Nº 186

NASH EQUILIBRIUM UNDER KNIGHTIAN UNCERTAINTY: BREAKING-DOWN BACKWARD INDUCTION

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Fevereiro de 1992

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by

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December 1991

ABSTRACT: We present two alternative definitions of Nash equilibrium for two person games in the presence of *uncertainty*, in the sense of Knight. We use the formalization of uncertainty due to Schmeidler and Gilboa. We show that, with one of the definitions, prudent behaviour (maxmin) can be obtained as an outcome even when it is not rationalizable in the usual sense. Most striking is that with the same definition we break down backward induction in the twice repeated prisoner's dilemma. We also link these results with the Kreps-Milgrom-Roberts-Wilson explanation of cooperation in the finitely repeated prisoner's dilemma.

ACKNOWLEDGEMENT: The authors are especially thankful to Tommy Tan, with whom this project started.

* London Business School. Part of this research was carried out while this author was visiting the Graduate School of Economics at the Getúlio Vargas Foundation (EPGE/FGV), under a grant from CNPq.

** EPGE/FGV

1. INTRODUCTION

Among the well-documented phenomena that economic models do not fully explain is the fact that players of a finitely repeated game do not backward induct. The careful experiments performed by Neelin, Sonnenschein and Spiegel (1988), and the well known prisoner's dilemma tournament of Axelrod (1980) show that subjects do not act according to the logic of backward induction. Several attempts to model this behaviour exist in the literature. Some of them are based on bounded rationality (Radner (1980), Chou and Geanakoplos (1985), Aumann (1981), Neyman (1985), Meggido and Widgerson (1986), and the references therein). Another type of explanation requires evolutionary behaviour with a lower bound on the number of "mutants", as in Nachbar (1981). Undeniably, however, the most successful model to justify cooperation in the finitely repeated prisoner's dilemma (which is, in fact, stronger than violation of backward induction) is due to Kreps, Milgrom, Roberts and Wilson (1982). Essentially their argument is that there is always a small chance that one of the players will not act rationally. In the game they consider, there is a small chance that one of the players will cooperate. This will lead to significant levels of cooperation on the part of both players.

In this paper, we propose to extend the notion of Nash equilibrium to incorporate agents who act in a game as if they faced uncertainty in the sense of Knight (1921). We propose two extensions. One of them, which we argue is the appropriate one, leads to the possibility of cooperation in the repeated prisoner's dilemma. We show that this has a close connection with the Kreps-Milgrom-Roberts-Wilson explanation. At the same time, given that the phenomenon has been modelled by departing from the Bayesian view of Savage (1954), it is also true that the bounded Bayesian rationality justifications are compatible with our explanation (the evolutionary model is of a completely unrelated type, more suitable for analysis of economic institutions).

Our definition also demonstrates the possibility of prudent behaviour even when this is inconsistent with the knowledge of the fact that the other player is Bayesian rational, as in the example in Werlang (1986, chapter 3). Finally, the definition of equilibrium allows for

the possibility that the same game may be played differently by the same player when facing different opponents (even if there is a unique rationalizable outcome).

The paper is organized as follows. The next section discusses the basic uncertainty model, which is due to Schmeidler (1982, 1989) and Gilboa (1987). Section 3 presents the definitions of equilibrium. Section 4 shows that the first definition may lead to non existence of (strict) Nash equilibria when the degree of uncertainty, as measured by the uncertainty aversion (defined in Dow and Werlang (1991a)) is high. In section 5, the second definition is used to obtain prudent behaviour which is incompatible with common knowledge of Bayesian rationality (as shown in Tan and Werlang (1988), this is the same as rationalizability in the sense of Bernheim (1984) and Pearce (1984)). Section 6 has the most striking example: the emergence of cooperation in a twice repeated prisoner's dilemma, under the second definition of equilibrium under uncertainty. Section 7 relates the equilibrium notion with the Kreps-Milgrom-Roberts-Wilson model. We argue that our notion endogenously generate the "irrationality" present in their explanation, thereby clarifying their results. Section 8 concludes and points to directions for further research.

2. UNCERTAINTY

Schmeidler (1982, 1989) and Gilboa (1987) have developed an axiomatic model of rational decision making in which agents' behaviour distinguishes between situations where agents know the probability distributions of random variables and situations where they do not have this information. We refer to the former as risk and the latter as uncertainty, or Knightian uncertainty. Synonyms that are used in the literature include roulette lottery, for risk, and horse lottery and ambiguity, for uncertainty. The traditional model of uncertainty used in economics is that of Savage (1954), which reduces all problems of uncertainty to risk under a subjective probability. The axiomatization of Schmeidler-Gilboa leads to very different behaviour: behaviour under uncertainty is inherently different from behaviour under risk.

We now give a brief exposition of the main aspects of their model. The reader is referred to the papers by Schmeidler and Gilboa cited above for a complete description and for the underlying axioms, and to Dow and Werlang (1991a) which contains an example and an application to portfolio choice (it also includes a mathematical appendix with the basic material on non-additive probabilities). Dow and Werlang (1991b) has an explanation of the excess volatility puzzle, and Simonsen and Werlang (1991) also describe the implications for portfolio choice. Also, Wakker (1989) has a model which is very similar to Gilboa (1987).

Bewley (1986) presents a similar model which is also designed to capture Knightian uncertainty. His model predicts that uncertainty leads to inertia, a tendency to favour the "status quo," while in Schmeidler-Gilboa there is a tendency to choose acts where the agent does not end up bearing uncertainty. In a decision problem, this will lead to different predictions unless the status quo is an act where the agent bears no uncertainty. In Game Theory, it is conventional not to distinguish any particular strategy as the status quo.

The Schmeidler-Gilboa model predicts that agents' behaviour will be represented by a utility function and a (subjective) non-additive probability distribution. A non-additive probability P reflecting aversion to uncertainty satisfies the condition

$$P(A) + P(B) \leq P(A \cap B) + P(A \cup B), \qquad (*)$$

rather than the stronger condition satisfied by (additive) probabilities:

$$P(A) + P(B) = P(A \cap B) + P(A \cup B).$$

In particular, $P(A) + P(A^c)$ may be less than 1; the difference can be thought of as a measure of the uncertainty aversion attached by the agent to the event A. The uncertainty aversion of P at event A is $c(P,A) = 1 - P(A) - P(A^c)$ (Dow and Werlang (1991a)).

All the non-additive probabilities considered in this paper will reflect uncertainty aversion, ie they will satisfy inequality (*). Also, we will restrict attention to the case of a finite set of states of the world.

The agent maximizes expected utility under a non-additive distribution, where the expectation of a non-negative random variable X is defined as:

$$E(X) = \int_{\mathbf{R}^+} P(X \geq x)\, dx.$$

Associated with a non-additive probability P is a set Δ of additive probabilities called the core of P, which is defined (analogously to the core in cooperative game theory) as the set of additive probability measures π such that $\pi(A) \geq P(A)$ for all events A. If the non-additive probability satisfies inequality (2) (reflecting aversion to uncertainty) the core is non-empty. (A closely related model of behavior under uncertainty is for the agent to act to maximize the minimum value, over the elements of the core, of expected utility (Gilboa and Schmeidler, 1989).)

The support of a non-additive probability P may be defined analogously to the additive case. However, there are several possible analogous definitions. One possible definition is the smallest event A such that $P(A) = 1$. Notice that if P reflects strictly positive uncertainty aversion at all events (other than the empty set and the set of all states) then the entire set of all states is the only event with probability one. Intuitively, the interpretation of an event of non-additive probability zero is the same as in the additive case: it is an event which will almost never happen. However, if a set has probability zero, that does not mean its complement is of probability one. But if the complement of a zero-probability set has positive probability (in principle it could be zero too) it has relatively infinitely more chance of happening than the original set. Hence, we are led to an alternative definition based on the idea that the complement of the support has zero probability.

DEFINITION: the *extended support* of a non-additive probability P is the smallest event A such that $P(A) = 1$.

DEFINITION: a *support* of a non-additive probability P is an event A such that $P(A^c) = 0$ and $p(B^c) > 0$ for all events $B \subset A$, $B \neq A$.

It should be clear that, while the extended support is unique, there may be several supports. Also note that any support must be contained in the extended support (otherwise a smaller set with the same property would be obtained by taking the intersection with the extended support).

EXAMPLE: Constant uncertainty aversion (see Dow and Werlang (1991a)). The extended support coincides with the (unique) support and also with the set of all states. If there are three states, say, we could have:

$p_1 = p_2 = p_3 = p \in (0, 1/3)$

$p_{12} = p_{13} = p_{23} = q \in (2p, 1-p).$

where p_i is the probability of state i and p_{ij} is the probability of state i or state j.

EXAMPLE: this example illustrates the difference between the support and the extended support. Again there are three states.

$p_1 = p_2 = p \in (0, ½)$

$p_3 = 0$

$p_{12} = q \in (2p, 1-p)$

$p_{13} = p_{23} = p.$

The extended support is the set of all states. The (unique) support is the event of state 1 or 2.

EXAMPLE: this example does not have a unique support. Again there are three states.

$p_1 = 0.5$

$p_2 = p_3 = 0$

$p_{12} = p_{13} = 0.6$

$p_{23} = 0.1.$

The extended support is $\{1, 2, 3\}$. The supports are $\{1, 2\}$ and $\{1, 3\}$.

3. TWO DEFINITIONS OF EQUILIBRIUM

We restrict attention to two-person finite games $\Gamma = (A_1, A_2, u_1, u_2)$, where the A_i are pure strategy sets and the u_i are utilities (payoffs).

In the standard theory, a mixed strategy Nash equilibrium is defined as follows. Let (μ_1, μ_2) be a pair of (additive) probability measures and let supp$[\mu_i]$ denote the support of μ_i. In Nash equilibrium, every $a_1 \in$ supp$[\mu_1]$ is a best response to μ_2, ie a_1 maximizes the expected utility of player 1 given that player 2 is playing the mixed strategy μ_2; conversely, every $a_2 \in$ supp$[\mu_2]$ is a best response to μ_1. A subjective interpretation can be given to the Nash equilibrium: the mixed strategy of player 1, μ_1, may be viewed as the beliefs that player 2 has about the pure strategy play of player 1. Conversely, the mixed strategy of player 2, μ_2, may be viewed as the belief player 1 has about the pure strategy play of player 2. This subjective definition is suitable for the generalization we will introduce in this paper.

DEFINITION: STRICT NASH EQUILIBRIUM UNDER UNCERTAINTY.

A pair (P_1, P_2) of non-additive probabilities P_1 over A_1 and P_2 over A_2 is a *Strict Nash Equilibrium under Uncertainty* if:

(i) for all a_1 in the extended support of P_1, a_1 maximizes the expected utility of player 1 given that P_2 represents player 1's beliefs about the strategies of player 2, and conversely;

(ii) for all a_2 in the extended support of P_2, a_2 maximizes the expected utility of player 2 given that P_1 represents player 2's beliefs about the strategies of player 1.

The definition above is an obvious generalization of the concept of Nash equilibrium under usual additive probabilities. We describe some of its properties below. However, we would argue that the definition is too strong. Intuitively, when we use the extended supports of the non-additive beliefs in the same way as the supports of (additive) probabilities, we are ignoring the great strength of the non-additive model: that an event may be infinitely more

likely than its complement, but still have probability less than one. Take, for example, the case of a strategy set with two elements, a and b. Suppose $P(a)=0.8$ and $P(b)=0$. If we want to be "sure" that an event is going to happen, then this event has to be the whole strategy set. However, the likelihood that strategy a will be used is infinite relative to b (ie the relative likelihood that strategy b is going to be used is zero). Thus in this case it would be fair to interpret that strategy b has no chance of happening. This leads to the second definition, where we use a support of the non-additive probabilities instead of the extended support. Later, we illustrate why we think this second definition is more appropriate.

DEFINITION: NASH EQUILIBRIUM UNDER UNCERTAINTY

A pair (P_1, P_2) of non-additive probabilities P_1 over A_1 and P_2 over A_2 is a *Nash Equilibrium under Uncertainty* if there exist a support of P_1 and a support of P_2 such that:

(i) for all a_1 in the support of P_1, a_1 maximizes the expected utility of player 1, given that P_2 represents player 1's beliefs about the strategies of player 2, and conversely;

(ii) for all a_2 in the support of P_2, a_2 maximizes the expected utility of player 2 given that P_1 represents player 2's beliefs about the strategies of player 1.

This definition above, like the preceding one, reduces to the standard definition of Nash equilibrium, whenever there is no uncertainty (which means that the P's are additive). It is also clear that: (i) a strict Nash equilibrium under uncertainty is always a Nash equilibrium under uncertainty; and (ii) a standard mixed strategy Nash equilibrium is also a strict Nash equilibrium under uncertainty. Therefore, both equilibria always exist. A more subtle problem is that of existence for any given maximum level of uncertainty aversion. In the next section we give an example which shows that strict Nash equilibria under uncertainty may not exist for high degrees of uncertainty aversion. Finally, note that it is easy to check that a strictly dominated pure strategy will never be in a support of a Nash equilibrium under uncertainty.

4. EXAMPLE: NON EXISTENCE OF STRICT NASH UNDER UNCERTAINTY WHEN UNCERTAINTY AVERSION IS GIVEN A PRIORI

This section shows that strict Nash equilibrium under uncertainty may not exist for high levels of uncertainty aversion. Consider the game in Figure 1 below, where there are two standard Nash equilibria in pure strategies: (a,b) and (b,a). Further, if each player tries to force the equilibrium which is favourable to him or her, they both get to their maxmin level. However, if one of them knew that the other was going to do this, he would rather play his Nash equilibrium action.

Player 2's beliefs, P_1, are of the form: player 2 believes player 1 will play a with probability p_a, b with probability p_b. Similarly, player 1's beliefs, P_2, are described by q_a and q_b with $q_a + q_b \leq 1$. It follows that the expected value under P_2 of player 1's strategies are:

$$u_1(a) = 2(q_b - 1)$$
$$u_1(b) = 2q_a - 1.$$

Similarly, under P_1, we have the expected values for the utility of player 2: $u_2(a) = 2(p_b - 1)$ and $u_1(b) = 2p_a - 1$. Thus the set of strict Nash equilibria under uncertainty is given by the set of all quadruples of positive numbers (p_a, p_b, q_a, q_b) such that $p_b = p_a + 1/2$ and $q_b = q_a + 1/2$. Hence, the uncertainty aversion of player 2 is $c_2 = 1 - p_a - p_b$ and that of player 1 is $c_1 = 1 - q_a - q_b$. Note that in this case we have not specified the event at which c is evaluated because the only relevant events are {a} and {b}, one being the complement of the other. Notice why the subscripts are interchanged: c_1 refers to P_2 and c_2 refers to P_1, because P_2 is a belief of player 1 and P_1 of player 2. Observe that since given the probabilities are positive and their sum cannot exceed 1, we have $c_1 \in [0, ½]$ and $c_2 \in [0, ½]$. For this game, high levels of uncertainty aversion of the players are incompatible with strict Nash equilibrium under uncertainty.

Notice however, that Nash equilibria under uncertainty exist even for very high uncertainty aversion. It may be verified that $p_a = 0$ and $p_b < ½$, and $q_a = 0$ and $q_b < ½$ are equilibria

according to the definition, so that uncertainty aversion c may be arbitrarily high (up to the maximum possible value of 1).

5. EXAMPLE: NON RATIONALIZABLE MAXMIN BEHAVIOUR MAY OCCUR IN NASH UNDER UNCERTAINTY

The game shown below, in Figure 2, has a unique rationalizable equilibrium (which, therefore, coincides with the (standard) Nash equilibrium), given by (u, a). Further, if player 1 knows that player 2 is rational (observe that this requires just one level of elimination of strictly dominated strategies) then she knows that player 2 can never play b, because it is a strictly dominated strategy. Thus she should play u. Let us imagine, however, that 10 stands for 10 million dollars, or, equivalently, that the parameter ϵ is very small. This game is very similar to one in Werlang (1986, chapter 3). Question: would you play u in this game? Note that strategy d gives a payoff very similar to the payoff obtained in the Nash equilibrium, without any "risk" that the other player does not play his part. Since the Schmeidler-Gilboa model favours prudent behaviour, one might expect that it would predict strategy d for player 1, in accordance with this intuition. However, under strict Nash under uncertainty we still get u as the only prediction. But this is not the case if we use the second (and preferred) definition.

As before, P_1 is described by probability p_u player 2 believes player 1 will play u, and with probability p_d will play d, with $p_u + p_d \leq 1$. Similarly, P_2 is described by q_a and q_b, with $q_a + q_b \leq 1$. Notice that from the point of view of player 2, the beliefs P_1 are irrelevant: player 2 always chooses a. Will that mean that player 1 will necessarily play u? The answer is no! We will look for Nash equilibria under uncertainty of the form $q_b = 0$ and $1 > q_a > 0$. In the case of strict Nash under uncertainty, these cannot be an equilibrium, because the extended support of P_2 is the set {a, b}, and we know that player 2 will never play b. However, using the concept of Nash under uncertainty, this is a perfectly reasonable equilibrium: since {a} is a support (in this simple case the only one) we only have to check that a is a best response, a fact that we already know. The strategy b, since it lies outside

the support, need not be a best response, again a fact that we already know. What we would like to obtain is the set of parameters which will yield d as the equilibrium action of player 1. Computing the expected values shows that she will play d if $q_a \leq 1 - \epsilon/20$. Notice that this is exactly consistent with the intuition that just a little bit ($\epsilon/20$) of uncertainty aversion is enough to make player 1 turn from the only rationalizable action u to the prudent, and intuitively sensible action d. Notice also that the smaller ϵ is, the less uncertainty averse player 1 has to be in order to play d. Finally, note that we could also justify this action by the argument of Kreps-Milgrom-Roberts-Wilson: if there were a small chance of player 2 being crazy, player 1 would behave cautiously. We will show that this relationship is more than simply intuitive in section 7 below.

6. Example: Breaking Down Backward Induction with Nash Equilibrium under Uncertainty

We now show that cooperation may arise in the twice repeated prisoner's dilemma, thereby demonstrating that (Knightian) rational agents may not backward induct. Consider the version of the prisoner's dilemma of Kreps, Milgrom, Roberts and Wilson (1982), as shown in Figure 3 below, where a = 1.25 and b = 0.5, so that a + b < 2 as they require. The strategies are F (for "fink") and C (for "cooperate").

In the twice repeated version of this game, there are eight strategies for each of the players. Four are the unconditional, or history independent strategies: F^2, FC, CF, C^2, which stand for, respectively, F in both rounds, F in the first round and then C in the second round, C and then F, and C in the both rounds. There are four history dependent strategies, which we name W, X, Y and Z, for convenience. W is: start with F. If the other player played C in the first round, then play C in the second round. Otherwise play F in the second round. The letter X stands for: start with F, and if the other player played C, then play F, otherwise play C. The letter Y stands for the tit-for-tat: start with C, and play C in the second round if the other player played C in the first round. Otherwise, play F. Finally, Z stands for: start with C, and play F in the second round if C was also played by the other player.

Otherwise, play C. To summarize:

Strategy	First Round	Second Round if other player's First Round move was: C	F
F^2	F	F	F
FC	F	C	C
CF	C	F	F
C^2	C	C	C
W	F	C	F
X	F	F	C
Y	C	C	F
Z	C	F	C

Consider the game without discounting (the payoffs are sums of payoffs of each one-shot game). It may be verified that the following is a non-additive probability P which reflects uncertainty aversion (inequality (*) above):

(1) The extended support is the set $\{F^2, CF, C^2, W\}$

(2) $P(\{F^2, CF, W\}) = 0.8$

(3) $P(\{C^2, CF, W\}) = P(\{F^2, CF, C^2\}) = 0.4$

(4) $P(\{F^2, CF\}) = P(\{CF, W\}) = P(\{CF, C^2\}) = 0.4$

(5) $P(W) = P(C^2) = P(F^2) = P(\{F^2, C^2\}) = P(\{C^2, W\}) = P(\{F^2, C^2, W\}) = 0$

(6) $P(CF) = 0.4$.

(7) For all events B, $P(B) = P(B \cap \{F^2, CF, C^2, W\})$

It is easy to see that the only support is {CF}. There is a Nash equilibrium under uncertainty in which both players have these beliefs. The expected payoffs for each of the players, given the belief P about the other player's actions is, for each strategy: $U(F^2)=0.5$, $U(FC)=0.2$, $U(CF)=0.6$, $U(C^2)=0.2$, $U(W)=0.3$, $U(X)=0.2$, $U(Y)=0.3$ and $U(Z)=0.4$. Thus, the equilibrium above has CF as the prediction, with the joint probabilities of all other strategies together being zero. This means that cooperation in the first round may occur, again with (Knightian) rationality.

7. Relation to Kreps-Milgrom-Roberts-Wilson

The four authors obtain cooperation in the finitely repeated prisoner's dilemma using an intuitively appealing argument. They consider a small "risk" that one of the players will always play tit-for-tat. They show that this generates cooperation in several stages of the repeated game. The irrational agent (tit-for-tat) that they "added" to the game is added exogenously, but clearly the modellers' choice of that specific type of irrationality was not exogenous. It was motivated intuitively in a way which was endogenous to the particular game under analysis.

A similar analysis is given in Kreps and Wilson (1982) and Milgrom and Roberts (1982) in their model of the chain-store paradox.

In the previous section, we showed that uncertainty can lead to cooperation. There we did not use the tit-for-tat strategy to achieve cooperation, but our point was just to show that cooperation could arise. In our model the potentially "irrational" behaviour of the other agent is generated by the fact that in the presence of uncertainty the players tend to give more weight to the cases where potentially non-realized profits are higher, or equivalently, potential losses from not taking an action are larger. Thus in our model, the players endogenously decide which sort of agent they are "afraid" of meeting. In other words, we have a theory that explains how the "irrationality" appears in their model.

In order to illustrate the connection more clearly, we examine a model which is simpler than the finitely repeated prisoner's dilemma. We analyze the example of section 5. To adapt the Kreps-Milgrom-Roberts-Wilson framework to that example, let us return to the realm of additive probabilities and standard Nash equilibria. Suppose that player 1 (in the game in Figure 2) believes that there is a small (additive) probability δ that the game she faces is different. In this new game, her payoffs do not change, but player 2's payoffs are inverted (or equivalently, assume that for player 2, b is a strictly dominant strategy with a small probability δ). It may easily be verified that as long as $\delta > \epsilon/20$ player 1 plays d in the Nash equilibrium of the incomplete information game. Notice that the parameter δ has

exactly the same bound as the uncertainty aversion of player 1 in the Nash equilibrium under uncertainty computed in section 5. This is not a coincidence. The theory of choice under uncertainty endogenously generates the incomplete information and the "irrationality" required for the Kreps-Milgrom-Roberts-Wilson model.

8. CONCLUSION

The definition of Nash equilibrium under uncertainty provided explains a number of economic phenomena which, to date, have not been modelled in a satisfactory way. For example, it is possible to provide a rationale, along the lines of the above examples, for the bargaining results of Neelin, Sonnenschein and Spiegel (1988). They ran bargaining experiments with iterated offers and complete information (similar to the models of Stahl (1972) and Rubinstein (1982)). They found that players behaved as if they applied only two rounds of backward induction, even when the game was repeated up to six rounds. The definition of Nash equilibrium under uncertainty provided here may readily be used to construct examples where backward induction breaks down.

Several interesting problems remain to be tackled. First, there is the definition of rationalizability under uncertainty. Rationalizable strategies should be defined as strategies which are compatible with common knowledge of *Knightian* rationality. Second, there is the relationship between this new rationalizability concept and Nash equilibrium under uncertainty: we conjecture that the outcomes are the same. Third, there is the extension to n players. Fourth, there is the question of existence of Nash equilibrium under uncertainty for any given level of uncertainty aversion on the part of the players. This last question is motivated by the possibility of modelling the game not only with utility functions and strategy sets, but also the uncertainty aversion of the players. We believe that the uncertainty aversion of the players is what is often referred to as the "missing parameter" in the description of a game.

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Player II

		a	b
Player I	a	-2, -2	0, 1
	b	1, 0	-1, -1

Figure 1

Player II

		a	b
Player I	u	10, 10	-10, 10-ε
	d	10-ε,10	10-ε,10-ε

Figure 2

COL

		F	C
ROW	F	0, 0	a, b
	C	b, a	1, 1

Figure 3

ENSAIOS ECONOMICOS DA EPGE

100. JUROS, PRECOS E DIVIDA PUBLICA - VOL I: ASPECTOS TEORICOS - Marco Antonio C.Martins e Clovis de Faro - 1987(esgotado).

101. JUROS, PRECOS E DIVIDA PUBLICA - VOL II: A ECONOMIA BRASILEIRA -1971/85 - Antonio Salazar P.Brandao, Clovis de Faro e Marco A.C.Martins - 1987 (esgotado).

102. MACROECONOMIA KALECKIANA - Rubens Penha Cysne - 1987(esgotado).

103. O PREMIO DO DOLAR NO MERCADO PARALELO, O SUBBFATURAMENTO DE EXPORTACOES E O SUPERFATURAMENTO DE IMPORTACOES - Fernando de Holanda Barbosa, Rubens Penha Cysne e Marcos Costa Holanda-1987 (esgotado).

104. BRAZILIAN EXPERIENCE WITH EXTERNAL DEBT AND PROSPECTS FOR GROWTH - Fernando de Holanda Barbosa and Manuel Sanches de La Cal - 1987 (esgotado).

105. KEYNES NA SEDICAO DA ESCOLHA PUBLICA - Antonio Maria da Silveira - 1987 (esgotado).

106. O TEOREMA DE FROBENIUS-PERRON - Carlos Ivan Simonsen Leal - 1987. (esgotado)

107. POPULACAO BRASILEIRA - Jesse Montelo - 1987 (esgotado).

108. MACROECONOMIA - CAPITULO VI: "DEMANDA POR MOEDA E A CURVA LM" - Mario Henrique Simonsen e Rubens Penha Cysne - 1987 (esgotado).

109. MACROECONOMIA - CAPITULO VII: "DEMANDA AGREGADA E A CURVA IS" - Mario Henrique Simonsen e Rubens Penha Cysne - 1987 (esgotado).

110. MACROECONOMIA - MODELOS DE EQUILIBRIO AGREGATIVO A CURTO PRAZO - Mario Henrique Simonsen e Rubens Penha Cysne - 1987 (esgotado).

111. THE BAYESIAN FOUNDATIONS OF SOLUTIONS CONCEPTS OF GAMES - Sergio Ribeiro da Costa Werlang e Tommy Chin-Chiu Tan - 1987 (esgotado).

112. PRECOS LIQUIDOS (PRECOS DE VALOR ADICIONADO) E SEUS DETERMINANTES; DE PRODUTOS SELECIONADOS, NO PERIODO 1980/1• SEMESTRE/1986 - Raul Ekerman - 1987. (esgotado)

113. EMPRESTIMOS BANCARIOS E SALDO-MEDIO: O CASO DE PRESTACOES - Clovis de Faro - 1988 (esgotado).

114. A DINAMICA DA INFLACAO - Mario Henrique Simonsen - 1988 (esgotado).

115. UNCERTAINTY AVERSIONS AND THE OPTMAL CHOISE OF OPRTFOLIO - James Dow e Sergio Ribeiro da Costa Werlang - 1988 (esgotado)

116. O CICLO ECONOMICO - Mario Henrique Simonsen - 1988 (esgotado)

117. FOREIGN CAPITAL AND ECONOMIC GROWTH - THE BRASILIAN CASE STUDY - Mario Henrique Simonsen - 1988 (esgotado)

118. COMMON KNOWLEDGE - Sergio Ribeiro da Costa Werlang - 1988 (esgotado).

119. OS FUNDAMENTOS DA ANALISE MACROECONOMICA - Mario Henrique Simonsen e Rubens Penha Cysne - 1988 (esgotado).

120. CAPITULO XII - ESPECTATIVAS RACIONAIS - Mario Henrique Simonsen 1988 (esgotado).

121. A OFERTA AGREGADA E O MERCADO DE TRABALHO - Mario Henrique Simonsen e Rubens Penha Cysne - 1988 (esgotado).

122. INERCIA INFLACIONARIA E INFLACAO INERCIAL - Mario Henrique Simonsen - 1988 (esgotado).

123. MODELOS DO HOMEM: ECONOMIA E ADMINISTRACAO - Antonio Maria da Silveira - 1988. (esgotado)

124. UNDERINVOICING OF EXPORTS, OVERINVOINCING OF IMPORTS, AND THE DOLLAR PREMIUN ON THE BLACK MARKET - Fernando de Holanda Barbosa, Rubens Penha Cysne e Marcos Costa Holanda - 1988 (esgotado).

125. O REINO MAGICO DO CHOQUE HETERODOXO - Fernando de Holanda Barbosa, Antonio Salazar Pessoa Brandao e Clovis de Faro - 1988 (esgotado).

126. PLANO CRUZADO: CONCEPCAO E O ERRO DE POLITICA FISCAL - Rubens Penha Cysne - 1988. (esgotado)

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128. CAPITULO II - MONETARY CORRECTION AND REAL INTEREST ACCOUNTING - Rubens Penha Cysne - 1988. (esgotado)

129. CAPITULO III - INCOME ANDDEMAND POLICIES IN BRAZIL - Rubens Penha Cysne - 1988. (esgotado)

130. CAPITULO IV - BRAZILIAN ECONOMY IN THE EIGHTIES AND THE DEBT CRISIS - Rubens Penha Cysne - 1988.(esgotado)

131. THE BRAZILIAN AGRICULTURAL POLICY EXPERIENCE: RATIONALE AND FUTURE DIRECTIONS - Antonio Salazar Pessoa Brandao - 1988.(esgotado)

132. MORATORIA INTERNA, DIVIDA PUBLICA E JUROS REAIS - Maria Silvia Bastos Marques e Sergio Ribeiro da Costa Werlang - 1988.(esgotado)

133. CAPITULO IX - TEORIA DO CRESCIMENTO ECONOMICO - Mario Henrique Simonsen - 1988. (esgotado)

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140. CONTABILIDADE COM JUROS REAIS - Rubens Penha Cysne - 1989.(esgotado)

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148. RETORNOS EM EDUCACAO NO BRASIL: 1976/1986. Carlos Ivan Simonsen Leal e Sergio Ribeiro da Costa Werlang - 1989.(esgotado)

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