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RATIONAL EXPECTATIONS, INCOME POLICIES AND GAME THEORY

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ABSTRACT

Rational expectations are viewed as a Nash equilibrium of a game.

If a change of regime occurs, it is argued that it is very unlikely that the economy is going to achieve the new equilibrium at once. In fact, one can show that if the public acts prudently endogenous inflation inertia arises. Hence, at this point, government intervention by means of income's policies is required, in the sense that it guides the economy to the new equilibrium, much faster than the market. One also analyses, the effects of income's policies on a staggered wage setting.

RESUMO

As expectativas racionais são vistas como um equilíbrio de Nash.

Ocorrendo uma mudança de regime, argumenta-se que é muito pouco provável que a economia atinja o novo equilíbrio de uma vez só. Pode-se mostrar que se o público age de maneira prudente, aparece uma inércia endógena na inflação. Neste instante a intervenção do governo, por meio de uma política de rendas é necessária para levar a economia para o novo equilíbrio, o que ocorrerá muito mais rapidamente do que no mercado. Analisam-se também os efeitos de políticas de renda numa situação de salários justapostos.

1. STRATEGIC INTERDEPENDENCE AND INCOME POLICIES

That aggregate demand discipline is a necessary condition for sustained price stability has long been known by economists and well advised policy-makers. Yet it may not be sufficient to successfully stop a big inflation, as evidenced by the failure of a number of IMF-supported programs that overlooked inflationary inert tia, thus leading to dismal stagflation. Not surprisingly, countries like Argentina, Israel and Brazil recently decided to focus on the supply side of inflation, attempting to stabilize prices by combining income policies with monetary reforms. Whether these experiments will yield success or not depends on a number of factors, including an adequate aggregate demand management. The interesting issue is that they were inspired on a sound game theoretical approach to the supply side of inflation. In fact, as we shall argue in the following discussion, income policies may be necessary to coordinate individual behavior according to rational expectations models. Moreover, monetary reforms may be used to dissipate the Fischer-Taylor inertia that is consistent with the rational expectations hypothesis.

The central question is to understand what causes inertial inflation and how income policies can break the dependence of the inflation rate on its past behavior. The traditional explanation of the late sixties, combining the natural unemployment rate hypothesis with adaptative expectations, was a benchmark in terms of interpreting inflation as an auto-regressive process, although it never made clear how income policies might affect expectations. In any case, it was soon eclipsed by the rational expectations revolution that dismissed adaptative expectations as "ad-hoc" assumptions usually inconsistent with optimal decision-making.

Under the aegis of the new classic economics, inertial inflation could only spring from two sources, auto-regressive expected rates of monetary expansion, and outstanding contracts based on previous inflationary expectations. The first source could be dried up by a credible monetary rule, once money supply was put under the control of respectable and independent Central Bankers. The second source could only be responsible for temporary and dampened inertia. Hence, painless inflation cure appeared as a strong possibility, perhaps not in a shock treatment, but at least in a gradualistic approach to price stability, in line with the maturity of old contracts. Since the length of non-indexed contracts is a decreasing function of the inflation rate, the transition from high to zero inflation could be quite rapid. For instance, if money wages were reset once a year, adjustment dates for different labor groups being uniformly distributed over time, inflation rates could be brought down from 20% in year 0 to 9% in year 1 and to zero in year 2 with no recession at all.

Anti-inflationary policies in the early eighties were painful enough to suggest that inflation rates might be held back by a force ignored in rational expectations models, namely, inertia caused by strategic interdependence among private economic agents. In fact, rational individual behavior, besides depending on anticipated economic policies, also depends on other individuals' decisions. Moreover, economic agents must make their own decisions without knowing how other players will act, which brings inflation theory into the field of games with imperfect information. Rational expectations literature explicitly recognizes the problems of strategic interdependence between the private sector and the government, activism meaning that the government decides to act as a dominant Stackelberg player. But, as noted by Tobin, this plainly ignores that, except in auction markets, there is no sense in treating the private sector as a single player. Summing up, the central weakness of the rational expectations macroeconomics is to describe as a two-person game what actually should be analysed as an $n+1$ person game. Tobin correctly guessed that, provided price-setting games were appropriately discussed in terms of strategic interdependence, inertia would emerge as a powerful inflationary force. The

same idea, incidentally, had been grasped by Modigliani in his classic contention as to the use of the rational expectations hypothesis in non-auction markets. What kept alive the academic prestige of the new classic economics was that none of these criticisms was explicitly formalized in terms of game theory.

The central question to be discussed is: what is rational decision doing in a non-cooperative n -person game with imperfect information? The new classic economics assumption is that all players should hit a Nash equilibrium in the first move. The new Keynesian school simply rejects such a narrow concept of rationality. In fact, the choice of a Nash strategy only means rational behavior if each player can be assured that all other players will choose consistent Nash strategies. In the absence of such a guarantee, it may be nothing but naively imprudent playing. Cautious economic agents might well prefer to start the game with the defensive maxmin strategy, which yields maximum pay-off in the worst conceivable scenario. In a few games, such as the prisoners' dilemma and zero-sum two-person games, Nash equilibria present strong stability properties. Except that, in all these cases, they are the outcome of maxmin strategic choices. Wage-price setting belongs to a much more complicated class of games, as will be shown in section 2, where rationality cannot be so narrowly defined.

Rational expectations literature uses two alternative approaches to by-pass these game theoretical complications. One is to bring on stage a Walrasian auctioneer that prevents any transaction as long as any player can improve his pay-off given the strategic choices of the remaining participants. The other is to assume that all available information is common knowledge, each player taking into account the reaction functions of other players before announcing his final decision. That the outcome is a Nash equilibrium should not be received as a surprise. In fact, it is a Nash equilibrium by definition. The problem is that even in auction markets there is some trading out of equilibrium, so that the Walrasian auctioneer can only be accepted as a first-order approximation. As to non-auction markets, the idea that a Nash equilibrium is hit at the first move as a result of a mental exercise involves some serious slips. First, to engage in such a mental exercise, every

economic agent must be convinced that the others will do the same, otherwise it will be a pure waste of time and money. Second, when the mental exercise is feasible, namely, in a game with a small number of participants, hitting a Nash equilibrium may be a poor achievement. Why not try Stackelberg dominance, or, in the case of repeated games, use signalling and threats to produce cooperation? Third, with a large number of players, each ignoring the other participants' pay-offs, the exercise may be impossible.

Moving back to our leit-motif, let us assume that after prolonged inflation the Central Bank announces that it will stop printing money so as to keep the nominal G.N.P. unchanged. Even if the general perception is that the nominal G.N.P. will be immediately stabilized by a new monetary constitution (an extreme hypothesis, since monetary austerity is not usually backed by a constitutional reform), prudent price setters should not take the lead in stopping price increases, as long as they consider the possibility of further price increases in other sectors. In fact, the first to jump has little to gain if he is followed by the remaining participants in the game, and much to lose if he jumps alone. In the latter case the penalty is concrete, a real income cut, while the reward is abstract: additional customers' orders that cannot be met since the day has only 24 hours. The fact that excess demand does not mean additional effective demand nor additional income helps to explain why prudent price-setters should stick to something such as a maxmin strategy.

Of course, in a repeated game with a large number of small players, leaving virtually no space for coalitions, threats, signalling or Stackelberg dominance, maxmin strategies that do not yield a Nash equilibrium are not likely to be indefinitely repeated. The price-setting model of section 2 indicates how prudent players in a repeated game may gradually narrow the conceivable range of other participants' strategies, triggering a "tatonnement approach" to a Nash equilibrium. Little can be said, however, about convergence speeds, which may be painfully slow after a prolonged period of high inflation rates.

The foregoing discussion provides the rationale for income policies: Governments should play the role of the Walrasian auctioneer, speeding up the location of Nash equilibria, namely, using the visible hand to achieve what rational expectations models assume to be the role of the invisible hand. It should be stressed that the central function of incomes policies is not to constrain individual decision making, but to clear externalities in a game with imperfect information, telling each actor how the others will play. This, incidentally, dismisses a traditional argument against income policies, i.e. that governments are not better equipped than private markets to identify individual Nash strategies. In fact, the central problem is not to discover such strategies, but to coordinate their simultaneous playing. This also explains why, in a second stage of the stabilization program, wage-prices controls should be removed gradually, at successive sectorial steps, and not in the one-shot manner. In fact, even if income policies are successful enough to bring the economy to a Nash equilibrium, there is no way to convey such information to the participants of the game. Each player finds himself in equilibrium, but does not know if the same applies to other players. If controls are lifted sector by sector, players will realize that no participant in the game will increase prices even when allowed to do so. On the other hand, the one-shot approach would simply bring back the uncertainty of individual players as to what other players will do, perhaps triggering large defensive wage-price increases.

Of course, the chances of hitting a Nash equilibrium through income policies are extremely remote, and the fact that wage-price controls yield some supply shortages should not come as a surprise. The central question is: what is worse in terms of social welfare, a few product shortages, that eventually may be overcome by imports, or massive unemployment, which is nothing but a shortage of jobs? From this point of view, objections to income policies should not be taken too seriously, especially when the problem is to fight a big inflation with strong inertial roots. This is all the more so because income policies can be managed with appropriate flexibility, substituting price administration for price freezes.

A more fundamental contention is that the temporary success of income policies may lead policy-makers to ignore that price stability cannot be sustained without aggregate demand discipline. That such misleading signals are a true risk has been known since the times of the Roman Emperor Diocletian, and the list of income policy failures is too long to be neglected. Yet the converse is also true. Trying to fight a big inflation only from the demand side may lead to such dismal stagflation that policy-makers may well conclude that life with inflation is preferable to life with a monetarist stabilization program.

2. A PRICE-SETTING GAME

Let us assume an economy with a continuum of goods, each one produced by an individual price-setter, and where the nominal output R is controlled by the Government¹. The nominal output is preannounced, but prices must be set simultaneously, each agent ignoring how the others will act. Production starts after prices have been set, according to consumers' orders.

All individuals have the same utility function:

$$U_x = L_x^b \int_0^1 q_{xy}^a dy \quad (0 < a < 1 ; b > 0) \quad (1)$$

where L_x stands for leisure time and q_{xy} for the consumption of good y by individual x ($0 \leq x \leq 1$). The supply S_x of good x equals the number of daily working hours of individual x :

¹ In line with the monetarist tradition, one may assume that the Central Bank controls some monetary aggregate M_n that determines the nominal output.

$$S_x = 24 - L_x \quad (2)$$

hence, indicating by P_y the price of good y ($0 \leq y \leq 1$), individual budget constraints are expressed by:

$$\int_0^1 P_y q_{xy} dy = R_x \quad (3)$$

where R_x is the nominal income of economic agent x .

Constrained utility maximization yields the demand Q_y for good y :

$$Q_y = \frac{R P_y^m}{P_y^{m+1}} \quad (4)$$

where:

$$R = \int_0^1 R_x dx \quad (5)$$

$$m = \frac{a}{1-a} \quad (6)$$

and where the consumer price index P is determined by:

$$P^{-m} = \int_0^1 P_x^{-m} dx \quad (7)$$

Let us first examine the conditions for market clearing. Since the quantity sold of good x cannot exceed Q_x :

$$R_x = \min \{P_x S_x ; P_x Q_x\} \quad (8)$$

Individual's x utility will then be expressed by:

$$U_x = \frac{R_x}{P} (24 - S_x)^b \quad (9)$$

Once all prices have been set, U_x is maximized for:

$$S_x = \min \left\{ Q_x ; \frac{24}{1+b} \right\} \quad (10)$$

since there is no incentive to increase supply either beyond demand or beyond the point that maximizes $S_x(24 - S_x)^b$. Hence all markets are cleared if and only if:

$$Q_x \leq \frac{24}{1+b} \quad (0 \leq x \leq 1)$$

or, equivalently, if and only if the inequality:

$$P_x^{m+1} \geq \frac{1+b}{24} R P^m \quad (11)$$

holds for every $x \in [0;1]$.

Let us now determine the optimum price-setting rule under perfect foresight. Each economic agent should choose P_x so as to maximize his or her utility U_x given R and P . Market clearing can be taken for granted, since individuals are encouraged to increase prices not only to the point of eliminating excess demand, but also to take advantage of their standing as monopolistic competitors. Hence, P_x should be chosen so as to maximize:

$$U_x = \frac{P_x Q_x}{P} (24 - Q_x)^b$$

Taking into account demand equation (4), the optimum price setting rule yields:

$$P_x^{m+1} = \frac{1+b'}{24} R P^m \quad (12)$$

where:

$$b' = \frac{m+1}{m} b \quad (13)$$

and leading to:

$$U_x^{\max} = \left\{ \frac{24}{1+b'} \right\}^{-\frac{m}{m+1}} \left\{ \frac{24b'}{1+b'} \right\}^b \left\{ \frac{R}{P} \right\}^{\frac{1}{m+1}} \quad (14)$$

Combining (12) and (7), the optimum price-setting rule under perfect foresight is expressed by:

$$P_x = -\frac{1+b'}{24} R \quad (15)$$

This corresponds to the unique Nash equilibrium of the price-setting game. It should be noted that equation (15) describes a monopolistic competition equilibrium, since each good is produced by one single individual. If all producers behaved as competitive price-takers, the supply of good x would maximize U_x given the price system. Since:

$$U_x = \frac{P_x S_x}{P} (24 - S_x)^b$$

the competitive supply of good x would be expressed by:

$$S_x = \frac{24}{1+b}$$

Introducing equations (4) and (7), competitive equilibrium prices would be given by:

$$P'_x = \frac{1+b}{24} R$$

The higher m , the lower the ratio between monopolistic competition and purely competitive prices, approaching 1 as m tends to infinity. This is to say that m indicates the degree of competitiveness in the economy.

Let us now turn to the dynamics of our price-setting game. We shall assume that for $t \leq 0$ our economy has adjusted for chronic inflation at a constant rate r per unit of time. This is to say that the government expands the nominal output according to $R_t = (1+r)R_{t-1}$ and that price-setters have already located the Nash equilibrium path according to equation (15), which also yields $P_t = (1+r)P_{t-1}$. Suddenly a new administration steps in and decides to stabilize the nominal output, announcing $R_t = R_0$ for $t \geq 1$.

According to rational expectations, if the new administration presents a credible stabilization program, inflation will stop immediately, with no side effects on output. This results from two assumptions:

i) equations (1) and (2), which describe individual preferences, endowments and production sets, are common knowledge. Hence, all their mathematical consequences, including equation (15), are also common knowledge;

ii) since all individuals believe that $R_t = R_0$, for $t \geq 1$, price-setting according to equation (15) will lead to immediate price stabilization with no side effects on output.

Now, two reasons make these assumptions highly questionable:

i) even if equations (1) and (2) are common knowledge and if all individuals believe that the government will actually stabilize the nominal output, this is not enough to convince price-setters to stop any further price increase. To use equation (15), each player must also be convinced that all other players do believe that the government will immediately stabilize the nominal output. This is to say, credibility must also become common knowledge;

ii) to assume that all preferences, endowments and production sets are common knowledge obviously sounds like science fiction. A more plausible assumption restricts individual knowledge to his own preferences, to production possibilities and to the demand function for his product. This is to say, individual x is well informed enough to derive the optimum price-setting equation (12) but not to solve the general equilibrium model that leads to equation (15).

Once these objections are taken into account, one must conclude that even if all individuals believe that the government will actually stabilize the nominal output, the only concrete information about the general price level in period 1 is that it must lie in some point of the interval $P_0 \leq P_1 \leq P_0(1+r)$. How to start the price-setting game at this point is a question that bears no single answer in terms of rational behavior.

Each P_1 in the interval $[P_0, P_0(1+r)]$ is a possible state of nature, and uncertainty may be faced through at least three different approaches:

a) individuals may attribute subjective probabilities to the different states of nature and then set prices so as to maximize the expected utility;

b) individuals may feel unable to attribute subjective probabilities to the different states of nature and prudently choose the pure maxmin strategy;

c) individuals may treat the problem as a game against nature and play the saddle-point mixed strategy.

We shall concentrate on approach b) which avoids the random choices of mixed strategies, as well as the need to create probability distributions without any concrete foundation, and which simplifies the mathematics of inflationary inertia. It should be stressed, however, that either of the other two approaches also leads to inertial problems. In the whole discussion we shall assume that each individual actually believes that $R_t = R_0$ for $t \geq 1$, and that the government does stabilize the nominal output.

What is the pure maxmin price-setting rule depends both on r and on the parameters of the model, as discussed in Appendix 2. In no hypothesis it corresponds to the Nash equilibrium, and a strong possibility is that it sets P_x according to equation (12) with the general price level at its upper bound $P_{\max}$:

$$P_x^{m+1} = \frac{1+b'}{24} RP_{\max}^m \quad (16)$$

The observed consumer price index, according to equation (7), will then be expressed by:

$$P^{m+1} = \frac{1+b'}{24} RP_{\max}^m \quad (17)$$

In our specific case, since $P_{\max} = P_0(1+r)$ and since $P_0 = \frac{1+b'}{24} R_0$:

$$P_1 = P_0(1+r) \frac{m}{m+1}$$

which shows that prices continue to increase in period 1 in spite of the credible nominal output stabilization. The counterpart, of course, is a recession, with real output falling below the full

employment level. To further describe the dynamics of inflation, an additional hypothesis is needed on how $P_{\max}$ is estimated for $t \geq 2$. A possible revision rule, assuming R to be kept unchanged, is provided by:

$$\frac{(P_{\max})_t}{P_{t-1}} = \frac{P_{t-1}}{P_{t-2}} \quad (18)$$

which assumes that, as a result of nominal output stabilization, economic agents estimate a non-increasing path for the inflation rate. Combining equations (17) and (18), taking into account that $P_0 = \frac{1+b'}{24} R_0$, and making $p_t = \ln P_t$, price dynamics will be described by the difference equation:

$$(m+1)p_t - 2mp_{t-1} + mp_{t-2} = p_0 \quad (19)$$

where p_t converges to the Nash equilibrium p_0 , but where little can be said about convergence speeds. Equation (19) apparently suggests that the higher m , the lower the convergence velocity to the Nash equilibrium; a striking conclusion, since in our model a high m means increased competition among price-setters. Yet equation (4) shows why this conclusion is not to be taken seriously. In fact, the higher m , the higher the costs of misestimating the consumer price index P , encouraging faster price resetting. Summing up, the time unit interval in equations (18) and (19) is also a decreasing function of m , so that no immediate relation can be found between the degree of competitiveness and inflationary inertia.

Equation (19) invites us to revisit the old-fashioned hypothesis of adaptative expectations. The traditional version of the fifties and sixties was obviously naive, in that it ignored the obvious fact that expectations respond to changes in economic policy. Yet it implicitly embodied some game theoretical common sense that has been completely overlooked by the rational expectations revolution. Of course, no simple backward-looking rule such as equation (19)

can be taken as a satisfactory description of how expectations are formed. It does serve, however, to recall an essential point, which is that there is no reason why in an n -person non-cooperative game a Nash equilibrium should be hit at the first move. Economic agents may be all forward-looking, but this does not make them naive enough not to know that uncertainties about other people's behavior are an essential ingredient in decision-making.

Summing up, credibility in rational expectations macroeconomics is a three-storey building. The first story, a strenuous but perhaps feasible construction, is a commitment to nominal output stabilization beyond any individual doubt. The second, perhaps only attainable through a dictatorial use of mass communication media, is to convince everybody that everybody has already been convinced that nominal output will be effectively stabilized. The third, which requires a formidable extension of common knowledge, is to teach price-setters that they can no longer benefit from further price increases. Rational expectations macroeconomics assumes that, once the first story has been built, the invisible hand will automatically build on the other two. The foregoing analysis indicates to what extent strategic interdependence brakes the power of the invisible hand.

The potential role for income policies is precisely to build up the upper stories of the rational expectations credibility model. Once the first storey has been completed, the other two may be immediately built on by a price freeze that convinces all economic agents to take $P_{\max} = P_0$ for $t \geq 1$, preventing the welfare losses of an interim recession. Game theory here tells us the true story behind the vague idea that income policies may reverse inflationary expectations. Their actual role is to coordinate the simultaneous playing of Nash strategies.

The obvious danger is the temptation to build the upper floors before having completed the first. In our model, this would mean freezing P_0 without preventing further expansion of the nominal output R . Initially the economy might experience a euphoric expansion in consumption, employment and output, as long as the increase in nominal incomes was matched by a cut in monopoly profit

margins. Then, in line with the long list of income policy failures, the program would collapse because of generalized shortages. In the preceding model general shortages would emerge if the nominal output expanded beyond:

$$\frac{R}{R_0} = \frac{1+b'}{1+b} \quad (20)$$

the right side above corresponding to the ratio between monopolistic competition and perfectly competitive equilibrium prices. The fact that after the price freeze some expansion in nominal output is possible without yielding supply shortages helps to explain why price controls are usually so successful in the very short run. Price-setters are converted into price-takers, thus being forced to accept some squeeze in their profit margins. As long as the latter do not fall below perfect competition margins, nominal output expansion leads to both output growth and increased welfare. Yet this euphoric start is nothing but an income policy trap. In fact, once price controls are lifted, producers will restore their previous margins by increasing prices and reducing quantities. Moreover, an ominous possibility is that policy-makers may misinterpret market signals and become convinced that, once prices have been frozen, growth can be promoted by relentless money printing.

3. STAGGERED WAGE SETTING AND INFLATIONARY INERTIA

Let us now examine the complications created by staggered wage setting. In the following discussion while labor is assumed to be homogeneous, workers are uniformly distributed among a continuum of classes, one for each real number $0 \leq x < 1$. Nominal wages of class x are reset at time $x+n$, for every positive, zero or negative integer n . This is to say that individual nominal wages move by steps of time length $T=1$ (the step curve below $S(\tau)$ in figure

1), but that adjustment dates for different classes are uniformly spread over time.

Under the above assumptions, the average nominal wage at time t will be given by the shaded area in figure 2:

$$W(t) = \int_{t-1}^t S(\tau) d\tau \quad (21)$$

$S(\tau)$ indicating the individual nominal wage of the class with resetting date τ ($t-1 \leq \tau < t$). To make sure that the integral on the right side of equation (21) does exist, we shall assume $S(\tau)$ to be a continuous function of τ .

Figure 1

Figure 2

Let us further assume that the price level $Q(t)$ is determined by multiplying unit labor costs by a constant mark-up factor:

$$Q(t) = (1+m)bW(t)$$

where m is the profit margin and b the labor input per unit output. In the following discussion we shall overlook supply shocks as well as both cyclical and long-term changes in productivity, thus

treating b as a constant. This makes the average real wage $W(t) / Q(t)$ a constant:

$$W(t) = zQ(t) \quad (22)$$

where $z^{-1} = (1+m)b$.

Staggered wage setting in an inflationary economy introduces two different real wage concepts, the peak and the average. The peak $k(\tau) = S(\tau)/Q(\tau)$ (OP in figure 3) is the individual purchasing power of the class with resetting date τ , immediately after the nominal wage increase. The average:

$$z(\tau) = k(\tau) \int_{\tau}^{\tau+1} \frac{Q(\tau)}{Q(\sigma)} d\sigma$$

indicated by the shaded area in figure 3, is the worker's average real wage in the time period where his nominal wage remains fixed. For example, if $Q(\sigma) = Q(0) e^{\pi\sigma}$, that is, if the inflation rate $Q'(\sigma)/Q(\sigma)$ is a constant π , the peak/average ratio will be given by:

$$k(\tau)/z(\tau) = \frac{\pi}{1-e^{-\pi}} \quad (24)$$

an increasing function of π that, for small inflation rates, can be approximated by

$$k(\tau)/z(\tau) = 1 + \frac{\pi}{2}$$

Figure 3

What really matters for both employers and employees is the average real wage $z(\tau)$. Yet the only objective element in each wage contract is the real wage peak $k(\tau)$. That the peak should be set so as to achieve a target average $z(\tau)$ seems pretty obvious. Under perfect foresight one may plausibly assume that $k(\tau)$ is calculated so as to make $z(\tau) = z$ in case of full employment, unemployment forcing workers to accept $z(\tau) < z$ and excess demand for labor leading to $z(\tau) > z$. Complications arise when strategic interdependence is brought on stage. In fact, the optimum $k(\tau)$ depends on both the nominal output path and the cost of living curve in the time interval $[\tau, \tau+1]$. Even if the former is credibly announced by the government, the problem remains that the cost of living path will depend on how other labor groups will react in terms of nominal wage increases. In fact, according to equations (21) and (22):

$$zQ'(t) = W'(t) = S(t) - S(t-1) \quad (25)$$

Let us first explore the mathematics of staggered wage setting. Three basic theorems are proved in the Appendix:

Theorem 1 (The constant inflation rate theorem): If, for $t \geq 0$, $Q(t) = Q(0)e^{\pi t}$, where π is a positive constant, then $z(\tau)$ converges to z and the peak/average ratio to:

$$\lim_{\tau \rightarrow \infty} k(\tau)/z(\tau) = \frac{\pi}{1 - e^{-\pi}}$$

Theorem 2 (The peak/average ratio theorem). If, for all $t \geq 0$, $k(t) = rz$, r indicating a constant greater than one, then the inflation rate converges to π , where:

$$r = \frac{\pi}{1 - e^{-\pi}}$$

Theorem 3 (The non-neutrality theorem): If the inflation rate decreases for $t-1 \leq \tau \leq t+1$, then $z(\tau) > z$ for some $t-1 \leq \tau \leq t$.

The first theorem explains why chronic inflation eventually leads to formal or informal indexation. If prices increase at approximately stable rates, real wage peaks will be recomposed at fixed time intervals. In practice this is achieved by periodic nominal wage increases for cost of living escalation:

$$S(\tau) = S(\tau-1) \frac{Q(\tau)}{Q(\tau-1)}$$

making $k(\tau) = k(\tau-1)$.

The second theorem proves the converse. Since indexation periodically resets real wage peaks above the real average the economy can afford to pay, inflation becomes predominantly inertial, being no longer the result of too much money chasing a few goods. The problem at this stage is no longer to explain why prices actually increase, but to explain how inflation can accelerate or decelerate.

Excess demand for labor is one possible source of inflation acceleration by making $k(\tau) > k(\tau-1)$, namely, by substituting indexation plus for simple cost of living escalation. Increased inflationary expectations can do the same, either because economic agents anticipate more expansive monetary policies or because each labor group expects other labor groups to move from simple indexation to indexation plus. Adverse supply shocks, lowering the equilibrium real wage average z , are another cause of inflation acceleration in an indexed economy. Recomposing the same real wage peaks now means increasing the coefficient r in Theorem 2, thus lifting the equilibrium inflation rate. Finally, if labor unions manage to reduce the wage adjustment interval, say, from twelve to six months, what was previously the annual inflation rate will become the six-month cost of living increase, as occurred in Brasil in late 1979. This, once again, is a consequence of the peak/average ratio theorem, where π is the equilibrium inflation rate for the wage resetting time interval.

One might argue that the foregoing analysis overstates the problem of inflationary inertia by ignoring that the length of nominal wage contracts is a decreasing function of the inflation rate. And that, once this fact is taken into consideration, a wage escalation clause is not a serious obstacle to fight inflation. A virtuous circle can be put in motion by a slight decrease in the inflation rate, which in turn will increase the length of the nominal wage contracts, further lowering the inflation rate and so on. The essence of the argument is that wage indexation at constant time intervals is not a convincing arrangement, since it makes average real wages a decreasing function of the inflation rate, a superior scheme being the trigger-point system where wages are adjusted at fixed inflation rate thresholds.

Both trigger-point and fixed time interval indexation schemes are found in inflationary economies, and one may argue that the second arrangement is nothing but a practical implementation of some implicit trigger-point, since wage adjustment intervals usually decrease as the inflation rate jumps up. Two points, however, should be stressed. First, once annual inflation rates reach a high two-digit figure or more, trigger points become highly impractical,

since nominal wages can be adjusted monthly, quarterly or every six months, but not at time intervals such as two months, seventeen days and three hours. Second, the response of the nominal wage contract length to changes in the inflation rate tends to be highly asymmetric. If inflation accelerates employers and employees may naturally agree on shorter indexation periods, leading to further inflation acceleration, and paving the road to hyperinflation. Yet, once inflation falls, strategic interdependence complications emerge again. No labor union will take the lead in accepting an extension of the indexation interval without the guarantee that other labor unions will do the same. Summing up, the implicit trigger point argument shows that inertia is an obstacle to inflation deceleration, but not to inflation acceleration.

Turning back to our staggered wage setting model, let us assume that for $t \leq 0$ inflation runs at a constant rate π with equilibrium wage indexation:

$$Q(\tau) \approx Q(0)e^{\pi\tau} \quad (\tau \geq 0) \quad (26)$$

$$S(\tau) = z \frac{\pi}{1-e^{-\pi}} Q(\tau) \quad (\tau \geq 0) \quad (27)$$

At time $t=0$ the government announces a stabilization program eventually intended to achieve zero inflation and full employment. This requires some spontaneous or managed de-indexation scheme where all peak/average ratios in nominal wage contracts are eventually reduced to one. Because of both strategic interdependence and lack of synchronization of wage adjustment dates, de-indexation cannot escape three complications.

First, in the absence of income policies, the government can, at best, determine the nominal output path, but not the price path, which is entirely dependent on nominal wage increases. Why then, in the absence of unemployment, any labor group should take the lead in accepting lower real wage peaks, can only be explained by

imprudent playing that only yields perfect foresight if the other players are equally imprudent. In a word, staggered wage setting with high inflation rates makes a strong case in favor of income policies.

Second, even if an ideal policy-mix leads to perfect foresight, the price path for $t \geq 0$ must be chosen so as to make $z(\tau) \geq z$ for all labor groups. In fact, under perfect foresight, workers will only accept $z(\tau) < z$ in case of unemployment. The neutrality dream, a declining inflation rate path with $z(\tau) = z$ for all, is a mathematical impossibility according to theorem 3. Yet, as will be shown below, perfect foresight price paths that yield $z(\tau) > z$ during the whole transition phase do exist. The problem is that they may be rather complicated, and that simpler paths may imbalance the real wage structure.

Third, unless nominal wages can be cut, gradualism is inevitable, some residual price increase being necessary to reduce from peak to average the purchasing power of the labor groups adjusted just before the announcement of the stabilization program. Prices can only be stabilized at a level $Q(f)$ such that:

$$\frac{Q(f)}{Q(0)} \geq \frac{\pi}{1 - e^{-\pi}} \quad (28)$$

As an example, if the inflation rate runs at 300% a year before the announcement of the stabilization plan and if wages are adjusted every six months (which makes $\pi = \ln 2 = 0,6931$), the residual price increase must be, at least:

$$\frac{Q(f) - Q(0)}{Q(0)} = 38,63\%$$

Even if nominal wages can be cut, a shock treatment intended to make $Q(t) = Q(0)$ is an awkward policy choice. In fact, according to equation (25), this would require:

$$zQ'(t) = W'(t) = S(t) - S(t-1) = 0 \text{ for all } t \geq 0$$

which, with stable prices, would perpetuate a disequilibrium wage structure, where nominal wages of class x ($0 < x \leq 1$) would be frozen at:

$$S(x) = \frac{\pi e^{(x-1)}}{1 - e^{-\pi}} z Q(0)$$

standing below the equilibrium level $zQ(0)$ for $e^x < \frac{\pi}{e^{\pi} - 1}$, and above $zQ(0)$ for higher values of x .

An attractive incomes-policy arrangement is the D-day proposal of former Brazilian Minister of Finance Octavio Gouveia de Bulhões. Nominal wages would be adjusted at the contractual dates for price increases before the announcement of the stabilization plan, but not for the residual inflation following that announcement. In our model, this would imply $S(t) = S(0)$ for all $t \geq 0$. Prices, according to equations (22) and (25), would move up so that:

$$Q'(t) = \frac{\pi}{1 - e^{-\pi}} (1 - e^{\pi(t-1)}) Q(0) \quad (0 \leq t \leq 1)$$

namely, along the curve:

$$Q(t) = \frac{Q(0)}{1 - e^{-\pi}} (1 + \pi t - e^{\pi(t-1)}) \quad (0 \leq t \leq 1)$$

$$Q(t) = \frac{\pi}{1 - e^{-\pi}} Q(0) \quad (t \geq 1)$$

The fact that the inflation gradually declines from π to zero without requiring nominal wage cuts and that prices stabilize with $z(\tau) = z$ for all labor classes after $t = 1$ only displays part of

the attractiveness of the D-day proposal. A still more interesting conclusion is that in the transition period, namely, for $-1 < \tau < 1$, the average purchasing power in every individual wage cycle is greater than the equilibrium level z . For contracts signed before the D-day and that mature after the beginning of the stabilization program, the conclusion is obvious. Such contracts would yield an average purchasing power z if the inflation rate was kept equal to π . Since the inflation rate declines after $t=0$, it follows that $z(\tau) > 1$ for $-1 < \tau \leq 0$.

For $0 < \tau < 1$, according to equation (23), and since $S(\tau) = k(\tau)Q(\tau) = S(0)$:

$$z(\tau) = S(0) \int_{\tau}^{\tau+1} \frac{1}{Q(\sigma)} d\sigma$$

Hence:

$$z'(\tau) = S(0) \left\{ \frac{1}{Q(\tau+1)} - \frac{1}{Q(\tau)} \right\}$$

Since $Q(\tau+1) > Q(\tau)$ for $0 < \tau < 1$, it follows that $z(\tau)$ is a decreasing function of τ for $0 < \tau < 1$. Since $z(1) = z$, one concludes that $z(\tau) > 1$ for $0 < \tau < 1$. The possibility of having $z(\tau) > 1$ in all individual wage cycles while, at any time, the average real wage of all workers stands at $W(t)/Q(t) = z$ should not be taken as a paradox. In fact, we are dealing with overlapping wage cycles, and the fact that $z(\tau) > 1$ for $-1 < \tau < 1$ does not imply that the purchasing power of all labor groups is greater than z in the transition period $0 \leq t \leq 1$.

Assuming perfect foresight, the D-day price path might be understood as a rational expectations response to the announcement of a policy that would gradually reduce the rate of expansion of nominal output from τ to zero. In fact, once strategic interdependence is taken into account, one can hardly believe that markets would spontaneously produce any arrangement such as the D-day wage

adjustment rule. It may be hard to implement, even as an incomes-policy decision, since labor unions are not expected to understand the mathematics of staggered wage setting. It may be politically much easier to dampen inflation through a fractional indexation arrangement:

$$\frac{S(t)-S(t-1)}{S(t-1)} = a \frac{Q(t)-Q(t-1)}{Q(t-1)} \quad (0 < a < 1)$$

technically inferior to the D-day proposal, but that can be understood without much intellectual sophistication.

The foregoing analysis describes how staggered wage setting complicates anti-inflationary policies, the same conclusions applying to staggered price setting. The central problem is that, since adjustment dates are not synchronized, instant wage-price stabilization becomes virtually impossible. In fact, at any instant some wages and prices are too high, some too low, equilibrium being reached across time by continuous rotation of relative positions. Of course, a tempting proposal is to immediately realign all wages and prices to equilibrium, cutting some and increasing others, depending on how recent the last adjustment was. Yet in most cases this will be found illegal, since it violates outstanding contracts. Moreover, it would involve nominal wage cuts, which politicians consider a nightmare. Gradualism then becomes inevitable, the D-day proposal indicating the quickest possible achievement.

Still, there is one escape route for a shock treatment, that of monetary reform. In fact, a currency change is not necessarily limited to a cut of a certain number of zeros in the country's monetary unit. It may well include a number of rules that establish how contracts should be translated from the old currency into the new one. Recent experiences such as the Austral Plan of Argentina and the Brazilian Cruzado Plan indicate how an imaginative use of such translation rules can lead to immediate relative wage and price synchronization.

4. THE ROLE OF MONETARY REFORMS

Attempts to stop a big inflation are often coupled with monetary reforms for two well known reasons. First, as a political move to signal a new age of price stability. Second, as a practical action to increase the purchasing power of the currency unit, which is usually achieved by cutting three, six or even twelve zeros in the old money bills. Needless to say, changing names and cutting zeros is a fruitless exercise unless if fiscal austerity is imposed and the inflationary expectations abated. As in the case of income policies, the list of failed monetary reforms is too long to be ignored.

How a monetary reform can erase the inflationary psychology is an intriguing question, since it involves a contradictory policy answer. Income policies may be essential to break inertia, but economic agents must also be convinced that the Central Bank will stop fueling inflation by money printing. Yet once the expected rate of inflation falls dramatically, fiat money must be generously printed for a period of time to meet the increase in money demand. The classic rule of thumb adopted by the German stabilization program of November, 1923, and borrowed by the Argentine Austral Plan, i.e., prohibiting money creation to finance public sector deficits, has a strong appeal, since it suggests that the money supply will be properly kept under control in the long run. How to bridge the short term problem, however, when renewed confidence in the domestic currency (based on the assumption that it will be no longer generously printed) requires a lot of additional printing, remains a complicated policy issue. Policy-makers must walk on a sharp knife-edge, avoiding so quick a reliquification that it might re-ignite inflationary expectations, as well as one so slow that it would lead to recession caused by a credit crunch.

The best solution might well be to set targets for total financial assets held by the public (M_4), allowing M_1 to expand as long as other financial assets decline. Yet, since M_4 statistics do not include foreign currency held by the public, the rule must be amended in case there is a significant shift from dollar assets

to domestic currency securities. Moreover, when the inflation rate falls from 400% or 2,000% a year to virtually zero, there is no guarantee that the demand for financial assets will be kept unchanged in real terms. Hence, besides tracking M_4 as a basic guide-line, the Central Bank should keep a close eye on what is actually happening to nominal GNP.

Monetary reforms can also perform a very important function, that of dissipating the Fischer-Taylor inertia. As previously discussed, a serious obstacle to overnight price stabilization is that outstanding non-indexed contracts reflect old inflationary expectations. Honoring such contracts when the inflation rates fall dramatically implies unanticipated wealth transfers between contracting parties that may lead to bankruptcies and recession. Of course, high inflation rates limit non-indexed contracts to short maturities, but even so the transfer effect may be sizeable. Let us assume, for instance, that the inflation rate falls abruptly from 15% a month to zero, as occurred in Brazil on February 28, 1986. In the absence of a monetary reform, a promissory note maturing three months after the stabilization day would cost the borrower a 52% unanticipated real surcharge. As to staggered wage-setting the obstacles it creates to immediate price stabilization were extensively discussed in the preceding section.

Monetary reforms can solve all these problems created by unanticipated stabilization and by lack of synchronization by establishing appropriate translation rules. In fact, once a new currency is substituted for the old one, the law must state how outstanding contracts should be rewritten in the new monetary unit. The imaginative innovation of the Argentine Austral Plan, borrowed by the Brazilian Cruzado Plan, was to recognize that there is no legal or economic reason why the translation rule for currency bills should hold for other obligations and contracts. In the case of non-indexed liabilities, the natural way to eliminate the effects of unanticipated stabilization is to establish conversion rates declining by a daily factor corresponding to the old inflation rate. As an example, the Cruzado reform immediately converted cruzeiro bills and demand deposits into cruzados by a cut of three zeros. Conversion rates for cruzeiro denominated liabilities with future maturities

were set to decline by a daily factor, starting February 28th, of 1.0045. Thus, a promissory note of one million cruzeiros maturing April 30th 1986 was worth 770.73 cruzados, not 1.000 cruzados. This specific translation rule was borrowed from the Argentine Austral Plan.

Neither Argentina nor Israel had to face problems of staggered wage setting, since in both cases wage adjustment dates were already synchronized. Brazil did, and the true innovation of the Cruzado Plan was to convert wages from cruzeiros into cruzados by computing their average purchasing power of the last six months with an increase of 8%. This actually meant reducing the nominal wages of the labor groups adjusted in January and February, 1986, and in all cases increasing wages substantially below the past inflation rate. The 8% bonus was intended to offset the inflationary effect of a number of price increases which were to be enforced on February 28th 1986, so that the plan was to synchronize all real wages at their average level of the previous six months, since the length of nominal wage contracts was exactly six months. At the last moment the ~~government~~ decided not to enforce these sectoral price increases, so that average real wages were actually increased by 8%. This may have been the original sin of the Cruzado Plan.

5. INCOMES POLICY MATCHING

The previous discussion on strategic interdependence among price-setters provides the rationale for combining monetary reforms with income policies and fiscal austerity. How incomes policies should be designed is a challenging issue. While a low calory menu seems enough to break inertia after a true hyperinflation, when prices expand by factors of millions or billions a year, much more sophisticated cooking is required when the problem is to stop a three or four digit inflation.

In fact, in a true hyperinflation (where, according to Cagan's definition, prices continuously increase above 50% a month) money

not only loses its function as a store of value, but also ceases to perform its role as an unit of account, wages and prices being usually set in some foreign currency. In this case, exchange-rate stabilization serves as a universal incomes policy tool. The classic success story is the German stabilization plan of November, 1923.

Complications arise when inflation is so high as to trigger widespread backward-looking indexation arrangements (since instantaneous price-index links never proved feasible), but not to the point of quoting wages and prices in foreign currencies. The unit of account, in such cases, is neither the domestic currency nor the exchange rate, but some lagged price index. Here imbalanced income policies may lead to a disaster, even if matched by fiscal austerity, as happened with Chile in 1979. The exchange rate was pegged at 39 pesos per dollar, the fiscal deficit was eliminated (actually it was turned into a surplus), but wages continued to be indexed for past inflation. As a result, the only force to dampen inflationary inertia was real exchange-rate overvaluation. Inflation rates actually fell from 40% in 1978 to 9% in 1981. Yet exchange-rate overvaluation led to dismal recession and to a wasteful increase in foreign debt. Summing up, income policies were seriously mismatched; once the exchange rate was frozen, wages should also have been frozen, perhaps with the political help of a temporary price freeze.

One may argue that once the exchange rate has been fixed and once nominal wages have been frozen, there is no need to freeze prices, provided both aggregate demand and monopoly profit margins are kept under control. This may be absolutely correct from the technical point of view, but an income policy package without price controls risks being a commodity with too much supply and no demand at all. In fact, the political marketing of income policies is largely dependent on its visible content, namely, on price freezes. It should be recognized, however, that it may be the poison-pill of the stabilization package. In fact, once prices are frozen, inflation falls to zero by definition. Yet there exists the risk that price inflation will be replaced by queue inflation, not necessarily a better achievement in terms of social welfare.

In a word, policy-makers and professional economists should agree on some basic principles of an anti-inflationary code: a) price stabilization cannot be sustained without aggregate demand management; b) fiscal austerity is the key issue as far as aggregate demand is concerned, except in a few countries with excess savings; c) inflationary inertia is a true problem, since common knowledge can never be so easily spread as assumed in the rational expectations macroeconomic literature; d) income policies, therefore, are an essential instrument to fight big inflations; e) there is no sense in discussing whether inflation should be tackled by the supply side or by the demand side, a concerted approach being necessary, at least when the problem is to stop a big inflation; f) stopping a big inflation involves the design of an equilateral triangle, the vertices of which are fiscal equilibrium, income policies and monetary reform; g) income policies can hardly be imposed without price controls, the political counterpart of a wage freeze being a general price freeze; h) price freezes may be the poison-pill of a stabilization package; i) wages, prices and exchange rates should never be frozen out of relative price equilibrium; j) the effectiveness of income policies can only be tested after they are abolished.

APPENDIX 1

THE MATHEMATICS OF STAGGERED WAGE SETTING

We shall now prove the three basic theorems on the mathematics of staggered wage setting mentioned in the preceding text. Key concepts and relations are provided by the following equations:

$$W(t) = \int_{t-1}^t S(\tau) d\tau \quad (\text{A.1})$$

$$W(t) = zQ(t) \quad (\text{A.2})$$

$$k(\tau) = \frac{S(\tau)}{Q(\tau)} \quad (\text{A.3})$$

$$z(\tau) = S(\tau) \int_{\tau}^{\tau+1} \frac{1}{Q(\sigma)} d\sigma \quad (\text{A.4})$$

From (A.4) it follows that, if $Q'(\sigma)/Q(\sigma) = \pi$ is a constant, then $Q(\sigma) = Q(\tau) e^{\pi(\sigma-\tau)}$, and:

$$z(\tau) = k(\tau) \frac{1 - e^{-\pi}}{\pi} \quad (\text{A.5})$$

Theorem 1: (The constant inflation rate theorem): If, for $t \geq 0$, $Q(t) = Q(0)e^{\pi t}$, π indicating a positive constant, then:

$$\lim_{t \rightarrow \infty} z(t) = z$$

and:

$$\lim_{t \rightarrow \infty} \frac{k(t)}{z(t)} = \frac{\pi}{1-e^{-\pi}}$$

Proof:

Taking derivatives in both sides of (A.1):

$$W'(t) = zQ'(t) = S(t) - S(t-1)$$

Dividing by (A.2):

$$\pi = \frac{Q'(t)}{Q(t)} = \frac{W'(t)}{W(t)} = \frac{S(t) - S(t-1)}{zQ(t)} = \frac{1}{z} \{k(t) - \frac{Q(t-1)}{Q(t)} k(t-1)\}$$

or, since $Q(t) = Q(t-1)e^{\pi}$:

$$k(t) - e^{-\pi} k(t-1) = \pi z$$

since $0 < e^{-\pi} < 1$, any solution of this difference equation converges to:

$$\lim_{t \rightarrow \infty} k(t) = \frac{z\pi}{1-e^{-\pi}}$$

It follows from (A.5) that:

$$\lim_{t \rightarrow \infty} z(t) = z$$

and, as a consequence:

$$\lim_{t \rightarrow \infty} \frac{k(t)}{z(t)} = \frac{\pi}{1-e^{-\pi}}$$

Before dealing with the peak/average ratio theorem, let us prove the:

Lemma 1: If $|r| < 1$, any solution of the differential-difference equation:

$$X'(t) = r(X(t) - X(t-1)) \quad (\text{A.6})$$

converges to a constant X .

Proof: For every integer n , let us define:

$$v_n = \max \{ |X'(t)| \mid n-1 \leq t \leq n \}$$

If $|X'(\tau)| = v_n$, then (A.6), combined with the Lagrange theorem, implies:

$v_n = |X'(\tau)| = |r X'(\tau-\theta)|$ where $0 < \theta < 1$. Since $|X'(\tau-\theta)| \leq \max \{v_n, v_{n-1}\}$ and since $|r| < 1$, this implies $v_n \leq |r| \max \{v_n, v_{n-1}\}$, or equivalently:

$$v_n \leq |r| v_{n-1} \quad (\text{A.7})$$

It follows that, $\lim_{t \rightarrow \infty} |X'(t)| \leq \lim_{n \rightarrow \infty} v_n \leq \lim_{n \rightarrow \infty} |r| v_{n-1} = 0$

According to (A.7), for any $0 \leq s < 1$

$$\sum_{n=1}^{\infty} |X'(s+n)| \leq \sum_{n=1}^{\infty} v_n \leq v_1 \frac{1}{1-r}$$

This implies that the series:

$$f(s) = \sum_{n=1}^{\infty} X'(s+n)$$

converges, since it is absolutely convergent.

From (A.6) it follows that:

$$\lim_{n \rightarrow \infty} X(s+n) = X(s) + \frac{f(s)}{r}$$

To complete the proof of the lemma it remains to prove that if $0 \leq s < s' < 1$, then $X(s') + \frac{f(s')}{r} = X(s) + \frac{f(s)}{r}$, or equivalently that:

$$\lim_{n \rightarrow \infty} X(s' + n) - X(s + n) = 0$$

Using once again Lagrange's theorem:

$$X(s' + n) - X(s + n) = (s' - s)X'(\alpha + n) \quad (s < \alpha < s')$$

which yields:

$$\lim_{n \rightarrow \infty} |X(s' + n) - X(s + n)| \leq \lim_{n \rightarrow \infty} v_n = 0$$

Theorem 2: (The peak/average ratio theorem): If, for all $t \geq 0$, $k(t) = rz$, r indicating a constant greater than one, then the inflation rate $Q'(t)/Q(t)$ converges to r , where:

$$r = \frac{\pi}{1 - e^{-\pi}}$$

Proof: $S(\tau) = rZQ(\tau)$, for $\tau \geq 0$. Hence, by equations (A.1) and (A.2), for $t \geq 1$:

$$Q(t) = r \int_{t-1}^t Q(\tau) d\tau \quad (\text{A.8})$$

Making:

$$X(t) = Q(t)e^{-\pi t} \quad (\text{A.9})$$

(A.8) is equivalent to:

$$X(t) = r \int_{t-1}^t e^{\pi(\tau-t)} X(\tau) d\tau \quad (\text{A.10})$$

From (A.9):

$$\frac{Q'(t)}{Q(t)} = \pi + \frac{X'(t)}{X(t)}$$

Hence, to prove the theorem, it suffices to prove that $X(t)$ converges to a positive constant.

Taking derivatives in both sides of (A.10):

$$X'(t) = re^{-\pi}(X(t) - X(t-1)) = \frac{\pi}{e^{\pi}-1}(X(t) - X(t-1)) \quad (\text{A.11})$$

Since $re^{-\pi} = \frac{\pi}{e^{\pi}-1} < 1$, it follows from lemma 1 that $X(t)$ conver

ges to a constant X . To complete the proof, we just need to show that $X > 0$.

For any integer n let us define:

$$m(n) = \min \{X(\tau) \mid n-1 \leq \tau \leq n\}$$

Since the price index $Q(t)$ is always positive, $m(n) > 0$. Let us assume that $X(t) = m(n)$, where $n-1 \leq t \leq n$. If $t=n$, equation (A.10) yields:

$$X(n) = m(n) = r \int_{n-1}^n e^{r(\tau-n)} X(\tau) d\tau \geq m(n)r \int_{n-1}^n e^{r(\tau-n)} d\tau = m(n)$$

the equality requiring $X(\tau) = X(n) = m(n)$ for all $n-1 \leq \tau \leq n$. In this case, (A.11) yields $X(t) = m(n) > 0$ for all $t \geq n-1$.

If $X(t) = m(n)$ and $n-1 \leq t < n$, equation (A.10) leads to:

$$X(t) = m(n) \geq r m(n-1) \int_{t-1}^{n-1} e^{r(\tau-t)} d\tau + r m(n) \int_{n-1}^t e^{r(\tau-t)} d\tau$$

which implies:

$$m(n) \geq m(n-1)$$

In either case, $m(n)$ is a non-decreasing sequence of positive real numbers, which proves that $X > 0$.

Theorem 3: (The non-neutrality theorem). If the inflation rate decreases for $t-1 \leq \tau \leq t+1$, then $z(\tau) > z$ for some $t-1 \leq \tau \leq t$.

Proof: We shall prove that the assumption of $z(\tau) \leq z$, for all $t-1 \leq \tau \leq t$, is inconsistent with that of a declining inflation rate.

In fact, to say that the inflation rate decreases for $t-1 \leq \tau \leq t+1$, is equivalent to saying that, in that interval, the function:

$$q(\sigma) = \ln Q(\sigma)$$

is strictly concave. Hence, if $m=q'(t)$, the curve lies below the tangent at the point $(t;q(t))$, namely:

$$q(\sigma) - q(t) \leq m(\sigma - t)$$

the equality holding only when $\sigma=t$. It follows that:

$$Q(\sigma) \leq Q(t)e^{m(\sigma-t)}$$

and as a consequence that, for $t-1 \leq \tau \leq t$:

$$\int_{\tau}^{\tau+1} \frac{1}{Q(\sigma)} d\sigma > \frac{1}{Q(t)} \frac{1-e^{-m}}{m} e^{m(t-\tau)}$$

Let us assume that $z(\tau) \leq z$ for all $t-1 \leq \tau \leq t$. It follows, from (A.2) and (A.4) that:

$$S(\tau) < \frac{m}{1-e^{-m}} W(t) e^{m(\tau-t)} \quad (\text{A.12})$$

45 Introducing (A.12) in (A.1) yields the contradiction $W(t) < W(t)$.

APPENDIX 2

Let us now determine the maxmin strategy for the price-setting game discussed in section 2, assuming that the general price level may lie in any point of the several $P_0 \leq P \leq P_1$, where $P_0 > 0$ and $P_1 = P_0(1+r)$.

According to equations (4), (9) and (10), individual's x utility function is expressed by:

$$U_x(P_x, P, R) = \begin{cases} \frac{RP^{m-1}}{P_x^m} \left\{ 24 - \frac{RP^m}{P_x^{m+1}} \right\}^b, & \text{if } \frac{RP^m}{P_x^{m+1}} \leq \frac{24}{1+b} \\ \frac{P_x}{P} \frac{24}{1+b} \left\{ \frac{24b}{1+b} \right\}^b, & \text{if } \frac{RP^m}{P_x^{m+1}} \geq \frac{24}{1+b} \end{cases}$$

If $m \leq 1$, given P_x and R , U_x is a decreasing function of P . It follows that, in this case

$$\min_P U_x(P_x, P, R) = U_x(P_x, P_1, R)$$

The maxmin strategy is to choose P_x so as to maximize $U_x(P_x, P_1, R)$. This, according to equation (16), yields:

$$P_x^{m+1} = \frac{1+b}{24} R P_1^{m+1} \quad (\text{A.13})$$

Leading to:

$$\max_{P_X} \min_P U_X(P_X, P, R) = \left\{ \frac{24}{1+b'} \right\}^{\frac{m}{m+1}} \left\{ \frac{24b'}{1+b'} \right\}^b \left\{ \frac{R}{P_1} \right\}^{\frac{1}{m+1}}$$

This is nothing but equation (14), which gives $\max_{P_X} U_X(P_X, P, R)$

taking $P = P_1$. Since the right side of equation (14) is a decreasing function of P it follows that, it $m \leq 1$:

$$\max_{P_X} \min_P U_X(P_X, P, R) = \min_P \max_{P_X} U_X(P_X, P, R)$$

which shows that the utility function has a saddle-point for $P = P_1$ and P_X given by (A.13)

Let us now discuss the case where $m > 1$. Given P_X and R , $U_X(P_X, P, R)$ increases until a certain value of P and decreases thereafter. It follows that:

$$\min U_X(P_X, P, R) = \min \{U_X(P_X, P_0, R), U_X(P_X, P_1, R)\} \quad (A.14)$$

Let us make:

$$P_X = \frac{1+b'}{24} RZ^m \quad (A.15)$$

which implies that $U_X(P_X, P, R)$ increases with P as long as $P < Z$, decreasing for $P > Z$. One immediately concludes that the maxmin strategy must set:

$$P_0 \leq Z \leq P_1 \quad (A.16)$$

since, outside this interval it would be possible to increase both $U_X(P_X, P_0, R)$ and $U_X(P_X, P_1, R)$. Making:

$$V(Z, P) = U_X(P_X, P_0, R)$$

we are led to a game in the square where both, Z and P must lie in the interval $[P_0, P_1]$. Equation (A.15) can be rewritten in the form:

$$\min_{P_X} U_X(P_X, P, R) = \min \{V(Z, P_0), V(Z, P_1)\} \quad (A.17)$$

Since $\max_Z V(Z, P) = V(P, P)$, it follows that $V(Z, P_0)$ is a decreasing function of Z and $V(Z, P_1)$ is an increasing function of Z . It should be noted, moreover, that, since $V(P, P)$ is expressed by equation (14), the second member of which is a decreasing function of P :

$$V(P_0, P_0) > V(P_1, P_1). \quad (A.18)$$

It follows that the maxmin strategy should set Z according to:

$$\max_{P_X} \min_P U_X(P_X, P, R) = \max_Z \{\min V(Z, P_0), V(Z, P_1)\} \quad (A.19)$$

Figures A.1 and A.2 indicate two possibilities: a) $V(P_1, P_0) \geq V(P_1, P_1)$, b) $V(P_1, P_0) < V(P_1, P_1)$. In the first case, the maxmin strategy sets $Z = P_1$, as in the previously discussed hypothesis $m \leq 1$.

The utility function has, in this case, a maddle-point for $Z = P = P_1$. In the second case the maxmin strategy sets Z so that:

$$V(Z, P_0) = V(Z, P_1) \quad (A.20)$$

In this case, $\max_Z \min_P V(Z, P) < \min_P \max_Z V(Z, P)$, opening the room for the choice of mixed strategies.

Let us finally determine the necessary and sufficient condition for the existence of a saddle-point equilibrium where $Z=P=P_1$. As previously noted, this happens if and only if $V(P_1, P_0) \geq V(P_1, P_1)$.

Making $K = (1+r)^{-1} = P_0/P_1$, elementary computations yield:

$$F(k) = \frac{V(P_1, P_0)}{V(P_1, P_1)} = \frac{k^{m-1} (1+b'-k^m)^b}{(b')^b}$$

which is to say that the saddle-point equilibrium exists if and only if $F(x) > 1$. Since $0 \leq k \leq 1$, the condition is immediately fulfilled if $m \leq 1$. The condition is also met in some left neighborhood of $k = 1$, since

$$\frac{F'(1)}{F(1)} = m-1 - \frac{b}{b'} m = -\frac{1}{m+1} < 0$$

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