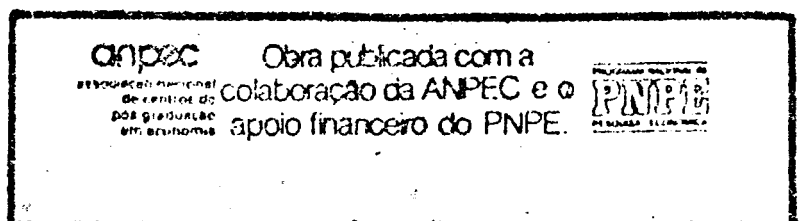


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A REVIEW ON THE THEORY OF COMMON KNOWLEDGE

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A REVIEW ON THE THEORY OF COMMON KNOWLEDGE

by

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## A REVIEW ON THE THEORY OF COMMON KNOWLEDGE

The concept of knowledge is central in economics as well as in many other sciences whose object of study is the human being, like psychology, linguistics, and artificial intelligence. Rational expectations models assume not only that economic agents form their expectations according to the model, but also that they all know the model. In the same lines, a game theorist always assume that the game is known by all the players to some extent. The famous Lucas' critique of the application of econometric models to policy, assumes the public knows the models that are being used by central planner. The examples above make it clear: knowledge is fundamental in economics.

In what follows, we will suppose the meaning of the concepts of knowledge of a fact, or knowledge about the truth of a proposition are well understood. A closely related notion is that of belief. If one person believes a fact is true, this does not imply this fact is true. People are allowed to hold false beliefs, but not false knowledge.

Since the knowledge of a fact by one person is itself a fact, we may think of the knowledge by an individual, say  $i$ , about the knowledge of another individual, say  $j$ , about a fact. In general one may repeat this iterative process as many times as wished. Hence, if we have several agents, we may say that a fact is known up to level  $m$  by these agents if: "everyone knows that (everyone knows that) $^{m-1}$  the fact" is true. That is, the words "everyone knows that" appear side by side  $m$  times, before the words "the fact". In the same fashion we define common knowledge of a fact if the fact is known up to level  $m$  for all  $m$ . In other words, a fact is common knowledge if everyone knows it,

everyone knows that everyone knows it, everyone knows that everyone knows that everyone knows it, ..., ad infinitum.

The concept of knowledge has been discussed for a long time by philosophers, at least since Plato. However, the ideas of common knowledge and of the importance of higher levels of knowledge are recent. They were introduced by David Lewis in 1966, so that he could study social conventions. Independently, John Harsanyi in his triad of papers on games with incomplete information (1967-1968) notices the importance of iterated guesses about the unknown parameter of the game. A game with incomplete information is such that the payoff functions of the players depend upon an unknown parameter (possibly multidimensional). Roughly, he argued that if a player is faced with a game of incomplete information, the action chosen by this player will depend on the beliefs she/he has about the values of the unknown parameter of the game. Therefore, this player would know that the same should occur with the other players. As the action of any player influences the payoff of any other player, a player should try to guess what other players are guessing about the unknown parameter, guess what other players are guessing about the guesses of the other players, and so on.

When we see these definitions of higher levels of knowledge, we promptly wonder what is the importance of levels greater than the first. In order to illustrate the point that higher levels of knowledge of a fact are linked with very different situations, we will refer to a well known puzzle. Long time ago a king decided to give amnesty to a large group of prisoners. The royal prison was a very special one: the prisoners were not allowed to speak to each other. In order to communicate his decision, the king summoned the prisoners. Each one was

put on a hat, which they were told they could not see the top, under any circumstance. If they did so, they would be killed. All but two hats had a white top. Those two hats were red topped. All the prisoners could see the hats of the others, but not his own.

The king delivered the following speech: "as you noticed, most of you have a white hat. But some have a red hat. From now on, every day you will be brought to this room. The day you find out the color of your hat, you are free to go. If any of you guess the wrong color, this person is going to be beheaded".

How many days did it take to the prisoners with the red hat to figure out the color of their hats?

Let us reason day by day. On the first day, after the speech, the prisoners with white hat see two red hats, and the prisoners with red hat see only one red hat. Hence, there is nothing they can conclude about the color of their own hats.

As a result, on the second day every prisoner is back in the main room. Again, the ones with a red hat see one red hat only, and the others two red hats. But there is an additional information: one day has passed. A prisoner with a red hat knows that the other prisoner with the red hat did not leave. As this other red hatted prisoner knows there is a red hat, this other prisoner must have seen another red hat. It follows that this other red hat has to be his own, since he sees no other. Therefore, the prisoners with a red hat leave the prison on the second day.

What would happen if we had three hats instead of two? A similar argument would apply, and the prisoners with red hat would leave on the third day. It should be clear that this reasoning could go on for any number of red hats, provided the number of white hats is greater

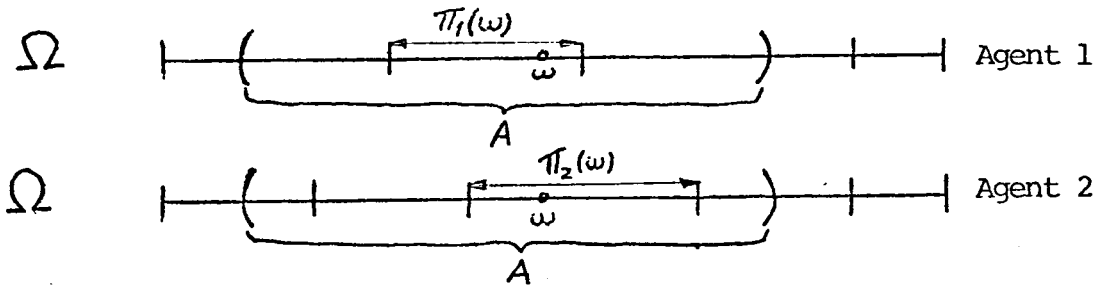
than or equal to the number of red hats.

This puzzle serves the purpose to show that different levels of knowledge of a fact are connected with very distinct situations. In fact, in the case of two hats, a person with a red hat sees at least one red hat, but does not know whether the other person sees at least one red hat. So, in the case of two hats, everyone sees a red hat, but it is not the case that everyone knows that everyone sees at least one red hat. In the case of three red hats, the person with a red hat sees two red hats, and knows that everyone sees at least one red hat. Thus, now everyone sees a red hat, and everyone knows everyone sees a red hat. We could go on inductively to show that each higher level of knowledge of the fact "everyone sees at least one red hat" is attained for versions of the puzzle with different numbers of red hats. As these puzzles with different numbers of red hats are very distinct problems (in particular, their solutions are different), we can see the importance of the higher levels of knowledge of facts.

The formalization of common knowledge is not an immediate task. To begin with, it is necessary to define what are the objects of knowledge. The first formalization was due to Robert Aumann in 1976. In his model, knowledge refers to events which are subsets of a set of states of the world. Call it  $\Omega$ . Aumann represents the agents by information partitions,  $\pi_1, \dots, \pi_n$ . These partitions work in the following way: if a state  $\omega \in \Omega$  occurs, then agent  $i$  observes the occurrence of the cell of his partition which contains  $\omega$ . We denote it by  $\pi_i(\omega)$ . The description of the set of states of the world and of the information structure above, has to be taken to be common knowledge in a primitive sense. Then, one agent knows an event  $A$  at the state  $\omega$  if the cell  $\pi_i(\omega)$  is contained in  $A$ . The iterations of the word "know"

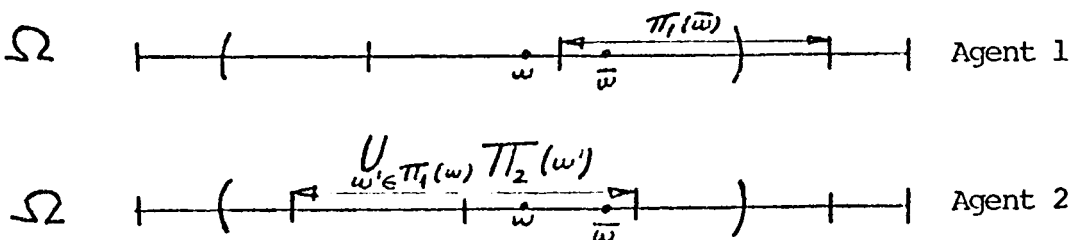
are a little more subtle. They depend entirely on the primitive common knowledge of the information structure. In the picture below, we can see an example with two agents. The set  $\Omega$  is an interval, the partitions of the agents are  $\pi_1$  and  $\pi_2$ , and the event is  $A$ . For convenience, we draw the set  $\Omega$  twice. In the first we see the information partition of the first agent, and in the second we see the information partition of the second agent.

FIGURE 1



We will verify the knowledge of the event  $A$  in its various layers, by both of the agents, when the state of the world that has occurred is  $\omega$ , as displayed. Because agent 1 has the information partition  $\pi_1$ , she "sees" the event  $\pi_1(\omega)$ . As  $\pi_1(\omega)$  is contained in  $A$ , agent 1 knows that the event  $A$  occurred. Similarly, for agent 2, as  $\pi_2(\omega)$  is contained in  $A$ , agent 2 knows  $A$ . As agent 1 sees  $\pi_1(\omega)$ , and she knows agent 2 has the partition  $\pi_2$ , she knows that agent 2 has observed the occurrence of  $\pi_2(\omega')$  for some  $\omega' \in \pi_1(\omega)$ . So, she knows that agent 2 has seen the occurrence of the union of the cells  $\pi_2(\omega')$  for  $\omega'$  in  $\pi_1(\omega)$ . As this set is contained in  $A$  (see Figure 2), agent 1 knows that agent 2 knows  $A$ .

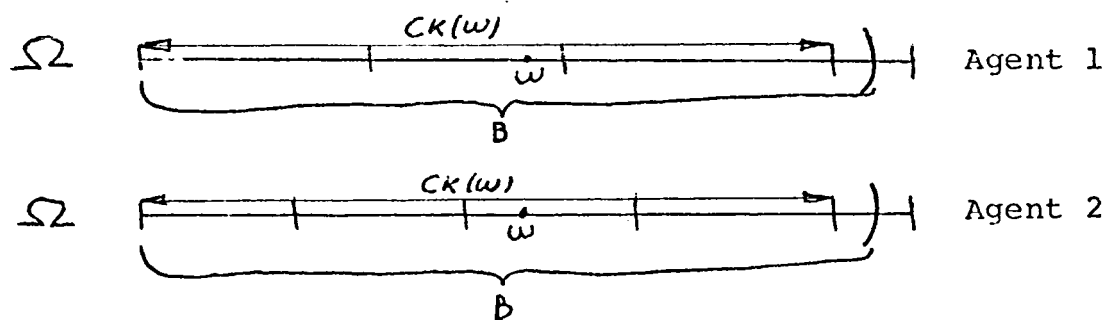
FIGURE 2



On the other hand, the same cannot be said about agent 2. In fact, as agent 2 sees  $\pi_2(\omega)$ , it turns out that the state  $\bar{\omega}$  shown in Figure 2 could be the real state of the world. As he knows agent 1 has partition  $\pi_1$ , he knows agent 1 could be observing  $\pi_1(\bar{\omega})$ , and hence she would be uncertain about the occurrence of the event A. It follows that agent 2 does not know whether agent 1 knows A.

Suppose now that we are considering the knowledge of the event B, drawn in Figure 3. The set  $CK(\omega)$  is observed by both agents. Furthermore, as the information partitions  $\pi_1$  and  $\pi_2$  are common knowledge to begin with, both agents know each other see  $CK(\omega)$ , know they know  $CK(\omega)$ , and so on. This set has a sort of fixed point property:  $CK(\omega)$  belongs at the same time to the set of events that agent 1 observes in her information structure and the set of events that agent 2 observes in his information structure. That is to say,  $CK(\omega)$  belongs to the meet of the two partitions, i.e., the finest common coarsening of the partitions, denoted by  $\pi_1 \wedge \pi_2$ . As the set B contains  $CK(\omega)$ , and it is common knowledge that the set  $CK(\omega)$  is observed by both, B is common knowledge.

FIGURE 3



Based on the intuition above, Aumann defines an event B to be common knowledge at  $\omega$ , if there is an element of  $\pi_1 \wedge \pi_2$  which contains  $\omega$  and is contained in B. Aumann's definition is elegant, and can be readily generalized to n agents. A further, and more complicated, step



allows for richer information structures, generated by  $\sigma$ -algebras. On this refer to Nielsen(1984) and Brandenburger and Dekel(1987).

The problem with his definition is the difficulty with its application. Imagine if we tried to formalize the following statement: "the model is common knowledge among the agents". The first thing we would have to do, would be to embed "the model" as an event on a large space of states of the world. Also it would be necessary to figure out what were the information structures which represented the agents in this world. Not only this, but this representation should be intuitively common knowledge a priori.

Aiming at avoiding these problems, and at applying knowledge and common knowledge to games, an alternative formalization was developed by Tan and Werlang(1985) (see also Brandenburger and Dekel(1985), Tan and Werlang(1984,1988,1988b), and Werlang(1986)). The idea behind is the direct modelling of layers of knowledge (to be precise, this is a model of belief with probability one). Suppose there are  $n$  agents, and each of the  $n$  agents are uncertain about some basic set of states of the world. To be consistent with the discussion up to now, say this set is  $\Omega$ . The Bayesian view holds that each agent has a first order belief over the possible occurrences of  $\Omega$ . Let  $s_i^1$  be the first order belief of agent  $i$ . That is,  $s_i^1$  is an element of  $\Delta(\Omega)$ . The set  $\Delta(\Omega)$  is the set of probability distributions on  $\Omega$ . We will avoid all complications, by assuming that all mathematical objects here are well defined. The very curious reader should refer to the papers cited above. Because beliefs about beliefs matter, agent  $i$  will have second order beliefs: if we call the set of first order beliefs  $S_i^1$ , a second order belief of agent  $i$  is a point  $s_i^2$  in  $\Delta(\Omega \times \prod_{j \neq i} S_j^1)$ , where  $\prod_{j \neq i} S_j^1$  indicates the cartesian product over all indices  $j \neq i$ . This means that a

second order belief is a probability distribution over the states of nature and the first order beliefs of all other agents. Call the set of second order beliefs of agent  $i$  by  $S_i^2$ . Inductively an  $m$ -th order belief,  $s_i^m$ , is a belief on  $\Omega \times \prod_{j \neq i} S_j^{m-1}$ , where  $S_j^{m-1}$  is the set of  $(m-1)$ -th order belief of agent  $j$ . Formally one has:  $s_i^m$  belongs to  $\Delta(\Omega \times \prod_{j \neq i} S_j^{m-1})$ . The "psychology" (or "type") of agent  $i$  is summarized by his infinite hierarchy of beliefs:  $(s_i^1, s_i^2, \dots) \in \prod_{m \geq 1} S_i^m$ . This set is too large. It allows the existence of extremely inconsistent beliefs. We impose a minimum consistency requirement: whenever there is an event whose probability can be evaluated by the  $m$ -th order belief, as well as by the  $p$ -th order belief, then the probability assessment given by both orders of beliefs must coincide. For example, this implies that the first order belief must be the marginal of the second order belief on  $\Omega$ :  $s_i^1 = \text{marg}_{\Omega}(s_i^2)$ . Recall that the marginal distributions, or simply the marginals of a probability measure  $P$  which is defined on a product space  $U \times V$  are defined by  $\text{marg}_U(A) = P(A \times V)$ , and similarly  $\text{marg}_V(B) = P(U \times B)$  for any event  $A$  in  $U$  and any event  $B$  in  $V$ . Hence, agent  $i$  is characterized by a point  $s_i$  in the set  $S_i = \{ (s_i^1, s_i^2, \dots) : \text{the beliefs satisfy the minimum consistency requirement} \}$ . By a theorem first proved by Böge(1974) (and later by Armbruster and Böge(1979), Böge and Eisele(1979), Mertens and Zamir(1985) and Brandenburger and Dekel(1985)), a point in this set of psychologies has a very important property: any  $s_i$  in  $S_i$  can be viewed as a joint probability distribution (belief) on the occurrence of nature and on the other agents' psychologies. In formal terms, there is an homeomorphism  $S_i \longrightarrow \Delta(\Omega \times \prod_{j \neq i} S_j)$ . Moreover, this homeomorphism has the property that the  $m$ -th order belief of agent  $i$ ,  $s_i^m$ , is the marginal of  $\phi_i(s_i)$  on the set  $\Omega \times \prod_{j \neq i} S_j^{m-1}$ , that is:  $s_i^m = \text{marg}_{\Omega \times \prod_{j \neq i} S_j^{m-1}} \phi_i(s_i)$ .

The interpretation given to the support of the marginal distribution of  $\phi_i(s_i)$  on  $S_j$ , denoted by  $\text{supp marg}_{S_j} [\phi_i(s_i)]$  is in the heart of the definition of knowledge. Recall that the support of a probability measure is the smallest closed set with probability one. We interpret  $\text{supp marg}_{S_j} [\phi_i(s_i)]$  as the set of psychologies of the  $j$ -th agent that agent  $i$  considers possible. In other words, it is the set of  $j$ -th agents which are possible in the eyes of agent  $i$ .

Let  $R_j$  be a subset of  $S_j$ ,  $j=1, \dots, n$ . Suppose one wants to formalize: "agent  $i$  knows that agent  $j$  is in  $R_j$ ". Making use of the interpretation given to  $\phi_i(s_i)$ , one has to say: every  $s_j$  which, in the eyes of agent  $i$ , could possibly be true must satisfy  $s_j \in R_j$ . Formally this means: for any  $j \neq i$ , for any  $s_j \in \text{supp marg}_{S_j} [\phi_i(s_i)]$ , one has  $s_j \in R_j$ . By extending this idea further, we can formalize longer sentences where the verb "know" appears many times. Given  $s_i$  and  $R_1, \dots, R_n$  as above, we define that the sets  $R_1, \dots, R_n$  are known up to level  $k$ , in the eyes of agent  $i$ , if:  $s_i \in \bigcap_{m=1}^k R_i^m$ , where  $R_i^1 = R_i$  and if  $m \geq 2$ ,  $R_i^m = \{ s_i \in R_i^{m-1} : \text{for all } j \neq i, \text{supp marg}_{S_j} [\phi_i(s_i)] \subset R_j^{m-1} \}$ . It is easy to see that by taking  $k=\infty$  we obtain the definition of common knowledge (in the eyes of agent  $i$ ).

This framework is more complex than Aumann's from the mathematical point of view. However, it allows the direct formalization of higher levels of knowledge. For example, an event  $A \subset \Omega$  may be common knowledge without any reference to the information partitions  $\pi_i$ , or the true state of the world  $\omega \in \Omega$ . In fact, let

$$A_i = \{ s_i \in S_i : \text{supp marg}_{\Omega} [\phi_i(s_i)] \subset A \} \text{ for all } i.$$

The set  $A_i$  represents the psychologies of agent  $i$  for which the first order beliefs are concentrated in a subset of  $A$ . This means that  $A$  is known for sure. We can apply the iterative procedure above to

define that  $A$  is common knowledge (in the eyes of agent  $i$ ) if  $s_i \in \bigcap_{m \geq 1} A_i^m$ , where  $A_i^1 = A_i$ , and for  $m \geq 2$

$$A_i^m = \{ s_i \in A_i^{m-1} : \text{for all } j \neq i, \text{supp marg}_{S_j} [\phi_i(s_i)] \subset A_j^{m-1} \}.$$

We may also define common knowledge of the partitions, which was implicitly assumed by Aumann. Given the partitions  $\pi_1, \dots, \pi_n$ , we say that agent  $i$  knows that agent  $j$  uses  $\pi_j$ , if  $s_i \in P_i$ , where the set  $P_i = \{ s_i \in S_i : \text{for } j \neq i, (\omega, s_j) \in \text{supp marg}_{S_j} [\phi_i(s_i)] \Rightarrow \text{supp marg}_{\Omega} [\phi_j(s_j)] = \pi_j(\omega) \}$ . Again, the interpretation is straightforward. If a state of the world  $\omega$  is thought possible to occur jointly with a psychology  $s_j$ , it must be the case that  $s_j$  has a belief over  $\Omega$  whose support coincides with the set of states of nature that agent  $j$  observes. As we have seen before, this set is  $\pi_j(\omega)$ , the element of the partition  $\pi_j$  which contains  $\omega$ . Inductively, we can define that the information partitions are common knowledge (in the eyes of agent  $i$ ) if  $s_i \in \bigcap_{m \geq 1} P_i^m$ , where  $P_i^1 = P_i$  above, and for  $m \geq 2$ ,

$$P_i^m = \{ s_i \in P_i^{m-1} : \text{for all } j \neq i, \text{supp marg}_{S_j} [\phi_i(s_i)] \subset P_j^{m-1} \}.$$

To compare this framework with Aumann's, we still need to formalize the fact that the state  $\omega$  has occurred in the eyes of agent  $i$ . Clearly this is the same as  $\text{supp marg}_{\Omega} [\phi_i(\Delta_i)] = \pi_i(\omega)$ . Tan and Werlang (1985) show the following theorem:

**Theorem.** Assume that in the eyes of agent  $i$  the information partitions  $\pi_1, \dots, \pi_n$  are common knowledge and that state  $\omega$  occurred. Then an event  $A$  is common knowledge at  $\omega$  in the sense of Aumann if, and only if, in the eyes of agent  $i$   $A$  is common knowledge.

Observe that we always mention that the knowledge or common knowledge occurs in the eyes of agent  $i$ . The reason for that was seen before. We formalized a belief with probability one. As a matter of fact, agent  $i$  could be completely wrong in his beliefs. This would not

invalidate any of the results above.

The infinite hierarchies of beliefs allowed us to state precisely the notions of knowledge and common knowledge of rationality in a game (see Tan and Werlang(1984,1988b) and Werlang(1986)).

There is a third way to model common knowledge: through the logic-theoretic framework. This consists basically of an iterative formalization as above, but with a less rich mathematical structure. The main source for that is Fagin, Halpern and Vardi(1984). We will not discuss it in further detail, mainly because of the difficulty with its applicability. Since the mathematical structure behind is not rich enough, it turns out to be very hard to formalize even simple statements like "rationality".

There are several applications of the concepts of knowledge and common knowledge in economics. Among them are the Bayesian foundations of solution concepts of games, no-trade results for speculative markets, rational expectations models, games with incomplete information, the mechanism of acquiring knowledge through information transmission, and the study of games whose payoffs depend on the knowledge about the outcomes. Outside the realm of economics, we can name some other applications: the theory of social conventions, the theory of communication and language, and the theory of information sharing in parallel processing. All the topics mentioned here are covered in the list of references. We will review only two of the most important consequences in economic theory.

Foundations of Solution Concepts of Games. There is a plethora of notions of equilibrium in games. The theory of knowledge and common knowledge applied to games helps the understanding of the implicit assumptions on the players which underlie some of these solution

concepts. We will concentrate on the study of normal form noncooperative games. The interested reader should refer to Tan and Werlang(1984,1988,1988b). The program of this literature is simply stated: given a solution concept, find the implicit assumptions about the knowledge and common knowledge of the players which will lead to this solution concept. One should be aware of another trend to investigate the foundations of solution concepts, the evolutionary view (see Samuelson(1988) and Maynard Smith(1982)). Also, there are some developments on the Bayesian foundations in the extensive form. On this see Reny(1988), Binmore(1988) and Bicchieri(1988).

The following solution concepts have been studied in the normal form: iterative elimination of strictly dominated strategies, rationalizable strategic behavior (defined by Bernheim(1984) and Pearce(1984)), correlated equilibrium (defined by Aumann(1974)), Nash equilibrium, perfect equilibrium (defined by Selten(1975)) and proper equilibrium (defined by Myerson(1978)). We will analyze two of them, namely iterative elimination of strictly dominant strategies and Nash equilibrium. We say that a player is Bayesian rational (or simply rational) when this player chooses an action which maximizes her expected utility given the beliefs she has about the actions of the other players. Thus, in the game of Figure 4,

		$l$	II	$\pi$
I	$\mu$	(1000, 1000)		(-1000, 900)
	$d$	(900, 1000)		(1000, 900)

FIGURE 4

player II has a strictly dominant strategy, which is 1. Hence, whatever player II thinks player I will do (i.e. given any belief about

player I's actions), player II will always play 1, if she is rational. If player I knows that player II is rational, then player I, being himself rational, will play u. This example shows us the links between layers of knowledge of rationality and iterations in the elimination of strictly dominated strategies. It is possible to show as in Tan and Werlang(1988b) that if Bayesian rationality is known up to level  $m$ , then the players will play strategies which survive  $m+1$  iterations of elimination of strictly dominated strategies. Carrying this argument to the limit, if it is common knowledge that players are Bayesian rational, then the outcome of the game has to survive iterative elimination of strictly dominated strategies. Converses of the results above are also true: in a game with  $n$  players, any  $n$ -tuple which survives  $m+1$  rounds of elimination of strictly dominated strategies is played by some  $n$ -tuple of players (represented by their "psychologies" or "types") for whom Bayesian rationality is known up to level  $m$ . The same goes for  $m=\infty$  (i.e., common knowledge).

When we come to Nash equilibrium, matters are not so nice. There are several ways to derive Nash behavior from the Bayesian point of view, none of them entirely satisfactory. In the case of two players, as in Armbruster and Böge(1979), we could require common knowledge of rationality as well as knowledge of each other's beliefs. Or as in Werlang(1986), Tan and Werlang(1988b) and Bacharach(1987), it is possible to show that if one requires a solution concept to be single valued, then the only solution concept which is consistent with common knowledge of rationality and of itself is a Nash equilibrium. This last explanation for Nash behavior shows the strong coordination requirements behind this solution concept. Not only rationality has to be common knowledge, but also the actions taken, before they are taken.

No-trade Results for Speculative Markets. It is known for a long time that in an economy with no asymmetry of information, there is no reason for the existence of speculative markets. In fact, consider the case of the stock market. A stock is a claim on future random profits of a firm. Since everyone knows the same probability distribution of the future profits (because there is no asymmetry of information), the only reason to trade stocks would be differences in risk aversion. Then the role of the stock exchanges should be taken by insurance companies.

Milgrom and Stokey(1982) show a much stronger result. They consider the trade of a risky asset in an economy with three periods: today, tomorrow and the day after tomorrow. Today, the state of the world is not realized yet, and we can trade the risky asset. Tomorrow, the state of the world occurs, and each trader has access to his own private information. The value of the asset is not yet disclosed, so the traders are allowed one more round of exchange. The day after tomorrow, the value of the risky asset becomes common knowledge, and no more trade makes sense. This is an economy with asymmetric information, where trade can occur before and after part of the information is revealed to the agents.

Their main result is: suppose tomorrow (after the state of the world is realized, but only partially revealed to the agents through their information structure) it is common knowledge that the agents want to trade. Then the only trade possible has to be indifferent to no-trade for every agent. The idea of the proof is simple: if it is common knowledge ex-post that the agents benefit from trade, then ex-ante there would be a feasible contract which could be written and would leave everyone better off, so that there would have been trade ex-ante. Here, the assumption of common knowledge is central. They give examples



where everyone wants to trade ex-post, but the fact they want to trade is not common knowledge.

Their result is a problem for the theory of speculative markets: asymmetric information alone cannot be responsible for the existence of large stock exchanges. A very important research project in the finance literature is to find where Milgrom-Stokey's model departs from reality. It is a point which is crucial for the understanding of the very complex speculative markets we see nowadays.

This essay did not pretend to be exhaustive. For surveys of the area which contain more diverse material, see Binmore and Brandenburger(1988), Tan and Werlang(1988) and Brandenburger(1987).

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