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Sacrifice and Efficiency of the Income Tax Schedule*

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Abstract

We investigate the efficiency of equal sacrifice tax schedules in an economy which primitives are exactly those in Mirrlees (1971): a continuum of individuals with identical preferences defined over consumption and leisure who differ with respect to their labor market productivity. Using a separable specification for preferences we derive the minimum equal sacrifice allocation and recover the tax schedule that implements it. The separable specification allows us to use the methodology developed by Werning (2007b) to check whether the schedule is efficient, that is, whether there is no alternative tax schedule that raises more revenue while delivering less utility to no one. We find that inefficiency does not arise for most parametrizations we use to approximate the US economy. For the few cases for which inefficiency does arise, it does so only for very high levels of income and marginal tax rates. **Keywords:** Equal Sacrifice; Efficiency.

J.E.L. codes: H2; D63.

1 Introduction

Following Mirrlees's (1971) seminal paper, it became standard practice to address the design of income tax schedules through the maximization of a social welfare functional under the constraints imposed by the informational structure of the environment. Despite its indisputable methodological advantages, the consensus regarding this approach may have obscured the fact that its adoption implies that a standing on Welfarism as the underlying principle of distributive justice has been made. This

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is not without loss for Welfarism is but one of the possible views of distributive justice that one may adopt. It fails, for example, to encompass the idea of ability-to-pay, or, at least, some of its variants like the 'equal sacrifice principle', which played a prominent role in the debate of distributive justice throughout most of the nineteenth and early twentieth centuries and found renewed interest, after a period of oblivion, thanks in great part to the work of H. P. Young.[1]

Young (1988) first proved that any method of apportioning taxes that satisfies a set of sensible properties is an equal sacrifice schedule for some utility function, while Young (1990) took real world distributions of before and after tax incomes and showed that one could find a common (and empirically sound) utility function that equalized the utility loss of all individuals, and such that this loss was minimal to finance the government revenue requirements.[2] Taken together these two works are suggestive that the simplicity of the notion of equal sacrifice along with the sensible properties of income taxes derived from it may have influenced the political debate and found its way into the actual design of tax schedules.

A shortcoming of Young's works and, for that matter, of all the early literature on equal sacrifice is that it (implicitly) takes taxable income to be independent of the tax schedule. Consequently, no discussion of efficiency can take place. There are several reasons why we might be interested in efficiency. Most important of all is the fact that many ideals of fairness yield to the notion of efficiency, in the sense that efficiency concerns may lead to deviations from a strict application of general fairness principles.[3] This possibility and the apparent empirical relevance of the equal sacrifice principle motivates the assessment of efficiency of equal sacrifice schedules which is the essence of this paper.

We address efficiency of equal sacrifice schedules in a Mirrlees's (1971) environment: an economy inhabited by a continuum of individuals with identical preferences defined over consumption and effort who differ with respect to their privately known labor market productivity, w. Let $T(.)$ be an equal sacrifice schedule derived in such environment. Associated to this schedule, is an equilibrium utility profile $v_T(.)$, where $v_T(w)$ is the utility attained by an individual with productivity w under this tax system. We ask whether there is a Bergson-Samuelson social welfare function, $W(v)$, increasing in v, such that this tax system is the one which maximizes

[1] The equal sacrifice principle is aptly described by John Stewart Mill's words "...whatever sacrifices the government requires should be made to bear as nearly as possible with the same pressure upon all" – cf. Mill (1844). Examples of recent works are Richter (1983); Young (1987, 1988, 1990); Berliant and Gouveia (1993); Ok (1995); Mitra and Ok (1996).

[2] Young (1988) tested and was not able to reject the hypothesis that almost all tax schedules that prevailed in the United States during the period 1957-1987 are based on the equal sacrifice principle. Similar results hold true for Germany, Italy, Japan, and, to a lesser degree, the United Kingdom.

[3] Young (1990), for example, suggests but cannot explore the possibility that efficiency concerns may explain the poor fit of equal sacrifice schedules at the high end of the distribution of income. In his words (Young (1990) p.264) "For high incomes, therefore, the departure from equal sacrifice may be due to efficiency considerations while for low income it is probably due to revenue requirements."

$W(v)$. If there is such a function, then the tax schedule is efficient and $W(.)$ rationalizes it. If not, the tax system is inefficient.[4] This is, of course, equivalent to asking whether there is an alternative tax schedule that induces a utility profile $v^*(.)$, such that $v^*(w) \geq v_T(w)$, $\forall w$, with strict inequality for a subset of positive measure of individuals and which raises at least as much revenue as $T(.)$.The first presentation of the problem is Bourguignon and Spadaro's (2008) while the second one is Werning's (2007b).

The first step toward answering this question is to derive incentive compatible allocations which generates excess resources that are sufficient to finance Government consumption while imposing an identical utility loss on all individuals. Among these allocations we pick the one for which such loss is minimal. The use of a truthful direct mechanism to derive tax schedules provides a useful strategy for incorporating incentive effects when very general budget sets are allowed. This is the approach used in Berliant and Gouveia (1993), which, to the best of our knowledge, was the first work to explicitly take into account labor supply responses in an equal sacrifice based tax problem.[5] In possession of $T(.)$, we follow the approach derived by Werning (2007b) to find efficiency bounds for the marginal tax rates and check whether equal sacrifice schedules respect those bounds.

Throughout the paper, we adopt a separable iso-elastic specification for preferences. Separability is very convenient for our discussions for two different reasons. First, it allows us to apply Werning's (2007a) methodology. Second, under separability, taxable income is invariant to the level of sacrifice as shown by Berliant and Gouveia (1993). This invariance of taxable income with respect to the level of sacrifice rationalizes the abstraction from labor supply responses in Young's (1990) empirical studies and all earlier works on equal sacrifice schedules — e.g. Samuelson (1947).

We first show that if utility of consumption is of the ln type and productivities follow a Pareto distribution simple back of the envelope calculations show that, whenever the distribution of productivities has a finite mean, there is a level of Government consumption above which equal sacrifice leads to Pareto inefficient allocations. Using a typical parametrization for the US economy, the level of expenditures as percentage of Gross Domestic Product - GDP - at which the tax schedule becomes inefficient is, however, above 50%.

Next we consider different parametrization for preferences that bring us closer to Young's (1990) finding that the equal sacrifice principle rationalizes the US income tax schedule for a coefficient of relative risk aversion between 1.5 and 1.7. For our pre-

[4] Note that by finding a SWF that rationalizes an observed schedule one cannot infer that a Mirrlees' approach is being used. For example, if a change in Government consumption requires a different SWF to rationalize the equal sacrifice schedule, then, a Mirrlees's approach is *not* being used.

[5] Although Berliant and Gouveia (1993) raise the issue of efficiency, they do not address it formally. Indeed, while declaring that "One of the aspects of the model we still need to clarify are its welfare properties" and suggesting that inefficiency should result since "The condition of a zero marginal tax rate at the top ability level, emphasized in Sadka (1976) and Seade (1977), is not generally satisfied." Berliant and Gouveia (1993) never produce a systematic discussion of the issue.

ferred specifications for preferences, we seldom find inefficiency. When we do find, it is only for very high levels of income and marginal tax rates. Finally, we consider the exact same coefficients of risk aversion and level of sacrifice studied by Young (1990). We show that, only for the 1957 income tax schedule we find inefficiency but at a level of income beyond the range considered by Young.

The rest of the paper is organized as follows. Section 2 describes the economy. Implementable allocations are described in Section 3. In Section 3.1 we derive the shape of equal sacrifice schedules for different parameters of risk aversion. The main results of this paper are found in Sections 4 and 5. Section 6 concludes. The appendix gathers the derivation of some of the main results.

2 The Environment

The economy is inhabited by a continuum of measure one of individuals with identical preferences defined over consumption, c, and effort, l. Preferences are represented by

$$U(c, l) = u(c) - h(l), \tag{1}$$

where u and h are smooth functions such that $u', -u'', h', h'' > 0$, $\lim_{c \to 0} u'(c) = \infty$ and $\lim_{l \to \infty} h(l) = \infty$.

Individuals differ from one another with respect to labor market productivity, $w \in W \subset R_+$, where W is a closed convex set. All the heterogeneity that exists across individuals is, therefore, captured by w. We assume that w is distributed according to $F(w)$ with associated density f, such that $f(w) > 0$ for all $w \in W$.

An individual with productivity w that makes effort l produces output $y = lw$ with y measured in units of the consumption good. Technology is, in this sense, very simple: one unit of output y is converted one for one into one unit of consumption. We assume that the economy is competitive so that each individual is paid his or her output. We shall, then, refer to y as output and taxable income, interchangeably.

Following Mirrlees (1971), we assume that w is private information. That is, neither w nor l are observed separately. We, too, focus on choices over (c, y) instead of (c, l)-bundles, noting that identical preferences over (c, l), $U(c, l) = u(c) - h(l)$, induce type dependent preferences, $\tilde{U}(c, y; w) = u(c) - h(y/w)$, over (c, y).

An allocation is a mapping $(c, y) : W \longmapsto R^2_+$ that associates to each type, w, a consumption/output pair $(c(w), y(w))$. Let $\Gamma(w)$ denote the set of choices available (budget sets) for an agent of productivity w. Each $\Gamma(.)$ induces an allocation, (c, y), through

$$(c(w), y(w)) \in \arg \max_{(c,y) \in \Gamma(w)} \{u(c) - h(y/w)\}$$

for all w.

The *no-sacrifice allocation*, in particular, is the allocation that results from $\Gamma(w) = \Gamma^0 \equiv \{(c,y); c \leq y\}\ \forall w$,

$$(c_0(w), y_0(w)) \equiv \arg \max_{(c,y)\in\Gamma^0} \{u(c) - h(y/w)\}.$$

We write $v_0(w) = u(c_0(w)) - h(y_0(w)/w)$ to denote the utility attained by type w in the no-sacrifice world. It is important to note that the specific representation for preferences that we use defines the standard by which sacrifice is to be measured; an issue to which we shall return later on.

In the economy there is also a government that must extract an exogenously given amount of resources, B, from the economy. This defines the economy's resource constraint,

$$B \leq \int [y(w) - c(w)] f(w) dw. \tag{2}$$

To induce an allocation satisfying (2), the government chooses the individuals' budget sets, $\Gamma(w)$. In its choice, however, the Government is restricted by the informational structure of the problem.

Let $T : R_+ \to R$ be a tax schedule, defined as $T(y) = \min_c \{y - c; (c,y) \in \Gamma\}$, and

$$v_T(w) \equiv \max_y \{u(y - T(y)) - h(y/w)\}.$$

We define the *sacrifice induced by the tax schedule on an individual of productivity* w, $s(w)$, by

$$s(w) \equiv v(w) - v_T(w),$$

where $v(w)$ is the utility attained by type w individual when the budget set is the one associated with a chosen reference point. In all that follows we take as a reference point the 'no-sacrifice world' for which $\Gamma(w) = \Gamma^0\ \forall w$.*Equal sacrifice tax schedules* are schedules that induce $s(w)$ constant in w, i.e., $s(w) = s\ \forall w$.

3 Incentive-compatible equal-sacrifice systems.

The first step in our study is to find, for a given economy, the associated equal sacrifice tax schedule. Since our goal is to investigate efficiency of the tax schedule it is crucial that we take into account the behavioral responses to taxation. Working directly with budget sets lead us to an intractable problem. We write, instead, a direct mechanism and use the taxation principle to recover the associated tax schedule. In thus proceeding we follow Berliant and Gouveia (1993), which, in turn, relies on Mirrlees (1971).

Let us, then, describe the direct mechanism associated with the minimum equal sacrifice problem.

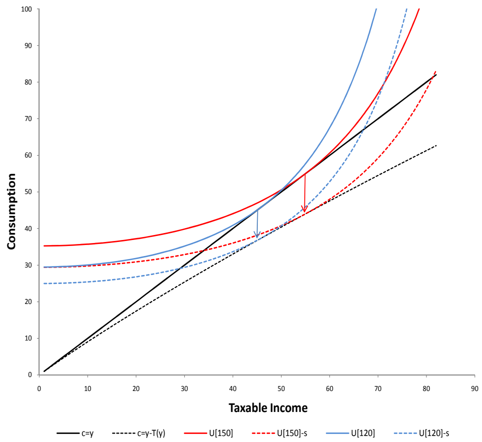

Figure 1: This figure displays an equal sacrifice schedule for iso-elastic preferences as well as optimal choices both at the reference point—indifference curves are straight lines—and under the equal sacrifice schedule—indifference curves are dotted lines. Individuals are of two different productivity levels: $w = 120$ and $w = 150$.

The Direct Mechanism By the revelation principle we can focus on a truthful mechanism in which the planner asks each individual his or her type, w, and uses the (possibly false) report $\hat{w}$ to assign a bundle $(c(\hat{w}), y(\hat{w}))$. To guarantee truthful revelation an allocation $(c, y) = (c(w), y(w))_{w \in W}$ must be such that

$$w \in \arg\max_{\hat{w} \in W} \left\{ u(c(\hat{w})) - h(y(\hat{w})/w) \right\}. \tag{3}$$

Define $v_1(w) \equiv u(c_1(w)) - h(y_1(w)/w)$ as the value of the solution to the problem above where we restrict $(c_1(.), y_1(.))$ to be such that $v_0(w) - v_1(w) = s \; \forall w$, and s, to be the minimum sacrifice for which $\int [y(w) - c(w)] \, f(w) dw \geq B$.

The global incentive compatibility condition (3) is satisfied if and only if the envelope condition,

$$v'(w) = h' \left(\frac{y(w)}{w} \right) \frac{y(w)}{w^2}, \tag{4}$$

and the monotonicity condition,

$$y(w) \text{ increasing in } w, \tag{5}$$

are satisfied.

Under the assumption that $h(.)$ is strictly increasing and strictly convex,

$$\frac{y(w)}{w} = \varphi(v'(w)w), \tag{6}$$

where φ is a strictly increasing function.

Nowhere in this discussion have we used the level of utility, only its variation. This is a very interesting consequence of separability: under incentive compatibility,

the cross-sectional variation in utility pins down the cross-sectional level of output produced by all individuals. Since equal sacrifice is all about preserving 'utility differences', the consequences for the cross-sectional distribution of income are very stark. Under equal sacrifice, differentiability of v_0, implies differentiability of v_1 and $v_0'(w) = v_1'(w)$, which, using (6) leads to $y_1(w) = y_0(w)$ for all w. Individuals must produce the exact same output they produce at the reference state![6]

Because everyone makes the same effort and produces the same output as in the reference state, it must be the case that all sacrifice is due to reduced consumption,

$$s = u(y_0(w)) - u(y_0(w) - T(y_0(w))). \tag{7}$$

Let $\xi(.) = u^{-1}$, then

$$T(y_0(w)) = y_0(w) - \xi(u(y_0(w)) - s). \tag{8}$$

3.1 The Shape of Equal Sacrifice Tax Schedules

Differentiating (7) with respect to y and rearranging terms we get

$$\frac{u'(y)y}{u'(y-T(y))[y-T(y)]} = \frac{1-T'(y)}{1-\varsigma(y)},$$

where $\varsigma(y) = T(y)/y$ is the average tax rate faced by someone who earns y. Using the fact that $T(y) \geq 0$, we have that $u'(y)y \geq u'(y-T(y))[y-T(y)]$ if the coefficient of relative risk aversion is greater than one. Since, for a smooth tax schedule, $T'(y) \geq \varsigma(y)$ is necessary and sufficient for average taxes to be increasing, one immediately connects risk aversion and progressivity. An equal sacrifice schedule is, therefore, progressive if and only if the coefficient of relative risk aversion of the chosen utility function is greater than one.

Samuelson (1947) derived this result disregarding incentives and assuming that utility depended only on consumption. If preferences depend not only on consumption but also on leisure and incentives are considered this need not hold, in general. What we have shown is that, for the special case of separable preferences, taxable income *is* invariant to the level of sacrifice and Samuelson's (1947) result remains valid.

Marginal and Average Tax Rate Progressivity We have used progressivity to describe a tax schedule for which *average* taxes weakly increase with income. Progressivity may refer also to increasing *marginal* tax rates. Progressivity in the former sense is an appealing notion for it implies that after tax income is more equally distributed than before tax income when the Lorenz criterion is used, while marginal tax rate progressivity is of interest for it is to marginal rather than average tax rates that dead

[6] This result, first derived by Berliant and Gouveia (1993) — see their Proposition 4 — is illustrated in Figure 1.

weight losses are associated. What we show next is that the two concepts of progressivity are intertwined in the case of equal sacrifice schedules and constant relative risk aversion preferences for consumption.

Let $u_0(w) = u(c_0(w))$ and use the fact that utility differences are the same for all w to see that

$$1 - \frac{\xi'(u_0(w) - s)}{\xi'(u_0(w))} = \tau(w),$$

where $\xi'(u)$ is the marginal cost in consumption terms of delivering utility u and $\tau(w) = T'(y(w))$. Note that ξ is an increasing convex function of u which means that $0 < \tau < 1$ for $u(y_0) - \underline{u} > s > 0$, where $\underline{u} = \lim_{c \to 0} u(c)$.

From now on, we restrict our analysis to an iso-elastic specification for preferences,

$$u(c) = \frac{c^{1-\rho} - 1}{1 - \rho},$$

for $\rho > 0$, $\rho \neq 1$, $u(c) = \ln c$ for $\rho = 1$ and

$$h(l) = l^{\gamma}/\gamma$$

for $\gamma > 1$. We do so not only for tractability, but also because equal sacrifice tax schedules have appealing properties regarding, among other things, invariance with respect to rescaling, when preferences are iso-elastic — See Young (1987),Young (1988).

Equal sacrifice in the case of iso-elastic separable preferences implies

$$1 - \varsigma(y_0(w)) = [1 - \tau(w)]^{1/\rho}, \tag{9}$$

which connects in a very stark way average and marginal tax rates.

Remarks To understand how the equal sacrifice schedule induces invariance of taxable income let us take the case $\rho > 1$ as an example. A decrease in net wage induced by an increase in the marginal tax rate would cause an increase in effort if taxes were linear. Consider the linear approximation of an individual's budget constraint at his or her optimal choice $y(w)$, $c(w) \leq y(w)(1 - \tau(w)) + I(w)$, where $I(w) = T(y(w)) - T'(y(w))y(w)$, is the virtual income as defined in Hausman (1985).

Using (9) we may rewrite the expression for virtual income as

$$I(w) = y(w) \left\{ [1 - \tau(w)]^{1/\rho} - [1 - \tau(w)] \right\}.$$

The term in curly brackets is a positive increasing function of $\tau(w)$. Virtual income introduces an additional income effect that adds to the traditional income effect, the one that results from the decrease in the 'price of leisure', in such a way as to exactly offset the substitution effect. As a result, taxable income is held constant.

4 Sacrifice and Efficiency

We now formalize the main question of this paper. Define an *environment*, $\mathcal{E}$, as a tuple (U, F, B) where $U(c, l) = u(c) - h(l)$ is the utility function representing the (identical across agents) preferences of all individuals, $F(w)$ is the distribution of skills and B the Government's revenue requirement.

Let $\Psi(.)$ be an arbitrary Bergson-Samuelson social welfare function, and define *the Mirrlees problem* at environment $\mathcal{E}$ for the social welfare function Ψ as

$$\max \int \Psi(u(c(w)) - h(y(w)/w)) f(w) dw$$

subject to

$$\int \{y(w) - c(w)\} f(w) dw \geq B$$

and

$$w \in \arg\max_{\hat{w}} \{u(c(\hat{w})) - h(y(\hat{w})/w)\}$$

We say that a tax schedule, $T(.)$, is *rationalizable at environment* $\mathcal{E}$ if there is a social welfare function Ψ such that the allocation that solves the Mirrlees' problem at environment $\mathcal{E}$ for the social welfare function Ψ is induced by $T(.)$. We then say that *the pair* $(\mathcal{E}, \Psi)$ *rationalizes* $T(.)$.

The question we ask is: given an environment, $\mathcal{E}$, and a tax schedule, $T(.)$, derived under the equal sacrifice principle, is it always the case that we may find a Paretian social welfare function Ψ such that the pair $(\mathcal{E}, \Psi)$ that rationalizes $T(.)$?

To address this question we need to describe the environment. We choose empirically sound preferences and a representation $U(c, l) = u(c) - h(l)$ for these preferences that captures the social norm we believe to represent those of the societies we study. Under this choice of preferences, we derive, for each level of productivity w, the income produced in the no-sacrifice world, $y_0(w) \equiv \arg\max_y \{u(y) - h(y/w)\}$, and the associated equilibrium utility profile, $v_0(w) = u(y_0(w)) - h(y_0(w)/w)$.

Assume that we somehow know the distribution of types, $F(.)$. Then, the description of the environment is complete once we define, B.

Given the environment, $\mathcal{E}$, define for each level of sacrifice, s, the consumption of a type w individual by $c(w) = u^{-1}(s + u(y_0(w)))$. Using $F(w)$ one links to each level of sacrifice, s, the revenue raised by the Government,

$$R = \int \left\{ u^{-1}(s + u(y_0(w))) - y_0(w) \right\} f(w) dw.$$

Finally, one finds the minimum equal sacrifice allocation by choosing s such that $R = B$. The associated tax schedule is given by (8).

The Efficiency Test Two recent works establish methodologies that allow us to answer the question we posed: Werning (2007b) and Bourguignon and Spadaro (2008).

The approach developed by Bourguignon and Spadaro (2008) consists in inverting optimal tax formulae that arise from the solution of a Mirrlees' program and to check whether $\Psi'(v) \geq 0$ for all v, i.e., to check whether the social welfare function is Paretian.

Werning (2007b) instead takes the allocation $(c(w), y(w))$ induced by the tax schedule $T(.)$ and the associated function $v(w)$ and solves the problem of maximizes Government revenue subject to delivering no less utility for any individual (or a positive measure of individuals). A tax schedule is efficient if and only if there is no alternative allocation $(\tilde{c}(w), \tilde{y}(w))$ that delivers no less utility for all agents and raises more revenue.

As it turns, Werning's (2007b) procedure is more convenient for our purposes. Hence, we start by replicating—see Appendix A.1—his findings for our setting.

Let $T(.)$ be a smooth tax schedule with associated marginal tax function $\tau(.)$, such that $\tau(w) > 0\ \forall w$, and define

$$\Phi(w) = (\gamma - 1)\frac{d \ln y}{d \ln w} - \frac{d \ln \tau}{d \ln w} - \frac{d \ln f}{d \ln w}. \tag{10}$$

Werning's (2007b) Proposition 4 adapted to our setting states that: *i)* if $\Phi(w) \leq 1 + \gamma$ the tax schedule is always efficient; *ii)* if, however, $\Phi(w) > 1 + \gamma$, $T(.)$ is efficient if and only if $\tau(w)$ is such that

$$\tau(w) \leq \frac{\gamma}{\Phi(w) - 1},\ \forall w. \tag{11}$$

Next, we use the properties of equal sacrifice schedules with separable and iso-elastic preferences to obtain Proposition 1, below. Before, however, it is important to remark that the term $d \ln y / d \ln w$ that appears in the definition of $\Phi(.)$, equation 10 is *not* the elasticity of taxable income with respect to w. Instead, it is the cross-sectional derivative of taxable taxable income with respect to w, i.e. the percentage change in taxable income when we compare individuals whose productivities differ by one percent for a given tax structure.[7] This makes the application of (11) quite simple under the separable iso-elastic specification for preferences, since $y(w) = y_0(w) = w^{\gamma/(\gamma+\rho-1)}$ which then implies $d \ln y / d \ln w = \gamma / (\gamma + \rho - 1)$.

Proposition 1. *For separable and iso-elastic preferences, an equal sacrifice labor income tax schedule $T(.)$ is efficient if and only if marginal tax rates τ are such that*

$$(1 - \tau(w))^{2-1/\rho}\frac{\rho\gamma}{\gamma + \rho - 1} - (1 - \tau(w))\left[\gamma - \frac{d \ln f}{d \ln w} - 1\right] \leq \frac{d \ln f}{d \ln w} + \frac{\gamma - 1 + \rho(1+\gamma)}{\gamma + \rho - 1}, \tag{12}$$

for all w.

[7] The two values differ if the virtual income varies as a percentage of total income across individuals, something that must happen for a non-linear tax schedule.

Proof. See Appendix A.2. *Q.E.D.*

The polynomial equation in Proposition 1 admits a closed form solution for a few cases of interest, e.g., $\rho = 1/2$, $\rho = 2$ and $\rho = 1$. For $\rho = 2$ a third degree polynomial defines the regions of efficiency. In the other two cases linear expressions obtain — see Appendix A.2. Note that this polynomial equation only involves structural parameters, which is in contrast with the optimal tax formulae found in Diamond (1998), Saez (2001) and Bourguignon and Spadaro (2008).

5 Sacrifice and Efficiency in Practice

We are now in a position to ask whether, at environment $\mathcal{E}$, it is possible to find Ψ such that $(\mathcal{E}, \Psi)$ rationalizes the equal sacrifice schedule $T(.)$.

Before we do it, however, let us consider a related question. Given an equal sacrifice tax schedule $T(.)$, is it possible to find a pair $(\mathcal{E}, \Psi)$ that rationalizes $T(.)$? The difference between the two questions is that, for this second question we are given the degree of freedom to choose the environment as well as the social welfare function.

The answer to this question is but a corollary to Proposition 2 in Werning (2007b), which states that "For any tax schedule, $T(y)$, and its resulting allocation, there is a set of skill distributions, $F(w)$, and net endowments, $-B$, for which the outcome is Pareto Efficient and another set of skill distributions, $F(w)$, and net endowments, $-B$, for which it is Pareto inefficient." Since Werning's (2007b) result holds for any smooth tax schedule, $T(.)$, the answer to our question is yes. We may *always* build an environment for which the tax schedule derived under the equal sacrifice principle is rationalizable.

Interestingly, the same proposition states that for any tax schedule it is always possible to find an environment for which the tax schedule is *not* rationalizable. The degrees of freedom one is given if allowed to choose the environment is sufficient to get this 'anything goes' type of result.

However interesting this result may be for highlighting the fact that equal sacrifice is not a particular instance of Welfarism, the last results are not what matters from a policy perspective; the environment is not an object of choice for the policy maker. What we really want to know is whether an equal sacrifice schedule is efficient for real world societies.

The $u(.) = \ln(.)$ case. Let us start our investigation with $u(.) = \ln(.)$, in which case the equal sacrifice principle yields a very simple tax schedule: a linear one.

To define the bounds of Proposition 1 we also need the values of γ and $d \ln f / d \ln w$.

When $\rho \neq 1$ we note that the cross-sectional elasticity of taxable income $d \ln y / d \ln w$ is equal to $\gamma / (\gamma + \rho - 1)$ and consider empirically sound values for this elasticity to

choose γ as a function of ρ.[8] This procedure does not work, however, when $u(.) = \ln$. Instead, we borrow from the literature values for the Frisch elasticity of taxable income, $\epsilon^f = \gamma/(\gamma-1)$, to choose γ.

As for $d\ln f/d\ln w$, assume for the moment that the tax system induces a Pareto distribution of income, $G(y) = 1-(\underline{y}/y)^{\alpha-1}$, with support $[\underline{y},\infty)$, $\underline{y}>0$, and associated density $g(y) = \kappa y^{-\alpha}$, $\alpha > 1$, where $\kappa = (\alpha-1)\underline{y}^{\alpha-1}$. Next, note that

$$\frac{d\ln f}{d\ln w} = \frac{d\ln g(y)}{d\ln y} = -\alpha,$$

since $y = w$ when $u(.) = \ln$, $h(l) = l^\gamma/\gamma$, and taxes are linear. The distribution of skills that generates the distribution of income $G(y)$ is, in this case,

$$F(w) = 1-\left(\frac{\underline{w}}{w}\right)^{(\alpha-1)},$$

for $w \in [\underline{w},\infty)$.[9]

Indeed, under these assumptions $d\ln y/d\ln w = 1$, and $d\ln\tau/d\ln w = 0$, for an equal sacrifice schedule. Therefore, $\Phi(w) = -2 - d\ln f/d\ln w$. Using, our choice for $f(w)$, (11) becomes

$$\tau \le \frac{\gamma}{\alpha+\gamma-2}, \tag{13}$$

as in Werning (2007b).

Equation 13 imposes bounds on the marginal tax rate when $\alpha > 2$ and allows for simple back of the envelope calculations. Moreover, with linear taxes, $B = \int T(y)dF(y) = \tau\int ydF(y)$, yields a one to one mapping from $B\left(\int ydF(y)\right)^{-1}$ to τ, which allows us to use Government consumption as a percentage of GDP as our reference for τ.[10]

The fact that we chose a commonly used specification for the distribution of income—e.g., Saez (2001); Diamond (1998)—allows us to borrow the value of the key parameter α from the literature. Saez (2001), for instance, considers the following values for α for the US economy: 1.5, 2 and 2.5, while Werning (2007b) considers $\alpha = 3$. For Saez's (2001) first two values, condition (13) does not have a bite, so we focus on $\alpha = 2.5$ and $\alpha = 3$.

When $\alpha = 2.5$, if we take a sensible value for ϵ^f, $\epsilon^f = 2$ for example, expenditures must be at least 75% for inefficiency to result. If we let $\epsilon^f \to \infty$, the maximum value for τ is close to 70%.

When $\alpha = 3$, inefficiency arises for lower levels of Government consumption. If $\epsilon^f = 2$, τ cannot exceed 67% for taxes to be efficient. The right hand side of (13) varies

[8] We refer to the elasticity of taxable income with respect to w, $d\ln y/d\ln w$. Some studies define it, instead, as $d\ln y/d\ln(1-\tau)$. We shall return to this point at the end of Section 5.1.

[9] For $\rho \neq 1$ we use, instead, $F(w) = 1-(\underline{w}/w)^{\varphi-1}$ where $\varphi = (\alpha\gamma+\rho-1)/(\gamma+\rho-1)$.

[10] Note also that a Pareto distribution does not have a finite mean if $\alpha < 2$, and it does not have a finite variance if $\alpha < 3$. Hence, this integral is only defined for $\alpha < 2$.

from 1, when $\epsilon^f = 0$ to 50%, when $\epsilon^f \to \infty$. The literature, seldom considers values for ϵ^f greater than 4, in which case, Government consumption of up to 55% of GDP may be efficiently financed under the equal sacrifice principle. Noting that transfers must be excluded from this calculation, the levels of expenditures as a share of GDP that leads to inefficiency is higher than that of most countries.

We conclude that, if one is willing to accept that $u(.) = \ln(.)$ reasonably describes the way the American society perceives ability to pay, then a tax schedule based on the equal sacrifice principle should be linear and would efficiently finance the current levels of Government consumption.

The $\rho \neq 1$ case. The case $u(.) = \ln(.)$ is an important benchmark. Preferences representable by this functional form induce inelastic labor supply, which does seem to adhere reasonably well to the data; for prime age males, at least.

It is important, however, to realize that ρ defines not only the elasticity of labor supply, but also the social norm of the society we aim at describing. That is, by choosing $\rho = 1$, for example, we commit ourselves not only to a world in which the elasticity of labor supply is zero, but also to a specific view of how society perceives the sacrifice born by different individuals.

We may, however, be interested in retaining some degrees of freedom to explore different social perceptions of equity, as captured by ρ, while holding the labor supply elasticity at an empirically relevant range. This is particularly relevant if we recall the results in Young (1990) which indicates that the equal sacrifice principle rationalizes the US tax schedule for the 1957-1987 period if ρ is in the range $[1.5, 1.7]$.

As we have seen a value of ρ greater than one is needed for the equal sacrifice tax schedule to be progressive, which is the best description of the US tax system for that period. Henceforth, we focus on this case by varying ρ in the range suggested by young and adjusting γ to compensate for changes in ρ. [11] Since $d \ln f / d \ln w = -\varphi$, $\forall w$, for a Pareto distribution, the bounds defined in (12) are independent of w. We assume that $\alpha = 3$ and that expenditures are 30% of GDP. Figure 3 displays our results for $\rho = 1.5$ and $\rho = 1.6$ holding $\gamma = 1.5$. These values for ρ are in the range deemed to represent well the perception of sacrifice for the US economy for the period studied by Young (1990).

[11] Although we do not have full flexibility for disentangling the perception of sacrifice from the elasticity of taxable income since ρ pins down the sign of the latter, provided that we accept the sign, we can use γ to hold it at a desired level. Because Young (1990) took taxable income as exogenous, the elasticity of taxable income was never of concern. The combination of exogenous taxable income and a progressive tax schedule, that we consider here, requires both separability and one's commitment to a specific sign for the cross-sectional labor supply elasticity. Moreover, if we want to match other specific empirical parameters, e.g., income (and compensated) elasticities of labor supply, then our degree of freedom is lost.

5.1 Main results for the US economy

We have so far been borrowing the relevant parameters of the distribution of w, $F(.)$, from the literature. In this section, we retrieve F from the data, using the actual US tax schedule, $T(.)$, and distribution of income, $G(y)$.

The Procedure Step by Step If $T(.)$ is the actual tax schedule of the economy we are studying, define

$$y_T(w) \equiv \arg\max_y \{u(y - T(y)) - h(y/w)\}$$

and assume that $y_T(w)$ is invertible. In this case, if $w_T(y)$ is the inverse of $y_T(w)$ — $y_T(w_T(y)) = y$ — then $F(w_T(y)) \equiv G(y)$ uniquely recovers $F(w)$.[12] An assumption that guarantees invertibility of $y_T(w)$ and greatly simplifies the procedure is that the actual tax system may be reasonably approximated by a linear one, $T(y) = \tau y$—e.g., Saez (2001). Under $T(y) = \tau y \; \forall y$,

$$w_\tau(y) = y^{\frac{\rho+\gamma-1}{\gamma}}(1-\tau)^{\frac{1-\rho}{\gamma}}.$$

Note that, by choosing the linear approximation we are either departing from the assumption that the current system is an equal sacrifice one, or we must restrict ourselves to the ln specification. Either view is in contrast with what Young (1990) has argued to be the best description of the data for the 1957-1987 period. We shall do so under the implicit assumption that, in contrast with those in the period analyzed by Young (1990), the current tax system is not based on the equal sacrifice principle.[13]

Figure 4 displays the distribution recovered through this procedure using labor income data from the Panel Study of Income Dynamic (PSID) for the year 2007. We chose $\rho = 1.5$ and $\gamma = 1.5$, which yields a cross-sectional elasticity of labor supply of $-.25$. We call the distribution of skills thus recovered, the 'empirical' distribution of skills. Next, to generate a well behaved $\bar{\tau}$ function, we adjust a parametric distribution to the empirical one. Figure 4 displays a Generalized Extreme Value distribution adjusted to the empirical distribution we recovered under the assumptions above. Because $d \ln f / d \ln w$ is decreasing in w after $\ln f$ reaches its maximum, the efficiency bound for marginal tax rates, $\bar{\tau}$, is decreasing in w in the same region. In the bottom part of figure 4 the dashed line is $\bar{\tau}$ and the solid line is the marginal tax rate associated with a minimum equal sacrifice schedule chosen to finance expenditures at 30% of GDP. $\bar{\tau}$ is only displayed for the region in which it is lower than 100%. It

[12] More generally, $y_T(w)$ is a selection from $y_T(w) \in \arg\max_y \{u(y - T(y)) - h(y/w)\}$. If $y_T(w)$ is not invertible, e.g., if the budget set induced by this tax system has concave kinks, which induces bunching an interval of skills is associated to some income levels, and $F(.)$ cannot be uniquely determined. da Costa and Pereira (2010) consider alternative procedures to deal with the issue.

[13] If the empirical schedule is itself an equal sacrifice one, the resulting schedule from our procedure will coincide with the empirical one. We still need $d \ln f / d \ln w$ to apply Proposition 1.

is apparent that the equal sacrifice schedule is efficient for the levels of income we investigate.

Figure 5 displays additional results for the Generalized Extreme Value distribution. The elasticity of taxable income from .720, in the right upper corner of the figure to .876 in the left lower corner. We consider only two values for ρ, 1.3 and 1.4, which yield a degree of progressivity more in line with current schedules in most developed countries than those values explored by Young. Inefficiency never arises for the range of income we study.

Next, we return to a Pareto distribution, motivated in part by the fact that this distribution has proven particularly useful in calculating optimal tax schedules — e.g., Saez (2001); Diamond (1998). Even more important for our purposes is the fact that the combination of progressive schedules and empirical distributions with decreasing values for $d \ln f / d \ln w$, which characterizes the data we use, implies that, if inefficiency is to arise, it will be in the upper part of the distribution, the one which a Pareto distribution fits best. In fact, because $d \ln f / d \ln w; -2$ is a necessary condition for $\bar{\tau} < 100\%$, the interval for which $d \ln f / d \ln w > -2$ may be neglected.

There is, however, a drawback in using a Pareto distribution. To fit the upper tail of the distribution well, one severely misses the bottom part, thus making the calculation of revenues unreliable. The left side of Figure 6 illustrates just this, while the right side shows the fit of two alternative distributions: a Generalized Extreme Value, which we have already seen and a Generalized Pareto Distribution — GPD. This latter distribution adds flexibility to fit the mode of the distribution and allows us to adopt the following procedure. We use the empirical distribution of skills to find the level of sacrifice needed to generate the the target level of revenue. We then fit a (scaled down) GPD starting at the distribution's mode, and use it to calculate $\bar{\tau}$. Figure 7 displays efficiency tests for the two distributions. In this example we do not take into account the bad fit of the GPD at the bottom part of the distribution and simply fit the distribution starting at the mode. Figure 8, in contrast, displays the results when we use the empirical distribution to calculate the level of sacrifice, following the procedure just suggested.

The remainder of our figures display our findings regarding the tax schedules analyzed by Young (1990). We use the same values for ρ and the same levels of sacrifice found in Young (1990) and test the efficiency of the associated schedules. Figure 9 displays our main results for this exercise. Since we borrow from Young's (1990) work the levels of sacrifice, the fit at the bottom of the distribution is no longer an issue. Hence, for the calculation of $\bar{\tau}$ we use a Pareto distribution and pick the values for γ by holding the elasticity of labor supply constant at -0.2. The graphs in the left side of the figure display results for $\alpha = -3$. We find inefficiencies only for the 1957 schedule and for levels of income around 250,000 dollars. This is above the maximum of the range found in Young's (1990) paper. The right side of 9 displays the same exercises but using $\alpha = -3.35$, in which case inefficiency arises for the 1957 at lower levels of income, but still out of the range considered by Young (1990).

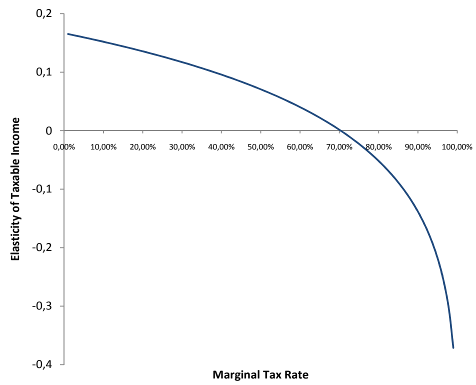

Figure 2: The elasticity of taxable income is defined as $d\ln y/d\ln(1-\tau)$. As marginal tax rates change, virtual income adjusts according to what is adequate to induce equal sacrifice.

In figure 10 we focus on 1957 and consider higher levels for the elasticity of taxable income more in line with the empirical evidence. It is apparent that taxable income must be well beyond the levels considered by Young for inefficiency to arise. For completeness, the year of 1987 is considered in Figure 11.

All in all, these results suggest that inefficiency is not likely the cause of deviations from equal sacrifice suggested by Young (1990). Note however that, because we only guessed a parameter to describe the descending part of the density of income, we did assess the revenues generated by the tax schedule.

Cautionary Note To make sense of Young's (1990) measure of sacrifice taxable income must be invariant to the tax schedule. This fact has led to our choice of a separable specification for preferences. We too focused on $\rho \geq 1$ to guarantee that the resulting equal sacrifice schedules were progressive. A consequence of these modeling choices is that the elasticity of taxable income with respect to the marginal tax rate that we obtain differs from most values used in the literature – e.g., Gruber and Saez (2002). In particular, we have positive values for most levels of income in our exercises, whereas most studies find the opposite to be true. It is important to note that we are no longer talking about cross-sectional elasticities but to elasticity defined as the percentage change in taxable income for each productivity type were his or her marginal tax rate to be increased by one percent.

Figure 2 plots the elasticity of taxable income against the marginal tax rate of an equal sacrifice schedule for $\rho = 1.5$ and $\gamma = 2.5$. Accounting for the variation in elasticity is the change in the relative importance of $y(w)(1-\tau(w))$ and $I(w)$ in an individual's disposable income, as marginal tax rates increase. Even though the elasticity of taxable income is substantially higher for high levels of income, the overall level is still lower than what most studies consider.

One possible reason why the elasticity of taxable income is higher in the 'real world' is the possibility of tax elision or evasion that our model does not allow for.

6 Conclusion

In a series of papers in the late 1980's Young (1987, 1988, 1990) has forcefully argued that the income US income tax schedule for the period 1957 to 1987 could be rationalized by direct applications of the equal sacrifice principle. The body of work that followed allowed us to better understand the restrictions imposed on observed tax schedules by the equal sacrifice system—Mitra and Ok (1996); Ok (1995)—and the consequences of taking incentives into account explicitly—Berliant and Gouveia (1993), among other things. This paper addresses an important issue that has not received a thorough analysis: the efficiency of such schedules.

We consider a separable iso-elastic specification for preferences that greatly facilitates the derivation of equal sacrifice schedules and allows for an explicit evaluation of efficiency using the methodology developed by Werning (2007b). We find that, if utility of consumption is logarithmic and the cross-sectional distribution of productivities is Pareto with a decay parameter above 3, there is always a level of per capita government spending above which an equal sacrifice tax schedule is inefficient. Back of the envelope calculations indicate that these thresholds are well above the average Government consumption for the United States.

We assume that risk aversion is greater than one, which yields a progressive equal sacrifice schedule, as shown by Samuelson (1947). For most parametrizations we have used, equal sacrifice schedules are either always efficient or become inefficient only at the far right of the distribution of taxable income, when marginal tax rates are urealistically high.

We finally check whether, for the levels of sacrifice found by Young (1990) to be compatible with the tax schedules that prevailed in the US for the period 1957-1987, inefficiency concerns could account for their relatively poorer fit at the very top of the income distribution.[14] If the elasticity of labor supply is relatively high in absolute value ($\leq -.2$) we find inefficiency only for the year of 1957 and for very high income levels. For all other periods and all other exercises we ran the equal sacrifice schedule is efficient within the income range considered by Young (1990).

With all the provisos that such stripped down environment requires, our findings are suggestive that, *if* the idea of equal sacrifice has really influenced the design of the US schedule, efficiency concerns are not likely to have imposed limits on marginal tax rates for the range of income studied by Young (1990).

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A Appendix

A.1 Deriving Expression (11)

In this appendix we provide, for sake of completeness, a sketch of the proof of necessity for efficiency condition (11). A complete proof of both necessity and sufficiency is found in Werning (2007b).

An allocation $(\bar{c}(w), \bar{y}(w))$ that generates a utility profile $\bar{v}(w)$ is Pareto efficient if and only if $(\bar{y}(w), \bar{v}(w))$ solves:

$$\max_{y(.),v(.)} \int [y(w) - e(v(w), y(w), w)] \, f(w)dw$$

s.t.,

$$v'(w) = h'\left(\frac{y(w)}{w}\right)\frac{y(w)}{w^2},$$

$$y(w) \text{ increasing,}$$

and

$$v(w) \geq \bar{v}(w) \; \forall w,$$

where $e(v(w), y(w), w)$ is implicitly defined by

$$v(w) = \frac{e(v(w), y(w), w)^{1-\rho}}{1-\rho} - \frac{y(w)^{\gamma}}{\gamma w^{\gamma}}.$$

Disregarding the monotonicity constraint, we may write the Lagrangian

$$\int \Bigg\{ [y(w) - e(v(w), y(w), w)]\, f(w) + \mu(w) \left[v'(w) - \frac{y(w)^{\gamma}}{w^{\gamma+1}} \right] \\ + \lambda(w)\, [v(w) - \bar{v}(w)] \Bigg\} dw.$$

Integrating by parts and using the transversality conditions, we re-write the Lagrangian as

$$\int \Bigg\{ [y(w) - e(v(w), y(w), w)]\, f(w) - \mu'(w) v(w) - \mu(w) \frac{y(w)^{\gamma}}{w^{\gamma+1}} \\ + \lambda(w)\, [v(w) - \bar{v}(w)]\, dw \Bigg\}$$

First order conditions are

$$\big(1 - e_y(v(w), y(w), w)\big)\, f(w) = \mu(w) \gamma \frac{y(w)^{\gamma-1}}{w^{\gamma+1}} \tag{14}$$

and

$$-\, e_v(v(w), y(w), w) f(w) = \mu'(w) - \lambda(w) \tag{15}$$

which implies,

$$-\, e_v(v(w), y(w), w) f(w) \leq \mu'(w) \tag{16}$$

Focusing on the case $1 - e_y = \tau > 0$, we have that $\mu > 0$ as well and (14) can be written in logs

$$\ln \tau + \ln f = \ln \mu + \ln \gamma + (\gamma - 1) \ln y - (\gamma + 1) \ln w$$

which implies

$$\frac{d \ln \tau}{d \ln w} + \frac{d \ln f}{d \ln w} = \frac{d \ln \mu}{d \ln w} + (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1)$$

Next, note that

$$\frac{d \ln \mu}{d \ln w} = \frac{\mu'}{\mu} w \geq -\frac{e_v f}{\mu} w = -\frac{\gamma y^{\gamma-1} e_v f w}{\left(1 - e_y\right) f w^{\gamma+1}} = -\frac{\gamma}{\tau} e_v \frac{y^{\gamma-1}}{w^{\gamma}} = -\frac{\gamma}{\tau}(1 - \tau)$$

Hence,

$$\frac{d \ln \tau}{d \ln w} + \frac{d \ln f}{d \ln w} \geq -\frac{\gamma}{\tau}(1 - \tau) + (\gamma - 1) \frac{d \ln y}{d \ln w} - (\gamma + 1). \tag{17}$$

Let

$$\Phi(w) = (\gamma - 1) \frac{d \ln y}{d \ln w} - \frac{d \ln \tau}{d \ln w} - \frac{d \ln f}{d \ln w}.$$

If $\Phi(w) = 1$ then (17) is always satisfied. Assume then $\Phi(w) \neq 1$. In this case, (17) becomes

$$\tau(w) \leq \frac{\gamma}{\Phi(w) - 1},$$

if $\Phi(w) > 1$, and

$$\tau(w) \geq \frac{\gamma}{\Phi(w) - 1},$$

if $\Phi(w) < 1$. Because the right hand side is negative and we have assumed $\tau(w) \geq 0$ the condition doesn't have a bite. Moreover, if $\Phi(w) \leq 1 + \gamma$, $\bar{\tau}(w) > 1$, which, once again does not restrict τ. As a result, regions of inefficiency may only exist if $\Phi(w) > 1 + \gamma$.

A.2 Proof of Proposition 1.

Proof. Assume that the tax function, $T(.)$, is twice continuously differentiable. Differentiating (7) and rearranging terms yields

$$1 - \frac{u'(y_0(w))}{u'(y_0(w) - T(y_0(w)))} = T'(y_0(w)). \tag{18}$$

That is, the marginal tax rate faced by any individual is (one minus) the ratio of his or her marginal utility of income before and after the introduction of taxes. Next, differentiate (18) to obtain

$$\begin{aligned}
\frac{T''(y_0(w))}{1 - T'(y_0(w))} &= \left\{ \frac{u''(y_0(w) - T(y_0(w)))}{u'(y_0(w) - T(y_0(w)))} \left[1 - T'(y_0(w))\right] - \frac{u''(y_0(w))}{u'(y_0(w))} \right\} \\
&= \frac{1}{y_0(w)} \left\{ r(c_0(w)) - r(c_1(w)) \frac{1 - T'(y_0(w))}{1 - \varsigma(y_0(w))} \right\},
\end{aligned} \tag{19}$$

where $\varsigma(y) = T(y)/y$ is the average tax rate and $r(c)$ is the coefficient of relative risk aversion at consumption level c.

For the case of CRRA preferences, $r(c) = \rho$ for all c, and expression (19) reduces to

$$- \left. \frac{d \ln (1 - T'(y))}{d \ln y} \right|_{y = y_0(w)} = \rho \left\{ 1 - \frac{1 - T'(y_0(w))}{1 - \varsigma(y_0(w))} \right\},$$

where we have also used the invariance property of taxable income.

Next, re-write the expression above as

$$\rho \left\{ \frac{\tau(w) - \varsigma(y_0(w))}{1 - \varsigma(y_0(w))} \right\} \frac{d \ln y_0(w)}{d \ln w} = \frac{d \ln \tau(w)}{d \ln w} \frac{\tau(w)}{1 - \tau(w)}. \tag{20}$$

Noting that $\varsigma(y_0(w)) = 1 - [1 - \tau(w)]^{1/\rho}$ and $d \ln y_0(w) / d \ln w = \gamma / (\gamma + \rho - 1)$, we get

$$\rho \left\{ 1 - (1 - \tau(w))^{1 - 1/\rho} \right\} \frac{d \ln y}{d \ln w} = \frac{d \ln \tau(w)}{d \ln w} \frac{\tau(w)}{1 - \tau(w)}. \tag{21}$$

Next, to simplify the algebra let $a = \frac{d \ln y}{d \ln w}$ and $b = \frac{d \ln f}{d \ln w}$. Then note that

$$\left\{1 - \tau(w) - (1 - \tau(w))^{2-1/\rho}\right\} \frac{\rho a}{\tau(w)} = \frac{d \ln \tau(w)}{d \ln w}$$

Using the expression above in (17) we get

$$\left\{1 - \tau - (1 - \tau)^{2-1/\rho}\right\} \frac{\rho a}{\tau} + b \geq -\frac{\gamma}{\tau}(1 - \tau) + (\gamma - 1)\, a - (\gamma + 1) \tag{22}$$

Assuming that $\tau > 0$,

$$\left\{1 - \tau - (1 - \tau)^{2-1/\rho}\right\} \rho a \geq -\gamma(1 - \tau) - [b - (\gamma - 1)\, a + (\gamma + 1)]\, \tau,$$

which we may rewrite as

$$(1 - \tau)\, [(\rho + \gamma - 1)\, a - b - 1] - (1 - \tau)^{2-1/\rho} \rho a \geq -\, [b - (\gamma - 1)\, a + (\gamma + 1)]\, .$$

Recalling that $a = \gamma / (\gamma + \rho - 1)$,

$$(1 - \tau) \left[\gamma - \frac{d \ln f}{d \ln w} - 1\right] - (1 - \tau)^{2-1/\rho} \frac{\rho \gamma}{\gamma + \rho - 1} \geq -\frac{d \ln f}{d \ln w} - \frac{\gamma - 1 + \rho(1 + \gamma)}{\gamma + \rho - 1}, \tag{23}$$

for all w. *Q.E.D.*

B Figures

Figure 3: Marginal Taxes Rate and $\bar{\tau}$: $\rho > 1$

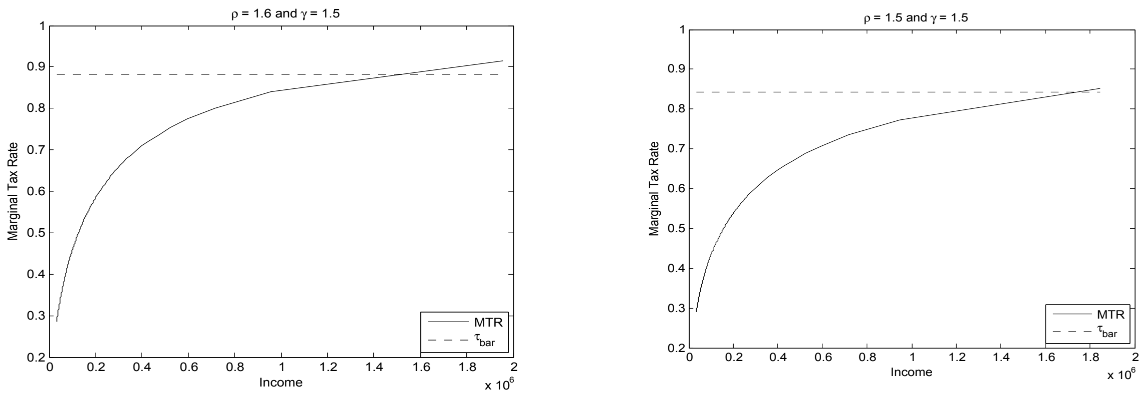
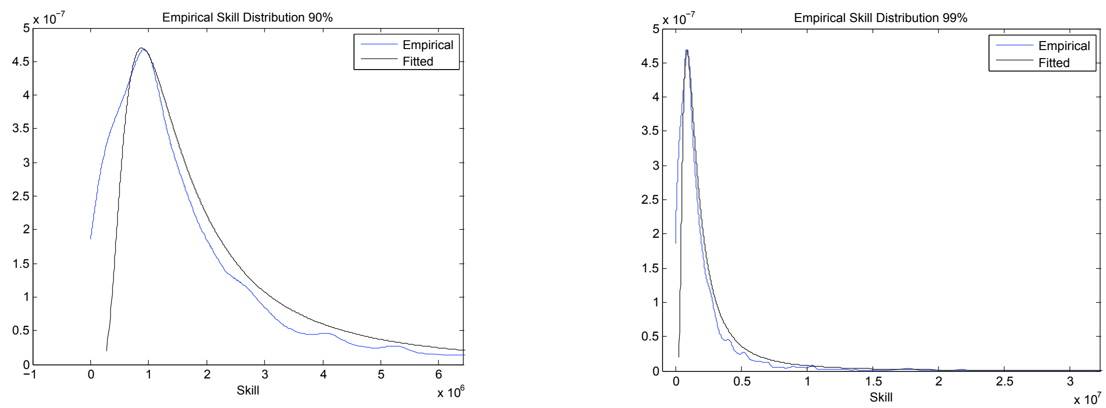


Figure 4: The adjusted GEV Distribution



Efficiency tests for the recovered distribution.

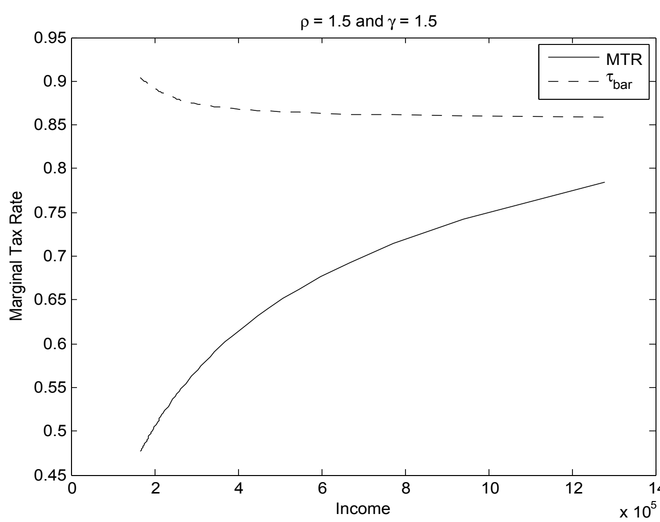


Figure 5: Marginal Taxes Rate and $\bar{\tau}$: $\rho > 1$

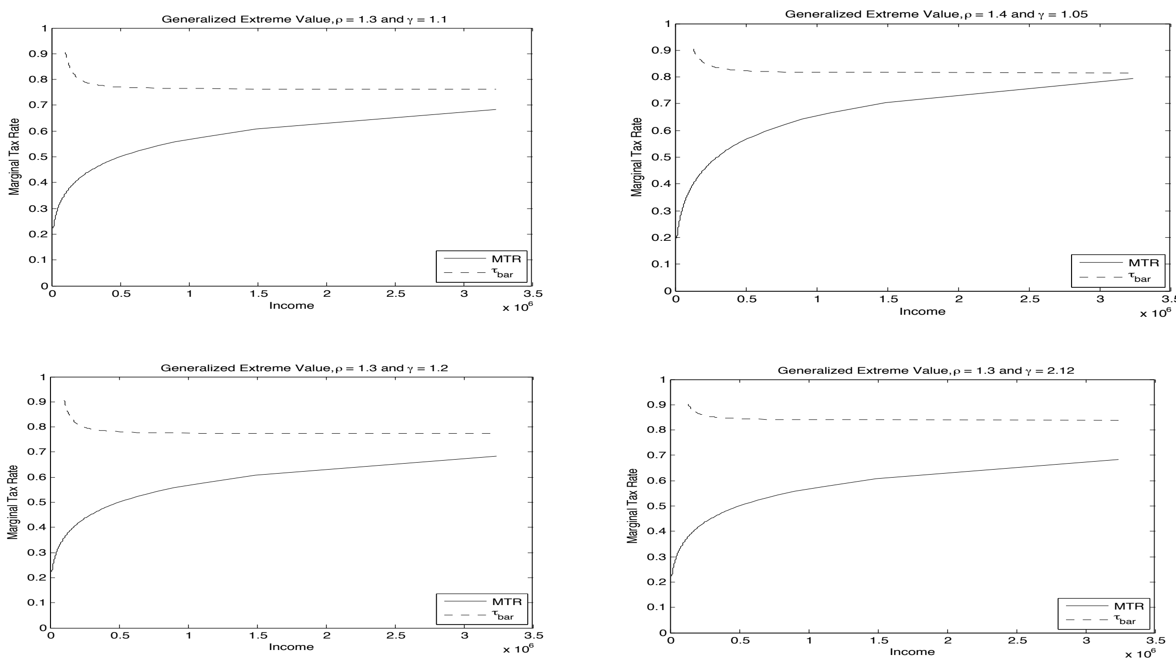


Figure 6: The Recovered Distribution of Skills

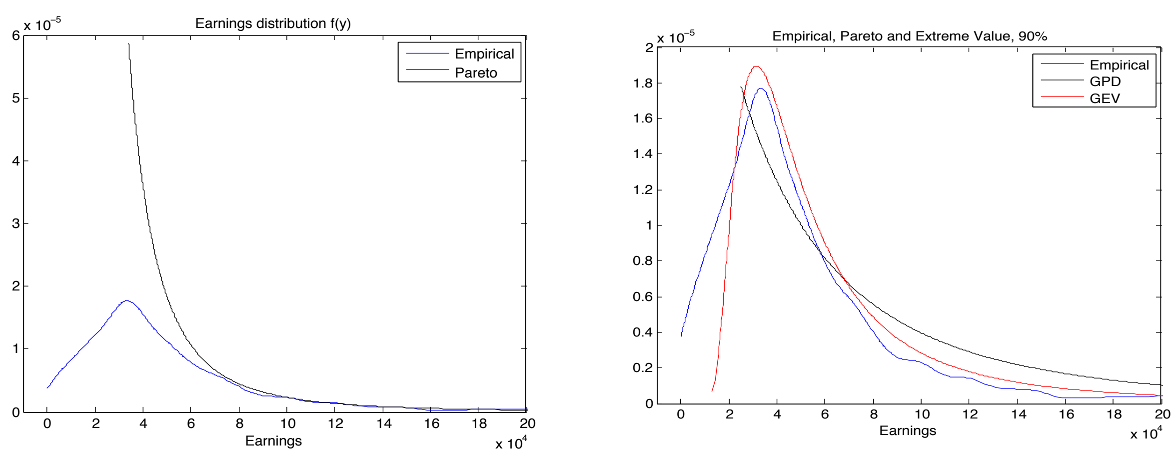


Figure 7: Efficiency Tests for GEV and GPD

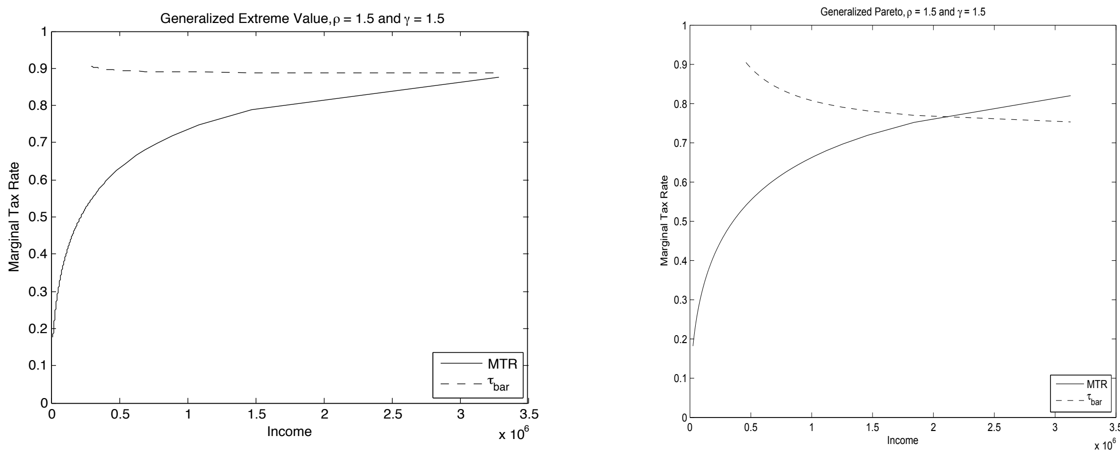


Figure 8: Efficeincy Tests for GPD

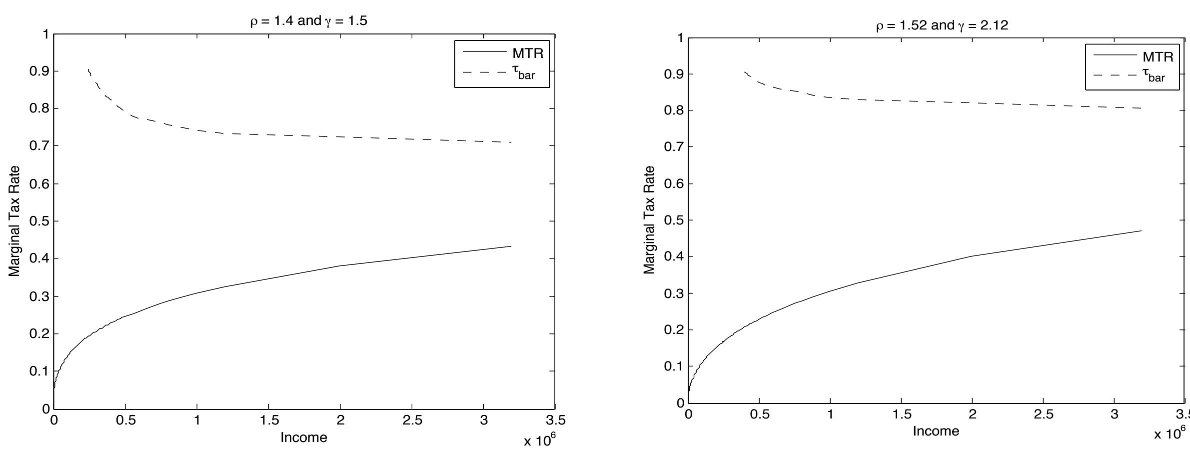


Figure 9: Efficiency in Young's (1990) Environment

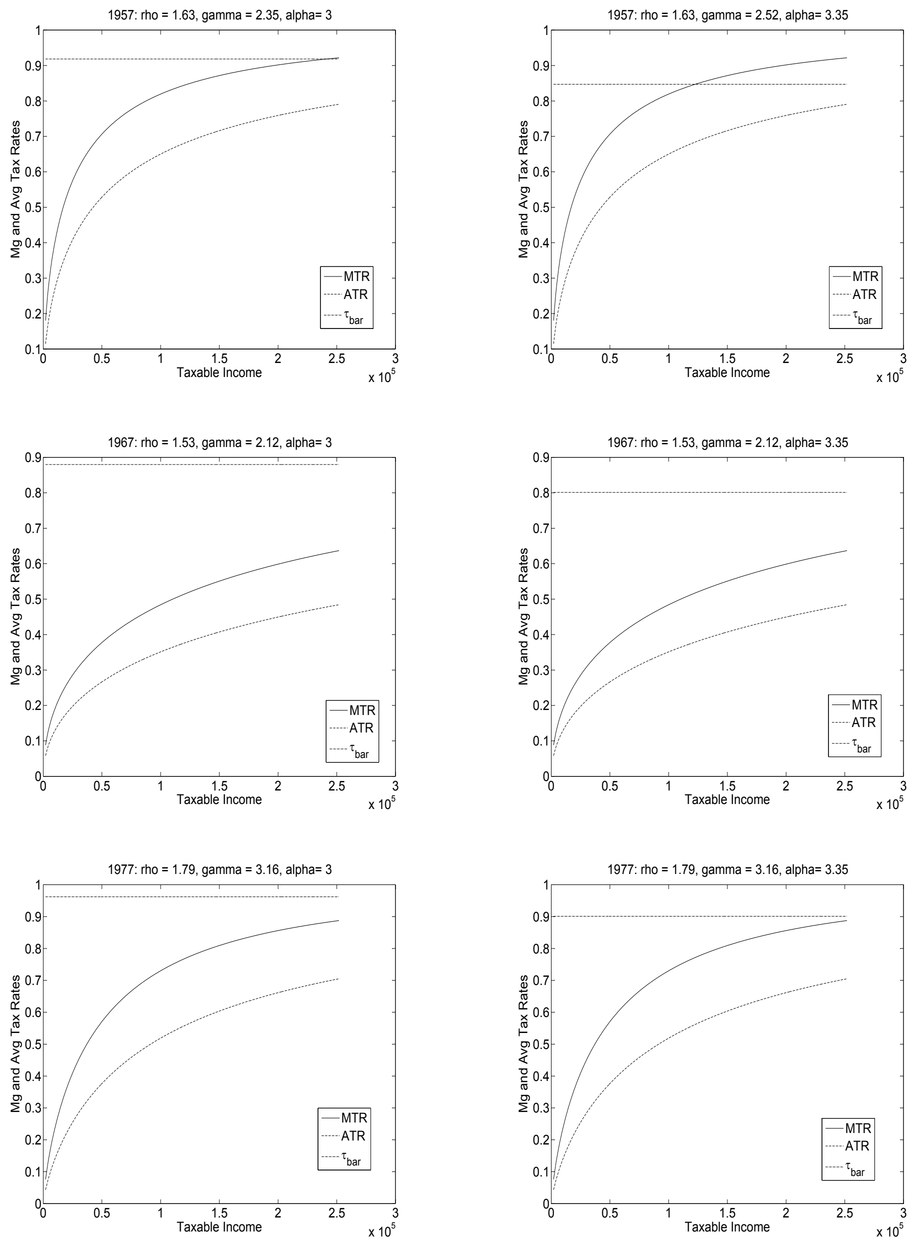


Figure 10: Efficiency in Young's (1990) Environment

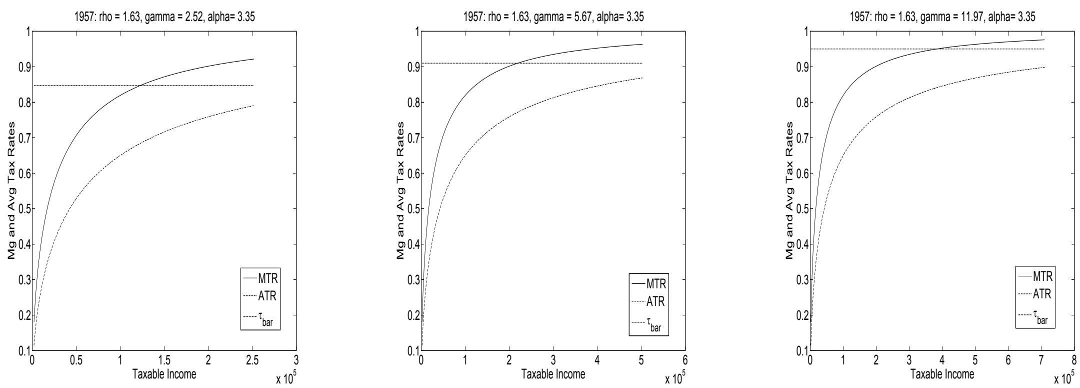


Figure 11: Efficiency in Young's (1990) Environment