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Thiago Neves Pereira

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Resumo

Esta tese dedica-se ao estudo dos sistemas tributários. Eu investigo como um sistema tributário afeta as escolhas dos indivíduos e consequentemente os recursos do país. Eu mostro como um sistema tributário induz as escolhas das pessoas, determinado assim as alocações de trabalho, produto e consumo da economia.

No primeiro e segundo capítulo eu examino a taxação sobre os indivíduos, enquanto que no terceiro e quarto capítulos analiso a incidîncia tributária sobre os diferentes agentes da sociedade.

No capítulo um, eu examino o sistema tributário ótimo, seguindo Mirrlees (1971) e Saez (2001). Eu mostro como seria este sistema tributário no Brasil, país com profunda desigualdade de renda entre os indivíduos. Ademais, eu investigo o sistema tributário afim, considerado uma alternativa entre os sistemas atual e o ótimo.

No segundo capítulo eu analiso o sistema tributário conhecido como sacrífício igual. Mostro como o sistema tribuária derivado por Young (1987), redesenhado por Berliant and Gouveia (1993), se comporta no teste de eficiência derivado por Werning (2007).

No terceiro e quarto capítulo eu examino como propostas de reforma tribuária afetariam a economia brasileira. No capítulo três investigo como uma reforma tributária atingiria as diferentes classes socias.

No capítulo quatro, eu estudo as melhores direções para uma reforma tributária no Brasil, mostrando qual arranjo de impostos é menos ineficiente para o país. Por fim, investigo os efeitos de duas propostas de reforma tributária sobre a economia brasileira. Explicito quais os ganhos de produto e bem estar de cada proposta. Dedico especial atenção aos ganhos/perdas de curto prazo, pois estes podem inviabilizar uma reforma tributária, mesmo esta gerando ganhos de longo prazo.

Abstract

This thesis is dedicated to study of tax schedule. I investigate how a tax schedule could affect the individuals' choice and consequently the resources of the country. I show how a tax schedule induce the individuals' choice, defining hence the allocations of labor, output and consumption of society.

In the first and the second chapters I examine the taxation of individuals, while in the third and the fourth chapter I analyze the incidence of levies on different agents of economy.

In the chapter one, I examine the optimal tax schedule, following Mirrlees (1971) e Saez (2001). I show how would be the optimal tax schedule in Brazil, charactering by a deeper income inequality among the individuals. Moreover, I investigate a affine tax schedule, that is considered an alternative tax schedule between the current and optimal tax schedule.

In the second chapter I analyze the tax schedule known as equal sacrifice. I show how the tax schedule derived by Young (1987), that was renewed by Berliant and Gouveia (1993), behavior itself in the efficiency test derived by Werning (2007).

In the third and the fourth chapter I examine how tax reform proposals would affect the Brazilian's economy. In the third chapter I investigate how a tax reform affects different social classes.

In chapter four, I study the better directions to a tax reform in Brazil, showing which rearrange of levies is the less inefficient to the country. In the end, I investigate the effects of two tax reform proposals in the Brazilian economy. I define the gains of output and welfare in each proposal. I call the special attention to gains/loses of short run, because they could make no possible to approve a tax reform, even though the reform could good effects in the long run.

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Introduction

This thesis is comprised of four chapters about macroeconomics and public finance. I focus on theoretical and empirical aspects of the economics of taxation. Taxation is a powerful device for affecting the behavior of consumers. Through taxation the government can encourage or discourage consumption, labor supply and the distribution of resources. Therefore, the government has to create a tax system that induces *good* allocations and consequently provides as higher as possible welfare to the society.

The debate about taxation is not limited to the economic research area but also attracts much attention in the political sphere and among individuals. Of course, there is an important component of political economy in this debate. At the center of debate there is the discussions about the equity-efficiency trade-off.

Along this thesis I analyze taxation in different ways. In all situations, I investigate alternative directions of changes for the current tax system. Although most of this thesis is dedicated to Brazilian economy, the results could provide useful academic and policy research discussion about tax system worldwide.

The current Brazilian tax system is too complex and is viewed as a barrier to the sustainable growth. It is responsible to many distortions in the economy. For this reason, I investigate the alternative allocations to the incidence of taxes into different tax base, i.e. consumption, investment, labor income and capital tax bases. Specifically to labor income, I also analyze a non-linear tax among individuals.

The first chapter computes the optimal non-linear income tax schedule for Brazil, following the lead of Mirrlees (1971) and Saez (2001). My first contribution is to take into account the non-linearity that is present in the current labor income tax system when we back up the distribution of skills from the labor income data. I use a separable specification for preferences and consider different levels of risk

aversion. By varying the level of risk aversion I capture both variations in the preferences for redistribution and variations in the elasticity of labor supply. I compare the welfare gains associated with changing the current tax system. I also consider the loss in efficiency that arises as a consequence of my restricting myself to affine income tax schedules.

The second chapter investigates efficiency properties of an income tax schedule derived under the minimum equal sacrifice criterion in a Mirrlees (1971) setting. The shape of equal sacrifice tax schedules does not depend on the underlying distribution of productivities which is in contrast with schedules resulting from the maximization of a social welfare functional. It is not hard, then, to find examples of economies for which equal sacrifice tax schedules are inefficient. This result is, however of little practical relevance since we do not 'choose' the underlying distribution of skills. Our focus is on the more relevant question of whether inefficiencies are likely to arise for preferences and distribution of productivities compatible with empirical distributions of income. We show that the methodology in Werning (2007) can be used to address this question. We find that, although under reasonable parameters for preferences and the distribution of productivities an equal sacrifice schedule may be inefficient, for our best specifications, inefficiency only arises for extremely high levels of income.

The third chapter analyzes the effects of a tax system that is built around income tax or consumption tax. Much of the interest in tax policy arises from the widespread belief that taxes on income and on savings tend to lower long-run income by retarding the creation and expansion of firms and by discouraging workers and investments. Following this belief, Brazilian government has proposed a tax reform which, basically, replaces tax on investment and on labor with tax on consumption. In this chapter, I develop a dynamic general equilibrium model with heterogeneous agents to guide my quantitative assessment of the economic and distributional implications of such tax reform. The model is calibrated in such a way that it matches some selected features of the Brazilian economy. I also use the calibrated model to calculate the deadweight loss of each type of taxation and thus provide some rationality for that rearrangement in the tax system. The main result of the chapter is that, even though the tax reform increases the asset accumulation, labor and output of economy, it also raises the welfare inequality as borrowing

constrained individuals cannot take advantage of the drop in tax on savings.

The last chapter analyzes two tax reform proposals for Brazil. The first proposal is the government's proposal and the second is the Brazilian National Confederation of Industry's (CNI) proposal. My focus is on the macroeconomics effects as measured by changes in consumption, stock of capital and output. More importantly I try to measure the welfare consequences of such proposals. In my analysis I consider the neoclassical capital accumulation model with a representative agent. I also calculate the Marginal Cost of Public Funds (MCF) from the different tax bases in Brazil. The MCF points to the most favorable directions for a tax reform. With regards to the specific proposals, I show that both proposals could increase the efficiency in the economy and cause a Pareto improvement to the society.

Chapter 1

The Optimal Labor Income Tax Schedule for Brazil

[1] We compute the optimal non-linear income tax schedule for Brazil, following the lead of Mirrlees (1971) and Saez (2001). Our first contribution is to take into account the non-linearity that is present in the current labor income tax system when we back up the distribution of skills from the labor income data. We use a separable specification for preferences and consider different levels of risk aversion. By varying the level of risk aversion we capture both variations in the preferences for redistribution and variations in the elasticity of labor supply. We compare the welfare gains associated with changing the current tax system. We also consider the loss in efficiency that arises as a consequence of our restricting ourselves to affine income tax schedules.
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Keywords: Optimal Non-linear Taxation, Distribution of Skills, Affine Tax System

1.1 Introduction

In this paper we analyze and propose some options of income tax reform. Our attention is on indirect (consumption taxes) and income taxation. Our first target

[1] This is joint work with Carlos Eugênio da Costa.

is the simulations of optimal income taxation in Brazil. Our main references are Mirrlees (1971) and Saez (2001).

Our first departure from Saez (2001) is the approach we use to create the skill distribution. In Saez (2001), the distribution of skills is backed up from the empirical distribution of income by approximating the labor income tax schedule by a linear one. That is, given an assumed utility function defined over labor and consumption one derives the level of skills that is compatible with the observed taxable income, under this linear tax schedule. We, instead, allow for the non-linearities that characterize the Brazilian income tax schedule.

We derive optimal income taxes for different values of risk aversion under a separable specification for preferences. By varying the degree of risk aversion we capture both changes in the willingness to redistribute income and the elasticity of labor supply.[2] We observe that the optimal tax schedule is sensitive to the risk aversion and labor supply elasticity. In our simulations, we consider a risk aversion parameter equal, smaller and bigger than those considered by Mirrlees (1971) and Saez (2001). Our labor supply elasticity parameter has a smaller value than Saez (2001).[3]

With these parameters, the compensated and uncompensated elasticities and the income effects vary across the skill distribution. As we see below, the compensated elasticity has an important role in the optimal income taxation formula. We show that the marginal tax rate is increasing in the risk aversion and labor supply elasticity parameters.

The second goal of this paper is propose and analyze a new *simple* income tax schedule in Brazil. This new tax schedule is completely defined by a constant marginal tax rate and a cash transfer per head. The idea is to find a feasible new tax schedule that is more efficient than the current tax schedules. Other important characteristic in the new schedule is to be easy to implement. Additionally, we want to investigate the effects of cash transfer program in the labor supply, consumption and utility.

The paper is organized as follows. In the section 2 the general model is introduced. First we present the benchmark model and after the Mirrlees model. In the

[2] Chetty (2006) uses labor supply elasticities to bound risk aversion.

[3] He considered the labor supply elasticity equal to 3 and 5 in his simulations

section 3 we explain our data and the methodology. The section 4 is dedicated to our results. We split this section in four parts. In the first part we present the results to benchmark model. In the second part we present optimal income taxation results in Brazil. In the third part we present the optimal taxation in United State according to our procedure. In the forth part we present the results to the new tax schedule in Brazil. Section 5 is devoted to concluding remarks. In the Appendix we show the complete derivation of our model and some useful figures.

1.2 Model

1.2.1 The Benchmark Model

Following Mirrlees (1971), consider a two good model in which a continuum measure one of individuals have identical preferences defined over consumption, c, and effort, l, represented by $\mathfrak{u}(c, l)$. $\mathfrak{u}(.)$ is a smooth strictly concave function, strictly increasing in c and decreasing in l.

Individuals differ with respect to their productivity, w, which we shall also refer to as their skill. The cross-sectional distribution of productivity is $F(.)$ with associated density $f(.)$. An individual with productivity n needs to put effort $l = y/w$ to generate output y whereas an individual with productivity w' needs to put effort $l' = y/w'$ to generate the same output. Because we consider a competitive economy, y is also an individual's earnings, $y = wl$, and we shall use the terms output and earnings interchangeably.

We may represent the preferences of an individual of productivity w in the consumption earnings space as $U(c, y; w) = \mathfrak{u}(c, y/w)$. We shall make extensive use of this latter representation when we derive the optimal income tax schedule. For now, however, note that given any labor income tax schedule $T(.)$ note that an individuals' budget constraint is $c \leq y - T(y)$.

Now, consider an individual who is earning y, and paying a total of $T(y)$ in taxes. If $T(y)$ is differentiable at y let $\tau = T'(y)$ be the marginal tax rate and $R = \tau y - T(y)$ be the virtual income.[4] We may use this linear approximation of a type-w individual's budget constraint around his optimal choice, y, $c \leq y(1-\tau) + R$, to

[4] If the tax schedule is not affine, then τ is itself a function of y.

define the following problem:

$$\max_{c_w, y_w} U(c, y/w) \tag{1.1}$$

$$\text{s.t. } c = y(1-\tau) + R. \tag{1.2}$$

The solution to this problem defines $y(1-\tau, R, w)$ and $c(1-\tau, R, w)$.

Following Saez (2001) we further define the compensated and uncompensated price elasticities as well as the income elasticity of earnings through

$$\zeta^c = \frac{1-\tau}{y} \frac{\partial y}{\partial(1-\tau)}\bigg|_u,$$

$$\zeta^u = \frac{1-\tau}{y} \frac{\partial y}{\partial(1-\tau)},$$

and

$$\eta = (1-\tau)\frac{\partial y}{\partial R},$$

respectively.

The Slutsky equation,

$$\zeta^c = \zeta^u - \eta, \tag{1.3}$$

relates the three elasticities.

The compensated elasticities is always non-negative. In all the discussion that follows we assume that leisure is not an inferior good. The functional forms that we use in the numeric exercises will also imply this property.

1.2.2 The Mirrlees Model

For sake of completeness, in this section we derive the optimal income taxation schedule, following Mirrlees (1971). We, then, relate the expressions found herein to those in Saez (2001). The main advantage of his representation is to relate optimal tax formulae to well known empirical parameters. Our focus on Mirrlees (1971) is due to the fact that, although Saez (2001) provides an interesting way of deriving the expressions for the optimal tax, his approach is not suited for the actual computational implementation.

As discussed before, an individual's consumption as a function of his or her earnings is $c(y) = y - T(y)$, where $T(y)$ is the tax function.

The government chooses a tax schedule to maximize a given social welfare function $G(u)$ subject to raising enough resources to meet an exogenously given expenditure level. Mirrlees (1971) tackled this very complex problem by solving the primal mechanism design program of maximizing

$$W = \int_{\underline{w}}^{\infty} G(u(w))f(w)dw,$$

where $G(.)$, is an increasing and concave function of utility $u(w) = U(c(w), y(w); w)$.

This maximization is subject to a resource constraint of the form

$$\int_{\underline{w}}^{\infty} (z(w) - c(w))\, f(w)dw \geq E, \tag{1.4}$$

where E is the government expenditures, and an incentive compatibility constraint,

$$n \in \arg\max_{\tilde{w}} U(c(\tilde{w}), z(\tilde{w}), w)\forall w. \tag{1.5}$$

Recall that $f(w)$ is the density associated with the cumulative distribution of skill $F(w)$, $w \in [\underline{w}, \infty]$, where $\underline{w} > 0$.

Under single-crossing, which obtains in our case if consumption is normal,[5] the local necessary conditions

$$U_c(c(w), z(w), w)c'(w) + U_y(c(w), y(w), w)y'(w) = 0 \tag{1.6}$$

and $y'(w) \geq 0$ may substitute for the global constraint (2.3).

As it turns, it is sometimes easier to work in the (u, y) space by defining function ψ implicitly through

$$U(\psi(u(w), y(w), w), y(w), w) = u(w).$$

Then, using the envelope condition

$$U_w(\psi(u(w), y(w), w), y(w), w) = u'(w),$$

to substitute for (1.6), we have completely eliminated $c(w)$ from the program.

This will be useful for us to provide an heuristic examination of the optimal tax formulae. Toward this end let us specialize to the case of separable iso-elastic

[5] Normality of consumption is sufficient, not necessary.

preferences and ignore for the moment the monotonicity condition. For lack of a better name, we call this problem without the monotonicity constraint, a *relaxed problem*.

Let

$$\mathfrak{u}(c,l) = \nu(c) - h(l)$$

where $h(l) = l^\gamma/\gamma$ and $\nu(c) = c^{1-\rho}(1-\rho)^{-1}$.

Next, define $\theta = w^{-\gamma}$ and let $\phi(\theta)$ is the density induced by $f(w)$. Abusing notation somewhat, let

$$u(\theta) = v(\psi(u(\theta), y(\theta), \theta)) - \theta y(\theta)^\gamma/\gamma,$$

then using the envelope condition[6]

$$u'(\theta) = -y(\theta)^\gamma/\gamma.$$

Now, the Lagrangian associated with the planner's problem Utilitarian criterion is

$$\int_0^{\bar{\theta}} \{u(\theta)\phi(\theta) + \mu(\theta)\left[u'(\theta) + y(\theta)^\gamma/\gamma\right] + \lambda\left[y(\theta) - \psi(u(\theta), y(\theta), \theta) - E\right]\phi(\theta)\}\, d\theta.$$

Integrating the second term by parts we get

$$\int_0^{\bar{\theta}} \left\{u(\theta)\phi(\theta) - \mu'(\theta)u(\theta) + \mu(\theta)\frac{1}{\gamma}y(\theta)^\gamma + \lambda\left[y(\theta) - \psi(u(\theta), y(\theta), \theta) - E\right]\phi(\theta)\right\} d\theta$$
$$-\mu(\bar{\theta})u(\bar{\theta}) + \mu(0)u(0).$$

It is then straightforward to see that

$$\mu'(\theta) = \phi(\theta) - \lambda\phi(\theta)\psi_u(u(\theta), y(\theta), \theta),$$

which, from $\mu(\bar{\theta}) = \mu(0) = 0$ implies

$$\lambda = \left\{\int_0^{\bar{\theta}} c(\theta)^\rho \phi(\theta) d\theta\right\}^{-1},$$

[6] Alternatively we could use Milgrom and Segal (2002) to write the incentive compatibility constraint as

$$u(\theta) = u(\bar{\theta}) + \frac{1}{\gamma}\int_\theta^{\bar{\theta}} \tilde{\theta} y(\tilde{\theta})^{-\gamma} d\tilde{\theta}.$$

and

$$\mu(\theta) = \int_0^\theta \left\{ 1 - c(\tilde{\theta})^\rho \left\{ \int_0^{\bar{\theta}} c(\theta)^\rho \phi(\theta) d\theta \right\}^{-1} \right\} \phi(\tilde{\theta}) d\tilde{\theta}.$$

Next, note that

$$\frac{\mu(\theta)}{\phi(\theta)} y(\theta)^{\gamma-1} = \lambda \left\{ \frac{\theta y(\theta)^{\gamma-1}}{c(\theta)^{-\rho}} - 1 \right\}$$

can be written

$$\frac{y(\theta)^{\gamma-1}}{\phi(\theta)} \int_0^\theta \left\{ \int_0^{\bar{\theta}} c(\hat{\theta})^\rho \phi(\hat{\theta}) d\hat{\theta} - c(\tilde{\theta})^\rho \right\} \phi(\tilde{\theta}) d\tilde{\theta} = \frac{\theta y(\theta)^{\gamma-1}}{c(\theta)^{-\rho}} - 1.$$

This expression fully describes how marginal tax rates are determined in this simple setting. A full derivation for the case with non-separable utility is found in Mirrlees (1971, 1976). In the appendix A, we discuss the problem that arises when the solution to this relaxed problem is not monotonic.

This expression is the one used in our computational exercise. Yet it conveys little about the forces at work. We shall, then, present (and show the derivation in the appendix A) a more intuitive expression found in both Diamond (1998) and Saez (2001). We maintain the separability assumption $U_{lc} = 0$, and write

$$\frac{T'(wl)}{1 - T'(wl)} = \frac{(1 + \zeta_w^u)}{\zeta_w^c} \left(\frac{1 - F(w)}{w f(w)} \right) u_c(w) \int_w^\infty \left[\frac{1}{u_c(\tilde{w})} - \frac{G'(u_{\tilde{w}})}{\lambda} \right] \left(\frac{f(\tilde{w})}{1 - F(w)} \right) d\tilde{w}. \tag{1.7}$$

The subscripts w represent the fact that a parameter is computed at the skill level w. There are also two transversality conditions, $\mu(\underline{w}) = \mu(\infty) = 0$, which allows us to derive the expression for λ for general social welfare function,

$$\lambda = \frac{\int_{\underline{w}}^\infty G'(u_{\tilde{w}}) f(\tilde{w}) d\tilde{w}}{\int_{\underline{w}}^\infty u_c(\tilde{w})^{-1} f(\tilde{w}) d\tilde{w}}.$$

Terms ζ^c and ζ^u are respectively the compensated and uncompensated elasticities of earned income.

Equation (1.7) can be rewritten as

$$\frac{T'(wl)}{1 - T'(wl)} = A(w)B(w)C(w)D(w), \tag{1.8}$$

where

$$A(w) = \frac{(1+\zeta_w^u)}{\zeta_w^c},$$
$$B(w) = \frac{1-F(w)}{wf(w)},$$
$$C(w) = u_c(w),$$

and

$$D(w) = \int_w^\infty \left[\frac{1}{u_c(\tilde{w})} - \frac{G'(u_{\tilde{w}})}{\lambda}\right] \left(\frac{f(\tilde{w})}{1-F(w)}\right) d\tilde{w}.$$

The $A(w)$ term expresses labor supply responses in uncompensated and compensated elasticities. Keeping others things constant, the marginal tax rate is decreasing in ζ^c and ζ^u. As described by Tuomala (2006), the greater income effect in absolute compared to uncompensated effect implies in an higher marginal tax rate. As the elasticity varies across population, it is important to know how the elasticity varies with the wage rate.

The second term, $B(w)$ is known as the inverse hazard ratio. It tells us how the shape of the distribution of skills affects the optimal marginal tax at the level w. When we increase the marginal tax rate at the some w^*, we collect more revenue from individuals whose productive is above w^*. Since these individuals are $1 - F(w^*)$ in number, the higher this term the largest the gain from increasing the marginal tax rate. The very same increase results in a lower output of individuals of skill n. According to equation (1.7), the marginal tax rate is higher when $f(w)$ is lower and $wf(w)$ is smaller.

Note that, if we raised the marginal rates on very low earnings, we would substantially raise the tax revenue, for most of the taxpayers have earnings higher than this low level. Moreover, the higher marginal rates at the bottom have very marginal effects to this group due to the low value of n.

The third term, $C(w)$, reflects both income effects, and differences in the social value of income. The higher the income effects (in absolute value), the higher is the marginal tax rate. In this kind of model, when there is the income effects, the government is concerned with the income inequality among the individuals, whilst in the quasi-linear case, the government is worried about the utility inequality.

The fourth term, $D(w)$, incorporates distributional concerns. The integral in $\hat{w}$ measures the social welfare gain from slightly increasing the marginal tax rate at

w and distributing as a poll subsidy to those below w. The integral increases in w until a skill level w^* and decrease after w^*. The turning point depends on the Lagrange multiplier, λ, also known as marginal social cost of public funds.

Since the integral affects the marginal tax rate positively, this means that the range over which the latter increases also stretches further. Therefore, more tax revenue leads to a less progressive tax structure. The intuition is that the lower is the revenue requirement, the more the government can afford to support the poor by a generous poll subsidy, recouping at least part of this by a pattern of rising the marginal tax rate on the better off.

The exact pattern of this term in the equation (1.7) follows as n rises depends on the social welfare function and the shape of the skill distribution. So the shape of the skill distribution is also important here. Moreover it is obvious in the integral term in the equation (1.7) that the functional form of u_c has the important role in the determining the shape of the schedule.

Note that if we specialize (1.7) to the preferences we have used,

$$\frac{T'(y(w))}{1-T'(y(w))} = \frac{1+\zeta_w^u}{\zeta_w^c}\frac{c(w)^{-\rho}}{wf(w)}\int_w^\infty \left[c(\tilde{w})^\rho - \int_{\underline{w}}^\infty c(w)^\rho f(w)dw\right] f(\tilde{w})d\tilde{w},$$

which is quite close to the expression we found.

1.3 Methodology

1.3.1 Income Data

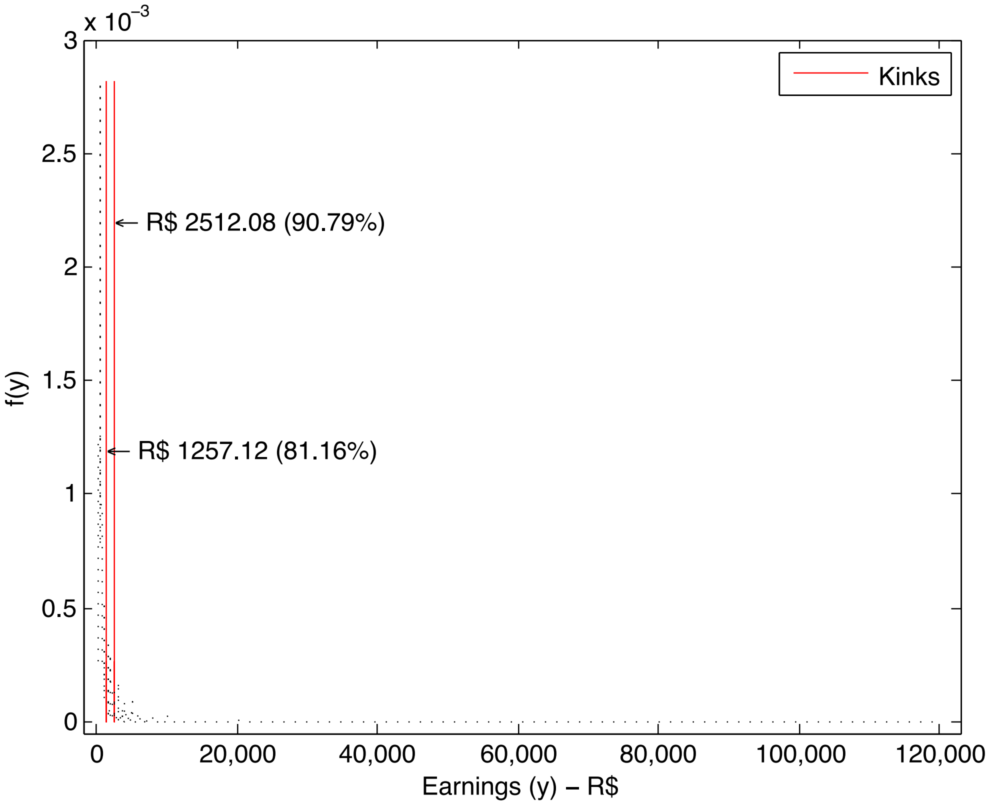
In this paper, one of our objective is to find the optimal income tax schedule for Brazil. Our first step is to consider the empirical income data of National Household Sample Survey (PNAD[7]) in 2006. As every survey, PNAD is subjected to errors in its collection or in different way. Trying to eliminate part of these troubles, we have refined the original income sample.

In the first procedure, we have chosen individuals that work between 30 and 60 hours per week. Additionally, we also have chosen individuals with income more or equal than $R\$100$ (one hundred reais) and less or equal than $R\$120,000$

[7]Collected by Instituto Brasileiro de Geografia e Estatística (2006).

per month. After these procedures, we have $133,118$ observations in our sample which we use to create distribution of skills.

Figure 1.1: Empirical Wage Income Distribution and Kinks



In 2006, the Brazilian's income tax system had three marginal tax rate. The first income marginal tax rate was 0% for individuals' income between $R\$0 - 1,257.12$ per month. The second group facing a marginal tax rate of 15% included individuals with income above $R\$1,257.12$ and up to $R\$2,512.08$. Finally, the individuals with incomes above $R\$2,512.08$ faced a marginal tax rate equal to 27.5%.

However, in our metric of taxation, besides the income tax, we also consider the indirect levy. The indirect levy or consumption tax has the same incidence on all groups. According to Pereira (2008), the indirect tax rate was 16.97% in 2006. Using the equivalence between taxes [8], the effective marginal tax rate for each group is respectively 15.25%, 27.97% and 38.56%. We use these values to create

[8] We can rewrite the equation $p(1+t_c)c \leq (1-t_l)y$ as $pc \leq (1-\tau)y$, where $\tau = 1 - \frac{(1-t_l)}{(1+t_c)}$.

the distribution of skills and to solve the benchmark model.

The empirical income distribution[9] is presented in Figure 1.1. We plot the empirical income distribution with the kinks of current tax system. According to our empirical income distribution, 81.16% of taxpayers are in the first group, that pay a marginal tax rate equal to 15.25%. In the second group we have 9.63% of taxpayers that pay 27.97% and in the last group, we have only 9.21% of taxpayers that pay a marginal tax rate equal to 38.56%. The modal value is equal to $R\$350.00$, the minimum wage in 2006. If an individual had earnings above $R\$3,000$, he or she would be in the top 5% of earning distribution and if an individual had earnings above $R\$7,000$ reais, he or she would be in the top 1% of earning distribution.

1.3.2 Distribution of Skills

We cannot directly apply equation (14) in Saez (2001) because the earnings distribution is affected by taxation. We, thus, need to back up the underlying distribution of skills from the distribution of income and the current tax schedule. That is, if we are to use the equation (1.7), and we have to create an exogenous distribution of skills that replicates the empirical earnings distribution. After this procedure, we can use the approximated distribution of skill to perform our numerical simulations.[10]

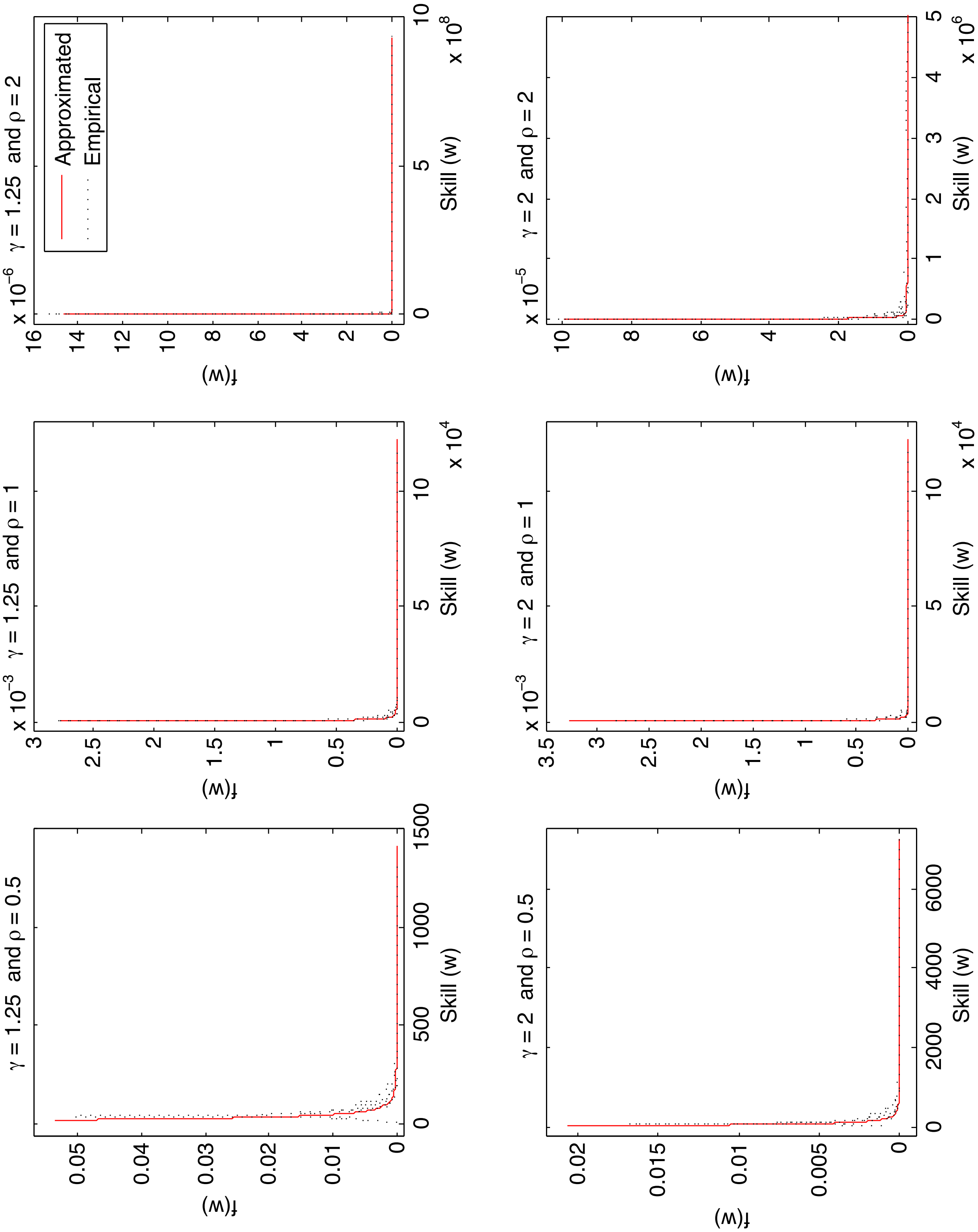
Given the income distribution, the current tax schedule and the utility function we can find the empirical distribution of skills, as in Saez (2001). Contrary to Saez (2001) in that we take into account the non-linearity of the current labor income tax schedule.

At this point it is worth noting one thing. Our procedure is a little different from Saez (2001). He approximated the empirical earnings by a parametric distribution. In other words, he supposed that the earnings in United States follows a Pareto distribution. Thus, given the smoothed earnings, the linear tax rate and the utility function, he recovered the skills and consequently the skill distribution. In our procedure, we consider the empirical earnings and the non-linearity of cur-

[9]The empirical income distribution was estimated by Gaussian kernel density distribution.

[10]Note also that given the empirical earnings distribution and the tax schedule, when we change the utility function parameters, we also generate a different distribution of skills. In Figure 1.2 we plot the empirical and approximated distribution of skills according to the risk aversion and labor supply elasticity parameters.

Figure 1.2: Empirical and Approximated Skill Distribution



rent labor income tax schedule to recover the empirical skills. Given the empirical skills, we smooth the empirical skills by a parametric distribution. In our case, we use a Generalized Pareto distribution.

To illustrate the procedure let us focus on a very convenient specification of preferences,

$$u(c, l) = \ln c + \gamma \ln(1 - l). \tag{1.9}$$

Assume that an individual has productivity n and faces a linear income tax schedule, $T(y) = \tau y$. The individual's maximization problem is to maximize (1.9) subject to $c \leq y(1-\tau)$. It is not hard to see that $y(w) = w/(\gamma+1)$. If we can observe the distribution of y, which is just the cross-sectional distribution of labor income in the society, we can, define $\mathfrak{w}(y) = y(\gamma + 1)$, the inverse of $y(w)$ and recover the distribution of w.

Roughly speaking, this is the procedure adopted by Saez (2001) in his seminal paper. This procedure works fine provided that the income tax schedule is reasonably well approximated by a linear tax function. The problem here is how to define what a reasonable approximation is in this context.

To try and handle this issue in the simplest possible manner, we recognize the non-linearity that is present in the Brazilian income tax schedule when we recover the distribution of skills. What makes this task feasible is the use of a linear approximation along each individual's choice.

Let us get back to our previous example, and recall our definition of virtual income, $R(y)$,

$$R(y) = \tau y - T(y).$$

where we have now made explicit its dependence on y.

First assume that the tax system is progressive in the sense of non-decreasing marginal tax rates and note that the current tax system is piecewise linear with a finite number of kinks. Let z_i, $i = 1, ..., w$, $y_i < y_j$ for $i < j$, be the income levels where there is a kink in the income tax schedule. Define τ_i as the marginal tax rate that applies to all individuals earning y such that $y_i < y < y_{i+1}$ and let us first focus on an individual that is not choosing in a kink of the tax system. In this case, we have that $R(y) = R_i$ for all $y \in (y_i, y_{i+1})$, where y_i is the highest income level that is less than y and for which there is a kink. That is $y_i < y$, and $y < y_j$ for all $j > i$.

Let us then get back to an individual's maximization problem. We can split it into two parts. First, for an individual with productivity w and for each pair (τ_i, R_i) define $\hat{y}_i(w)$ as the solution for this problem.

So the solution for the individual's problem is[11]

$$\hat{y}(w) = \frac{w(1-\tau_i) - \gamma R_i}{(\gamma+1)(1-\tau_i)}.$$

Next define $y_i(w)$ through:

$$\begin{cases} y_i(w) = y_{i-1} & \text{if, } \hat{y}_i(w) \leqslant y_{i-1} \\ y_i(w) = \hat{y}_i(w) & \text{if, } y_{i-1} < \hat{y}_i(w) \leqslant y_i \\ y_i(w) = y_{i-1} & \text{if} \hat{y}_i(w) > y_i \end{cases}$$

This will define

$$V(\tau_i, R_i; w) \equiv \ln(y_i(w)(1-\tau_i) + R_i) + \gamma \ln(1 - Y_i(w)/w)$$

Then, the second stage is simply that of choosing

$$u(w) \equiv \max_i \left\{ V(\tau_i, R_i; w) \right\}_{i=1}^{W}.$$

When there are concave kinks in the budget set defined on the $Y \times C$ space, then we have that $y_i(w) = y_i$ for all individuals such that $V(\tau_i, R_i; w) = V(\tau_{i-1}, R_{i-1}; w)$. If this is the case, define, the set

$$\mathbf{W}_i \equiv \{w \in [\underline{w}, \infty]; V(\tau_i, R_i; w) = V(\tau_{i-1}, R_{i-1}; w)\}, \tag{1.10}$$

the set of all types that are bunched at y_i. Concave kinks in the budget set means that these sets are not singletons, i.e. the presence of a concave kink leads to a mass point in all y_i's.

The main difficulty that this situation imposes on us is that, in principle, we cannot distinguish these individuals from observing their incomes. A simple way for us to deal with this issue would be to assume that the density is flat along these

[11] It is now easy to see that the affine function

$$\mathfrak{w}_i(y) = (\gamma+1)y + \frac{\gamma}{1-\tau_i} R_i$$

generalizes $\mathfrak{w}_i$ for all income levels $y \in (y_i, y_{i-1})$.

intervals. Alternatively, we could use a spline to try to complete the empirical distribution.

There are two reasons why we believe that this should not pose too much of a problem for us. First it should however have little impact on the tax schedule we derive since we use a parametrized distribution that we shall adjust to this empirical distribution of skills. Second, as it turns we do not observe these bunching points in the data.

The Parametrized Distribution We choose the Generalized Pareto (GP) distribution to fit the empirical distribution of skills. The GP distribution fits well at the upper tail. Previous papers has used the log-normal distribution, despite the fact that it approximates very poor the empirical distributions at the top and bottom tails.

The parameters of Generalized Pareto distribution are chosen so that the skill distribution replicates the empirical earnings distribution. The GP has three parameters, σ, κ and μ. Specifically we choose these parameters to as much as possible recreate the income distribution and the *empirical* distribution of skills and to match the empirical ratio expenditure/GDP. The GP probability density function is given by

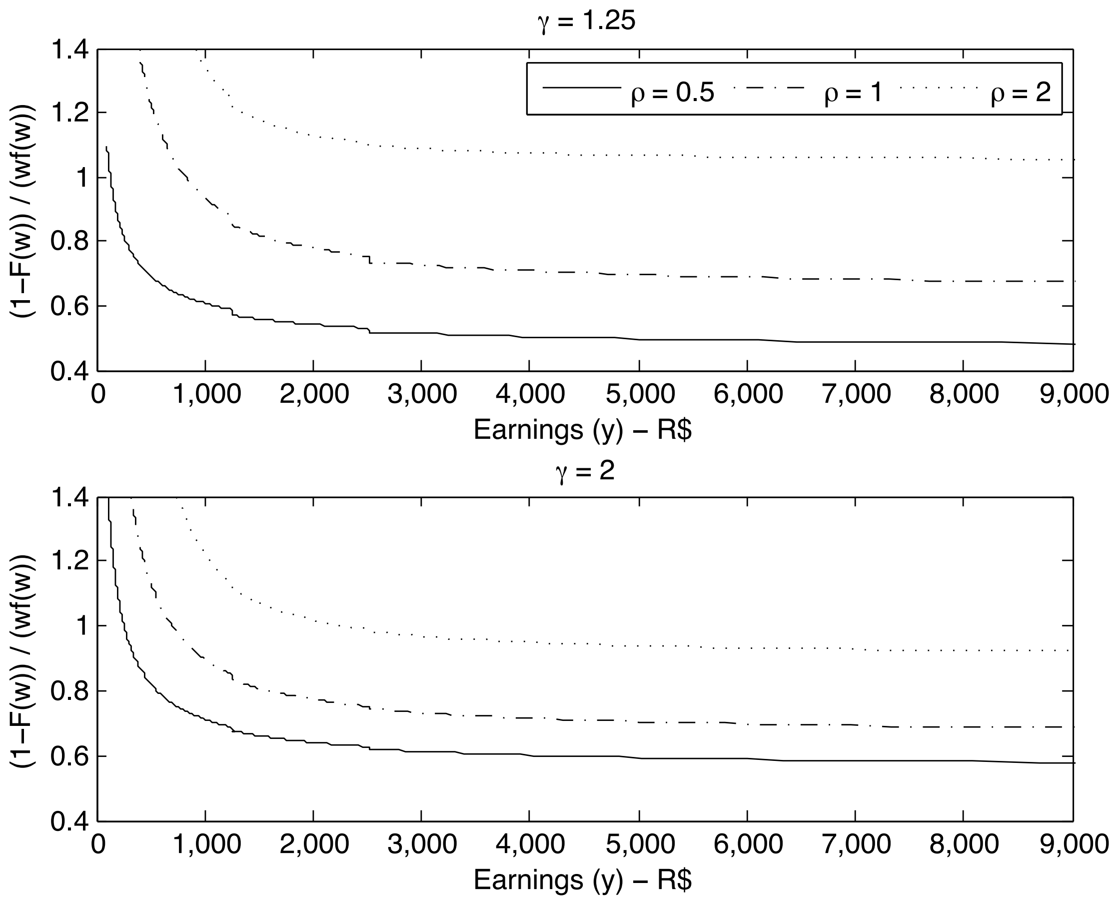
$$f(w \mid \kappa, \sigma, \mu) = \left(\frac{1}{\sigma}\right)\left(1 + \frac{\kappa(w-\mu)}{\sigma}\right)^{-1-\frac{1}{\kappa}} \tag{1.11}$$

where the Generalized Pareto distribution with the tail index (shape) parameter κ, scale parameter σ and threshold (location) parameter, μ, evaluated at the values in w. If $\kappa = 0$ and $\mu = 0$, the GP distribution is equivalent to the exponential distribution. If $\kappa > 0$ and $\mu = \sigma/\kappa$, the GP distribution is equivalent to the Pareto distribution.

We plot the empirical and approximated skill distribution in the Figure 1.2 to all parametrization that we consider in this paper. In the appendix B, the Figure 1.11 displays one case to make easier to observe the fit between empirical and approximated skill distribution.

In the Figure 1.3 we observe that the hazard ratio is very high at the bottom of income distribution because $wf(w)$ is close to zero while $1 - F(w)$ is close to one. At the top, the hazard ratio converge to levels that depend on the parameters

Figure 1.3: Hazard Ratio - 99% of Individuals



of risk aversion and the elasticity of labor supply. In the appendix B, we plot the hazard ratio function to 99.9% individuals in Figure 1.13. It is easier to observe that the hazard ratio is almost stable for earnings above $R\$9,000$ reais.

Holding the Frisch elasticity of labor supply constant the hazard ratio at the top is lower when risk aversion is smaller and it is bigger when risk aversion is higher. In all cases considered, the hazard ratio is L-shaped while in Saez (2001), the hazard ratio is U-shaped.

According to equation (1.7), the optimal marginal tax rate is increasing in the hazard ratio. Thus, we expect the higher hazard ratio (risk aversion) to induce a higher optimal marginal tax rate. Saez (2001) found a hazard ratio around 0.5 while we found a range between 0.45 and 1.1. It is easy to observe that, at the top, the hazard ratio has a smaller variance when $\gamma = 2$ than in the case where $\gamma = 1.25$.

Convex Kinks Before closing this section, a word is due on convex kinks in the budget sets, which is induced by regressivity in the income tax schedule. Now, let y_j be a point where the tax system induces a convex kink in the agents' budget set. The issue now is that there will be a productivity level n such that any individual with productivity n will be indifferent between to levels of earning y' and y'' where $y' < y_i < y''$. Nobody would be observed choosing between these two values. It should be noted that this observation does not create any difficulty for our procedure since the set of agents that are indifferent between two levels of earnings is of measure zero.

1.4 Results

In our simulation, we calibrate our model along the lines of Mirrlees (1971) and Saez (2001) to represent the Brazilian economy in 2006 The government's expenditures E are set at 24.91% of output. It is important to remember that we consider both consumption and labor income taxes rates in our simulations.

The utility function is of form $u(c,l) = \frac{c^{1-\rho}}{1-\rho} - \frac{l^\gamma}{\gamma}$, where ρ is the parameter of risk aversion and γ is the parameter of labour supply. In our simulations we use the following values for risk aversion, $\rho = \{0.5, 1, 2\}$ and for the parameter of labor

supply, $\gamma = \{1.25, 2\}$.[12]

We consider two social welfare criteria. The first is the Utilitarian criterion, where $G(u) = u$, respectively $G'(u) = 1$. The second criteria is the Rawlsian criterion, where $G'(u) = 0$.

1.4.1 The Benchmark Model

The benchmark model is the one that results from applying the current tax system to the distribution of skills that results from solving problem (1.1). The marginal taxes rate, τ, are the values described in the last section. The virtual income is the empirical values in 2006[13]. Ideally the model would exactly replicate the Brazilian economy in 2006. The approximations we use, however make it diverge somewhat from the actual data.[14]

Table 1.1 summarizes some statistics for the data and the benchmark model, like value per percentile, the Expenditure/GDP ratio, mean, variance, etc. It also displays the empirical and the benchmark earnings according to label supply elasticity and risk aversion parameters. For example, in the percentile 50, the individual earns $R\$500$ reais, while in the benchmark model with $\gamma = 1.25$, the earnings are $R\$292.49$, $R\$368.63$ and $R\$509.92$, respectively to $\rho = 0.5$, $\rho = 1$ and $\rho = 2$.

The large difference in the standard deviation is explained by the peeks in the empirical distribution. As the approximated distribution is smoother, the values are spread following a parametric distribution. In contrast, the empirical has many peeks along the distribution. There are cluster points in the empirical distribution, that could be explained because the wages are discontinuous.

In Figure 1.4 we plot the empirical and the approximated (benchmark) earnings associated with different parameters for the cumulative function of individuals. The graphics in the left side are plotted to 90% of individuals that are sorted by earnings (skills). In the right side we have the individuals that belong to the range between the percentile 90% and 99% of earnings.

[12]When $\rho = 1$ the utility function has the form $u(c, l) = \ln(c) - \frac{l^\gamma}{\gamma}$. γ is directly associated with the Frisch elasticity of labor supply η^f. When $\gamma = 1.25$ the Frisch elasticity is equal to 4, $\eta^f = 4$ and $\eta^f = 1$ when $\gamma = 2$.

[13]The empirical virtual income to the first group was zero (0), to the second group was $R\$159.70$ reais and to the last group was $R\$422.15$ reais.

[14]For example, in the benchmark model, individuals *do* bunch at kinks.

Table 1.1: Statistic Details: Empirical and Approximated Earnings (R$)

Percentile	*Empirical*	$\gamma = 1.25$			$\gamma = 2$		
		$\rho = 0.5$	$\rho = 1$	$\rho = 2$	$\rho = 0.5$	$\rho = 1$	$\rho = 2$
0.10	250	120.47	139.14	189.80	121.78	133.43	169.12
0.25	350	162.09	213.79	307.16	165.31	197.12	275.23
0.50	500	288.51	414.82	541.64	292.49	368.63	509.92
0.75	900	658.73	902.51	992.01	644.95	790.32	992.56
0.90	1,850	1,348.00	1,823.70	1,795.89	1,387.66	1,643.03	1,916.68
0.95	3,000	2,512.08	2,847.46	2,684.30	2,512.93	2,649.42	2,997.25
0.99	7,000	9,033.67	9,769.16	7,933.94	8,707.01	8,815,51	8,916.41
1.00	120,000	128,929.43	122,232.96	118,992.10	119,491.94	122,634.78	120,292.48
E^*	0.2028	0.2491	0.2492	0.2490	0.2492	0.2491	0.2493
$Mean$	912.18	914.29	1,065.24	1,072.40	886.44	977.18	1,107.41
$St.Deviation$	1,522	4,679	4,456	4,141	4,322	4,372	4,280

Source: Elaborated by Authors

* $E = Expenditure/GDP$

Figure 1.4: Earnings versus Cumulative Function

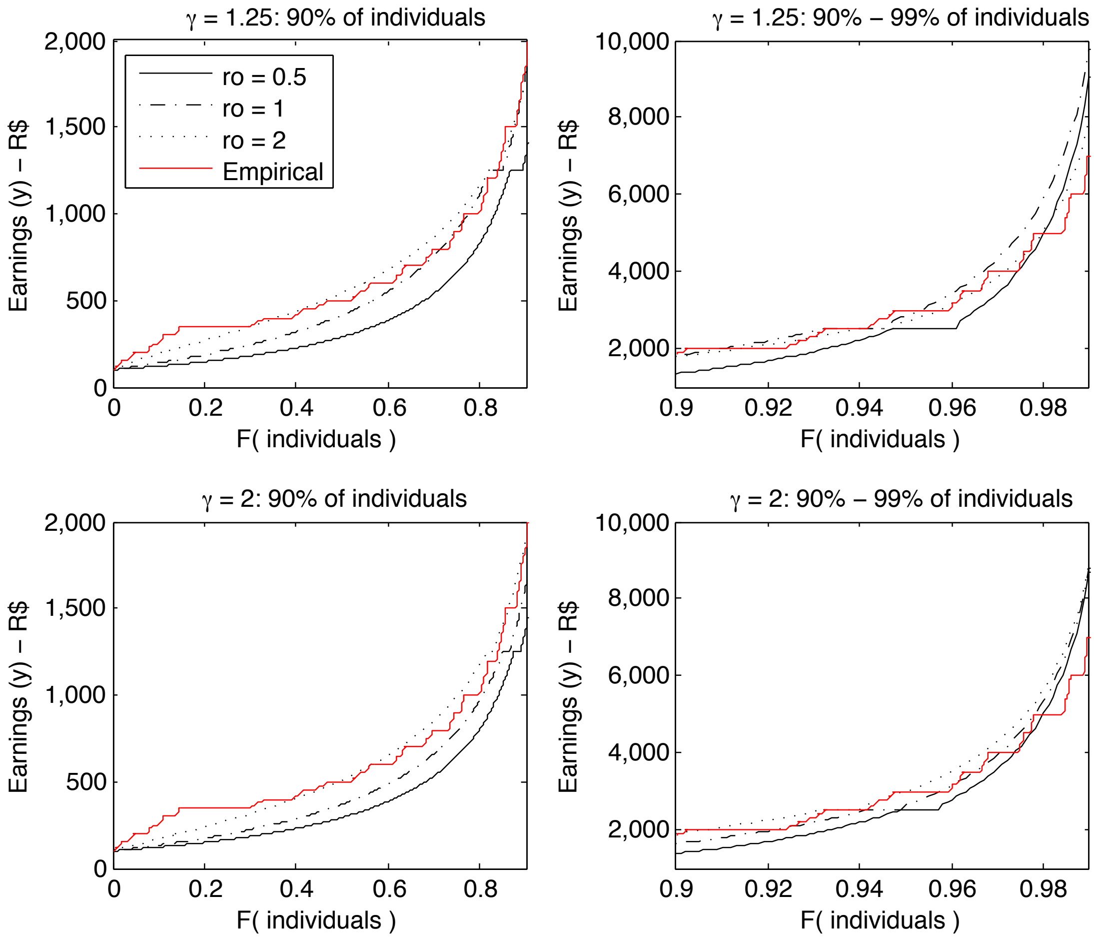
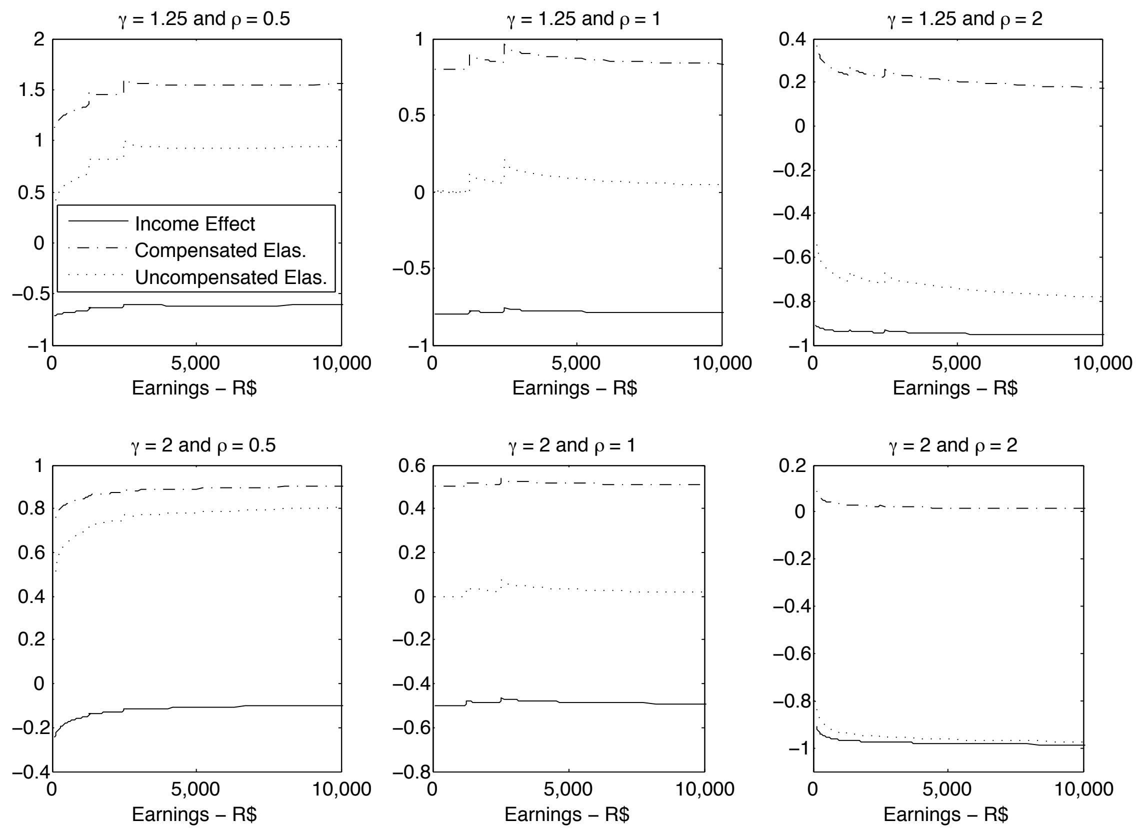


Figure 1.4 and the Table 1.1 suggest that model with $\rho = 2$ fits the empirical earnings better than the models when $\rho = 0.5$ and $\rho = 1$. In general, our approximation fits the empirical earnings very well. The exception could be made after the percentile 98%. The benchmark earnings grow faster than the empirical earnings after the 98% of sample. The approximated earnings values are higher than the empirical earnings. We explain the "bad" fit at the bottom due to the behavior of empirical earnings distribution at the bottom. The empirical earnings data have an increase in the density function at the bottom before the model (see Figure 1.10 in appendix B). We can also observe that there is bunching in the kinks in our benchmark (approximated) data, which is contrast with the actual data.

Figure 1.5: Compensated Elasticity, Uncompensated Elasticity and Income Effect - 99% of Individuals



Other important result is the direction and magnitude of the uncompensated

and compensated elasticities and the income effect. Figure 1.5 we plot the direction and magnitude of each elasticity to 99% of population. The compensated elasticity is decreasing in ρ and is always positive. The estimated income effect depends on risk aversion and labor supply elasticities. Keeping the labor supply elasticity constant, the resulting income effect is increasing, in absolute values, in earnings (skills). In Figure 1.12 in appendix B we plot the elasticities to 99.9% of population.

According to equation (1.7), the optimal marginal tax rate is decreasing in compensated elasticity and increasing in the uncompensated elasticity. The uncompensated elasticity is decreasing in risk aversion and it changes its signal from positive to negative.

The term $A(w)$ in equation (1.8) may be written as $\frac{1}{\zeta^c} + \frac{\zeta^u}{\zeta^c}$. Because ζ^u is always smaller than 1, $\frac{1}{\zeta^c} > \frac{\zeta^u}{\zeta^c}$. Therefore, the compensated elasticity's effect dominates the term $\frac{1+\zeta^u}{\zeta^c}$. Although the sign of ζ^u changes from positive to negative when we increase risk aversion, the uncompensated elasticity's effect could only attenuate or intensify the effects of compensated elasticity on term $A(w)$ but never change its sign.

1.4.2 The Optimal Taxation Model

In this section, we present our simulations for the optimal non-linear tax schedule for Brazil. As we said before, we consider different values for ρ and γ. These utility functions generate a compensated elasticity that varies with earnings.

Table 1.2 summarizes the welfare measure according to different parameters for the utility function and different social welfare criteria. As expected, at the optimal schedule there are some individuals that are better off and individuals that are worse off than in the benchmark case.[15] The gains are located at the bottom and at the top and the loses in the center of distribution. As we consider the Generalized Pareto (GP) Distribution, the number of individuals at the bottom is very large when compared to number of individuals at the top. One reading is that an optimal tax schedule increases the utility to the poor individuals.

[15] This should be the case if we consider the findings in Mattos (2008) which shows that the Brazilian tax system is Paretian. However, because he considers different functional forms and data sets, it is not obvious that his result would apply to our setting.

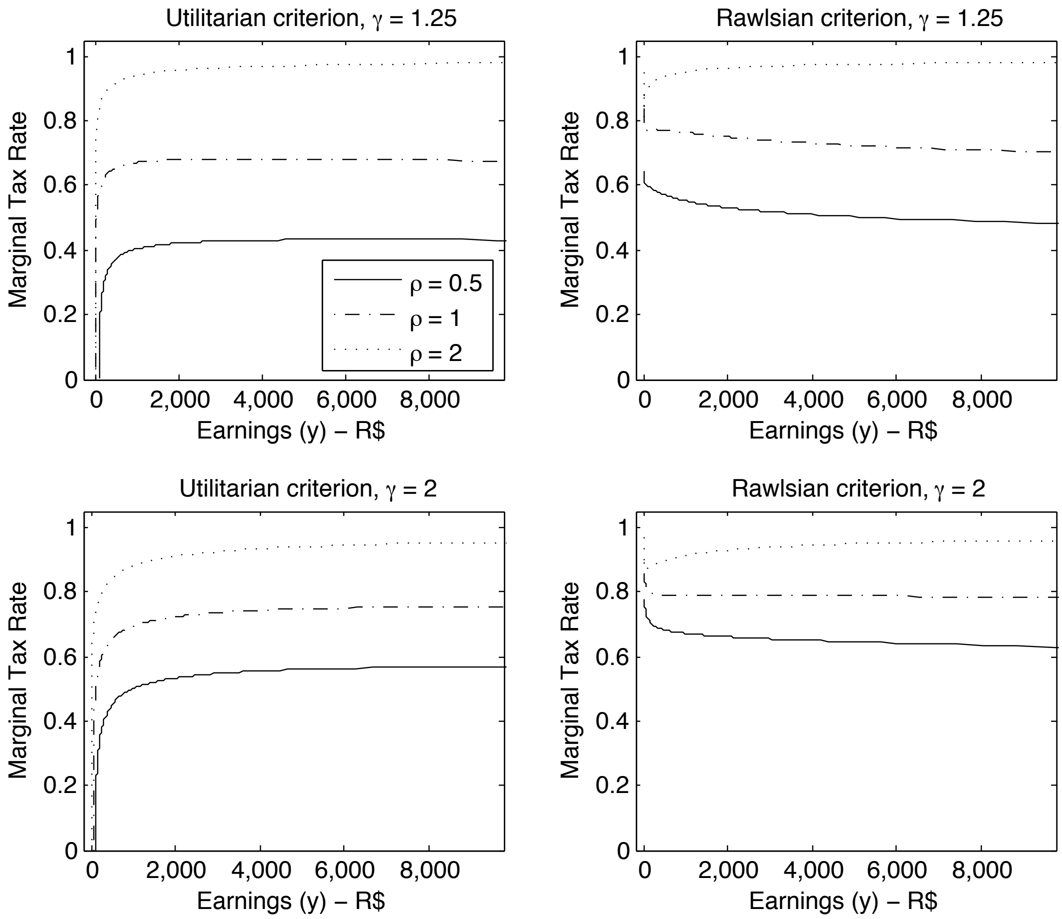
Table 1.2: Utility: Benchmark and Optimal

	Utilitarian criterion				Rawlsian criterion			
	$\gamma = 1.25$		$\gamma = 2$		$\gamma = 1.25$		$\gamma = 2$	
	Benchmark	Optimal	Benchmark	Optimal	Benchmark	Optimal	Benchmark	Optimal
$\rho = 0.5$	25.3105	28.1190	31.3462	35.6434	11.0570	25.3003	13.8219	31.5701
$\rho = 1$	5.2526	5.9092	5.4586	6.0783	3.6434	5.7902	3.9426	5.9197
$\rho = 2$	-0.0052	-0.0011	-0.0047	-0.0014	-0.0210	-0.0011	-0.0176	-0.0014

Source: Elaborated by Authors

A possible reading of our result is that the social welfare function implicit in the Government's decision places more weight on richer individuals than what is implied by a utilitarian SWF. This is not in contradiction with Mattos (2008) since we are allowing for a direct desire for redistribution through the concavity of the individuals' utility functions.

Figure 1.6: Optimal Marginal Tax Rate - 99% of Individuals



The results for the optimal marginal rates in our simulations are plotted in Figure 1.6 for monthly earnings in reais ($R\$$). Within this range of earnings we find about 99% of individuals. In Appendix B we plot the same picture for 99.9% of our population (see Figure 1.14).

The range of values of skill distribution is bounded at the bottom, i.e., $[\underline{w}, \infty]$, with $\underline{w} > 0$ and the distribution is unbounded distribution at the top. Our values for the Marginal Tax Rate (MTR) at the bottom of distribution are smaller than those in Saez (2001) and Tuomala (2006). As expected, the level of the optimal marginal rate depends on the risk aversion and labor supply parameters.

In all situations, the optimal tax schedule has the form of inverted U-shape for the Utilitarian criterion. We verify that the optimal tax schedule leads to increasing earnings, $y'(w) > 0$. It is the necessary and sufficient condition for individual second-order conditions explained in details in Mirrlees (1971).

When $\gamma = 1.25$, the optimal marginal rate is increasing in earnings for most of the range. It falls dramatically at the top and is equal to 0 for the most productive individual. This 0 (zero) marginal tax rate at the top is necessary for efficiency as shown by Sadka (1976) and Seade (1977). We do not focus on this result that depends on the boundedness of the skills distribution. We believe this is not a good representation of the relevant distribution of skills.

The maximum optimal marginal rate is around 43%, 68% and 98% respectively to the risk aversion parameters equal to $\rho = 0.5$, $\rho = 1$ and $\rho = 2$.

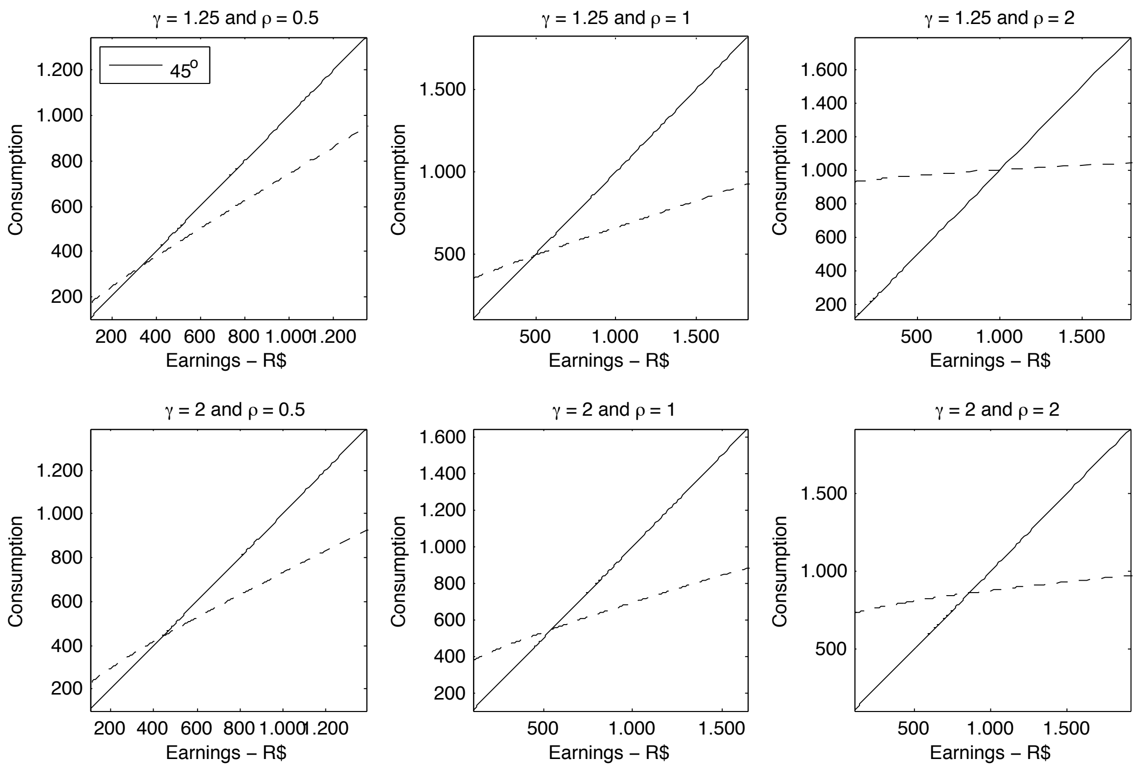
When $\gamma = 2$, the marginal tax rates is very close to the case $\gamma = 1.25$. However, the marginal tax rate is a little higher than in the first case, the exception is in the case $\rho = 2$. The maximum marginal tax rate for $\rho = 0.5$, $\rho = 1$ and $\rho = 2$ are, respectively, 54%, 75% and 96%.

The Rawlsian criterion yields higher marginal rates at low earnings. However, the results when $\rho = 2$ are a bit surprising. Independently of the labor supply parameter, the marginal rate in the Rawlsian criterion falls in the begin and subsequent increases before the $R\$1,000$ reais. The values of MTR reach about 97% and keep constant until the $R\$90,000$ reais, when the MTR starts to decrease and converges to zero. To the others values of risk aversion, the MTR is monotone decreasing in the earnings and converges to the marginal tax rate equal to zero at the top.

The marginal tax rate does not have the U-shape found elsewhere in Utilitarian criterion. Two forces seem to be at play here. First, the hazard ratio in the our paper has the L-shape instead of the U-shape found in Saez (2001). Second, we do not have bunching at the bottom. When $z'(n) < 0$ in the relaxed problem, one must

change the initial condition. When we change this initial condition, the marginal rate at the bottom becomes higher. As we do not have this problem, the initial MTR is zero at the bottom. This procedure is described in great detail in Mirrlees (1971).

Figure 1.7: Optimal Consumption: Utilitarian criterion - 90% of Individuals



The high marginal tax rates for those at the top of the distribution in the case $\rho = 2$ are due to the fact that the compensated elasticity (ζ^c) is small and close to zero. As we can see in equation (1.7), the compensated elasticity has an important role in the determination of optimal marginal rate. Other important point is the hazard ratio, that is increasing in the risk aversion.

With a large value of risk aversion, the government has great incentives to smooth consumption and, consequently, increase the redistribution of earnings. Thus, the high value for the marginal tax rate to support the redistributive goals of the government. This effect is apparent in Figure 1.7.

Figure 1.7 displays consumption against earnings for various levels of ρ and γ. Transfers can be inferred by the distance to the 45 degrees line. The higher risk aversion implies in the lower the consumption inequality. Figure 1.7 represents 90% of individuals. Note that the level of guaranteed consumption, i.e., the intersection with the vertical axis is increasing in risk aversion. This illustrates the redistributive goals of the government.

There are some differences in the results obtained here and those obtained elsewhere. Brazil is a country characterized by a very high level of income inequality, which is reflected in a very disperse distribution of skills. The lower marginal tax rates found at the bottom of the may be explained by the distribution of skills. The other important point is the hazard ratio $(1 - F(w))/(wf(w))$. Due to the Generalized Pareto distribution, the hazard ratio has L-shape. However, the it does not tend to zero like in log-normal distribution.

Mirrlees (1971) found a lower marginal tax rates than we did. The reasons for that are the utility function and the log-normal distributions of skills considered by him. Saez (2001) found a high rates at top like us. He considered a compensated elasticities equal to $\zeta^c = 0.25$ and $\zeta^c = 0.5$. The other important point is the hazard ratio, that has a U-shape. The shape of hazard ratio is responsible to the U-shape in the marginal tax rate in his paper.

1.4.3 The Optimal Marginal Tax Rate in United State

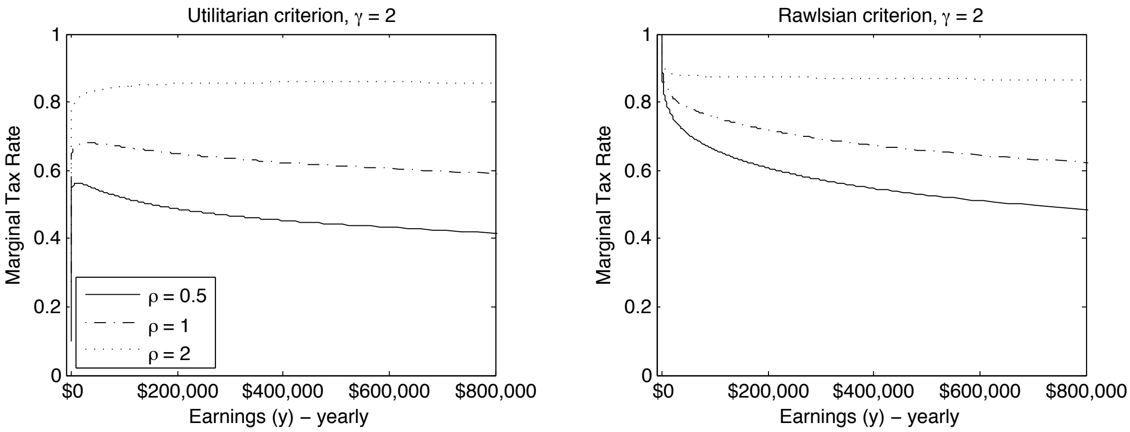
In this subsection we simulate the optimal marginal tax rate in United State (US). There are many articles that analyze the optimal marginal tax rate in US, some examples are Tuomala (1990, 2006) and Saez (2001). Like Brazilian case described above and contrary to Saez (2001), we take account the non-linearity of the United State labor income tax schedule. We recognize the non-linearity that is present in the US income tax schedule when we recover the distribution of skills. In present paper, the labor income tax schedule is composed by income and consumption taxes schedules.

To simulate the US marginal tax rate we consider the panel study of income dynamic (PSID) data in 2007. We use the head's income from wages and salaries

in 2006[16]. We also drop all head that declare with no wife, husband or first-year cohabitor and individuals with income equal to zero.

In 2006 the income tax schedule to head of household filing status was 10% on the yearly income between $0\$ - \$10,750$, 15% on $\$10,750 - \$41,050$, 25% on income between $\$41,050 - \$106,000$, 28% on $\$106,000 - \$171,650$, the individuals pay 33% on the income $\$171,650 - \$336,550$ and 35% on the income over $\$336,550$. To consumption tax, we consider the mean among US states of the sale tax in 2006, where the value was 5.1%.

Figure 1.8: Optimal Marginal Tax Rate in US - 99% of Individuals



As we drop the individuals with income equal to zero, when we recover the distributions of skills we have the range of skills, $[\underline{w}, \infty]$, with $\underline{w} > 0$. In our simulations, we consider only the case where $\gamma = 2$. To compare the optimal tax rate in Brazil and in United State, we calibrate the government expenditures (E) to be around 0.2491 of product. We also consider a Generalized Pareto distribution to fit the empirical distribution of skills.

To the Utilitarian criterion, we find the inverted U-shape to MTR in US. The Figure 1.8 we plot the earnings versus MTR in US to 99% of individuals. Likewise the previews simulations, the MTR in US is increasing in the risk aversion. As we said above, the MTR is increasing in the risk aversion because the individuals do

[16]The variable index is ER40903.

not tolerate the income inequality among them.

The MTR in US is a little lower compared to Brazil in all situation. When $\rho = 0.5$ and $\rho = 1$, the MTR begins to fall after the $\$100,000$ dollars and keeps almost constant between $\$100,000$ and $\$800,000$. To $\rho = 2$ the MTR is increasing in the earnings and it is constant around the rate 84%. In this case, the MTR in US is very similar to Brazil, because it is stable at the top taxpayers.

To the Rawlsian criterion, as expected, the MTR is higher compared to the Utilitarian criterion. As in Brazilian case, the largest difference between two criteria is at low earnings. At the bottom, ours values are closed to Saez (2001), however we do not observe the increase in MTR like him. Our marginal tax rate is decreasing toward 0.

Compared to Saez (2001), the values at the top are similar to his article. In the logarithm case, ($\rho = 1$), the value of MTR at the top belongs to the range of values found to him, i.e. between 0.6 and 0.8, though we consider a different compensated elasticity.

Ours simulations differ completely from Saez (2001) at the bottom in the Utilitarian criterion. We do not find the fall in the begin of the earnings. As we said above, our distribution of skills is bounded to a lower bound, $\underline{w} > 0$. For this reason, we do not observe the fall of MTR in the begin of earnings. The other important point is the empirical hazard ratio function (see Figure 1.16 in appendix B). Our hazard ratio has the L-shape while Saez (2001) has the U-shape. As we show in equation (1.7), the hazard ratio has a very important role to define the shape of optimal marginal tax rate.

Figure 1.17, in the appendix B, we plot the marginal tax rate for yearly earnings between $0 and $2,000,000. In this range we have around 99.9% of our sample. In appendix B we also plot the distribution of earnings in 2006 (see Figure 1.15).

1.4.4 Affine Tax Schedule Proposal

In this section we compute the optimal affine tax schedule for Brazil. The idea here is to see if we can get close to the optimum with a simple tax schedule. In fact, in a static setting, this schedule is equivalent to a poll tax (or subsidy) and a consumption tax, which is a very appealing system in practice.

As we saw in the previous section, at the bottom, the optimal tax schedule is characterized by a cash transfer to individuals with low earnings. This transfer depends on the redistributive goals of the government that guarantee a consumption level to the individuals. Grossly speaking, a cash transfer program could have the same function at the bottom than optimal tax model. The most famous cash transfer program in Brazil is the *Bolsa Família* Program. The *Bolsa Família* is conditional to the family's structure[17].

Our proposal consists in a simplification of the number of marginal tax rate and the incorporation of any kind of the cash transfer program to the affine tax schedule. This way, the taxation and the transfers would be made by the same institution.

The affine tax schedule is composed by a pair (t, B), where t is the marginal tax rate and B is the transfer. The transfers are financed by the tax. The individual's problem may then be written

$$\max_{c_w, z_w} U(c, y/w) \tag{1.12}$$
$$\text{s.t. } c = y(1-t) + B.$$

As in the problem describes by equation (1.1), the solution is defined by $y(1-t, B, w)$ and $c(1-t, B, w)$.

Beyond financing the program the government must also finance the other expenditures, E. Therefore the government's budget constraint is

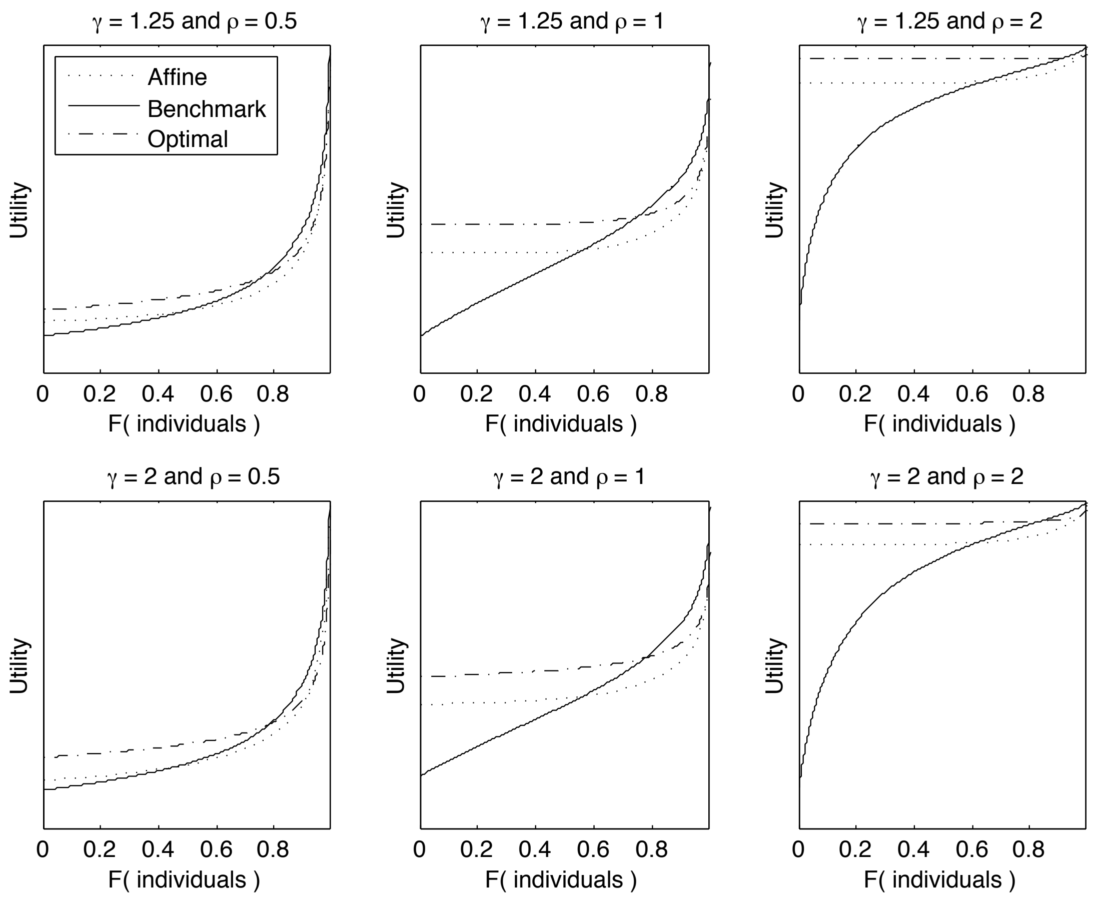
$\int_{\underline{w}}^{\infty} (z_{\tilde{w}} - c_{\tilde{w}} - B)\, f(\tilde{w}) d\tilde{w} \geq E$, where B is independent of w.

As we want to compare the current tax schedule to the affine tax schedule, we have chosen the same level of government expenditure, E, that we consider in the previews sections.

To find the affine tax schedule, we define a vector of marginal taxes rates, $\tilde{t}$, and a vector of cash transfers, $\tilde{B}$. In the first step, we start with a big range of values to t and B. In the second step we refine and reduce the range of t and B. We repeat this procedure until to find the global maximizer. In this section we consider only the Utilitarian criterion. Observe that, for each pair (ρ, γ) of the parameters of risk aversion and labor supply elasticity, we have to look for the global maximizer point in a different range of t and B.

[17]For details on Bolsa Família, Program (2009) http://www.mds.gov.br/bolsafamilia

Figure 1.9: Individual Utility - Benchmark and Affine Schedule : 99% of Individuals



We find the following results, when $\gamma = 1.25$, for $\rho = 0.5$, we have $t = 0.3894$ and $B = 54.73$ reais. To $\rho = 1$, we have $t = 0.6786$ and $B = 176.84$ reais. To $\rho = 2$, we find $t = 0.7929$ and $B = 342.63$ reais. When $\gamma = 2$, for $\rho = 0.5$, we find $t = 0.4845$ and $B = 70.52$ reais. To $\rho = 1$, we have $t = 0.7143$ and $B = 191.57$ reais. To $\rho = 2$, we find $t = 0.8418$ and $B = 367.36$.

Figure 1.9 shows who is better off and who is worse off if we substitute the affine tax schedule for the benchmark tax schedule. We also plot the utility in the optimal tax schedule. We plot individual utility in the y-axis and the cumulative function of number of individuals in x-axis. The figure is plotted to 99% of individuals. It is easy to see that the affine schedule provides an increase in the utility for a large fraction of the population. The utility increases for at least 45% of the individuals in all simulations.

We can observe that in some situations the affine tax schedule provides an utility close to the optimal schedule.

In Table 1.3 we summarize the aggregate utility to the benchmark and to the affine schedules models. According to our results, except when $\rho = 0.5$, the affine tax schedule leads to an increase in the social welfare in the Utilitarian criterion.

Table 1.3: Aggregate Utility: Benchmark and Affine Tax Schedule

	$\gamma = 1.25$		$\gamma = 2$	
	Benchmark	Affine Schedule	Benchmark	Affine Schedule
$\rho = 0.5$	25.3105	24.3819	31.3462	30.1774
$\rho = 1$	5.2526	5.4644	5.4586	5.6093
$\rho = 2$	-0.0052	-0.0027	-0.0047	-0.0024

Source: Elaborated by Authors

We explain the result when $\rho = 0.5$ due to the low risk aversion. It implies in a greater earnings inequality among the individuals compared to the case when $\rho = 1$ and $\rho = 2$. Although the aggregate utility is smaller in the affine schedule than in the benchmark schedule, looking at Figure 1.9 we observe an increase in the utility for individuals with low earnings. However, this increase is not enough to compensate for the losses imposed on such a large fraction of individuals.

As the optimal tax schedule is extremely difficult to implement in reality, we

have to look for some alternatives to the optimal schedule. In the section we present one of this alternatives. We show that is possible to find a feasible affine schedule that is better than the current one under the Utilitarian metric.

1.5 Conclusions

In present paper we investigate alternatives ways to the current tax schedule in Brazil.

In the first part of this paper, we back the distribution of skills for Brazil using the distribution of income and the tax schedule for the year 2006 for various parameters for individuals' preferences. Our approach is similar to Mirrlees (1971) and Saez (2001). We differ from Saez (2001) in the way we create the empirical distribution of skills. Saez (2001) approximates the current tax system in the US using a flat tax to create the skill distribution whilst we take into account the non-linearities of the Brazilian system.

In the second part, we derive optimal tax schedules for different preferences for distribution. As the optimal taxation is difficult to implement in reality, we also calculate the optimal affine tax schedule. This allows us to focus on the effect of cash transfers programs, like *Bolsa Família*, in labor supply and utility. The idea is to find a simple tax system which is better than the current system and verify how close we get to the optimal tax system.

Our results of optimal taxation are in line with others papers in the literature in some dimensions. However, we find inverted U-shaped marginal tax rate function instead of U-shaped, which is in contrast with Saez (2001). The fact that we investigate optimal policy in a developing country with a very high inequality in earnings may be the underlying reason for our findings.

1.6 Appendix A

In this appendix we derive equation (1.7) and the equations of the differential system that we solve numerically in the present paper.

The equations were found with blind Hamiltonian optimization and under $u_{lc} = 0$ assumption. Following Mirrlees (1971), we consider that $u(w)$ is a state variable, $l(w)$ is the control variable and $c(w)$ is determined implicitly as a function of $u(w)$ and $l(w)$ from equation $u(w) = u(c(w), l(w))$. Others examples are Sadka (1976), Seade (1977), Tuomala (1990) and Diamond (1998).

The government's problem is described by equation (1.13). The terms λ and $\mu(w)$ are respectively the Lagrange multiplier of resources constraint and the multiplier of incentives constraint. E is the government expenditure.To obtain the incentive constraint we derive $u(c(w), l(w))$ into w, i.e., $\frac{du}{dw} = -\frac{lu_l}{w}$.

$$H = \int_{\underline{w}}^{\infty} G(u(w)f(w)dw + \lambda \left[\int_{\underline{w}}^{\infty} (wl(w) - c(w)) f(w)dw - E\right] + \mu(w)\left[\frac{du(c(w), l(w))}{dw}\right] \tag{1.13}$$

To solve the problem we derive the eq.(1.13) into u and l. The first order condition into $u(n)$ is equation

$$\frac{d\mu}{dw} = \frac{\partial H}{\partial u} = -\left[G'(u(w)) - \frac{\lambda}{u_c}\right] f(w) \tag{1.14}$$

The first order condition into l is given by:

$$\lambda\left[w + \frac{u_l}{u_c}\right] f(w) + \mu(w)\frac{\varphi_l}{w} = 0 \tag{1.15}$$

where $\varphi(u, l) = -lu_l$. Integrating the equation (1.14) from w to infinity and we also consider the transversality condition, $\mu(\underline{w}) = \mu(\infty) = 0$. Then we have a new equation that is given by:

$$\mu(w) = \int_{w}^{\infty} \left[G'(u(\tilde{w})) - \frac{\lambda}{u_c(\tilde{w})}\right] f(\tilde{w})d\tilde{w} \tag{1.16}$$

Now we substitute the equation (1.16) into the equation (1.15) and we have equation (1.17):

$$\lambda\left[w+\frac{u_l}{u_c}\right]f\left(n\right)=\frac{\varphi_l}{w}\int_w^\infty\left[\frac{\lambda}{u_c(\tilde{w})}-G'(u(\tilde{w}))\right]f(\tilde{w})d\tilde{w} \tag{1.17}$$

Define the variable $\upsilon(n)$ that is continuously differentiable functions of c and l and has the form, $\upsilon(w)=\left(1+\frac{u_l}{wu_c}\right)\diagup\varphi_l$. We can rewrite the equation (1.17) as

$$\upsilon(w)=\frac{1}{w^2f(w)}\int_w^\infty\left[\frac{1}{u_c(\tilde{w})}-\frac{G'(u(\tilde{w}))}{\lambda}\right]f(\tilde{w})d\tilde{w} \tag{1.18}$$

As c and l can be expressed as continuously differentiable function of u and υ, therefore themselves differentiable of w. The differentiation of equation (1.18) with respect to w is:

$$\begin{aligned}\frac{d\upsilon}{dw}=&-\left[2wf(w)+w^2f'(w)\right]\frac{1}{(w^2f(w))^2}\int_w^\infty\left[\frac{1}{u_c(\tilde{w})}-\frac{G'(u(\tilde{w}))}{\lambda}\right]f(\tilde{w})d\tilde{w}\\&-\frac{1}{n^2f(w)}\left[\frac{1}{u_c}-\frac{G'(u(w))}{\lambda}\right]\end{aligned} \tag{1.19}$$

The equation (1.19) can be written as

$$\frac{d\upsilon}{dw}=-\frac{\upsilon}{w}\left[2+\frac{wf'(w)}{f(w)}\right]-\frac{1}{w^2}\left[\frac{1}{u_c(w)}-\frac{G'(u_w)}{\lambda}\right] \tag{1.20}$$

The relationship between utility and skill is given by the equation (1.21):

$$\frac{du}{dw}=-\frac{l_wu_l}{w} \tag{1.21}$$

Then our system of differential equations is given by the equations (1.20) and (1.21). The complete description of conditions to solve this system see Mirrlees (1971). Like Saez (2001), $f(w)$ is derived from the empirical distribution of wage income. However, we use the current taxes schedules to matches the empirical distribution.

Following Mirrlees (1971), we have to do some transformations in equations (1.20) and (1.21) to solve our system. Consider the utility function $u(c,l)=\frac{c^{1-\rho}}{1-\rho}-\frac{l^\gamma}{\gamma}$ and its partial derivate with respect to c and l are $u_c=c^{-\rho}$ and $u_l=-l^{\gamma-1}$. According to definition, the variable υ could be write as:

$$\upsilon(\tilde{w})=\frac{1-\frac{l(\tilde{w})^{\gamma-1}}{wc(w)^{-\rho}}}{\gamma l(w)^{\gamma-1}} \tag{1.22}$$

where $\varphi_l = \gamma l^{\gamma-1}$. The equation (1.22) could also be rewritten as $c = \left[wl^{1-\gamma} - \gamma w \upsilon\right]^{1/\rho}$. The equation (1.20) has the form:

$$\frac{d\upsilon}{dw} = -\frac{\upsilon}{w}\left[2 + \frac{wf'(w)}{f(w)}\right] - \frac{1}{w^2}\left[\frac{1}{c^{-\rho}} - \frac{G'(u(w))}{\lambda}\right] \tag{1.23}$$

If we substitute the consumption expression above into equation (1.23), we finally have the equation (1.24)

$$\frac{d\upsilon}{dw} = -\frac{\upsilon}{w}\left[2 + \frac{wf'(w)}{f(w)}\right] - \frac{1}{w^2}\left[wl^{1-\gamma} - \gamma w\upsilon - \frac{G'(u(w))}{\lambda}\right] \tag{1.24}$$

The term λ comes from the equation (1.14) and the transversality conditions $\mu(\underline{w}) = \mu(\infty) = 0$. Then the λ expression is given by:

$$\lambda = \frac{\int_{\underline{w}}^{\infty} G'(u(\tilde{w}))f(\tilde{w})d\tilde{w}}{\int_{\underline{w}}^{\infty} 1/u_c(\tilde{w})f(\tilde{w})d\tilde{w}} \tag{1.25}$$

More details about this expression see Mirrlees (1971). Now we have to do the transformation in equation (1.21) to eliminate the utility. The idea is, instead of has a equation in du/dw, we would have a equation in dl/dw. With this in mind, the first step is to substitute the consumption relation above in the utility function. The new equation is:

$$u(c,l) = \frac{\left[w(l^{1-\gamma} - \gamma\upsilon)\right]^{\frac{1-\rho}{\rho}}}{1-\rho} - \frac{l^\gamma}{\gamma} \tag{1.26}$$

Now we derive the equation (1.26) with respect to w, the new relation is given by:

$$\frac{du}{dw} = \frac{1}{\rho}\left[w(l^{1-\gamma} - \gamma\upsilon)\right]^{\frac{1-2\rho}{\rho}}\left[(l^{1-\gamma} - \gamma\upsilon) + w(1-\gamma)l^{-\gamma}\frac{dl}{dw} - w\gamma\frac{d\upsilon}{dw}\right] - l^{\gamma-1}\frac{dl}{dw} \tag{1.27}$$

From equation (1.21), we know that $\frac{du}{dw} = \frac{l^\gamma}{w}$. If we substitute this relation into equation (1.27) and reorganize the terms we have the equation:

$$\frac{dl}{dw} = \frac{\frac{l^\gamma}{w} - \frac{1}{\rho}\left[w(l^{1-\gamma} - \gamma\upsilon)\right]^{\frac{1-2\rho}{\rho}}\left[(l^{1-\gamma} - \gamma\upsilon) - w\gamma\frac{d\upsilon}{dw}\right]}{\frac{1}{\rho}\left[w(l^{1-\gamma} - \gamma\upsilon)\right]^{\frac{1-2\rho}{\rho}} w(1-\gamma)l^{-\gamma} - l^{\gamma-1}} \tag{1.28}$$

Now we substitute the equation (1.24) into the equation (1.28). Finally we have the system of two differential equations in $l(w)$ and $\upsilon(w)$ that are describe below:

$$\frac{dl}{dw} = \frac{\frac{l^{\gamma}}{w} - \frac{1}{\rho}\left[w(l^{1-\gamma} - \gamma\upsilon)\right]^{\frac{1-2\rho}{\rho}}\left[l^{1-\gamma} + \gamma\left(\upsilon\frac{wf'(w)}{f(w)} + \left[l^{1-\gamma} - \frac{1}{w}\frac{G'(u(w))}{\lambda}\right]\right)\right]}{\frac{1}{\rho}\left[w(l^{1-\gamma} - \gamma\upsilon)\right]^{\frac{1-2\rho}{\rho}} w(1-\gamma)l^{-\gamma} - l^{\gamma-1}} \tag{1.29}$$

$$\frac{d\upsilon}{dw} = -\frac{\upsilon}{w}\left[2 + \frac{wf'(w)}{f(w)}\right] - \frac{1}{w^2}\left[wl^{1-\gamma} - \gamma w\upsilon - \frac{G'(u(w))}{\lambda}\right] \tag{1.30}$$

To solve this system according to Utilitarian and Rawlsian criterions, we have to change the derivate of social welfare condition, $G'(u(w))$. In the Rawlsian criterion, the Lagrange multiplier (λ) is not defined.

Derivation of the equation (1.7):

We can write the equation (1.17) as

$$\left(w + \frac{u_l}{u_c}\right) f(w) = \frac{\varphi_l}{w}\int_w^{\infty}\left[\frac{1}{u_c(\tilde{w})} - \frac{G'(u(\tilde{w}))}{\lambda}\right] f(\tilde{w})d\tilde{w} \tag{1.31}$$

The first order condition of the individuals is $n\,(1-T')\,u_c + u_l = 0$ which implies that $w + u_l/u_c = wT'$. The first order condition can be write as $w = \frac{-u_l}{(1-T')u_c}$. If we substitute this relation into the last relation we have

$$w + \frac{u_l}{u_c} = \left(\frac{-u_l}{u_c}\right)\frac{T'}{(1-T')} \tag{1.32}$$

Now we multiply and divide the the equation (1.31) by the term $(1 - F(w))$. The manipulation of the equation (1.31) we can write

$$\frac{T'}{1-T'} = \frac{-\varphi_l}{u_l}\frac{(1-F(w))}{wf(w)}\int_w^{\infty}\left[1 - \frac{G'(u(\tilde{w}))u_c(\tilde{w})}{\lambda}\right]\frac{u_c(w)}{u_c(\tilde{w})}\left(\frac{f(\tilde{w})}{1-F(w)}\right)d\tilde{w} \tag{1.33}$$

As described by Saez (2001), the term $-\varphi_l/u_l = (1+\zeta^u)/\zeta^c$. Then, finally we derive the equation (1.7)

$$\frac{T'}{1-T'} = \frac{(1+\zeta_w^u)}{\zeta_w^c}\left(\frac{1-F(w)}{wf(w)}\right)u_c(w)\int_w^{\infty}\left[\frac{1}{u_c(\tilde{w})} - \frac{G'(u(\tilde{w}))}{\lambda}\right]\left(\frac{f(\tilde{w})}{1-F(w)}\right)dm \tag{1.34}$$

The equation (1.34) shows that the optimal income taxation is function of density function, accumulative function, compensated and uncompensated elasticities, the social welfare function, the marginal utility of consumption and the parameter of marginal cost of funds, λ.

1.7 Appendix B

In this appendix we include some graphics to help us illustrate ours results and conclusions along the this paper.

Figure 1.10: Earnings Density Function - Brazil: 2006

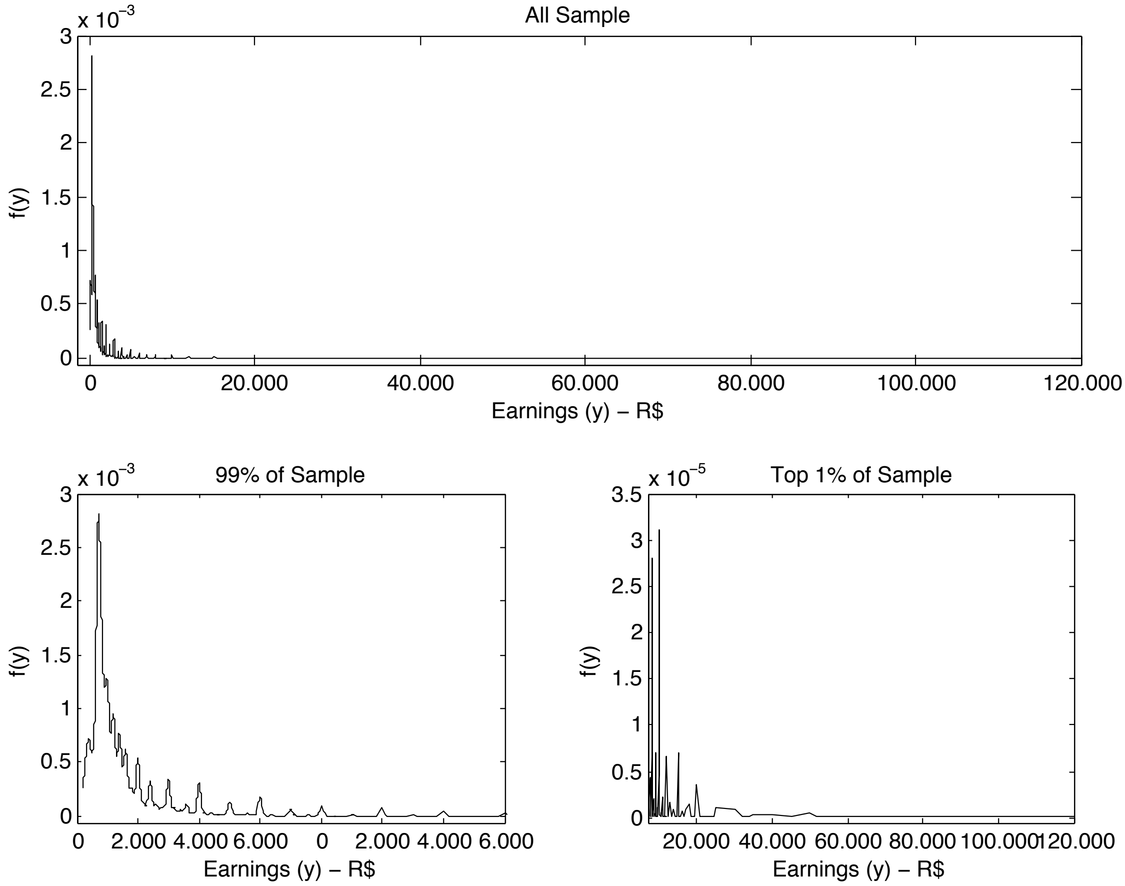


Figure 1.11: Empirical and Approximated Skill Distribution - Model 1

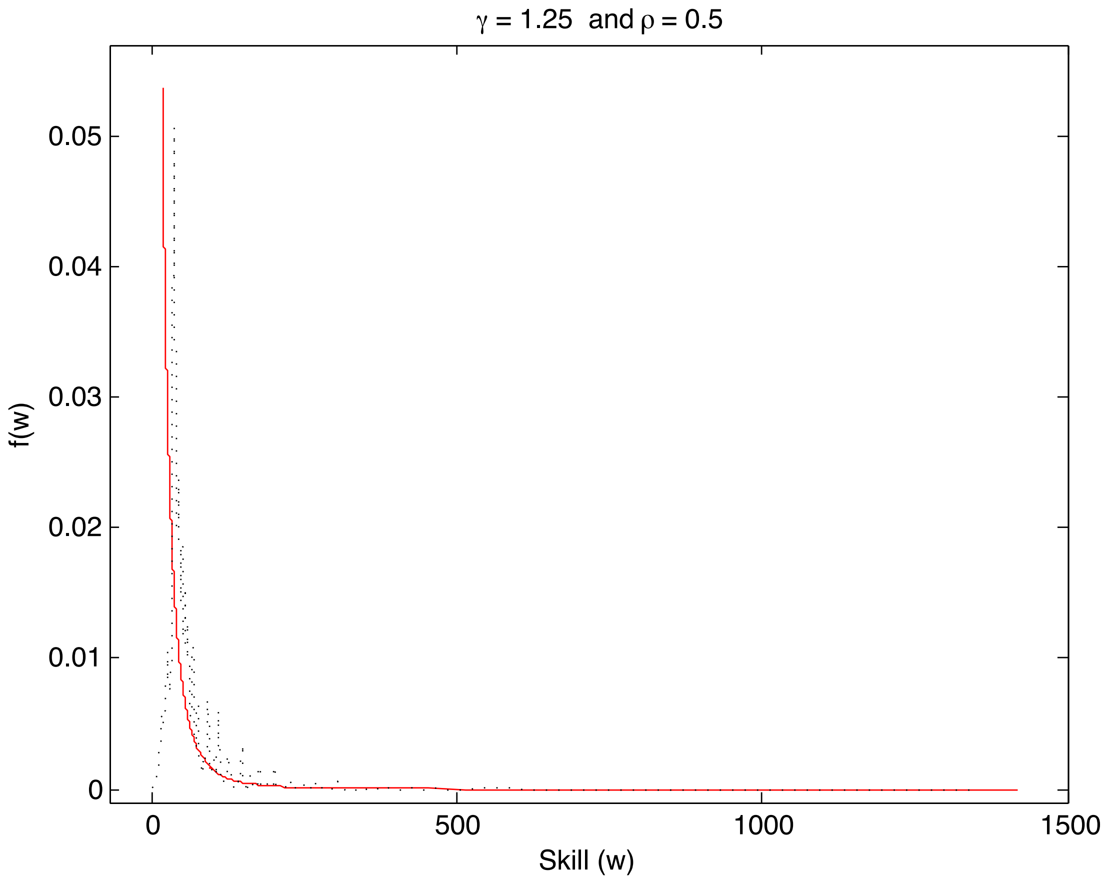


Figure 1.12: Elasticity - 99.9% of Individuals

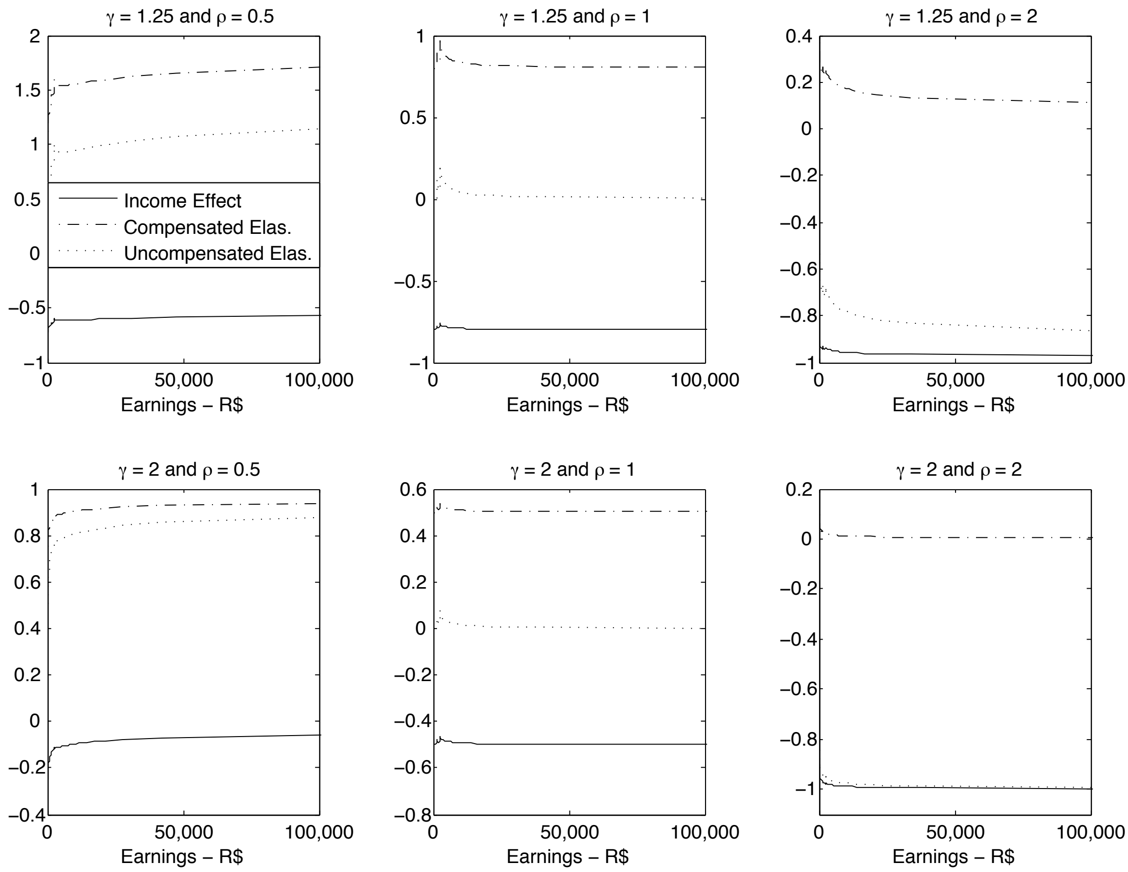


Figure 1.13: Hazard Ratio - 99.9% of Individuals

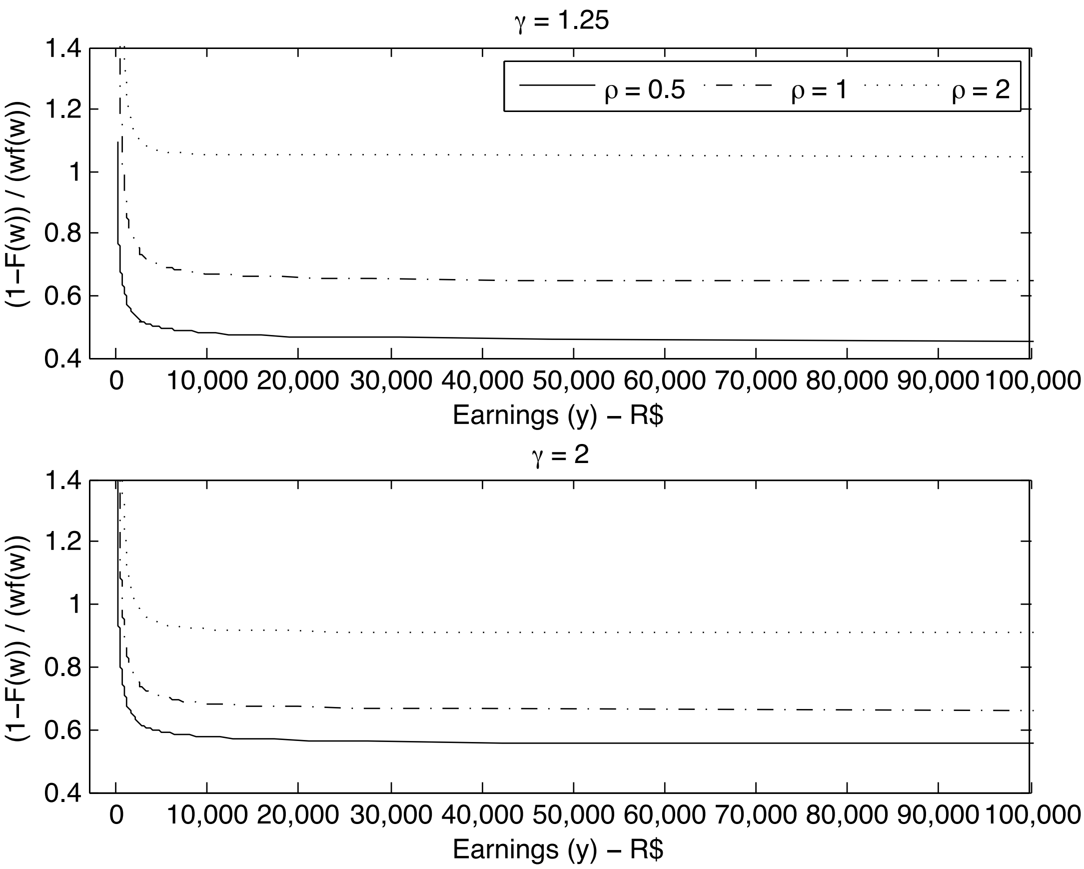


Figure 1.14: Optimal Marginal Rate in Brazil - 99.9% of Individuals

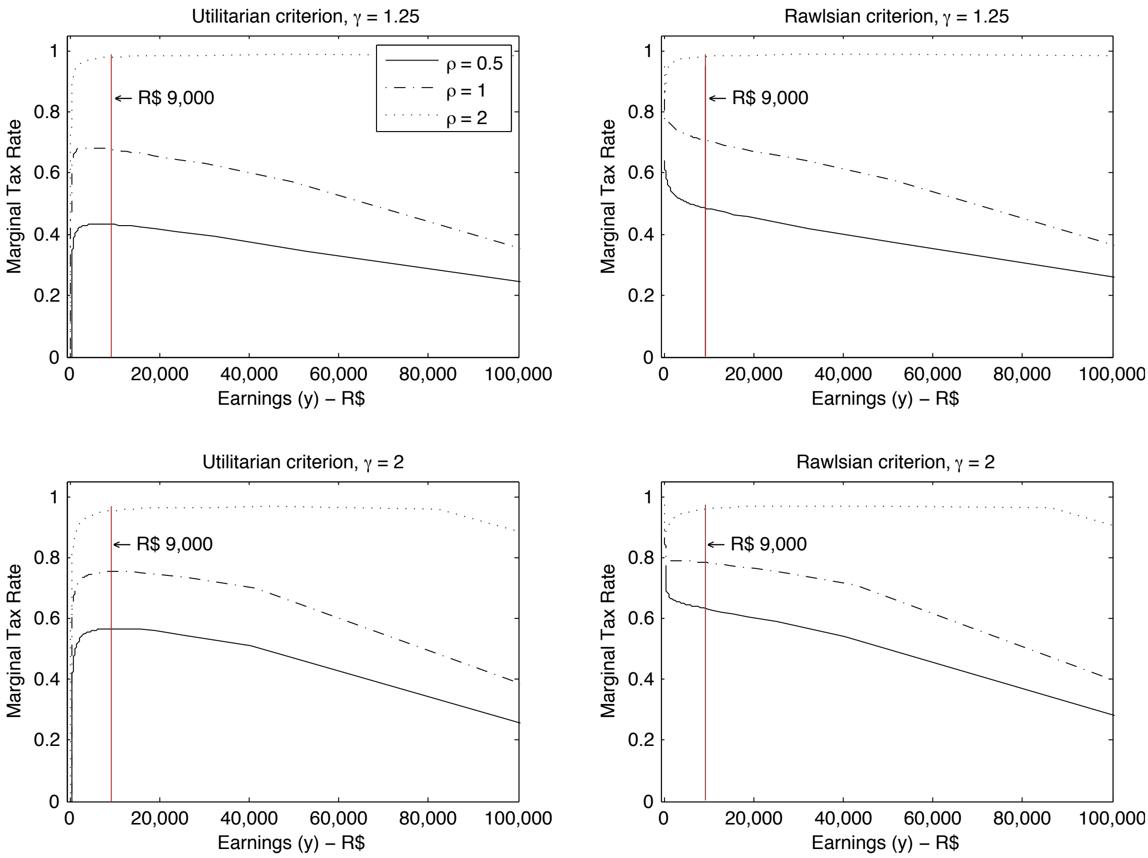


Figure 1.15: Earnings Density Function - United State: 2006

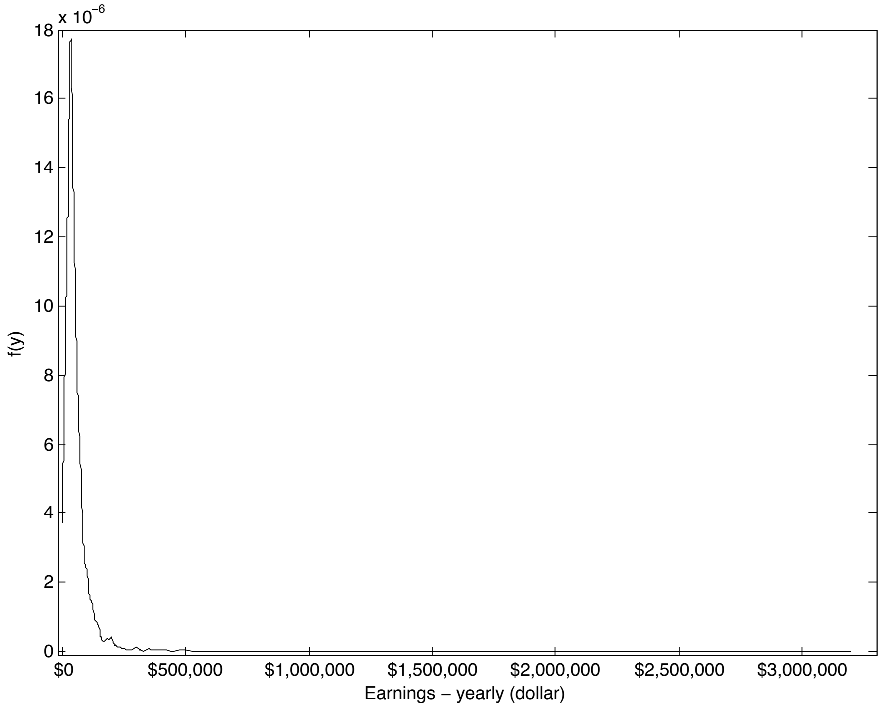


Figure 1.16: Hazard Ratio Function - (1-F(y))/(yf(y)) - United State: 2006

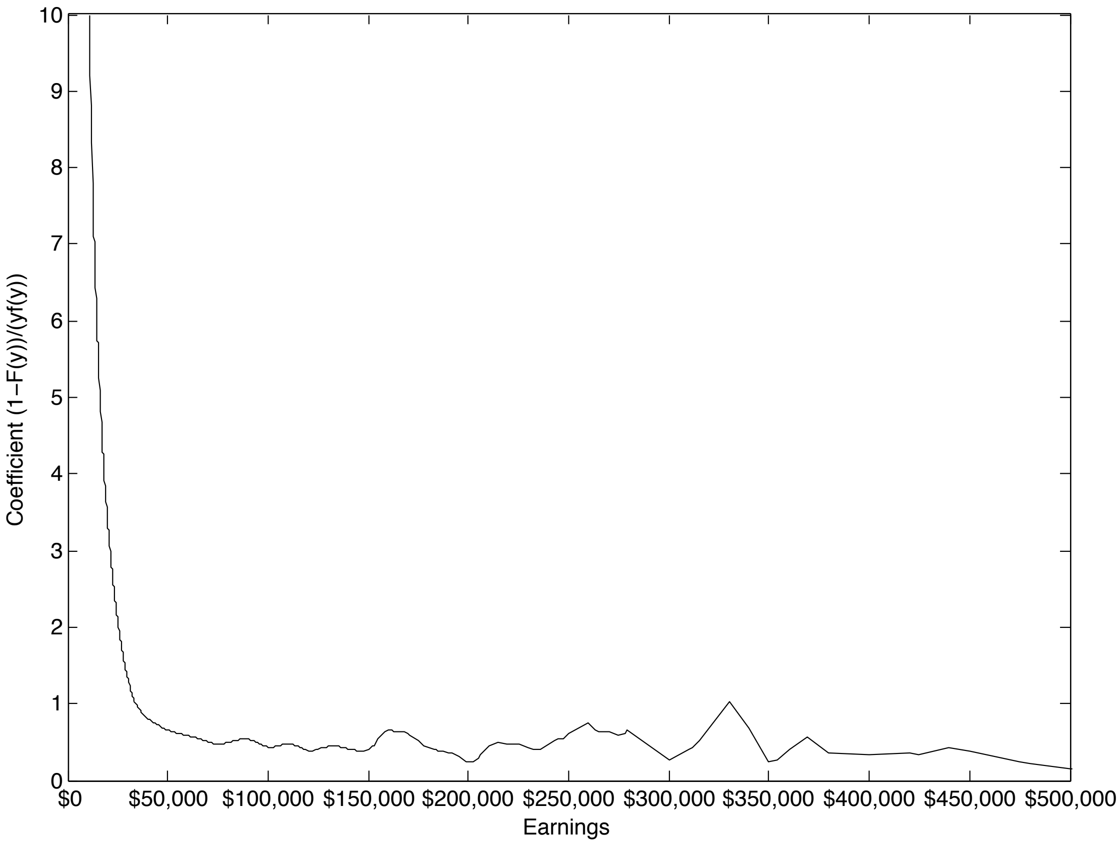
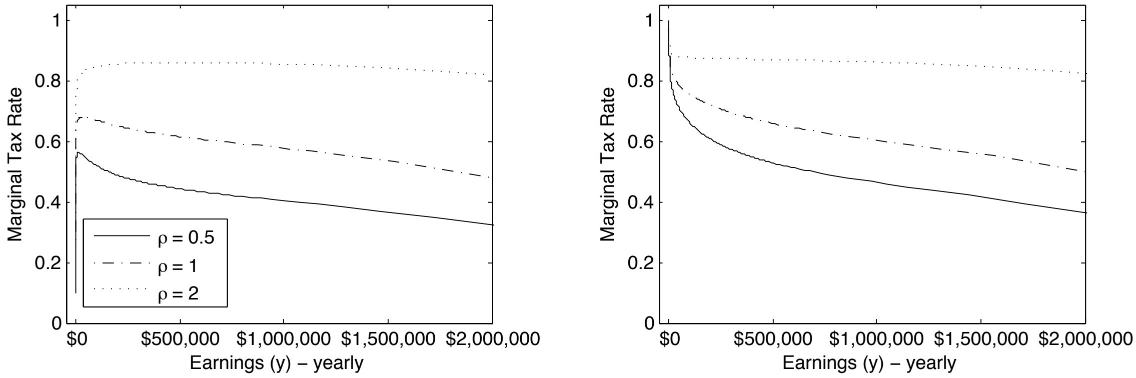


Figure 1.17: Optimal Marginal Rate in US - 99.9% of Individuals



Chapter 2

Sacrifice and Efficiency of the Income Tax Schedule

[1] [2]We investigate the efficiency of labor income tax schedules derived under the equal sacrifice principle. The primitives of our economy are exactly those in Mirrlees (1971): a continuum of individuals with preferences defined over consumption and leisure differ along a single dimension, their labor market productivity. We derive the minimum equal sacrifice allocation, under a separable specification for preferences and recover the tax schedule that implements it. We then use the methodology developed by Werning (2007) to check whether there is an alternative tax schedule that raises more revenue while delivering less utility to noone. For our preferred parametrization of preferences, we find that the equal sacrifice principle induces inefficient allocations when revenue compatible with the US current level of government consumption must be raised. In all interesting cases, inefficiencies arise only at the top of the income distribution.

JEL Classification: H2, D63.

Keywords: Equal Sacrifice, Efficiency.

[1]This is joint work with Carlos Eugênio da Costa.

[2]We thank Luis Braido, Ricardo Cavalcanti, Bev Dahlby and participants at the 2011 PET Meeting for their invaluable comments and retain full responsibility for all remaining errors. Carlos da Costa thanks the hospitality of MIT, and gratefully acknowledges financial support from CNPq. First Version: July, 2010.

2.1 Introduction

Ask any public finance economist who has graduated in the last forty years how to think about income tax schedules and you will get the Mirrlees' approach for an answer. By Mirrlees' approach we mean the optimization of a social welfare function under implementability constraints. Such consensus is, however, new. For a long time it was the ability to pay doctrine and the idea of 'equal sacrifice' that was dominant.[3]

Although formal discussions of tax schedules based on the equal sacrifice principle may be found as early as Samuelson (1947), the concept has experienced renewed academic interest in the 1980's thanks to the work of Peyton Young —Young (1987, 1988, 1990); Richter (1983) — who has shown that many actual tax systems can be approximated by an equal sacrifice schedule.

For a given observed distribution of before and after tax incomes, Young (1990) asks whether one may find a common (and empirically sound) utility function that equalizes the utility loss of all individuals, and such that this loss is minimal to finance the government revenue requirements. He tests and is not able to reject the hypothesis that almost all tax schedules that prevailed in the United States during the period 1957-1987 are based on the equal sacrifice principle. Similar results hold true for Germany, Italy, Japan, and, to a lesser degree, the United Kingdom. Young (1990) work, thus, provides some indirect evidence suggesting that the equal principle may have influenced actual policy making in different moments in time and different places.

A shortcoming of Youngs' works and, for that matter, of all the early literature on equal sacrifice is that it (implicitly) takes taxable income to be independent of the tax schedule. Because incentive effects are not included, efficiency could not be examined. This is unfortunate since Young (1990) himself suggests but cannot explore the possibility that efficiency concerns may underlie the poor fit of equal sacrifice at the high end of the distribution of income. In his words (Young (1990)

[3]John Stewart Mill's words "whatever sacrifices the government requires should be made to bear as nearly as possible with the same pressure upon all" are an early statement of the equal sacrifice principle—see Mill (1844). A competing view of fairness took the form of the benefit doctrine for which one's contribution should be based on one's benefit from the existence of a state.

p.264) "For high incomes, therefore, the departure from equal sacrifice may be due to efficiency considerations while for low income it is probably due to revenue requirements."

In this paper, we use the informational structure in Mirrlees (1971) to investigate efficiency of an equal sacrifice tax system. To be precise, we consider an economy inhabited by a continuum of individuals with identical preferences defined over consumption and work. As in Mirrlees (1971), individuals differ with respect to their labor market productivity, w, which is private information. This informational asymmetry precludes the use of lump sum taxes thus imposing welfare losses. The question we ask is whether under equal sacrifice these costs are minimal.

To a given tax system let $v_1(.)$ be the associated equilibrium utility profile. Here, $v_1(w)$ is the utility attained by an individual with productivity w under this tax system. Assume that the tax system is designed under the equal sacrifice principle. The question we aim at answering is whether there is a Bergson-Samuelson social welfare function $W(v)$, increasing in v, such that this tax system is the one which maximizes $W(v)$. If there is such a function we that the tax schedule is efficient and $W(v)$ rationalizes it. If not, there is no such a function we shall say that the tax system is inefficient.

Our procedure is, of course, equivalent to asking whether there is an alternative tax schedule that generates at least as much revenue and which induces a utility profile $v^*(.)$ such that $v^*(w) \geq v_1(w)$, $\forall w$, with strict inequality for a subset of positive measure of individuals.[4]

The first step toward answering this question is to derive the equal sacrifice schedule for the economy we want to study. We do so by finding incentive compatible allocations which generate a surplus not inferior than the revenue the government needs to raise while imposing the same utility loss on all individuals and such that this loss is minimal. This, which is a natural way of incorporating incentive effects when very general budget sets are allowed, is the same approach used in Berliant and Gouveia (1993)—to the best of our knowledge, the first work to explicitly take into account labor supply incentives in an equal sacrifice based

[4]The first presentation of the problem is the one in Bourguignon and Spadaro (2005) and the second, that of Werning (2007).

tax problem.[5]

Throughout the paper we adopt a separable specification for preferences. This assumption is particularly convenient for the discussion of equal sacrifice tax schedules for it induces a tax schedule with the property that taxable income is invariant to the level of sacrifice. Using this invariance property of labor income we are able to derive the associated tax schedules as a function of the curvature of the term in the utility function associated with consumption only. We then follow the characterization of Pareto efficient tax schedules derived by Werning (2007) to assess the efficiency of the schedule we derived.

Because the shape of efficient tax schedules depends on the underlying distribution os skills while the shape of equal sacrifice schedules does not, it is not hard to construct examples of equal sacrifice schedules that induce constrained inefficient allocations.[6] Although interesting from a theoretical perspective, this latter result is of limited practical value: the actual distribution of types is not a choice variable. We then concentrate our analysis on real world economies. We first show that, if preferences are of the $\ln$ type and if productivities follow a Pareto distribution with a fast enough decay,[7] then there is a level of government above which equal sacrifice leads to Pareto dominated allocations. Using a typical parametrization for the US economy, the level of expenditures at which the tax schedule becomes inefficient is, however, above 50% of GDP.

Next we try different parametrization for preferences, that bring us closer to the findings by Young (1990) regarding the shape of the tax schedule. That is, to rationalize the progressivity of the US income tax schedule using the equal sacrifice

[5]Although Berliant and Gouveia (1993) raise the issue of efficiency, they do not address it formally. Indeed, while declaring that "One of the aspects of the model we still need to clarify are its welfare properties" and suggesting that inefficiency should result since "The condition of a zero marginal tax rate at the top ability level, emphasized in Sadka (1976) and Seade (1977), is not generally satisfied." Berliant and Gouveia (1993) never produce a systematic discussion of the issue.

[6]A trivial example obtains with $\ln$ utility and a distribution of productivity with bounded support. Most recent works in applied optimal taxation, however, depart from the assumption of a bounded support—e.g., Saez (2001)—by viewing this as a rather unrealistic description of the data. As it turns, even with unbounded support, counterexamples of inefficient schedules are always possible if one is allowed to choose the distribution of productivities.

[7]To be precise, a coefficient greater than 2.

principle a coefficient of relative risk aversion greater than one is needed.[8] We derive regions of inefficiency for marginal tax rates for different parametrizations of preferences and levels of Government expenditures and ask whether they may justify the poor fit for high incomes suggested by Young (1990) in the quotation above. For the best specifications for preferences associated with levels of risk aversion aligned with those estimated by Young (1990) we find that inefficiency arises at the high end of the distribution of income.

The rest of the paper is organized as follows. Section 2.2 describes the economy. Implementable allocations are described in Section 2.3. In Section 2.3.1 we derive the shape of equal sacrifice schedules for different parameters of risk aversion. The main results of this paper are found in Sections 2.4 and 2.5. Section 2.6 concludes. The appendix gathers the derivation of some of the main results.

2.2 The Environment

The economy is inhabited by a continuum of measure one of individuals with identical preferences defined over consumption, c, and effort, l. Preferences are represented by

$$U(c,l) = u(c) - h(l), \tag{2.1}$$

where u and h are smooth functions such that $u', -u'', h', h'' > 0$. We also impose $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{l\to\infty} h(l) = \infty$.

Individuals differ from one another along a single dimension: labor market productivity, $w \in W \subset R_+$, where W is a closed convex set. w captures, in this sense, all heterogeneity across individuals. We assume that w is distributed according to $F(w)$ with associated density $f(w) > 0$ for all $w \in W$.

An individual with productivity w that makes effort l produces an output $y = lw$, with y measured in units of the consumption good. Technology is very simple: one efficient unit of effort lw is converted one for one into one unit of consumption. We assume that the economy is competitive so that each individual is paid his or

[8] Young (1990) finds that a coefficient between 1.5 and 1.7 provides the best fit. Our focus on a coefficient of relative risk aversion greater than one is, then, due to its empirical content. Still, for sake of completeness we explore regressive tax schedules induced by equal sacrifice when the coefficient of relative risk aversion is less than one.

her output. We shall, then, refer to y as output and taxable income, interchangeably.

Following Mirrlees (1971), we assume that w is private information. That is, neither w nor l are observed separately, only the product $lw = y$ is observed by all. Our environment is, therefore, exactly that of Mirrlees (1971) with the specialization for the case of separable preferences.

In what follows we focus on choices over (c, y) bundles instead of (c, l) bundles, noting that identical preferences over (c, l) bundles, $U(c, l) = u(c) - h(l)$, induce type dependent preferences, $\tilde{U}(c, y; w) = u(c) - h(y/w)$, over (c, y) bundles.

An allocation is a mapping $(c, y) : W \longmapsto R^2_+$ that associates to each type, w, a consumption/output pair $(c(w), y(w))$. Let $\Gamma(w)$ denote the set of choices (budget sets) available for an agent of productivity w. Each $\Gamma(.)$ induces an allocation through

$$(c(w), y(w)) \in \arg \max_{(c,y)\in\Gamma(w)} \{u(c) - h(y/w)\}$$

for all w.

An example is the allocation that results from $\Gamma(w) = \Gamma^0 \equiv \{(c, y); c \leq y\} \forall w$,

$$(c_0(w), y_0(w)) \equiv \arg \max_{(c,y)\in\Gamma^0} \{u(c) - h(y/w)\},$$

the *no-sacrifice allocation*. We write $v_0(w) = u(c_0(w)) - h(y_0(w)/w)$ to denote the utility attained by type w in this no-sacrifice setting. Under the assumptions adopted, it is not hard to see that $v_0(w)$ is differentiable and $v_0'(w) = h'(y_0(w))y_0(w)/w^2$.

In the economy there is also a government that must extract from the economy an exogenously given level of resources, B,

$$B \leq \int [y(w) - c(w)]f(w)dw. \tag{2.2}$$

To try to and induce an allocation that satisfies (2.2) it is assumed that the government has the power to choose individuals' budget sets, $\Gamma(w)$. In its choice, however, the Government is restricted by the informational structure of the problem. Indeed, asymmetric information regarding w precludes the imposition of type-dependent budget sets. Therefore, $\Gamma(w) = \Gamma$ for all w.

To this unique budget set we associate a tax schedule, i.e., a function, $T(.)$, that maps an individual's output y into his tax obligations $T(y)$, through $T(y) = \min_c \{y - c; (c, y) \in \Gamma\}$.

We next introduce the notion of *sacrifice induced by a tax schedule*, $T(y)$. Let $T : R_+ \to R$ be a tax schedule, as defined above. Note that we do not impose $T(y) > 0$. If we let $v_T(w) \equiv \max_y \{u(y - T(y)) - h(y/w)\}$, it is possible to define the sacrifice, imposed by the tax schedule $T(.)$ on an individual of productivity w, $s(w)$, by

$$s(w) \equiv v(w) - v_T(w),$$

where $v(w)$ is the utility attained by type w individual when the budget set is the one associated with a chosen reference point. The focus of this paper is on tax schedules that induce $s(w)$ constant in w, we call a tax schedule with such property, an *equal sacrifice tax schedule*.

The first step in deriving the equal sacrifice schedule is, then, to find a reference point in the sense of an initial utility level from which sacrifice will be defined. In all that follows we shall take as a reference point the 'no-sacrifice world' for which $\Gamma(w) = \Gamma_0 = \{(c, y); c \leq y\} \forall w$.[9]

2.3 Incentive-compatible equal-sacrifice systems.

The environment we consider is, therefore, exactly that of Mirrlees (1971). Contrary to Mirrlees (1971), we do not consider a social welfare functional that will be maximized by the planner to obtain the tax schedule. Instead, we try to find a tax schedule, $T(.)$, that induces an allocation satisfying (2.2) while imposing an equal utility loss on all agents, and such that this loss is as small as possible.

At this point it is worth noting that U, as defined in (2.1), ought to be viewed as a 'social norm' that will ultimately reflects how society perceives what sacrifice is being made by each individual.

Since our goal is to investigate efficiency of the tax schedule it is crucial that we take into account the behavioral responses to changes in budget sets. It is apparent

[9]Note that in the no-sacrifice world, $T(y) = 0$ for all y. We do not want to interpret this as a no-Government world, for it may be the very presence of a Government that allows individuals to make the best use of their productivity, thus introducing the possibility of differentiated 'benefits' from taxation. We might, in this case, be lead to the contractarian view that underlies the benefit doctrine instead of the ability to pay doctrine to which the equal sacrifice principle is associated. Instead, we think of the no-sacrifice world as a counterfactual one in which all activities necessary to sustain the economic activity are carried on without costs. SeeMusgrave (1985) for an exposition.

from the presentation above that focusing directly on $T(.)$ is bound to create an intractable problem. How do we characterize all tax schedules that induce equal sacrifice across agents?

As it turns, it is very straightforward to attack the problem if we focus directly on implementable allocations, using a truthful direct mechanism. In thus proceeding we follow Berliant and Gouveia (1993), which, in turn, relies on Mirrlees (1971).

Let us, then, describe the direct mechanism associated with the minimum equal sacrifice problem.

The Direct Mechanism By the revelation principle we can focus on a truthful mechanism in which the planner asks each individual his or her type, w, and uses the (possibly false) report $\hat{w}$ to assign a bundle $(c(\hat{w}), y(\hat{w}))$. To guarantee truthful revelation an allocation $(c, y) = (c(w), y(w))_{w \in W}$ must be such that

$$w \in \arg\max_{\hat{w} \in W} \{u(c(\hat{w})) - h(y(\hat{w})/w)\}. \tag{2.3}$$

Define $v_1(w) \equiv u(c_1(w)) - h(y_1(w)/w)$ as the value of the solution to the problem above where we restrict $(c_1(.), y_1(.))$ to be such that $v_0(w) - v_1(w) = s \; \forall w$, and s, to be the minimum sacrifice for which $\int [y(w) - c(w)] f(w)dw \geq B$.

Our procedure will be to characterize $(c_1(w), y_1(w))$ and use it to find $T(y)$. Toward this end note that the global condition of incentive compatibility (2.3) is satisfied if and only if the envelope condition,

$$v'(w) = h' \left(\frac{y(w)}{w} \right) \frac{y(w)}{w^2}, \tag{2.4}$$

and the monotonicity condition,

$$y'(w) \geq 0, \tag{2.5}$$

are satisfied.

Under the assumption that $h(.)$ is strictly increasing and strictly, we have that

$$\frac{y(w)}{w} = \varphi(v'(w)w), \tag{2.6}$$

where φ is a strictly increasing function. Nowhere in this discussion have we used the level of utility, only its variation. This is a very interesting consequence of

separability: under incentive compatibility, the cross-sectional variation in utility pins down the cross-sectional level of output produced by all individuals.

Next, note that equal sacrifice, and differentiability of v_0, imply that v_1 is also differentiable and $v_0'(w) = v_1'(w)$.

Using (2.6) and $v_0'(w) = v_1'(w)$, it is apparent that $y_1(w) = y_0(w)$ for all w. Individuals must produce the exact same output they produce at the reference state![10]

Because everyone makes the same effort and produces the same output as in the reference state, it must be the case that all the sacrifice is due to reduced consumption:

$$\begin{aligned} s &= u(c_0(w)) - u(c_1(w)) \\ &= u(y_0(w)) - u(y_0(w) - T(y_0(w))) \end{aligned} \tag{2.7}$$

Let $\xi(.) = u^{-1}$, then

$$T(y_0(w)) = y_0(w) - \xi(u(y_0(w)) - s). \tag{2.8}$$

2.3.1 The Shape of Equal Sacrifice Tax Schedules

In what is probably the first attempt to relate equal sacrifice and progressivity, Samuelson (1947) has shown that an equal sacrifice schedule is progressive if and only if the coefficient of relative risk aversion of the chosen utility function is greater than one. To arrive at this conclusion Samuelson (1947) disregarded incentives and used a utility function that only depends on income. If preferences depend not only on consumption but also on leisure and incentives are considered this need not hold, in general. Yet, for the special case of separable preferences the result still obtains.

Risk Aversion and Progressivity Equation (2.8) provides an explicit solution for the income tax at income level $y_0(w)$. Differentiating (2.7) with respect to y and

[10] This result was first derived by Berliant and Gouveia (1993) — see their Proposition 4. Figure 2.2 displays an equal sacrifice schedule for iso-elastic preferences along with the optimal choices at the reference point and under the equal sacrifice schedule for individuals of two different productivities: $w = 120$ and $w = 150$. Note how $y_0(w) = y_1(w)$ for both individuals.

rearranging terms we get

$$\frac{u'(y)y}{u'(y-T(y))[y-T(y)]} = \frac{1-T'(y)}{1-\varsigma(y)},$$

where $\varsigma(y) = T(y)/y$ is the average tax rate faced by someone who earns y.

Using the fact that $T(y) \geq 0$, we have that $u'(y)y \geq u'(y-T(y))[y-T(y)]$ if the coefficient of relative risk aversion is greater than one.[11] Since $T'(y) \geq \varsigma(y)$ is necessary and sufficient for a smooth tax schedule to be progressive in the sense of increasing average taxes, one immediately connects risk aversion and progressivity.

Although we have used progressivity to describe a tax schedule for which *average* taxes weakly increase with income, one often takes the term progressivity to refer to increasing *marginal* tax rates. Progressivity in the former sense is a very appealing notion for it has been shown to imply that after tax income is more equally distributed than before tax income,[12] but marginal tax rate progressivity is also of much interest since it is to marginal rather than average tax rates that dead weight burden is associated.

The question is what does equal sacrifice imply for marginal tax rates? What we show next is that the two concepts of progressivity are intertwined in the case of equal sacrifice schedules and constant relative risk aversion preferences for consumption.

Marginal and Average Tax Rates Let $u_0(w) = u(c_0(w))$ and use the fact that utility differences are the same for all w to see that

$$1 - \frac{\xi'(u_0(w)-s)}{\xi'(u_0(w))} = \tau(w),$$

where $\xi'(u)$ is the marginal cost in consumption terms of delivering utility u and $\tau(w) = T'(y(w))$. Note that ξ is an increasing convex function of u which means that $0 < \tau < 1$ for all $s > 0$.

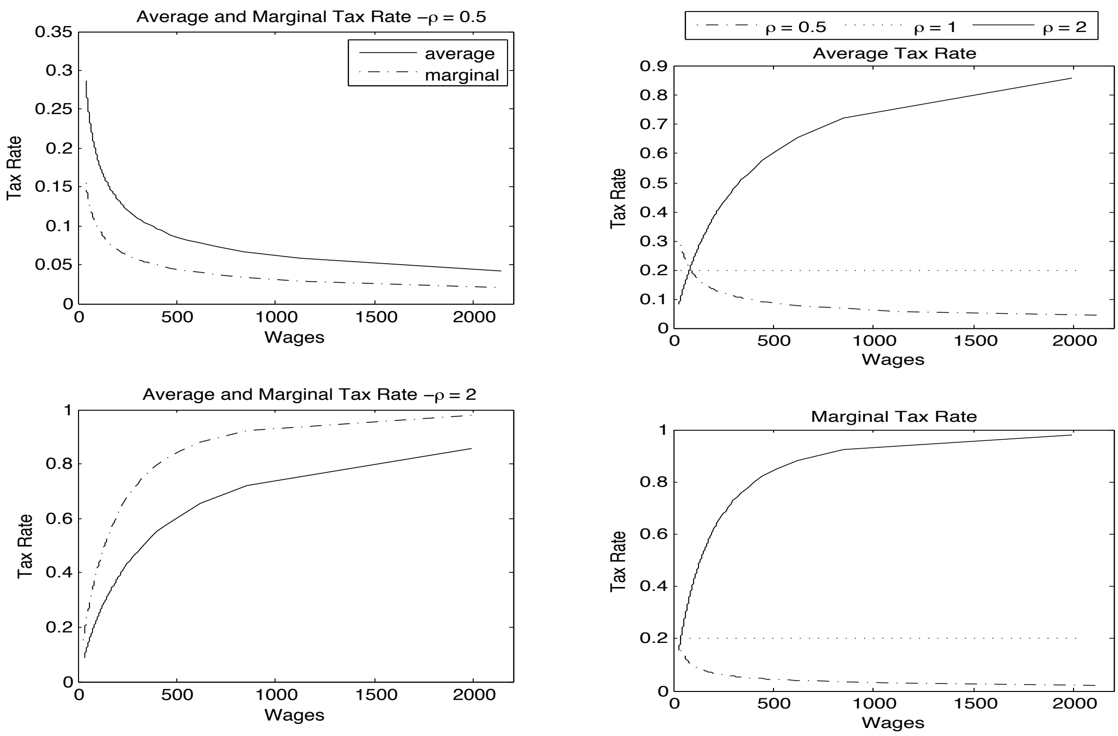
From now on, we focus on the iso-elastic specification for preferences,

$$u(c) = \frac{c^{1-\rho}}{1-\rho},$$

[11] $u'(y)y \geq u'(y-T(y))[y-T(y)]$ if and only if the coefficient of relative risk aversion is greater than one if preferences exhibit constant relative risk aversion.

[12] To compare two distribution of incomes the Lorenz criterion is used.

Figure 2.1: Marginal Tax Rate and Marginal Tax Rate



for $\rho > 0$, $\rho \neq 1$, $u(c) = \ln c$ for $\rho = 1$ and

$$h(l) = l^{\gamma}/\gamma$$

for $\gamma > 1$.[13]

Equal sacrifice in the case of iso-elastic separable preferences implies

$$1 - \varsigma(y_0(w)) = [1 - \tau(w)]^{1/\rho}. \tag{2.9}$$

Equation (2.9) connects in a very stark way average and marginal tax rates. It also shows how any possible ambiguity disappears: equal sacrifice schedules are progressivity in one sense if and only if they are progressivity in the other.

Figure 2.1 displays marginal and average tax rates derived from the use of the equal sacrifice principle for different values of risk aversion. The level of sacrifice

[13] For the iso-elastic case, a clearer characterization obtains if we define $\theta \equiv w^{-\gamma}$ and note that (2.4) may also be written $v'(\theta) = h(y(\theta))$. Hence, given a path for $v(\theta)$ we uniquely define $y(\theta)$. Because there is a one to one relationship between θ and w, from the path for $v(w)$ we recover a unique path for $y(w)$.

is chosen in such a way as to generate total tax revenues of 20 percent of GDP. This first figure makes explicit the fact that progressivity in both senses characterizes tax schedules when $\rho \geq 1$ and regressivity characterize them otherwise. Figure 2.1 also superimposes average and marginal taxes for the two cases. The case $\rho = 1$, i.e., $u(c) = \ln(c)$ yields linear tax rates.

Remarks The fact that taxable income is invariant to the level of sacrifice is somewhat surprising for all but the $\ln$ case.

Take $\rho > 1$, for example. A decrease in net wage induced by an increase in the marginal tax rate would cause an increase in effort if taxes were linear. However, due to the non-linearity of the tax system the linear approximation of an individual's budget constraint, is $c(w) \leq y(w)(1 - \tau(w)) + I(w)$, where $I(w)$, is the virtual income defined as $I(w) = T(y(w)) - T'(y(w))y(w)$.[14] Using (2.9) we may rewrite the expression for virtual income as

$$I(w) = y(w) \left\{ [1 - \tau(w)]^{1/\rho} - [1 - \tau(w)] \right\} .$$

The term in curly brackets is a positive increasing function of $\tau(w)$. The virtual income introduces an additional income effect that combined with the one that arises from the traditional one generated by the decrease in the 'price of leisure' exactly offsets the substitution effect. As a results the optimal effort level is held constant — see Figure 2.2.

A consequence of this invariance property is that the Laffer curve associated with increased sacrifice has no decreasing regions when preferences are separable.

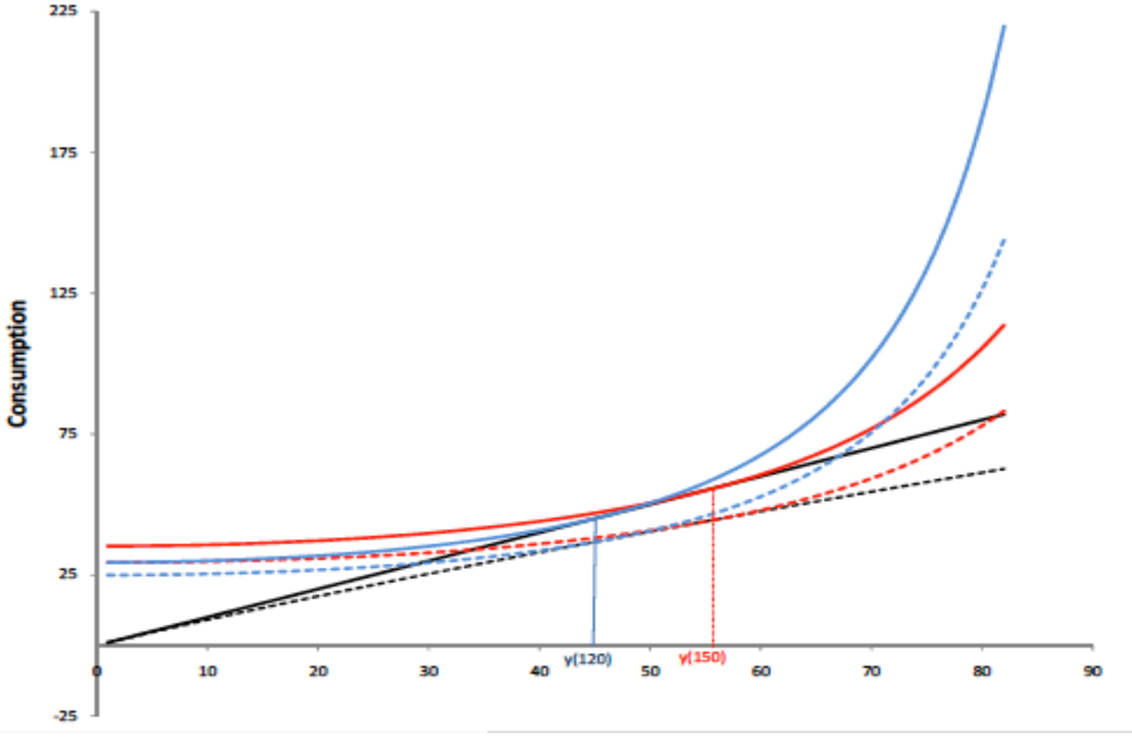
2.4 Sacrifice and Efficiency

The next question we address, the focus of our work, is whether for a tax schedule derived from the use of the equal sacrifice principle is it always the case that we may find a Paretian social welfare function that rationalizes it as the optimal choice?

To formalize our question, let us define an *environment*, $\mathcal{E}$, as a tuple (U, F, B) where $U(c, l) = u(c) - h(l)$ is the utility function representing the (identical across

[14] See Hausman (1985).

Figure 2.2: Invariance Property



agents) preferences of all individuals, $F(w)$ is the distribution of skills and B is the resource requirement by the Government.

Let $\Psi(.)$ be an arbitrary Berson-Samuelson social welfare function, and define the Mirrlees problem at environment $\mathcal{E}$ for the social welfare function Ψ as

$$\max \int \Psi(u(c(w)) - h(y(w)/w))f(w)dw$$

subject to

$$\int \{y(w) - c(w)\} f(w)dw \geq B$$

and

$$w \in \arg\max_{\hat{w}} \{u(c(\hat{w})) - h(y(\hat{w})/w)\}$$

We say that a tax schedule, $T(.)$, is *rationalizable at environment* $\mathcal{E}$ if there is a social welfare function Ψ such that the allocation that solves the Mirrlees problem at environment $\mathcal{E}$ for the social welfare function Ψ is induced by $T(.)$. We then say that *the pair* $(\mathcal{E}, \Psi)$ *rationalizes* $T(.)$.

The question we ask is: given an environment $\mathcal{E}$ and a tax schedule $T(.)$ derived under the equal sacrifice principle, is it always the case that we may find a social welfare function Ψ such that the pair $(\mathcal{E}, \Psi)$ that rationalizes $T(.)$?

Before we tackle this question let us consider a related question. Given a tax schedule $T(.)$ derived under the equal sacrifice principle is it possible to find a pair $(\mathcal{E}, \Psi)$ that rationalizes $T(.)$? Note that for this second question we are given the degree of freedom to choose the enviroment as well as the social welfare function. To answer it, let us consider Proposition 2 in Werning (2007), which states that "For any tax schedule $T(y)$ and its resulting allocation, there is a set of skill distributions $F(w)$ and net endowments $-B$ for which the outcome is Pareto Efficient and another set of skill distributions $F(w)$ and net endowments, $-B$ for which it is Pareto inefficient." This holds for any tax schedule $T(.)$, which means that the answer to our question is: yes, we may always build an environment for which the tax schedule derived under the equal sacrifice principle is rationalizable.

Note, however, that the same proposition yields an answer to a counterpart for our question. Namely, that for any tax schedule it is possible to find an environment for which the tax schedule is not rationalizable. That inefficiency could arise for *some* environment was to be expected. Consider for example a tax schedule

derived under the equal sacrifice principle for iso-elastic preferences and $\rho \geq 1$. We have seen that this implies a progressive marginal tax rate. Now, if F has a compact support, $W = [\underline{w}, \overline{w}]$, then efficiency requires a non-positive marginal tax rate for $\overline{w}$ which is not compatible with $B > 0$.

The result above is useful for highlighting the fact that, because equal sacrifice schedules are independent of the distribution of types, there will always be a distribution of types that makes the induced schedule inefficient for a positive level of sacrifice. This is not, however, the question we are interested in from a policy perspective. What we really want to know is whether an equal sacrifice schedule is inefficient given the actual distribution of productivities for the society we study.

Let us then get back to our original question. More precisely let us consider 'real world' environments. We build these real world environments using the following procedure. We first pick empirically sound preferences and a representation $U(c, l) = u(c) - h(l)$ that captures the social norm we believe to represent those of the societies we study. This choice of preferences suffice to derive $y_0(w) \equiv \arg\max_y \{u(y) - h(y/w)\}$ as well as $v_0(w) = u(y_0(w)) - h(y_0(w)/w)$.

Let $T(.)$ be the actual tax schedule of the economy we are studying. Use $y(w) \equiv \max_y \{u(y - T(y)) - h(y/w)\}$ to recover F through $F(w) = G(y(w))$, where $G(y)$ is the empirical distribution of taxable income.

Next, for each level of sacrifice, s, define the consumption of a type w individual by $c(w) = u^{-1}(s + u(y_0(w)))$ and use $F(w)$ to associate to each level of sacrifice the revenue raised by the Government, $R = \int \{u^{-1}(s + u(y_0(w))) - y_0(w)\} f(w)dw$.

Finally, define the minimum equal sacrifice allocation, along with the associate tax schedule, the by choosing s such that $B = R$. We are now in a position to ask whether, for a tax schedule, $T(.)$, thus derived it is possible to find Ψ such that $(\mathcal{E}, \Psi)$ rationalizes $T(.)$.

The Efficiency Test Two recent works establish methodologies that allow us to answer the question we posed: Werning (2007) and Bourguignon and Spadaro (2005).

The approach developed by Bourguignon and Spadaro (2005) consists in inverting optimal tax formulae that arise from the solution of a Mirrlees' program and to check whether $\Psi'(v) \geq 0$ for all v, i.e., to check whether the social welfare

function is Paretian.

Werning (2007) instead takes the allocation $(c(w), y(w))$ induced by the tax schedule $T(.)$ and the associated function $v(w)$ and solves the problem of minimizing the resource uses subject to delivering no less utility for all (or a positive measure of) individuals. The minimization problem is, naturally, being subject to incentive compatibility constraints as well. If there is an alternative allocation $(\tilde{c}(w), \tilde{y}(w))$ that delivers no less utility for all agents and uses fewer resources, then, the associated tax schedule is inefficient.

As it turns, Werning (2007) procedure is more convenient for our purposes. Hence, we start by replicating—see appendix—his findings to our setting: a smooth tax schedule is efficient if and only if $\tau(w)$ is such that

$$\tau(w) \leq \frac{\gamma}{\Phi(w) - 1}, \ \forall w, \tag{2.10}$$

for $\Phi(w) \geq 1$, with the opposite being needed when $\Phi(w) \leq 1$, where

$$\Phi(w) = (\gamma - 1) \frac{d \ln y}{d \ln w} - \frac{d \ln \tau}{d \ln w} - \frac{d \ln f}{d \ln w}. \tag{2.11}$$

Because taxes are always positive with a positive level of sacrifice, we need only to worry with the inequality in (2.10). It is important to remark that $d \ln y / d \ln w$ is *not* the elasticity of taxable income with respect to w. Instead, it is the cross-sectional derivative of taxable taxable income with respect to w, i.e. the percentage change in taxable income when we compare individuals whose productivities differ by one percent for a given tax structure. The two values will differ under a non-linear schedule since on the cross-section since the virtual income will also differ across individuals. This makes the application of (2.10) quite simple under the separable iso-elastic specification for preferences, since $y(w) = y_0(w) = w^{\gamma/(\gamma+\rho-1)}$ which then implies $d \ln y / d \ln w = \gamma/(\gamma + \rho - 1)$. Note, by contrast, that the optimal tax formulae found in Diamond (1998) or Saez (2001) and used by Bourguignon and Spadaro (2005) use elasticities which are not 'structural' parameters of the problem.

As for $d \ln \tau / d \ln w$, it too refers to cross sectional changes in marginal tax rates. It depends on the difference between average and marginal tax rates, as expression (2.19) in the appendix shows. Using (2.9) it is then possible to show that the

polynomial equation

$$(1-\tau)\left[\gamma - \frac{d\ln f}{d\ln w} - 1\right] - (1-\tau)^{2-1/\rho}\frac{\rho\gamma}{\gamma+\rho-1} \geq -\frac{d\ln f}{d\ln w} + \frac{\rho(\gamma+1)+\gamma-1}{\gamma+\rho-1}.$$

establishes bounds for τ as a function of exogenous parameters only.

This polynomial equation admits a closed form solution for a few cases of interest, e.g., $\rho = 1/2$, $\rho = 2$ and $\rho = 1$. [15]We turn to this latter case, $\rho = 1$, next.

When $u(.) = \ln(.)$, the equal sacrifice principle yields a very simple tax schedule: a linear one.

Before we start the investigation, it is worth making explicit $v(w)$:

$$\begin{aligned} v(w) &= \ln y_0(w)\,[1-\tau] - h(y(w)/w) \\ &= \ln\left(\frac{A(v(\underline{w}))}{A(v_0(\underline{w}))}\right) + \underbrace{\ln y_0(w) - h(y_0(w)/w)}_{v_0(w)}. \end{aligned}$$

Next, assume that the tax system induces a distribution of income that has a Pareto distribution with support $[\underline{y}, \infty)$, $\underline{y} > 0$, and associated density $g(y) = \kappa y^{-\alpha}$, $\alpha > 1$, where $\kappa = (\alpha-1)\underline{y}^{\alpha-1}$ [16]. This is a commonly used specification for the distribution of income, at least for descending part of the distribution—e.g., Saez (2001); Diamond (1998).

To apply the result above, note that

$$\frac{d\ln f}{d\ln w} = \frac{d\ln g(y)}{d\ln y} = -\alpha,$$

since with $u(.) = \ln$, $h(l) = l^\gamma/\gamma$ and linear taxes $y = w$.[17] The Pareto distribution is

$$F(w) = 1 - \left(\frac{\underline{w}}{w}\right)^{(\alpha-1)},$$

for $w \in [\underline{w}, \infty)$.

It is not hard to see that there is a level of government per capita expenditure, $\overline{B} < \int y f(y) dy$, such that the use of the equal sacrifice principle leads to an inefficient tax schedule for all $B \geq \overline{B}$. Indeed, under these assumptions $d\ln y/d\ln w = 1$,

[15] See appendix.

[16] The distribution is $G(y) = 1 - \left(\frac{\underline{y}}{y}\right)^{\alpha-1}$

[17] For $\rho \neq 1$ we use, instead, $F(w) = 1-(\underline{w}/w)^{\varphi-1}$ for $\varphi = [\alpha(\gamma+\rho-1)-(\rho-1)]/\gamma$. This simple calculation is possible for an equal sacrifice schedule when preferences are separable, since $y_1(w) = y_0(w)$.

and $d \ln \tau / d \ln w = 0$, for an equal sacrifice schedule. Therefore, $\Phi(w) = -2 - d \ln f / d \ln w$. Using, our choice for $f(w)$, (2.10) becomes

$$\tau \leq \frac{\gamma}{\alpha + \gamma - 2}, \tag{2.12}$$

as in Werning (2007).

Inequality (2.12) imposes an upper bound on the marginal tax rate when $\alpha > 2$.

Equation (2.12) allows for simple back of the envelope calculations that illustrates actual applications of our procedure. Let us then try to find some numbers to use in equation (2.12). If a government must raise a given value B where $B < \int y dF(y)$,[18] then $B = \int T(y) dF(y) = \tau \int y dF(y)$, which gives the value of τ that we will put to test in (2.12).

With regards to α, Saez (2001) considers the following values: 1.5, 2 and 2.5 for the US economy, whereas Werning (2007) considers $\alpha = 3$. As we have seen, for the first two values, the condition does not have a bite, so we focus on $\alpha = 2.5$.[19] Next, note that $\epsilon = \gamma/(\gamma - 1)$ is the Frisch elasticity of labor supply. In the case $\alpha = 2.5$, taking $\epsilon \to \infty$, the maximum value for τ is close to 70%, and it is 75% for $\epsilon = 2$, for example. If, instead, we use $\alpha = 3$, then, depending on the value of the Frisch elasticity of demand, ϵ the right hand side will vary from 1, when $\epsilon = 0$ to 0.5 when $\epsilon \to \infty$. Most studies consider values for ϵ not greater than 4, in which case, the highest fraction of GDP that can be efficiently financed using an equal sacrifice principle is 55%.

Because transfers must be excluded from this calculation, for inefficiency to result, expenditures as a share of GDP need to be too high, when compared to what one observes in the US. The message seems to be that, if one is to accept that $u(.) = \ln(.)$ reasonably captures how the American society perceives ability to pay, then a tax schedule based on the equal sacrifice principle should be linear and would finance efficiently the current levels of expenditures of the US economy.

[18] We restrict $B < \int y dF(y)$ to guarantee that the result is non-trivial. An issue regarding this type of restriction is that $F(y)$ is the distribution of income induced by the tax system. Still, for the separable specification of preferences we use, $y_1(w) = y_0(w)\ \forall w$, meaning that we may define B before knowing the resulting tax schedule.

[19] Recall that for values of $\alpha < 2$, the Pareto distribution does not have a finite variance.

2.5 Sacrifice and Efficiency in Practice

The discussions in the previous sections were based on a specification for the utility of consumption of the form $u(.) = \ln(.)$. This is an important benchmark since preferences representable by this functional form induce inelastic labor supply, which does seem to adhere reasonably well to the data for prime age males, at least.

Taking taxable income as independent of the tax schedule. In trying to rationalize with the equal sacrifice principle the US tax schedule for most of the period that ranges from 1957 to 1987 Young (1990) has shown that a value for ρ in the range $[1.5, 1.7]$ was needed.[20] Under this parametrization for preferences the equal sacrifice principle induces a progressive tax schedule that provides a better description of the US tax system for that period.

More importantly for our investigation is to realize that ρ defines not only the elasticity of labor supply, but also the social norm of the society we aim at describing. That is, by choosing $\rho = 1$, for example, we commit ourselves not only to a world in which the elasticity of labor supply is zero, but also to a specific view of how society perceives the sacrifice born by different individuals.

If we choose, instead, $\rho \neq 1$, we retain some degree of freedom to explore different social perceptions of equity while still holding the labor supply elasticity fixed at an empirically relevant level. Indeed, because our model is static the parameter γ can be used to provide us with some flexibility to vary the social norm while holding the elasticity of labor supply constant at an empirically relevant range.[21]

When $\rho \neq 1$ a closed form solution for allowable marginal tax rates is only possible in a few cases: e.g., $\rho = 1/2$ and $\rho = 2$, with the latter involving a third degree polynomial. Instead of exploring these two cases we shall use (2.10) directly.

The first step for implementing the test is to back the distribution F up somehow from the data. In Section 2.4 we borrowed the relevant parameters of F from other papers in the literature. In this section we apply the procedure we described in Section 2.4 to the distribution of w, $F(w)$, using the actual distribution of in-

[20] One should however bear in mind that a linear specification is often used as an approximation of the actual system for many purposes–e.g., Saez (2001).

[21] Unfortunately, we do not have full flexibility for disentangling the two since ρ pins down the sign of the elasticity independently of the value of γ. Moreover, if we want to match other specific empirical data, e.g., income (and compensated) elasticities of labor supply, then our degree of freedom is lost.

come $G(y)$ and income tax schedule $T(.)$ under our preferred parametrization of preferences.

The second step, which is to solve for $y_0(w)$, is trivial whereas the first step requires us to take into account the effect of a potentially complex tax system on labor supply.

An assumption that greatly simplifies the procedure is that the tax system may be reasonably approximated by a linear one, $T(y) = \tau y$—e.g., Saez (2001).[22] Under $T(y) = \tau y$, $\forall y$, we recover $F(w)$ from $G(y)$, using $w_\tau(y) = y^{\frac{\rho+\gamma-1}{\gamma}}(1-\tau)^{\frac{1-\rho}{\gamma}}$. This function associates to each output, y, the productivity, w, of an individual who is supplying it, given the approximated tax system. We can recover $F(w)$, hence $G(y_0)$.[23]

Note that, by choosing the linear we are either departing from the assumption that the current system is an equal sacrifice one, or we must restrict ourselves to the $\ln$ specification. Either view is in contrast with what Young (1990) has argued to be the best description of the data for the 1957-1987 period. We shall do so under the implicit assumption that, in contrast with those in the period analyzed by Young (1990), current tax systems, the ones that induce the distribution of income from which our distribution of types is recovered, are not based on the equal sacrifice principle.[24]

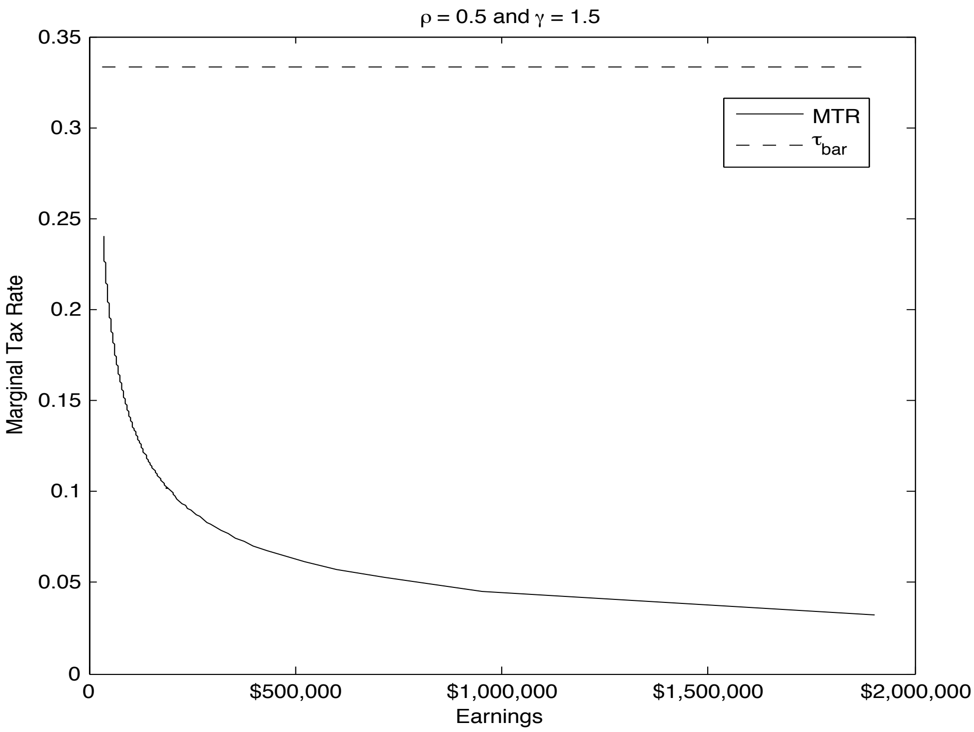
Main results for the US economy To check whether equal sacrifice tax schedules are efficient for the US economy at the current levels of Government expenditures, we first choose a level of expenditures as a percentage of GDP that reasonably describes the pattern we see in the data. Note however that the logic of an equal sacrifice schedule means that we should exclude from our calculations all transfers. We have chosen to set expenditures at 30% of GDP.

[22] da Costa and Pereira (2010) incorporate the non-linearities of the Brazilian tax system in recovering the distribution of types.

[23] For our purposes, of course, one needs not recover $F(w)$ to derive the equal sacrifice distribution of income under our specification for preferences and a linear tax system. Indeed, it is clear that $y_0 = y_\tau(1-\tau)^{\frac{\rho-1}{\rho+\gamma-1}}$, where y_τ is the level of income induced by the tax system $T(y) = \tau y$. In possession of $y(w)$ we need only use (2.8) to derive $T(y)$.

[24] If the empirical schedule is itself an equal sacrifice one, the resulting schedule from our procedure will coincide with the empirical one. This does not eliminate our need to recover F that enters Φ through $d\ln f/d\ln w$.

Figure 2.3: Marginal Taxes Rate and $\bar{\tau}$ - $\rho < 1$



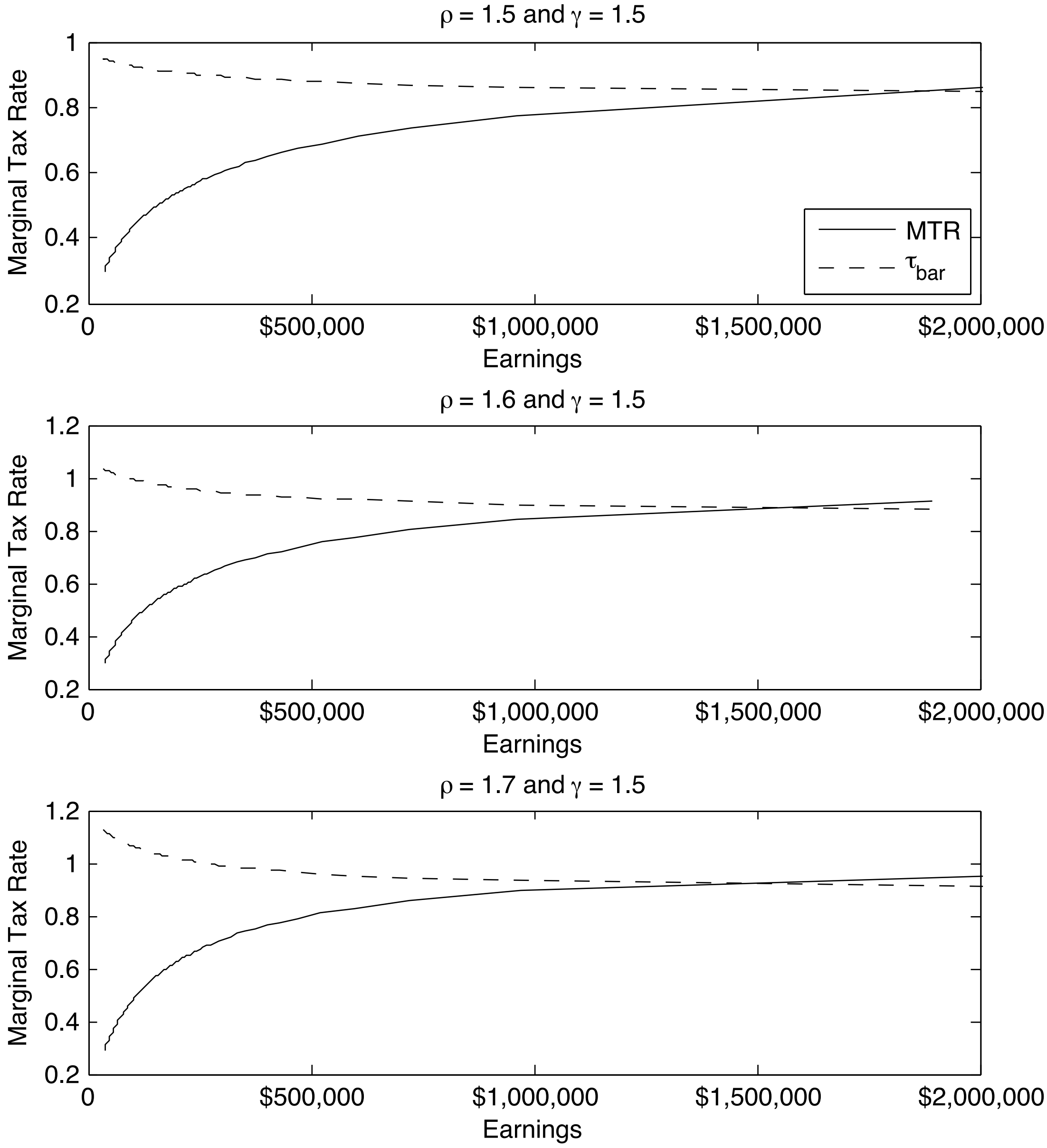
To make our results comparable to the ones found in then optimal taxation literature we follow Saez (2001) and Werning (2007) in considering that earnings, y, has a Pareto distribution, and use $\alpha = 3$. According to Saez (2001), a Pareto distribution fits the US empirical earnings very well from the mode value to the top of distribution. Although the Pareto distribution misses the ascending part of the density, given s, this is unimportant for us. In this region, $\Phi(w) < 1$, and the efficiency condition does not have a bite for an equal sacrifice schedule. In our economy, the poorest individual earns at least $\$34,000$ dollars per year.[25]

Figure 2.3 displays our results for the case $\rho < 1$ and $\gamma = 1.5$. The dashed line is the upper bound and it is given by equation (2.10). The solid line is the marginal tax rate according to equal sacrifice to finance expenditures at 30% of GDP.

Given the level of expenditures as a fraction of GDP, the tax schedule obtained by applying the equal sacrifice principle is Pareto efficient. The condition to be a Pareto efficient tax schedule is that the $\bar{\tau}$ does not cross the tax schedule to anybody.

[25]We collect the data of labor income from Panel Study of Income Dynamic (PSID) in 2007.

Figure 2.4: Marginal Taxes Rate and $\bar{\tau}$ - $\rho > 1$



According to Figure 2.3, the tax schedule would be inefficient if any individual paid a marginal tax rate equal or above 33%. This does not happen for the equal sacrifice schedule at the level of government consumption we study.

Although useful for illustrating the method, the case $\rho < 1$ is uninteresting from the empirical point of view due to the regressivity of the tax schedule. Thus, in all that follows we consider $\rho > 1$.

Figure 2.4 shows the earnings versus marginal tax rate and upper bound tax rate. We vary the risk aversion and keep γ uncharged. We consider the same range to risk aversion parameter than Young (1990) to US economy. Again, the dashed line is upper bound tax rate and and solid line is the marginal tax rate.

In all situations, the tax schedules derived from the use of the equal sacrifice principle is Pareto inefficient. As the tax schedule is progressive, the inefficiency occurs on the top of distribution. It is easy to observe that when we increase ρ the inefficient earning value is dislocated gradually from the top to the center of distribution.

An increase in risk aversion, affects the possibility of finding regions of inefficiencies through both changes in the marginal tax rate and through changes in $\bar{\tau}$. In fact, beyond the dependency of $\Phi(w)$ on the marginal tax rates through $d \ln \tau / d \ln w$, two other terms are affected by ρ, $d \ln y / d \ln w$ and $d \ln f / d \ln w$. The reason for the former is apparent. As for the latter, note that $F(w)$ is backed up from the data under a specific assumption about U. If we change our assumption about U we must redefine the whole environment to make it internally consistent.

In the case $\rho = 1.5$, $\bar{\tau}$ is around 95% at the bottom and 84% at the top of distribution. For $\rho = 1.6$ these numbers are 103% and 86%, respectively, while for $\rho = 1.7$, $\bar{\tau}$ is around 113% at the bottom and 91% at the top. We can see the impact of increasing risk aversion into marginal tax rate and $\bar{\tau}$ throughout equations (2.17) and (2.10).

To summarize, using parameters that approximate those for the US economy, we find that, to finance the expenditures at 30% of GDP, the tax schedules derived from the equal sacrifice principle is Pareto inefficient for ρ defined in the range studied by Young (1990). Moreover, inefficiency arises at the high end of the distribution of skills.

2.6 Conclusion

In a series of papers in the late 1980's Young (1987, 1988, 1990) has forcefully argued that the income tax schedule for the US for the period from 1957 to 1987 could be rationalized by direct applications of the equal sacrifice principle. The body of work that followed allows one to pin down the restrictions imposed on observed tax schedules by the equal sacrifice system—Mitra and Ok (1996); Ok (1995)—and to understand the consequences of taking incentives into account explicitly—Berliant and Gouveia (1993).

We use a separable iso-elastic specification for preferences to derive a tax system built using the principle of equal-sacrifice. The use of a separable specification for preferences greatly facilitates the characterization of the shape of equal sacrifice schedules. It, thus, allows for an explicit evaluation of efficiency of such schedules for it lends itself to efficiency tests based on the methodology developed by Werning (2007).[26]

We find that if utility of consumption is logarithmic and the cross-sectional distribution of productivities is Pareto with a decay parameter above 3, there is always a level of per capita government spending above which an equal sacrifice tax schedule is inefficient. Back of the envelope calculations indicate that these threshold values are much higher than the average expenditures for the United States for normal times.

When the parameter of risk aversion is greater than one a progressive income tax schedule results from the equal sacrifice principle. For most parametrizations we have used equal sacrifice schedules only become inefficient for very high levels of income. Whether this explains the poor fitting of equal sacrifice schedules at the very top of the income distribution suggested by Young (1990) is debatable. It is however suggestive that *if* the idea of equal sacrifice has really influenced the design of the US schedule, efficiency concerns may have imposed limits on top marginal tax rates.

[26]Werning (2007) offers an interesting interpretation of these efficiency tests—see, p. 9. He shows that bounds of efficiency may be found by comparing the income distribution induced by the equal sacrifice system to that induced by an optimal Rawlsian tax schedule. That is, one calculates the optimal Rawlsian tax schedule for the relevant level of government expenditures, and check whether the equal sacrifice income distribution is first order stochastically dominated by the one induced by the Rawlsian tax schedule or its multiples.

2.7 Appendix

Deriving Expression (2.10)

In this appendix we provide, for sake of completeness, a sketch of the proof of necessity for efficiency condition (2.10). A complete proof of both necessity and sufficiency is found in Werning (2007).

An allocation $(\bar{c}(w), \bar{y}(w))$ that generates a utility profile $\bar{v}(w)$ is Pareto efficient if and only if $(\bar{y}(w), \bar{v}(w))$ solves:

$$\max_{y(.),v(.)} \int [y(w) - e(v(w), y(w), w)]\, f(w)dw$$

s.t.,

$$v'(w) = h'\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^2},$$

$$y(w) \text{ increasing,}$$

and

$$v(w) \geq \bar{v}(w)\, \forall w.$$

Disregarding the monotonicity constraint, we may write the Lagrangian

$$\int \left\{ [y(w) - e(v(w), y(w), w)]\, f(w) + \mu(w)\left[v'(w) - \frac{y(w)^\gamma}{w^{\gamma+1}}\right] + \lambda(w)\left[v(w) - \bar{v}(w)\right] \right\} dw,$$

where

$$v(w) = u\left(e(v(w), y(w), w)\right) - \frac{y(w)^\gamma}{\gamma w^\gamma}$$

Integrating by parts and usign the transversality conditions, we re-write the Lagrangian as

$$\int \Bigg\{ [y(w) - e(v(w), y(w), w)]\, f(w) - \mu'(w)v(w) - \mu(w)\frac{y(w)^\gamma}{w^{\gamma+1}} \\ + \lambda(w)\left[v(w) - \bar{v}(w)\right] dw \Bigg\}$$

First order conditions are

$$\left(1 - e_y(v(w), y(w), w)\right) f(w) = \mu(w)\gamma\frac{y(w)^{\gamma-1}}{w^{\gamma+1}} \tag{2.13}$$

and

$$-e_v(v(w), y(w), w)f(w) = \mu'(w) - \lambda(w) \tag{2.14}$$

which implies,

$$-e_v(v(w), y(w), w)f(w) \leq \mu'(w) \tag{2.15}$$

Focusing on the case $1 - e_y = \tau > 0$, we have that $\mu > 0$ as well and (2.13) can be written in logs

$$\ln \tau + \ln f = \ln \mu + \ln \gamma + (\gamma - 1) \ln y - (\gamma + 1) \ln w$$

which implies

$$\frac{d \ln \tau}{d \ln w} + \frac{d \ln f}{d \ln w} = \frac{d \ln \mu}{d \ln w} + (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1)$$

Next note that

$$\frac{d \ln \mu}{d \ln w} = \frac{\mu'}{\mu} w \geq -\frac{e_v f}{\mu} w = -\frac{\gamma y^{\gamma-1} e_v f w}{(1 - e_y) f w^{\gamma+1}} = -\frac{\gamma}{\tau} e_v \frac{y^{\gamma-1}}{w^\gamma} = -\frac{\gamma}{\tau}(1 - \tau)$$

Hence,

$$\frac{d \ln \tau}{d \ln w} + \frac{d \ln f}{d \ln w} \geq -\frac{\gamma}{\tau}(1 - \tau) + (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1). \tag{2.16}$$

That is, let

$$\Phi(w) = (\gamma - 1) \frac{d \ln y}{d \ln w} - \frac{d \ln \tau}{d \ln w} - \frac{d \ln f}{d \ln w},$$

then we require

$$\tau(w) \leq \frac{\gamma}{\Phi(w) - 1},$$

if $\Phi(w) > 1$, and

$$\tau(w) \geq \frac{\gamma}{\Phi(w) - 1},$$

if $\Phi(w) < 1$.

Condition (2.10) for the equal sacrifice schedule.

We now derive a polynomial equation that defines ranges of efficiency for the marginal tax rate. Assume that the tax function, $T(.)$, is twice continuously differentiable. Differentiating (2.7) and rearranging terms yields

$$1 - \frac{u'(y_0(w))}{u'(y_0(w) - T(y_0(w)))} = T'(y_0(w)). \tag{2.17}$$

That is, the marginal tax rate faced by any individual is (one minus) the ratio of his or her marginal utility of income before and after the introduction of taxes. Next, differentiate (2.17) to obtain

$$\begin{aligned}\frac{T''(y_0(w))}{1-T'(y_0(w))} &= \left\{\frac{u''(y_0(w)-T(y_0(w)))}{u'(y_0(w)-T(y_0(w)))}\left[1-T'(y_0(w))\right]-\frac{u''(y_0(w))}{u'(y_0(w))}\right\} \\ &= \frac{1}{y_0(w)}\left\{r(c_0(w))-r(c_1(w))\frac{1-T'(y_0(w))}{1-\varsigma(y_0(w))}\right\}, \qquad (2.18)\end{aligned}$$

where $\varsigma(y) = T(y)/y$ is the average tax rate and $r(c)$ is the coefficient of relative risk aversion at consumption level c.

For the case of CRRA preferences, $r(c) = \rho$ for all c, and expression (2.18) reduces to

$$-\left.\frac{d\ln(1-T'(y))}{d\ln y}\right|_{y=y_0(w)} = \rho\left\{1-\frac{1-T'(y_0(w))}{1-\varsigma(y_0(w))}\right\}.$$

Next, re-write the expression above as

$$\rho\left\{\frac{\tau(w)-\varsigma(y_0(w))}{1-\varsigma(y_0(w))}\right\}\frac{d\ln y_0(w)}{d\ln w} = \frac{d\ln\tau(w)}{d\ln w}\frac{\tau(w)}{1-\tau(w)}. \qquad (2.19)$$

Noting that $\varsigma(y_0(w)) = 1-[1-\tau(w)]^{1/\rho}$ and $d\ln y_0(w)/d\ln w = \gamma/(\gamma+\rho-1)$, we get

$$\rho\left\{1-(1-\tau(w))^{1-1/\rho}\right\}\frac{\gamma}{\gamma+\rho-1} = \frac{d\ln\tau(w)}{d\ln w}\frac{\tau(w)}{1-\tau(w)}. \qquad (2.20)$$

Next, to simplify the algebra let $a = \frac{d\ln y}{d\ln w}$ and $b = \frac{d\ln f}{d\ln w}$. Then note that

$$\left\{1-\tau(w)-(1-\tau(w))^{2-1/\rho}\right\}\frac{\rho a}{\tau(w)} = \frac{d\ln\tau(w)}{d\ln w}$$

Using the expression above in (2.16) we get

$$\left\{1-\tau-(1-\tau)^{2-1/\rho}\right\}\frac{\rho a}{\tau}+b \geq -\frac{\gamma}{\tau}(1-\tau)+(\gamma-1)\,a-(\gamma+1) \qquad (2.21)$$

Assuming that $\tau > 0$,

$$\left\{1-\tau-(1-\tau)^{2-1/\rho}\right\}\rho a \geq -\gamma(1-\tau)-\left[b-(\gamma-1)\,a+(\gamma+1)\right]\tau,$$

which we may rewrite as

$$(1-\tau)\left[(\rho+\gamma-1)\,a-b-1\right]-(1-\tau)^{2-1/\rho}\rho a \geq -\left[b-(\gamma-1)\,a+(\gamma+1)\right].$$

Recalling that $a = \gamma/(\gamma + \rho - 1)$,

$$(1-\tau)\left[\gamma - \frac{d\ln f}{d\ln w} - 1\right] - (1-\tau)^{2-1/\rho}\frac{\rho\gamma}{\gamma+\rho-1} \geq -\frac{d\ln f}{d\ln w} + \frac{(\rho-1)(\gamma+1)}{\gamma+\rho-1}. \quad (2.22)$$

The expression above defines a condition which is very easy to check. More interestingly, for some choices of ρ, analytical solutions for the roots of (2.22) are available.

Indeed, let $\rho = 1/2$, then

$$1-\tau \geq \left\{-\frac{d\ln f}{d\ln w} + \frac{(\rho-1)(\gamma+1)+\gamma\rho}{\gamma+\rho-1}\right\}\left\{\gamma - \frac{d\ln f}{d\ln w} - 1\right\}^{-1}.$$

When $\rho = 2$, define $z = (1-\tau)^{1/2}$, and we may write (2.22) as

$$z^2\left[\gamma - \frac{d\ln f}{d\ln w} - 1\right] - z^3\frac{\rho\gamma}{\gamma+\rho-1} \geq -\frac{d\ln f}{d\ln w} + \frac{(\rho-1)(\gamma+1)}{\gamma+\rho-1},$$

a cubic equation, which has also closed form solutions for its roots.

Chapter 3

Moving to a Consumption-Tax Based System: A Quantitative Assessment for Brazil

[1]For many years, it has been a primary issue in tax policy whether the tax system ought to be built around income tax or consumption tax. Much of the interest in tax policy arises from the widespread belief that taxes on income and savings tend to lower long-run income by retarding the creation and expansion of firms and by discouraging workers and investments. Following this belief, Brazilian government has proposed a tax reform which, basically, replaces tax on investment and labor with tax on consumption. In this paper, we develop a dynamic general equilibrium model with heterogeneous agents to guide our quantitative assessment of the economic and distributional implications of such tax reform. The model is calibrated in such a way that it matches some selected features of the Brazilian economy. We also use the calibrated model to calculate the deadweight loss of each type of taxation and thus provide some rationality for that rearrangement in the tax system. The main result of the paper is that, even though the tax reform increases the asset accumulation, labor and output of economy, it also raises the welfare inequality as borrowing constrained individuals cannot take advantage of

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the drop in tax on savings.
JEL classification: E2, H2, H3
Key words: Tax Reform, Welfare Distribution, Deadweight Loss, Brazil

3.1 Introduction

Brazilian economy has improved significantly in many dimensions over the last decade. Inflation continues to be under control and inflation expectations remain low. Brazil's external position is solid, with a strong current account surplus—about 2 percent of GDP in 2007—and international reserves around $201 billion, equivalent to more than 200 percent of its short-term debt. The fiscal adjustment since the floating of the Real in 1999 has been impressive, allowing the consolidation of macroeconomic stability in the period and ensuring that the dynamics of public indebtedness is sustainable.

Nevertheless, the fiscal adjustment has taken place on the back of revenue hikes and, to a lesser extent, a compression of public investment, rather than the retrenchment of current outlays. As a result, Brazil's tax-to-GDP ratio has increased almost ten percentage points over the last two decades and now it is much higher than that of countries with comparable income levels, being close to the OECD average and nearly twice as high as that of the rest of Latin America, see de Mello (2008).

This increase in the tax burden in Brazil has been pointed out as an important explanation for the unsatisfactory performance of the economy in terms of growth. Much of the interest in tax policy arises from the widespread belief that taxes on income and savings tend to lower long-run income by retarding the creation and expansion of firms and by discouraging workers and investment. The empirical evidence that come from, for example, Helms (1985), Mofidi and Stone (1990) and Slemrod (1999) suggests a significant effect of taxation on the economic performance, particularly over the long term.

Following this belief, Brazilian's government has submitted a constitutional amendment to the examination of the congress, which, among others changes, proposes a significant reduction of tax on investment and on labor income. However, the proposal will not change the tax burden in terms of GDP since it contains a

provision which ensures the neutrality of the tax reform. Thus, the government's proposal ultimately entails a rearrangement of the current tax system in such way that it cuts taxes on investment and labor and offsets it by increasing tax on consumption.

The goal of the present analysis is to quantitatively assess the economic and distributional implications of this rearrangement in the tax system. To guide our assessment, we develop a dynamic general equilibrium model with overlapping generations in line with Auerbach and Kotlikoff (1987) and Rios-Rull (1996). The model economy considered in this paper is populated by a large number of agents who have preferences over consumption and leisure and face idiosyncratic productivity shocks. Individuals cannot insure directly against these shocks, but they can trade an asset subject to an exogenous lower bound on asset holdings. Following Aiyagari (1994), this asset takes the form of capital. Thus, savings may be precautionary and allow partial insurance against the idiosyncratic shocks. Because of the lack of full insurance, this model generates an endogenous distribution of wealth across consumers. Indeed, at any point in time, households differ in their current shock, in their asset holdings (that somehow summarizes all their past luck) as well as in terms of their age. We calibrate the model in such a way it matches selected statistics of the Brazilian economy and then use the calibrated model to study many issues related with the tax reform proposed by the government.

First, we calculate the welfare cost due to each type of taxation. This is done by calculating the deadweight loss of taxes of raising extra revenues from an already existing distorting tax. An extensive literature has estimated the deadweight loss and it now is understood as being an important guide to design the fiscal policy.[2] Overall, however, this literature has focused on portions of the tax system such as tax on capital and labor income or its estimates are based on data from the U.S. economy. The model used in this paper, in contrast, is calibrated using data from Brazilian economy and, besides tax on capital and labor income, it takes into account tax on consumption and investment. As a consequence, the framework in this paper can be used to assess, for example, how much distortionary the tax on

[2]See, for example, Browning (1987), Hansson (1984), Ballard et al. (1985), Judd (1998), McGrattan (1994) and Feldstein (1997).

investment is and whether or not government should move towards consumption taxation in order to reduce the inefficiency associated with the tax system.

Second, given that individuals in our model face uninsured idiosyncratic shocks of productivity, wealth accumulation in this class of models is higher than when there is no uncertainty,[3] as individuals react to the uncertainty regarding to their future labor income by reducing current consumption and increase savings.[4] As it will turn out, this over accumulation may play an important role in determining the welfare cost of taxes.

Furthermore, we measure the long-run impact of the tax reform on the aggregate variables of the model economy such as output, capital accumulation, hours worked in order to document the potential economic gains that can be obtained with the reduction of the distortions caused by the taxation of capital and labor. This assessment is important because proponents of the reform argue that its positive effects on those variables are expected to be of considerable magnitude and, as a consequence, the shift towards a consumption-based tax system is viewed as highly desirable.

Finally, by calibrating the income distribution in the model in such a way that it matches the income inequality as measured by Gini index, our framework is also useful to address the distributional implications of the tax reform, an issue that has not been fully analyzed yet. The assessment of the distributional effects of the type of reform proposed by the government is especially important in Brazil because tax on consumption is strongly regressive. This is so because individuals at low levels of the income distribution spend the most part of their income on consumption and many of them do not save at all, whereas those at high levels of the income distribution save the most part of their income. A tax reform which, for example, replaces tax on investment by tax on consumption improves the situation of the richer individuals and may worsen the situation of the poorer ones. This latter effect is due to the fact that, even though that reform may increase the per capita income of economy, the increase in tax on consumption may be even higher, and thereby it can negatively impact on the welfare of the poorer individuals. Therefore, the tax reform can adversely affect the cross-sectional welfare distribution.

[3]See, for example, Kimball (1990), Deaton (1991) and Carroll (1997).

[4]As long as utility function is increasing, concave and has a positive third derivative.

Feenberg et al. (1997), using data from the U.S. tax system, carry out an analysis of distributional tables to investigate the effects of replacing tax on income with tax on consumption on the welfare distribution. Even though they conclude it could increase the welfare inequality, their approach faces with many shortcomings associated with the absence of general equilibrium feedback effects, which may play an important role in the assessment of the distributional impacts of the tax reform. This paper, in contrast, uses a dynamic general equilibrium model, and thereby we are able to take into account, for example, the response of individuals to the changes in the tax system.

Pereira and Ferreira (2010) have also investigated the economic impacts of changes in the tax system in Brazil. They rely on the representative-agent abstraction to evaluate how the tax reform affect the output, capital, labor and individuals welfare. However, given that their model economy behaves as if it is inhabited by a single type of consumer, the framework used in that paper is not suitable for addressing the distributional implications of the reform, which is a major point in the present analysis. Moreover, they implicitly assume that there are complete insurance markets for consumers' idiosyncratic risks, which is an assumption hard to defend (Aiyagari (1994).

Besides this introduction, this paper is organized as following. In section 2, we present the model which will be used to guide our quantitative assessment. In section 3, the data and calibration procedures are described. In section 4, we discuss the main results. Section 5 concludes.

3.2 The model

3.2.1 Demography

The economy is populated by a continuous of mass one agents that can live to a maximum of T periods. There is uncertainty regarding the time of death, each individual faces a probability ψ_t of dying in the age t. The age profile of the population $\{\mu_t\}_{t=1}^{T}$ is modeled assuming that the fraction of agents at the age t in the population is given by $\mu_t = \frac{\psi_t}{(1+g_n)}\mu_{t-1}$ and $\sum_{t=1}^{T}\mu_t = 1$, where g_n denotes the population growth rate.

3.2.2 Preferences

Each individual maximizes the discounted expected utility throughout life:

$$E\left[\sum_{t=1}^{T}\beta^{t-1}\left(\prod_{k=1}^{t}\psi_k\right)u(c_t+\sigma G_t, 1-h_t)\right] \tag{3.1}$$

where β is the discount factor and E is the expectation operator. The utility function of each period is assumed to take the form:

$$u(c_t, 1-h_t) = \frac{\left[(c_t+\sigma G_t)^{1-\nu}(1-h_t)^{\nu}\right]^{1-\rho}}{1-\rho} \tag{3.2}$$

where ρ denotes the risk aversion parameter and ν denotes share of leisure in the utility.

The specification in equation (3.2) assumes that, besides consumption c_t and leisure $(1-h_t)$, the household's utility is also influenced by the government expenditures if $\sigma \neq 0$. If $\sigma < 0$, the marginal utility of consumption increases with an increase in G and if $\sigma > 0$, the opposite meaning is true. Thus, our framework allow for substitutability or complementarity between public and private goods.

3.2.3 Technology

The technology of production of the economy is given by a Cobb-Douglas function with constant returns to scale, $Y_t = BK_t^{\alpha}(A_tH_t)^{1-\alpha}$ where α is the capital share and Y, K, H and $B > 0$ denote aggregate output, capital, labor and a scale parameter, respectively. The variable A_t denotes a labor augmenting productivity index that grows at a constant rate g_A. The problem of the firms is standard, that is, they pick capital and labor optimally in order to solve:

$$\max_{K_t,H_t} BK_t^{\alpha}(A_tH_t)^{1-\alpha} - wH_t - rK_t \tag{3.3}$$

Capital and labor services are paid by their marginal products, i.e.,

$$r = \alpha B\left(\frac{K}{AH}\right)^{\alpha-1} \tag{3.4}$$

$$w = (1-\alpha)BA\left(\frac{K}{AH}\right)^{\alpha} \tag{3.5}$$

where r denotes the net rate of return on capital and w the wage.

3.2.4 The Government Budget

Government expenditures (G_t) are assumed to be a certain constant fraction θ of output of economy Y_t. To finance its stream of expenditures, we also assume that government has access to a set of fiscal instruments. This set of instruments available consists of proportional taxes on consumption τ_c, labor τ_w, capital τ_k and investments τ_i. Moreover, it is assumed that the government does not have debt, so that its budget constraint can be written as following:

$$\tau_c C_t + \tau_i I_t + \tau_w r K_t + \tau_k w H_t = G_t = \theta Y_t \tag{3.6}$$

where C_t, I_t, K_t and H_t denote the aggregate consumption, investment, capital and labor, respectively.

Note that the government does not directly control the set of taxes $\{\tau_c, \tau_w, \tau_k, \tau_i\}$, since it is assumed to be endogenously determined. What the government controls, indeed, is how the total amount of revenue needed to finance its expenditures is split into those sources. Let θ_m with $m \in \{c, w, k, i\}$ be the share of government's revenue due to tax on the source m, so that $\theta = \sum\limits_m \theta_m$ which denotes the tax burden of economy. Thus, the set of taxes is given by:

$$\tau_c = \theta_c \frac{Y_t}{C_t},\ \tau_w = \theta_w \frac{Y_t}{wH_t},\ \tau_k = \theta_k \frac{Y_t}{rK_t},\ \tau_i = \theta_i \frac{Y_t}{I_t} \tag{3.7}$$

Inasmuch as the distribution is determined by the production function, the taxes τ_w and τ_k are given exogenously by the technology of economy and by the fiscal policy. Moreover, if the government reduces θ_i and for example, increases θ_c in order to keep the tax burden unchanged, the equations in (3.7) can be used to estimate the taxes τ_c and τ_i in the new equilibrium.

We use the formulation in (3.7), instead of assuming taxes to be exogenous, because C/Y and I/Y are endogenous and thereby they are expected to change as the tax reform takes place. As a consequence, we do not know in advance what tax should be levied in order to keep the tax burden in terms of GDP unchanged. By using the formulation in (3.7), the taxes are endogenously determined, along with C/Y and I/Y and other endogenous variables of the model.

3.2.5 Individuals' budget constraints

In this economy, individuals make decisions about their labor supply h_t and capital k_{t+1}. At the beginning of each period, they face idiosyncratic shocks z on their labor productivity. We assume that z is a random variable which follows a first order auto-regressive process $\ln z_t = \pi \ln z_{t-1} + \varepsilon_t$, where $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$. In addition, the labor productivity of individuals also depends on an age-efficiency component $\bar{y}_t$. Let $e_t(z, \bar{y}_t)$ denote the labor productivity of an individual at age t so that the labor income after tax is given by $y_w(z,t) = (1 - \tau_w) w h_t e_t(z, \bar{y}_t)$.

Each period, individuals purchase consumption c_t and investment goods i_t with their after-tax income, which is obtained by renting the factors of production that they own to the firms. The capital income after-tax is $(1 - \tau_k) r k_t$, where k_t denotes the asset holdings. Individuals are price-takers and the price of consumption and investment goods are normalized to 1. The final source of income is the accidental bequest transfers ξ. Thus, an individual's budget constraint can be written as following:

$$(1 + \tau_c)c_t + (1 + \tau_i)i_t = r(1 - \tau_k)k_t + y_w(z,t) + \xi \tag{3.8}$$

The investment i_t is determined by the law of motion for asset accumulation, that is:

$$i_t = (1 + g_A)k_{t+1} - (1 - \delta)k_t \tag{3.9}$$

where $(1+g_A)k_{t+1}$ denotes the asset holdings in the next period, taking into account the technological progress g_A, and $(1-\delta)k_t$ is the current asset stock after applying the depreciation rate δ.

Given that we are going to focus on the state steady of the economy under study, we have divided consumption, asset holdings, lump sum transfers and wage rate by A_t in order to eliminate the effect of economic growth. This transformation accounts for the term $(1 + g_A)k_{t+1}$ in (3.9).

In this economy, there is no private market for insurance against the risk from the shocks on labor productivity or from living longer than expected. Additionally, we assume that individuals are liquidity constrained and set the restriction of assets carried over from age t to $t+1$ to be $k_{t+1} \geq 0 \; \forall t$. As it will turn out, liquidity

constraints tend to play an important role in determining the results of some simulations that will be carried out in this paper. This is so because individuals who are liquidity constrained tend to be more affected by a tax reform which increases the tax on consumption.

Given that there is no altruistic bequest motive and death is certain after age T, an agent who reached age T will consume all his assets at that age, so that we have $k_{T+1} = 0$.

3.2.6 Equilibrium

Let s denote the individual states. It depends on the asset holdings k at the beginning of the period and on the idiosyncratic shock z so that $s = (k, z)$. Let $V_t(s)$ denote the value function of an agent age t. The value functions $V_t(s)$ are defined by the following dynamic programs:

$$V_t(s) = \max_{h,a'} \{u(c, 1-h) + \beta\psi_{t+1} E_{z'} V_{t+1}(s')\} \tag{3.10}$$

$$\text{subject to } (1+\tau_c)c_t + (1+\tau_i)i_t = r(1-\tau_k)k_t + y_w(z,t) + \xi$$

where $s' = (k', z')$ and $\psi_{t+1} = \prod_{k=1}^{t+1} \psi_k$.

Suppose $K \subset R_+$ and $Z \subset R$, are the sets of possible values that k and z can take, so that we can define the state space as $S = K \times Z$. Let $g_t : S \to R_+$ and $n_t : S \to [0, 1]$ be the policy functions associated with k' and h in the dynamic programs above.

At each point of time, agents are heterogeneous in regard to age t and to state $s \in S$ and, as a consequence, we need some way of describing this heterogeneity. The agents' distribution at age t among the states s is represented by a measure of probability λ_t defined on subsets of the state space S. Let $(S, \Omega(S), \lambda_t)$ be a space of probability, where $\Omega(S)$ is the Borel $\sigma-$algebra on S. Thus, for each $\omega \subset \Omega(S)$, we have that $\lambda_t(\omega)$ denotes the agents' fraction at age t that are in ω.

The transition from age t to age $t + 1$ is governed by the transition function $Q_t(s, \omega)$, which depends on the decision rule $g_t(s)$ of assets and on the realization of the idiosyncratic productivity shock z. The function $Q_t(s, \omega)$ gives the probability of an agent at age t and state s to transit to the set ω at age $t + 1$.

Thus, a recursive competitive equilibrium for this economy can be defined as following:

Definition 1 *Given the policy parameters, a recursive competitive equilibrium for this economy is a collection of value functions* $\{V_t(s)\}$, *decision rules for individual asset holdings* $g_t(s)$, *for labor supply* $n_t(s)$, *prices* $\{w, r\}$, *age dependent, time-invariant measures of agents* $\lambda_t(s)$, *transfers* ξ *such that:*

1) $g_t(s)$, $n_t(s)$ *solve the dynamic problem (3.11)*

2) *The individual and aggregate behaviors are consistent, that is:*

$$\tilde{K} = \sum_{t=1}^{T} \mu_t \int_S g_t(s) d\lambda_t$$

$$N = \sum_{t=1}^{T} \mu_t \int_S n_t(s) e(z,t) d\lambda_t^w$$

3) $\{w, r\}$ *are such that they satisfy the optimum conditions, (eq.3.4) and (eq.3.5);*

4) *The final good market clears:*

$$\sum_{t=1}^{T} \mu_t \int_S \{c_t(s) + [(1+g_A)g_t(s) - (1-\delta)g_{t-1}(s)]\} = B\tilde{K}^{\alpha} N^{1-\alpha}$$

5) *Given the decision rule* $g_t(s)$, $\lambda_t(\omega)$ *satisfies the following law of motion:*

$$\lambda_{t+1}(\omega) = \int_S Q_t(s,\omega) d\lambda_t \ \ \forall \omega \subset \Omega(S)$$

6) *The distribution of accidental bequests is given by:*

$$\xi = \sum_{t=1}^{T} \mu_t \int_S (1 - \psi_{t+1}) g_t(s) d\lambda_t$$

7) *The set of taxes* $\{\tau_c, \tau_w, \tau_k, \tau_i\}$ *is given by (3.7).*

3.3 Data and calibration

In this section, we describe the data used to calculate the model and the calibration procedures[5]. Initially, the model is calibrated using data from the Brazilian econ-

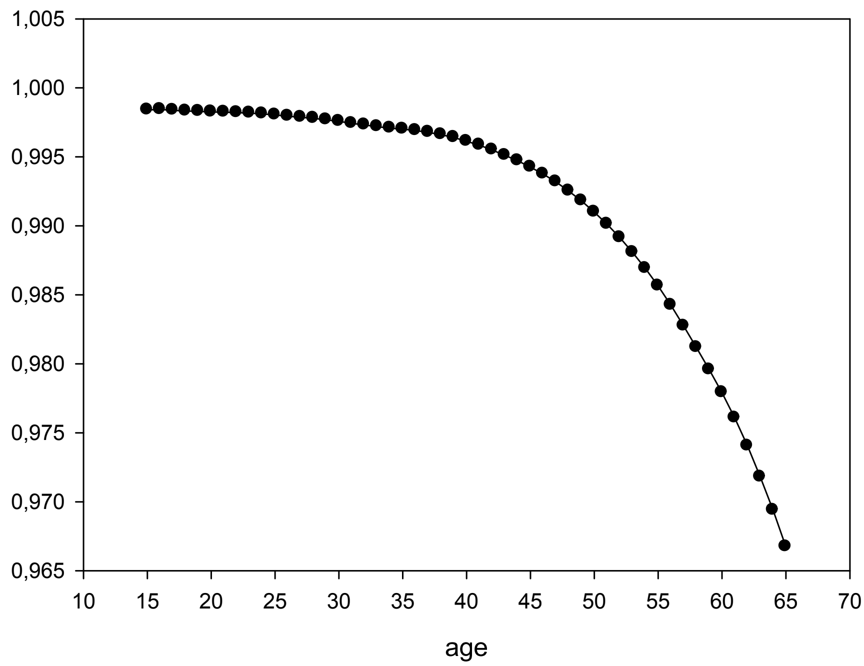
[5]The standard calibration procedure of overlapping generations models can be found in Auerbach and Kotlikoff (1987) and in Rios-Rull (1996), which we follow here.

omy for 2006. Afterwards, we use the calibrated model to measure the welfare cost of each type of taxation in Brazil and to carry out simulations to have the macroeconomic and distributional implications of the tax reform that has been proposed by the government.

3.3.1 Demography

The population age profile $\{\mu_t\}_{t=1}^{T}$ depends on the population growth rate g_n, on the survival probabilities ψ_t and on the maximum age T that an agent can live. In this economy, a period corresponds to 1 year and an agent can live at the most 51 years, so that $T = 51$. Additionally, we assumed that an individual is born with 15 years old, so that the real maximum age is 65 years old.

Figure 3.1: Survival probability



Data on survival probability for each cohort were extracted from IBGE and are shown in Figure 3.1. The population growth rate is chosen based on the average population growth from 1997 to 2007. This yields a g_n equal to 0.0145.

3.3.2 Preferences and Technology

The values of the parameters related with the individual preferences (β, ρ, ν) are summarized in Table 3.1. The value of the relative risk aversion parameter ρ along with the share of leisure in the utility ν determines the elasticity of intertemporal substitution of consumption, which is given by $1/[1-(1-\rho)(1-\nu)]$. Using the values for (ρ, ν) reported in Table 3.1, we obtain a value of 0.46 for the intertemporal substitution of consumption, which is in the rage of the estimates of the microeconomic studies revised by Auerbach and Kotlikoff (1987).

In representative agent models, given the capital income share and the depreciation rate, there is a one to one relationship between the parameter ν and the fraction of time that individuals spend working in the stationary state. In overlapping generation models, however, such relation is more complicated because of the heterogeneity among agents. In this case, the procedure that is used to choose ν is such that the average fraction of time that individuals in our model spend working is consistent with the empirical evidence, which suggests a value near 33%.[6]

The calibration of the parameter σ, which governs the effect of government on the individuals' utility, is more difficult because the empirical literature either do not provide a plausible range of values for it or even a robust evidence of substitutability or complementarity between public and private consumption.[7] Thus, we decide to carry out simulations in which $\sigma = 0$, $\sigma < 0$ and $\sigma > 0$.

Table 3.1: Technological and Preferences Parameters

β	ρ	ν	B	α	δ	g_A	g_n
0.997	4.00	0.62	0.90	0.42	0.041	0.0135	0.0145

Source: Elaborated by Authors

In our model, since there is technological progress, the discount factor is given by $\beta = \widetilde{\beta}(1+g_A)^{(1-\nu)(1-\rho)}$. Given g_A, ρ and ρ the parameter $\widetilde{\beta}$ is calibrated so that the capital-output ratio in the benchmark economy is equal to 2.75.

[6]See, for instance, Juster and Stafford (1991).

[7]See, for example, Aschauer (1985), Bean (1986), Ahmed (1986), Campbell and Mankiw (1990) and Ni (1995).

The values of technological parameters (B, α, δ) are also summarized in Table 3.1. We chose a value for α based on the Brazil time series data from the Brazilian Institute for Geography and Statistics (IBGE).

The depreciation rate is given by:

$$\delta = \frac{I/Y}{K/Y} - g_A - n - ng_A$$

We set the investment-product ratio I/Y equal to 0.1916 and the capital-product ratio K/Y equal to 2.75. The productivity growth rate g_A is constant and consistent with the average growth rate of GDP per capita over the second half of the last century. Based on the data from IBGE, we set g_A equal to 1.35%. Thus, the above equation yields a δ consistent with the value shown in Table 3.1.

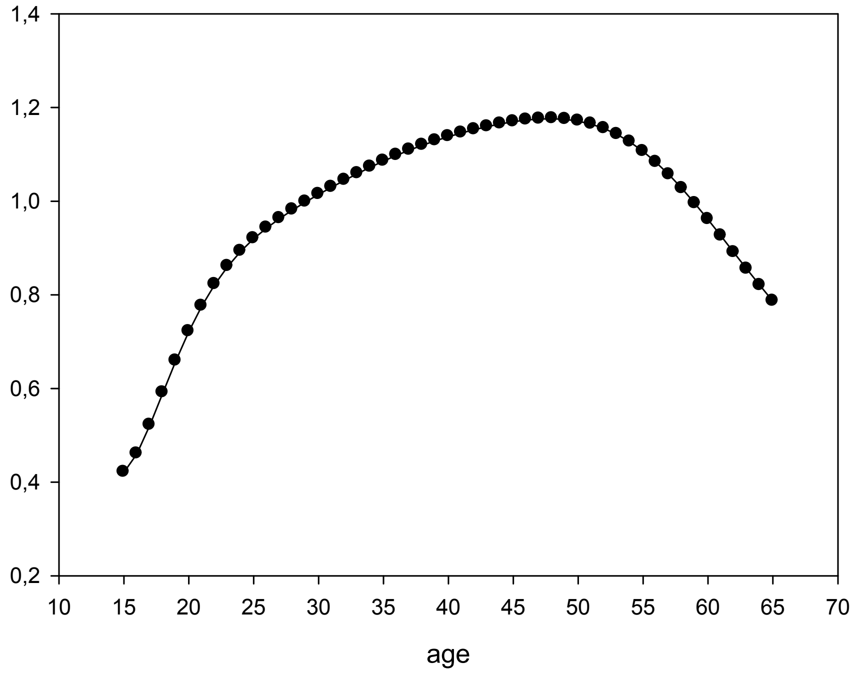
Rios-Rull (1996) normalizes the value of parameter B, which measures the total factor productivity, to 1. In this paper, we follow Huggett (1996) so that we chose B to normalize the wage rate w in the benchmark economy. Thus, given a capital-product ratio of 2.75 and $\alpha = 0.42$, the value of B is such that $w = 1$, that is, 0.9.

3.3.3 Individual productivity

Each agent in this economy is endowed with an individual productivity level $e(z_t, t) = \exp(z_t + \bar{y}_t)$, where $\bar{y}_t$ denotes a permanent component without risk and z_t denotes a temporary component, which follows an auto-regressive process of first order with parameters $(\pi, \sigma_\varepsilon^2)$. Several authors have estimated similar stochastic processes for the labor productivity.[8] Controlling for the presence of measurement errors and/or effects of such observable characteristics as education and age, the literature provides a range of values for π that goes from 0.88 to 0.96 and for σ_ε from 0.12 to 0.25. The procedure used in this paper to calibrate these parameters was to set $\pi = 0.90$ and adjust σ_ε in such a way that the income distribution in the model matches the observed income distribution. As a measure of the income distribution on data, we use the Gini index which was 0.52 for Brazil in 2006. Thus, choosing a value of 0.07 for σ_ε^2, the model yields a Gini index of 0.46 in the benchmark case.

[8] A revision of this literature can be found on Atkinson (1971) and Floden and Lindé (2001).

Figure 3.2: Age-efficiency profile



The values for $\bar{y}_t$ are constructed following Huggett (1996). We used data from Brazil's National Household Sample Survey (PNAD) on median earnings of full-time workers for each cohort. We divided these values by the total median earnings and, then, interpolated to get the individual productivity component by age $\bar{y}_t$. In Figure 3.2, we show the age-efficiency profiles that are used in our calculations.

For computational reasons, we have approximated the $AR(1)$ process which describes the idiosyncratic productivity shock z by a finite Markov chain. First, we discretized the state space Z using a grid of 19 points equally spaced in the interval $[-3\sigma_z, 3\sigma_z]$, where σ_z denotes the unconditional standard deviation of z, that is, $\sigma_\varepsilon/\sqrt{(1-\pi^2)}$. The transition probabilities are computed using the algorithm described in Tauchen (1986). After calculating the matrix of stochastic transition among the states in Z, we calculated the invariant distribution associated with this matrix and, then, took this result to describe the agents' initial distribution in the economy.

3.3.4 Fiscal Policy

The fiscal policy in our model can be summarized by the set of parameters $\{\theta_c, \theta_w, \theta_k, \theta_i\}$. To calibrate these parameters we use data from the Brazilian Ministry of Finance for 2006, our benchmark year[9]. Many taxes in Brazil are cumulative in such a way that they end up being levied on both consumption and investment. When this is the case, we multiply the amount collected by the share of consumption and investment on GDP in order to account for the share collected due to taxation on consumption and investment, respectively.[10] In the second column of Table 3.2, we show the calibrated values for $\{\theta_c, \theta_w, \theta_k, \theta_i\}$ in the benchmark case.

In the third column of Table 3.2, we show the values for $\{\theta_c, \theta_w, \theta_k, \theta_i\}$ that must prevail after the reform. One can see that, the main goal of the tax reform is to reduce the amount collected from taxes on labor income and savings and increase the amount collected from taxes on consumption. In fact, under the current tax system, about 41% of the total government revenue is due to consumption taxes and 10.15% is due to tax on investment. After the tax reform and given that the government wants to keep the tax burden in terms of GDP unchanged, the former share will increase to 53%, while the latter will decrease to 3.70%.

Table 3.2: Fiscal Policy Parameters

	Current Tax System	After Reform
θ_c	0.1365	0.1765
θ_i	0.0337	0.0123
θ_w	0.1008	0.0822
θ_k	0.0611	0.0611
Total Tax Burden	0.3321	0.3321

Source: Elaborated by Authors

[9]For more details about the fiscal parameters see Pereira (2008) and appendix.

[10]For example, only 26.51% of the amount collected from tax on imported goods is due to consumption. Thus, we multiply the amount collected from tax on imported goods by 0.2651 in order to calculate the share of θ_c due to this type of taxation.

3.4 Results

3.4.1 The welfare cost of taxes

In order to assess the welfare cost of taxes, we calculate the deadweight loss due to an increase of one percentage point in each type of taxation. The deadweight loss of taxation (also called the excess burden) is based on a conceptual experiment where the government imposes (or increases) taxes, thereby distorting prices, and returns the revenue to the taxpayer in a lump-sum base. Thus, in order to assess the welfare cost of taxation, we measure how much money individuals require to be indifferent between an increase in taxation and no change in the tax system. Our calculations are carried out taking into account different values of the parameter σ, which governs the effect of government on household's utility. As a consequence, we can also investigate how the calculation of the welfare cost of taxes may depend on the way that government expenditures affect individuals' marginal utility.

Table 3.3: Deadweight Loss with Uncertainty

	$\sigma = -0.05$	$\sigma = 0.00$	$\sigma = 0.05$
$\Delta\tau_c$	0.4514	0.3880	0.3306
$\Delta\tau_i$	2.1150	1.8827	1.7364
$\Delta\tau_w$	0.8432	0.7755	0.7171
$\Delta\tau_k$	0.7752	0.7141	0.6587

Source: Elaborated by Authors

In the first column of Table 3.3, we show what type of taxation has been changed and in the other three columns we show the deadweight loss calculated for different values of σ. As one can see in the Table 3.3, tax on consumption is the least distortionary taxation no matter what value σ takes, whereas tax on investment is the largest one. In fact, our simulations suggest that the welfare cost due to an increase in tax on investment can be more than twice as larger as that due to labor income and 4.5 times higher than that due to tax on consumption. Thus, this result supports a tax reform which replaces taxation on saving with taxation on con-

sumption. However, this argument is based on efficiency issues and the findings presented in Table 3.3 cannot tell us anything about the distributional implications of such reform. In the next subsection, we provide an assessment of this subject.

Moreover, note that the welfare cost of each taxation is higher when we set $\sigma < 0$ and lower for $\sigma > 0$ than its benchmark value $\sigma = 0$. To understand this result, remember that for $\sigma = -0.05$, private and public goods are substituted. Hence, as an increase in taxation lowers the output and thereby the government expenditure, the private consumption tends to increase. As a consequence, individuals require a higher compensation in order to agree with the change in the tax system.[11]

Table 3.4: Deadweight Loss without Uncertainty ($\sigma = 0.00$)

$\Delta\tau_c$	0.3306
$\Delta\tau_i$	1.7364
$\Delta\tau_w$	0.7171
$\Delta\tau_k$	0.6587

Source: Elaborated by Authors

In Table 3.4, we show the results for the experiment carried out in Table 3.3, but leaving aside the uncertainty regarding to labor income. In this case, as one can see in the table, the deadweight loss is lower for all taxations, although the difference is more significant for taxes on savings. This is so because in the uncertain environment, individuals build up precautionary savings to protect themselves against low incomes and thereby low consumption levels in future periods. Thus, asset holdings are used to smooth out idiosyncratic shocks and, as a result, asset accumulation is less responsive to changes in the rate of return. This prudent behavior entails that the elasticity of savings is lower under uncertainty, so that the distortion caused by taxation tends to be higher. We omit the results for others values of σ because they have the same way as in Table 3.3.

[11] A similar argument can be used to explain the result for $\sigma > 0$.

3.4.2 Macroeconomic and distributional impacts

The analysis carried out in the last subsection suggests that, by replacing tax on investment and labor with tax on consumption, the tax reform will reduce the distortion caused by the tax system and thereby put forward capital accumulation and the increase of income. In order to asses this, we show in Table 3.5 the results of simulation of the tax reform using the calibrated model.

As one can see in Table 3.5, the macroeconomic impacts of such reform are quite positive. In fact, when $\sigma = 0$ the capital stock and labor increase by 23.43% and 2.10%, respectively, whereas the output increases by 11.05%. These values entail an increase in investment-output ratio and government revenue by 12.38% and 10.98%, respectively.

As shown in the last three columns of Table 3.5, when $\sigma = -0.05$ the impacts of the tax reform is even higher, with the capital stock, hours worked and income growing to 24.17%, $2,40\%$ and 11.68%, respectively. This is so because, when $\sigma = -0.05$, the distortion caused by the tax system tends to be higher as argued in the last subsection. Thus, the gains due to the tax reform tend to be higher too.

This increase in the long-run income is higher than that found by other papers. For example, Kotlikoff (1992), using a life-cycle model without uncertainty, estimates an increase in output by 8%, while the econometric study carried out by Slemrod (1999) provides values that range from 4% to 6%. This significant impact on the capital accumulation, income and government revenue is the main reason for which this tax reform has been proposed.

In Table 3.5 we also show the taxes before and after the reform. Note that for the benchmark case, $\sigma = 0$, the model estimates a decrease in the tax on investment and labor income by 67.75% and 18.30%, respectively. As government increases the share of its revenue due to consumption taxation in order to keep the tax burden in terms of GDP unchanged, the tax on consumption is expected to increase by 33.21%. Along with the increase of average income of economy, the large fall in tax on investment and labor income more than offset this increase in tax on consumption, so that the tax reform tends to increase the consumption and thereby the individuals' welfare.

Nevertheless, this is not the case for a significant number of agents in the econ-

Table 3.5: Simulation Results of the Tax Reform

	$\sigma = 0.0$		$\sigma = -0.05$	
	Benchmark	After Reform	Benchmark	After Reform
Capital Stock	2.0538	2.5350	2.0705	2.5710
Hours Worked	0.3087	0.3152	0.3202	0.3279
Output	0.7454	0.8273	0.7601	0.8489
Investment/Output	0.1916	0.2154	0.1901	0.2140
Consumption/Output	0.8084	0.7846	0.8099	0.7960
Government Revenue	0.2475	0.2747	0.2524	0.2819
τ_c	0.1668	0.2222	0.1663	0.2205
τ_i	0.1749	0.0564	0.1772	0.0579
τ_w	0.1748	0.1428	0.1748	0.1418
τ_k	0.1442	0.1442	0.1442	0.1442

Source: Elaborated by Authors

omy. This is so because in our model individuals face idiosyncratic shocks and, as borrowing constraints prevent them from insuring themselves against those shocks, some agents who faced a bad sequence of shocks will save a small amount of their disposable income or will not save at all. Therefore, these agents will not take advantage of the fall in tax on investment directly and thereby the high increase in tax on consumption may prevent their consumption from increasing or, at least, from growing at the same pace as the consumption of the average individual. As a matter of fact, the consumption of the poorest 20% of individuals rises by 1.9%, while the average consumption goes up by 7.5%. The situation is even worse when we compare with the increase observed for the richest 20% of the population, 18.23%. As a consequence, the standard deviation of consumption increases from 0.2473 to 0.2834.

Needless to say, the welfare of the poorer individuals falls significantly in regard to the welfare of the richer ones, increasing the welfare inequality. In fact, before reform, the welfare of the richest 20% of individuals in regard to the poorest

20% ones is 22% and after the reform that ratio increases to 28.8%.

In Table 3.6, we show the share of individuals who worsen after the tax reform, controlling for their income. This assessment is done by comparing the individual value function $V_t(s)$ in each state s before the reform with that observed after the change in the tax system. As one can see in the Table 3.6, nearly 90% of the individuals whose income is lower than 20% of the average income would not vote for the tax reform. This proportion decreases as we raise the income of the individuals in the sample. Indeed, 63.55 % of agents whose income is lower than 40% of the average worsen as the tax system change. If we take into account all individuals in economy, 37.45% would prefer the economy before reform. Needless to say that the reform does not bring about a Pareto improvement since it is unable to improve the welfare of all individuals.

Table 3.6: Share of Individuals who worsen with the Reform(%)

$< 0.20y_m$	$< 0.40y_m$	$< 0.60y_m$	$< 0.80y_m$	All economy
89.59	63.55	51.67	40.84	37.45

Source: Elaborated by Authors

The problem with the change in the tax system carried out in this paper lies in the fact that it has a twofold effect on individuals' budget constraint: one positive due to the reduction of taxes on investment and labor, and another negative due to the increase in tax on consumption. Thus, for those who save the most part of their disposable income, the former effect tends to more than offset the latter, while the opposite tends to be true for those who spend almost all their disposable income on consumption.

These findings entail that the market incompleteness is crucial to determine the distributional impacts of a tax reform that replaces taxes on savings and labor income with consumption taxation. As a matter of fact, the results above diminish the attractiveness of a consumption-based tax system, at the same time that it strengthens the arguments in favor of saving and labor income taxation. This is so because taxation on income and savings levies a higher tax on savers and, as a

consequence, could redistribute welfare from the better off (not credit constrained) to the worse off (credit constrained).

Furthermore, it should be stressed that the trade-off between economic growth and inequality associated with the tax reform analyzed in this paper tends to be more important in poor countries or emerging economies in which the share of low income individuals, those who are likely to worsen with the reform, is high. Our results suggest that a tax reform toward a consumption-based tax system should be carried out after the country had reached a certain stage of economic development in which there is more equality, and not the opposite. As a matter of fact, governments from developed countries usually have a higher share of their revenue based on consumption taxation than poor countries and emerging economies have.

However, the negative impacts on welfare inequality presented above can be diminished by adopting some mechanisms which avoid the increase of taxation on consumption for low income individuals. As an example, we carried out simulations in which taxation on consumption is kept at its benchmark level for the poorest 20% of individuals. In this case, although the capital accumulation and output grow by 10.01% and 21.05% (less than the previous cases), the share of individuals who worsen after the change in the tax system drops to 28.8%. Thus, even though it is possible to avoid some increase in welfare inequality, it cannot be done without undermining the growth of output since it usually entails some kind of compensation to low income individuals to the detriment of richer ones.

3.4.3 The social welfare criterion

When one has to compare inequality or welfare of distributions across agents, there is still some disagreement about the method to be used. In this subsection, we take a stand on two specific social welfare functions in order to assess the welfare implications of the tax reform further. We start with a welfare criterion based on an utilitarian social welfare function among all generations currently alive in the steady state. Under this criterion, well-being of the heterogeneous population is aggregated by the weighted sum of individual utilities. In particular, given the decision rules $g_t(s)$ and $n_t(s)$ and the invariant cross-sectional distribution $\lambda_t(s)$ of

the generation t across the states s, the average steady-state utility for a taxation policy arrangement $\Omega = \{\theta_c, \theta_w, \theta_i, \theta_k\}$ can be written as:

$$SWF_1(\Omega) = \sum_{t=1}^{T} \mu_t \beta^{t-1} \psi_t \int_S u(c_t(g_t(s), n_t(s)), 1 - n_t(s)) d\lambda_t(s) \tag{3.11}$$

Here, the average utility for each generation t, $\int_S u(c_t(g_t(s), n_t(s)), 1-n_t(s)) d\lambda_t(s)$, is weighted by the unconditional probability of being alive at age t, $\psi_t = \prod_{k=1}^{t} \psi_k$, by the intertemporal discount factor, β, and by the share of individuals aged t in the population, μ_t.

Alternatively, one could argue that a better way to evaluate the distributional implications of the tax reform would be to consider a person who had to choose a particular distribution in a complete ignorance of what his own relative position would be within the system over her lifetime. In this case, the social welfare should be measured by the ex-ante expected (with respect to labor income shocks) lifetime utility of a newborn in a stationary equilibrium. Thus, given that all newborns start with zero assets, the social welfare function implied by the policy arrangement $\Omega = \{\theta_c, \theta_w, \theta_i, \theta_k\}$ is the expected value of the value function in regard to shock level at birth, that is:

$$SWF_2(\Omega) = E_z V_1(0, z) \tag{3.12}$$

where $V_1(0, z)$ is the solution of the dynamic program in (10).[12]

Table 3.7 shows the values of $SWF_1(\Omega)$ and $SWF_2(\Omega)$ for the benchmark and after-reform cases. It can be seen that the social welfare falls nearly 4.76% with the reform when we use $SWF_1(\Omega)$, whereas it increases by 5.20% based on $SWF_2(\Omega)$. Intuitively, the first result is associated with the curvature of the social welfare function, which in our context is determined by the curvature of the individuals utility function. The concavity of the social welfare function reflects the marginal social value of equality - the extent to which a dollar is deemed to be worth more to a poorer individual than to a richer one.[13] A high curvature of $SWF_1(\Omega)$ means

[12] We used a quadrature numerical integration method to compute the expectancy in (12).

[13] See, for example, Atkinson (1970, 1973) and Atkinson (1980).

a high aversion to inequality and, thereby changes in consumption for low income agents have a larger impact on social welfare than for high income individuals. Thus, a tax reform that makes poor agents worse off is more likely to decrease $SWF_1(\Omega)$ since it is given by the weighted sum of individuals' utility.

Table 3.7: Social Welfare implications of the Tax Reform

	$SWF_1(\Omega)$	$SWF_2(\Omega)$
Benchmark	-94.77	-114.37
Reform	-99.28	-108.42

Source: Elaborated by Authors

On the other hand, if we ask a newborn under what tax arrangement she would like to live, Table 3.7 shows that she would prefer the steady state after reform. In order to understand this result, one should initially consider that at the beginning of the first period, individuals draw a realization of z from a normal probability distribution with mean zero and variance σ_z^2 and, as a consequence, they face a relatively large probability to start with a labor productivity not far away from the average labor productivity. Considering that individuals in this situation or better improve with the tax reform, we have that the social welfare measured by SWF_2 increases. In other words, since labor income trajectories in which a newborn will be better off after the reform are more likely to be realized, she prefers the steady state implied by the new tax system.[14]

3.4.4 The relevance of accidental bequest transfers

So far we have assumed that accidental bequests are transferred in a lump-sum fashion to all agents in the economy. These transfers are small, amounting about 1.5% of the average income under the baseline calibration and, as a consequence,

[14]Nevertheless, it should be stressed that our model abstracts from any kind of link across the generations. In particular, if the productivity shocks of the newly born agent was correlated with her parent's productivity, we would have to use the appropriate conditional distribution in the calculation of (12). In this case, it is not clear beforehand if SWF_2 would increase.

Table 3.8: Descriptive statistics with and without accidental inheritances

	with transfers		without transfers	
	Benchmark	Reform	Benchmark	Reform
Capital Stock	2.0538	2.5350	2.0599	2.5448
Hours Worked	0.3087	0.3152	0.3116	0.3182
Output	0.7454	0.8273	0.7542	0.8351
τ_c	0.1668	0.2222	0.1699	0.2268
τ_i	0.1749	0.0564	0.1762	0.0561
τ_w	0.1748	0.1428	0.1748	0.1424
τ_k	0.1442	0.1442	0.1442	0.1442

Source: Elaborated by Authors

it should not largely affect the reported results. The relatively small size of the accidental inheritances is due to the fact that individuals in our model live up to age 65. Given that the conditional survival probability falls faster for the elderly, those transfers would play a more important role in a model with a longer lifespan. The potential importance of the lump-sum transfers in an economy with market incompleteness and borrowing constraints relies on the fact that it provides individuals a partial insurance against labor income shocks. In this subsection, we briefly assess an alternative approach to the uniform distribution of bequests.

The literature has attacked this problem of accidental bequests in a variety of ways. For example, Rios-Rull (1996) eliminates accidental inheritances by imposing the existence of a market for one-period annuities, where agents can perfectly insure themselves against lifetime uncertainty. In Imrohoroglu et al. (1998), agents receive accidental bequests only once, at a pre-determined age.[15] In particular, their experiments consist in first giving all bequests to the agents at age 45, which they consider a plausible age to receive inheritances and, second, giving all bequests to the newborns. They find that the aggregate capital decreases in the first

[15]In their model, agents may live up to age 85 and the accidental bequest transfers amounts to nearly 4.5% of the average income.

case due to the fall in savings on the young agents, while it increases under the second scenario because of the monotonicity of the policy function for assets. A similar experiment is carried out in Hubbard and Judd (1987), who find that the results are slightly sensitive to changes in the timing of the receipt of the accidental inheritances.

Here we propose an alternative approach, which consists in simply eliminating the accidental bequest of the system altogether. In particular, we assume that the government neither transfers the collected amount of bequests to individuals nor uses it to finance its current expenditures. In Table 3.8, we show the results for selected statistics when the transfers are left aside. In order to make the comparison easier, we also show in Table 3.8 the results that include the accidental inheritances.[16] As one can see in the Table, the absence of the lump-sum transfers increases hours worked and asset holdings, thereby enhancing the output of economy in both benchmark and after-reform steady states. This is so because agents vary their labor supply and savings in order to (partially) keep the previous life cycle levels of consumption and utility. Overall, however, it has limited impact on the results reported previously, which should be expected given the small magnitude of those transfers.

3.5 Conclusions

This paper develops and calibrates a dynamic general equilibrium model with uncertainty to assess the macroeconomic and distributional implications of a tax reform, which replaces tax on investment and labor income with tax on consumption. We use data from the Brazilian economy to carry out our analysis of the effects of such reform since it has been proposed in Brazil as a way of accelerating the economic growth thereby increasing the long-run income. We also use our model to calculate the deadweight loss of each type of taxation and compare the results with the case in which the uncertainty is left aside.

Our results suggest that tax on savings is the most inefficiency taxation, while the tax on consumption is the least one. Thus, the reduction of the former and the increase of the latter, in order to keep the tax burden in terms of GDP unchanged,

[16] All results in Table 3.8 are calculated with a $\sigma = 0$.

reduce the inefficiency of the tax system and thereby increase capital accumulation and the output of economy. The expansion of the output estimated in this paper is higher than that estimated in some econometric studies such as Slemrod (1999) among others.

Nevertheless, the results that come from the analysis carried out in this paper about the impacts of the tax reform on the welfare distribution cast doubt on the capability of such reforms in improving the economic environment. This is because that reform raises welfare inequality in the cross-section. As a matter of fact, the consumption of the richest 20% of individuals increases in relation to that of the poorest 20% and the ratio of welfare between the former and the latter goes up by nearly 23%. As a consequence, nearly 37% of individuals would not prefer the economy before reform with we look in the cross-section.

Therefore, even though the tax reform proposed by the brazilian's government reduces the distortions associated with the high income and capital tax rates that are paid by households mainly in the top income brackets, it also reduces the redistributive properties of the current tax system and, as a consequence, this policy switch implies a trade-off between efficiency and equality.

Another interesting result that comes from our assessment is that the uncertainty in the economic environment may be important to the calculation of the welfare cost of taxes, mainly for taxation on savings. In fact, as individuals increase their savings in an uncertain environment in order to build up precautionary savings to protect themselves against future negative income shocks, asset accumulation is less responsive to changes in the rate of return. As a consequence, the elasticity of savings is lower under uncertainty and the distortion caused by taxation tends to be higher as well.

There are some dimensions in which the theory of wealth differences used in this paper can be enriched, which is a goal of ongoing research. First, we want to investigate the consequences of allowing the households in our model economy to engage in activities such as human capital accumulation that change the characteristics of stochastic process that underlay the income paths of individuals. Second, in line with Castaneda et al. (1999), we also aim to study the implications of altruism, which provides a reason for households to accumulate significantly larger amounts of wealth than those that are needed to maintain a high standard of living

during the life-cycle. This feature, along with the existence of lifespan uncertainty, introduces another dimension of heterogeneity that can be helpful to better assess the impacts of the tax reform on wealth distribution.

3.6 Appendix

The table 3.9 summarizes description of the Brazilian tax system. The first column shows the name of each tax or contribution, column 2 presents the amount collected in terms of GDP. Finally, column 3 shows the tax jurisdiction and column 4 contains the incidence of each type of taxation.

The government tax reform proposes to eliminate the COFINS, PIS and CIDE and creation of federal value-added tax, IVA-F. The COFINS, PIS and CIDE are cumulative taxes, with incidence on both consumption and investment. By definition, the IVA-F has incidence only on consumption.

Additionally, the proposal intends to create a new ICMS. As one can see in Table 7, the ICMS is responsible for 7.10% of the total tax burden. The tax rate of the ICMS is 7% in some states and 12% in others. The government wants to replace these rates with a single 2% new one. We find the value of new ICMS multiplying the old value by $2/9.5$, where 9.5 is the mean between 7% and 12%.

The tax reform also intends to eliminate the education wage - a contribution made by employers. The amount collected with this contribution will be added to IVA-F. Moreover, the employer contribution to the Social Security system will decrease from 20% to 14%. To capture this change, we multiply the total amount collected with the Social Security by $14/20$.

Table 3.9: Description of Brazilian Tax System in 2006

Taxes and Contributions	Tax Burden	Jurisdiction	Incidence
Labor's Income - IRPF	0.0037	Federal	Labour
Firms' Profits - IRPJ	0.0240	Federal	Capital
Gains with Labor - IRRF-T	0.0168	Federal	Labour
Gains with Capital - IRRF-C	0.0091	Federal	Capital
Gains to Foreign - IRRF-E	0.0032	Federal	Capital
Others Gains - IRRF-O	0.0020	Federal	Capital
Financial Operations- IOF	0.0029	Federal	Cons/Invest
VAT on Manufac. Products - IPI	0.0115	Federal	Cons/Invest
Import and export taxes - (II and IE)	0.0042	Federal	Cons/Invest
Bank Account Deposits - CPMF	0.0138	Federal	Cons/Invest
Social Contribution - COFINS/PIS/PASEP	0.0499	Federal	Cons/Invest
Fuels - CIDE	0.0034	Federal	Cons/Invest
Firms' Net Profits - CSLL	0.0120	Federal	Capital
Others - Others Revenue	0.0019	Federal	Capital
Contri. to Agriculture- FUNDAF	0.0001	Federal	Consumption
VAT on Goods & Services - ICMS	0.0710	State	Cons/Invest
Vehicle License Registration - IPVA	0.0052	State	Capital
Property transmissions - ITCD and ITBI	0.0013	State/Local	Capital
Sales Tax on Service - ISS	0.0062	Local	Cons/Invest
Urban (IPTU) and Rural Property	0.0042	Loc/Fed	Capital
Payroll - Contrib. Employer/Employee*	0.0784	Fed/Sta/Loc	Labour
Fees	0.0053	Fed/Sta/Loc	Consumption
Total tax burden	0.3321		

Source: Ministry of Finance, * FGTS, INSS, Transfer to Others, S-System, Social Contribution

Chapter 4

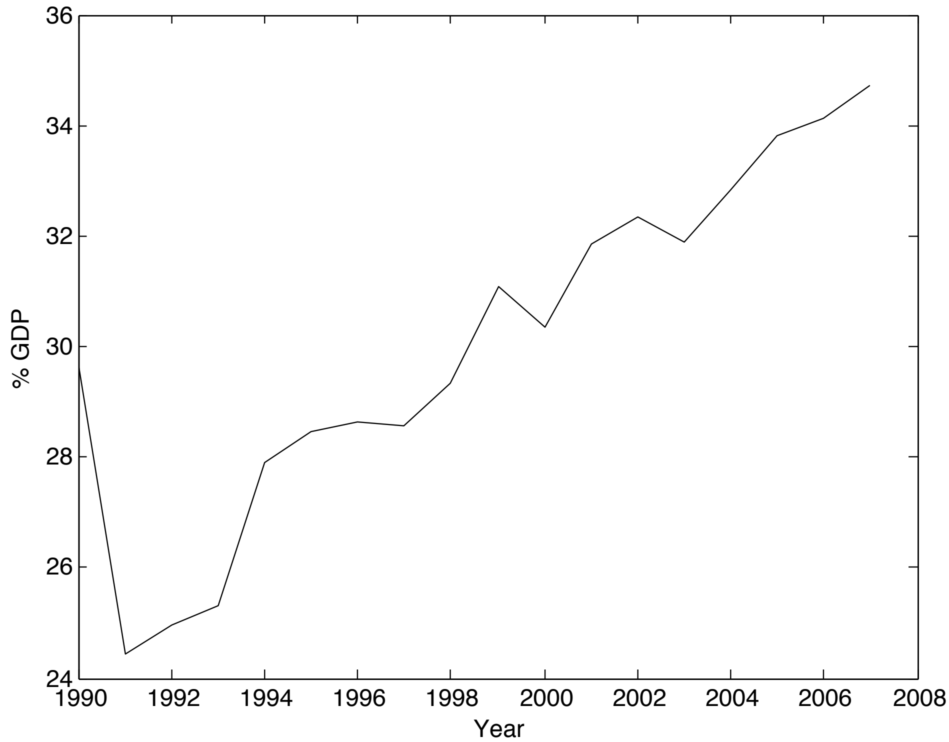
Tax Reform: Theory and Analysis of Proposals to Brazil

I analyze two tax reform proposals to Brazil, Government's proposal and the Brazilian National Confederation of Industry's (CNI) proposal. My focus is on the macroeconomics effects of each tax reform. To assess these effects, I consider the neoclassical capital accumulation model with a representative agent. I also assess the Marginal Cost of Public Funds (MCF) of the current Brazilian tax system. The MCF shows the better direction to a tax reform that want to reduce the distortion of current tax schedules.
JEL Classification: C15, C68, H20, H30;
Key words: Tax Reform, Marginal Cost of Funds, Macroeconomics Effects, Brazil

4.1 Introduction

In the last two decades, the tax revenue as percentage of Gross Domestic Product (GDP) has increased more than ten percentage points in Brazil. Figure 4.1 shows the evolution of this ratio between 1990 and 2007. There is a consensus that current tax system is too complex[1] and causes many distortions. These complexities and distortions cause a great deal of inefficiency in economy and create some barriers to firms and households.

Figure 4.1: Evolution of Tax Revenue (In percentage of GDP)



In the nineties, Brazil adopted a stabilization plan to control inflation. Brazil had success in bringing inflation down with low costs to the economy. The stabilization plan was based on increasing total tax revenue to support the government's plan. Nowadays, inflation is controlled and Brazil has a good macroeconomic stability. However, the current tax system is considered one obstacle to a

[1]See Ministry of Finance (2008) and Brazilian National Confederation of Industry (2008).

sustainable growth of the country.

The distortions in current system are so many that they can be easily enumerate. One example is the enormous numbers of federal levies and social contributions, which creates large administrative costs. Another example is the cumulatively levies, when a tax paid at one stage of productive chain, does not generate a credit in the following stages. This non-efficient tax credit system is responsible to inefficient organization of productive structure, additionally, it increases the investment and export costs. However, the deeper problem is caused by the ICMS. The ICMS is value-added tax on goods and services. Nowadays, the ICMS has 27 different legislations, each state has its own law. Due to the uncertainty caused by the legislation, the current system creates a environment that induce a fiscal war among states, with negative impact on growth for every one.

Although I recognize the role of these inefficiencies in the discussion, I focus on a choice of distribution of burden among the classic tax base. Hence, the main goal of this paper is to assess the costs of current Brazilian tax system and examine in details some proposals of tax reform to Brazil. I also measure the macroeconomics gains of the reforms. I do a normative and positive analysis of two tax reform proposals.

After defining the relevant equilibrium concept I calibrate the model to reproduce the Brazilian economy. Hence, I can measure the marginal costs of public funds (MCF) from the different tax base. When the MCF differs among the tax base, I can say that there is money in the table, it is possible to increase welfare and keep tax revenues constant by switching from taxes with a high to one with a low MCF.

In the first part of this paper, I assess the MCF in Brazil. I show that value of MCF for the tax base with the lowest MCF in Brazil is within the same range as the one found for others countries; e.g., Browning (1987), Hansson (1984) and Feldstein (1997). It suggests that a well designed reform may bring down the inefficiency of the Brazilian system to the levels found elsewhere.

The MCF is crucial in that regard for it allows me to determinate the distortions that are caused by each kind of taxation. Using the MCF I get a hint as of what directions of reform are bound to work better. I can also right away pass judgement about the proposals of tax reform that are bound to make things worse.

In the second part, I examine two concrete tax reform proposals. The first proposal is the Government's proposal[2] and the second is the Brazilian National Confederation of Industry's (CNI) proposal[3] .

I consider a standard neoclassical capital accumulation model with representative agent. I consider two cases. In the first case, there is no government spending to be financed and all tax revenue return to households in a lump-sum transfer in the same period. In the second case, there is the government spending to be financed by the taxes. I calibrate the government spending in such a way that there is no transfers to households. In this context, households and government are rival in the use of resources. I can think the second case as if there was total waste of resources, because the government spending does not provide any benefit to the society. I consider that both situations are extreme, the real world is a situation between in these two cases.

The references to this paper are Hall (1971), Atkinson and Stiglitz (1972), Araújo and Ferreira (1999), Ljungqvist and Sargent (2004) and Lledo (2005). The present paper is organized as follows. In section 2, I present the model. In section 3 the proposals of the government and CNI are presented. In section 4, I demonstrate the calibration of my parameters. In the section 5, I estimate the MCF and present the results to my simulations of the macroeconomics effects of each proposal of tax reform. The section 6 has the main conclusions. Additionally, there are two appendix to provide a support to my analysis.

4.2 The Model

I use non-stochastic version of the standard growth model with a deterministic fiscal shocks on equilibrium outcomes. This model allows me to simulate the allocative effects of a tax reform in the Brazilian economy. In my model, the time is discrete, the economy is closed and there are no technological progress and population growth. Additionally, there is no debt in my economy.

As my focus is on the macroeconomics gains (or loses) of a tax reform, I prefer a

[2]see the proposal of Ministry of Finance in the references.

[3]Mine mains references are the Brazilian National Confederation of Industry (CNI) working papers available at www.cni.org.br, details on the references.

simple model to assess the effects of a tax reform. For this reason, I do not consider productivity shocks, debt and an open economy.

Besides the government, the economy is inhabited by representatives households and firms. The households optimally choose their consumption, stock of capital and leisure. The firms operate in a competitive market. The government levies the households in four ways. The households are levied on their consumption, investment, labor income and capital income.

The allocation of collect tax is an important variable in the determination of the equilibrium. As I said above, I suppose two cases to the taxes collected. In the first case, there is no the government spending to be financed by taxes. All resources return to the households by a lump-sum transfer in the same period. In the second case, there is the government spending to be financed by taxes. The government spending consists in purchases of goods and services. The government does not invest in this economy and the government spending does not provide any productivity benefit to society. In this situation, households and government are rivals in the resources allocations.

4.2.1 Household

In this model there is no uncertainty and households have perfect foresight. The representative household maximizes his/her lifetime utility, equation (4.1), subject to his/her budget constraint. He/She chooses the sequence of consumption, capital and leisure, $\{c_t, k_t, l_t\}_{t=0}^{\infty}$, that maximize his/her utility. In the initial period, t_0, household has $k_0 > 0$ units of capital. In each period t, the household has one time endowment and he/she chooses the time allocated to leisure (l_t) and or work $(h_t = 1 - l_t)$.

The household discounts the future with rate $\beta \in (0, 1)$ and χ is the weight of leisure in the instantaneous utility function. As we said above, there is no population growth in this economy. The utility function, $U(c_t, l_t)$, is assumed to be increased, concave and twice differentiable. The lifetime utility is given by:

$$U[c_0, c_1, ..., l_0, l_1, ...] = \max \sum_{t=0}^{\infty} \beta^t [\ln c_t + \chi \ln l_t] \tag{4.1}$$

The household's income consists in selling labor, renting capital to the firm and

the transfer from government, when there is it. As I said above, I consider two situations to the taxes collected. In the first case, there is a lump-sum transfer and there is no government spending, i.e. $g_t = 0$[4]. The government levies and redistributes the taxes in the same period to the households. In the second case, there is government spending to be financed by taxes, i.e. $g_t \neq 0$. In this situation I want to investigate the behavior of the households when there is perfect rivalry among households and government.

The representative household's budget constraint has the form:

$$(1+\tau_{ct})c_t + (1+\tau_{it})i_t = (1-\tau_{ht})w_t h_t + (1-\tau_{kt})r_t k_t + \xi_t \tag{4.2}$$

where the variables c_t, i_t, k_t and ξ_t represent respectively *per capita* consumption, investment, capital and government's transfer[5], where $\xi_t \neq 0$ or $\xi_t = 0$. The terms $\tau_{ct}, \tau_{it}, \tau_{ht}$ and τ_{kt} are respectively the time varying taxes rates on consumption, investment, labor income and capital income. The investment follows the condition $i_t = k_{t+1} - (1-\delta)k_t$, where δ is the depreciation rate.

The household's problem is given by:

$$\begin{aligned} U[c_0, c_1, ..., 1-h_0, 1-h_1, ...] &= \max_{\{c_t, h_t, k_t\}} \sum_{t=0}^{\infty} \beta^t [\ln c_t + \chi \ln(1-h_t)] \qquad (4.3)\\ \text{subject to} (1+\tau_{ct})c_t + (1+\tau_{it})i_t &= (1-\tau_{ht})w_t h_t + (1-\tau_{kt})r_t k_t + \xi_t \\ &\text{and } k_0 > 0 \end{aligned}$$

The first order conditions of the problem described by equation (4.3) are given by:

$$c_{t+1} = \beta c_t \frac{(1+\tau_{ct})}{(1+\tau_{ct+1})} \left\{ \left[\frac{(1-\tau_{kt+1})}{(1+\tau_{it})} \right] r_{t+1} + \left[\frac{(1+\tau_{it+1})}{(1+\tau_{it})} \right] (1-\delta) \right\} \tag{4.4}$$

$$c_t \chi (1+\tau_{ct}) = (1-h_t)(1-\tau_{ht}) w_t \tag{4.5}$$

The complete description of the solution of household's problem is in appendix A.

[4] g_t is the per capita government spending, the aggregate government spending is $G_t = Lg_t$, where L is the population.

[5] $\xi_t = \tau_{ct}c_t + \tau_{it}i_t + \tau_{ht}w_t h_t + \tau_{kt}r_t k_t - g_t$. If $g_t \neq 0$, then $\xi_t = 0$.

4.2.2 Equivalence between Taxes

In this subsection I show that some combinations of taxes do not change the consumption possibility set. For each period t, I can rewrite the budget constraint, equation (4.2), according to the example below.

Let's define the others taxes in terms of consumption tax. I can write the new taxes as:

$$1+\tau_{it}^{'} = \frac{1+\tau_{it}}{1+\tau_{ct}}, 1-\tau_{ht}^{'} = \frac{1-\tau_{ht}}{1+\tau_{ct}}, 1-\tau_{kt}^{'} = \frac{1-\tau_{kt}}{1+\tau_{ct}}, \tau_{\xi_t}^{'} = \frac{1}{1+\tau_{ct}}$$

The budget constraints above could be rewritten as following:

$$c_t + (1+\tau_{it}^{'})i_t = (1-\tau_{ht}^{'})w_t h_t + (1-\tau_{kt}^{'})r_t k_t + \tau_{\xi t}^{'}\xi_t \tag{4.6}$$

After the transformation of taxes, I can observe that consumption possibility set is the same before the transformation. In other words, the budget constraints has the same slope as before the transformation. Furthermore, in each period, I can normalize the taxes in terms of one tax. As described by Salanié (2003), when the consumption possibility set keeps uncharged, I can claim that there is a equivalence between taxes in the model. Therefore, although I can write the budget constraint in different ways, the results of my model are the same.

4.2.3 Government

I suppose two situations to tax revenue. In the first situation, there is no government spending to be financed, $G_t = 0$, in equation (4.7). The government levies the households and returns the taxes collected in the same period by a lump-sum transfer. In this case, I investigate a situation where there is no second order distortions caused by the government spending. In others words, the distortions are caused only by taxation.

In the second case, there is the government spending, $G_t \neq 0$, which is calibrated to represent in steady state the same amount of taxes collected. When I do that, it means that there is no transfer to the households, $\Xi_t = 0$. Then, in this situation there is a competition among households and the government for the resources. I can think this case as the government spending does not provide

any benefit to households. The government wastes the resources of economy. The government's budget constraint in the period t in the general case is given by:

$$\tau_{ct}C_t + \tau_{it}I_t + \tau_{ht}w_tLh_t + \tau_{kt}r_tK_t - G_t = \Xi_t \tag{4.7}$$

where $C_t, I_t, K_t, G_t, \Xi_t$ and L are respectively, aggregate consumption, investment, capital, lump-sum transfer and population (normalized to be 1). The terms w_t, r_t and h_t are the wage, the interest rate and the labor supply respectively.

Following Hall (1971), I take the government behaviors as exogenous, it means that the government only lists the sequences of spending, G_t, to $t \geq 0$ and the taxes $\{\tau_{ct}, \tau_{it}, \tau_{ht}, \tau_{kt}\}_{t=0}^{\infty}$ as the sets of fiscals instruments.

Note that the government does not control directly the value of fiscals parameters, τ. It controls the tax revenue among four tax base. Let θ_m with $m = \{c, i, h, k\}$ the share of each tax base m in the government's tax revenue. The total tax revenue in the economy in period t is represented by $\theta_t = \sum\limits_{m=\{c,i,h,k\}} \theta_{mt}$. Thus, the determination of the set of fiscals instruments is given by:

$$\tau_{ct} = \theta_{ct}\frac{Y_t}{C_t}, \tau_{it} = \theta_{it}\frac{Y_t}{I_t}, \tau_{ht} = \theta_{ht}\frac{Y_t}{w_tLh_t}, \tau_{kt} = \theta_{kt}\frac{Y_t}{r_tK_t} \tag{4.8}$$

I use the equation (4.8) to calculate the marginal tax rate by each tax base. The equation (4.8) comes from a manipulation of equation (4.7). A tax reform implies in a change in the parameters θ_m. After a tax reform, I have new parameters thetas, θ_m^*. These new parameters θ_m^* imply in a new marginal tax rate τ_m^* to each tax base. Besides the thetas ($\theta' s$), the relations Y_t/C_t, Y_t/I_t, Y_t/K_t, w_t and r_t define the marginal tax rate by each tax base. The relations Y_t/C_t, Y_t/I_t and Y_t/K_t are exogenous in my model and the parameters w_t and r_t represent the technology.

4.2.4 Firms

The output, Y_t, is produced by competitive firms using the Cobb-Douglas technology. The firms work in a competitive market. The share of capital and labor in the output are respectively α and $1 - \alpha$. This technology holds Inada condition. The output is represented by:

$$Y_t = K_t^{\alpha}(Lh_t)^{1-\alpha} \tag{4.9}$$

In this economy, all firms have the same problem, they hide workers and rent households' capital. The aggregate problem of the firms is described by equation (4.10). They choose $\{K_t, Lh_t\}_{t=0}^{\infty}$ to maximize their profits,

$$\max_{\{K_t,h_t\}} K_t^{\alpha}(Lh_t)^{1-\alpha} - w_t Lh_t - r_t K_t. \tag{4.10}$$

I normalize the price of good to be 1. The first order conditions are described by:

$$w_t = f_h(K_t, Lh_t) = (1-\alpha)k_t^{\alpha}h_t^{-\alpha} \tag{4.11}$$

$$r_t = f_K(K_t, Lh_t) = \alpha k_t^{\alpha-1}h_t^{1-\alpha} \tag{4.12}$$

4.2.5 Equilibrium

The assumptions by the form of utility function and the technology assure that the first order conditions are necessary and sufficient for my problem and that allocations are interior. Now I can define a competitive equilibrium:

Definition 2 *A Competitive Equilibrium with taxes consists of, given a policy rule $\pi = \{G_t, \Xi_t, (\tau_{jt})_{j=\{c,i,h,k\}}\}_{t=0}^{\infty}$, an allocation $\{c_t, h_t, k_t\}_{t=0}^{\infty}$ for the household and a price system $\{w_t, r_t\}_{t=0}^{\infty}$ such that:*

i) given the price system and the policy, the allocations$\{c_t, h_t, k_t\}_{t=0}^{\infty}$ solve the household's problem (4.3);

ii) competitive pricing i.e., (4.11) and (4.12), holds for firms' problem;

iii) the government budget balances, i.e., equation (4.7) holds;

iv) the fiscal parameters are defined by (4.8);

v) market clearing conditions hold.

4.3 Tax Reform Proposal

In this section I explain in details the government's and CNI's proposals that I examine in this paper. There are some common points in both proposal, one of that

is the hypothesis of revenue guarantee. The implication of this hypothesis is that the both taxes reforms are neutral, i.e., the tax revenue in percentage of GDP would be kept unchanged after reform. Table 4.1 shows the Brazilian tax system structure in 2006, organized according to the allocation of the taxes and contributions in my model.

The structure below is my benchmark. In the appendix B, I summarize all taxes, with complete name, tax base, value and percentage of GDP of each levy (see Table 4.14).

Table 4.1: Tax Revenue (% GDP) According to Tax Base - Benchmark

Consumption	Investment	Labor	Capital	Total
IE,Others Revenues	II*, IOF**	IRPF,	IRPJ,	
Fees,IPI-Cigar,	IPI - Import*,	IRRF-T	IRRF-C	
IPI-Beverages,	IPI-Vehicle**	IRRF-O	IRRF-F	
FUNDAF, II*	IPI-Others**,	Employer/	IPTU	
IPI - Import*, ISS**,	PIS/PASEP**	Employee	IPVA,	
IPI-Vehicle**	CPMF**		ITCD,	
IPI-Others**, CSLL**,	ICMS**,	Transfer	ITR	
CIDE**,COFINS**	CSLL**,ISS**,	to		
PIS/PASEP**,IOF**	COFINS**	Others		
CPMF**, ICMS**	CIDE**			
θ_c =0.1448	θ_i =0.0375	θ_h =0.1004	θ_k =0.0491	θ =0.3317

Source: Elaborated by authors

The parameters θ_c, θ_i, θ_h and θ_k are respectively the share of each tax base in the tax revenue in percentage of GDP and $\theta = \sum_{m=\{c,i,h,k\}} \theta_m$.

The cumulative taxes are marked with $(**)$. They have incidence on consumption and on investment, this fact is responsible to increase the distortion of the current tax system. One way to split the values that have incidence in each tax base, I multiply the total value of taxes marked with $(*)$ and $(**)$ by some ratio. For example, in the case of II, marked with $(*)$, only 26.51%[6] of its total value has

[6]The II has an incidence on consumption and on investment. The consumption share is compounded by

incidence on consumption. In the case of taxes marked with $(**)$, they have an incidence on consumption and on investment, thus their total value are multiplied by ratio C/Y in the case of consumption and by $I/Y = 1 - C/Y$ in the case of investment.

I have to comment some details about both proposals. First, the both proposals consider that there is a necessity to change the financing of the current regional development policy structure. The second point is about the changes in the federative relations and government's transfer to states. In the present paper, I do not analyze these points, I leave these topics to others forward works. I keep my focus in the simplification of federal and state taxes and in the reduction of levies, specifically on investment and on payroll taxes.

4.3.1 Government's Proposal

In February 28, 2008 Brazilian's government sent to the National Congress a tax reform proposal[7]. In present paper I will analyze some details of this proposal. My focus will be on the simplification and elimination of federal taxes, reduction on payroll contribution and simplification of a value-added state tax, ICMS.

About simplification and elimination of federal taxes, government's tax reform proposal suggests the elimination of the following taxes, COFINS, PIS and CIDE. To substitute these taxes, a federal value-added (IVA-F) tax would be created. COFINS, PIS and CIDE are cumulative levies, they have an incidence on consumption and on investment. By definition, the IVA-F has incidence only on consumption. Besides this changed, this proposal also suggests the extinction of the CSLL, which it would be merged into the corporate income tax (IRPJ). This change would create a new IRPJ. According to this proposal, all changes in the tax system would happen one year after the tax reform has been approved.

Additionally, the proposal intends to create a new ICMS. The ICMS is a state value-added tax. As we can see in Table 4.14 in appendix B, the ICMS is 7.10% of GDP or 21.40% of total tax burden. The marginal tax rate of the ICMS is 7%[8] in

consumption goods and 80.45% of oil and fuel imported. The investment share is compounded by capital goods, raw goods and 19.55% of oil and fuel, the investment share is equal to 73.49%.

[7] Details on the working paper of Ministry of Finance (2008).

[8] The states, MG, PR, RJ, RS, SC, SP, have this tax. They are responsible for about 69% of ICMS.

some states and 12% in others. The government wants to replace these marginal rates for only one single rate equal to 2% for every states.

Table 4.2: ICMS Rate and Payroll Reduction

Period	ICMS Rate		Payroll
t = 0	7%	12%	20%
t = 1	6.5%	11%	19%
t = 2	6%	10%	18%
t = 3	5%	8%	17%
t = 4	4%	6%	16%
t = 5	3%	4%	15%
t = 6	2%	2%	14%

Source: Elaborated by authors

The changes in the ICMS would be gradual to the new value. The gradual process of reduction is described in Table 4.2. I find the final value of new ICMS multiplying the old value by ($2/8.55$), where 8.55 is the weighted[9] average between 7% and 12%. The values of transition periods are found in the similar way. Due to transition period, the reform in the ICMS would begin in $t = 1$ and would finish in $t = 6$. The $t = 0$ is the benchmark value.

Although ICMS is a value-added tax, I consider that it has an incidence on consumption and on investment in current tax system. The reason is the system of charging of tax in interstate operations. In interstate transactions, part of ICMS is collected in the state where the merchandise originated and the other part in the state of destination. As there are different marginal rate among states, based on the argument that states would have to refund a tax debt in a different state, many states avoid refunding cumulative ICMS credits. In this situation, the firms have to wait a long period to receive the refund of ICMS credit. In government's proposal, ICMS would be paid only in the state of the destination of the product. Hence, after the tax reform, government expects to eliminate this problem among states. In new ICMS, it would be an incidence only on consumption.

[9] The weighted is given by share of states in the total value of ICMS.

Table 4.3: Tax Revenue (% GDP) According to Tax Base - Government's Reform

Consumption	Investment	Labor	Capital	Total
IE,Others Revenues	II*,	IRPF,	new IRPJ,	
Fees,IPI-Cigar,	ISS**	IRRF-T	IRRF-C	
IPI-Beverages,	IOF**	IRRF-O	IRRF-F	
FUNDAF, II*	CPMF**	Employer/	IPTU	
ISS**,IPI-Vehicle**	ICMS**	Employee	IPVA,	
IPI - Import**,	IPI-Vehicle**	Transfer	ITCD	
IPI-Others**	IPI - Import**	to	ITR	
CPMF**, ICMS**	IPI-Others**	Others		
IOF**, IVA-F				
θ_c =0.1696	θ_i =0.0141	θ_h =0.0869	θ_k =0.0611	θ =0.3317

Source: Elaborated by authors

The other important topic that I analyze is the reduction on payroll taxes. Government's proposal would eliminate "Education Wage" - a contribution that it is paid by employer. The amount collected with this contribution will be incorporated into IVA-F. Besides the elimination of the "Education Wage", government's proposal suggests a reduction in the employer contribution to the Social Security system, that it would decrease from 20% to 14%.

Likes ICMS changes, the reduction of Social Security contribution would be gradual, one percentage point per year. This reduction would begin in the second year following approval of the reform. The Table 4.2 describes the transition period into the contribution achieves the value of 14%. To capture this change, I multiply the total contribution collected from the Social Security by $(14/20)$ in the end of the reform. Looking at the data, I observe that 70% of Social Security contribution correspond to the employer/employee contribution and that the $2/3$ of this total value is due to the employer contribution.

The new tax system structure after government's reform is described in Table 4.3. The values in Table 4.3 refer to the new equilibrium, after the transitory period.

4.3.2 National Confederation of Industry's (CNI) Proposal

Brazilian National Confederation of Industry's (CNI) proposal[10] has some common points to government's proposal. The reason is because CNI and government have had some discussions about how to define a *good* tax reform to Brazil. There is a consensus that any tax reform has to keep the tax revenue in terms of GDP uncharged.

In government's proposal, it prefers to avoid some controversies topics because these topics could delay the approval of reform. Therefore, some important points are postponed to another opportunity. Then, in this subsection I present some of these topics. They are considered very important to growth and competitiveness of the country.

Table 4.4: Tax Revenue (% GDP) According to Tax Base - CNI's Reform

Consumption	Investment	Labor	Capital	Total
IE,	II*	IRPF,	new IRPJ,	
Others Revenues	IOF**	IRRF-T	IRRF-C	
Fees, II*	CPMF**	IRRF-O	IRRF-F	
FUNDAF,		Employer/	CSLL,	
CPMF**,		Employee	IPVA,	
IOF**		Transfer	ITCD,	
IVA-F, IVA-E		to Other	ITR, IPTU	
$\theta_c = 0.1744$	$\theta_i = 0.0063$	$\theta_h = 0.0869$	$\theta_k = 0.0611$	$\theta = 0.3317$

Source: Elaborated by authors

In CNI's proposal, my focus will be on the simplification and elimination of some federal levies. Moreover, I also examine how a reduction on payroll and a new state value-added tax, IVA-E, affect the economy. Although CNI's proposal has similar suggestions than government's proposal, CNI wants a deeper change in the current tax structure. As I show below, CNI wants to reduce the period to implement all changes in tax system. According to CNI, the sort period to imple-

[10]My references are working papers of Brazilian National Confederation of Industry (CNI) in February and Brazilian National Confederation of Industry (CNI) in May.

ment the changes in the tax system would cause already effects in the short run.

One common topic in both proposals is the reduction of payroll tax. CNI also suggests a reduction on the employer's contribution to Social Security from 20% to 14%. However, it suggests an immediate reduction while government suggests a transition period. Other common point is the incorporation of CSLL into IRPJ.

Besides COFINS, PIS, CIDE, CNI's proposal also wants to eliminate IPI. COFINS, PIS, CIDE and IPI would be substituted by IVA-F. CNI wants a reform with a deeper reduction on investment tax. Table 4.4 summarizes the new tax structure according to CNI's proposal.

The most important difference between CNI's and government's reforms is due to ICMS and ISS, two cumulative levies. CNI suggests the extinction of ICMS and ISS and the creation of state value-added, IVA-E. IVA-E would have an incidence only on the consumption, furthermore it would provide a higher reduction of the taxation on the investment.

Table 4.5: Share of each Tax Base in Tax Revenue (% GDP)

Period	Government				CNI			
	θ_{ct}	θ_{it}	θ_{ht}	θ_{kt}	θ_{ct}	θ_{it}	θ_{ht}	θ_{kt}
t = 0*	0.1448	0.0375	0.1004	0.0491	0.1448	0.0375	0.1004	0.0491
t = 1	0.1513	0.0236	0.0957	0.0611	0.1774	0.0063	0.0869	0.0611
t = 2	0.1541	0.0226	0.0939	0.0611	0.1774	0.0063	0.0869	0.0611
t = 3	0.1580	0.0204	0.0922	0.0611	0.1774	0.0063	0.0869	0.0611
t = 4	0.1619	0.0183	0.0904	0.0611	0.1774	0.0063	0.0869	0.0611
t = 5	0.1658	0.0162	0.0887	0.0611	0.1774	0.0063	0.0869	0.0611
t = 6	0.1696	0.0141	0.0869	0.0611	0.1774	0.0063	0.0869	0.0611

Source: Elaborated by authors

$*$: $t = 0$ is the benchmark

Table 4.5 summarizes the share of each tax base in the tax revenue in percentage of GDP. The table shows the transition path for both reforms into achieve the new equilibrium. As I said above, CNI's proposal suggests a faster implementation of reform. Thus, the transition period is only one year while government's proposal

has five years to be implemented. I consider the values of θ's in Table 4.5 to find marginal tax rate to each tax base in the next section.

4.4 Calibration

In this section I find the technological and preference parameters. I use the equation (4.8), the values in Table 4.5 and the first orders conditions of households and firms to calibrate the parameters. The complete description of taxes and contributions is in the appendix B.

4.4.1 Technology and Preference Parameters

Table 4.6 summarizes the values of technology and preference parameters that I consider in the present paper.

Table 4.6: Technology and Preference Parameters

β	χ	α	δ	r	K/Y	C/Y	I/Y
0.9848	1.9298	0.3456	0.0738	0.1227	2.8166	0.7922	0.2078

Source: Elaborated by authors

I consider Morandi and Reis (2004) to find the share of capital (α). The share of capital (α) in the output is calculated using the capital-output ratio following Morandi and Reis (2004). I consider the average between 1970-2000, $K/Y = 2.8166$. The interest rate (r) is the average between 2006 and 2010. It values is equal to $r = 12.27\%$ per year. The share of capital is given by marginal productivity of capital that could be write as:

$$\begin{aligned} f_k(k,h) = \alpha Y/K = r \\ \alpha = 0.3456 \end{aligned} \tag{4.13}$$

The depreciation rate (δ) is defined by steady state relations between (I/Y) and (K/Y). As we said above, the relation (I/Y) is defined by the expression $I/Y = 1 - C/Y$. Thus, the depreciation rate is given by:

$$\begin{aligned} I &= \delta K \\ \delta &= 0.0738 \end{aligned} \tag{4.14}$$

The intertemporal discount rate (β) is given by the equilibrium condition in the steady state $(t = 0)$:

$$\begin{aligned} \beta &= \frac{(1+\tau_i)}{(1-\tau_k)r + (1+\tau_i)(1-\delta)} \\ \beta &= 0.9848 \end{aligned} \tag{4.15}$$

I choose the weight of leisure in utility (χ), such that the household works $1/3$ of his time endowment in the steady state. According to this assumption, the value of χ is:

$$\begin{aligned} \chi &= \frac{(1-\alpha)(1-\tau_h)\left[\frac{1}{h} - 1\right]}{(1-\alpha)(1-\tau_h) + (1-\tau_k)\alpha - \frac{(1-\tau_k)\alpha\beta}{1-(1-\delta)\beta}} \\ \chi &= 1.9298 \end{aligned} \tag{4.16}$$

4.4.2 Fiscal Policy Parameters

In this subsection I find the marginal tax rate to each tax base. I use the values in Table 4.5, the conditions describe above and the relations that are given by equations (4.8) to determine the marginal tax rate to each tax base. According to these conditions, I find the new marginal tax rate to each tax reform. The values that I find are summarized in Table 4.7.

According to Table 4.7, I observe an increase in the taxation on consumption and on capital income and a reduction of the marginal taxes rates on investment and on labor income in both proposals. As CNI is the productive sector, its proposal intends to eliminate more cumulative levies. For this reason, I observe a more significant reduction in the taxation on investment and an increase in the taxation on consumption.

Table 4.7: Marginal Tax Rate

Period	Government's Proposal				CNI's Proposal			
	τ_{ct}	τ_{it}	τ_{ht}	τ_{kt}	τ_{ct}	τ_{it}	τ_{ht}	τ_{kt}
t = 0	0.1828	0.1803	0.1534	0.1420	0.1828	0.1803	0.1534	0.1420
t = 1	0.1910	0.1137	0.1462	0.1768	0.2239	0.0306	0.1328	0.1768
t = 2	0.1946	0.1086	0.1435	0.1768	0.2239	0.0306	0.1328	0.1768
t = 3	0.1995	0.0934	0.1409	0.1768	0.2239	0.0306	0.1328	0.1768
t = 4	0.2044	0.0881	0.1382	0.1768	0.2239	0.0306	0.1328	0.1768
t = 5	0.2092	0.0779	0.1355	0.1768	0.2239	0.0306	0.1328	0.1768
t = 6	0.2141	0.0677	0.1328	0.1768	0.2239	0.0306	0.1328	0.1768

Source: Elaborated by authors

4.5 Results

The mains goals of this paper are to assess the costs of current Brazilian tax system and examine in details some proposals of tax reform to Brazil. To calculate the costs of current tax system I consider the Marginal Cost of Public Funds (MCF) to measure the distortion caused by each tax base.

The MCF allows me to determine the distortions that are caused by each kind of taxation. In this way, I can use the MCF to define the better direction to a tax reform. Using the MCF I can do judgement about the proposals of tax reform.

Our second goal is to measure the macroeconomics gains or loss of tax reform proposals. To do that I consider a standard neoclassical capital accumulation model with representative agents.

4.5.1 Marginal Cost of Funds

According to Dahlby (2008), the marginal cost of public funds (MCF) measures the loss incurred by society in raising additional revenues to finance government spending. The MCF is the key component in evaluations of tax reform, public expenditure programs and others public policies.

Before defining the MCF I have to calculate the dead weight loss (DWL). The

definition of DWL that I consider is

$$DWL = \frac{|U_{(t=0)} - U_{(t=T)}|}{\lambda_{(t=0)}} \tag{4.17}$$

where the numerator is the absolute value of the difference between the utilities. The term $U_{(t=0)}$ is the utility in the benchmark and $U_{(t=T)}$ is the utility in the new steady state, after increasing the tax revenue in percentage of GDP in 1%, i.e. the tax revenue would be 34.17%. The term $\lambda_{(t=0)}$ is the marginal utility of consumption in the benchmark[11].

There are many ways to express the marginal cost of public funds. In the present paper I consider the following definition of Marginal Cost of Funds (MCF):

$$MCF = 1 + \frac{\Delta DWL}{\Delta R / Y_{(t=T)}} \tag{4.18}$$

where ΔR is an increase of 1% in the tax revenue and $Y_{(t=T)}$ is the output in the new steady state. I find the MCF to each kind of tax base. For example, to find the MCF of consumption I increase the consumption tax rate until that the tax revenue reaches an increase of 1%, kept the others marginal tax rate uncharged. I repeat this procedure to each tax base.

When I calculate the marginal cost of funds (MCF), I want to assess the marginal costs of each tax base in the current Brazilian tax system. In others words, MCF defines the better way to increase or reduce a tax. Therefore, MCF can be used to determinate the better direction to a tax reform in Brazil.

Table 4.8 presents the values of MCF to each tax base. As I said above, I consider two situations to the taxes collected. In the first situation, the total taxes return to households in a lump-sum transfer. In the second situation there is no transfer to households. When there is transfer, the values of MCF are smaller than in the otherwise situation. The reason is due to the second order distortion caused by the government spending. In others words, when there is no transfer, the government and households are rivals in the use of resources. Therefore, no productive spending increases the MCF.

[11] In the next subsection, the DWL will be assessed in terms of marginal utility of consumption in the Pareto Optimal situation.

Table 4.8: Marginal Costs of Funds

	With Transfer	No Transfer
Consumption	1.3224	2.8508
Investment	2.4326	2.9500
Labor	1.5550	2.8394
Capital	2.1704	3.0683

Source: Elaborated by authors

In the case with transfer, the lowest value is in the consumption and the highest is in the investment. The higher value means a more inefficient taxation. According to the results, if government wants to finance a project using an increase in taxes, the least distortionary levy is the taxation on consumption.

Let's suppose a project with an expected cost of 1 million of dollars, government should finance this project by taxation on consumption only if this project creates benefits to society more than 1.3224 million of dollars. In the same way, a project that would be financed by investment tax should provide benefits higher than 2.4326 million of dollars. According to mine results, the taxation on consumption is the least inefficient taxation while the taxation on investment is the most inefficient taxation.

When there is no transfer, the values of MCF are higher. The taxation on investment and capital are the more inefficient. In this economy the households accumulate too many capital, what increase the MCF. The values of taxation on consumption on labor income are closed and they are the less inefficient taxation. The reason for these results is because the households have to work and save more. As I show in the next subsection, the stock of capital is higher in this situation compared to the situation with transfer. In the same way, the labor supply is higher in the case without transfer.

In the literature, there is no consensus about the value of MCF. Ballard et al. (1985) estimated a value between $\$1.17$ and $\$1.56$ dollars, Browning (1987) calculated a value between $\$1.10$ and $\$4.00$ dollars. In a more recent paper, Feldstein (1997) affirmed that the MCF was $\$2.65$ in the United States. Given these results, I could claim that MCF is higher than in United States.

If MCF differs among tax bases, I can say that government is not rational. When this situation happens, it suggests that there is available resources in economy. Therefore, it is possible to increase welfare and keep tax revenues constant by switching from taxes with a high to one with a low MCF.

As I said above, using the MCF I get a hint as of what directions of reform are bound to work better and can also right away pass judgement about the proposals of tax reform. Thus, I can take two conclusions of my results. First, if government and CNI want to increase the efficiency in economy, they have to decrease the most inefficiency taxation and also have to increase the least distortionary taxation. In this way, a simple reorganization of taxes may cause a Pareto improvement to the society.

The second conclusion is, the both proposals want to reduce the taxation on investment and on labor income and increase the taxation on consumption. As I show above, it is a good direction to reduce the inefficiency in Brazilian tax system. Therefore, the both taxes reforms proposals may increase the efficiency of tax system and consequently they could increase the output and welfare in society.

4.5.2 Macroeconomics Effects

As I said above, one goal of the present paper is to examine two tax reform proposals. Using a standard neoclassical accumulation model with representative agents I simulate the effects of each proposal. Additionally, I also simulate the effects of Pareto Optimal reform, i.e. all levies are equal to zero.

To determine the effects of a tax reform, I compare the present value of the sequence path of each variable in each reform to the value of benchmark situation. Given the initial steady state, I consider the present value of whole path of this situation. The values in level of the initial steady state are described in Table 4.9 for the case when there is or no transfer.

The steady state value of the variables are defined by equations (4.11), (4.12), (4.29), (4.30), (4.31) and (4.32). It is easy to see that the labor supply, the stock of capital and output are higher in the economy no transfers. However, the consumption is higher in level when there in no transfer.

The results are split in two subsection. One subsection is dedicated to the econ-

Table 4.9: Variables in the Steady State (in level)

Variables	With Transfer	No Transfer
Labor	0.2441	0.3333
Consumption	0.3340	0.2946
Capital	1.1870	1.6208
Output	0.4216	0.5757
Wage	1.1304	1.1304
Interest Rate	0.1227	0.1723
DWL^{**}	105.34	361.20
C/Y	0.7923	0.7922+
I/Y	0.2077	0.2078
K/Y	2.8155	2.8154
Tax Revenue/Y	0.3318	0.2805

Source: Elaborated by authors

+: I aggregate the households and the government.

$**$: Present value.

omy with transfer and the other to the economy when there is no transfer. In the subsection with transfer, I also investigate the short run effects of a tax reform. The effects in the short run are important because they could make not possible to approve the reform in the Congress. Although a tax reform has good effects in the long run, in the short run economy could dive in a recession, reducing the welfare.

Economy - Transfer

i) Long run

In this subsection I show the macroeconomics long run effects when there is transfer to individuals. Figure 4.2 displays the transition path from the initial steady state to the new steady state in each proposal. As I said above, government's proposal suggests a transitory period to implement all changes in the tax schedule while CNI's proposal suggests that all changes would be made in one period.

Figure 4.2: Transition Path: Government's and CNI's Proposals - With Transfers

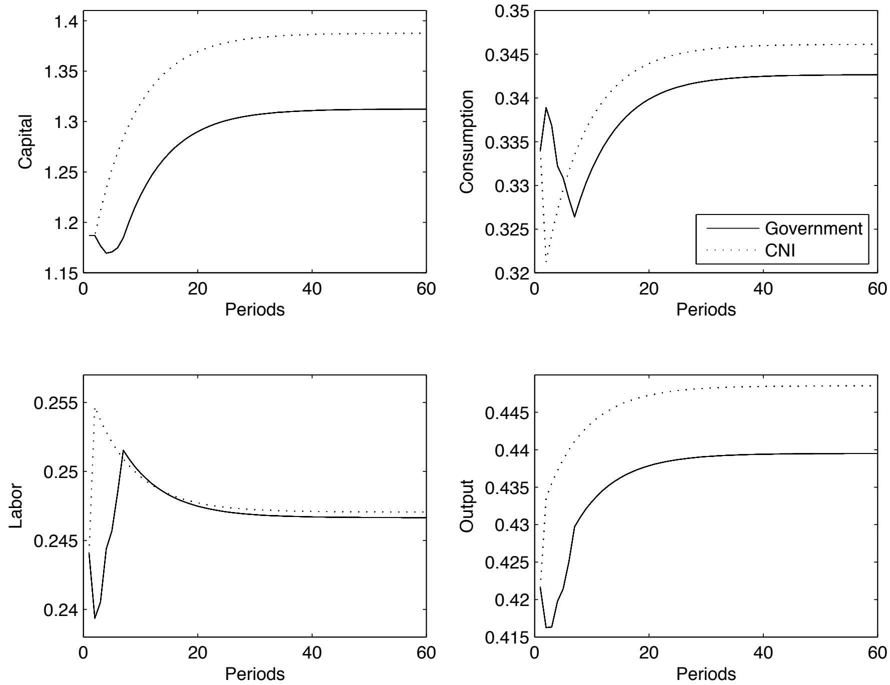


Figure 4.2 shows that the variance of consumption in government's proposal is smaller than in CNI's proposal. However, the new steady state level is lower in government's proposal than CNI's proposal. My model achieves the new steady state in less than 60 periods, where one period is equivalent to one year.

Table 4.10 compares the difference among each reform. To do that, I compare the present value of the sequence path of each variables in each reform to present value of benchmark situation. Table 4.10 shows the percentage deviation from the initial steady state. I observe that both proposals have relevant impact on variables. The most important topic in both proposals is the reduction of the taxation on investment, the most inefficiency tax. Due to the deeper reduction of taxation on investment in CNI's proposal, this proposal has a better macroeconomics results than government's proposal.

According to my results, the consumption increases 1.87% in government's proposal and 2.78% in CNI's proposal. In the Pareto Optimal, the consumption would increase 36.87% compared to the benchmark situation. The capital and output

Table 4.10: Effects of Tax Reform Proposals (%) - With Transfer

Variables	Government	CNI	Pareto Optimal
Labor	1.15	1.54	38.27
Consumption	1.87	2.78	39.68
Capital	8.50	14.81	105.97
Output	3.62	5.92	58.04
Wage	2.44	4.32	14.47
Interest Rate	-4.43	-7.65	-21.6
DWL	-13.10	-19.40	0
C/Y^*	0.7789	0.7688	0.7002
I/Y^*	0.2211	0.2312	0.2998
K/Y^*	2.9482	3.0518	3.6693
Tax Revenue/Y*	0.3298	0.3278	0

Source: Elaborated by authors

$*$: Variables in level

would increase 8.50% and 3.62% in government's proposal and around 14.5% and 6.00% in CNI's proposal. Therefore, I expect a good macroeconomic effect in the economy in both reforms.

The wage goes up and the interest rate goes down in both proposals. The others macroeconomics variables have the expected behavior according to the economic theory. I explain the better results in CNI's reform due to the deeper reduction in the taxation on investment, as describe above, the most inefficient levy.

ii) Short Run

In this section, I analyze the short run effects of a tax reform. I consider that the short run has less than thirty two periods. Due to the effects in the short run, a Congress could not choose to implement a tax reform because the loses in terms of utility, output and consumption. As in the previews section, to assess the effects of sort run, I get the difference between the steady state and the new sequence path after the reform. I consider the cumulative sum in present value of each variable.

According to table 4.13, in the first year after Government's proposal the output

Table 4.11: Short Run Effects of Each Proposals (%) - With Transfer

	Government								CNI							
Periods	1	2	4	6	8	12	16	32	1	2	4	6	8	12	16	32
Consumption	0.72	0.76	0.17	-0.41	-0.61	-0.46	-0.15	0.79	-1.89	-2.22	-2.01	-1.58	-1.13	-0.37	0.21	1.49
Capital	0	-0.28	-0.72	-0.69	-0.18	1.18	2.43	5.39	0	0.68	2.25	3.71	4.97	7.00	8.50	11.65
Output	-0.62	-0.83	-0.59	-0.05	0.45	1.17	1.67	2.66	1.40	2.02	2.75	3.23	3.59	4.11	4.48	5.21
Utility	0.79	0.88	0.30	-0.36	-0.68	-0.73	-0.57	0.01	-2.00	-2.42	-2.39	-2.10	-1.78	-1.22	-0.78	0.16

Source: Elaborated by authors

would be reduce in -0.62% and the consumption and utility would increase 0.72% and 0.79% respectively. However, the gains of utility and consumption would be transformed in loses in the six period. In the six period, the individuals would be 0.36% worse in the new equilibrium than in the initial steady state, in terms of utility. Moreover, in the eighth period, the individual would be worst than in the fourth period, the utility falls 0.73% in terms of initial steady state. Only in the thirty second period the utility registers gains in terms of welfare.

In CNI's proposal, I find a higher utility cost than government's proposal in the beginning of reform. I find a deeper reduction in consumption and utility, but a faster increase in capital and output. Even though CNI's proposal has bad results in terms of utility in the initial period, it registers gains before government's proposal. As I know from the previews section, CNI's proposal provides better results in the long run.

Although a tax reform has good macroeconomic effects in the long run, the effects in the short run could make impossible to implement it. I have a simple problem of intertemporal inconsistency. Hence, to approve a tax reform, it is important to have collusion among political forces. They need to know that the tax reform is important to country, even though the short run cost.

Economy - No Transfer

i) Long run

Figure 4.3 displays the transition path in the economy when there is no transfer. Again, the variance of the variables in government's proposal is smaller than in CNI's proposal.

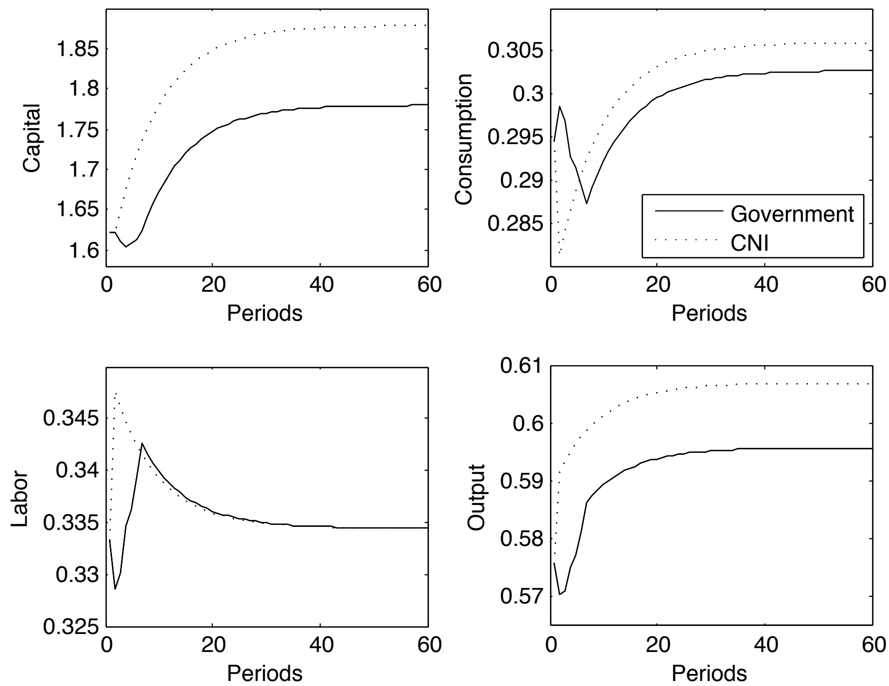
In general, the variables have a higher change when there is no transfer than in the case with transfer. The reason is because households are poorer than in the situation with transfer. As there is no transfer, households have to work and save more. I can observe in Table 4.9 and in Figure 4.3 that the level of capital is higher in the case without transfer.

According to the results, the consumption, capital and the output would increase respectively 1.93%, 7.84% and 3.02% in the government's proposal and 2.81%, 13.75% and 5.08% in CNI's proposal. The DWL is huge in this case because the gov-

ernment spending does not provide any benefit to the society. In this subsection, the DWL is in terms of marginal utility of consumption in the Pareto Optimal case. Higher DWL implies in a lower welfare.

When there is no transfer and the government spending does not provide any benefit to the society, the taxation has a second order effect. Households and government are rivals in using resources. Therefore, I can suppose that the government wastes the resources in a no productive direction.

Figure 4.3: Transition Path: Government's and CNI's Proposals - No Transfer



Observing Figures 4.2 and 4.3 and Tables 4.10 and 4.12, I can conclude that the both proposals of tax reform may cause a positive macroeconomics effects in the economy. There is an increase in consumption, capital, output and welfare. The two reforms are able to reduce the inefficiency in the economy.

CNI's proposal has the better results because it suggests a deeper reduction of the taxation on investment, the most inefficient levy. The reduction of the taxation on investment is following by an increase of the taxation on consumption. In the

Table 4.12: Effects of Tax Reform Proposals (%) - No Transfer

Variables	Government	CNI	Pareto Optimal
Labor	0.59	0.80	1.12
Consumption	1.93	2.81	60.17
Capital	7.84	13.75	54.58
Output	3.02	5.08	16.87
Wage	2.42	4.25	15.67
Interest Rate	-4.39	-8.53	-24.62
DWL	-4.71	-6.81	0
C/Y^{*+}	0.7732	0.7612	0.6945
I/Y^{*}	0.2268	0.2388	0.3155
K/Y^{*}	2.9027	2.9940	3.5658
Tax Revenue/Y*	0.2804	0.2615	0

Source: Elaborated by authors

$*$: Variables in level

$+$: I aggregate the households and the government.

model, the taxation on consumption is the least distortionary tax. Moreover, CNI's proposal also suggests a short period to implement all changes in the tax system.

Due to the short period to implement the changes, CNI's proposal provides the better benefit to the society in the short run. The stock of capital, the supply of labor and the output increase faster than in government's proposal. The consumption also has a higher and faster falls in the begin of transition period in CNI's proposal. Nevertheless, in few periods after reform, the consumption in CNI's proposal catches up and passes the consumption in government's proposal.

Although government's proposal does not have as good results as CNI's proposal, the increase in the efficiency is quite impressive. I guess that the two situations are extremes, the real world belongs to a situation between these two extremes cases. For example, an increase in the GDP about 5% means more than 100 billion of reais in the economy. Therefore, even though government's proposal does not provide as good results as CNI's proposal, it already has a positive

macroeconomic effects in the economy.

There is no question that Brazil needs a tax reform. The current tax system is very complex and creates many distortions. I show above that there are goods proposals to change the current system. In my opinion, in the first stage, it is important to define and approve one proposal of tax reform. In the second stage, some adjusts would be made according to the society intentions.

CNI's proposal predicts a better results to the economy due to the deeper reduction of the taxation on investment. Furthermore, there is a short transition period to implement all changes in current tax schedule, which it implies in a short run effects of the tax reform.

The most important point is to approve one proposal of tax reform and gradually make the right adjustments. The advantage of my model is that I consider a very simple model to analyze a tax reform proposal. Government takes as reference the paper of Fernandes et al. (2004). However, my paper is simpler and predict the same effects of a tax reform than Fernandes et al. (2004).

ii) Sort run

The short run effects in the economy no transfer have the same direction than in the economy with transfer. The magnitude of effects is smaller than in the case with transfer and take a longer period to produce positive effects in the utility. Due to the transition period, government's proposal produces positive effects among periods 1 and 4. However, among periods 6 and 32 the utility would be lower than the benchmark situation. In CNI's proposal, there is a deeper reduction on utility in the beginning of reform, but gradually the utility would increase, with good results in the long run.

As in the economy with transfer, the effects in the short run could suggest that a Congress could not decide to approve the reform, even though, it produces good long run effects in the long run.

Table 4.13: Short Run Effects of Each Proposals (%) - No Transfer

	Government								CNI							
Periods	1	2	4	6	8	12	16	32	1	2	4	6	8	12	16	32
Consumption	0.66	0.69	0.08	-0.52	-0.75	-0.62	-0.31	0.70	-2.25	-2.69	-2.57	-2.15	-1.70	-0.88	-0.23	1.24
Capital	0	-0.19	-0.48	-0.40	0.04	1.20	2,27	4.91	0	0.57	1.91	3.18	4.29	6.12	7.50	10.55
Output	-0.45	-0.57	-0.30	0.16	1.15	1.17	1.53	2.28	1.39	1.96	2.57	2.95	3.23	3.63	3.91	4.49
Utility	0.66	0.71	0.14	-0.49	-0.79	-0.85	-0.70	-0.09	-2.18	-2.67	-2.70	-2.45	-2.14	-1.57	-1.12	-0.08

Source: Elaborated by authors

4.6 Conclusions

In the present paper, my mains goals are to assess the costs of current Brazilian tax system and examine in details some proposals of tax reform in Brazil. I examine the government's proposal and the Brazilian National Confederation of Industry's (CNI) proposal. I also measure the macroeconomics gains of a tax reform. I do a normative and positive analysis of the tax reform proposals.

I consider a standard neoclassical capital accumulation model with representative households. I consider two cases. In the first case there is no government spending to be financed and all tax revenue returns to households in a lump-sum transfer in the same period. In the second case there is the government spending to be financed by taxes. I calibrate the government spending such that there is no transfer to households. In this context, households and the government are rival in the use of resources. As the government spending does not provide any benefit to the society, I can think this case as if there is total waste of resources. I consider that both situations are extreme, the real world is a situation between these two cases.

In the first part of present paper I measure the marginal cost of funds (MCF) to the different tax bases. MCF is crucial in that regard for it allows me to determine the distortions that are caused by each kind of taxation. Using the MCF I get a hint as of what directions of reform are bound to work better. Therefore I can also right away pass judgement about the proposals of tax reform that are bound to make things worse.

I show that the value of MCF to all tax base is higher than one found to other countries. I also show that the value of MCF differs for different tax bases. A rational government should not allow this difference in the MCF. Because when this is happened, I can say that there is available resources in the economy. Thus it is possible to increase welfare and keep tax revenues constant by switching from taxes with a high to one with a low MCF.

I find that the taxation on consumption has the lowest MCF while the taxation on investment has the highest value. It means that a good tax reform should increase the taxation on consumption and reduce the taxation on investment. If I do that, a good reform may bring down the inefficiency of the Brazilian tax sys-

tem. The both proposals that I examine in the present paper suggest reforms in this direction.

In the second part I measure the macroeconomics gains of each tax reform. According to the model, both proposals may provide a positive impact in the economy. The model predicts an increase in consumption, stock of capital, output and welfare. CNI's proposal has the better results because there is a bigger reduction of the taxation on investment than government's proposal.

Looking into the results, I could claim that both tax reform proposals are in correct direction to cause a Pareto improvement. They could reduce the distortions in the economy, increase the efficiency of allocations and have positive effects in the welfare.

4.7 Appendix A

In this appendix I solve household's problem describes by equation (4.3). Let's consider the case where there is the government's spending to be financed, i.e. $g_t \neq 0$, in this case there is no transfers to the households, $\xi_t = 0$. The household's problem can be written as:

$$U[c_0, c_1, ..., 1-h_0, 1-h_1, ...] = \max_{\{c_t, h_t, k_t\}} \sum_{t=0}^{\infty} \beta^t [\ln c_t + \chi \ln(1-h_t)] \tag{4.19}$$

$$\text{s.t}(1+\tau_{ct})c_t + (1+\tau_{it})i_t = (1-\tau_{ht})w_t h_t + (1-\tau_{kt})r_t k_t + \xi_t \tag{4.20}$$

$$\text{and } k_0 > 0 \tag{4.21}$$

The government's budget constraint to each household is:

$$\tau_{ct}c_t + \tau_{it}i_t + \tau_{ht}w_t h_t + \tau_{kt}r_t k_t - g_t = \xi_t \tag{4.22}$$

The first orders conditions (FOC) of household's problem are:

$$U_{ct} = \mu_t(1+\tau_{ct}) \tag{4.23}$$

$$\chi U_{lt} = \mu_t(1-\tau_{ht})w_t \tag{4.24}$$

$$\beta\mu_{t+1}\left\{\left[\frac{(1-\tau_{kt+1})}{(1+\tau_{it})}\right] r_{t+1} + \left[\frac{(1+\tau_{it+1})}{(1+\tau_{it})}\right](1-\delta)\right\} = \mu_t \tag{4.25}$$

$$\lim_{t\longrightarrow\infty} \mu_t k_t = 0 \tag{4.26}$$

where μ_t is the Lagrange multiplier on the budget constraint.

The equation (4.26) is the transversality condition and expresses that the value of marginal utility of capital is zero in the infinity. I can manipulate the equations above and I find the equations (4.4) and (4.5) in text. They are written as:

$$c_{t+1} = \beta c_t \frac{(1+\tau_{ct})}{(1+\tau_{ct+1})}\left\{\left[\frac{(1-\tau_{kt+1})}{(1+\tau_{it})}\right] r_{t+1} + \left[\frac{(1+\tau_{it+1})}{(1+\tau_{it})}\right](1-\delta)\right\} \tag{4.27}$$

$$c_t\chi(1+\tau_{ct}) = (1-h_t)(1-\tau_{ht})w_t \tag{4.28}$$

The interest rate (r), labor supply(h), stock of capital (k) and consumption (c)

in steady state are given by the following equations:

$$r^{SS} = \left(\frac{1+\tau_i}{1-\tau_k}\right)\left[\frac{1}{\beta} - (1-\delta)\right] \tag{4.29}$$

$$h^{SS} = \frac{\left(\frac{1+\tau_i}{1-\tau_k}\right)\frac{1}{\chi}(1-\alpha)\left(\frac{r^{SS}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - g}{\left(\frac{r^{SS}}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}\left[1 + \frac{1-\tau_h}{1+\tau_c}\frac{1}{\chi}(1-\alpha)\right] - \delta\left(\frac{r^{SS}}{\alpha}\right)^{\frac{1}{\alpha-1}}} \tag{4.30}$$

$$k^{SS} = \left(\frac{r^{SS}}{\alpha}\right)^{\frac{1}{\alpha-1}} h^{SS} \tag{4.31}$$

$$c^{SS} = k^{SS\alpha} h^{SS1-\alpha} - \delta k^{SS} - g^{SS} \tag{4.32}$$

where g could be $g = 0$ or $g \neq 0$.

Remember that when $g = 0$ there is transfers to the households and when $g \neq 0$ there is no transfers to the households. In the last situation, the government spending does not provide any benefit to society. It means that the government and the households are rival in allocation of resources.

The aggregate consumption, capital, investment, labor and government spending in steady state can be written respectively as $C^{SS} = Lc^{SS}$, $K^{SS} = Lk^{SS}$, $I^{SS} = Li^{SS}$, $H^{SS} = Lh^{SS}$ and $G^{SS} = Lg^{SS}$, where L is the population that it is normalized to be one.

4.8 Appendix B

We collect the datas on the website of Ministry of Finance (www.fazenda.gov.br) and Instituto de Pesquisa Econômica Aplicada (IPEA) (www.ipeadata.gov.br). According to the government's agency, the Brazilian Gross Domestic Product (GDP) was $R\$ \ 2,332,935,544,000.00$ reais, in currency value of 2006. The other variable that we use in the present paper is the annual average of nominal interest rate. The value is $r = 12.27\%$ per year.

The table 4.14 summarizes the description of the Brazilian's tax system. The first column shows the name of each tax and contribution. The column 2 presents the amount collected in reais (R$) and in the column 3 we have taxes in terms of GDP. Finally, the column 4 shows the tax jurisdiction and the column 5 contains the incidence on each type of taxation.

Table 4.14: Complete Description of Taxes - Economy 2006

Taxes and Contributions	Value - R$ Millions	Tax Burden	Jurisdiction	Tax Base
labor's Income - IRPF	8.525.144.251,41	0.0037	Federal	Labor
Firms' Profits - IRPJ	56.080.219.057,98	0.0240	Federal	Capital
Gains with Labor - IRRF-T	39.172.575.177,73	0.0168	Federal	Labor
Gains with Capital - IRRF-C	21.321.839.561,53	0.0091	Federal	Capital
Gains to Foreign - IRRF-E	7.448.585.342,85	0.0032	Federal	Capital
Others Gains - IRRF-O	4.694.273.238,74	0.0020	Federal	Capital
Financial Operations- IOF	6.784.082.199,75	0.0029	Federal	Cons/Invest
VAT on Manufac. Products - IPI	26.780.064.494,21	0.0115	Federal	Cons/Invest
Import Taxes - II	9.814.826.097,71	0.0042	Federal	Cons/Invest
Export Taxes - IE	42.309.714,80	0.0000	Federal	Consumption
Rural Property - ITR	284.023.315,98	0.0001	Federal	Capital
Bank Account Deposits - CPMF	32.081.789.761,64	0.0138	Federal	Cons/Invest
Social Contribution - COFINS	92.235.608.490,32	0.0395	Federal	Cons/Invest
Fuels - CIDE	7.816.026.081,48	0.0034	Federal	Cons/Invest
Social Contribution - PIS/PASEP	24.228.444.727,89	0.0104	Federal	Cons/Invest
Firms' Net Profits - CSLL	28.070.606.880,29	0.0120	Federal	Capital
Others - Others Revenue	4.414.271.876,36	0.0019	Federal	Capital
Contr. to agriculture- FUNDAF	347.289.898,48	0.0001	Federal	Consumption
VAT on Goods & Services - ICMS	165.666.357.598,86	0.0710	State	Cons/Invest
Vehicle License Registration - IPVA	12.064.105.158,21	0.0052	State	Capital
Property transmissions - ITCD	966.978.258,03	0.0004	State	Capital
Property transmissions- ITBI	2.098.945.204,64	0.0009	Local	Capital
Sales Tax on Service - ISS	14.541.132.561,67	0.0062	Local	Cons/Invest
Urban Property - IPTU	9.528.350.475,61	0.0041	Local	Capital
Payroll - Employer/Employee*	181.856.778.231,19	0.0780	Fed/Sta/Loc	Labor
Fees	12.386.145.280,23	0.0053	Fed/Sta/Loc	Consumption
Total Tax Burden	770.229.626.274,82	0.3317		

Source: Ministry of Finance $*$: Transfer to Others, S-System and Social Security Contribution

Conclusions

In this thesis I analyze some aspects of the economics of taxation. The discussion about the equity-efficiency trade-off has an important role in this thesis. As the taxation causes distortions in the economy, I want to provide useful essays to reduce part of these distortions. Thus, these essays could help the government to create a tax system that induces the allocative efficiency and provides as higher as possible welfare to the society.

Depending on the chapter, I consider or static or dynamic model. I have model to heterogenous and homogenous individuals. These models allow me to get equity-efficiency trade-off conclusions about different taxes schedules. I show that there are some good alternative directions to the incidence of levies. Some alternatives are feasible to implement and could provide faster macroeconomics and welfare effects.

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