A note on regional voting power and budget allocation in the Brazilian Congress*

Ana Dolores Novaes**
David Rosenblatt**

A disputa política regional sobre o orçamento da União é freqüentemente citada como um dos empecilhos à efetivação dos cortes fiscais necessários à consolidação de um efetivo plano de combate à inflação brasileira. Este artigo analisa a representação regional no Congresso Brasileiro e calcula índices de poder (o valor de Shapley e o índice de Banzhaf-Coleman) para as diferentes regiões brasileiras. Os resultados mostram que não é óbvio que a região nordestina é sobre-representada contrariamente à crença popular.

1. Introduction

The motivation for this paper comes from observations often made in the Brazilian press that the Northeast region of Brazil is not only overrepresented in the National Congress relative to other regions but also the beneficiary of many subsidies that are inefficiently and corruptly used. This viewpoint has even reached the international press. In its analysis of the recent stabilization plan announced by the new Brazilian administration, the London based magazine *The Economist* said: "In Congress the fight could be tougher yet. An election is due in October... Businessmen – especially Mr. Collor's original fans, from the subsidy-hooked Northeast – will be lobbying and bribing as usual" *[The Economist,* 24-30 Mar. 1990, p. 44].

Among the measures taken by the new administration is the elimination of fiscal incentives directed to the Northeast region. It was commonly commented in the Brazilian Press that the Northeast representation in Congress would not approve the plan as it was. It is said that the states in the region vote in block, regardless of ideology, when voting on the fiscal incentives directed to the region. The question we want to answer in this paper is the following: Is it true that the Northeast or any other region is overrepresented in Congress, and consequently able to reap economic advantage over other regions?

* We would like to thank Alessandra Casella and an anonymous referee from this journal for valuable comments. The remaining errors are only ours.

** Professor of economics at the Federal University of Pernambuco and Ph.D. candidate at the University of California, Berkeley, respectively.

In order to answer this question we will borrow concepts from cooperative game theory that have already been applied by political scientists to analyze regional voting power. We believe that in our particular case the use of cooperative game concepts fits better our purposes than the noncooperative approach currently popular in economics. It will be assumed that regional political coalitions do form and act in block. We calculate two indices of power for the Brazilian regional representation in the National Assembly: the Shapley Value and the Banzhaf-Coleman Index of Power.

The criterion for deciding whether a region is over or underrepresented is the principle of one person (one citizen), one vote. If every legislative issue were decided by direct popular vote, this principle would be maintained. In modern representative democracies, an attempt is made to preserve this principle by having one of two legislative bodies with the number of representatives proportional to state populations. With the indexes of power used in this paper, the principle of one person, one vote is best preserved by forming legislatures where a region's index of power is equal to its proportion of total population. This point is more carefully explained later in the paper.

The results are sensitive to whether the state of Minas Gerais is considered part of the Northeast or not.[1] Under the official political regional division, the calculations show that the Norteast region is actually underrepresented in the Assembly (*Câmara dos Deputados*) in comparison with the South and Southeast regions. Only when Minas Gerais is separated from the Southeast region the Northeast becomes slightly overrepresented while the states from the Southeast (excluding Minas Gerais) are underrepresented. The main difference from the former case is the gain in power by the North region. If there were issues voted on simultaneously by the Assembly and the Senate, our results show that the Northeast would be overrepresented. An analysis of the situation where the Assembly votes first and the Senate votes second is beyond the scope of this paper.

The paper is organized as follows. Section 2 presents a brief overview and some data to illustrate the regional problem in Brazil. The cooperative game concepts we use are explained in section 3. Section 4 shows the results and section 5 concludes.

2. The regional question: income, tax revenues and expenditures

The regional economic conflict in Brazil has intensified by the creation of four regional development agencies. With the exception of the more wealthy Southeast, each region has its own agency in charge of its economic development and planning.[2] Among these agencies, the largest one, Sude-

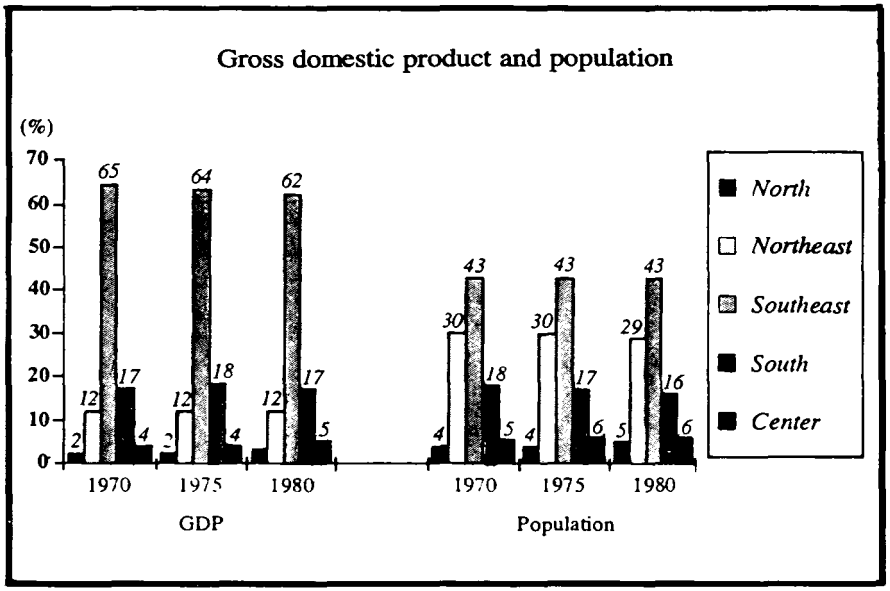
[1] The north of the state of Minas Gerais is part of the Northeast for matters related to fiscal incentives.

[2] The agencies are Sudam (for the North), Sudeco (for the Center), Sudesul (for the South), and Sudene (for the Northeast). The Southeast does have with an agency, but as the other regions, it has especial programs like Programa Especial do Norte Fluminense (Prodenor), Programa de Desenvolvimento dos Cerrados (Polocentro). Programa de Desenvolvimento do Vale do Jequitinhonha (Prodevale) e Programa de Desenvolvimento Microrregional do Norte do Espírito Santo (Prodesp).

ne, is from the Northeast. The Northeast region, predominantly agricultural and plagued by periodic droughts, is the most impoverished region of Brazil. Very often, Sudene has been labelled inefficient and corrupt. In addition, the agency and its fiscal incentive schemes, along with other regional incentives, are accused of creating vested interests that make it extremely difficult to cut public expenditures and to implement stabilization plans.

Unfortunately, the national accounts-type of data is available only at five-year intervals and the figure for 1985 has not yet been released. Figure 1 shows the regional profile of gross domestic product and the population for each region.

Figure 1



Source: Estatísticas Históricas do Brasil.

Despite having about 30% of the population, the Northeast generates only 12% of the national income and its per capita income in 1980 was only 40% of the national average.

With regard to tax revenues, the Southeast generates the most – more than three-quarters of the total – followed by the South region's 12% share (table 1). The federal government expends more in the North and Northeast than it collects in those regions and in this sense the claim that the Northeast takes a large bite is true.

It is interesting to compare the tax expenditure figures from table 1 with those from the Participation Fund of States and Cities. In Brazil, the income tax and the tax on industrialized goods are collected by the Federal Government. By law, the Federal Government transfers a certain percentage of the revenues to states and districts according to criteria invol-

ving income and population.[3] Notice the distortion that may arise when someone looks only at the participation fund and claims that one region is clearly the beneficiary over others. There is no doubt that the North and Northeast regions receive a relatively larger transfer of the tax revenues. However this difference does not give the whole picture: the disparity is reduced when expenditures of the Federal Government in each state is also taken into consideration (columns 3,4 and 5 from table 1).

It is not our purpose here to discuss at length the regional question in Brazil. It should be mentioned that it is very difficult to evaluate the regional consequences of many other programs and subsidies that the Federal Government provides, *e.g.* favorable interest rates for long term investment (BNDES system), subsidies to the export sector (Befiex) etc. These incentives are not tied to a particular region.

Table 1
Tax revenue and expenditure per region

Region	Federal tax revenue (%)		Federal expenditure (%)			Participation fund (%)
	1986	1987	1985	1986	1987	1987
North	2,13	2,26	4,57	4,56	6,02	12,7
Northeast	6,93	8,09	10,07	10,17	15,39	43,8
Southeast	78,93	76,36	47,85	53,45	68,65	23,6
South	10,84	11,93	5,18	4,03	6,91	12,4
Center[1]	1,17	1,37	1,80	1,77	3,04	7,4
Unidentified[2]	0,00	0,00	30,53	26,02	0,00	-
Brazil	100,00	100,00	100,00	100,00	100,00	100,00

Sourse: Anuário Estatístico do Brasil – 1987, pp. 577-578 and Boletim Mensal do Banco Central do Brasil, July 1989, pp. 134-135.

[1] Excluding Brasilia.

[2] Until 1987 some expenditures that were directed for more than one state were not disentangled to allow identification of final destination.

3. The Shapley value and the Banzhaf-Coleman index

In the vast literature on the theory of games, one branch of research has been based on cooperative games in characteristic function form. In these games, enforceable agreements enable players to cooperate and achieve payoffs as a group which are greater than the sum that could be achieved by individuals acting on their own. Games in characteristic form have been used to model both economic (Shubik, 1984) and political issues (Ordeshook, 1986).

[3] Until 1988, the Union used to transfer 14% and 17% of its revenue (net of legal deductions for fiscal incentives and special programs) to state and cities, respectively. After the approval of the new constitution in 1988, this percentage will rise to 21,5% for the states and 22,5% for the cities.

3.1. A simple game

Assuming trans erable utility, the characteristic function (ϑ) is simply a real valued function which assigns the maximin payoff to any player, (i), or coalition of players,[4,5] For a "simple game" where there are only winners and losers, and no single player can win by playing alone, we have:

$$\vartheta(i) = 0, \text{ for } i = 1,\ldots,n$$
$$\vartheta(S) = 0 \text{ if } S \text{ is losing.}$$
$$= 1 \text{ if } S \text{ is winning, and}$$
$$\vartheta(N) = 1, \text{ where } N \text{ is the grand coalition of all players.}$$

An obvious application for the above game is a weighted majority voting system, where a coalition is winning if the sum of the players votes in that coalition is greater than or equal to the majority needed for victory (and the utility from victory is normalized to one). For this paper, then, one can define each player i as one of the regional voting blocks in the Brazilian Assembly (assuming that these voting blocks vote as a unit). The voting weight associated with each is the sum of the deputies (*deputados*) from the states in that region. No single region has a majority in Congress, so no single region can win a vote on its own; however, certain coalitions of regions do form a majority and can pass legislation, receiving one unit of utility for that coalition.

3.2 The Shapley value

There exist a number of different approaches to finding a solution, or stable payoff allocation, for games in characteristic function form. Shapley (1953) derived a unique "value" for such games from a set of only three axioms. The first axiom uses the idea of a permutation, π, of the players of a game. If we can reshuffle players, and the roles they play in coalitions, then we can reshuffle the function ϑ to get us back to the original game. In other words, for any π and S in the set of players, we can define $\pi\vartheta$ as: $\pi\vartheta(\pi S) = \vartheta(S)$. Given this definition, the Shapley "value", $\varphi_i(\vartheta)$, can be derived from the following three axioms:

(1) For any permutation π,

$$\varphi_{\pi(i)}[\pi\vartheta] = \varphi_i(\vartheta) \text{ for each player i}$$

[If we reorder the Shapley values in the same way in which we reorder (reshuffle) the players and their payoffs, then we get back to the original Shapley values.]

[4] If there exist a commodity (money) which is linearly separable in the agent's utility functions, then outcomes of the game measured in terms of that commodity are identical to the utilities derived from those outcomes. This assumption allows us to discuss the division of outcome (payoff) for any coalition without referring to the individual player's utility functions. We can then define the characteristic function as real-valued, rather than as a set of utility vectors for the players of the coalition.

[5] This maxmin value can be thought of as the most a coalition S, acting on its own, can be assured of receiving from the game. In other words, assume the players not in S follow the strategies which harm S the most (minimizes S's payoff). The maxmin value is the best S can do under such circumstances.

(2) For any carrier[6] S of S,

$$\sum_{s} \varphi_i[\vartheta] = \vartheta(S)$$

[Dummy players have a Shapley value of zero, and the sum of Shapley values for the non-dummy players in any coalition is equal to the payoff to that coalition.]

(3) For any two independent games μ and ϑ,

$$\varphi_i[\ \mu + \vartheta\ ] = \varphi_i(\mu) + \varphi_i(\vartheta)$$

[In two independent games, the payoff to any player in the joint game is the sum of the payoffs from the two individual games. The same additive property holds for the Shapley value of each player.]

From these three axioms, it is possible to derive the unique Shapley value for each player i in the game ϑ:[7]

$$\varphi_i[\vartheta] \sum_{T \subset N\ i \in T} \frac{(t-1)!\ (n-t)!}{n!} \quad [\ \vartheta\ (T) - \vartheta\ (T - \{i\})\ ]$$

which for the simple game defined above can be simplified to the formula:

$$\varphi_i[\ \vartheta\ ] = \sum_{T \subset N\ i \in T} \frac{(t-1)!\ (n-t)!}{n!}$$

where the summation is taken over all winning coalitions T such that $T - \{i\}$ is not winning; t is the number of players in coalition T and n is the total number of players in the game.

3.3 Interpreting the Shapley value

Suppose coalitions form as players arrive on by one at different times, and suppose that as player arrives he is given the marginal amount he adds to the coalition already formed, *i.e.* $[\vartheta(T) - \vartheta(T - \{i\})]$ with $T - \{i\}$ already formed. There are $n!$ possible orders of arrival. There are $(t\text{-}1)!\ (n\text{-}t)!$ different combinations in which the coalition $T - \{i\}$ might form, leaving out the remaining players in N-T. So, $\frac{(t-1)!\ (n-t)!}{n!}$ is the probability of finding the coalition $T - \{i\}$ already formed when i arrives to play the game. One can interpret the Shapley value $\varphi_i[\mu]$ then, as the expected payoff of player i under this randomization scenario.

In a simple game, the marginal contribution of any player to any coalition is either 0 or 1. The marginal contribution is one whenever that pla-

[6] A carrier of a game is a coalition whose members are all essential to determining the payoff $[\vartheta(S)]$ of some coalition S. More formally, a coalition T is a carrier of a game U, if for *any* S $\vartheta\ (S) = \vartheta\ (S \cap T)$. Any players that do not belong to any carrier are called dummies. *i.e* they contribute nothing to any coalition.

[7] See Shapley (1953) or Owen (1982) for the precise derivation of this formula.

yer is a "swing" player, *i.e.* the addition of that player to the coalition converts a losing coalition into a winning coalition. On average, a player with a bigger voting weight will be a swing player more often than a player with a small voting weight. the Shapley value captures the fact that the large players (regional voting blocks in our particular case) have a power at the margin that is greater than the power represented by the fraction of total votes belonging to that player.

For example, imagine a country with five states which is trying to establish a representative voting body, *i.e.* a congress, with ten total votes. State. A has 60% of the population, while states B,C,D, and E each have 10% of the population. As a result, state A is given 6 votes in congress, while B,C,D and E are each given 1 vote. Assume that on regional issues, representatives vote in block on a state by state basis. As a result, we can think of one representative from state A, call him Deputy A, with 6 votes as being equivalent to 6 representatives with one vote each.

If a regional issue were to be voted upon by direct popular vote, then each Deputy (*deputado*) from state A, as a voting citizen, would have the same voting power as Deputy B from state B, and every other deputy. As representatives in the Assembly, however, Deputy A with his 6 votes becomes a dictator in deciding any regional issues before the Assembly, Deputy B and the other deputies. Would have absolutely no power.

It might be more "fair" then to give state A 4 votes (and 1.5 vote each to States B through E). Deputy A would still have considerable power (as he should since he comes from the most populated state). The only way the other states could pass legislation that Deputy A disapproved of would be is all the other 4 states formed a united coalition, but at least Deputy A would no longer be dictator.

The rationale behind the Shapley value is that State A should receive enough votes so that it belongs to 60% of the possible majority coalitions. Similarly, States B through E should be given enough votes so that they belong to 10% of all possible majority coalitions. In this way, for issues voted on only by the Assembly, one might be able to better preserve the principle of one person, one vote.

3.4 The Banzhaf-Coleman index power

The Banzhaf-Coleman index is a different measure of power which is built upon the concept of the swing player. More formally, a swing for player i can be defined as a set $S \subset N$, such that $i \in S$ than S wins and $S - \{i\}$ loses.

Let Θ_i be the number of swings for player i, the Banzhaf-Coleman index is defined as:

$$\beta_i[\vartheta] = \frac{O_i}{\sum_{j=1}^{n} \Theta_j}$$

Both Shapley value and Banzhaf-Coleman indices have been applied to voting schemes in the United States and Canada. Owen (1975) calculated both indices for the power of individual states in the Electoral College process for selecting the President of the United States. Miller (1973) and Straffin (1978) calculated Shapley value and Banzhaf-Coleman indices for regional voting power in a proposed scheme to change the voting procedure for constitutional amendments in Canada.

4. Results

We calculated the Shapley value and the Banzhaf-Coleman (BC hereafter) indices for the geographical regions of Brazil. We considered two different regional divisions. The first one matches the political division; the second one takes into consideration that the state of Minas Gerais, which belongs to the Southeast, is actually under Sudene's jurisdition with respect to fiscal incentives. In other words, Minas Gerais may be motivated to vote in block with the Northeast.

Table 2 shows the results for the traditional geographic division. According to the results, comparing with the population and the actual votes in the House of Representatives, the Northeast is underrepresented relativel to the Sountheast and South.[8]. The North region is also overrepresented. This is a result of the electoral rules in Brazil. According to the Constitutional Law, the number of state representatives is proportional to the population, but each unit must have at least eight federal deputies and no more than eighty deputies. Since the states in the North are scarcely populated, they ended up overrepresented with the minimum quota of eight.[9]

Table 2
Indices of power

Region	Population (%)	Votes in assembly[1] (%)	Shapley value	Banzhaf-Coleman index
North	4,92	9,96	6,66	7,69
Northeast	29,27	31,50	23,33	23,07
Southeast	43,46	35,36	40,00	38,46
South	15,98	16,06	23,33	23,07
Center	6,33	7,11	6,66	7,69
Total	100,00	100,00	100,00	100,00

[1] Câmara dos Deputados Federais.

Table 3 presents the indices of power taking into account that Minas Gerais may vote in block with the Northeast in regional matters. Under this

[8] Recall that based on the principle of one person, one vote, congressional representation should be such that each region's φ_i is equal to its share of population. Otherwise the region is either over or underrepresented.

[9] See article 45 of the Brazilian Constitutional Law.

alternative arrangement the Northeast becomes overrepresented while the Southeast exluding Minas Gerais becomes underrepresented. This is not a surprising result given the magnitude of the representation of Minas. Notice the shift of importance between the North and Center regions. The North region here has indices of power twice as large as its actual population.

Table 3
Indices of power - excluding Minas Gerais from the Southeast

Region	Population (%)	Votes in assembly[2] (%)	Shapley value	Banzhaf-Coleman index
North	4,92	9,96	10,00	10,71
Northeast	29,27	31,50	33,33	32,14
Southeast[1]	32,23	24,39	26,67	25,00
Minas Gerais	11,23	10,97	10,00	10,71
South	15,98	16,06	16,67	17,86
Center	6,33	7,11	3,33	3,57
Total	100,00	100,00	100,00	100,00

[1] Excludes Minas Gerais.

[2] Câmara dos Deputados Federais.

According to the Constitutional Law, the discussion and voting of law projects that are initiative of the Presidency or Supreme Court are initiated by the House of Deputies. After approval by that house, the project is sent to the Senate for voting. We also calculated the Shapley value for the 5-region version of the composite game of winning in the House of Deputies *and* simultaneously winning in the Senate.[10,11]. The results are the following:

North 8,33
Northeast..... 41,67
Southeast..... 25,00
South 16,67
Center..... 8,33

Because the Northeast is formed by 9 of the 23 states – 39% of the Senate House – its Shapley value is substantially higher than the other regions. Recall that since the majority of projects have to first be approved by the assembly, the assembly has an additional power that is not represented in the simple game analysed above.[12]

[10] Each state, except the territories, has three senators.

[11] A winning coalition is then defined as any coalition that has enough votes to secure a majority in *both* the House of Deputies and the Senate. A coalition which has a majority in one and not the other is defined as a losing coalition.

[12] See articles 60 to 69 of the Brazilian Constitution.

5. Conclusion

Based on the assumption that regions vote in block and the assumption that Representative Assemblies should attempt to preserve the principle of one vote per citizen, we can draw the following conclusions from our analysis.

1. If the assembly (*Câmara dos Deputados*) were the only legislative body, the Northeast would be underrepresented, the Southeast is slightly underrepresented while the other regions are all overrepresented.[13]
2. If the Asssembly and the Senate were to vote simultaneously on a given issue, then the North and the Northeast are overrepresented, the South and the Center are slightly overrepresented and the Southeast is strongly underrepresented.

A more complicated game is needed to model the process whereby the Assembly votes *first* and if approved, the proposed law is sent on to the Senate for approval. There is an implicit power to the House that votes first (the Assembly) given that the other House (the Senate) takes on a revisory role in the legislation process. The simple modelling process used in this paper does not capture this subtlety.

3. The results above are, of course, sensitive to the inclusion/ exclusion of Minas Gerais in the Southeast region or many other redefinitions of regions. Given the nature of Brazil's political institutions, the regional definitions used in this paper seem reasonable.

We should mention some caveats of this analysis. We are not incorporating strategic voting associated with agenda setting, nor vote trading, nor the political expertise of individual players. The methodology does not explicitly consider the difficult of changing the level of real expenditures due to the past creation of vested interests. Future research will expand the game to the state level, in addition to exploring these caveats.

[13] This result is not surprising to the extent that the main reason for establishing two houses of Congress in the United States was to balance power between larger population States and smaller population states. The compromise reached was to create a senate, where all states have canal representation (favored by small states), and a House of Representatives, with proportional representativ (favored by the larger states).

Appendix

Table 1
State representation in National Congress and population

State	Population (%) 1980	Number of representatives in Assembly[1]	Participation (%)
North	*4,92*	*49*	*9,96*
Rondônia	0,41	8	1,62
Acre	0,25	8	1,62
Amazonas	1,20	8	1,62
Roraima	0,07	4	0,81
Pará	2,86	17	3,45
Amapá	0,15	4	0,81
Northeast	*29,27*	*155*	*31,50*
Maranhão	3,36	19	3,86
Piauí	1,80	10	2,03
Ceará	4,44	22	4,47
Rio Grande do Norte	1,60	8	1,62
Paraíba	2,33	12	2,44
Pernambuco	5,16	26	5,28
Alagoas	1,67	9	1,83
Sergipe	0,96	8	1,62
Bahia	7,94	41	8,33
Southeast	*43,47*	*174*	*35,36*
Minas Gerais	11,24	54	10,97
Espírito Santo	1,70	10	2,03
Rio de Janeiro	9,49	46	9,35
São Paulo	21,04	64	13,01
South	*15,98*	*79*	*16,05*
Paraná	6,41	31	6,30
Santa Catarina	3,05	17	3,45
Rio Grande do Sul	6,53	31	6,30
Center	*6,33*	*35*	*7,11*
Mato Grosso do Sul	1,15	18	3,65
Mato Grosso	0,96	9	1,83
Goiás	3,24	8	1,62
Brasília-DF	0,99	0	0,00
Brazil	*100,00*	*492*	*100,00*

Source: Quem é quem na Constituinte. Estatísticas históricas.

[1] Number of federal deputies.

Abstract

Regional political disputes over the federal budget have frequently been cited as one of the reasons for the failure to enact fiscal reforms in the Brazilian stabilization plans. This paper looks at the regional representation in the Brazilian Congress and calculates power indices (the Shapley value and the Banzhaf-Coleman index) for the regional voting blocks. The results show that it is not clear that the Northeast is overrepresented, contrary to popular belief.

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ERRATA

Republicação de equações devidamente corrigidas do artigo Projetos de irrigação: uma nota metodológica sobre a determinação de sua eficiência relativa e de sua rentabilidade, de Juan Hersztajn Moldau, publicado na *RBE*, *45* (1): 127-43, jan./mar. 1991:

p. 134, eq. (3)*:*

$W^A.t = f\,(K.t, H_r.t, L_{i,j,k,l}.t, W'.t; E, W; t)$

p. 135, eq. (4)*:*

$W^A_m.t = F_m\,(K.t, H_r.t, L_{i,j,k,l}.t, W'.t; E, W; t)$

p. 137, eq. (6)*:*

Custo/método m/ u. de área/ u. de tempo = $C_m\,(W^A_m, P)$

p. 137, eq. (7)*:*

Custo/método m/ u. de área/ u. de tempo = $C_m(A, W, i, j, k, l, P)$

p. 139, eq. (8)*:*

$C_{m,n}$/u. de área/ u. de tempo = $C_{m,n}\,(A, W, i, j, k, l, P)$