A Note on Growth and Inflation

Pedro Cavalcanti Ferreira*
Graduate School of Economics,
Fundação Getulio Vargas

December, 1997

Abstract

A simple exercise on growth and inflationary financing of public expenditures is presented in this note. In a parameterized overlapping generations model where government expenses positively affects the growth rate of human capital, steady state capital and output increase with inflation, reproducing the so called Tobin effect. For large inflation rates, however, government authorities cannot affect real variables and there are only nominal effects. It is also shown that the optimal policy implies some inflation but not growth maximization.

1 Introduction

This note presents a simple exercise on inflation and growth using a parameterized overlapping generations model where government expenses positively affects the growth rate of human capital (following Boldrin(1993) and Glomm and Ravikumar(1992) among others) and consequently productivity growth rate. Endemic inflation is thus explained because of, in one hand, the inability of the government to enlarge the tax base (which is not modeled here) and, in the other hand, the essential role that infrastructure and education, mostly public financed, play in the economy. The lack of alternative financial sources forces the government to use money to pay for its projects, which are instrumental for economic growth.

The model is simulated and the results show that monetary equilibria exist in this economy for large intervals of parameters and also that steady state capital (per efficiency units of labor) increases with the rate of money creation. This last result reproduces the so called Tobin effect (e.g., Tobin (1965) and Mundell (1965)). However, when inflation is too large, the monetary authorities cannot affect real variables and there are only nominal effects. It is also shown that for

*The author would like to thank the financial support of CNPq and PRONEX.

reasonable values of parameters growth maximization is never optimal. In the next section the model is presented while in section 3 the simulation results are presented. In section four some brief concluding remarks are made.

2 The Model

Consider an overlapping generations economy with no population growth. Each generation is composed of a large number of individuals who live for two periods, except the first generation that only lives for one period. In the first period of their lives, "youth", the individuals are endowed with one unit of labor which they supply inelastically. When young the individuals work, receive a wage, consume the only good of this economy and save. In the second and last period of their lives, "old" people do not work, but consume the proceedings of their savings.

There are two different assets in the economy. One is fiat money issued by the government and the other is a capital asset issued by the firms. Money may or may not be valued in equilibrium and in this last case individuals will hold only capital in their portfolios. The capital and money levels in the first period (time zero) are given by history. The problem of the consumer is to maximize his life-time utility by choosing the saving level as well as its distribution between capital and money:

$$Max_{C_t^y, M_t, C_{t+1}^o} \beta \ln(C_t^y) + (1-\beta)\ln(C_{t+1}^0) \quad (1)$$

$$s.t.: C_{t+1}^0 + R_{t+1}C_t^y + (\Pi_t - R_{t+1}) M_t/P_t = w_t R_{t+1} \quad (2)$$

where R_{t+1} is the gross return on capital. Π_t the gross return on money (the inverse of inflation factor). C_t^y and C_{t+1}^0 are the consumption of a young person and a old person born at period t. respectively. M_t is the nominal money demand. P_t the price level and w_t wages. The first order conditions for this problem are:

$$\frac{\beta}{C_t^y} = \frac{(1-\beta)}{C_{t+1}^0} R_{t+1} \quad (3)$$

$$R_{t+1} \geqslant \Pi_t, \quad = \ if \quad M_t > 0 \quad (4)$$

Equation 3 is the usual Euler equation while equation 4 is a non-arbitrage condition. Competitive firms maximize profits choosing optimally capital and labor. It is assumed a Cobb-Douglas production function, $Y_t = K_t^{\alpha} L_t^{1-\alpha}$, where K_t is capital stock at time t and L_t the flow of efficiency units of labor of a worker born at time t. The first order conditions of this problem. in efficient units, are given by:

$$w_t = (1-\alpha)k^{\alpha} \tag{5}$$

$$q_t = \alpha k_t^{\alpha-1} \tag{6}$$

where q is the rental price of capital. In equilibrium, and under the hypothesis of full depreciation, q_t is equal to R_t.

In this economy, the government budget constraint is $P_t G_t = M_t - M_{t-1}$, where G_t is real government expenditures at time t. Assuming a constant and preannouced rate of money creation (μ). we obtain $M_t = (1+\mu)M_{t-1}$. which implies

$$g_t = \frac{\mu}{1+\mu} m_t \tag{7}$$

where g_t is real government expenditures per efficient unit of labor.

In this economy public expenditures enhance the productivity of labor, increasing the flow of labor services per unit of time. The idea is that by investing in public education, infrastructure, health services, sanitation, and so on, the government increases the quality of the labor force. In particular, assume that:

$$L_{t+1} = \lambda(g_t) L_t \tag{8}$$

where the function $\lambda(g_t)$ is the government expenditure function. It transforms each unit of public investment in infra-structure, by a relative increase of L_{t+1}/L_t in labor productivity. The government expenditure function λ will be of the form $\lambda(g_t) = 2 - exp(-g_t/\phi)$, where ϕ is a real number greater than one[1].

The equilibrium saving function of this economy does not depend on interest or inflation rate, but only on income, and is given by $(1-\alpha)(1-\beta)k$. The equilibrium in the asset market in efficient units is given by:

$$(1-\beta)(1-\alpha)k_t = m_t + \lambda(g_t)k_t \tag{9}$$

The dynamic system given by equations (3)-(7) and (9) can be reduced to two equations:

$$\alpha k_t^{\alpha-1} = \frac{2 - \exp\left(-\frac{\mu}{1+\mu}\frac{m_t}{\phi}\right)}{(1+\mu)} \tag{10}$$

$$(1-\beta)(1-\alpha)k_t = m_t + \lambda(g_t)k_t \tag{11}$$

[1] In Cavalcanti Ferreira(1996) the existence of monetary equilibrium was proved for general functional forms. It was assumed that λ was positive, increasing and differentiable, $\lambda(0)$ was assumed to be one so that if the public infra-structure remains the same the labor productivity does not change. It was also assumed that λ was bounded so that the marginal gain in labor productivity was close to zero when government expenditures are very large. The assumed form for λ in the present paper respects all these properties.

In Cavalcanti Ferreira(1995) the dynamic properties of the above system were studied and the existence of monetary equilibrium for this economy was proved under more general assumptions. At the steady state, equations (10) and (11) become:

$$m = (1-\alpha)(1-\beta)k^{\alpha} - \left[2 - \exp\left(-\frac{\mu}{1+\mu}\frac{m}{\phi}\right)\right]k \tag{12}$$

$$k = \left(\frac{2-\exp\left(-\frac{\mu}{1+\mu}\frac{m}{\phi}\right)}{(1+\mu)\alpha}\right)^{\frac{1}{\alpha-1}} \tag{13a}$$

3 Results

Simulations were concentrated on steady state equilibria. More specifically, it will be investigated the behavior of the monetary steady state, assuming different combinations of the parameters α, β and ϕ, when the government changes the monetary policy.

Figure 1 below shows, for different values of the rate of money creation, the behavior of equilibrium levels of capital, money holdings and savings, together with the inflation rate[2]. For this particular figure we used a capital share of one quarter, β equal 0.35 and ϕ equal to 10.

FIGURE 1

Although the corresponding values are different, the behavior of money and capital vis-a-vis the inflation rate for a wide range of parameters coincides with the one displayed in the above figure. For μ equal or close to zero the agents hold similar quantities of capital and money on their portfolios. For successively higher rates of inflation, the steady state level of money decreases until it reaches zero while the capital per efficiency unit of labor increases until the economy reaches the non-monetary steady state. The positive correlation between inflation and capital reproduces the so called Tobin effect. Overall, savings are higher for higher rates of inflation.

In other worlds, through an inflationary financing scheme the government can stimulate the economy and higher levels of steady state inflation correspond to higher levels of capital stock. At higher stationary inflation rates, money demand is very small or null. Thus, there is a bound on the ability of the government to use inflation to finance growth and capital accumulation.

Tables I and II below present the steady state level of money and capital, respectively, for different values of money growth rates, using six combinations

[2] Note that in this model the inflation rate at the steady state is given by $\{(1+\mu)/\lambda(g) - 1\}$, which is equal to μ only at g = 0. However, it follows μ very close.

of parameters: α equal to one third and one quarter, and β equal to 0.5, 0.45 and 0.35. For all combinations ϕ was set to be equal to 10. as changes in this parameter did not significantly affect the results.

Table I
Steady State Money Stock Levels

μ	m					
	α=1/4			α=1/3		
	β=0.35	β=0.45	β=0.50	β=0.35	β=0.45	β=0.50
0.00	0.1496	0.1021	0.0788	0.0611	0.0223	0.0029
0.01	0.1485	0.1010	0.0774	0.0595	0.0205	0.0010
0.02	0.1475	0.1005	0.0761	0.0578	0.0187	1E-06
0.03	0.1463	0.0990	0.0761	0.0562	0.0168	0
0.04	0.1452	0.0973	0.0734	0.0545	0.0150	0
0.11	0.1369	0.0887	0.0636	0.0422	0.0013	0
0.12	0.1357	0.0870	0.0622	0.0404	1E-06.	0
0.20	0.1294	0.0751	0.0502	0.0251	0	0
0.30	0.1117	0.6000	0.0344	0.0043	0	0
0.32	0.1087	0.0574	0.0311	1E-07	0	0
0.40	0.0968	0.0440	0.0177	0	0	0
0.50	0.0811	0.0276	1E-05	0	0	0
0.60	0.0644	0.0095	0	0	0	0
0.64	0.0575	1E-05	0	0	0	0
0.70	0.0469	0	0	0	0	0
0.90	0.0097	0	0	0	0	0
0.95	1E-07	0	0	0	0	0

The steady state value of money holdings falls monotonically with μ for all the combinations of parameters. However, the level at which it will reach zero depends crucially on the values of α and β. The smaller their values, the higher the inflation rates for which monetary steady state will exist. For the lowest combination (capital share of one quarter and beta equal 0.35) there will be a monetary steady state for rates of money creation up to 0.95, which corresponds roughly to a inflation rate of 94% per period in the model. On the other hand, for α and β equal to one third and one half, respectively, there is no monetary steady state for μ larger than 0.02. This is not unexpected because the higher the β the higher the importance for the individual utility function of the consumption in the first period of his life. and so. the lower the propensity to save. Furthermore. as we can see from table II below. capital increases with α so that the participation of money in the total savings, everything being the same, decreases with this parameter.

Table II
Steady State Capital Stock Levels

μ	k					
	α=1/4			α=1/3		
	β=0.35	β=0.45	β=0.50	β=0.35	β=0.45	β=0.50
0.00	0.1575	0.1575	0.1574	0.1911	0.1910	0.1911
0.01	0.1596	0.1596	0.1596	0.1940	0.1930	0.1940
0.02	0.1617	0.1616	0.1616	0.1969	0.1960	0.1968
0.03	0.1637	0.1638	0.1637	0.1997	0.1990	0.1968
0.04	0.1658	0.1658	0.1658	0.2026	0.2020	0.1968
0.10	0.1785	0.1787	0.1786	0.2202	0.2200	0.1968
0.11	0.1807	0.1808	0.1808	0.2232	0.2230	0.1968
0.12	0.1828	0.1829	0.1830	0.2262	0.2260	0.1968
0.30	0.2227	0.2230	0.2232	0.2821	0.2260	0.1968
0.32	0.2273	0.2276	0.2278	0.2893	0.2260	0.1968
0.40	0.2458	0.2463	0.2464	0.2893	0.2260	0.1968
0.50	0.2694	0.2701	0.2704	0.2893	0.2260	0.1968
0.60	0.2938	0.2946	0.2704	0.2893	0.2260	0.1968
0.64	0.3037	0.3046	0.2704	0.2893	0.2260	0.1968
0.70	0.3187	0.3046	0.2704	0.2893	0.2260	0.1968
0.90	0.3443	0.3046	0.2704	0.2893	0.2260	0.1968
0.95	0.3837	0.3046	0.2704	0.2893	0.2260	0.1968

The behavior of capital is, in certain sense, a mirror image of the behavior of money: its level on the steady state is higher for higher values of α and β, for a given μ, and it increases monotonically with μ for given capital share and beta. The later fact, as commented before, reproduces the Tobin effect. Another result easily observed from table II is that the effect of α is much more strong and dominant over the value of capital in the steady state than variations in β. This is not entirely unexpected because. from equations (12) and (13a) above, we can see that β affects capital only indirectly through its effects over money holdings, while α is a parameter of the production function.

As α and β fall, there will exist monetary steady states for increasingly higher inflation rates. Consequently, the capital stock level at the non-monetary steady state decreases with alphas and betas: it is 0.386 for α equal to 0.25 and β equal to 0.35 and 0.197 for α equal to one third and beta equal to one half.

The outcome of one last exercise is shown in figure 2 below. where steady state levels of the utility[3] of one generation and growth rates were plotted against rates of money creation. $\alpha = 1/4$ and $\beta = 0.35$ were used in this exercise.

FIGURE 2

[3]For a matter of scale we subtracted 0.15 from the utility level, otherwise the utility line would be out of scale. Utility* means utility minus 0.15.

The above result has interesting implications. In a model where government expenditures are not "wasted" or lump sum transferred, but directly affect the productivity growth of the economy, the best policy implies always some inflation. Inflation is always optimal because, up to a certain level, the benefits of money creation over accumulation are larger than the distortion costs. We can thus explain endemic inflation cases: if the government is unable to enlarge tax collection (say, for social or political reasons), the importance for the economy of government expenses and its effect over productivity growth, forces the authorities to resort to money creation.

The second and more significant implication is that the inflation rate which maximizes the utility of the consumer is lower than that which maximizes growth (0.33 and 0.5, respectively). As the rate of money creation rises, the increase in seigniorage becomes progressively small, and so do the welfare gains from economic growth. At a certain point, the loss due to the distortionary effect of inflation tax overcomes the gains from the growth effect. So, if the government wants to maximize the welfare of the present generation he should, consequently, operate on the left side of the Laffer curve, below the maximum revenue it can get from money creation, and he should not maximize economic growth. This result holds for a large number of combinations of β and α.

For discount rates other than zero. this result has to be qualified. It is still true that the optimal money growth rate will be smaller than that which maximizes growth, unless the present government gives equal weight to the present and all future generations. However, as the discount rate increases, the optimal rate approaches 0.5, the rate that maximizes growth. It is still 0.33 for discount rates equal to one half and it goes to 0.37, 0.44, 0.49 for discount rates equal to 0.8, 0.9, and 0.99 respectively. The intuition behind this result is that if the government cares about the future generations it will be willing to accept more distortion and inflation in order to achieve higher rates of growth with the larger expenditures he will be able to finance.

4 Concluding remarks

This simple exercise may be used to explain particular historical experiences in which inflation acceleration apparently helped to spur a period of economic growth, as in certain countries of Latin America in the fifties or Russia in the nineties. If public expenditure is an important factor for growth and seigniorage an important source of revenue, for inflation rates not large enough. money creation positively affects steady state levels of real variables such as income and capital stock and also the growth rate of this economy. For high inflation rates, however, there are no real effect but only nominal effects. This seems to be the case for these countries. Another important result is that growth maximization is never optimal in this economy.

References

[1] Boldrin, M., 1993, Public education and capital accumulation, Manuscript, Northwestern University.

[2] Cavalcanti Ferreira, 1996, Sustained growth, government expenditure and inflation, Ensaios Econômicos 278, EPGE, Fundação Getulio Vargas.

[3] Glomm, G. and B. Ravikumar, 1992. Public versus private investment in human capital: endogenous growth and income inequality," Journal of Political Economy 100, 818-834.

[4] Mundell, R., 1965, Growth, stability and inflationary finance, Journal of Political Economics 73, 97-109.

[5] Tobin, J., 1965, Money and economic growth, Econometrica 33, October, 671-684.

Figure 1: Steady State Equilibria Levels

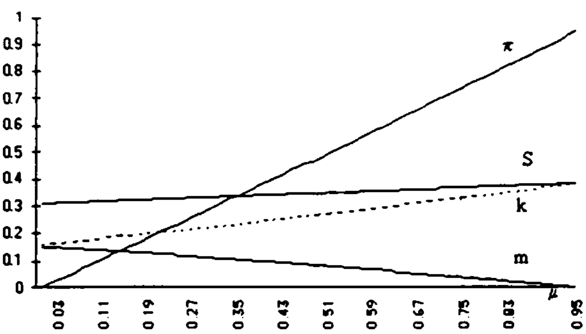
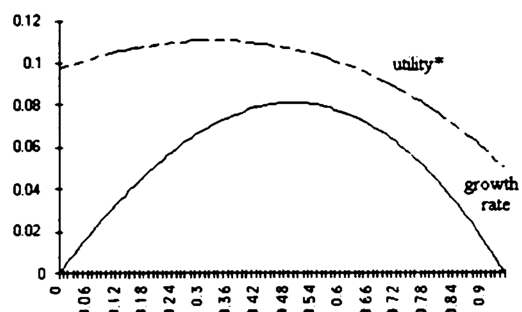


Figure 2: Welfare and Growth



ENSAIOS ECONÔMICOS DA EPGE

300. CRESCIMENTO ENDÓGENO, DISTRIBUIÇÃO DE RENDA E POLÍTICA FISCAL: UMA ANÁLISE CROSS-SECTION PARA OS ESTADOS BRASILEIROS - Victor Duarte Lledó e Pedro Cavalcanti G. Ferreira - Fevereiro 1997 - 41 pág. (esgotado)
301. SISTEMA FINANCEIRO BRASILEIRO: DIAGNÓSTICO E REFORMAS REQUERIDAS - Rubens Penha Cysne e Lauro Flávio Vieira de Faria - Março 1997 - 68 pág. (esgotado)
302. DESEMPREGO REGIONAL NO BRASIL: UMA ABORDAGEM EMPÍRICA - Carlos Henrique Corseuil, Gustavo Gonzaga e João Victor Issler - Abril 1997 - 40 pág.
303. AS DELIBERAÇÕES SOBRE ATOS DE CONCENTRAÇÃO: O CASO BRASILEIRO - Luiz Guilherme Schymura - Abril 1997 - 20 pág.
304. EFFECTS OF THE REAL PLAN ON THE BRAZILIAN BANKING SYSTEM - Rubens Penha Cysne e Sérgio Gustavo Silveira da Costa - Maio 1997 - 30 pág.
305. INFRA-ESTRUTURA, CRESCIMENTO E A REFORMA DO ESTADO - Armando Castelar Pinheiro - Maio 1997 - 25 pág. (esgotado)
306. PUBLIC DEBT SUSTAINABILITY AND ENDOGENOUS SEIGNORAGE IN BRAZIL: TIME-SERIES EVIDENCE FROM 1947-92 - João Victor Issler e Luiz Renato Lima - Junho de 1997 - 38 pág. (esgotado)
307. THE BRAZILIAN 1994 STABILIZATION PLAN: AN ANALYTICAL VIEW - Eduardo Felipe Ohana - Julho de 1997 - 24 pág.
308. A INDETERMINAÇÃO DE SENIOR E O CURRÍCULO MÍNIMO DE ECONOMIA - Antonio Maria da Silveira - Julho de 1997 - 39 pág.
309. HIPERINFLAÇÃO: IMPOSTO INFLACIONÁRIO E O REGIME DE POLÍTICA ECONÔMICA - Fernando de Holanda Barbosa - Julho de 1997 - 31 pág.
310. TAXA DE CÂMBIO E POUPANÇA: UM ENSAIO SOBRE O EFEITO HARBERGER-LAURSEN-METZLER - Fernando de Holanda Barbosa - Julho de 1997 - 23 pág.
311. A CONTRIBUIÇÃO ACADÊMICA DE MÁRIO HENRIQUE SIMONSEN - Fernando de Holanda Barbosa - Julho de 1997 - 22 pág. (esgotado)
312. ANAIS DO III ENCONTRO NACIONAL SOBRE MERCADOS FINANCEIROS, POLÍTICA MONETÁRIA E POLÍTICA CAMBIAL - Parte I - BALANÇA COMERCIAL E FLUXO DE CAPITAIS - Rubens Penha Cysne (editor) - Agosto de 1997 - 103 pág.
313. ANAIS DO III ENCONTRO NACIONAL SOBRE MERCADOS FINANCEIROS POLÍTICA MONETÁRIA E POLÍTICA CAMBIAL - Parte II - MERCADO DE CAPITAIS - Rubens Penha Cysne (editor) - Agosto de 1997 - 85 pág.

314. ANAIS DO III ENCONTRO NACIONAL SOBRE MERCADOS FINANCEIROS POLÍTICA MONETÁRIA E POLÍTICA CAMBIAL - Parte III - MERCADOS FINANCEIROS E POLÍTICA MONETÁRIA - Rubens Penha Cysne (editor) - Agosto de 1997 - 73 pág.

315. O IMPACTO DA INFRA-ESTRUTURA SOBRE O CRESCIMENTO DA PRODUTIVIDADE DO SETOR PRIVADO E DO PRODUTO BRASILEIRO - Pedro Cavalcanti Ferreira e Thomas Georges Malliagros - Agosto de 1997 - 34 pág. (esgotado)

316. REFORMA TRIBUTÁRIA NO BRASIL: EFEITOS ALOCATIVOS E IMPACTOS DE BEM-ESTAR - Pedro Cavalcanti Ferreira e Carlos Hamilton Vasconcelos Araújo - Setembro de 1997 - 40 pág. (esgotado)

317. A CAPM WITH HIGHER MOMENTS: THEORY AND ECONOMETRICS - Gustavo M. de Athayde e Renato G. Flôres Jr. - Outubro de 1997 - 32 pág.

318. MANDATORY PROFIT SHARING, ENTREPRENEURIAL INCENTIVES AND CAPITAL ACCUMULATION - Renato Fragelli Cardoso - Dezembro de 1997 - 43 pág.

319. A NOTE ON GROWTH AND INFLATION - Pedro Cavalcanti Ferreira - Dezembro de 1997 - 11 pág.