Minimal identification of dynamic rational expectations systems*

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This paper presents a further view on the identification of rational expectations (RE) models. Its main point is the establishment of necessary and sufficient conditions for identification on the structural form of static and dynamic models, which extends the results obtained till now; no specific assumptions being made on the stochastic processes generating the endogenous and exogenous variables. As a consequence, a clearer view of the cost/benefit of further selections in the solution set is gained. In the RE context, the concept of identification can be enlarged, depending on the past information possessed by the econometrician. The main previous results are discussed under the light of the proposition.

1. Introduction; 2. Definitions; 3. Minimal identification: theorems; 4. Two comparisons; 5. Final remarks.

1. Introduction

Rational expectations (RE) models have brought up new aspects of the identification problem, most of them due to the fact that some of the variables in the model are not observable. This paper introduces the concept of minimal identification for RE models and obtains operational results on the structural form, for the general dynamic case.

As raised by Salemi (1986) and Broze and Szafarz (1991), the model solution and the model identification are different questions that are not necessarily related. The identification of the structural parameters can be dealt with previously, before choosing a specific subset of solutions. This allows for a clearer perception of the compromise faced by the

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econometrician between stronger hypotheses on the solution set and less initial identification conditions, on the one hand, and weaker assumptions on the model processes and solutions together with more *a priori* conditions, on the other hand.

After Wallis (1980)'s seminal contribution, Pesaran (1981)'s is probably the first approach to the problem concentrating on conditions on the structural form — the natural and operational way of solving it. The precise characterisation of the complete set of solutions of dynamic RE models initiated by Gourieroux et alii (1982) and developed by Broze et alii (1985), Broze and Szafarz (1991), has provided the tools for a more general approach to identification. Our developments have some similarity with those in Wegge and Feldman (1983), but, beyond supposing observability of all exogenous — a point further developed in Wegge (1984*a*, *b*) — only the static model is treated there. The idea that, with RE models, identification requires a new definitional framework was originally mentioned in Broze et alii (1987).

A unifying concept for the identification of RE models is presented and discussed in the next section, and the main results are proved in section 3. Section 4 compares them with those in Pesaran (1981) and Wegge and Feldman (1983), while in the final section some concluding remarks are made.

2. Definitions

We shall consider the following dynamic model:

$$Ay_t + B_0 y_t^e + B_1 y_{t+1}^e y_{t+1}^{e} + \ldots + B_k y_{t+k}^e + C x_t + \overset{\approx}{D} \overset{\approx}{x}_t^e = u_t \qquad (2.1)$$

where:

A, B_0, B_1, ..., B_k are nxn matrices and C and $\overset{\approx}{D}$ are, resp., nxm and nxm_2 matrices; column vectors y_t, $nx1$ and x_t, $mx1$ account, resp., for the endogenous and exogenous variables, vector x_t being equal to the piling up of vectors $\tilde{x}_t$, m_1x1 and $\overset{\approx}{x}_t$, m_2x1, $m = m_1 + m_2$, the first being a member of the information set at the beginning of time t, I_{t-1}, defined below; u_t accounts for the stochastic residuals, a multivariate white noise with zero means and contemporaneous dispersion matrix Σ_u. The residuals are an innovation[1] with respect to I_{t-1}; all RE $y_t^e, y_{t+1}^e, \ldots$ are equal to the conditional expectation

[1] See, for instance, Hendry (1989), chap. 2.

on the (same) σ-algebra generated by set $I_{t-1} = \{y_{t-1}, x_{t-1}, y_{t-2}, x_{t-2}, \dots\}$, $\tilde{x}_t$, as said, being measurable in this algebra.

Different normalization conditions may be imposed on matrices A or B_i, $0 \le i \le k$, depending on how the model is treated. Also, matrix C will sometimes be split into its two components $\breve{C}$ and $\tilde{C}$, related, resp., to $\tilde{x}_t$ and $\breve{x}_t$.

Identification can be viewed in different ways, depending on the information possessed by the econometrician, *i.e.*, the σ-algebra available to him. The model definition reflects the agents' information set I_{t-1}, however the econometrician can work with a different information set, up to a maximal one comprising all exogenous till time t (or rather, providing perfect forecasts of all current exogenous).

Calling Ω_{t-1} the information set available to the econometrician and L the set of initial constraints on the structural parameters of (2.1), we have the following definition:

Definition 1

A set of admissible values (*i.e.*, satisfying the constraints in L) A^0, B_i^0 (i=0, ..., k), C^0, $\breve{D}^0$ are Ω-identifiable iff for any other admissible values A, B_i (i=0, ...,k), C, $\breve{D}$, and for every j, $0 \le j \le k$:

$$E\,[y_{t+j} \mid \Omega_{t-1}] = E^0\,[y_{t+j} \mid \Omega_{t-1}] \quad \rightarrow \quad A = A^0,\ B_i = B_i^{\,0}\,(\,i = 0,\dots,k\,)$$
$$C = C^0,\ \breve{D} = \breve{D}^0$$

Definition 1 constrains identification, beyond the given *a priori* conditions L, by an equality defined by a specified projection.[2]. As this equality relates the means of feasible conditional probability laws $y_{t+j} \mid \Omega_{t-1}$, the definition characterizes a first-order identification. Broze and Szafarz (1991) have worked with two special choices for Ω_{t-1}, namely $I_{t-1} + \{\,\breve{x}_t\,\}$ and I_{t-1}, calling them, respectively, weak and strong identification, as the last one is based on a smaller information set and is thus harder to achieve.

In the above framework rests the key idea of our approach, as it implies that, for identifying (2.1), one resorts to the auxiliary equations obtained by taking expectations with respect to the chosen σ-algebra. This provides a kind of reduced form — a pseudo-reduced

[2] The right-hand member of the equality means the conditional expectation when A^0, $B_i^{\,0}$ (i = 0, ..., k), C^0, $\breve{D}^{\,0}$ are substituted for the parameters, while in the left-hand member the other admissible values A, B_i (i = 0, ... , k), C, $\breve{D}$ are considered.

form — and establishes a link with the results available for parametric identification.

The minimal identification case supposes that the parameters of the corresponding (forward) distributed lags model

$$y_{t+k} = \pi_1 y_{t+k-1} + \dots + \pi_{k-1} y_{t+1} +$$

$$+ \pi_k y_t + \pi_{k+1} \tilde{x}_t + \pi_{k+2} \tilde{\tilde{x}}_t + v_{t+k} \tag{2.2}$$

are a set of identifiable reduced form parameters in the sense of Rothenberg (1971).

The motivation for the name minimal comes from the fact that if Ω_{t-1} in definition 1 is the (minimal) σ-algebra generated by the constants, the conditional expectation operator becomes the mean and, if the y_t process is non stationary, from (2.1) one can write:

$$A\, Ey_t + B_0\, Ey_t + B_1\, Ey_{t+1} + \dots + B_k\, Ey_{t+k} + C\, Ex_t + \tilde{\tilde{D}}\, E\tilde{\tilde{x}}_t = 0$$

or, collecting the common terms:

$$(A+B_0)\, Ey_t + B_1\, Ey_{t+1} + \dots + B_k\, Ey_{t+k} + \tilde{C}\, E\tilde{x}_t + (\tilde{\tilde{C}} + \tilde{\tilde{D}})\, E\tilde{\tilde{x}}_t = 0 \tag{2.3}$$

The left hand side of (2.3) provides a limit case for evaluating both members in the equality in definition 1. Moreover, supposing B_k invertible, there is a correspondence between the coefficients in (2.2) and those in (2.3) normalized by B_k:

$$\pi_i = B_k^{-1} B_{k-i}\ , \quad 1 \le i \le k-1, \qquad \pi_k = B_k^{-1}(A+B_0)$$

$$\pi_{k+1} = B_k^{-1}\tilde{C}, \qquad \pi_{k+2} = B_k^{-1}(\tilde{\tilde{C}} + \tilde{\tilde{D}}) \tag{2.4}$$

Thus, resorting to the parameters of the associated stochastic difference equation corresponds, in a certain sense, to use the smallest information set available for helping the (first order) identification. It might then seem that minimal identification is a strongest case, harder to reach than the I_{t-1} case. However, for all σ-algebras Ω_{t-1} up to I_{t-1}, use of the conditional expectation operator will produce a rearrangement of the structural parameters exactly

like the one in (2.3). In this way, the minimal case reflects an identification that can be checked also by the agents when formulating the expectations model (and before investigating its solutions).

Estimation of (2.2) is not the issue here; it will depend on the properly chosen assumptions on v_{t+k}, and a few words will be devoted to it in the final section of the paper.

The econometrician will be fitting the underlying distributed lags model based on the available past information; consequently his "sample" will start at the k+1$^{\text{th}}$ observation. The important point however is that if one works with an information set smaller or equal to that of the agents, the v_{t+k} error term, built up out of the forecasting errors in the original expectations, does not bring in any additional information for identifying the parameters in (2.1). Consequently, (2.2) should be viewed as a tool for solving the (minimal) identification problem — a tool not to be confounded with the solution to (2.1).

Invertibility of matrix B_k is not insured in some practical situations. This, apart from encumbering the algebra, does not put a major problem. Actually, as in Chow (1983), the system can then be broken into separate blocks, the first containing the largest invertible square matrix in B_k that excludes all endogenous whose furthest expectation does not appear in the model. Solution of this part allows substitution of the corresponding y_{t+k} in the remaining equations and a continuation of the process.

Finally, in the special case when $B_i = 0$, $i \geq 1$, (2.3) becomes:

$$(A + B_0)\, Ey_t + \tilde{C}\, E\tilde{x}_t + (\tilde{\tilde{C}} + \tilde{\tilde{D}})\; E\tilde{\tilde{x}}_t = 0$$

and by supposing $(A + B_0)$ invertible, this gives:

$$Ey_t = -\,(A + B_0)^{-1}\,[\,\tilde{C}\, E\tilde{x}_t + (\tilde{\tilde{C}} + \tilde{\tilde{D}})\, E\tilde{\tilde{x}}_t\,]$$

so that "true" reduced form parameters are obtained and easily estimated in the static case (see, for instance, Wallis, 1981).

We now turn to the identification conditions related to the proposed approach.

3. Minimal identification: theorems

The central result is the necessary and sufficient condition stated in theorem 1. Although it is later translated to the case of *a priori* zeros, it is important to remark that these, in the context of RE models, can have fairly restrictive

consequences. Corollary 2 below rules out, from each equation, the simultaneous appearance of any variable and the RE of its current value. Nevertheless, in many models, the error made in the expectation is an explanatory variable, what amounts to a linear restriction on the parameters, but not to their exclusion.

Although the basic concern is with the structural form coefficients, we shall include in the *a priori* restrictions conditions on the (constant) contemporaneous covariance matrix of the noise process. Following Rothenberg (1971), set L is then defined through q initial constraints in the form of continuously differentiable functions:

$$\Phi_i\,(B_k, B_{k-1}, \ldots, B_0, A, \;\tilde{C}, \overset{\approx}{C}, \overset{\approx}{D}, \Sigma_u) = 0,\; 1 \leq i \leq q \tag{3.1}$$

Moreover, if π is the matrix formed by placing sideways the "pseudo-reduced form parameters" in (2.2), inverting the order of π_k with that of π_{k+1}, we have the additional equation:

$$B_k\pi + \mathring{A} = 0 \tag{3.2}$$

where:

$$\mathring{A} = [\, B_{k-1} \ldots B_1\, \tilde{C}\;\; (A + B_0)\;\; (\overset{\approx}{C} + \overset{\approx}{D})\,] \tag{3.3}$$

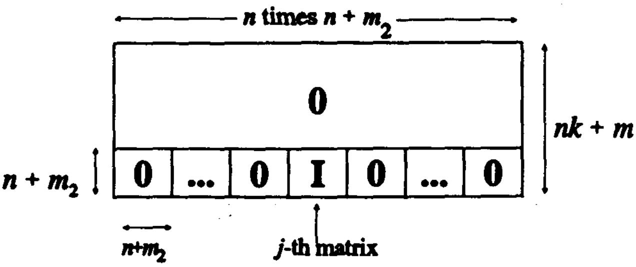
Calling β_k, σ the vectorizations of B_k, Σ_u, and $\alpha = (\alpha^*, (b_0, \overset{\approx}{d}))$ the union of those of $[\, B_{k-1} \ldots B_1\, \tilde{C}\; A\;\; \overset{\approx}{C}\,]$ and $[\, B_0\, \overset{\approx}{D}\,]$, the matrix to be analysed at a regular point $(\beta_k,\; \alpha,\; \sigma)'$ is:

$$\begin{bmatrix} (I_n \otimes \pi)' & I^* & 0 \\ \dfrac{\partial\Phi}{\partial\beta_k} & \dfrac{\partial\,\Phi}{\partial\alpha} & \dfrac{\partial\Phi}{\partial\sigma} \end{bmatrix} \tag{3.4}$$

where: $\Phi = (\,\Phi_1, \Phi_2, \ldots, \Phi_q\,)$ and I^* is a rectangular zero-one matrix that in the lines corresponding to $A+B_0$ and $C+D$ entries has two ones, so that it can be written as:

$$I^* = [\; I_n 2_{k+nm} \qquad M \;] \tag{3.5}$$

matrix M being the piling up of n matrices $(nk+m) \times (n^2+nm_2)$, each having zeroes everywhere, except in the last $n+m_2$ rows where the identity matrix successively appears. The j-th element (starting from the top) of this array being like this:



This allows matrix (3.4) to be decomposed into:

$$\begin{bmatrix} 0 & (I_n 2_{k+nm}) & M & 0 \\ W^* & \dfrac{\partial\Phi}{\partial\alpha^*} & \dfrac{\partial\Phi}{\partial(b_o,\tilde{\tilde{d}})} & \dfrac{\partial\Phi}{\partial\sigma} \end{bmatrix} \begin{bmatrix} (I_n \otimes B_K^{-1})' & 0 & 0 & 0 \\ (I_n \otimes \pi)' & I_n 2_{k+nm} & 0 & 0 \\ 0 & 0 & I_n 2_{+nm2} & 0 \\ 0 & 0 & 0 & I_n 2 \end{bmatrix} \tag{3.6}$$

where:

$$W^* = \frac{\partial\Phi}{\partial\beta_k}(I_n \otimes B_k') + \frac{\partial\Phi}{\partial\alpha^*}(I_n \otimes \mathring{A})$$

Relying on Rothenberg (1971), we can now state:

Theorem 1.

Under regularity conditions, for locally identifying the $n^2(k+2) + nm + nm_2 + n(n+1)/2$ parameters of model (2.1) it is necessary and sufficient that:

$$\text{rank}\begin{bmatrix} W^* = \dfrac{\partial\Phi}{\partial(b_0,\tilde{\tilde{d}})} - \dfrac{\partial\Phi}{\partial(a,\tilde{\tilde{c}})} & \dfrac{\partial\Phi}{\partial\sigma} \end{bmatrix} = 2n^2 + n(n+1)/2 + nm_2 \tag{3.7}$$

where:
a and $\tilde{\tilde{c}}$ account, resp., for the vectorizations of A and , $\tilde{\tilde{C}}$, and the derivatives in the difference are evaluated at corresponding entries in matrices A, B_0 and $\tilde{\tilde{C}}$, $\tilde{\tilde{D}}$.

Proof

As the right term of (3.6) is a full rank matrix, the rank of (3.4) depends on that of the rectangular matrix at the left. This, by rearrangement, can be viewed as a four blocks' matrix with a n^2k+nm identity matrix at the upper left corner. Simple rank properties then imply that its total rank will be n^2k+nm plus that of the matrix formed by its lower right block less the product of the remaining two, *i.e.*:

$$\left[W^* = \frac{\partial \Phi}{\partial (b_0, \tilde{\tilde{d}})} \quad \frac{\partial \Phi}{\partial^{\sigma}} \right] - \left[0 \quad \frac{\partial \Phi}{\partial \alpha^*} \quad M \quad 0 \right] \tag{3.8}$$

so that the theorem follows from the definition of M and Rothenberg (1971)'s results.

The above condition reflects the fact that identification of the error covariance structure, and separation of the current time parameters from the ones in the corresponding expectation is totally dependent on the *a priori* restrictions. This is better seen in the following corollary:

Corollary 1

Under the hypotheses of theorem 1, a necessary condition for the identification of the structural parameters in (2.1) is that the following three equalities hold:

i) rank $(W^*) = n^2$

$$\text{ii) rank } \frac{\partial \Phi}{\partial (b_0, \tilde{\tilde{d}})} - \frac{\partial \Phi}{\partial (a, \tilde{\tilde{c}})} = n(n + m_2) \tag{3.9}$$

$$\text{iii) rank } \frac{\partial \Phi}{\partial \sigma} = n(n+1)/2$$

Proof

Immediate if one takes into account the number of columns in each block forming (3.7).

Equality i is similar to the standard rank condition for identification of the coefficients of the general linear distributed lags model. Condition ii is satisfied under some particular assumptions, like the ones given in corollary 2 below. It requires a set of prior restrictions allowing to separate

A from B_0 and $\tilde{C}$ from $\tilde{D}$. The number of such restrictions is equal to the number of coefficients in matrix $[A \ \tilde{C}]$. Finally, equality iii requires a full though indirect specification of the matrix of contemporaneous covariances of the noise process. This is because the identifiable parameters of the pseudo-reduced form considered in theorem 1, as shown by (3.2), are only combinations of the structural coefficients.

Together these conditions clearly show the added complexity and the limits between identification, estimation and solution in the RE context. Actually, inference in models like (2.1) is possible either by knowing beforehand how to separate the parameters at stake — as shown by *i* to *iii* — or by making special selections or assumptions in the solution set.

As usual, a global identification result can be stated:

Theorem 2

Under the same hypotheses of theorem 1, if the Φ_i are linear functions of the structural parameters, then the system is globally identified.

Proof

The proof follows from an easy adaptation of Rothenberg (1971)'s results, for Σ_u does not appear in (3.2) and the $(A+B_0)$ and $(\tilde{C} + \tilde{D})$ terms still contribute with constant terms to its derivative.

In the case of only *a priori* zeroes, a simpler, classical treatment is possible, identification being almost equivalent to examining, in a first step, that of the distributed lags model on $y_{t+k-1}, y_{t+k-2}, \ldots, y_{t+1}, y_t, x_t$:

Corollary 2

Under the hypotheses of theorem 1, but supposing only *a priori* zeroes, the structural form coefficients of system (2.1) are identified

if

a) for every i, $1 \leq i \leq n$,

$$\operatorname{rank}\,[\,(A + B_0)_i\, B_{1i}\ B_{2i}\ \ldots\ B_{ki}\ \ \tilde{C}_i\ (\tilde{C} + \tilde{D}\,)_i] = n - 1,$$

where the subscript i means the matrix obtained by removing row i from the original matrix, as well as the columns corresponding to non-zero elements in this row; and

b) no endogenous or unpredictable exogenous appears simultaneously with its (first) expectation.

Proof

Identification is obtained via a model like (2.3), assuming, to avoid purely formal results, that matrix $(A + B_0)$ has a diagonal of 1's. The classical (algebraic) theorem for identification on the structural form says that such a model will be identified iff for every line/equation i, $1 \leq i \leq n$:

$$\text{rank}\,[\,B_{ki} \dots B_{1i}\ (A + B_0)_i \quad \tilde{C}_i\ (\approx{C} + \approx{D})_i\,] = n - 1.$$

The proof follows by use of the additional condition:
→ If (2.1) is identified then the coefficients in (2.3) are identified and by the classical theorem we obtain the rank condition a) By applying it in the reverse side we have that, for $1 \leq i, i' \leq n$, and $m_1 + 1 \leq 1 \leq m$, parameters:

$$\beta_{ii'} = A_{ii'} + Bo_{ii'} \text{ and } \delta_{il} = \approx{C}_{il} + \approx{D}_{il}$$

are uniquely defined. But then, for fixed $\beta_{ii'}$, and δ_{il} there will exist several values of $A_{ii'}$, $Bo_{ii'}$ and $\approx{C}_{il}$, $\approx{D}_{il}$ which satisfy the above equations, what contradicts the hypothesis. As the only restrictions are *a priori* zeros, the second statement has to be valid.
← The rank condition implies that the coefficients in (2.3) are identified. This fact plus the exclusion condition entails the identification of the structural form coefficients of (2.1).

4. Two comparisons

4.1 Pesaran (1981)

In the static case, let matrix Φ of homogeneous linear restrictions in his theorem 1 be a juxtaposition of r canonical (column) vectors in R^{2n+m}; Pesaran's result, in our notation, says that:

$$\text{rank}\,[\,A_i\,(B_0)_i\,C_i\,C_i\,] + \text{rank}\ \approx{C} = 2n - 1 \tag{4.1}$$

a condition due basically to the fact that his "pseudo-reduced form parameters" are:

$$-(A + B_0)^{-1}\tilde{C}\ ;\ -A^{-1}\ \approx{C} \text{ and } A^{-1} B_0 (A + B_0)^{-1}\ \approx{C}, \tag{4.2}$$

the last one being a consequence of considering $\tilde{x}_t^e$ e as "observable". This forces the number of non-predictable exogenous (m_2) to be at least equal to

the number of equations, and his first result is a sort of variation of our corollary 2, when rank $\tilde{\tilde{C}} = n$.

In his other formulation of the static case, a (linear) hypothesis on the stochastic process generating the exogenous is added, differing from our approach.

The dynamic formulation treats only the one-step-ahead case, *i.e.*, $k=1$, assuming $B_0 = 0$. But, as he has to deal with a pseudo-reduced form which at the same time is a solution to the model, the problem of *solving* the RE model hinders the study of its identification. For obtaining (at least) one solution, hypotheses on the stochastic processes generating the exogenous variables have again to be added. Pesaran (1981, 1988) also says that models with future expectations are not identifiable, *i.e.*, one can never be sure on which kind of process the observed data is following. Nevertheless, this has to do with the solution to the model. It is an issue for the study of the applications of RE models, but not one for identification purposes.

4.2 Wegge and Feldman (1983)

As only the static case is treated, they still try, as other previous attempts, to find a reduced form by solving the system for the RE. Nevertheless, due to the $\tilde{x}_t$ term, computation of the reduced form parameter related to it needs an extra hypothesis on the stochastic process for x_t, what is not mentioned.

The assumption of observability of the unpredictable exogenous, beyond providing matrix π_1 in their reduced form, adds to it an extra term for the errors dispersion matrix.[3] With this, a value for the rank condition, smaller than that given by our theorem 1 in this case, can be obtained, as they get rid of one n^2 and of the nm_2 term in (3.7), arriving at a value of $2n^2$. If all exogenous variables are predictable, the criterion matrix in formula (9) of their paper coincides with ours in (3.7).

Further developments have been presented by Wegge (1984*a*, 1984*b*), considering a dynamic term in the model. However, his new approach is linked to the strategies for solving the model.

5. Final remarks

Minimal identification can be viewed as being based on the minimal information sequence, namely the σ-algebra generated by the constants. It sets up a clear starting point for determining the identification of RE models, as it appears as the strongest case among all types of Ω-identifica-

[3] Condition (5*b*) in Wegge & Feldman (1983).

tion allowed for in definition 1. Every model identified with respect to the conditions in theorem 1 will so continue when the σ-algebra is enlarged. Moreover, definition 1 shows that ideally the econometrician can be placed at any point of the continuum ranging from the minimal to the weak case to solve the identification problem. Information sets larger than I_{t-1}, or rather, perfect knowledge by the econometrician of some exogenous forecasted by the agents, as in Wegge & Feldman (1983), will provide more information for at least separating $\tilde{C}$ from $\tilde{D}$ in (2.3). The fact that the $\{u_t\}$ process will not be an innovation any more with respect to these sets can also be explored. The name minimal thus is twofold: identification will continue to be valid either under any adjunction of information or under new hypotheses on the data generating process.

A final word should be said on the estimation of model (2.1). The method immediately suggested is the one which takes advantage of the relations in (3.2), *i.e.*, a sort of indirect least-squares (*i.l.s.*) solution. For estimating (2.2), depending on the situation, a transfer function technique or a stochastic difference equations approach (see, for instance, Harvey, 1983) can be used. Assumption of a moving average structure on v_{t+k} will most of the times be mandatory. Of course, simultaneous estimation of the parameters in (2.2), and consequently (2.1), may become technically involved if high structures on v_{t+k} are combined with the existence of lagged endogenous variables among the exogenous. Also, as in the classical *i.l.s.* case, the final structural form estimators may be of cumbersome obtention. Use in the original model of a limited information method — including MacCallum (1976)'s suggestion — can then be competitive.

The above outline of estimation strategies brings back the central point of this paper: by supposing the pseudo-reduced form identifiable, minimal identification leads to a set of workable conditions on the structural form, obtained with a minimum of hypotheses. If, by any chance, a serious estimation problem hinders its application, the role of the needed extra information — in terms of identifiability — can be easily evaluated against the minimal identification background.

Resumo

O artigo apresenta uma nova abordagem para a identificação dos modelos dinâmicos a expectativas racionais, sem recorrer a hipóteses específicas sobre os processos estocásticos seguidos pelas endógenas e exógenas. Condições necessárias e suficientes são estabelecidas para a forma estrutural, estendendo os resultados obtidos até agora. Para isso, o conceito de identificação é claramente separado da questão da multiplicidade de soluções. Em conseqüência, ganha-se uma melhor visão dos custos e benefí-

cios advindos de hipóteses suplementares sobre o conjunto de soluções. Além do mais, no contexto dos modelos a expectativas racionais, a identificação varia conforme o conjunto de informações sobre o passado de que o econometrista possa dispor.

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