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On the Welfare Costs of Business-Cycle Fluctuations and Economic-Growth Variation in the 20th Century*

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Abstract

Lucas(1987) has shown a surprising result in business-cycle research: the welfare cost of business cycles are very small. Our paper has several original contributions. First, in computing welfare costs, we propose a novel setup that separates the effects of uncertainty stemming from business-cycle fluctuations and economic-growth variation. Second, we extend the sample from

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which to compute the moments of consumption: the whole of the literature chose primarily to work with post-WWII data. For this period, actual consumption is already a result of counter-cyclical policies, and is potentially smoother than what it otherwise have been in their absence. So, we employ also pre-WWII data. Third, we take an econometric approach and compute explicitly the asymptotic standard deviation of welfare costs using the Delta Method.

Estimates of welfare costs show major differences for the pre-WWII and the post-WWII era. They can reach up to 15 times for reasonable parameter values – $\beta = 0.985$, and $\phi = 5$. For example, in the pre-WWII period (1901-1941), welfare cost estimates are 0.31% of consumption if we consider only permanent shocks and 0.61% of consumption if we consider only transitory shocks. In comparison, the post-WWII era is much quieter: welfare costs of economic growth are 0.11% and welfare costs of business cycles are 0.037% – the latter being very close to the estimate in Lucas (0.040%). Estimates of *marginal* welfare costs are roughly twice the size of the *total* welfare costs. For the pre-WWII era, marginal welfare costs of economic-growth and business-cycle fluctuations are respectively 0.63% and 1.17% of per-capita consumption. The same figures for the post-WWII era are, respectively, 0.21% and 0.07% of per-capita consumption.

1. Introduction

From the perspective of a representative consumer, who dislikes systematic risk, it makes sense for macroeconomic policy to try to reduce the variability of pervasive shocks affecting consumption. The best known welfare-cost approach to this issue was put forth by Lucas (1987, 3), who calculates the amount of extra consumption a rational consumer would require in order to be indifferent between an infinite sequence of consumption under uncertainty (aggregate consumption) and a consumption sequence with the same deterministic growth and no cyclical variability. Here, business-cycle shocks are the only source of variation for aggregate consumption. Thus, Lucas' measure is known as the *welfare cost of business cycles*. For 1983 figures, using a reasonable parametric utility function (CES or Power utility function), and post-WWII data, the extra consumption is about \$ 8.50 per person in the U.S., a surprisingly low amount.

Several papers have been written just after Lucas first presented his results. For example, Imrohoroglu (1989) and Atkeson and Phelan (1995) recalculated welfare costs using models with a specific type of market incompleteness. Van Wincoop (1994), Pemberton (1996), Dolmas (1998), and Tallarini (2000) have either changed preferences or relaxed expected utility maximization. In some of them, welfare costs of business cycles reached up to 25% of per-capita consumption. On that matter, Otrok (2001) notes that "it is trivial to make the welfare cost of business cycle as large as one wants by simply choosing an appropriate form for preferences."

Regarding the original setup, as in Zellner's (1992) version of the KISS principle, Lucas *Keeps It Sophisticatedly Simple*: if only transitory shocks hit consumption, the best a *macroeconomist* can hope to achieve in terms of welfare improvement is to eliminate completely its cyclical variation, which is equivalent to eliminating all systematic risk. Of course, the implicit counter-factual exercise

being performed is rather extreme, since no one really believes that this trained *macroeconomist* can indeed eliminate all cyclical variation in consumption. Shutting out completely the uncertainty behind shocks to consumption in computing welfare costs forces the counter-factual exercise to be of limited practical importance. Moreover, it dismisses any sources of uncertainty affecting long-term growth. Indeed, Lucas recognizes that the setup could also include permanent shocks, which lead Obstfeld (1994) to compute welfare costs in this context; see also Dolmas, Tallarini, Issler, Franco, and Guillén (2008) and Reis (2009), the latter showing explicitly the importance of properly measuring the persistence in aggregate consumption.

In a very interesting paper, Alvarez and Jermann (2004) generalized the setup in Lucas by proposing a more realistic counter-factual exercise, where the representative consumer is offered a convex combination of consumption and its conditional mean, but not a deterministic sequence *a priori*. Their setup includes the *total* and the *marginal* welfare costs of business cycles. Total welfare costs are computed when, in the counter-factual exercise, all the weight goes to the conditional mean as in Lucas[1]. Marginal costs are obtained when we consider small changes in welfare costs in the neighborhood of observed consumption, which has a more practical appeal.

More recently, the literature has focused on rare disasters – Barro (2009); on the effects of model uncertainty on the welfare cost of business cycles – Barillas, Hansen, and Sargent (2009); on how the stochastic properties of aggregate consumption affects welfare cost estimates – Reis; on the distinction between individual and aggregate consumption risk in computing welfare costs – De Santis (2009); and on the difference between welfare costs based on preference-parameter values that fit or not asset-pricing data – Melino (2010).

In our view, despite the existence of a seemingly mature literature, there are still important issues to be discussed in it. Consider models where aggregate consumption is hit by permanent shocks (shocks affecting economic growth) and transitory shocks (typical business-cycle shocks). The nature and sources of these shocks are completely different and they can arise in the real-business-cycles tradition, e.g., King, Plosser and Rebelo (1988), and King et al. (1991), or in new-keynesian tradition, e.g., Galí (1999). As we note in a previous paper (Issler, Franco, and Guillén), the welfare impact of permanent and transitory shocks is completely different: for the former, its conditional variance increases without bound with time, whereas it is bounded for the latter. Hence, separating the effects of these two type of shocks in a sensible way requires thinking deeper about the counter-factual exercise being performed. An easy solution is to lump all uncertainty together, computing the welfare costs of what we have labelled *macroeconomic uncertainty*. However, this approach is clearly limited in scope, given the very different roles that these two types of shocks play and their potentially different sources. Indeed, this dichotomy has been key in macroeconomics

[1] To be equivalente to the exercise in Lucas, all the weight should go the unconditional mean instead. However, Alvarez and Jermann want to take into account the possibility that consumption is non-stationary. Thus, they focus on the conditional mean, which is still well defined in this case.

and in macro-econometrics since the seminal work of Phelps (1967, 1968, 1970).

Another important issue that deserves further attention is the fact that (almost) all of the previous literature has computed welfare costs for the post-WWII period[2]. Although this is interesting on its own right, it helps little in measuring the welfare benefits of counter-cyclical policies, for the simple reason that they were already in place during this period. Borrowing ideas from the *treatment-effect* literature, post-WWII aggregate consumption reflects already the *treatment* from counter-cyclical policies, thus it cannot serve as a benchmark to compute the welfare benefits associated with them. One candidate to compute the latter is to use pre-WWII consumption data, which lead us to compute here "The Welfare Costs in the 20th Century." We recognize that the match is not perfect, since the pre-war period may include policies (or lack thereof) that hurt welfare. Despite that, it is interesting on its own right: separating the samples in pre-WWII and post-WWII allows to measure by how much welfare costs have changed over time, something that could serve as a guide for current and future macroeconomic policy.

Our paper has three original contributions. First, while the whole literature makes no effort to construct a setup that separates the effects of uncertainty stemming from business-cycle fluctuations and economic-growth variation, we explicitly make an effort to do so. In addition to that, uncertainty is computed in a bivariate model containing consumption and income, which enlarges the conditioning set used by the representative consumer in extracting consumption shocks, something that is not seen in the literature. Here, permanent shocks to consumption arise from the unit-root component in its trend. There are empirical reasons for that, e.g., Hall (1978), Nelson and Plosser (1982), Engle and Granger (1987), King et al. (1991), Issler and Vahid (2001), and Reis. There are also theoretical reasons: in the consumption literature – e.g., Hall (1978) and Flavin (1983) – it is shown that consumption should follow a martingale; in the stochastic discount factor literature – e.g., Alvarez and Jermann (2005), and Hansen and Scheikman (2009) – it is shown that the limit stochastic discount factor must entail permanent shocks. Indeed, as stressed by Alvarez and Jermann, "for many cases where the pricing kernel is a function of consumption, innovations to consumption need to have permanent effects." Thus, we model the trend in consumption as martingale process to accommodate this need. The fluctuations about the trend (the cycle) are modelled as a stationary and ergodic zero-mean process. Trend and cyclical innovations are assumed to be independent, which allows the joint measurement of welfare costs of business cycles and of economic-growth variation.

Second, we depart from Lucas in changing the sample from which to compute the moments of consumption: the whole of the literature chose primarily to work with post-WWII data. However, for this period, actual consumption is already a result of counter-cyclical policies, and is potentially

[2] The only exception is Alvarez and Jermann, who also estimated welfare costs including the pre-WWII period (1889-2001 and 1927-2001), although they do not present separate pre- and post-WWII results. In any case, their emphasis is on the post-WWII period (1954-2001).

smoother than what it otherwise have been in their absence. It would be desirable to measure the welfare cost of business cycles observed in times with no (or little) counter-cyclical policy. Despite the caveat raised above, that is why we use pre-WWII data.

Third, we take an econometric approach, and compute explicitly the asymptotic standard deviation of welfare costs using the Delta Method. This allows us to compute confidence bands for welfare costs. Indeed, we go back to the idea behind the original exercise done by Lucas, where he notes that: "It is worth re-emphasizing that these calculations rest on assumptions about preferences *only*, and not about any particular mechanism – equilibrium and disequilibrium – assumed to generate business cycles." In other words, we need not specify a full structural model to investigate the welfare costs of business cycles in the presence of trend and cyclical shocks, which is exactly our approach.

The paper is divided as follows. Section 2 provides a theoretical and statistical framework to evaluate the welfare costs of business cycles. Section 3 provides the estimates that are used in calculating them. Section 4 provides the calculations results, and Section 5 concludes. There is also an Appendix providing the econometric background necessary to implement the calculations carried out in the paper.

2. The Problem

Lucas (1987) proposed the following way to evaluate the welfare gains of cycle smoothing (or the welfare costs of business cycles). Suppose that consumption (c_t) is *log-Normally* distributed about a deterministic trend:

$$c_t = \alpha_0 \left(1 + \alpha_1\right)^t \exp\left(-\frac{1}{2}\sigma_z^2\right) z_t, \tag{2.1}$$

where $\ln\left(z_t\right) \sim N\left(0, \sigma_z^2\right)$ is the stationary and ergodic cyclical component of consumption.

Cycle-free consumption is the sequence $\{c_t^*\}_{t=0}^{\infty}$, where
$c_t^* = \mathbb{E}\left(c_t\right) = \alpha_0 \left(1 + \alpha_1\right)^t \exp\left(-\frac{1}{2}\sigma_z^2\right) \mathbb{E}\left(z_t\right) = \alpha_0 \left(1 + \alpha_1\right)^t$. Notice that $\{c_t^*\}_{t=0}^{\infty}$ is the resulting sequence when we replace the random variable c_t with its unconditional mean. Hence, for any particular time period, c_t represents a mean-preserving spread of c_t^*.

An intuitive way of thinking about c_t^* is realizing that:

$$c_t^* = \lim_{\sigma_z^2 \to 0} c_t = \lim_{\sigma_z^2 \to 0} \alpha_0 \left(1 + \alpha_1\right)^t \exp\left(-\frac{1}{2}\sigma_z^2\right) z_t = \alpha_0 \left(1 + \alpha_1\right)^t.$$

Hence, c_t^* is a degenerate random variable with all the mass of its distribution at $\alpha_0 \left(1 + \alpha_1\right)^t$, obviously risk free.

Risk averse consumers prefer $\{c_t^*\}_{t=0}^{\infty}$ to $\{c_t\}_{t=0}^{\infty}$. Then, to evaluate the welfare costs of business cycles, amounts to calculating λ, which solves the following equation[3]:

$$\mathbb{E}\left(\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left((1+\lambda) c_t\right)\right) = \sum_{t=0}^{\infty} \beta^t u\left(c_t^*\right), \tag{2.2}$$

where $\mathbb{E}_t(\cdot) = \mathbb{E}(\cdot \mid \mathcal{I}_t)$ is the conditional expectation operator of a random variable, using $\mathcal{I}_t$ as the information set, $u(\cdot)$ is the utility function of the representative agent who discounts future utility at the rate β. Then, the welfare cost is expressed as the compensation λ, that consumers would require at all dates and states of nature, which makes them indifferent between the uncertain stream $\{c_t\}_{t=0}^{\infty}$ and the risk-free stream $\{c_t^*\}_{t=0}^{\infty}$.

Notice that uncertainty here comes in the form of stochastic business cycles alone, since the trend in consumption is purely deterministic. One important limitation of this setup is that it prevents the existence of permanent shocks to consumption. Of course, at least since Nelson and Plosser (1982), macroeconomists have benefitted from the dichotomy of having econometric models with permanent and transitory shocks, the first being associated with permanent factors influencing economic growth - such as productivity, population, etc., and the second being associated with transient factors - such as monetary policy.

Since Lucas modelled consumption trend as deterministic, eliminating *all the cyclical variability* in $\ln(c_t)$ is equivalent to eliminating *all* its variability. Under difference stationarity for (log) consumption, where the econometric model now entails a permanent-transitory decomposition for shocks, this equivalence is lost, since uncertainty comes both in the trend and the cyclical component of $\ln(c_t)$. Moreover, $\mathbb{E}(c_t)$ is not defined, since the stochastic component of $\ln(c_t)$ is neither stationary nor ergodic. This led Obstfeld (1994) to use $\mathbb{E}_0(\cdot)$ in defining welfare costs:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left((1+\lambda) c_t\right) = \sum_{t=0}^{\infty} \beta^t u\left(\mathbb{E}_0(c_t)\right). \tag{2.3}$$

Here, λ is the welfare cost associated with all the uncertainty in consumption, not just the uncertainty associated with the business-cycle component of consumption. Thus, it cannot be labelled the welfare cost of business cycles. Indeed, on an earlier paper (Issler, Franco, and Guillén (2008)), we have labelled it the *welfare cost of macroeconomic uncertainty* as opposed to the *welfare cost of business cycles.*

An interesting generalization of the setup in Lucas is due to Alvarez and Jermann (2004), who proposed offering the consumer a convex combination of $\{c_t^*\}_{t=0}^{\infty}$ and $\{c_t\}_{t=0}^{\infty}$: $(1-\alpha) c_t + \alpha c_t^*$, where

[3] Notice that Lucas (1987) uses the unconditional mean operator instead of the conditional mean operator in (2.2). The same problem can be proposed using the conditional expectation instead. This is exactly how we proceed in this paper.

$c_t^* = \mathbb{E}_0(c_t)$. They make the welfare cost to be a function of the weight α, $\lambda(\alpha)$, which solves:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(\left(1+\lambda(\alpha)\right) c_t\right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(\left(1-\alpha\right) c_t + \alpha c_t^*\right). \tag{2.4}$$

In their setup $\lambda(0) = 0$, and λ, as defined by Lucas, is obtained as $\lambda = \lambda(1)$, when using $\mathbb{E}(\cdot)$ instead of $\mathbb{E}_0(\cdot)$ in (2.4). They label $\lambda(1)$ as the *total cost of business cycles* and define the *marginal cost of business cycles*, obtained after differentiating (2.4) with respect to α as[4]:

$$\lambda'(0) = \frac{\mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta^t u'(c_t) \times \mathbb{E}_0(c_t)\right]}{\mathbb{E}_0 \sum_{t=0}^{\infty} \left[\beta^t u'(c_t) \times c_t\right]} - 1. \tag{2.5}$$

As stressed by Alvarez and Jermann, there is a straightforward interpretation for $\lambda'(0)$. Consider a Taylor-expansion argument for $\lambda(\alpha)$ around zero. We have: $\lambda(\alpha) \approxeq \lambda(0) + \lambda'(0)\alpha$. Recall that $\lambda(0) = 0$. Thus, $\lambda(\alpha) \approxeq \lambda'(0)\alpha$, which makes $\lambda'(0)$ the first-order approximation of $\lambda(1)$ around zero, recalling that $\lambda(1)$ is Lucas' measure. Their setup relies solely on asset-pricing data to compute $\lambda'(0)$, which avoids completely the specification of preferences. However, as seen in (2.5), there is a preference counterpart of their formulas which will be used here as we show below.

From (2.2), (2.3), and (2.5), notice that the total and the marginal cost of business cycles can be computed if consumption is stationary and ergodic and also when it is not. The only difference is whether we employ $\mathbb{E}(\cdot)$ or $\mathbb{E}_t(\cdot)$, respectively, in defining it. Despite that, the choice of how to model consumption is an important one for several reasons. As is well known, unless consumption has a unit root, we cannot consider the existence of shocks with a permanent effect on it. The arguments in Reis (2009) in favor of consumption containing a unit root, what he has labelled *Hall's (1978) consumption process*, are convincing. Moreover, as stressed by Alvarez and Jermann (2005), "for many cases where the pricing kernel is a function of consumption, innovations to consumption need to have permanent effects." A permanent-transitory decomposition of consumption shocks allows to explicitly isolate transient and permanent sources of welfare fluctuations, which could, in principle, be associated with the welfare costs of business cycles and the welfare costs of growth components. If one does not separate the welfare costs associated with permanent and transitory components, there is the risk of inconsistent estimation of business-cycle costs alone.

As stressed in Issler and Vahid (2001), "theoretical models are rarely built in terms of permanent or transitory shocks. Rather, they are built in terms of real (e.g., productivity) or nominal (e.g., monetary) shocks." Here, in the original spirit of Lucas, we will link transitory shocks to sources of business cycles. Permanent shocks will be linked to sources of economic growth. Moreover, we impose independence between them. To go one step further would be to link these shocks, respectively, to monetary policy and to productivity, something we refrain from doing here. We rely

[4]We have to assume that the usual regularity conditions hold in exchanging the integral and derivative signs; see the conditions in Amemiya (1985, Theorem 1.3.2).

on the argument put forth by Issler and Vahid who point or that not all "permanent" shocks are "productivity" shocks, since there may be permanent demand shocks to taste, for example. One could also think of transitory productivity shocks as well, challenging the link between "transitory" and "monetary." With that in mind, we now expose our own setup.

To start the discussion of difference-stationary consumption, we first assume that the utility function is in CES class, with risk aversion coefficient ϕ:

$$u(c_t) = \frac{c_t^{1-\phi} - 1}{1-\phi}, \tag{2.6}$$

where $u(c_t)$ approaches $\ln(c_t)$ as $\phi \to 1$.

As shown in Beveridge and Nelson (1981), every linear difference-stationary process can be decomposed as the sum of a deterministic term, a random walk (martingale) trend, and a stationary cycle ($ARMA$ process). The analogue of (2.1) when consumption is difference stationary is:

$$\ln(c_t) = \ln(\alpha_0) + \ln(1+\alpha_1) \cdot t - \frac{\omega_t^2}{2} + \sum_{i=1}^{t} \varepsilon_i + \sum_{j=0}^{t-1} \psi_j \mu_{t-j} \tag{2.7}$$

where $\ln\left[\alpha_0 (1+\alpha_1)^t \cdot \exp\left(-\omega_t^2/2\right)\right]$ is the deterministic term, $\sum_{i=1}^{t} \varepsilon_i$ is the random walk component, $\sum_{j=0}^{t-1} \psi_j \mu_{t-j}$ is the $MA(\cdot)$ representation of the stationary part (cycle), which entails $\psi_0 = 1$ and $\sum_{j=0}^{\infty} \psi_j^2 < \infty$. The permanent shock ε_t and the transitory shock μ_t are assumed to have a bivariate Normal distribution as follows:

$$\begin{pmatrix} \varepsilon_t \\ \mu_t \end{pmatrix} \sim i.i.d.\mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{11} & 0 \\ 0 & \sigma_{22} \end{pmatrix} \right), \tag{2.8}$$

i.e., shocks are uncorrelated across time and are contemporaneously uncorrelated. This implies independence across time for both shocks and independence among them too. Thus, $\omega_t^2 = \sigma_{11} \cdot t + \sigma_{22} \sum\limits_{j=0}^{t-1} \psi_j^2$ is the conditional variance of $\ln(c_t)$, where it becomes clear that ε_t and μ_t have two very different roles in terms of uncertainty: the uncertainty of ε_t grows without bound with t $(\sigma_{11} \cdot t)$, whereas that of μ_t also increases with t $\left(\sigma_{22} \sum\limits_{j=0}^{t-1} \psi_j^2\right)$ but is bounded from above by the unconditional variance $\sigma_{22} \sum\limits_{j=0}^{\infty} \psi_j^2$.

As noted by Reis (2009), the degree of persistence imposed in the process $\{\ln(c_t)\}_{t=1}^{\infty}$ is critical to determine the welfare costs of business cycles. As an example, suppose we use a first-order autoregressive $AR(1)$ assumption for $\ln(c_t)$ about a deterministic trend, i.e., $\ln(c_t) = \ln(\alpha_0) + \ln(1+\alpha_1) \cdot t - \frac{\omega_t^2}{2} + \sum_{j=0}^{\infty} \psi^j \mu_{t-j}$, where $\omega_t^2 = \sigma_{22} \sum\limits_{j=0}^{t-1} \psi^{2j}$ and ψ is the first-order autoregressive coefficient, with $|\psi| < 1$. Then, the variance of $\ln(c_t)$ about its trend is $\frac{\sigma_{22}}{1-\psi^2}$. Making $\{\ln(c_t)\}_{t=1}^{\infty}$

more persistent implies letting $|\psi|$ approach unity from below and $\frac{\sigma_{22}}{1-\psi^2}$ to grow without bound. Since the consumer dislikes risk, the welfare cost of business cycles is an increasing function of the persistence in $\{\ln(c_t)\}_{t=1}^{\infty}$.

Additionally, there is a discontinuity of the asymptotics for the least-square estimate of ψ, $\widehat{\psi}$, at $|\psi| = 1$, when one uses a sample size of T observations in estimation. If $|\psi| < 1$, $\widehat{\psi}$ is $\sqrt{T}$-consistent, whereas, at $\psi = 1$, it is T-consistent and downward biased in small samples[5]. Reis applies two alternative methods to compute welfare costs if consumption has a unit root. The first is a local-to-unity approach, where the unit root only shows up in the limit. Alternatively, based on the results of several tests, Reis also imposes a unit root to consumption, avoiding the downward-bias problem in estimation. As can be seen from equation (2.7), we chose to impose a unit root to consumption as well[6]. However, we go one step further since we separate the welfare effects of permanent and transitory shocks to $\ln(c_t)$ given the structure underlying (2.8).

A main objective of this paper is to isolate the welfare costs of business cycles and the welfare costs of economic growth. As stressed by Issler, Franco, and Guillén (2008), one way to study the welfare cost of business cycles in a difference-stationary world is to work with independent shocks responsible for trend and cyclical movements in $\ln(c_t)$. If one does not separate the effects of these shocks, she/he is forced to examine the welfare cost of *all* macroeconomic uncertainty, or to work with a tainted measure of welfare cost of business cycles which encompasses some or all of the cost associated with economic-growth factors. A previous attempt to deal with this issue includes only examining consumption fluctuations at business-cycle horizons; see, e.g., Alvarez and Jermann (2004). In our view, this strategy is best viewed as an approximation, since some business-cycle variation in consumption can be due to permanent shocks: recall that one of the main features of the real-business-cycle literature was that permanent shocks could indeed generate business-cycle fluctuations; see, *inter alia*, Kydland and Prescott (1982), King, Plosser and Rebelo (1987), King, Plosser, Stock and Watson (1991), and Issler and Vahid (2001).

In the framework above, because of independence of shocks, it is natural to evaluate the welfare cost of business cycles using μ_t, and to evaluate the welfare cost of economic growth using ε_t. To do so, consider the two processes below, where we start with (2.7) and shut out permanent and

[5] Moreover, the effect of uncertainty is very different for welfare costs. As $\psi \to 1$, the autoregressive process becomes a random walk, for which the conditional variance is $\sigma_{22} \cdot t$, i.e., increases without bound with time.

[6] Reis claims that "Consumption growth is positively serially correlated, a fact that has inspired most modern research on consumption." Indeed, the models we entretain below have this character.

transitory shocks, respectively, as follows:

$$c_t^T = \alpha_0 (1+\alpha_1)^t \cdot \exp\left[-\frac{\sigma_{22} \sum_{j=0}^{t-1} \psi_j^2}{2}\right] \sum_{j=0}^{t-1} \psi_j \mu_{t-j}, \text{ and,} \tag{2.9}$$

$$c_t^P = \alpha_0 (1+\alpha_1)^t \cdot \exp\left(-\frac{\sigma_{11}}{2} t\right) \sum_{i=1}^{t} \varepsilon_i. \tag{2.10}$$

From (2.7), we can think of c_t^T and c_t^P as limit cases, respectively:

$$\lim_{\sigma_{11}\to 0} c_t = c_t^T, \text{ and } \lim_{\sigma_{22}\to 0} c_t = c_t^P.$$

We propose measuring the welfare cost for the representative consumer of bearing the uncertainty associated with $\{\mu_t\}$ alone (business cycles) through the use of c_t^P. Notice that the conditional means of c_t^P and c_t are identical: $\mathbb{E}_0\left(c_t^P\right) = \mathbb{E}_0(c_t) = \alpha_0(1+\alpha_1)^t$. However, the uncertainty of the consumption stream $\{c_t\}_{t=1}^{\infty}$ is larger than that of $\left\{c_t^P\right\}_{t=1}^{\infty}$. Thus, c_t is a mean-preserving spread of c_t^P. Risk averse consumers prefer the stream $\left\{c_t^P\right\}_{t=1}^{\infty}$ over $\{c_t\}_{t=1}^{\infty}$. Thus, we measure the welfare cost associated with $\{\mu_t\}$ alone using λ_P, which solves:

$$\mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty} \beta^t u\left((1+\lambda_P) c_t\right)\right] = \mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty} \beta^t u\left(c_t^P\right)\right], \tag{2.11}$$

i.e., we can think of λ_P as the welfare cost of bearing the risks associated with transitory shocks alone. Thus, we label it the welfare cost of business cycles.

In order to implement the computation of λ_P, we specialize the utility function to in the CES class as in (2.6). After straightforward but tedious algebra we get,

$$\lambda_P = \exp\left(\phi \widetilde{\sigma}_{22}/2\right) - 1, \tag{2.12}$$

where, for the sake of simplicity in computation, we replace $\sigma_{22} \sum_{j=0}^{t-1} \psi_j^2$ by its respective unconditional counterpart $\widetilde{\sigma}_{22} = \sigma_{22} \sum_{j=0}^{\infty} \psi_j^2$. We also assume that the convergence condition $\beta \cdot (1+\alpha_1)^{(1-\phi)} \cdot \exp^{(-\phi(1-\phi)\sigma_{11}/2)} < 1$ holds. Notice that the welfare cost of business cycles does not depend on the uncertainty associated with permanent shocks. However, it depends on σ_{22} – the uncertainty behind transitory shocks – as well as on the degree of persistence of these shocks, captured by $\sum_{j=0}^{\infty} \psi_j^2$, and on the relative risk-aversion coefficient ϕ.

Analogously, we propose measuring the welfare cost for the representative consumer of bearing the uncertainty associated with $\{\varepsilon_t\}$ alone (economic growth) through the use of c_t^T. Recall that

$\mathbb{E}_0\left(c_t^T\right) = \mathbb{E}_0\left(c_t\right) = \alpha_0\left(1+\alpha_1\right)^t$, and c_t is a mean-preserving spread of c_t^T. Hence, we measure the welfare cost associated with $\{\varepsilon_t\}$ alone by using λ_T, which solves:

$$\mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty}\beta^t u\left(\left(1+\lambda_T\right)c_t\right)\right] = \mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty}\beta^t u\left(c_t^T\right)\right] \tag{2.13}$$

Hence, we can think of λ_T as the welfare cost of economic growth. Using (2.6), one can show that:

$$\lambda_T = \begin{cases} \left[\frac{\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\cdot\exp(-\phi(1-\phi)\sigma_{11}/2)\right)}{\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\right)}\right]^{\frac{1}{(1-\phi)}} - 1, \quad \text{for } \phi \neq 1 \\ \\ \exp\left(\frac{\beta\sigma_{11}}{2(1-\beta)}\right) - 1, \quad \text{for } \phi = 1 \end{cases}, \tag{2.14}$$

where we assume that the convergence condition $\beta\cdot(1+\alpha_1)^{(1-\phi)} < 1$, holds. Notice that λ_T does not depend on σ_{22} – i.e., on how uncertain transitory shocks are. However, it depends on β, ϕ, σ_{11} and α_1.

Finally, we can compute welfare costs for the representative consumer of bearing the uncertainty associated with both $\{\varepsilon_t\}$ and $\{\mu_t\}$ by introducing c_t^D:

$$\lim_{\sigma_{11}\to 0, \sigma_{22}\to 0} c_t = c_t^D = \alpha_0\left(1+\alpha_1\right)^t. \tag{2.15}$$

Here, $\mathbb{E}_0\left(c_t^D\right) = \mathbb{E}_0\left(c_t\right) = \alpha_0\left(1+\alpha_1\right)^t$, making c_t a mean-preserving spread of c_t^D. We measure the welfare cost associated with both $\{\varepsilon_t\}$ and $\{\mu_t\}$ using λ_D, which solves:

$$\mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty}\beta^t u\left(\left(1+\lambda_D\right)c_t\right)\right] = \mathbb{E}_0\left[\sum\nolimits_{t=0}^{\infty}\beta^t u\left(c_t^T\right)\right]. \tag{2.16}$$

Using (2.6), we obtain:

$$\lambda_D = \begin{cases} \left[\frac{e^{\phi(1-\phi)\tilde{\sigma}_{22}/2}\cdot\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\cdot e^{(-\phi(1-\phi)\sigma_{11}/2)}\right)}{\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\right)}\right]^{\frac{1}{(1-\phi)}} - 1, \quad \text{for } \phi \neq 1 \\ \\ e^{\frac{\beta\sigma_{11}+(1-\beta)\tilde{\sigma}_{22}}{2(1-\beta)}} - 1, \quad \text{for } \phi = 1 \end{cases}, \tag{2.17}$$

where we assume that the convergence condition $\beta\cdot(1+\alpha_1)^{(1-\phi)} < 1$, holds.

Measures λ_P, λ_T, and λ_D are what Alvarez and Jermann have labelled measures of *total welfare costs*. Here, we are also interested in measures of *marginal welfare costs*, i.e., $\lambda_P'(0)$, $\lambda_T'(0)$, and $\lambda_D'(0)$. Starting from (2.4), and using (2.6), we measure marginal welfare costs of business cycles,

economic growth, and macroeconomic uncertainty by using c_t^P, c_t^T, and c_t^D, respectively:

$$\lambda_P'(0) = \exp\left(\phi\widetilde{\sigma}_{22}\right) - 1, \tag{2.18}$$

$$\lambda_T'(0) = \begin{cases} \dfrac{\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\cdot e^{\frac{-\phi(1-\phi)\sigma_{11}}{2}}\right)}{\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\cdot e^{\frac{\phi(1+\phi)\sigma_{11}}{2}}\right)} - 1, & \text{for } \phi \neq 1 \\ \\ \frac{1-\beta}{1-\beta e^{\sigma_{11}}} - 1, & \text{for } \phi = 1 \end{cases}, \text{ and,} \tag{2.19}$$

$$\lambda_D'(0) = \begin{cases} \dfrac{e^{\phi\cdot\widetilde{\sigma}_{22}}\cdot\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\cdot e^{\frac{-\phi(1-\phi)\sigma_{11}}{2}}\right)}{\left(1-\beta\cdot(1+\alpha_1)^{(1-\phi)}\cdot e^{\frac{\phi(1+\phi)\sigma_{11}}{2}}\right)} - 1, & \text{for } \phi \neq 1 \\ \\ \frac{e^{\widetilde{\sigma}_{22}}(1-\beta)}{1-\beta e^{\sigma_{11}}} - 1, & \text{for } \phi = 1 \end{cases}, \tag{2.20}$$

where we assume that the usual specific convergence conditions apply in computing $\lambda_P'(0)$, $\lambda_C'(0)$ and $\lambda_T'(0)$, respectively, and replace $\sigma_{22}\sum_{j=0}^{t-1}\psi_j^2$ by its respective unconditional counterpart $\widetilde{\sigma}_{22} = \sigma_{22}\sum_{j=0}^{\infty}\psi_j^2$ in computing $\lambda_C'(0)$. As in the case of total welfare costs, we interpret $\lambda_P'(0)$, $\lambda_T'(0)$, and $\lambda_D'(0)$ as being the marginal welfare costs of business cycles, of economic growth, and of all macroeconomic uncertainty, respectively.

Finally, we give some intuition behind the measures of welfare costs proposed above. One way to think about (2.10) is:

$$\lim_{\sigma_{22}\to 0} c_t = c_t^P = \mathbb{E}\left[c_t \mid \mathcal{I}_0, \{\varepsilon_t\}_{t=0}^{\infty}\right],$$

which shows that c_t^P is the conditional expectation of c_t when we have *perfect foresight* of the sequence $\{\varepsilon_t\}_{t=0}^{\infty}$ of permanent shocks. Thus, in computing the welfare costs of business cycles, we *control* for the existence of permanent shocks to consumption. This shows that the welfare-cost measures λ_P and $\lambda_P'(0)$ only take into account the uncertainty that goes beyond permanent shocks, i.e., transitory shocks alone.

Using (2.9) and (2.15), a similar reasoning applies to c_t^T and c_t^D, respectively:

$$\lim_{\sigma_{11}\to 0} c_t = c_t^T = \mathbb{E}\left[c_t \mid \mathcal{I}_0, \{\mu_t\}_{t=0}^{\infty}\right],$$

$$\lim_{\sigma_{11}\to 0,\sigma_{22}\to 0} c_t = c_t^D = \mathbb{E}\left[c_t \mid \mathcal{I}_0, \{\mu_t\}_{t=0}^{\infty}, \{\varepsilon_t\}_{t=0}^{\infty}\right].$$

3. Identification and Estimation of Structural Parameters used in Computing λ_T, λ_P, λ_D, $\lambda_T'(0)$, $\lambda_P'(0)$, and $\lambda_D'(0)$

Next, we discuss the reduced form and the structural form used in estimating λ_T, λ_P, λ_D, $\lambda_T'(0)$, $\lambda_P'(0)$, and $\lambda_D'(0)$. For the reduced form, we borrow heavily from the discussion in Issler, Franco,

and Guillén (2008). This is especially important regarding possible long-run constraints in the data. Our starting point is a vector autoregression (VAR), where possible cointegrating restrictions are used in estimation. We show how a simple identification strategy can be used in this setup, although it does not impose the restriction that $\mathbb{E}(\varepsilon_t \mu_t) = 0$. For that reason, we also discuss structural time-series models based on Harvey (1985b) and Koopman et al. (2009) where $\mathbb{E}(\varepsilon_t \mu_t) = 0$ is imposed under joint Normality for shocks.

3.1. Reduced Form: Long-Run and Short-Run Constraints

A full discussion of the econometric models employed here can be found in Beveridge and Nelson (1981), Stock and Watson (1988), Engle and Granger (1987), Campbell (1987), Campbell and Deaton (1989), Vahid and Engle (1993), and Proietti (1997). Denote by $y_t = (\ln(c_t), \ln(I_t))'$ a 2×1 vector containing respectively the logarithms of consumption and disposable income per-capita. We assume that both series contain a unit-root and are possibly cointegrated as in $[-1,1]' y_t$ because of the Permanent-Income Hypothesis (Campbell(1987)). A vector error-correction model $(VECM(p-1))$ is:

$$\Delta y_t = \Gamma_1 \Delta y_{t-1} + \ \ldots \ + \Gamma_{p-1} \Delta y_{t-p+1} + \gamma [-1,1] y_{t-p} + \eta_t. \tag{3.1}$$

Here, long-run constraints in the VAR are imposed through the error-correction mechanism $[-1,1] y_{t-p}$. As discussed in Vahid and Engle (1993), short-run restrictions in the form of common cycles can be imposed in (3.1). Let $[-1,1]' = \alpha$ and consider the following restrictions on the parameters of (3.1):

$$\widetilde{\alpha}' \Gamma_i = 0, \text{ for all } i = 1, 2, \cdots p-1, \text{ and } \widetilde{\alpha}' \gamma = 0.$$

Then, we can represent (3.1) as having *common-cyclical-feature* restrictions in a 2×1 system:

$$\begin{bmatrix} 1 & \widetilde{\alpha}^{*\prime} \\ \mathbf{0} & 1 \end{bmatrix} \Delta y_t = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ A_1^* & \cdots & A_{p-1}^* & \gamma^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-p} \end{bmatrix} + \eta_t, \tag{3.2}$$

where $A_1^*, \cdots, A_{p-1}^*, \gamma^*$ represent partitions of $\Gamma_1, \ldots \Gamma_{p-1}, \gamma$, respectively. Notice that $\begin{bmatrix} 1 & \widetilde{\alpha}^{*\prime} \\ \mathbf{0} & 1 \end{bmatrix}$ is non-singular, which allows to recover the parameters in (3.1) from the ones in (3.2) as we pre-multiply the latter by $\begin{bmatrix} 1 & \widetilde{\alpha}^{*\prime} \\ \mathbf{0} & 1 \end{bmatrix}^{-1}$. Indeed, (3.2) is just a more parsimonious representation than (3.1).

3.2. Structural Time-Series Models with Long-Run Constraints

Regarding our purposes here, the main problem of the reduced-form approach described in the previous section is that it does not impose the constraint that permanent and transitory shocks to $\ln(c_t)$ are orthogonal. Under Normality, this would imply independence of these shocks. For that reason, we now turn to the discussion of *structural* time-series models, where possible long- and short-run restrictions are still kept in a different setup. Here, we present a brief summary of the *structural* time-series model of Harvey (1985b) and Koopman et al. (2009). We start the discussion using a univariate framework. There, the main objective is to decompose a single integrated series $(I(1))$ in a trend and a cycle, treating both as latent variables to be estimated by maximum likelihood, which guarantees consistent and asymptotically Normal parameter estimates, a key property in our case.

For a single economic series x_t, we decompose it as:

$$x_t = \tau_t + \varphi_t$$

where τ_t is the $I(1)$ trend, φ_t is the cycle. Shocks to each of these two components are independent of each other and also across time. The trend evolves as:

$$\tau_t = \tau_{t-1} + \delta + \upsilon_t, \tag{3.3}$$

where υ_t has variance given by σ_υ^2, whereas the cyclical component evolves as a bivariate $VAR(1)$:

$$\begin{bmatrix} \varphi_t \\ \varphi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} \varphi_{t-1} \\ \varphi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix} \tag{3.4}$$

where the component φ_t^* shows up by construction; see Harrison and Akran (1983). Both ω_t and ω_t^* are orthogonal white noise errors with variances given by σ_ω^2 and σ_ω^{*2}, respectively. Harvey (1985b) argues that very little is lost in terms in terms of fit if we impose the restriction that $\sigma_\omega^2 = \sigma_\omega^{*2}$, representing an advantage in terms of parsimony. Finally, some restrictions on parameter values should be observed:

$$0 \leq \lambda^* \leq \pi \text{ and } 0 < \rho \leq 1,$$

where λ^* is the frequency of the cycle and ρ is the discount factor for its amplitude. The last restriction makes the cyclical component stationary.

One can also show that the cyclical component obeys:

$$\varphi_t = \frac{(1 - \rho\cos\lambda L)\,\omega_t + (\rho\sin\lambda L)\,\omega_t^*}{1 - 2\rho\cos\lambda L + \rho^2 L^2}$$

where L is the lag operator, $L^k x_t = x_{t-k}$. Under $\sigma_\omega^2 = \sigma_\omega^{*2}$, we can put the last equation in an $ARMA$ format as:

$$\left(\rho^2 L^2 - 2\rho\cos\lambda L + 1\right)\varphi_t = (1 + \Phi L)\,\omega_t$$

where it becomes clear that φ_t follows an $ARMA\,(2,1)$, with $\Phi = \rho\,(\sin\lambda - \cos\lambda)$. This is a restriction into the $ARMA$ class of models, since not every cycle of an economic series will be well modelled as an $ARMA\,(2,1)$.

Following the notation for the univariate class of models, in a multivariate setting, we can represent $y_t = (\ln(c_t), \ln(I_t))'$ as having a common trend and a common cycle, respectively, as:

$$\begin{bmatrix} \ln(c_t) \\ \ln(I_t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tau_t + \begin{bmatrix} 1 \\ \theta \end{bmatrix} \varphi_t, \tag{3.5}$$

where the $I\,(1)$ trend component τ_t follows (3.3) and the stationary cyclical component φ_t follows (3.4). Here, the bivariate system in y_t is modelled with just a single stochastic trend and a single cycle, respectively. The trend affects identically the two series in y_t, whereas the cycle affects them differently. The vector $[-1, 1]'$ removes the common trend and that the vector $[-\theta, 1]'$removes the common cycle, where there is the additional restriction that $\theta \neq 1$[7]. The structural time-series model in (3.5) is analogous to its reduced-form counterpart (3.2), in which it imposes identical long- and short-run restrictions. Despite that, they differ in which (3.5) imposes independence for the shocks to τ_t and φ_t, whereas (3.2) does not.

As stressed by Issler and Vahid (2001), there are several theoretical reasons why consumption and income should cointegrate (Campbell (1987)) and have common cycles (King, Plosser, and Rebelo (1988), Campbell and Mankiw (1989), and King et al. (1991)). Despite that, one may be more willing to impose long-run restrictions than short-run restrictions, meaning that the two variables in y_t have two distinct cycles[8], but still a common trend as in (3.5). This can be easily accommodated by the structure in (3.5), where $\begin{bmatrix} 1 \\ \theta \end{bmatrix} \varphi_t$ is replaced by the 2×1 vector φ_t,

$$\begin{bmatrix} \varphi_t \\ \varphi_t^* \end{bmatrix} = \left[\rho \begin{pmatrix} \cos\lambda & \sin\lambda \\ -\sin\lambda & \cos\lambda \end{pmatrix} \otimes \mathbf{I}_2\right] \begin{bmatrix} \varphi_{t-1} \\ \varphi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix}, \tag{3.6}$$

where now φ_t, φ_t^*, ω_t and $\omega_t^*$are 2×1 vectors, λ is a 1×2 vector, and we impose the restriction that $\mathbb{E}\,(\omega_t \omega_t') = \mathbb{E}\,(\omega_t^* \omega_t^{*\prime}) = \Sigma_\omega$, making $\mathrm{VAR}\begin{bmatrix} \omega_t \\ \omega_t^* \end{bmatrix} = \mathbf{I}_2 \otimes \Sigma_\omega$.

The univariate and multivariate models discussed above can be easily put in state-space form with Normal disturbances, where the Kalman Filter can be used to compute the likelihood function through the one-step prediction error decomposition. Consistent and asymptotically Normal estimates of parameter values are thus obtained, which is a critical step to construct our estimates of λ_T, λ_P, λ_D, $\lambda_T'(0)$, $\lambda_P'(0)$, and $\lambda_D'(0)$, as well as to construct their respective asymptotic confidence intervals; more details on state-space forms, the likelihood function, and the use of the Kalman Filter can be found in Koopman et al. (2009, Chapter 9).

[7] Testing for common cycles in a multivarite framework is discussed by Carvalho, Harvey, and Trimbur (2007).

[8] See the discussion and proposed tests in Carvalho, Harvey, and Trimbur (2007).

Finally, we discuss the identification of the key parameters in the welfare-cost formulas of Section 2 by using τ_t and φ_t: the variances σ_{11} and $\widetilde{\sigma}_{22}$ and the instantaneous growth rate of consumption, α_1. The parameter σ_{11} can be identified using $\text{VAR}(\Delta\tau_t)$, whereas $\widetilde{\sigma}_{22} = \sigma_{22}\sum_{j=0}^{\infty}\psi_j^2$ can be identified by using $\text{VAR}(\varphi_t)$. If one uses the model with a common trend, but idiosyncratic cycles as in (3.6), identification of $\widetilde{\sigma}_{22}$ is still straightforward by using $\text{VAR}([1,0]\times\varphi_t)$. It is easy to identify $\ln(1+\alpha_1)$ employing $\mathbb{E}(\Delta\tau_t)$.

The identification strategy outlined above suggests how to estimate consistently α_1, σ_{11}, and $\widetilde{\sigma}_{22}$, as well as how to compute the variances of these estimates. These are based on Phillips and Solo (1992), who discuss how to compute consistent estimates of parameters of linear processes transformed using the Beveridge and Nelson (1981) filter. First, running a regression of $\Delta\tau_t$ on a constant provides a consistent estimate of $\ln(1+\alpha_1)$: $\ln\widehat{(1+\alpha_1)} = \frac{1}{T}\sum_{t=1}^{T}\Delta\tau_t$, where T is the sample size used in estimation. Using Slutsky's Theorem, it is straightforward to find a consistent estimate for α_1. Since the cycle is a zero-mean stationary and ergodic linear process with serial dependence, $\widehat{\widetilde{\sigma}_{22}} = \frac{1}{T}\sum_{t=1}^{T}\varphi_t^2$ is a consistent estimate of $\widetilde{\sigma}_{22}$. On the other hand, the first difference of the trend, $\Delta\tau_t$, is still a linear process, but serially independent. Hence, $\widehat{\sigma_{11}} = \frac{1}{T}\sum_{t=1}^{T}\left(\Delta\tau_t - \ln\widehat{(1+\alpha_1)}\right)^2$ is a consistent estimate of σ_{11}.

As long as the serial dependence is not too strong – as is the case for all estimates above – it poses no problem to estimate consistently α_1, σ_{11}, and $\widetilde{\sigma}_{22}$. But we must account properly for the existence of serial dependence in order to estimate consistently the variance of $\widehat{\alpha_1}$, $\widehat{\sigma_{11}}$, and $\widehat{\widetilde{\sigma}_{22}}$, which are all sample means. In our context, if the elements in these sample means have serial dependence and heterogeneity of unknown form, their variances can still be consistently estimated using the concept of long-run variance, which is given by $\gamma_0 + 2\cdot\sum_{i=1}^{\infty}\gamma_i$, where γ_i is the i-th auto-covariance of the terms in the sample mean[9]. Based on the fact that $\sqrt{T}\left(\widehat{\sigma_{11}} - \sigma_{11}\right) \xrightarrow{d} \mathcal{N}(0, V_{11})$, $\sqrt{T}\left(\widehat{\widetilde{\sigma}_{22}} - \widetilde{\sigma}_{22}\right) \xrightarrow{d} \mathcal{N}(0, V_{22})$, and $\sqrt{T}\left(\widehat{\alpha_1} - \alpha_1\right) \xrightarrow{d} \mathcal{N}(0, V_\alpha)$ it is straightforward to estimate consistently V_{11}, V_{22} and V_α. In our context, the only sample mean for which the elements are serially dependent is $\frac{1}{T}\sum_{t=1}^{T}\varphi_t^2$, whereas those in $\frac{1}{T}\sum_{t=1}^{T}\Delta\tau_t$ and $\frac{1}{T}\sum_{t=1}^{T}\left(\Delta\tau_t - \ln\widehat{(1+\alpha_1)}\right)^2$ are independent. Implementing a long-run-variance estimate for V_{22} can be easily accomplished by using Newey and West's (1987) non-parametric procedure, which relies on consistent estimates of the auto-covariances of φ_t^2 and a truncation window for computing a weighted average of them using a Bartlett kernel.

[9] For any sample average $\frac{1}{T}\sum_{t=1}^{T}x_t$, of satationary and ergodic linear series x_t, serially dependent, the long-run variance is $\gamma_0 + 2\cdot\sum_{i=1}^{\infty}\gamma_i$, where γ_i is the i-th auto-covariance of x_t, i.e., $\gamma_i = \mathbb{E}\left[(x_t-\mu)(x_{t-i}-\mu)\right] = \mathbb{E}\left[(x_t-\mu)(x_{t+i}-\mu)\right]$.

3.3. Computing Asymptotic Confidence Intervals for Welfare Costs λ_T, λ_P, λ_D, $\lambda'_T(0)$, $\lambda'_P(0)$, and $\lambda'_D(0)$

In this section, we show how to compute asymptotic confidence intervals for welfare-cost estimates based on (2.12), (2.14), (2.18), (2.19), and (2.20). As discussed in the previous section, we are able to identify σ_{11}, $\widetilde{\sigma}_{22}$, and α_1, based on consistent and asymptotically Normal estimates (maximum likelihood) obtained for the unobserved-component model proposed by Harvey (1985b) and Koopman et al. (2009). Given these estimates, asymptotic confidence intervals can be obtained using the Delta Method.

Consider first the set of parameters $\theta^* = (\beta, \phi, \sigma_{11}, \widetilde{\sigma}_{22}, \alpha_1)'$. All welfare costs λ_T, λ_P, λ_D, $\lambda'_T(0)$, $\lambda'_P(0)$, and $\lambda'_D(0)$ can be expressed as specific non-linear functions of θ^*. Here, we follow the literature in treating β and ϕ as known (fixed), whereas the remaining parameters in θ^*, stacked in $\theta = (\sigma_{11}, \widetilde{\sigma}_{22}, \alpha_1)'$, are estimated consistently employing a sufficiently large sample of $t = 1, 2, \cdots, T$ observations. In this setup, the uncertainty in estimating λ_T, λ_P, λ_D, $\lambda'_T(0)$, $\lambda'_P(0)$, and $\lambda'_D(0)$ will be a function of the uncertainty in estimating the components of θ alone, and the Delta Method can be used to compute asymptotic standard errors (and asymptotic confidence intervals) for welfare-cost estimates.

Suppose that a generic welfare measure λ^* relates to θ as:

$$\lambda^* = G(\theta),$$

where $G(\theta)$ is a continuous and continuously differentiable function. Here, the function $G(\cdot)$ is specific to each welfare cost in equations (2.12), (2.14), (2.18), (2.19), and (2.20), and it can be verified that all the assumptions required to use the Delta Method are valid, case by case.

Given that a Central Limit Theorem holds for $\widehat{\theta}$,

$$\sqrt{T}\left(\widehat{\theta} - \theta\right) \xrightarrow{d} \mathcal{N}(0, V),$$

the Delta Method can be employed to compute asymptotic confidence intervals for $\widehat{\lambda^*}$, which are based on:

$$\sqrt{T}\left(\widehat{\lambda^*} - \lambda^*\right) \xrightarrow{d} \mathcal{N}\left(0, \frac{\partial G(\theta)}{\partial \theta} V \frac{\partial G(\theta)}{\partial \theta'}\right).$$

In practice, we have to replace V with a consistent estimator, $\widehat{V}$, and evaluate $\frac{\partial G(\theta)}{\partial \theta}$ and $\frac{\partial G(\theta)}{\partial \theta'}$ at $\theta = \widehat{\theta}$. In this context, the estimated variance of $\widehat{\lambda^*}$ in finite samples is given by $\frac{1}{T} \frac{\partial G(\theta)}{\partial \theta}\Big|_{\theta=\widehat{\theta}} \widehat{V} \frac{\partial G(\theta)}{\partial \theta'}\Big|_{\theta=\widehat{\theta}}$, and the 95% confidence interval for testing $H_0 : \lambda^* = 0$, is given by $\widehat{\lambda^*} \pm 1.96 \times \sqrt{\frac{1}{T} \frac{\partial G(\theta)}{\partial \theta}\Big|_{\theta=\widehat{\theta}} \widehat{V} \frac{\partial G(\theta)}{\partial \theta'}\Big|_{\theta=\widehat{\theta}}}$.

4. Empirical Results

Data for consumption of non-durables and services were obtained from DRI from 1929 through 2010. Data for consumption of perishables and services from 1901 to 1929 were obtained from Kuznets (1961) in real terms, and then chained with DRI data, resulting in a long-span series for consumption of non-durables and services from 1901-2011. Data for real GNP were also extracted from DRI from 1929 through 2010 and from Kuznets from 1901 through 1929. Data on population were extracted from Kuznets and DRI, and then chained. Figure 1 presents the data on consumption and income per-capita for the whole period 1901-2010. The peculiar features are first the magnitude of the great depression in both consumption and income behavior, and second the fact that pre-WWII data present much more volatility than post-WWII data.

We fitted a bivariate vector autoregression for the logs of consumption and income. Lag length selection indicated that a VAR(2) with an unrestricted constant term was an appropriate description of the dynamic system. This was true not only in terms of minimizing information criteria but also because this specification did not fail diagnostic testing.

Table 1 presents results of the cointegration test using Johansen's (1988, 1991) technique. The *Trace Statistics* for the null of no cointegration and of at most one cointegrating vector were respectively 16.43 and 0.18. At 5% significance, we conclude that there is one cointegrating vector, which estimate is given by $(-1.000, 1.005)'$. Conditioning on the existence of one cointegrating vector, we tested the restriction that it was equal to $(-1, 1)'$. We used the likelihood-ratio test in Johansen (1991), which yields a p-value of 0.831, not rejecting the null at usual levels of significance. An interesting by-product of cointegration analysis is testing the significance of the error-correction term in each regression of the system. The *t-statistic* associated with this test are -0.07 and 3.16, for the regression involving consumption and income respectively. Hence, the error-correction term affects income but not consumption, and the latter is long-run *weakly exogenous* in the sense of Engle, Hendry, and Richard (1983) and Johansen (1992). Despite that, we find Granger (1969) causality from income to consumption – the coefficient of lagged income is significant in consumption's equation – a t-statistic of -3.67. This shows the usefulness of the bivariate setup employed here, since conditioning in income's past helps predicting consumption today beyond what past consumption would have allowed.

Given the cointegration vector found in the empirical analysis, we implemented the multivariate structural time-series model in the form suggested by Harvey (1985b) and Koopman et al. (2009). Figure 1 shows the result of this exercise. The consumption series and the trend are very close throughout the whole period, reflecting the fact that agents do update their beliefs about future income, and that the permanent-income theory is probably a reasonable approximation to consumption behavior; see Cochrane (1994) *inter alia*. Also, the cyclical component of consumption varies much more in the pre-WWII era than afterwards.

Next, we present the results of the structural time-series model discussed in Section 3.2, where trends and cycles are estimated imposing that their shocks are independent. Table 2 displays the description of the data in terms of the parameters estimates associated with the log of consumption (2.7) under alternative periods in it. Estimates are obtained for four distinct periods: pre-WWII data – 1901-1941, post-WWII data – 1947-2000, 20th Century data – 1901-2000, and the whole period 1901-2010. It is obvious that uncertainty in the pre-WWII period is much larger than in the post-WWII period. In the pre-WWII era, the variance of the permanent component is about three times that of the post-WWII era. Results for the transitory component are even more striking: about four times.

The estimates of the *total* welfare costs are presented in Table 3. First, there are major differences in results for the pre-WWII and the post-WWII era. This is true regarding the welfare cost of business cycles (associated with transitory shocks), the welfare cost of economic growth (associated with permanent shocks), and the welfare costs of macroeconomic uncertainty (associated with both shocks). These differences can reach up to 15 times for reasonable parameter values – $\beta = 0.985$, and $\phi = 5$, for example. Second, regarding the welfare costs of business cycles in the post-WWII period, our results are very similar to those of Lucas, although the methods of estimation are completely different. Third, the welfare costs of economic growth can be twice or three times those of business cycles, while welfare costs of macroeconomic uncertainty can be about 50% larger than those of economic growth.

We now turn our attention to the analysis of the pre-WWII period (1901-1941). For reasonable preference parameter and discount values ($\beta = 0.985, \phi = 5$), welfare costs are 0.31% of consumption if we consider only permanent shocks and 0.58% of consumption if we consider only transitory shocks, which roughly translates into US$ 60.00 a year and US$ 120.00 a year, respectively, in current value. In comparison, the post-WWII era is much quieter: welfare costs of economic growth are 0.106% and welfare costs of business cycles are 0.037% – the latter being very close to the estimate in Lucas (0.040%). Results for the whole period 1901-2010 are a combination of those of pre- and post-WWII eras. For reasonable preference parameter and discount values ($\beta = 0.985, \phi = 5$) we get a compensation of 0.48% and 0.27% of consumption, respectively.

We now compare our empirical results with those in Reis (2009). He does not separate the effects of transitory and permanent shocks, i.e., he computes the welfare cost of *all* macroeconomic uncertainty. We compare Table 4 in Reis (sample 1947-2003), where a unit root is imposed for consumption, with our results for λ_D for post-WWII data (sample 1947-2000). Using an ARMA model for the instantaneous growth rate of consumption, Reis finds welfare costs to be roughly between 0.5% and 5% of consumption, whereas we find much lower estimates – between 0.05% and 0.15%. When Reis compared his results to those in Obstfeld (1994), there is also a large difference in estimates, which he attributed to the use of the calibrated effective discount rate $\rho = \beta^* + (\phi - 1) \ln (1 + \alpha_1)$, instead of the subjective discount rate β^*, where $\beta = \exp(-\beta^*)$. Since

ρ and β^* are identical for $\phi = 1$, results in this case are directly comparable: when $\rho = \beta^* = 0.03$, and thus $\beta = 0.97$, Reis reports a welfare cost of 0.31% of consumption, whereas we find 0.083%, roughly 1/4 of his estimate; for $\rho = \beta^* = 0.015$, and thus $\beta = 0.985$, we find 0.16%, whereas Reis finds 1.25% for $\rho = \beta^* = 0.01$, and 0.61% for $\rho = \beta^* = 0.02$, both much higher than our estimate. Thus, there must be an additional source of differences at work here[10].

Table 4 presents estimates of marginal welfare costs. They are roughly twice the size of welfare costs reported in Table 3. For the pre-WWII era, and reasonable preference parameter and discount values ($\beta = 0.985, \phi = 5$), the marginal welfare costs of economic growth and of business cycles are respectively 0.627% and 1.169% of per-capita consumption. The same figures for the post-WWII era are, respectively, 0.212% and 0.074% of per-capita consumption. The latter can be compared to marginal costs found by Alvarez and Jermann (2004) for 1954-2001: between 0.08% and 0.49% of consumption, when computed at business-cycle frequencies alone. As we argued above, if one does not disentangle the effects of permanent and transitory shocks to consumption, there is the risk of upward biasing the estimate of the welfare costs of business cycles alone. Notwithstanding the slight difference in sample periods in both cases, the estimates in Alvarez and Jermann are higher than our estimate for the welfare costs of business cycles – 0.074%.

Results for the whole period 1901-2000 are indeed a combination of those of pre- and post-WWII eras. For reasonable preference parameter and discount values ($\beta = 0.985, \phi = 5$) we get a compensation of 0.972% if we consider only permanent shocks. If we take into account only transitory shocks we get 0.54% of per-capita consumption. Extending the sample period up to 2010, which includes the last global recession, makes little difference in welfare-cost estimates.

Testing whether welfare costs are statistically significant can be done for all sub-samples employed here. With the exception of welfare costs of economic-growth variation for the 1947-2000 period, all other welfare costs are significantly different from zero.

From the discussion above we can conclude the following. First, current *marginal* and *total* welfare cost of business cycles are small – 1947-2010. Hence, it makes little sense to deepen current counter-cyclical policies. Second, from the point of view of a pre-WWII consumer, the marginal welfare costs of business cycles were fairly large. Indeed, for reasonable parameter values ($\beta = 0.985, \phi = 5$) they were 1.169% of consumption in all dates and states of nature. Therefore, from her (his) point of view, it made sense to have had counter-cyclical policies implemented in the post-WWII era.

Last, but not least, a comparison between the welfare costs of business cycles in the pre-WWII and post-WWII periods can give some idea of the effectiveness of counter-cyclical policies which

[10] One possible source is the fact that Reis fits an ARMA model which uses as information set only lagged consumption growth. We use a bi-variate model comprised of consumption and income. Given the evidence of Granger (1969) causality from income to consumption (a t-statistic of -3.67 for income growth in consumption's equation) with post-WWII data, this reduces the variance of shocks to the latter.

were implemented in the latter period. Considering reasonable parameter values such as $\beta = 0.985$ and $\phi = 5$, the welfare cost of business cycles (λ_P) decreased from 0.583% to 0.037% of consumption – roughly a factor of 15. The reduction in the marginal welfare costs of business cycles ($\lambda'_P(0)$) are even more impressive: from 1.169% to 0.074% of per-capita consumption. Indeed, if we could credit these reductions in welfare costs to post-WWII counter-cyclical policies – which, by the way, is a big if – it is hard to find any type of implemented economic policy in the name of which it could be claimed such an impressive impact on welfare.

5. Conclusion

Using only standard assumptions on preferences and an econometric approach for modelling consumption, we separate the effects of uncertainty stemming from business-cycle fluctuations and economic growth variation. We model the trend in consumption as a martingale process, while fluctuations about the trend are a stationary and ergodic zero-mean process. Trend and cyclical innovations are assumed to be independent sources of uncertainty. This hypothesis allows the measurement of welfare costs of business cycles and also of economic growth variation.

The whole of the literature chose to work primarily with post-WWII data. However, for this period, actual consumption is already a result of counter-cyclical policies, and is potentially smoother than what it otherwise would have been in their absence. Because of this, we use four distinct sample periods: pre-WWII data – 1901-1941, post-WWII data – 1947-2000, 20th Century data – 1901-2000, and the whole sample – 1901-2010.

For the estimates of the total welfare costs ($\lambda_P, \lambda_T, \lambda_D$), there are major differences in results for the pre-WWII and the post-WWII era. This is true regarding the welfare cost of business cycles (associated with transitory shocks), the welfare cost of economic growth (associated with permanent shocks), and the welfare costs of macroeconomic uncertainty (associated with both shocks). These differences can reach up to 15 times for reasonable parameter values – $\beta = 0.985$, and $\phi = 5$, for example. In pre-WWII period (1901-1941), for reasonable preference parameter and discount values ($\beta = 0.985, \phi = 5$), we get welfare costs of 0.310% of consumption if we consider only permanent shocks and 0.608% of consumption if we consider only transitory shocks, which roughly translates into US\$ 60.00 a year and US\$ 120.00 a year, respectively, in current value. In comparison, the post-WWII era is much quieter: welfare costs of economic growth are 0.106% (not significant) and welfare costs of business cycles are 0.037% – the latter being very close to the estimate in Lucas (0.040%).

The estimates of *marginal* welfare costs ($\lambda'_P(0), \lambda'_T(0), \lambda'_D(0)$) are roughly twice the size of the *total* welfare costs. For the pre-WWII era, and reasonable preference parameter and discount values ($\beta = 0.985, \phi = 5$), the marginal welfare costs of economic growth and of business cycles are respectively 0.627% and 1.169% of per-capita consumption. The same figures for the post-WWII

era are, respectively, 0.212% and 0.074% of per-capita consumption. The latter can be compared to welfare costs estimated by Alvarez and Jermann (2004). For the 1954-2001 period, they find it to be between 0.08% and 0.49% of consumption, when computed at business-cycle frequencies alone. As we argued above, if one does not disentangle the effects of permanent and transitory shocks to consumption, there is the risk of over-estimating the welfare costs of business cycles alone.

We can conclude the following. First, current marginal and total welfare costs of business cycles are small. Hence, it makes little sense to deepen current counter-cyclical policies. This is true even including in our sample the data for the last global recession. Second, from the point of view of a pre-WWII consumer, marginal and total welfare costs of business cycles were fairly large. Therefore, from her (his) point of view, it made sense to have had counter-cyclical policies implemented then. Last, a comparison between the welfare costs of business cycles in the pre-WWII and post-WWII period can give some idea of the effectiveness of counter-cyclical policies implemented in the latter period. Considering reasonable parameter values such as $\beta = 0.985$ and $\phi = 5$, the welfare cost of business cycles (λ_P) decreased from 0.583% to 0.037% of consumption – roughly a factor of 15. Notice that the reduction in the marginal welfare costs of business cycles ($\lambda'_P(0)$) are even more impressive: from 1.169% to 0.074% of per-capita consumption. Indeed, if we could credit these reductions in welfare costs to post-WWII counter-cyclical policies – which, by the way, is a big if – it is hard to find any type of implemented economic policy in the name of which it could be claimed such an impressive impact on welfare.

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Table 1: Cointegration test – Johansen (1988, 1991) Technique

Cointegrating Vectors under H_0	Eigenvalues	Trace Stat.	5 % Crit. Value	$\lambda_{\max}$ Stat.	5 % Crit. Value
None	0.150	16.44	15.41	16.26	14.07
At most 1	0.0018	0.18	3.76	0.18	3.76

Estimate of the cointegrating vector is: $(-1, 1.005)$.

$H_0 : \beta' = (-1, 1)$, conditional on $r = 1$, p-value $= 0.831$.

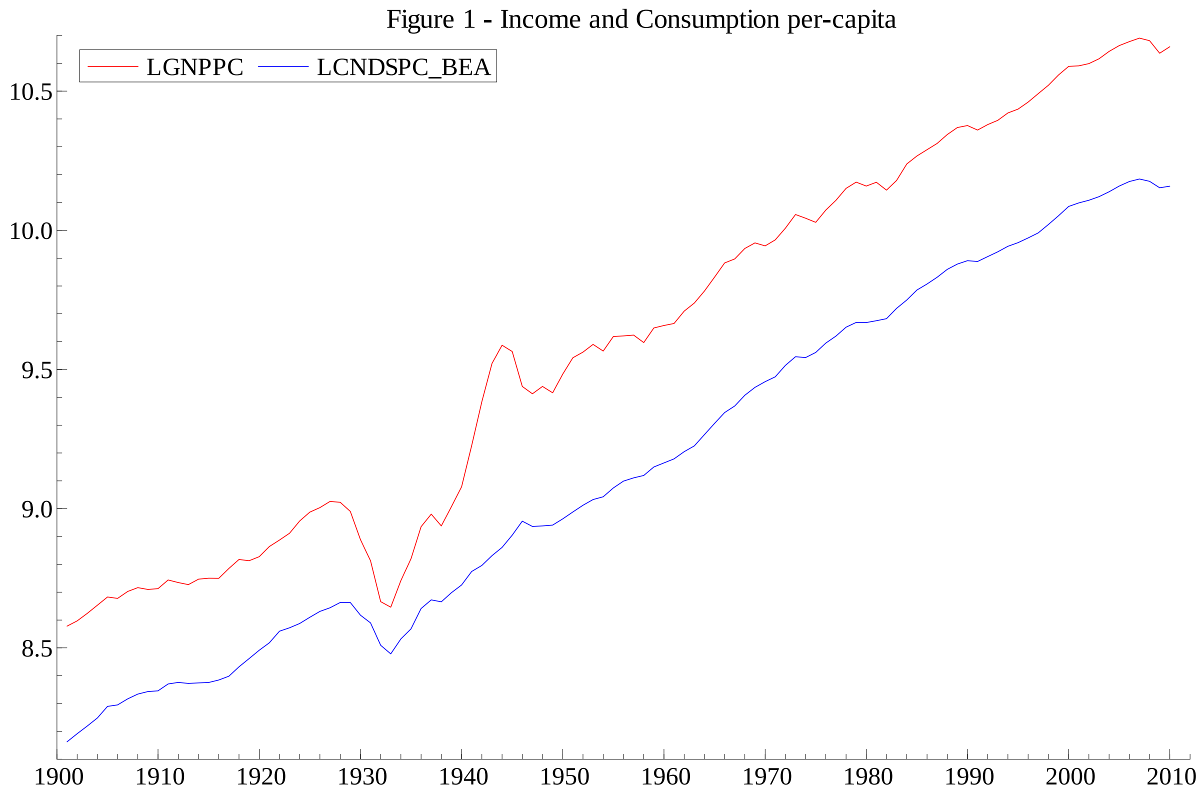

Figure 1: Real Consumption and Income per-capita

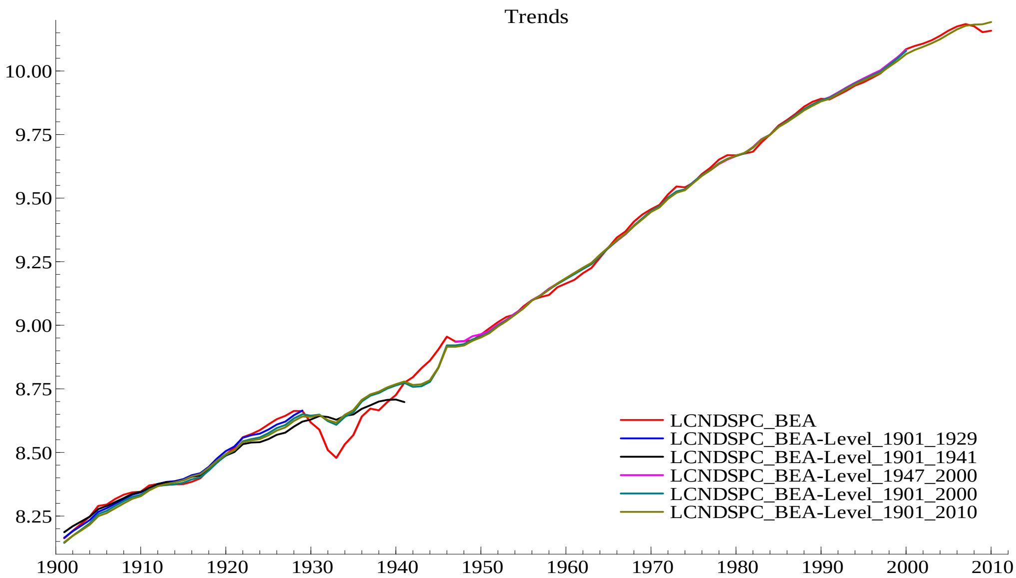

Figure 2: Consumption and Consumption Trends Computed in Different Sub-samples (in logs)

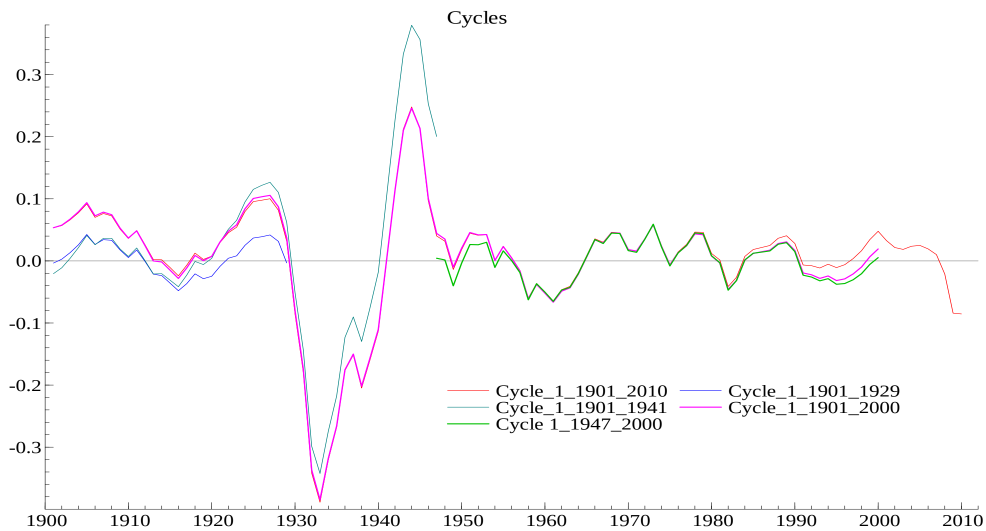

Figure 3: Consumption Cycles Computed in Different Sub-samples (in logs)

Table 2: Consumption – Parameter Estimates in Equations (2.7) and (2.8)

	1901-2000	1901-1941	1947-2000	1901-2010
$\ln \widehat{(1+\alpha_1)}$	0.0195	0.0152	0.0217	0.0188
	(0.0013)	(0.0027)	(0.0009)	(0.0011)
$\widehat{\sigma_{11}}$	0.0001843	9.71885E-05	4.51548E-05	0.000140286
	(0.0000854)	(4.06191E-05)	(3.88781E-05)	(0.0000663)
$\widehat{\widetilde{\sigma}_{22}}$	0.0010802	0.0023237	0.0001482	0.0011765
	(0.0004640)	(0.0010908)	(0.0000257)	(0.0004671)

Note: Standard errors in parenthesis.

Table 3: Welfare Costs of Business Cycles, Economic Growth, and Macroeconomic Uncertainty (%) for different Values of (β, ϕ)

	1901-1941				1947-2000				1901-2000				1901-2010			
	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$
$\lambda_T(\cdot)$																
$\beta = 0.950$	0.0924	0.2058	0.2378	0.2477	0.0429	0.0764	0.0812	0.0776	0.1752	0.3364	0.3695	0.3699	0.1334	0.2623	0.2898	0.2906
	(0.0386)	(0.0865)	(0.1008)	(0.1071)	(0.0366)	(0.0653)	(0.0696)	(0.0670)	(0.0813)	(0.1574)	(0.1752)	(0.1811)	(0.0631)	(0.1248)	(0.1394)	(0.1433)
$\beta = 0.971$	0.1628	0.2587	0.2725	0.2683	0.0756	0.0919	0.0902	0.0824	0.3090	0.4101	0.4144	0.3957	0.2351	0.3212	0.3260	0.3113
	(0.0681)	(0.1089)	(0.1157)	(0.1163)	(0.0645)	(0.0786)	(0.0773)	(0.0712)	(0.1435)	(0.1922)	(0.1969)	(0.1942)	(0.1112)	(0.1531)	(0.1571)	(0.1538)
$\beta = 0.985$	0.3196	0.3101	0.3007	0.2835	0.1484	0.1056	0.0971	0.0859	0.6069	0.4773	0.4496	0.4142	0.4617	0.3754	0.3545	0.3262
	(0.1338)	(0.1307)	(0.1279)	(0.1230)	(0.1267)	(0.0903)	(0.0833)	(0.0742)	(0.2822)	(0.2241)	(0.2140)	(0.2037)	(0.2187)	(0.1792)	(0.1711)	(0.1614)
$\lambda_D(\cdot)$																
$\beta = 0.950$	0.2087	0.7896	1.4092	2.6044	0.0503	0.1135	0.1554	0.2260	0.2293	0.6078	0.9131	1.4600	0.1923	0.5576	0.8815	1.4775
	(0.0670)	(0.2883)	(0.5624)	(1.1246)	(0.0366)	(0.0657)	(0.0708)	(0.0718)	(0.0846)	(0.1963)	(0.2930)	(0.5051)	(0.0673)	(0.1717)	(0.2742)	(0.4956)
$\beta = 0.971$	0.2793	0.8429	1.4443	2.6255	0.0830	0.1290	0.1644	0.2308	0.3632	0.6816	0.9582	1.4861	0.2941	0.6167	0.9179	1.4984
	(0.0874)	(0.2960)	(0.5655)	(1.1258)	(0.0645)	(0.0789)	(0.0784)	(0.0757)	(0.1454)	(0.2254)	(0.3067)	(0.5101)	(0.1137)	(0.1933)	(0.2837)	(0.4990)
$\beta = 0.985$	0.4362	0.8945	1.4729	2.6410	0.1558	0.1427	0.1712	0.2343	0.6613	0.749	0.9936	1.5048	0.5208	0.6711	0.9466	1.5135
	(0.1447)	(0.3049)	(0.5684)	(1.1267)	(0.1267)	(0.0906)	(0.0843)	(0.0786)	(0.2833)	(0.2533)	(0.3181)	(0.5140)	(0.2201)	(0.2147)	(0.2919)	(0.5015)
$\lambda_P(\cdot)$																
$\beta = 0.950$	0.1163	0.5826	1.1686	2.3509	0.0074	0.0371	0.0741	0.1483	0.0540	0.2704	0.5416	1.0861	0.0588	0.2946	0.5900	1.1834
	(0.0546)	(0.2743)	(0.5518)	(1.1165)	(0.0013)	(0.0064)	(0.0128)	(0.0255)	(0.0232)	(0.1163)	(0.2332)	(0.4690)	(0.0234)	(0.1171)	(0.2349)	(0.4726)
$\beta = 0.971$	0.1163	0.5826	1.1686	2.3509	0.0074	0.0371	0.0741	0.1483	0.0540	0.2704	0.5416	1.0861	0.0588	0.2946	0.5900	1.1834
	(0.0546)	(0.2743)	(0.5518)	(1.1165)	(0.0013)	(0.0064)	(0.0128)	(0.0255)	(0.0232)	(0.1163)	(0.2332)	(0.4690)	(0.0234)	(0.1171)	(0.2349)	(0.4726)
$\beta = 0.985$	0.1163	0.5826	1.1686	2.3509	0.0074	0.0371	0.0741	0.1483	0.0540	0.2704	0.5416	1.0861	0.0588	0.2946	0.5900	1.1834
	(0.0546)	(0.2743)	(0.5518)	(1.1165)	(0.0013)	(0.0064)	(0.0128)	(0.0255)	(0.0232)	(0.1163)	(0.2332)	(0.4690)	(0.0234)	(0.1171)	(0.2349)	(0.4726)

Note: Standard Errors are in parenthesis (multiplied by 100). Formulas for λ_P, λ_T, and λ_D are given, respectively, in equations (2.12), (2.14), and (2.17).

Table 4: Marginal Welfare Costs of Business Cycles, Economic Growth, and Macroeconomic Uncertainty (%) for different Values of (β, ϕ)

	1901-1941				1947-2000				1901-2000				1901-2010			
	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$	$\phi = 1$	$\phi = 5$	$\phi = 10$	$\phi = 20$
$\lambda'_T(\cdot)$																
$\beta = 0.950$	0.1850	0.4148	0.4837	0.5144	0.0859	0.1533	0.1634	0.1572	0.3514	0.6818	0.7595	0.786	0.2673	0.5300	0.5922	0.6092
	(0.0775)	(0.1757)	(0.2085)	(0.2308)	(0.0733)	(0.1315)	(0.1409)	(0.1375)	(0.1635)	(0.3233)	(0.3701)	(0.4079)	(0.1266)	(0.2549)	(0.2909)	(0.3144)
$\beta = 0.971$	0.3265	0.5226	0.5555	0.5585	0.1514	0.1844	0.1815	0.1671	0.6209	0.8333	0.8542	0.8432	0.4720	0.6503	0.6676	0.6540
	(0.1369)	(0.2221)	(0.2404)	(0.2517)	(0.1294)	(0.1583)	(0.1567)	(0.1463)	(0.2897)	(0.3968)	(0.4182)	(0.4400)	(0.2241)	(0.3139)	(0.3293)	(0.3390)
$\beta = 0.985$	0.6423	0.6274	0.6142	0.5911	0.2974	0.2121	0.1955	0.1742	1.2251	0.9721	0.9287	0.8846	0.9298	0.7616	0.7271	0.6866
	(0.2702)	(0.2675)	(0.2666)	(0.2672)	(0.2544)	(0.1821)	(0.1689)	(0.1526)	(0.5749)	(0.4648)	(0.4565)	(0.4634)	(0.4435)	(0.3687)	(0.3598)	(0.3570)
$\lambda'_D(\cdot)$																
$\beta = 0.950$	0.4181	1.5883	2.8460	5.296	0.1007	0.2276	0.3119	0.4545	0.4599	1.2271	1.8539	2.9871	0.3853	1.1231	1.7826	3.0046
	(0.1343)	(0.5819)	(1.1420)	(2.3099)	(0.0734)	(0.1322)	(0.1435)	(0.1471)	(0.1702)	(0.4010)	(0.6027)	(1.0426)	(0.1352)	(0.3486)	(0.5591)	(1.0146)
$\beta = 0.971$	0.5599	1.6973	2.9195	5.3422	0.1663	0.2587	0.3301	0.4644	0.7297	1.3793	1.9496	3.0456	0.5902	1.2442	1.8589	3.0505
	(0.1757)	(0.5985)	(1.1493)	(2.3132)	(0.1294)	(0.1589)	(0.1590)	(0.1554)	(0.2937)	(0.4631)	(0.6344)	(1.0566)	(0.2292)	(0.3944)	(0.5808)	(1.0233)
$\beta = 0.985$	0.8765	1.8034	2.9795	5.3763	0.3123	0.2863	0.3441	0.4715	1.3345	1.5189	2.0248	3.0879	1.0487	1.3560	1.9192	3.0838
	(0.2923)	(0.6177)	(1.1560)	(2.3159)	(0.2545)	(0.1827)	(0.1711)	(0.1614)	(0.5774)	(0.5233)	(0.6610)	(1.0673)	(0.4465)	(0.4400)	(0.5993)	(1.0300)
$\lambda'_P(\cdot)$																
$\beta = 0.950$	0.2326	1.1686	2.3509	4.7571	0.0148	0.0741	0.1483	0.2968	0.1081	0.5416	1.0861	2.1839	0.1177	0.5900	1.1834	2.3809
	(0.1343)	(0.5518)	(1.1165)	(2.2854)	(0.0734)	(0.0128)	(0.0255)	(0.0511)	(0.1702)	(0.2332)	(0.4690)	(0.9482)	(0.1352)	(0.2349)	(0.4726)	(0.9564)
$\beta = 0.971$	0.2326	1.1686	2.3509	4.7571	0.0148	0.0741	0.1483	0.2968	0.1081	0.5416	1.0861	2.1839	0.1177	0.5900	1.1834	2.3809
	(0.1343)	(0.5518)	(1.1165)	(2.2854)	(0.0734)	(0.0128)	(0.0255)	(0.0511)	(0.1702)	(0.2332)	(0.4690)	(0.9482)	(0.1352)	(0.2349)	(0.4726)	(0.9564)
$\beta = 0.985$	0.2326	1.1686	2.3509	4.7571	0.0148	0.0741	0.1483	0.2968	0.1081	0.5416	1.0861	2.1839	0.1177	0.5900	1.1834	2.3809
	(0.1343)	(0.5518)	(1.1165)	(2.2854)	(0.0734)	(0.0128)	(0.0255)	(0.0511)	(0.1702)	(0.2332)	(0.4690)	(0.9482)	(0.1352)	(0.2349)	(0.4726)	(0.9564)

Note: Standard Errors are in parenthesis (multiplied by 100). Formulas for λ'_P, λ'_T, and λ'_D are given, respectively, in equations (2.18), (2.19), and (2.20).