

Country risk index for emerging economies: A dynamical proposal with a case study*

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Abstract

We introduce a dynamical country risk index for emerging economies. The proposal is based on the intensity approach of credit risk, i.e. the default is the first jump of a point process with stochastic intensity. Two different models are used to estimate the yield spread. The dynamics of the interest rates is modeled through a multidimensional affine model, and the Kalman filter with an Expectation-Maximization algorithm is used to calibrate it. The USD interest rates constitute part of the input of the model, while prices of relevant domestic bonds in the emerging market complete the input. For an application, we select the Uruguayan bond market as the emerging economy.

Keywords: country risk index, emerging markets, sovereign debt, Kalman filter

JEL Codes: H63, C58, G15

1 Introduction

In the last decades the sovereign credit risk analysis has gained importance due to the high globalization of the financial markets. The sovereign credit risk is the risk associated with those factors that determine or affect a country's ability and willingness to pay scheduled interests and the repayment of its sovereign debt. There is a large number of macroeconomic variables that measure the credit risk of a given economy. In our work we focus on the *country risk index*. Generally speaking, the country risk index is related to the sovereign yield spread that measures the risk premium required by investors to hold securities issued by country borrowers (affected by credit risk, market risk and liquidity risk). We define the country risk index at any point in time as the average of the yield

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spread of sovereign bond portfolio of the issuing country with respect to yield rates from instruments issued by a country that (in theory) does not present credit risk. This index, related to the implied default probability, is therefore crucial for credit portfolio risk management and for pricing credit derivatives. In order of importance it can be established that the index is related to the sovereign credit rating, and of course the credit rating of national banks and local companies with access to the international capital market. Furthermore, it affects foreign investment in the country and may lead to big movements of capital.

The country risk index is very useful in emerging economies. Emerging markets are those developing countries with high potential growth. In recent years, these economies greatly increased their participation in the global market. In detail, during the eighties, developed countries accounted for two thirds of global production; however, actually the emerging countries account for more than 50% of the global economy. In the last decade, low interest rates in developed countries, led to a strong sovereign yield spread in bonds. As a consequence, their bond markets grown substantially, and the investment universe became broader. This situation led to investment portfolios with a significant portion in fixed income investments from emerging markets. However, these economies are characterized by irregular economic cycles and high currency volatility. The greatest risks of emerging countries are related to the financial market, exchange rate and inflation. During their economic crisis, different macroeconomic variables are affected, such as the GDP, balance of payments, and prices of sovereign instruments (the focus of the present work), which causes a financial and economical downturn and abrupt changes. Due to these risks, the sovereign assets present credit risk, which leads to greater risk premium required by investors.

The purpose of this work is then to provide a methodology for country risk index measurement in emerging markets. Our proposal uses extensive financial information related to the sovereign debt of the country, and is based on the intensity approach. In order to produce a reliable assessment we include in our model a large amount of information related to sovereign bonds, its capitalization, and place and type of issuance.

The rest of the paper is organized as follows. [Section 2](#) discusses the model in the context of the literature and the methodologies used in the market by the credit rating agencies. In [section 3](#) we establish the basic ideas of the intensity approach to value the credit risk. We introduce the econometric

model based on stochastic differential equations governing the different states that constitute the free-risk default rate and the intensity process of default. Particular attention is paid to two situations: in the first one the rate and the intensity are modeled as independent stochastic processes, in the second, a correlation factor is introduced. In [section 4](#), we present an empirical application to the Uruguayan bond market. This study includes a selection of the most relevant bonds according to capitalization and issuance place and the parameter estimation of the model. As a conclusion, a *dynamical country risk index* is introduced and estimated. In [section 5](#) we summarize the results of the work and discuss some possible extensions. In the appendix, we briefly present the development of the model and an explanation of the statistical method used to calibrate the model is also included.

2 Credit risk model

The credit risk modelation is a well developed area of research. The different methodologies for its estimation applied by financial agencies can broadly be classified as qualitative or quantitative. However, several models combine the two methodologies to produce a single value. When considering methods of credit risk estimation, different kinds of information is used, generally based on a set of relevant macroeconomic variables and financial information on the country's sovereign debt. Basically, there are two approaches to model the credit risk: the structural approach and the intensity approach. The structural approach, initially proposed in [Merton \(1974\)](#), uses option pricing to study the relationship between the default risk and the asset structure of the country. A risky bond is modeled as a contingent claim over some measure related to the economic or financial conditions of the country that triggers the default event. The intensity approach, initially proposed in [Lando \(1998\)](#), takes the default as an unpredictable event governed by a stochastic process. In our work, the sovereign yield spread is not explicitly related to the finance state or economic conditions of the country.

Empirical research has found relationships between sovereign credit risk and various macroeconomic variables. The determinants of this risk were analyzed in [Cantor and Packer \(1996\)](#) using macroeconomic variables such as growth of GDP, inflation, fiscal balance, external debt balance, external debt, and economic growth level and default history. In [Bissoondoyal-Bheenick \(2005\)](#), the study in this field was extended with a more number of countries involved. In [Chee, Fan Fah, and Nassir \(2015\)](#), the authors examine nine macroeconomic

variables with the same objective. In general, the literature related to empirical sovereign credit risk models is reviewed in [Hoty and McAleer \(2004\)](#). In [Sachs \(1985\)](#), the author investigates the role of various macroeconomic policies and fundamentals for the debt crisis and provides the empirical rationale for using certain economic fundamentals in the determination of the sovereign yield spread in international capital markets at countries of Latin America and East Asia. Another relationship well analyzed in the literature is between sovereign yield spread and credit ratings. Among them stand out [Gande and Parsley \(2005\)](#) and [Sy \(2002\)](#). In this area, the work by [Hartelius, Kashiwase, and Kodres \(2008\)](#) investigates the relationship between the investment grade status and the sovereign yield spread. This work captures the benefits of investment grade status by allowing for differences in the impact of each rating grade on yield spread. Particularly in the case of emerging economies, [Jaramillo and Tejada \(2011\)](#) studies the impact in the sovereign yield spread by increasing the rating.

In recent decades, credit rating agencies (CRAs) became a source of information in the financial market. These agencies play an important role in the decisions of international financial market agents, by regularly publishing reports in which they identify the different aspects of the sovereign credit rating. The main role of these financial agencies is to measure and quantify the credit quality debts via Sovereign Credit Rating. The agencies assess many factors ranging from solvency factors to socio-political factors that affect the ability to pay off debt. However, the actual degree of importance of the different variables and their change over time is not known in the rating. Information regarding the criteria used by the three most important credit rating agencies can be found at [Fitch \(2014\)](#), [Moody's \(2013a, 2013b\)](#), [Standard and Poor's \(2004, 2017\)](#). Besides, some credit rating agencies produce their own quantitative estimates via country risk index. Country risk index is the surcharge that a country pays for its bonds in relation to the rate paid by the treasury of the United states. In other words, it is the difference between the performance of a public security issued by the national government and a similar title issued by the Treasury of the United States. The estimation of the index includes US dollar denominated Brady bonds, Eurobonds, and traded loans issued by the country. Instruments to be used meet conditions in the outstanding amount, maturity and type of rate and amortization. The yield spread over Treasury is calculated as the difference between the yield to maturity of each bond and the yield to maturity of the corresponding point on the US Treasury spot curve. The index most widely used in the financial industry is called J.P. Morgan's Emerging Market Bond

Index (EMBI), and its extensions EMBI+ and EMBI– Global.

Our proposal index is constructed as a weighted average of bond returns, as in the EMBI case. The difference is the utilization of historical prices of sovereign bonds. The advantages of this approach are: it gives consistent results across time; in non-liquid markets it avoids artificial volatility produced by a single bond transaction; it gives the possibility of simulation forward in time to obtain projections of the index with corresponding confidence intervals. In the empirical financial literature, the intensity approach has been applied to value defaultable assets but has been not used to propose a country risk index.

3 Econometric Model

In the work, we proposed the intensity approach to quantify the default risk of the sovereign debt in an emerging country. In this approach, the event of default is typically produced exogenously. In other words, it is not modeled in terms of the assets and liabilities of the economy as is the case in the structural approach. The intensity approach is used to analyze a variety of contracts in which the credit risk is part of the contractual setup. In particular for defaultable bond prices—the case that concerns us—this approach leads to tractable valuation formulas, similar to those arising in ordinary default-free term structure modeling. One of the first proposals to use point processes to model default events was made by Lando (1998). Afterwards, Duffie and Singleton (1999) apply this approach in different situations, and Duffie, Filipović, and Schachermayer (2003) extend the theory to general affine process with jumps. In the case of emerging economies, Duffie, Pedersen, and Singleton (2003) applied an affine model to analyze the Russian debt market. Regarding Latin American markets, in the paper by Ochoa (2006) default probabilities from Chilean bonds are estimated, and Meres and Almeida (2008) analyzes the Brazilian global bonds market through intensity models.

3.1 The intensity approach

We consider an underlying probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{Q})$, endowed with a filtration $\mathbb{F} = \{F_t: t \geq 0\}$. This filtration is completed with the null sets and assumed to be right continuous. The probability measure \mathbf{Q} is a martingale measure for the market model. The probability space is rich enough to support a continuous d -dimensional stochastic process $\{y_t: t \geq 0\}$. The evolution of relevant economic factors, such as macroeconomic variables and financial information of the issuing country, is contained in this state variable y_t .

An essential role is played by the default-free instantaneous interest rate process $\{r_t: t \geq 0\}$, that depends on the process y_t , in such a way that the savings account $\{B_t: t \geq 0\}$, is given by the usual expression

$$B_t = \exp\left\{\int_0^t r_u du\right\}, \quad \forall t \in \mathbb{R}_+,$$

and is well defined. For more information see Bjork (2009), Brigo and Mercurio (2006) and Musiela and Rutkowski (2005).

In order to define the default time, we assume that there exists an exponential random variable ϵ with parameter one, which is independent of the y_t process. In addition, we assume that exist a process $\lambda: \mathbb{R}^d \rightarrow \mathbb{R}$, called the intensity process, that depends on the process y_t , is non-negative and continuous. Therefore, it is possible define the default time τ as

$$\tau = \inf\left\{t \in \mathbb{R}: \int_0^t \lambda(y_s) ds \geq \epsilon\right\}.$$

The default time is then the first jump of a Cox Process with intensity process λ . Cox processes are a generalization of non-homogeneous Poisson process, having stochastic intensity process. This process is also called a conditional Poisson process or a doubly-stochastic Poisson process.

In the framework of an arbitrage-free financial market model, a general defaultable zero-coupon bond pricing formula with zero recovery is then given

$$P^*(t, T) = \mathbb{1}_{\{\tau > t\}} \mathbf{E}_{\mathbf{Q}}\left(e^{-\int_t^T (r_v + \lambda_v) dv} \mid \mathcal{F}_t\right) = \mathbb{1}_{\{\tau > t\}} \mathbf{E}_{\mathbf{Q}}\left(e^{-\int_t^T \hat{r}_v dv} \mid \mathcal{F}_t\right),$$

where the introduced process $\{\hat{r}_t: t \geq 0\}$, is interpreted as the default risk adjusted instantaneous interest rate process $\hat{r}_t = r_t + \lambda_t$. This new discount interest rate takes the same form as the discount of default-free instantaneous interest rate in risk-neutral term structure models. For more information see Bielecki and Rutkowski (2004).

In conclusion, the approach is based on the existence of two stochastic processes, one for the dynamics of the default-free instantaneous interest rate and the other for the dynamics of the intensity process that is interpreted as the yield spread that is generated when analyzing the market that presents default risk. Besides, it is important to note that the default time does not appear explicitly in the pricing formula.

3.2 The model

The multivariate term structure model used in the present paper specifies that the default risk adjusted instantaneous interest rate process \hat{r}_t is a deterministic function of an N -dimensional state vector that satisfies a stochastic differential equation (SDE) driven by a multidimensional Brownian motion. More precisely, we have selected the Gaussian Affine Term Structure model (GATS) to model the state vector. This selection is based on using a parsimonious structure model where there is a compromise between generality and tractability: due to the form the logarithm bond price formula is a linear function of the state variables. Due to this characteristic, these models have received much attention in the theoretical finance literature.

The risk neutral dynamics of the state vector is then

$$dy_t = A(B - y_t)dt + \Sigma dW_t, \quad (1)$$

where B is a $N \times 1$ vector, A and Σ are $N \times N$ matrices, and $\{W_t: t \geq 0\}$ is an N -dimensional Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbf{Q})$. In our work, we denote $B = (b_i)$ and the matrices, by parsimony, are chosen to be diagonal, therefore denoted by $A = \text{diag}(a_i)$ and $\Sigma = \text{diag}(\sigma_i)$ (i.e. a non-correlated volatility matrix). For more information in the admissibility restrictions in general affine processes, see [Dai and Singleton \(2000\)](#), and for an application of admissible models in the Brazilian market see [Almeida \(2004\)](#).

3.3 Risk adjusted interest rate

We propose two alternative models of default risk adjusted instantaneous interest rate process \hat{r} defined in [subsection 3.1](#). Firstly, we consider the independence case between the default-free instantaneous interest rate process and the intensity process. However, the empirical evidence shows that there is correlation between both processes. For this reason, we establish another model that considers that the default risk adjusted instantaneous interest rate process affects the intensity process. To support our proposal an information based model selection approach is performed. To establish precisely the two proposed models it is possible to divide the state vector into two components $y_t = (y_t^{(r)}, y_t^{(\gamma)})$. Each coordinate is called a state.

In the first model denominated independent risk free rate and intensity process (i.e. the independent model), we consider both process given by

$$r_t = r(y_t^{(r)}) = y_t^1 + \dots + y_t^M,$$

$$\lambda_t = \lambda\left(y_t^{(\gamma)}\right) = y_t^{M+1} + \dots + y_t^N,$$

where $M \geq 1$ and $N > M$. Due to the independence in the processes, the valuation of defaultable zero-coupon bond is the same as in the case of a single factor except that each component of the process vector adds a term to the bond price formula.

In the second model denominated correlated risk free rate and intensity process (i.e. the correlated model), we consider both process given by

$$r_t = r\left(y_t^{(r)}\right) = y_t^1 + \dots + y_t^M,$$

$$\lambda_t = \lambda\left(y_t^{(r)}, y_t^{(\lambda)}\right) = \psi r_t + y_t^{M+1} + \dots + y_t^N,$$

where $M \geq 1$ and $N > M$. In the previous model, one more parameter ψ is added in the process that measures the relationship between both processes. Obviously, the case $\psi = 0$ in this model gives the previous model.

4 A case Study: Uruguayan sovereign bond market

In order to apply our results, the US bond market is chosen as a reference for the risk-free interest rate, and the Uruguayan bond market as the emerging economy. It is clear that a similar analysis can be carried out for any other emerging economy.

4.1 Data

The data is extracted from two different sources because the parameters of the processes are identifiable from information on default-free instruments and defaultable bond prices. The dynamics of sovereign yield spread need to be analyzed at a high frequency to incorporate the movements in the market. For this reason, we use daily data for the time period 01/02/2014–07/31/2016. In the case of the default-free market, the data are the interest rates denominated Constant Maturity Treasury released by the Federal Reserve for ten different maturities. In the case of the default market, we use the prices of negotiated bonds issued in USD, released by the Central Bank of Uruguay. These bonds comprises approximately three-quarters of the capitalization of the Uruguayan sovereign debt issued in USD. They include medium and long term maturities. In particular, the six global bonds active during the whole period are considered. Besides, two local bonds are incorporated in order to obtain good approximation

in short-term maturities. The selected bonds are detailed in [Table 1](#). It is important to note that in the Uruguayan market there are no interest rate derivatives. This entails the need to use only prices of bonds.

Table 1. Sovereign bonds denominated in US dollars.

Instrument	Issue date	Maturity date	Coupon
Local Bond 2019	03/29/03	03/23/19	7.500
Local Bond 2020	05/29/03	02/28/20	9.750
Global Bond 2022	11/18/05	11/18/22	8.000
Global Bond 2024	08/14/13	08/14/24	4.500
Global Bond 2025	09/28/09	09/28/25	6.875
Global Bond 2033	05/29/03	01/15/33	7.875
Global Bond 2036	03/21/06	03/21/36	7.625
Global Bond 2045	11/20/12	11/20/45	4.125

4.2 Results

The two models described in [subsection 3.3](#) are applied with vectors with different number of states, i.e. different values of M and N . In the work, the yield rate of the Uruguayan debt is decomposed as the sum of the yield rate of the US debt (assumed to be default-free risk) plus the yield spread that causes the credit risk in the domestic debt. This difference is model by intensity process of the jump. Therefore, the parameter estimation in the model is performed in two steps: in the first one, we estimate the parameters of the US debt based only on its yields for different maturities; in the second one, the parameters of the intensity process are estimated based on Uruguayan bond yields and the parameters obtained in the first step.

In [Table 2](#) we present the results of the chosen model, that corresponds to four states: $M = 2$ and $N = 4$. It should be noticed that, according to the literature, more than one state is necessary to explain the changes over time in the slope and shape of the term structure curves, see for instance [Brigo and Mercurio \(2006\)](#).

In what concerns the results of this parameter estimation in the Uruguayan market, some comments are in order. First, the parameter values obtained for the US debt are in accordance with the bibliography for all the chosen models, see [Chen and Scott \(1993\)](#).

Table 2. Parameter estimation in both models.

		Independent model	Correlated model
	r	λ	λ
a_1	0.319650	1.313931	1.190081
a_2	0.037311	0.103552	0.103065
b_1	0.017365	0.016763	0.008009
b_2	0.016145	0.020187	0.005950
σ_1	0.008863	0.015253	0.014160
σ_2	0.008112	0.014022	0.019631
ψ	–	–	0.681380
ϵ_1	0.000749	0.000622	0.000509
ϵ_2	0.000391	0.000939	0.000967
ϵ_3	0.001054	0.000986	0.000846
ϵ_4	0.001178	0.000808	0.000847
ϵ_5	0.000827	0.001202	0.001081
ϵ_6	0.001574	0.001357	0.000967
ϵ_7	0.001729	0.000853	0.000689
ϵ_8	0.001085	0.000750	0.000693
ϵ_9	0.000974	–	–
ϵ_{10}	0.001077	–	–
$\ln L$	–	14,082.74	14,179.85
AIC	–	–28,105.48	–28,297.70
BIC	–	–27,902.56	–28,088.01

Second, comparing the parameters of the intensity process between the independent model ($\psi = 0$) and the correlated one ($\psi \neq 0$), we obtained that the asymptotic interest rates given by the parameters b_i in equation (1) are lower in the correlated model than in the independent one. The reason is that these parameters are influenced by the additional parameter ψ .

Third, the values of the likelihood function of the correlated model are higher than the ones of the independent one. To take into account the difference in the number of parameters, two information criteria are used to quantify the quality of the statistical model for the database: the Akaike and the Bayesian information criteria (AIC and BIC respectively). In both cases, the correlated model is preferred, because the change in the likelihood function is larger than the change in the penalty factor when adding the additional parameter ψ .

To conclude the discussion of the results, we note that the analysis performed

above gives the evolution of the intensity process for each model. This intensity process is interpreted as the instantaneous probability of default risk of the Uruguayan sovereign debt. Figure 1 shows the evolution of the intensity process for both models.

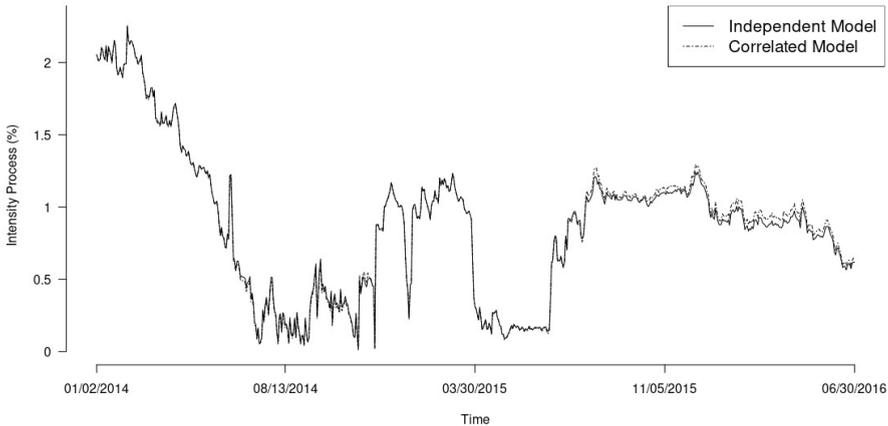


Figure 1. Estimation of the intensity process for both models.

4.3 Application: Proposed dynamical country risk index

In the Uruguayan bond market two local and one international country risk indices are daily released: the IRUBEVSA (Indice de Riesgo Uruguay) by BEVSA, the UBI (Uruguay Bond Index) by República AFAP, and the EMBI (for more information see Section 2). These three indexes are constructed departing from daily estimated term structure curves of both the Uruguayan and the United States debt, based on sovereign assets. The differences between the two curves at different maturities are weighted, depending on the capitalization and the liquidity of the respective bonds.

As an application of the results obtained in subsection 4.2 a *dynamical country risk index* is proposed, both in the correlated and non correlated models, based on historical information of both the emerging and the reference markets. As in other cases they are weighted averages of differences of term structures from the Uruguayan and the US markets, for maturities corresponding to the eight Uruguayan sovereign bonds used in the estimation, computed at a given

date. The weights take into account capitalization. A correlation analysis (see Table 3) of the five described instruments shows that the three released indices and the two dynamical indices have very high correlation (above 0.98 in all cases). This means that any of the three released indices, or any of the two dynamical indices produces equivalent information. Nevertheless, the correlation between the released and dynamical indices are not so high, exhibiting some differences. In the statistical balance between signal and noise, the dynamical index is more stable, capturing a more parsimonious signal, as can be seen in the comparison of Figure 2.

Table 3. Correlation between the different country risk indexes.

	Irubevsa	Ubi	Embi	Indep. M	Correl. M.
Irubevsa	1.0000	0.9888	0.9866	0.9495	0.9687
Ubi	—	1.0000	0.9968	0.9180	0.9449
Embi	—	—	1.0000	0.9174	0.9453
Indep. M.	—	—	—	1.0000	0.9926
Correl. M.	—	—	—	—	1.0000

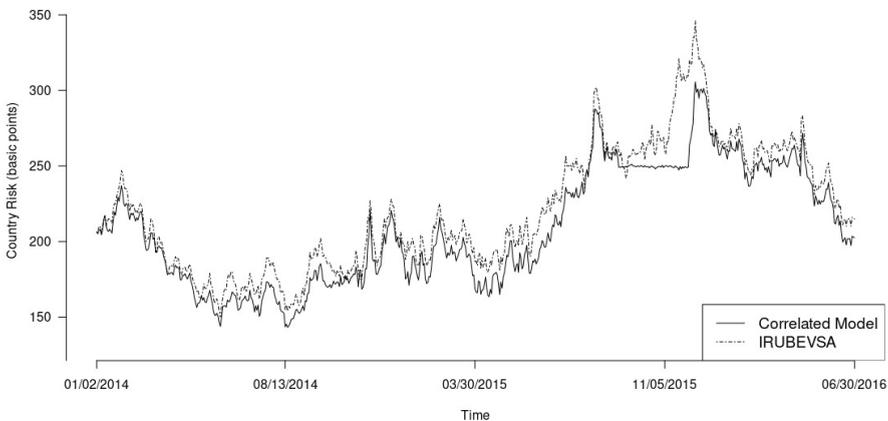


Figure 2. Country risk indices for correlated model (—) and IRUBEVSA (---).

5 Conclusions

The preferences of global investors are influenced by the level of the credit risk implicit in the different available investment options. Regarding sovereign debt, the yield rate depends mainly on economic factors of the country. An important macroeconomic variable in the analysis of sovereign debt in emerging markets is the country risk index. In this work, a new *dynamical country risk index* is proposed. The computation of this index follows the approach of the most used index on the market, therefore it is a weighted mean of differences of yields of the US bond market w.r.t the yields of an emerging economy. The calibration in the US debt market follows the classical approach based on a multidimensional Vasicek model, while in the emerging economy a yield spread (modeled as the intensity of a jump process) is added. Therefore, this article contributes by providing a parametric arbitrage-free dynamic model to estimate defaultable term structures of sovereign bonds. To model the term structure of the emergent market, the yield spread and the US interest rates are firstly considered independent, and secondly correlated. One advantage of the proposed country risk index is that it involves historical information of both markets, producing a more robust estimation than the usual released indexes. To handle this amount of data, an expectation-maximization algorithm is implemented in the framework of the Kalman filter methodology. An application to the Uruguayan market is then performed with data from January 2014 to July 2016. The dynamical index is relatively close to the local and international quantitative indexes released by different financial institutions, being more parsimonious and comprising more information.

The proposed work raises several questions for future development. One of them is the application of this methodology to other emerging markets, in order to compare the relative performances of these economies. This comparison can be carried out from the analysis of the respective estimated parameters of the models. Another question is whether the consideration of more sophisticated models (as correlated Vasicek models, or even other classes of Lévy driven multidimensional affine processes) would produce better results, mainly taking into account the disposal of large international databases, both on bond prices and on derivative prices.

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Appendix: Estimation with Kalman filter and EM procedure

The aim of the work is to obtain the dynamics of the risk adjusted interest rate, that depends of the state vector. This state vector, on its turn, depends on parameters that have to be estimated. The data for the estimation consists in two sets of bond prices: the first one observed in the US market and a second one observed in a emergent market.

The estimation procedure is then not direct: the observed data (bond prices) is a function of the state vector, that gives the parameters of the adjusted interest rate dynamics. As the equation (1) is a generalization of the classic model by Vasicek (1977), the estimation procedure follows the same steps, but in a multidimensional setting. An adequate tool for this situation is the Kalman filter (see Kalman, 1960). This method is optimal in the selected family of state vector models (see Harvey, 1992). In order to implement the estimation procedure following the Kalman filter methodology we assume that bond prices are observed at discrete time points. Therefore, it is possible represented the model in state-space form. We then begin by discretizing the process y_t the dimension N given by equation (1) dividing the time interval into equally spaced intervals of length $\Delta_t = t_{i+1} - t_i$.

Given that the state process follows a multidimensional Ornstein-Uhlenbeck dynamics, the Euler approximation gives a transition matrix equation:

$$\underbrace{\begin{pmatrix} y_{t_i}^1 \\ y_{t_i}^2 \\ \vdots \\ y_{t_i}^N \end{pmatrix}}_{y_{t_i}} = \underbrace{\begin{pmatrix} b_1(1 - e^{-a_1\Delta t}) \\ b_2(1 - e^{-a_2\Delta t}) \\ \vdots \\ b_N(1 - e^{-a_N\Delta t}) \end{pmatrix}}_{\alpha} + \underbrace{\begin{pmatrix} e^{-a_1\Delta t} & 0 & \cdots & 0 \\ 0 & e^{-a_2\Delta t} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-a_N\Delta t} \end{pmatrix}}_{\phi} \underbrace{\begin{pmatrix} y_{t_{i-1}}^1 \\ y_{t_{i-1}}^2 \\ \vdots \\ y_{t_{i-1}}^N \end{pmatrix}}_{y_{t_{i-1}}} + \underbrace{\begin{pmatrix} \mu_{t_i}^1 \\ \mu_{t_i}^2 \\ \vdots \\ \mu_{t_i}^N \end{pmatrix}}_{\mu_{t_i}}, \quad (2)$$

where $\mu_{t_i} | F_{t_{i-1}} \approx \mathcal{N}(0, Q)$ and

$$Q = \text{diag}\left(\frac{\sigma_1^2}{2a_1}(1 - e^{-2a_1\Delta t}), \dots, \frac{\sigma_N^2}{2a_N}(1 - e^{-2a_N\Delta t})\right)$$

On the other hand, the model states that the logarithm of the bond price is a linear function of the state vector of the form

$$z(\tau) := -\frac{\log P(\tau)}{\tau} = -\frac{A(\tau)}{\tau} + \frac{\sum_{i=1}^N B_i(\tau)y_t^i}{\tau}, \quad (3)$$

where

$$A(\tau) = \sum_{i=1}^{i=N} \left(b_i - \frac{\sigma_i^2}{2a}\right) (B_i(\tau) - \tau) - \frac{\sigma_i^2 B_i^2(\tau)}{4a_i}$$

and

$$B_i(\tau) = \frac{1}{a_i}(1 - e^{-a_i\tau}).$$

We furthermore assume that there are $n > N$ sovereign bonds in the market with maturities T^1, \dots, T^n and respective yield rates $z(\tau_t^1), z(\tau_t^2), \dots, z(\tau_t^n)$, where $\tau_t^j = T^j - t$ is the time to maturity. However, to avoid identification problems we allow differences between observed rates and modeled rates, introducing error variables (ν_{t_i}) . This procedure was first introduced by [Chen and Scott \(1993\)](#) to match the number of bonds with the dimension of the state vector. The measure

matrix equation is derived from (3), and we obtain

$$\underbrace{\begin{pmatrix} z(\tau_{t_i}^1) \\ z(\tau_{t_i}^2) \\ \vdots \\ z(\tau_{t_i}^n) \end{pmatrix}}_{z_{t_i}} = \underbrace{\begin{pmatrix} -A(\tau_{t_i}^1) \\ \frac{\tau_{t_i}^1}{\tau_{t_i}^2} \\ -A(\tau_{t_i}^2) \\ \frac{\tau_{t_i}^2}{\tau_{t_i}^2} \\ \vdots \\ -A(\tau_{t_i}^n) \\ \frac{\tau_{t_i}^n}{\tau_{t_i}^2} \end{pmatrix}}_{A_{t_i}} + \underbrace{\begin{pmatrix} B_1(\tau_{t_i}^1) & B_2(\tau_{t_i}^1) & \dots & B_N(\tau_{t_i}^1) \\ \frac{\tau_{t_i}^1}{\tau_{t_i}^2} & \frac{\tau_{t_i}^1}{\tau_{t_i}^2} & \dots & \frac{\tau_{t_i}^1}{\tau_{t_i}^2} \\ B_1(\tau_{t_i}^2) & B_2(\tau_{t_i}^2) & \dots & B_N(\tau_{t_i}^2) \\ \frac{\tau_{t_i}^2}{\tau_{t_i}^2} & \frac{\tau_{t_i}^2}{\tau_{t_i}^2} & \dots & \frac{\tau_{t_i}^2}{\tau_{t_i}^2} \\ \vdots & \vdots & \ddots & \vdots \\ B_1(\tau_{t_i}^n) & B_2(\tau_{t_i}^n) & \dots & B_N(\tau_{t_i}^n) \\ \frac{\tau_{t_i}^n}{\tau_{t_i}^2} & \frac{\tau_{t_i}^n}{\tau_{t_i}^2} & \dots & \frac{\tau_{t_i}^n}{\tau_{t_i}^2} \end{pmatrix}}_{B_{t_i}} \underbrace{\begin{pmatrix} y_{t_i}^1 \\ y_{t_i}^2 \\ \vdots \\ y_{t_i}^N \end{pmatrix}}_{y_{t_i}} + \underbrace{\begin{pmatrix} \nu_{t_i}^1 \\ \nu_{t_i}^2 \\ \vdots \\ \nu_{t_i}^n \end{pmatrix}}_{\nu_{t_i}}, \quad (4)$$

where $\nu_{t_i} \approx \mathcal{N}(0, R)$ and $R = \text{diag}(\epsilon_1^2, \dots, \epsilon_n^2)$. The statistical problem is to find the default-free instantaneous interest rate and the intensity process given access only to daily yield bonds, to subsequently estimate the parameter set. The equations (2) and (4) conform a dynamical system with two classes of variables: the unobservable variables that describe the dynamics of the default risk adjusted instantaneous interest rate through a Markov process; and the observable variables that correspond to the yield rates of sovereign bonds. Both sets of variables are related through a parametric linear function.

The Kalman Filter methodology is a set of equations that implement a predictor-corrector type estimator to be updated the system once a new observation of the variable becomes available. This prediction is obtained using the distribution of the state variable, conditional on the previous estimated values. These estimates are updated using the information provided by the observed yield. This estimation we allow construct the log-likelihood function to find the optimal parameter set. For the optimization, it is necessary to use nonlinear techniques due to the complexity. In this work, we choose the algorithm of expectation maximization (EM). There is an extensive literature related to this subject, see for instance Linke et al. (2014). In particular, the Kalman filter is used in estimation of term structure models in Chen and Scott (2003) and Bolder (2001).