AN EQUILIBRIUM THEORY OF THE BUSINESS CYCLE UNDER CERTAINTY*

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Abstract

This paper develops a full-employment theory of the business cycle in the context of a simple equilibrium dynamic monetary model with complete information, overlapping generations and production processes. This theory is founded on three basic notions: (a) economic agents plan to limit their own freedom of action; (b) it takes time to produce one unit of output; the productive input can be employed on alternative discrete overlapping production processes at each point in time. According to this theory, fully anticipated monetary shocks cause the kind of movements in outputs, prices and nominal incomes, which are well known as the business cycle.

1. Introduction.

The aim of this paper is to present a full-employment theory of the business cycle in the context of a simple equilibrium dynamic monetary model with complete information and overlapping production processes.

This theory rests (a) on the assumption that individuals and firms hold to their nominal economic decisions for at least a number of periods and (b) on the notion that it takes time to obtain one unit of output.

We do not intend to discuss in detail any of the prominent literature on the business cycle. Nevertheless, we shall attempt to indicate concisely how the framework to be developed here relates to the leading model presented by Lucas (1975, 1977), from which it springs.

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Lucas' goal is to derive a Hayekian equilibrium theory of the business cycle (Hayek 1933, p. 33n) according to which all profit (or utility increasing) opportunities are fully exploited by the economic agents. To accomplish this task Prof. Lucas (1975) develops an exploratory theory in which serially correlated, "cyclical" changes in real output about trend are triggered by unsystematic monetary shocks in the context of a competitive equilibrium model with efficiently processed, though incomplete information. However, as he illustrates it, the mere introduction of noise into the monetary policy is "not sufficient to induce the sort of responses in real and nominal variables which occur during the observed business cycle. The problem is that in economy in which all trading occurs in a single competitive market, there is too much information in the hands of traders for them ever to be fooled into altering real decision variables" (Lucas 1975, p. 1120). And so it is *crucial*, for his argument, that the economic agents make a confusion between relative and general price movements, between real and nominal changes, as these changes and movements occur (see Lucas 1977, p. 21).

To get an analytically convenient kind of ambiguous informational environment, Lucas then thinks of "production and trade as occurring in a large number of markets which are imperfectly linked both physically and informationally", with traders distributed in some way over these markets, at the beginning of a period. "This... analytical device first proposed by Phelps (1969)... leads to a real response to a purely nominal disturbance"... However, these real movements are of no longer duration than the duration of the shock: no forces are present to account for the persistence of accumulation of the effects of the initial disturbance (Lucas 1975, p. 1114). So, to get away from this analytical insufficiency he introduces two of these forces: "informational lags, such as to prevent even relevant past variables from becoming perfectly known, and physical capital, introducing a form of the familiar accelerator effect" (Lucas 1975, p. 1114).

The goal of this paper is also to present a competitive equilibrium theory of the business cycle. However, we shall attempt to fulfill this design in the context of a perfectly forecasted, certainty model. As it seems to us, the inclusion of uncertainty does not alter significantly the theoretical content of the economic models. It serves basically for imposing negative real income effects on eco-

nomic agents (risk is costly) and substitution effects against the risky states, with the purpose of obtaining analytically convenient results.

But uncertainty is just a device. If we could obtain the same basic results out for a certainty model we might at least gain in analytical simplicity. The central economic ingredient of Lucas's theory, for instance, is not the existence of risk, but the existence of informational lags, which are imposed from outside the model [see Lucas (1975, p. 1121)]. In this respect, as he (*ibid.* p. 1143) has hinted and as Edi Karni (1981) has shown, the introduction of a market capable of conveying speedy information about relevant aggregate economic variables, and thus, capable of circumventing these lags, defuses the cycle effect presented in Lucas's model, despite the persistence of uncertainty.

In this sense, the fundamental task of this paper is to attempt to show how that central ingredient can be substituted by another one, under certainty, yielding the same basic results.

The thrust of our argument is as follows: observe that the business cycle is well described by serially correlated movements in output about trend. These movements do not exhibit uniformity of either period or amplitude (Lucas 1977, p. 9). The key characteristic of this picture is the *recurrent* nature of these movements (Lucas 1977, p. 14).

One way of considering this characteristic is to treat the economic agents as reacting to cyclical changes as if they were facing a risk, as Lucas has done (1975, p. 1121 and 1977, p. 15).

Another course, which will be adopted in this paper, is to think of the recurrent character of the business cycle as reflecting mainly the fact that the relevant concept of real resources and of full employment is related to a length of time, not to a point in time. The cyclical pattern would then, essentially, result from the economy's reallocation of some given amount of employment, and so of real output, among points in time within the cycle, in response to external shocks. In this context, an equilibrium theory of the business cycle should then be able to explain how that reallocation of output over time can be *efficiently* set forth.

In order to set the appropriate stage for this type of analysis we shall depart from Lucas in some important respects:

Firstly, we shall introduce into the analysis the following notions: a) it takes time to produce one unit of output; b) produc-

tion processes overlap. This will make current economic decisions entirely dependent upon future prices and incomes.

Secondly, instead of thinking of traders (a) as distributed over a large number of "islands" which are imperfectly linked both physically and informationally and (b) as making their economic decisions simultaneously, at the beginning of each period, we will think of them (a) as located in a single competitive market and (b) as making their production-consumption decisions at different points in time, not simultaneously.

Finally, instead of assuming that (a) there are informational lags and (b) that all traders can review simultaneously their decisions at the beginning of each period, we will assume that (a) there is complete information and that (b) as in Martins (1980), individuals stick to the nominal economic decisions taken at each point in time, for at least a number of periods. In addition we will also assume that firms plan ahead but not too far ahead of time; that the length of their relevant planning period is limited. This means that although they may now worry about the next five, they will hardly begin to bother about the entire next ten crops of wheat.

Given this environment, we shall attempt to show that the time paths of nominal prices, of wages and of income, conform to the money supply. However this will occur differentially, both in timing and degree. As a result, fully anticipated monetary disturbances will bring about real effects and will induce firms to reallocate output towards the points characterized by relatively higher prices and incomes.

In the next two sections we present the theoretical framework for deriving the individuals's demands for consumption and for the monetary asset, and for deriving the general price level and the aggregate demand of the economy. This framework will be entirely based on Samuelson (1958) and on Martins (1975 and 1980).

In the forth section we will present the production side of the model. It is based upon the notions that it takes time to produce one unit of output and that production processes overlap at each point in time. As we shall argue briefly, in the next section, these two basic assumptions alone are enough give rise to Okun's law.

In the fifth section we will derive the demands for labor and the supply of output as functions of past decisions and of the anticipated aggregate demands of the economy. As we shall demon-

strate, the time path of this last variable can be neatly represented by a difference equation of low order.

Finally, in the sixth section we present the market solution for the entire model, and in the seventh section we use it for simulating the response of output, prices and of the conventionally measured income velocity of the demand for money, to a once-and-for-all and to a temporary, fully anticipated increase in the supply of money. These simulations exhibit both a high degree of conformity among these variables, and trade-offs between present and future productions, as implied by our simple equilibrium theory of the business cycle.

2. The consumption decisions.

The purpose of this section is to present a theoretical framework for deriving a demand function for money and a theory of the general price level. This framework will be entirely based on Samuelson's 1958 exact consumption-loan model of interest (Samuelson 1958, pp. 467-82).

Samuelson's model deals with the divergence between the competitive and the social optimum opportunity set for consumption, in a simple growth model with overlapping generations and selfish individuals. The source of the divergence lies in the fact that in the context of the model it is impossible to motivate any voluntary private transfer of real resources from incoming (potential savers) to outgoing generations (potential dis-savers) and that leads to the shrinking of the per capita consumption below the social optimum level. This level could nonetheless be attained by a Hobbes-Rousseau type of social contract, under which the old generation would have a claim on part of the output produced by the younger one living in the same period. This contract could be materialized by the issuance of a "contrivance money" by a central authority.

As we have argued elsewhere (Martins 1975, pp. 14-21, 61-66) Samuelson's model is highly useful for monetary analysis, since it generates a demand for securities not backed up by physical capital, only by public trust. Moreover, it gives rise to a theory according to which, the price level exhibits high conformity with the supply of government securities, as we have shown [Martins (1980 p. 183, and 1988 p.7)]. It commands then a great deal of appeal

for serving as foundation for deriving the aggregate demand side of a theory for the business cycle.

However, in the context of the model's original version, the path of the market output, and thus the summation at a point in time of the paths of individual consumptions is exogenously set by assuming that each individual earns one unit of output in the first period of his life. This fact rules out from the outset any type of theoretically interesting cyclical phenomenon, both at the market and at the individual level.

To get the adequate set up, it is necessary, instead, to let the market output respond to demand pressures. This will be essentially accomplished by assuming, in this paper, that the quantity of output earned by each individual in the first period of his life will be endogeneously determined by the model. To simplify, we assume that there exists only one tradeable asset, government fiat money. To highlight the role of time we shall use the three-periods version of Samuelson's framework. The aggregate demand side of the model is as follows.

Generations overlap. An individual's life only covers three periods. Each individual earns Q_t units of output in the first period of his life, nothing afterwards. Output cannot be stored; it melts away in one period. At each point in time t , exactly one member of each generation is alive simultaneously. Let $C_{t,j}$, for the j -th period of life consumption by a member of the i -th generation. A "representative" member of the i -th generation is assumed to value his consumption plan according to the value of a "regularly shaped" utility function $U_t(C_{t,1}, C_{t,2}, C_{t,3})$, nonnegative consumptions $C_{t,j}$, twice differentiable, strictly concave, strictly monotonic increasing function, and the marginal utility of consumption in any period goes to infinity as the afore-mentioned consumption level in that period goes to zero. This last condition guarantees that, if income is positive, consumption in any period is also positive.

In the absence of a social contract, each individual would consume all his income in the first period of his life. However, government intervenes: It issues fiat money which is trustingly held by the public on their own free will, to provide for consumption during the second and the third periods of their lives. The aggregate demand side of the model can then be represented by the following problem: Maximize $U_t(C_{t,1}, C_{t,2}, C_{t,3})$

to $(M_{t,2}, M_{t,3})$, subject to

$$P_t C_{t,1} + M_{t,2} = P_t Q_t + G_t \quad (1a)$$

$$P_{t+1} C_{t,2} + M_{t,3} = M_{t,2} \quad (1b)$$

$$P_{t+2} C_{t,3} = M_{t,3} \quad (1c)$$

where $M_{t,2}$ and $M_{t,3}$ are the nominal quantities of money carried over periods $t+1$ and $t+2$; P_t, P_{t+1}, P_{t+2} are the nominal prices of consumption in periods $t, t+1, t+2$. At the beginning of period t , the individual earns $P_t Q_t$ units of nominal income and anticipates G_t units of nominal government transfer payments. To solve his problem, he takes $P_t, P_{t+1}, P_{t+2}, Q_t, G_t$ as given and chooses $M_{t,2}$ and $M_{t,3}$. The supply side of the model is represented by government policy with respect to $G_t, M_{t,2}$ and $M_{t,3}$. We shall integrate this side of the model into the analysis at a later stage. Let us now solve the demand functions for government fiat money and the aggregate nominal income. In order to simplify, we shall specialize our utility function.

3. The demand for money and the aggregate nominal income.

The solution to the problem mentioned above, yields the demand functions for $M_{t,2}$ and $M_{t,3}$. With "regularly shaped" utility functions, $C_{t,2} > 0$ and $C_{t,3} > 0$; then $M_{t,2} > 0$ and $M_{t,3} > 0$. The first-order conditions are:

$$\frac{\partial U_t}{\partial C_{t,1}} = \frac{P_t}{P_{t+1}} \cdot \frac{\partial U_t}{\partial C_{t,2}}, \quad (2a)$$

$$\frac{\partial U_t}{\partial C_{t,1}} = \frac{P_t}{P_{t+2}} \cdot \frac{\partial U_t}{\partial C_{t,3}}, \quad (2b)$$

$$P_t C_{t,1} = P_t Q_t + G_t - M_{t,2}, \quad (2c)$$

$$P_{t+1} C_{t,2} = M_{t,2} - M_{t,3}, \quad (2d)$$

$$P_{t+2} C_{t,3} = M_{t,3}. \quad (2e)$$

For given values of $P_t, P_{t+1}, P_{t+2}, Q_t$ and G_t the system above determines the consumption plan $(C_{t,1}, C_{t,2}, C_{t,3})$ and the demands for $M_{t,2}$ and $M_{t,3}$. Together with government policy for $M_{t,2}$ and $M_{t,3}$ it becomes a theory of simultaneous determination

for the current price level P_t . Aiming toward simulations, we shall investigate in this paper only a simple, particular solution. Assume that the utility function is represented by

$$U_t = \log C_t, \quad (3)$$

the marginal first-order conditions then become:

$$\frac{C_{t,1}}{C_{t,1}} = \frac{P_t}{P_{t+1}} \quad (4a)$$

$$\frac{C_{t,3}}{C_{t,1}} = \frac{P_t}{P_{t+2}} \quad (4b)$$

Inserting (2c), (2d) and (2e) into (4a) and (4b), and simplifying, we obtain:

$$2(M_{t,2} \quad (4a')$$

$$(M_{t,2} \quad (4b')$$

By collapsing them, and also by inserting the results into (2a), we find the demand functions for $M_{t,2}$

$$M_{t,2} \quad 2 \quad (5a)$$

$$M_{t,3} \quad 1 \quad (5b)$$

$$P_t C_{t,1} \quad 1 \quad (5c)$$

As can be readily observed, equations (5a) and (5b) are perfectly analogous to the quantity theory of money, at the individual level. However, as it will become clear, along this paper, the value of the income velocity of the demand for money, at the market level, which will be presented in the sixth section, is not generally a good guess for the individuals's demand for money.

Together with (5c) those equations imply that the individual spends M_t ,
So his perceived nominal income is always equal to three times M_t ,
how this nominal income is split between prices and quantities.

But now we just want to use the properties of the solution above, to obtain a very simple representation of the time path of the economy's aggregate nominal expenditures.

To do that, note that at the beginning of period t the member of the old generation spends all his available stock of money, $M_{t-2,3}$, the new generation spends $M_{t-1,2}$, the new generation accumulates $M_{t,2}$, the total stock of money by G_t . Transactions equal deaccumulations plus net additions; so we can readily assert that

$$M_{t,2} = G_t - M_{t-1,2} + M_{t-2,3} \quad (6)$$

which shows how individual money holdings are linked together throughout time. It can also be seen that, given the past history of these holdings, government policy with respect to G_t determines $M_{t,2}$ income.

Now, note that in view of (5a) and (5b),

$$M_{t,2} = G_t - M_{t-1,2} + M_{t-2,3} \quad (7)$$

for all t . By inserting this equality, for t and for $t - 1$, into (6), we obtain:

$$M_{t,3} = M_{t-1,2} - M_{t-2,3} + G_t - G_{t-1} \quad (8)$$

Finally, by inserting $G_t = 3M_{t,3}$ we get

$$P_t Q_t = M_{t,3} \quad (9)$$

which shows how the current aggregate nominal income conforms to the past history of the individuals' money holdings; or else, to the sum of their current expenditures.

In the next section we shall present the production side of our theoretical framework.

4. The production decisions.

The aim of this section is to lay down the output supply side of our theoretical framework. To do that, we shall depart

from Samuelson's 1958 pure consumption-loan model of interest (Samuelson 1958, pp. 467-82), by assuming that the stream of non-storable goods is produced by employment of non-storable labor services bought in competitive markets, rather than freely provided by the heavens.

But that is not enough. To get the kind of cyclical behavior we have in mind, it is not sufficient to let output vary over time; not even to assume that it takes time to complete the production process.

For instance, the assumption that today's output is a function of yesterday's flow of factor services only retards in a "mechanical" way, the time path of the former with respect to the latter. By "mechanical" we mean that the resulting lag structure of models encompassing this assumption is not a matter of economic choice, which would take into account present and future prices. It is just built from outside these models and so it does not bring about any relevant economic trade-off, between present and future.

In order to circumvent this problem we shall introduce below, as one of the most important concepts in this paper, the notion that the non-storable flow of labor services available at the beginning of each period can be alternatively employed, depending on choice, to start new production processes or to reinforce processes which are underway or about to be completed. Therefore we shall assume below not only that it takes time to produce one unit of output but also that production processes overlap.

This device is essentially analogous to the one built by Shleifer (1986, p. 1168) to generate a cyclical supply of innovations within his model of business cycle. According to him, overlapping firms produce inventions costlessly at different times, and at an ad-hoc rate of production which is independent of any market forces. These inventions do not need, however, to be transformed into innovations immediately after they have been made. On the contrary, each firm can "save" its own invention to use it latter on, to maximize the present value of its profit, in response to market forces. Consequently, Shleifer obtained an aggregate supply of innovations over time, by all firms, which is both responsive to entrepreneurs's expectations about the future state of the economy, and capable of displaying cyclical behavior.

Observe that the key assumption, here, is not that the production processes overlap. In his model, as well as in ours, the

key assumption is that the utilization of the theoretically relevant input (inventions in his, and labor in our case) can be shifted from current to future productions (of innovations in his, and of goods in our case) to maximize entrepreneurs's profits over time.

To illustrate the analytical potentialities of this hypothesis, let us assume for a moment that an infinite lived unit of labor is capable of delivering one unit of non-storable output at the end of a three-period production process. If that unit is fully employed, its contribution to the time path of the market output would look like: ... 0,0; 1,0,0; 1,0,0; 1... .

Now let us assume that there are nine of such units of labor and ask what would the time series of the market output look like. The answer is that it depends on how the beginning of the individual productions were originally distributed throughout time.

For instance, if all individuals started their production activities together, the path of the market output would look like: ... 0,0; 9,0,0; 9,0,0; 9,0,0; 9, If instead, the origin of these activities were evenly distributed throughout time, the resulting series of output would be: ... 3,3; 3,3,3; 3,3,3; 3,3,3; 3, Still another possibility would be: ... 5,2; 2,5,2; 2,5,2; 2,5,2; 2, In all these cases the labor force would have to be fully employed all the time. The corresponding time path for the market employment level would then be given by: ... 9,9; 9,9,9; 9,9,9; 9,9,9; 9,

This very simple set up gives us, we believe, some important messages. One, is that full employment alone is far from being a good guess for social optimality. If the individuals in the economy above, need badly to consume during all periods of their lives, they will then prefer a positive output series, over any one containing points of zero total production.

Another is that the employment level at any point in time is not sufficient to forecast output flow at any further date. In order to predict the time path of the output we first need to know how the origins of the individual production activities over time, were distributed.

Another, still more important matter is that it is impossible to change from one to another steady state position without first causing some unemployment during the transitional period. As an example, assume that we want to move from a steady state position like: ... 5,2; 2,5,2; 2,5,2; 2,5,2; 2,5,2; 2; ... , to another one given by: ... 3,3; 3,3,3; 3,3,3; 3,3,3 3; Assume also that

the first term of the former series is the current outcome. Hence there are five units of labor which have just completed another production cycle. Under these circumstances, and disregarding consumer's preferences, the most efficient way of performing this change is by leaving one unit of this labor force unemployed by one period, and another by two periods, as the reader can easily confirm by himself. There is no way to perform such a task without generating at least this amount of unemployment during the transition.

Now, as a final illustration of the kind of insights brought about by the explicit consideration of overlapping production processes, let us consider the following less-than-full-employment sequences of output, delivered by the above mentioned nine units of labor: ... 1,2,3,2,1; 1,2,3,2,1; 1,2,3,2,1 ; As can be readily checked, the sum of the quantities of output over any sequence of five successive points is equal to 9.

In that respect observe first that unemployment will basically be reflected as an increase in the number of periods it takes to obtain the full-employment market production, which is equal to 9 units of output over three periods.

Moreover, remember that, by assumption, it takes exactly three periods to produce one unit of output. Thus, the total number of individuals employed at any point in time has to be equal to the sum of the output market production over the three next points in time. This allows us to construct easily, the path of the economy's employment level, for the mentioned sequences of output, which are under consideration. It will be: ... 6,7,6,4; 4,6,7,6,4; 4,6,7,6,4; 4; Many other examples could be built. If we did that, we would observe that:

a) employment lags with respect to output. This lag is equal to the (average) number of periods in which each individual remains unemployed throughout the cycle — two periods, in the above case;

b) the level at which employment peaks is less than full-employment; and

c) the employment cycle is smoother than the market output cycle, a phenomenon which, according to current macroeconomics, is known as "Okun's law".

Despite all the possibilities for discussion, in this paper we shall concentrate our attention only on a simple full-employment

business cycle. The production side of the model is as follows.

Output is produced by firms in competitive markets. It takes three periods to complete a production process. There is only one variable factor of production, labor services, which cannot be stored; it melts away in one period. At each point in time t there is only one source of supply for labor services, the member of the t -th generation, who also inherits the non-negotiable shares of the firms. Production processes overlap. At each point in time t there is one process underway which was initiated one period earlier, at $t-1$; another process was initiated two periods before, at $t-2$; one more, just completed, commenced three periods earlier, at $t-3$; and another one is just about to be initiated. So, the labor services available at each point in time t can be alternatively allocated, or to continue the production processes initiated at $t-2$ and $t-1$, or to start a new production process. We also assume that people do not begin looking for the robin in the middle of winter; or in other words, that people only begin planting crops to harvest four periods later after they have planted the ones to ripen three periods later. This means that the only new process in which the labor available at t can be applied is the one to be completed three periods later, and that the t -th firm will be concerned only with the productions to be finished at $t+1$, $t+2$ and $t+3$, leaving the longer range ones, to be dealt with by future generations.

So let L_{it} be the labor services employed in the production processes to be completed j periods later, $j = 1, 2, 3$. A "representative" firm of the t -th generation is assumed to obtain its output according to a "regularly shaped" production function $f_t(L_{t-3,t}, L_{t-2,t}, L_{t-1,t}, L_t)$, where $L_{t-3,t}$ is negative quantities of labor services. The production function has the usual monotonic and curvature properties. The production side of the model can then be represented by the following problem. Maximize $P_{t+1}Q_{t+1} + P_{t+2}Q_{t+2} + P_{t+3}Q_{t+3} - W_t(L_{t,t+1} + L_{t,t+2} + L_{t,t+3})$ subject to

$$Q_{t+1} = f_{t+1}(L_{t-2,t+1}, L_{t-1,t+1}, L_{t,t+1}) \quad (10a)$$

$$Q_{t+2} = f_{t+2}(L_{t-1,t+2}, L_{t,t+2}, L_{t+1,t+2}) \quad (10b)$$

$$Q_{t+3} = f_{t+3}(L_{t,t+3}, L_{t+1,t+3}, L_{t+2,t+3}) \quad (10c)$$

where S_t is the nominal wage rate at the beginning of period t . To solve this problem the firm takes as given $L_{t-2,t+1}$, $L_{t-1,t+1}$, $L_{t,t+1}$, $L_{t-1,t+2}$, $L_{t,t+2}$, $L_{t+1,t+2}$, $L_{t,t+3}$, $L_{t+1,t+3}$, $L_{t+2,t+3}$.

L_{t-1} ,
chooses the employment plan ($L_{t,t+1}$, L_t ,
order conditions are:

$$P_{t+1} \frac{\partial f_{t+1}}{\partial L_{t,t+1}} = W_t \quad (11a)$$

$$P_{t+2} \frac{\partial f_{t+2}}{\partial L_{t,t+2}} = W_t \quad (11b)$$

$$P_{t+3} \frac{\partial f_{t+3}}{\partial L_{t,t+3}} = W_t \quad (11c)$$

which simply means that the value of the marginal productivity of labor of the t -th generation must be the same across all the production processes which overlap at the beginning of period t , and equal to the nominal wage rate, W_t , at this point in time. This means, ultimately, that for a given wage rate W_t the t -th generation firm will attempt to shift its available amount of resources, and thus of output, towards the points, within its decision period, for which the price levels are relatively larger.

This firm treats the problem of allocating a given amount of labor input among different points within its decision period as basically analogous to the problem of distributing this same amount of resources among alternative uses at a point in time within a space. However, while in space it is possible to go back and forth, there is no way to go back in time.

For given values of $L_{t-2,t+1}$, $L_{t+2,t+3}$, P_{t+1} , P_{t+2} , P_{t+3} , W_t , the above mentioned system, together with the set of equations (10a), (10b) and (10c), determine the labor input plan (L_t , (Q_{t+1} , Q_{t+2} , Q_{t+3}). In the next section we shall investigate the corresponding solutions for the demands for labor and for the supplies of output. To simplify, and pointing to simulations, we shall specialize our production function.

5. The demand functions for labor and the supplies of output.

Assume that the production function is represented by

$$\frac{1}{\alpha} \log Q_t = \log L_{t-3,t} + \log L_{t-2,t} + \log L_{t-1,t}, \quad (12)$$

for all t , where the degree of homogeneity, given by 3α , is assumed to be smaller than one. The marginal first-order conditions then become:

$$\alpha P_{t+1} \frac{Q_{t+1}}{L_{t,t+1}} = W_t \quad (13a)$$

$$\alpha P_{t+2} \frac{Q_{t+2}}{L_{t,t+2}} = W_t \quad (13b)$$

$$\alpha P_{t+3} \frac{Q_{t+3}}{L_{t,t+3}} = W_t \quad (13c)$$

according to which in order to solve for the labor input plan $(L_{t,t+1}, L_{t,t+2}, L_{t,t+3})$, needs only to know the wage rate at point t , W_t , and the values of the nominal incomes $P_{t+1}Q_{t+1}$, $P_{t+2}Q_{t+2}$, $P_{t+3}Q_{t+3}$. It can, for this purpose, completely disregard the decisions with respect to $(L_{t+1,t+2}, L_{t+1,t+3}, L_{t+2,t+3})$, to be taken by the two next generations. In our specialized world, the knowledge of only a few key macroeconomic variables is enough to orient employment decisions at the level of individual firms.

With respect to this solution observe that doubling W_t and all the prices P_{t+1} , P_{t+2} , and P_{t+3} within the decision period, would leave the distribution of the labor services unchanged among the overlapping production processes.

However, a doubling of only one or two of these prices would have distributional effects toward the points of relatively larger prices, and with relatively larger initial labor inputs.

There is no way of knowing what is going to be the response of tomorrow's output to a fall in current wage rates without considering the after-tomorrow's price level.

Moreover, note that according to the above solution, the time path of the nominal income, over any three successive periods, can be easily described by a difference equation of very low order. To see that, just remember that the total amount of labor services demanded by the t -th firm at the beginning of period t , is given by:

$$L_t = L_{t,t+1} \quad (14)$$

By plugging (13a), (13b) and (13c) into the above expression, and manipulating it we obtain immediately:

$$P_{t+1}Q_{t+1} + P_{t+2}Q_{t+2} + P_{t+3}Q_{t+3} = \frac{1}{\alpha} W_t L_t \quad (15)$$

as stated. Although we do not plan to deal with equilibrium unemployment in this paper, we cannot help, but register, that according to our intuition less-than-full-employment cycles would also be indicated by the same general type of expression, the only difference being that the summation would have to be taken over a number of periods larger than three, for each amount of labor services demanded.

Now let us solve (12), (13a), (13b) and (13c) regarding the demand functions for labor, and for the supplies of output. To do that, let us expand the set of first-order equilibrium conditions (13a), (13b) and (13c) to encompass the past. Assuming perfect-foresight we can affirm that:

$$\alpha P_{t+1} \frac{Q_{t+1}}{L_{t-2,t+1}} = W_{t-2} \quad (16a)$$

$$\alpha P_{t+1} \frac{Q_{t+1}}{L_{t-1,t+1}} = W_{t-1}. \quad (16b)$$

By collapsing (13a), (16a) and (16b) we easily find that

$$W_{t-2} L_{t-2,t+1} = W_{t-1} L_{t-1,t+1} = W_t L_{t,t+1} \quad (17a)$$

which says that the nominal wage outlays made by the firms at the beginning of periods $t-2$, $t-1$ and t , to obtain a given amount of output Q_{t+1} , at the end of period $t+1$, are all equivalent. Moreover, it also says that the input labor ratio between any of these points in time must be equal to the ratio between the corresponding wages. That is, that

$$\frac{W_{t-2}}{W_t} = \frac{L_{t,t+1}}{L_{t-2,t+1}}; \quad \text{---} = \frac{L_{t,t+1}}{L_{t-1,t+1}} \quad (17b)$$

By plugging the production function (12), and (17b), into (13a), and manipulating, we find

$$(L_{t,t+1})^{1-3\alpha} = \frac{\alpha P_{t+1}}{W_{t-2}^\alpha W_{t-1}^\alpha W_t^{1-2\alpha}} \quad (18)$$

which is the expression of the demand for labor L_t , of the past wages W_{t-2} , W_{t-1} , of the current W_t , and of the future

price P_{t+1} . Since it can be directly generalized for $L_{t,t+2}$ and $L_{t,t+3}$, we pass immediately to the solution for the level of output Q_{t+1} . This can also be readily found by expressing the labor inputs $L_{t-2,t+1}$, $L_{t-1,t+1}$ and $L_{t,t+1}$ as functions only of wage rates W_{t-2} , W_{t-1} , W_t and of the price level P_{t+1} , by using (17b) and (18), and plugging the result into the production function (12). The result, which may also be straightforwardly generalized for Q_{t+2} and Q_{t+3} , is given by:

$$(Q_{t+1}) = \left[\frac{\alpha P_{t+1}}{W_{t-2}W_{t-1}W_t} \right]^{\frac{\alpha}{1-3\alpha}} \quad (19)$$

We now pass to the market solution.

6. The market solution.

At the beginning of period t the member of generation $t - 2$ wants to spend all his money holdings, $M_{t-2,2}$; the member of generation $t - 1$ wants to retain $M_{t-1,3}$, and the member of generation t wants to accumulate $2M_{t,3}$. The sum of these last two figures adds up to the total nominal stock of money, M_t , available at the beginning of period t . The demand for $M_{t,3}$ is given by equation (5b) and the supply of money, M_t , is given by the government policy with respect to G_t . For given values of $M_{t-2,3}$ and of M_{t-1} , the money market of our model can be fully represented by the following set of equations:

$$M_{t,3} = \frac{1}{3}(P_t Q_t + G_t) \quad (5a)$$

$$2M_{t,3} + M_{t-1,3} + M_t \quad (20)$$

$$G_t = M_t - M_{t-1} \quad (21)$$

which determines $M_{t,3}$ for a given value of the current aggregate nominal income $P_t Q_t$.

The consumption goods market is composed of the aggregate demand and of the supply of output at each point in time. The first one, given by equation (9), is entirely described by a history of the individuals's money holdings; the second is given by the production function (12). This market can then be fully represented

by:

$$P_t Q_t = M_{t,3} + M_{t-1,3} + M_{t-2,3} \quad (9)$$

$$\frac{1}{\alpha} \log Q_t = \log L_{t-3,t} + \log L_{t-2,t} + \log L_{t-1,t}. \quad (12)$$

At the beginning of period t , the government's policy with respect to G_t , in the money market, determines the total nominal quantity of money $M_{t,3}$ to be retained by the member of generation t . So for given values of $M_{t-1,3}$ and $M_{t-2,3}$, it also determines the current aggregate demand of the economy. Moreover, for given values of $L_{t-3,t}$, $L_{t-2,t}$ and $L_{t-1,t}$, it determines the current quantity of output, Q_t . Thus, equation (9) and (12) determine both current price level and current output.

Moreover, by using (9) and (20) above, the conventional income velocity of the demand for money, V_t , at the market level, will be given by:

$$V_t = \frac{P_t Q_t}{M_t} = \frac{M_{t,3} + M_{t-1,3}}{2M_{t,3}} \quad (22)$$

Clearly, the value of V_t depends not only on the demand for money at the individual level, but also on the state of the economy, represented by past values of the money holdings, and on current government policy. It is not generally, a trustful guess for the individuals' income velocity.

The labor market can be entirely represented by

$$P_{t+1}Q_{t+1} + P_{t+2}Q_{t+2} + P_{t+3}Q_{t+3} = \frac{1}{\alpha} W_t L_t \quad (15)$$

$$L_{t,t+1} = \alpha \frac{P_{t+1}Q_{t+1}}{W_t} \quad (13a')$$

$$L_{t,t+2} = \alpha \frac{P_{t+2}Q_{t+2}}{W_t} \quad (13b')$$

$$L_{t,t+3} = \alpha \frac{P_{t+3}Q_{t+3}}{W_t} \quad (13c')$$

where (13a'), (13b') and (13c') are just (13a), (13b) and (13c) conveniently re-written. By plugging the value of W_t given by

(15) into (13a'), (13b') and (13c'), and by generalizing the results, we obtain:

$$L_{t-3,t} = \frac{P_t Q_t}{P_{t-2} Q_{t-2} + P_{t-1} Q_{t-1} + P_t Q_t} \times L_t \quad (23a)$$

$$L_{t-2,t} = \frac{P_t Q_t}{P_{t-1} Q_{t-1} + P_t Q_t + P_{t+1} Q_{t+1}} \times L_t \quad (23b)$$

$$L_{t-1,t} = \frac{P_t Q_t}{P_t Q_t + P_{t+1} Q_{t+1} + P_{t+2} Q_{t+2}} \times L_t \quad (23c)$$

which can be inserted into (12) to yield

$$Q_t^{\frac{1}{\alpha}} = \frac{(P_t Q_t)^3 L_t^3}{(P_{t-2} Q_{t-2} + \dots + P_t Q_t)(P_{t-1} Q_{t-1} + \dots + P_{t+1} Q_{t+1})(P_t Q_t + \dots + P_{t+2} Q_{t+2})} \quad (24)$$

Expression (24) is the one we have been looking for. It concisely synthesizes the theory exposed in this paper. It shows that in the context of an economy in which the current labor input can be alternatively employed, depending on choice, to start new production processes or to reinforce processes which are underway or about to be finished, then the current supply of output, Q_t , depends not only on that labor input, L_t , but also on the anticipated distribution of nominal income over time. Moreover, as it will become clear along the simulations that shall be performed, output tends to migrate toward those periods for which the density of the nominal income are the highest over time. For that reason, the supply of output over time can also display cyclical behavior.

Finally observe that the values of $P_{t+2} Q_{t-2}, \dots, P_{t+2} Q_{t+2}$ that enter (24) can be easily found by the definition of the government policy with respect to $G_{t-2}, \dots, G_{t+2}$, and by the utilization of (21), (20) and (9), in this order. This solves the model.

To conclude this paper we shall present some very simple simulations. However, before that let us show how the nominal income is split between prices and productions in the context of our simple world. To do that we start by inserting the term $P_{t+1} Q_{t+1}$ in the equation (19), in a convenient way, re-writing it as:

$$Q_{t+1} = \left[\frac{\alpha \frac{P_{t+1} Q_{t+1}}{Q_{t+1}}}{W_{t-2} W_{t-1} W_t} \right]^{\frac{\alpha}{1-\alpha}}$$

and then as:

$$\frac{1-2\alpha}{\alpha} \log Q_{t+1} = \log \frac{\alpha}{W_{t-2}W_{t-1}W_t} \log(P_{t+1}Q_{t+1}). \quad (19')$$

By differentiating this last expression with respect to $(P_{t+1}Q_{t+1})$, and re-arranging the terms, we find:

$$d \log Q_{t+1} = (1 - \delta_{t+1}) \frac{\alpha}{1 - 2\alpha} d \log(P_{t+1}Q_{t+1}) \quad (25)$$

where δ_{t+1} is the elasticity of the current wage rate W_t with respect to $(P_{t+1}Q_{t+1})$. It is equal to the share of the nominal income at the beginning of period $t+1$, $P_{t+1}Q_{t+1}$, in the sum of the nominal incomes over periods $t+1$, $t+2$ and $t+3$, as can be readily checked by inspecting (15). It is then smaller than one.

According to (25) a change in the aggregate nominal income of period $t+1$, *while keeping constant the values of the incomes at periods $t+2$ and $t+3$* , will generally have an expansionary effect in the output of period $t+1$. Also note that δ_{t+1} is positive and smaller than one, and that $3\alpha < 1$. The elasticity of Q_{t+1} with respect to $P_{t+1}Q_{t+1}$ is then smaller than one. This is exactly what we should expect in view of the fact that the sum of this elasticity with the elasticity of P_{t+1} with respect to the same $P_{t+1}Q_{t+1}$ must be equal to one. Then, in the context of this simple experiment we find that anticipated changes in a future nominal income within the firm's decision period, are accompanied by less than proportionate changes in the output of the same point in time.

The magnitude of that effect will increase with the elasticity of output with respect to the labor input, α , and it will decrease with the increase of δ_{t+1} . Clearly, the size of such an effect depends on production conditions, on the state of the economy and on government policy. It cannot be predicted only on the basis of theoretical considerations.

In view of the above discussion, the effect of changes in nominal incomes upon price levels can be readily expressed as

$$1 - (1 - \delta_{t+1}) \frac{\alpha}{1 - 2\alpha}.$$

The factors that tend to increase (decrease) the effect of the above experiment on output tend to decrease (increase) it on prices. We now turn to some basic simulations.

7. Simulations.

To illustrate the working of the model we shall now present some simulations. To smooth out the time paths of the variables, we assumed that the t -th generation lives for ten periods and that it takes ten periods to complete each production process. Since the model summarized in the last section can be promptly generalized in that direction, this requires no additional explanation. Care should however be exercised in defining the total stock of money which, under the 10 periods assumption, will be given by

$$M_t = 9M_{t,10} \quad (26)$$

rather than by (20).

To start the simulations we supposed that at the beginning of period zero the economy was resting upon the following initial conditions: $\alpha = 0,075$, $L_0 = 10$, $Q_0 = 1$, $M_0 = 450$, $P_0 = 100$ and $W_0 = 7,5$. We then supposed that the economic agents correctly anticipated that the government would change the time path of the money supply at the beginning of the 10-th period.

Figure 1 exhibits the response of the nominal income, of the price level, of the real output and of the market income velocity of the demand for money, when that change is represented by a once-and-for-all 30% increase in the quantity supplied of money. In the case of Figure 2 the change is given by a temporary one-period increase of this stock, by the same percentage.

Both Figures speak for themselves. Except in the case of the behavior of the market income velocity of money in Figure 2, the movements of $P_t Q_t$, P_t , Q_t and V_t are mainly pro-cyclical and exhibit a high degree of conformity among themselves. It is also evident, in both cases, that the increase in productions over some periods, induced by monetary shocks, have to be compensated by decreases, which must occur over other periods of time.

8. Concluding remarks.

In this paper we have developed a perfect forecast full-employment equilibrium theory of the business cycle in the context of a simple dynamic monetary model with overlapping generations and production processes. This theory is based on the notions that: (a) the economic agents plan to limit their own freedom of

Figure 1.

Figure 2.

action; (b) it takes time to produce one unit of output; and (c) the productive input of the economy can be employed in alternative overlapping production activities, at each point in time.

According to this theory, fully anticipated monetary shocks produce the kind of movements in output, prices and nominal incomes, that are widely known as business cycles.

The notion that productive activities overlap at a point in time imparts a great deal of flexibility to the production side of the model. A more complete framework could for instance include activities of various maturities and technologies. In that respect we believe, that in the context of the mentioned framework, it is plausible to expect that a fully anticipated monetary shock would have differential effects and so it would affect the relative costs of production of these activities, at a point in time. In that sense we basically agree with Prof. Hayek's view that it is wrong to believe "that monetary changes affect only the *general* level of prices", and so to conclude that they are harmful chiefly for disrupting the creditor-debtor relationship, "as if they raised or lowered all prices *simultaneously* and by the *same* percentage" while "the real harm they do is due to the differential effect on different prices, which change successively in a very irregular order and to a very different degree, so that as result the whole structure of relative prices becomes distorted and misguides productions into the wrong direction" (Hayek 1978, p. 74).

References

- Hayek, Friedrich A. *Monetary Theory and the Trade Cycle*. London, pp 33n, Jonathan Cape, (1933).
- Hayek, Friedrich A. *Denationalization of Money – The Argument Refined*. The Institute of Economic Affairs, London, (1978).
- Karni, Edi. A Note on Lucas's Equilibrium Model of the Business Cycle. *Journal of Political Economy*, **88**, no. 6: cember (1980).
- Lucas, Robert E., Jr. An Equilibrium Model of the Business Cycle. *Journal of Political Economy*, **83**, no. 6: 1113–44; December (1975).
- ucas, Robert E., Jr Understanding Business Cycle. Suppl. on Stabilization of the Domestic and International Economy. *Journal of Monetary Economics*, **3**: 7–21, (1977).

- Martins, Marco A.C. The Private and the Social Optimum Quantity of Public Debt in a Pure Consumption-Loans Model. (unpublished Ph.D. Dissertation, University of Chicago, June 1975).
- Martins, Marco A.C. A Nominal Theory of the Nominal Rate of Interest and the Price Level. *Journal of Political Economy*, 88 no. 1: 174-185, February (1980).
- Martins, Marco A.C. A Nominal Theory of the Nominal Rate of Interest and the Price Level with the Integration of both the Fisher Effect and the Gibson Paradox. *Revista de Econometria*, VIII no. 1: 7-20, January (1988).
- Phelps, Edmund S., et al. *Microeconomic Foundations of Employment and Inflation Theory*. Norton, New York, (1969).
- Samuelson, P.A. An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money. *Journal of Political Economy*, 66: 467-82, December (1958).
- Shleifer, Andrei. Implementation Cycles. *Journal of Political Economy*, 94 no. 6: 1163-1189, December (1986).

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