefficient of x_t in table 1 is significantly less than 1, i.e., 0.93, a conclusion that the DF and ADF tests have corroborated.

One explanation for the coefficient being less than 1 is the existence of a time-varying risk premium. Equation (2) illustrates this point. If the conditional covariance term is negative and declines as maturity approaches, regression (5) of the one day ahead futures price on the previous day's futures price will generate a coefficient below 1. Intuitively, under risk neutrality, $F_{t,T} \leq E_t(P_T)$ because movements from the expected price level to low price levels generate a much greater gain than the loss generated by moving from the expected price level to high price levels. However, if greater marginal utility is obtained at high price levels than at low price levels, this downward bias can be reversed. For this to be true, inflation must be a risk, e.g., the endowment process must be such that the representative agent has a greater marginal utility of consumption when inflation is high.

As maturity approaches, the risk premium declines, and so does the futures price, approaching the expected value of the price level from above. Therefore, when the number of contracts grows large, the time invariant estimate of α_t in regression (5) converges to $E\left(\frac{\sum F_{t+1\,T}F_{t\,T}}{\sum F_{t\,T}^2}\right) < 1.$

The effect described by the model of section 2 is still present, although it is more than offset by the presence of risk aversion, represented by the covariance term in equation (2). This conditional covariance between the marginal utility of consumption and the inverse of the price level (normalized by conditional expectation of the marginal utility of consumption) represents a time-varying risk premium.

Several models have been proposed in the literature to deal with time-varying risk premiums (for references, see Attanasio, 1990). The specific difficulty for the Brazilian futures market is that the "spot" price of this market, the daily price level, is non-observable. Therefore, this is a case of a time-varying risk premium with unobserved components. In the last section, this point, among others, is listed for future research. The next section uses the *term structure* of the futures contracts for the price level to infer the stochastic structure of the expected inflation process.

4. Term structure of the price level futures contracts

This section uses the *term structure* of the futures contracts for the price level to infer the stochastic structure of the *expected* inflation process. On any given day, several futures contracts maturing at the end of subsequent months are traded. These contracts pay off according to the price level at the end of the previous month. If P_t is the price level at t, and π_{t+1} the inflation rate between t and $t+1$, then $P_{t+1} = (1 + \pi_{t+1}) P_t$. Hence, if an informational shock to the economy raises π^e_{t+1} by 1 percent, P^e_{t+1} will also rise by 1 percent. What happens to the subsequent expected prices, P^e_{t+2}, P^e_{t+3}, ..., will depend on the stochastic process followed by expected inflation. For example, if expected inflation follows a martingale difference, that is, if price increases today do not imply further price increases or decreases in the future, an increase of 1 percent in P^e_{t+1} is associated with a 1 percent increase in all expected future price levels, P^e_{t+2}, P^e_{t+3}, and so on. At the other extreme, if expected inflation follows a martingale, that is, a price increase today implies a similar price increase in the future, an increase of 1 percent in P^e_{t+1} is associated with a 2 percent increase in P^e_{t+2}, 3 percent in P^e_{t+3}, and so on.

Figures 5 and 6 clarify the above points. Figure 5 displays the evolution of the futures prices of all contracts that were open on August 7, 1987, when the OTN market started operating. The futures prices of all contracts were normalized to 100 percent in August 7, 1987. On August 28 the futures price of the September contract rose to almost 102 percent. Assume for the moment that the futures price is the expected value of the price level at maturity, or that the difference between the two does not vary with the number of days to maturity. Then, the evolution of the futures price means that the inflation forecast for the month of August rose by 2 percent between August 7 and August 28. If the rise of August inflation were uncorrelated with September inflation, the futures price of the September contract should also have risen by the same amount, i.e., by 2 percent. On the other hand, if the inflation process was a martingale, 2 percent more inflation in August would have meant

Figure 5
OTN futures market - settlement prices normalized by *Initial* prices for different contracts

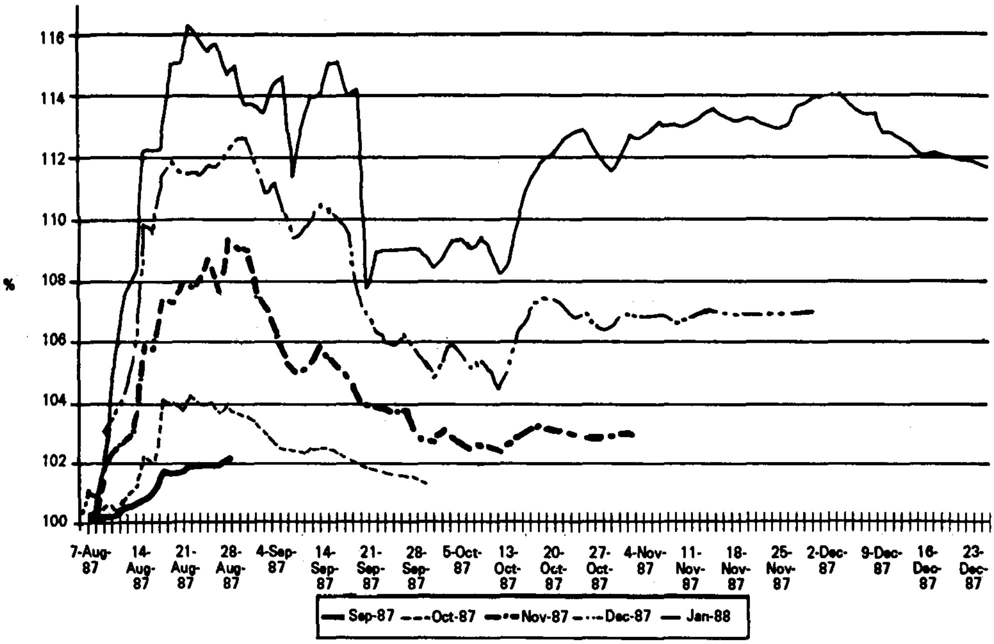
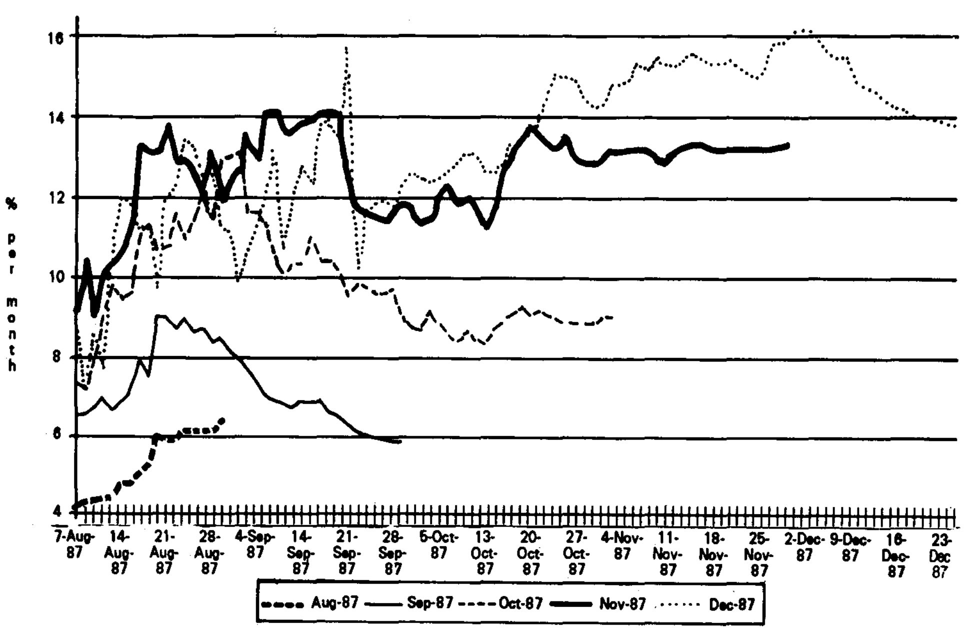


Figure 6
OTN futures market — monthly inflation forecasts



2 percent more inflation in September. This would have made the September futures price on August 28 rise to 104 percent. This latter characterization is what figure 5 suggests. The same reasoning can be applied to the other contracts which appear in figure 5. However, as already mentioned, the trade on the contracts very far from maturity was very thin, and became more so as inflation escalated. Therefore, in the econometric estimations performed in this section, only the two contracts with earliest maturity dates are used.

Figure 6 displays the same information contained in figure 5 in the form of inflation forecasts. Under the simplifying assumptions spelled out in the previous paragraph, the growth rate computed by dividing the October futures price by the same day's September futures prices is what the market it forecasting for the inflation in September. One can easily see the point explained in the previous paragraph: Both the August and the September inflation rates increased by around 2 percent between August 7 and August 28, suggesting that monthly expected inflation could be a martingale.

If expected monthly inflation contains a unit root, shocks affecting the current month's price level by 1 percent should affect the next month's price level by 2 percent or more. Of course this a general proposition; there could be shocks affecting one of the inflation rates but not the other. However, in a large sample one would expect to find a coefficient of 2 in a regression of the daily percentage variations of the contract with the second earliest maturity date on the daily percentage variations of the nearby contract. Formally:

$$\Delta f_{t,T+30} = \alpha + \beta \, \Delta f_{t,T} + \varepsilon_t \tag{8}$$

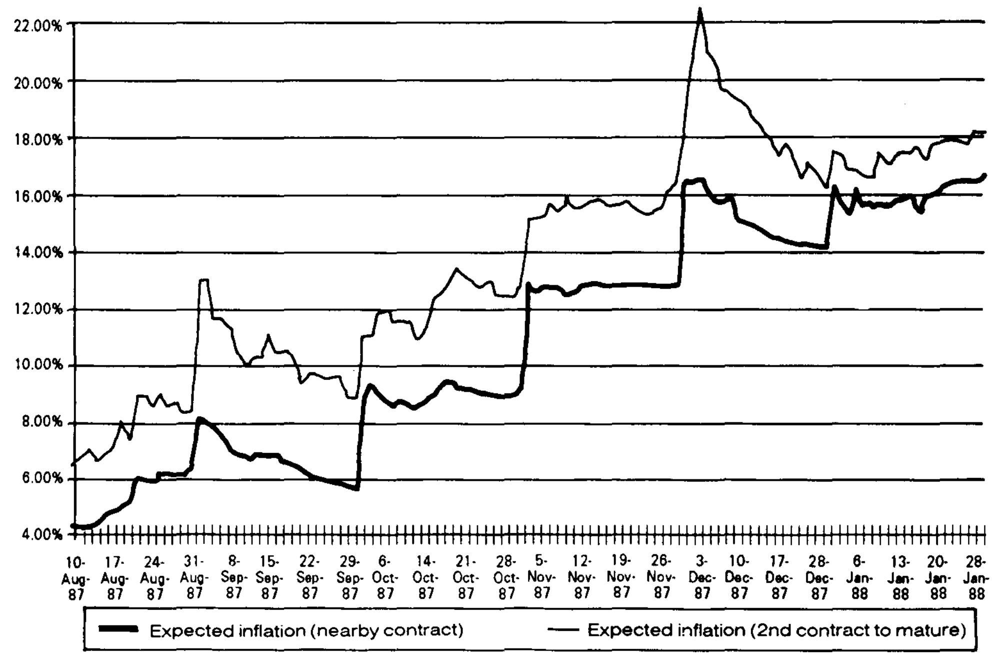
where $\Delta f_{t,T} = ln\,(F_{t,T}) - ln\,(F_{t-1,T})$.

The null hypothesis that monthly expected inflation contains a unit root corresponds to $\alpha = 0$ and $\beta = 2$. If expected monthly inflation is a martingale difference, $\beta = 1$. The error term in regression (8) could be interpreted as the effect of shocks that are idiosyncratic to each contract.

The dataset for regression (8) is composed of the data from the OTN futures market. During the period of the BTN futures market, trade on the second contract was too thin for long periods of time, so I decided to drop it altogether. However, the inclusion of the data from the BTN futures market which presented a minimum degree of liquidity did not change the results that are presented below. I now explain how the dataset was constructed. For each month, the first-differences of the natural logarithms of the futures prices are computed for the nearby contract and for the contract with the second earliest maturity date. When the nearby contract matures, the one with one month until maturity becomes the nearby contract and the one with two months until maturity becomes the contract with the second earliest maturity date. In this fashion, a series of $\Delta f_{t,T}$ and $\Delta f_{t,T+30}$ is constructed from August 10, 1987 to January 13, 1989 (354 observations).

Figure 7 helps one understand the idea behind regression (8). The dark squares mark the series of the current month's "expected inflation", computed by dividing the futures price of the OTN for the nearby contract by the known value of the current month's OTN. The white squares mark the series of "expected inflation" for the following month, computed by dividing the futures price of the OTN for the contract with the second earliest maturity date by the same day's futures price of the nearby contract. Notice that by the end of each month (when the nearby contract matures), the contract with the second earliest maturity date becomes the

Figure 7
Expected inflation — August, 1987 to January, 1988



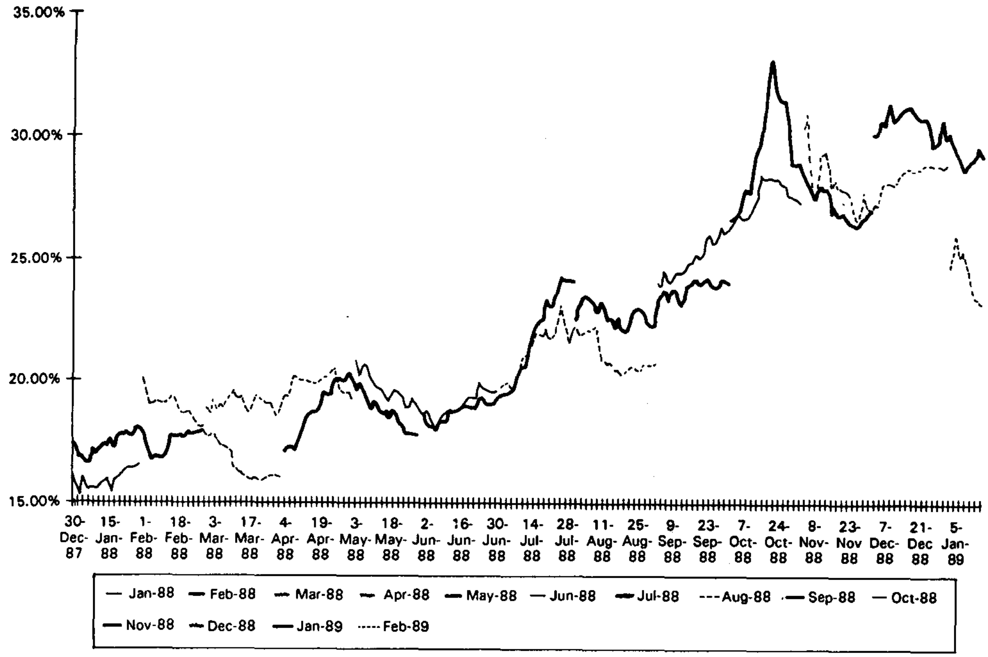
nearby contract. Therefore, the sequence of "forecasts" for each month's inflation is given by a sequence of aproximately 20 white squares, followed by another of approximately 20 dark squares, covering two calendar months. For given initial "forecasts", the sequence of daily "forecasts" for the current month's inflation is obtained by adding $\Delta f_{t,T}$ to the previous day's "inflation forecast". The daily variation in the following month's "expected inflation" is given by $[\Delta f_{t,T+30} - \Delta f_{t,T}]$. If expected monthly inflation is a martingale, then $\Delta f_{t,T+30}$ must be twice $\Delta f_{t,T}$.

Figure 8 shows the remaining period of the OTN futures market not shown in figure 7. In figure 8, each sequence of "inflation forecasts" for the same month is a different segment. Note that the regularity that prevailed until the end of 1987, by which the following month's inflation forecast was given by the current month's forecast plus some percentage points, is lost in 1988 as inflation escalated.

Table 6 displays the results of regression (8). The method of estimation was OLS with White's (1980) correction for heteroscedasticity. When the full sample is used, the results indicate that the constant is indeed zero, and the coefficient of $\Delta f_{t,T}$ is 1.52. The χ^2 tests show that this value is significantly different from both 1 and 2. Therefore, the stochastic process of expected monthly inflation cannot be characterized as either a martingale or a martingale difference. Since the inflation process at this time in Brazil had a unit root, it is somewhat puzzling that one does not find a coefficient greater or equal to 2 in regression (8).

Several simplifying assumptions made so far may be driving the above result. A more careful analysis must distinguish between the two periods in which shocks hit the economy. This is because prices are usually collected from the 16th of one month to the 15th of the following month. Therefore, in the last 15 (calendar) days of the nearby contract, no current

Figure 8
Expected inflation — January, 1988 to January, 1989



price increase should impact its futures price, only those of contracts with later maturities. Therefore, the regression above would be mixing two different "regimes" given by the two fortnights of each month. In order to test whether this is true, the sample was divided into two: before and after the 15th of each month. In the first sub-sample I also included observations until the first day of trade after the 15th, which were excluded from the second sub-sample. This criterion aims to include in the first sub-sample all observations in which current price increases affect both open contracts. It implies that the second sub-sample has fewer observations.

When only the first 15 days of each month are included in the sample $\hat{\beta}$ drops to 1.40, while when only the last 15 days are included, it rises to 1.89. The Chow test (not shown in

Table 6
Tests for $\alpha = 0$ and $\beta = 1$ or 2 in the regression $\Delta f_{t,T+30} = \alpha + \beta\ \Delta f_{t,T}$

Sample	$\hat{\alpha}$	$\hat{\beta}$	R^2	D. W.	χ^2 for $\beta = 1$	χ^2 for $\beta = 2$
Full sample	0.92E-5	1.52	0.49	1.91	26.6	20.4
	(0.05)	(14.4)			[0.00]	[0.00]
First 15 days	0.18E-3	1.40	0.49	1.80	12.8	28.2
	(0.70)	(12.5)			[0.00]	[0.00]
Last 15 days	-.24E-3	1.89	0.51	1.98	20.9	0.35
	(-0.97)	(9.73)			[0.00]	[0.56]

The tests are computed using White's (1980) standard errors corrected for heteroscedasticity. There are 354 usable observations. The t-statistics are in parentheses under the coefficients. The χ^2 statistics have 1 degree of freedom, and the significance levels are in brackets under the statistics.

table 6) for the stability of β rejects the hypothesis that β is the same for the two sub-samples at the 6 percent significance level. Therefore, the characterization of the expected inflation stochastic process as a martingale cannot be rejected when only the last fortnight of each month is considered, but can be rejected for the first fortnight. In both sub-samples, the explanatory power of the regression seems to be the same, i.e., around 50 percent.

Before the implications of the analysis are discussed, a few other results will attest to the robustness of the conclusions. An alternative way of constructing the dataset yielded similar results. These results are not reported here. To guard against the possibility that outliers could be driving the regression's results, the regressions were rerun with robust estimation (see Huber, 1973). Table 7 reports the results. The main findings of the original regressions (table 6) remain: the characterization of the expected inflation stochastic process as a martingale cannot be rejected when only the last fortnight of each month is considered, but can be rejected for the first fortnight and for the whole sample. As before, the explanatory power of the regressions seems to be the same, i.e., around 50 percent.

Table 7

Tests for $\alpha = 0$ and $\beta = 1$ or 2 in the regression $\Delta f_{t,T+30} = \alpha + \beta \Delta f_{t,T}$

Sample	$\hat{\alpha}$	$\hat{\beta}$	R^2	D. W.	χ^2 for $\beta = 1$	χ^2 for $\beta = 2$
Full sample	-.58E-4	1.44	0.48	1.88	27.94	46.72
	(-0.36)	(17.4)			[0.00]	[0.00]
First 15 days	0.15	1.34	0.49	1.78	16.02	59.16
	(0.71)	(15.70)			[0.00]	[0.00]
Last 15 days	-.30E-3	1.72	0.51	1.98	16.43	2.38
	(-1.31)	(9.64)			[0.00]	[0.12)

The method of estimation is minimum absolute deviations (MAD), computed through iterated least squares (Huber, 1973). There are 354 usable observations. The t-statistics are in parentheses under the coefficients. The χ^2 statistics have 1 degree of freedom, and the significance levels are in brackets under the statistics.

I also investigated another possible source of distortion of the results. Agents may take positive and negative shocks to inflation differently. If positive shocks are taken as more permanent than the negative shocks, which are perceived as more transitory, then $\hat{\beta}$ will underestimate the impact of positive shocks to inflation. This could reconcile the existence of a unit root in the Brazilian inflation with a coefficient less than 2 in the regressions. In order to test this hypothesis, I ran regression (8) with $\Delta f_{t,T}$ constituted only of either positive or negative shocks. Table 8 summarizes the results.

The Chow tests for the stability of β between the positive and negative shocks sub-samples are in the margin of rejection at the 10 percent level. I interpret table 8 as providing weak indication that the reason why we are not able to find $\beta = 2$ may be due to the perceived "temporariness" of the negative shocks.

Another possible explanation for the failure in finding a coefficient of 2 in regression (8) may be related to the great instability of the Brazilian inflation process in 1988, when it moved from very high inflation to almost hyperinflation, until it was stopped for a few months by the monetary reform of January, 1989 (Plano Verão). Figure 7 displays the "inflation forecasts" for this period. As noted above, the regularity of the last months of 1987 contrasts remarkably with the extreme volatility of 1988 (figure 8). Table 9 summarizes the results

Table 8

Tests for $\alpha = 0$ and $\beta = 1$ or 2 in the regression $\Delta f_{t,T+30} = \alpha + \beta \, \Delta f_{t,T}$

Sample	$\hat{\alpha}$	$\hat{\beta}$	R^2	D. W.	χ^2 for $\beta = 1$	χ^2 for $\beta = 2$
$\Delta f_{t,T} > 0$	0.30E-3	1.51	0.37	2.02	8.37	7.47
	(0.75)	(8.52)			[0.00]	[0.01]
Chow test for the stability of $\beta : \chi^2(2)$			-3.96	[0.14]		
$\Delta f_{t,T} < 0$	-.80E-3	1.17	0.19	1.86	0.62	13.91
	(-1.99)	(5.31)			[0.43]	[0.00]
Chow test for the stability of $\beta : \chi^2(2)$			-4.62	[0.10]		
$\Delta f_{t,T} > 0$	0.28E-3	1.38	0.36	2.01	6.83	18.08
Robust est.	(0.85)	(9.48)			[0.01]	[0.00]
Chow test for the stability of $\beta : \chi^2(2)$			-4.30	[0.12]		
$\Delta f_{t,T} > 0$	-.68E-3	1.14	0.19	1.85	0.49	17.38
Robust est.	(-1.79)	(5.57)			[0.48]	[0.00]
Chow test for the stability of $\beta : \chi^2(2)$			-3.07	[0.21]		

The methods of estimation were OLS with White's (1980) standard errors corrected for heteroscedasticity (first two rows) and minimum absolute deviations (MAD), computed through iterated least squares (Huber, 1973) (last two rows). There are 354 usable observations. The t-statistics are in parentheses under the coefficients. The χ^2 statistics have 1 degree of freedom, and the significance levels are in bracket under the statistics.

Table 9

Tests for $\alpha = 0$ and $\beta = 1$ or 2 in the regression $\Delta f_{t,T+30} = \alpha + \beta \, \Delta f_{t,T}$

For 1987 only

Sample	$\hat{\alpha}$	$\hat{\beta}$	R^2	D.W.	χ^2 for $\beta = 1$	χ^2 for $\beta = 2$
Full sample	0.11E-3	2.11	0.45	2.14	15.62	0.16
	(0.28)	(7.50)			[0.00]	[0.69]
First 15 days	0.41E-3	2.02	0.39	1.85	9.69	0.01
	(0.64)	(6.15)			[0.00]	[0.94]
Last 15 days	-23E-3	2.31	0.57	2.63	9.61	0.54
	(-0.55)	(5.47)			[0.00]	[0.46]
Chow test for the stability of $\beta : \chi^2(2)$			-1.08	[0.58]		
Full sample	-.19E-3	1.78	0.47	1.99	6.42	0.49
Robust est.	(-0.57)	(5.77)			[0.01]	[0.48]
First 15 days	0.63E-4	1.64	0.47	1.90	4.57	1.44
Robust est.	(0.01)	(5.47)			[0.03]	[0.23]
Last 15 days	-.30E-3	2.32	0.49	1.97	2.26	0.13
Robust est.	(-0.75)	(2.64)			[0.13]	[0.71]
Chow test for the stability of $\beta : \chi^2(2)$			-0.78	[0.68]		

The methods of estimation were OLS with White's (1980) standard errors corrected for heteroscedasticity (first three rows) and minimum absolute deviations (MAD), computed through iterated least squares (Huber, 1973) (last three rows). There are 97 usable observations. The t-statistics are in parentheses under the coefficients. The χ^2 statistics have 1 degree of freedom, and the significance levels are in bracket under the statistics.

obtained when only the *well-behaved* period of the last four months of 1987 is used in the regressions.

For 1987, the null hypothesis of $\beta = 2$ can no longer be rejected in any sub-sample or method of estimation. The Chow test for the stability of β pre-January 1, 1988 (not shown in table 9) rejects the null hypothesis of stability at the 7 percent significance level. The finding

that the shocks have a larger effect in the last fortnight of each month also shows up, but the large standard errors do not allow a rejection of the stability of β (see table 9). Note too that the explanatory power of regression (8) is also higher in the last fortnight of the month.

The non-rejection of $\beta = 2$ for the 1987 sub-sample together with the rejection for the full sample looks paradoxical *a posteriori*, since inflation never stopped trending upwards. These results suggest that the previous rejection of the null hypothesis of $\beta = 2$ is related to the extreme volatility of the Brazilian inflation during 1988, when the government tried several (ultimately unsuccessful) anti-inflationary policies.

All the conclusions drawn so far in this section were based on the simplifying assumption that the futures price is the expected value of the price level at maturity, or that the difference between the two does not vary with the number of days to maturity. The results in section 3, however, do not support this hypothesis. They suggest the existence of a risk premium, which declines as maturity approaches. If this is the case, it is possible that a variable representing the number of days to maturity might capture the effect of this declining upward bias in futures prices. I included the variable days to maturity for the nearby contract (and its square and natural logarithm) in regression (8), for different methods of estimation and samples. The results, however, were not significantly altered, and this variable or its variations did not show up significantly in the regressions. This result, however, does not mean that the declining bias is not important in the determination of the futures price. Further modeling is needed to justify the use of the variable days to maturity in regression (8).

5. Conclusion

This paper has examined the formation of inflation expectations in Brazil by exploring the data of the futures market for the price level from August 1987 to March 1990. These data are uniquely suited for this purpose because sources of extraneous noise always present in financial assets returns, like variations in the *ex ante* real interest rate, are much less important in the determination of these futures prices. Therefore, tests using these data shall perform better than those relying either on actual inflation data, which contain forecast errors, or on survey data, which are liable to the criticism that respondents may not have the right incentive to reveal their true inflation expectations.

Section 2 developed an asset pricing model *à la* Lucas (1978) where risk neutrality is assumed. Even with this assumption, the futures prices were shown to have a downward bias *vis-à-vis* the conditional expectation of the price level. As the maturity of the contract approaches, and more information regarding inflation is revealed, this downward bias declines. Therefore, the one-day-ahead futures price should, on average, be greater than the current futures price. In other words, one implication of the model was that a unit root test should not reject the existence of a unit root. If, however, the downward bias is not empirically relevant, futures prices should be a martingale, as Samuelson (1965) suggested.

Section 3 tested for the existence of a unit root in futures prices and whether or not these futures prices were a martingale. The results overwhelmingly rejected the existence of a unit root in futures prices, and, consequently, also rejected the martingale hypothesis. The rejection of the existence of a unit root in futures prices suggests that the downward bias is more than offset by a risk premium, which also declines as maturity approaches, and more information about inflation is uncovered. Therefore, risk neutrality is not a good characterization of this futures market. The introduction of risk aversion in the model and in the estimation would

require the use of techniques involving a time-varying risk premium and unobserved components. This was not attempted in this paper, and was left for future research.

One practical importance of the existence of a positive bias of the futures price is that the Brazilian Central Bank might have paid too high an interest in government bonds. This is because the Central Bank set the daily overnight rate by putting a given real interest rate on top of expected inflation.[10] Since the futures price of the price level was taken as an unbiased predictor of the price level, an excessively high real rate might have resulted.

Section 4 used the *term structure* of the futures contracts for the price level to run tests regarding the stochastic structure of the expected inflation process. The empirical results confirmed that expected inflation in Brazil had a unit root during 1987, when inflation was extremely high, although still *well-behaved.* This was the end of Finance Minister Bresser Pereira's term. Mr. Bresser Pereira was a firm believer that the indexation of prices to previous inflation had nothing to do with the inflationary phenomenon in Brazil. Therefore, he did not fight it. Not surprisingly, inflation escalated smoothly during his term, and the test for inflation inertia was easily accept. In 1988 inflation escalated towards hyperinflation (although at a lower rate than in Bresser's term), and the unit root in expected inflation was rejected. This *a posteriori* puzzling result is associated with the extreme unpredictability of inflation during this period, when the new Finance Minister Mailson da Nóbrega insuccessfully tried several times to stop inflation from escalating. It implies that the anti-inflationary measures tried during 1988, while ultimately insuccessful, had the temporary effect of partially breaking down the expected inflation inertia.

When interpreting the above results, one has to bear in mind that fiscal adjustment was never undertaken by any of the above mentioned Finance Ministers. Therefore, it comes as no surprise that inflation eventually escalated toward hyperinflation. Nevertheless, those results show that short term inflation expectations are very sensitive to government's interference in the economy's price setting mechanism. Given that the government debt had a very short maturity, the futures market for the price level brought invaluable information for the pricing of government bonds.

Very robust evidence was found that shocks affecting the current inflation forecast have a more powerful effect in the subsequent month's inflation during the second fortnight of the month, when data for computing the current inflation has already been gathered. There was also weak evidence that positive shocks to the current month's inflation forecasts affected the following month's inflation forecast more than negative shocks. This suggests that negative shocks were perceived as temporary, whereas positive shocks, as permanent. The above results were robust to different estimation techniques. The variable days to maturity was also introduced in the regressions to proxy for the declining risk premium, but it was not significant. However, further modelling is needed in this area.

Further research in this area should contemplate the introduction of time-varying risk premium in the model of section 2. The estimation of such a model would rely on the Kalman filter and ARCH techniques. Tools for this task are developed in Harvey, Ruiz & Sentana (1990). Regarding the estimation performed in section 4, it may be possible to derive a reduced-form equation in which the risk premium vanishes or depends on a variable related to the number of days to maturity. This could shed more light on how expectations were formed when the economy was reaching hyperinflation.

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[10] During the period studied, a Brazilian government bond (LBC, LFT) paid an *ex post* return equal to the average return paid by private banks to overnight deposits during the bond's maturity.

Another interesting topic that could be explored with these data is how information affects expectations — in particular, whether or not the publication of the official index induces any "excess variance" in futures prices. The same could be done with other price information usually released during the course of the month. Finally, it would be desirable to study how the movements of futures prices were related to the other Brazilian financial markets, in particular the ones of fixed-income instruments (indexed and non-indexed to inflation).

Appendix

Derivation of equation (1) of the article

Disregard the difference between a future contract and a term contract, that is, suppose that a future contract negotiated at price $F_{t,T}$ agreed at t for liquidation at T, only gives the buyer the cash flow (S_T - $F_{t,T}$) at T, where S_T is the price of the good at T in the sight market.

Any investment must obey the intertemporal condition of equilibrium

$$1 = E_t\,(Q_{T,t}\gamma_{T,t}) \tag{1}$$

where:

$\gamma_{T,t}$ = the nominal rate of return between t and T, and $Q_{T,t} = \dfrac{E_T(UM_T)}{P_T} \cdot \dfrac{P_t}{E_t(UM_t)}$

is the marginal rate of intertemporal substitution between monetary units at T and monetary units at t. (Note that $E_t\ (UM_t)$ is the conditional expectation of marginal utility of the money *good* in a utility function that may not be temporally separable.)

Imagine the following operation:

invest $\$F_{t,T}$ at t at a known $R_{t,T}$ rate, and buy $R_{t,T}$ future contracts. This operation costs $\$F_{t,T}$ at t and pays $[F_{t,T}\ R_{t,T} + R_{t,T}(S_T - F_{t,T})]$ at T. That is, for this operation

$$\gamma_T = \frac{R_{t,T}S_T}{F_{t,T}} \tag{2}$$

Substituting (2) in (1), and using an intertemporally separable utility function so that $U_{ti}(C_{ti}) = \beta^i\, U(C_{t+i})$, we have:

$$1 = E_t\left[\beta^{T-t}\frac{U'(C_T)}{U'(C_t)}\frac{P_t}{P_T}\frac{R_{t,T}S_T}{F_{t,T}}\right] \tag{3}$$

$$0 = \frac{\beta^{T-t}P_t}{U'(C_t)}E_t\left[\frac{U'(C_T)}{P_T}\frac{R_{t,T}S_T}{F_{t,T}}\right] - 1 \tag{3'}$$

Noting that by (1), $1 = \dfrac{\beta^{T-t}P_t}{U'(C_t)}E_t\left[\dfrac{U'(C_T)}{P_T}R_{t,T}\right]$ and multiplying both sides of (3') by $F_{t,T}$, we have:

$$0 = \frac{\beta^{T-t} P_t}{U'(C_t)} E_t \left[\frac{U'(C_T)}{P_T} (S_T - F_{t,T}) R_{t,T} \right] \tag{4}$$

Dividing both sides by $R_{t,T}$, which is known at t, we obtain the equation (1) of the article, that is:

$$0 = \frac{P_t}{U'(C_t)} E_t \left[\beta^{T-t} U'(C_T) \frac{S_T - F_{t,T}}{P_T} \right]$$

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