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## Inflationary Financing of Public Investment and Economic Growth

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# Inflationary Financing of Public Investment and Economic Growth.\*

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Abstract: a theoretical model is constructed in order to explain particular historical experiences in which inflation acceleration apparently helped to spur a period of economic growth. Government financed expenditures affect positively the productivity growth in this model so that the distortionary effect of inflation tax is compensated by the productive effect of public expenditures. We show that for some interval of money creation rates there is an equilibrium where money is valued and where steady state physical capital grows with inflation. It is also shown that zero inflation and growth maximization are never the optimal policies.

Key words: Inflation; Growth; Public Investment.  
JEL classification: O41,E31.

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## 1. Introduction

During the fifties and sixties, inflation and growth was a recurrent theme in growth theory as shown in the influential articles by Tobin (1965) and Mundell (1965). It was also part of the political discussion and in Latin America it was perhaps the most important economic topic and source of controversy between "monetarists" and "structuralists"<sup>1</sup>. This debate took place not only in the economic journals but also in the newspapers and among political parties. However, in much of the recent growth literature this phenomenon is ignored<sup>2</sup>, as most economists believe that, in general, distortion, and specially inflation, can only hurt economic growth. For instance, Jones and Manuelli(1995) and Chari, Jones and Manuelli(1996) proposes several models where monetary policy affects negatively economic growth.

The purpose of this paper is to develop a theoretical model to explain particular historical experiences in which inflation acceleration apparently helped to spur a period of economic growth. In this model, public expenditures financed by inflation tax help private capital accumulation, so that the model displays, within limits, the so called Tobin effect.

Unlike the already cited papers by Tobin and Mundell, as well as articles by Weiss(1980) and Summers(1981), in the present model government expenditures cause not only inflation and growth but inflation and sustained growth, because they introduce an externality by raising the quality of labor services. This is not a linear effect and too much inflation can be detrimental to the economy, as the flight from money can destroy the inflation tax base. In other words, money can affect the rate of growth of real variables while the inflation rate is not too high ( in a sense that will be explained later), after this point there is only nominal effects over real variables, and the economy halts at some *level* of per capita income<sup>3</sup>.

Another important result is that even employing a distortionary tax like inflation, it may be the case that government expenditures can improve the welfare of the economy because of the spillover effect of the public investment on education and

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<sup>1</sup> See Kaldor ( 1974) and the comments following his article for a good presentation of this debate and the position of both groups in it.

<sup>2</sup>Some exceptions are De Gregorio(1993), Roubini and Sala-i-Martin (1992), Jones and Manuelli(1995) , Chari, Jones and Manuelli(1996)and Fischer(1993).

<sup>3</sup>In a certain sense, this model is reminiscent of models of public investment in infrastructure where the government can raise only one type of tax (for instance, Barro(1990), Barro and Sala-i-Martin(1992) and Glomm and Ravikumar(1994)). In those models, the growth rate of product rises with the tax rate when the tax rate is small and falls when it is large.

production. It is proved in the paper that, unless government expenditures cannot influence productivity at all as in the conventional models, it is always optimal to introduce some inflation in the economy. However, maximization of growth is never the optimum policy. The rate of money creation that maximizes economic growth is the same that maximizes seigniorage. As the rate of money creation rises, the increase in seigniorage becomes progressively small, and so do the welfare gains from economic growth. At some point below the rate that maximizes seigniorage and growth, the loss due to the increasing distortionary effect of inflation tax overcome the gains from growth. The economy needs too much inflation to pay for too little growth.

In models like Lucas (1988) and Azariadis and Drazen (1990) sustained growth is achieved through investment in human capital. By replacing physical labor in the production function, and by advocating a linear function for human capital investment, there will be no fixed factor in these models. As human capital grows it raises the marginal productivity of physical capital, stimulating firms to increase investment. The final result (assuming enough homotheticity) is a constant rate of growth in income and physical capital.

This basic framework is used here but some important features are modified. In these models the equation of human capital accumulation is linear in its current stock and grows with the fraction of total human capital devoted to training and learning. The investment decision and the path of human capital depend thus on the decision of the individuals as to how much training they are willing to undergo. As everything else in the model depends on the path of human capital, the dynamic behavior of these economies is completely determined by the way individuals decide to allocate their time.

This hypothesis ignores the fact that in most countries education is provided, at least up to the high school, by the government and in general is mandatory. Moreover it also does not deal with the fact that the effective productivity of labor does not depend only on the amount of investment in education. It is also related, among other factors, to investments in health and infrastructure, which do not depend entirely on individual choices. Finally, it ignores problems of credit rationing, due to moral hazard factors, that individuals may face when deciding how much to invest in education. In this article it is postulated, as in Boldrin(1993) and Glomm and Ravikumar(1992), that the quality of labor in a given economy depends on government expenditures so that labor productivity varies with increases in public investments. Individuals are not provided with an investment function in education. By adopting the opposite view of what is generally accepted, we hope to stress that differences in growth rates in per capita income among countries may be explained by government intervention in the form of investment in education, health, sanitation, transportation, etc.

A second departure from the literature is the assumption that the government finances its expenditures through seigniorage. In doing so we are thinking of particular economic experiences in which government involvement was essential for the promotion of growth but also prompted an inflationary process.

The idea is not that the government resorts to inflation taxes rather than other taxes because it is less distortionary (which is probably not true), but because it is the easiest and least costly way to overcome restrictions and rigidities in the tax structure. In other words, once the political decision for the government intervention in the economy through investment in education and infrastructure is taken, it is necessary to establish how to finance these expenditures. More often than not the fiscal system is obsolete and too weak for this task - the tax base is too small, there is widespread tax evasion and, what maybe the most important reason, the underground economy is disproportionate large - and it would take a long time to modernize it. Moreover, political pressures from different groups concerning which of them will pay for the increase in taxes can postpone a fiscal reform for years and even threaten the survival of the government. Given that even today there is a large number of countries without an independent central bank so that monetary policy will be conducted, directly or indirectly, by the central government it is easy to see that in many situations inflationary finance is the fastest way for the government to achieve its aims and avoid the problems associated with broad tax reforms<sup>4</sup>.

This is not an unfamiliar idea for economists around the world, and we think there are plenty of historical examples and theoretical arguments that can be used to justify this model. In Latin America, for instance, from the fifties to the seventies a large number of countries experienced high rates of growth sustained in part by government investments. In this region, public investment reached, on average, more than 44% of private investment in the fifties (Cepal, 1963) and in Argentina, Brazil, Chile, Colombia, Venezuela and Mexico it was, on average, 31%, 43%, 82%, 22%, 69% and 83% of private investment, respectively. Education expenditures also grew, and in Argentina total enrolment increased by 77% between 1938 and 1955, while population increased only by 37% in the same period. In many of these cases - Brazil, Chile and Argentina, for instance - prices rose uninterruptedly during the period at rates greater than ten per cent per year<sup>5</sup>.

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<sup>4</sup> Russia may be a point in case here, as many of the previous state owned firms resist or simply do not pay their taxes since they have been privatized, and there is a large number of unregistered companies in the shadow economy which do not pay taxes too. After reaching 300% in 1994 and 190% in 1995, annual inflation in Russia today is near 15%.

<sup>5</sup> Annual inflation from 1951 to 1960 in Argentina, Brazil, Chile, Colombia, Venezuela and Mexico was, on average, 29.5%, 21.5%, 38.8%, 7.55%, 1.8%, 7.6%, respectively. Seigniorage in Brazil, was in 1963 not far from 20% of all government revenues (Holanda-Barbosa, Brandão e Faro, (1991)).

Note also that the relationship between public investment and growth is well documented in the literature. Easterly and Rebelo(1993) show that the share of public investment in transport and communication is robustly correlated with growth for a cross-section of countries, a result replicated by Ingran(1994), among others. Aschauer(1989), Munnell(1989), Nadiri and Mamuneas(1994) and many others, using different techniques and data sets, estimate a positive relationship between private productivity and public investment and/or public capital for American time series and panel data<sup>6</sup>. The positive relationship between education and growth, and also between public spending on education and growth, is documented in Barro and Sala-i-Martin(1995).

The paper is organized as follows. In the next section the model is presented. In the third section we prove the existence of monetary equilibria and study the effect over capital and money demand of changes in monetary policy. The fourth section studies welfare effects of inflation and in section five some concluding remarks are made. In a separated appendix, the dynamic behaviour of the model is studied.

## 2. The Model

Consider an overlapping generations economy with no population growth. Each generation is composed of a large number of individuals who live for two periods, except the first generation that only lives for one period. In the first period of their lives, "youth", the individuals are endowed with one unit of labor. Individuals do not value leisure, so that they supply their labor inelastically. When young the individuals work, receive a wage, consume the only good of this economy and save. In the second and last period of their lives, "old" people do not work, but consume the proceedings of their savings.

There are two different assets in the economy competing for the savings of the young generation. One is fiat money issued by the government and the other is a capital asset issued by the firms. Money may or may not be valued in equilibrium. In this case individuals will hold only capital in their portfolios (and the equilibrium will be called non-monetary). The capital and money levels in the first period (time zero) are given by history.

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<sup>6</sup>Two exceptions are Holtz-Eakin(1992) and Hulten and Swchartz(1992) who could find no evidence of public capital affecting productivity. Gramlich(1994) is a comprehensive review essay on this subject.

The consumers' utility function follows standard assumptions: it is concave, increasing and (at least) twice differentiable in all of its components. We will also assume that it is homothetic. The problem of the consumer is to maximize his utility by choosing the saving level as well as its distribution between capital and money. The budget constraint of a young person born at time  $t$  is given by:

$$C_t^t = w_t - S_t \quad (1)$$

where  $w_t$  are the wages and  $S_t$  is the total saving.

Letting  $R_{t+1}$  be the gross return on capital,  $\Pi_t$  the gross return on money (the inverse of inflation factor) and  $M_t$  nominal balances, the consumer's budget constraint when he is old is:

$$C_{t+1}^t = S_t R_{t+1} + (M_t / P_t) [\Pi_t - R_{t+1}], \quad (2)$$

where  $C_{t+1}^t$  represents the consumption at time  $t+1$  of a person born at time  $t$  (the old) and  $P_t$  the price level. Applying (1) in (2), we have the lifetime budget constraint, so that the consumer's problem can be expressed as:

$$\text{Max}_{C_t^t, M_t, C_{t+1}^t} U(C_t^t, C_{t+1}^t) \quad (3)$$

$$\text{s.t.} \quad C_{t+1}^t + R_{t+1} C_t^t + [\Pi_t - R_{t+1}] (M_t / P_t) = w_t R_{t+1} \quad (4)$$

It is easily seen that the solution of this problem is given by:

$$U_1(C_t^t, C_{t+1}^t) = U_2(C_t^t, C_{t+1}^t) R_{t+1} \quad (5)$$

$$R_{t+1} \geq \Pi_t, \quad \text{if } M_t > 0 \quad (6)$$

Expression (5) equates the marginal rate of substitution with the marginal rate of transformation between consumption when young and capital. It says that the loss in utility by giving up one unit of consumption when young has to be equal, in equilibrium,



to the gross yield of capital (in utility terms) in the next period. Equation (6) is a no-arbitrage condition expressing the fact that in equilibrium the returns on the assets must be the same in order for consumers to be willing to hold both of them in their portfolios, otherwise it will hold only capital.

Government expenditures perform a very particular task in this economy: they enhance the productivity of labor. The idea is that by investing in public education, health services, sanitation, transport system and so on, the government can increase the quality of the labor force. As work services are measured in efficiency units in the model, public investment increases the flow of labor services per unit of time. In particular, calling  $L_t$  the flow of efficiency units of labor of a worker born at time  $t$  and  $g_t$  government expenditures per efficiency units (note that lower case is used for variables in efficiency units of labor), it is assumed that

$$L_{t+1} = I(g_t)L_t \quad (7)$$

The function  $\lambda(g_t)$  is the government expenditure function. It transforms each unit of public investment in infra-structure, by a relative increase of  $L_{t+1}/L_t$  in labor productivity. It is assumed that  $I$  is positive, increasing, concave, differentiable and that  $\lambda(0)$  is one (so that if the public infra-structure remains the same the labor productivity does not change). Finally,  $\lambda$  is assumed to be bounded so that when government expenditures are very large the marginal gain in labor productivity is close to zero.

Government budget constraint is  $P_t G_t = M_t - M_{t-1}$ , where  $G_t$  is real government expenditures at time  $t$ <sup>7</sup>. Assuming a constant and preannounced rate of money creation ( $\mu$ ), we obtain  $M_t = (1 + \mu) M_{t-1}$ , which implies

$$g_t = \left( \frac{\mu}{1 + \mu} \right) m_t \quad (8)$$

where  $m_t$  are real money holdings per efficiency units of labor.

The production side of the economy is represented by competitive firms with a technology that is homogeneous of degree one in capital and efficiency labor. They use capital, that fully depreciate with use, and labor services to produce a homogeneous

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<sup>7</sup> Although the main objective of the paper is to analyze the effects of money creation on growth, taxes and loans could be introduced as sources of government revenue. If, at the same time, an upper bound on tax collection was introduced, this would not change the behavior of the economy. Hence, if the government wanted to spend above this bound it would be less costly to use money creation, so that only for small levels of public expenditures seigniorage would not be necessary, a fact not unfamiliar for most of Latin America countries or countries with high inflation experiences

good. Both inputs are rented at market prices. These hypotheses imply that output per efficiency units of labor,  $y_t$ , depends only on the capital-labor ( in efficiency units) ratio  $k_t$ :

$$y_t = f(k_t) \quad (9)$$

where  $f$  is the neoclassical production function endowed with the standard assumptions:

$$f(0) = 0; f' > 0; f'' < 0; f \text{ concave} \quad (10)$$

From the solution of the problem of the firms, given the homogeneity and competitiveness assumptions, we can derive both the labor and the capital demand schedules:

$$r_t = f'(k_t) \quad (11)$$

$$w_t = f(k_t) - k_t \cdot f'(k_t) \quad (12)$$

where  $w_t$  in this model is wage rate per efficient unit of labor.

In this economy the labor market is always in equilibrium, as workers supply inelastically their services. So, there are two markets that need to be checked for equilibrium, the goods market and the asset market. Only the latter will be examined, making use of Walras' law. This market will clear when the total demand for assets, represented by the total savings of the young, equates the total supply. The supply of assets is composed by the stock of money held by the old plus money created in this period and the debt issued by firms to finance capital investment.

The saving function is derived from the first order conditions of the consumer problem, more precisely from equation (5). Using the implicit function theorem we can write it as:

$$S = S(w_t, r_{t+1})$$

It will be assumed that consumption in the first and second period of life are both normal goods, so that savings are increasing in wages. We are able now to write the equilibrium condition in the asset market:

$$S(w_t, r_{t+1}) = K_{t+1} + M_t / P_t,$$

which can be rewritten in efficiency units of labor, after some manipulations, as

$$s(w_t, r_{t+1}) = I(g_t)k_{t+1} + m_t \quad (13)$$

An *equilibrium* in this economy is a *capital sequence*  $\{k_t\}$  and a *money holdings sequence*  $\{m_t\}$  such that, for given initial  $m_0$  and  $k_0$ , in every period the dynamic system given by equations (6), (8), (11), (12) and (13) is satisfied.

This system can be reduced to two equations. First, note that the return to fiat money, which is the inverse of the inflation rate  $(P_t/P_{t+1})$ , is equal to  $[I(g_t)/(1+m)] \cdot [(m_{t+1})/m_t]$  in this economy. Applying this expression in equation (6) and then, applying (8), (11) and (12) in the remaining two, the following two-dimensional first order dynamical system is obtained:

$$f'(k_{t+1}) = \frac{I\left(\frac{m}{1+m}m_t\right)m_{t+1}}{1+m} \quad (14')$$

$$s(w(k_t), r(k_{t+1})) = I\left(\frac{m}{1+m}m_t\right)k_{t+1} + m_t \quad (15')$$

Given the homotheticity property of the utility function<sup>8</sup>, the above dynamical system becomes:

$$f'(k_{t+1}) = \frac{I\left(\frac{m}{1+m}m_t\right)m_{t+1}}{1+m} \quad (14)$$

$$sw(k_t) = I\left(\frac{m}{1+m}m_t\right)k_{t+1} + m_t \quad (15)$$

where  $s$ , a constant between zero and one, is the saving rate.

In what follows only stationary equilibria will be studied, as there can be several capital and money sequences satisfying the above two equations. However, it is worth noting that steady states in this economy are not constant sequences, but a balanced growth path in which capital, consumption and income grow at the same rate  $\lambda(g)$ .

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<sup>8</sup> It is shown in a previous version of this article that this hypothesis does not change the dynamic behavior of the monetary steady state, the possible numbers of stationary equilibria and the general structure of the dynamic system.

For a given money growth rate there are three types of stationary equilibrium satisfying the above system. The first one is a trivial one given by  $(k_t, m_t) = (0, 0)$ : with zero capital, there is no production and so no savings and money demand in any period. The second type of steady state is such that individuals hold only capital in their portfolio, and is given by  $(k_t, m_t) = (k_d, 0)$ , where  $k_d > 0$  solves  $sw(k_t) = k_t$ , for all  $t$ .

The third type of stationary equilibrium is a monetary equilibrium in which both assets are held by the individuals:  $(k_t, m_t) = (k^*, m^*)$ , such that  $k^*$  and  $m^*$  solve, for all periods

$$m^* = sw(k^*) - l\left(\frac{m}{1+m}m^*\right)k^* \quad (16)$$

$$f'\left(\frac{sw(k^*) - m^*}{l\left(\frac{m}{1+m}m^*\right)}\right) = \frac{l\left(\frac{m}{1+m}m^*\right)}{1+m} \quad (17)$$

Equation (16) represents the locus in the economy where  $k$  is constant while equation (17) is the locus where money balances per efficiency units of labor is constant.

Note that for the first two types of equilibrium the no-arbitrage condition holds as an inequality, it does not bind. With zero money holdings the above planar system turns into the standard Diamond (1965) one-sector-growth-model with only one asset. It is clear thus that these two types of steady states correspond to the steady states of the Diamond model. In what follows we will call  $(k_t, m_t) = (k_d, 0)$  the "Diamond equilibrium".

Under mild and standard assumptions equation sixteen can be represented by a parabola that grows initially with money, reaches a single peak and then falls, cutting the capital axis at zero and  $k_d$ . If we rewrite it as

$$\frac{m_t}{k_t} = \frac{sw(k_t)}{k_t} - l\left(\frac{m}{1+m}m_t\right) \quad , \quad (18)$$

it is easy to see that as capital goes to infinity the limit of the money-capital ratio goes to minus  $I(g)$  that is negative number<sup>9</sup>. On the other hand, when capital approaches zero there can be no production and savings, so that  $m_t$  also approaches zero, and we have

$$\lim_{k_t \rightarrow 0} \frac{m_t}{k_t} = \frac{sw(k_t)}{k_t} - I(0) = \frac{sw(k_t)}{k_t} - 1$$

In order for this limit to be positive it has to be assumed that the limit of the saving-capital ratio is greater than one close to the origin. This is not a strong assumption. In fact, it is the exact same one needed for the existence of a stationary equilibrium in the standard one sector OLG model.

Finally, given the assumption of monotonicity of the function  $I$  and of normality of consumption ( which imply a monotone saving function), the condition for a maximum in this curve,  $sw'(k) = I$ , is met only once - for a given  $\mu$  - at some capital level  $k_\mu$ . Rewriting equation (17) as  $m_t = G(k_t, \mathbf{m})$ , it is easy to see that for a given  $\mu$  the maximum is reached at a  $(m_m, k_m)$  point such that<sup>10</sup>

$$G'_k = \frac{dm_t}{dk_t} = \frac{sw' - I}{1 + \frac{I' k_t \mathbf{m}}{1 + \mathbf{m}}} = 0 \Leftrightarrow sw' = I$$

For the system to have a monetary steady state, the curve given by equation 17 has to be positive for some combination of parameters and has to cut the curve given by equation 16 at  $k$  smaller than  $k_d$ . In this case slightly stronger assumptions will be needed. It is shown in appendix A that when inflation is not too large, this is in fact the case. In the next section we give further intuition for this result when proving the existence of a monetary steady state. The condition is equivalent of assuming that  $\mu$  is not large enough (  $\mu$  such that  $I/\mathbf{m} > \{ I' k [ \sigma/(1-\alpha) - I ] - I \}$ , where  $\sigma$  is the elasticity of substitution between capital and labor inputs and  $\alpha$  is the capital share of output), which is always true for small  $k$  and  $m$ , as proved in the appendix<sup>11</sup>. In addition to that, this curve will cross the capital line at  $k_\mu$  defined by

<sup>9</sup> By L'Hôpital rule it is easy to show that the limit of the ratio  $sw(k_t)/k_t$  goes to zero when capital stock goes to infinity.

<sup>10</sup> Note that to the left of  $k_\mu$ ,  $sw' > \lambda$  ( for small  $k$  and  $m$ ,  $\lambda$  is close to one while  $sw'$  is large) so that  $G_k$  is positive. On the other hand, to the right of  $k_\mu$ ,  $G_k$  is negative as  $sw'$  decreases to zero as  $k$  increases, while  $\lambda$  is bounded away from zero.

<sup>11</sup> In the case of Cobb-Douglas technology, log-utility and  $\lambda$  function linear or given by  $\lambda(g_t) = 2 - \exp(-g_t)$ , for instance, the slope of equation 17 is positive for any  $\mu$  and any capital share. Another sufficient condition for positive slope is  $\sigma/(1-\alpha) < 1$ , but this is too strong. It is not true, for instance,

$$f'(sw(k_m)) = \frac{1}{(1+m)} \quad (19)$$

while its intercept with the money axis will be negative. This can be easily seen because when capital is zero, the money phase line becomes

$$f' \left( \frac{-m}{l \left( \frac{m}{1+m} \right)} \right) = \frac{l \left( \frac{m}{1+m} \right)}{1+m} \quad (20)$$

Note that the derivative of the production function is only defined in  $\mathbb{R}_+$ , so that  $m$  has to be negative for  $\{-m/\lambda\}$  be positive. Putting the two curves together we have, for  $\mu$  not very large, the following diagram:

**figure 1**

Figure 1 above also displays the phase diagram of the system. By visual inspection, the three steady states of the system,  $(k_t, m_t) = (0, 0)$ ,  $(k_d, 0)$  and  $(k^*, m^*)$ , seen to be a source point, an attractor and a saddle point, respectively. Appendix B studies the dynamic properties of the system composed by equations 16-17 and prove that this is in fact the case under mild assumptions.

### 3. The Monetary Equilibrium

In this section the existence of a monetary equilibrium is proved. The effect of the rate of money creation on the steady state levels of capital and money holdings are also investigated.

**Proposition 1** : There exists a monetary equilibrium for  $\mu$  sufficiently small. Furthermore, for  $\mu$  such that  $1/m > \{ l' k [ s/(1-a) - l ] - l \}$  there can also exist a monetary steady state as long as  $k_\mu$  is to the left of  $k = k_d$  which solves  $sw(k_d) = k_d$ .

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for the case of Cobb-Douglas technology. Note also that  $\sigma/(1-\alpha) = -f'/(kf'')$ , which is the elasticity of the demand for capital with respect to the rental rate  $f'(k)$ . Hence, the condition above is equivalent to the assumption that this elasticity is close to one in absolute valued in the relevant capital interval.

*Proof*: For  $\mu = 0$  the dynamical system given by equations 16-17 becomes

$$k_{t+1} = sw(k_t) - m_t, \quad (16')$$

$$m_{t+1} = f'(k_{t+1}).m_t. \quad (17')$$

This is the standard OLG model with national debt, bubbles or fiat money that is studied by Azariadis (1993), Tirole (1985) and Wallace (1980). We know from these authors that a monetary steady state exists and is a saddle point.

A continuity argument can be used to prove existence of equilibrium for  $\mu$  different from zero. As  $\mu$  increases away from zero the money phase line will start to shift down and to the right while its slope will eventually become negative for large  $\mu$ . First, note that by expression (19) above, as  $\mu$  increases, the capital intercept of the money phase line also increases. And by equation (20) the intercept on the money axis eventually shifts down<sup>12</sup>. On the other hand the slope of this curve, given by expression (19), will be positive for  $\mu$  not very large ( $\mu$  such that  $1/m > \{ I' k [ S/(1-a) - 1 ] - 1 \}$ ) while for intermediate values of  $\mu$ ,  $k$  and  $m$  it can even be negative (for large  $\mu$ , given the boundness of  $\lambda$ , the slope will be  $sw'$  again). The important point, however, is that the slope does not jump but moves smoothly.

The intercepts of equation 16 at the capital axis will not change as  $\mu$  departs from zero, because when  $m_t$  is zero  $g_t$  does not change with  $\mu$ . However the level of money holdings will be lower for a given capital stock, and the parabola shifts down keeping the same intercepts: for any  $m_t$  positive, given that  $I(g_t) > I(0) = 0$ , for  $g_t > 0$ , the curve given by  $m_t = sw - I(g_t)k_t$  is always below  $m_t = sw - k_t$ .

In other words, as  $\mu$  increases from zero there will be no jumps in either curves. This fact implies that as the economy departs from a situation depicted by equations (16')-(17') to the more general model of equations (16)-(17), there still exists a positive intercept between both phase lines, and the economy will have (at least) one stationary monetary equilibrium  $(k^*(\mu), m^*(\mu))$  for each  $\mu$ . For large money growth rates ( $\mu$  such that  $1/m < \{ I' k [ S/(1-a) - 1 ] - 1 \}$ ), the slope of equation (17) becomes negative. It may also be the case that for  $\mu$  large enough, equation (17) shifts "too much" to the

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<sup>12</sup> Note that for large  $\mu$  equation (20) becomes  $f'(-m/I(m)) = I(m)/(1 + \mu) = 0$ , which implies that  $m$  goes to minus infinity. For  $\mu$  close to zero, given the assumption that  $\lambda'$  is not large at  $g_t = 0$ , we can expect that  $I(m)$  increases less than  $(1+\mu)$  initially. This implies that  $f'$  also decrease, so that  $-m$  increases and  $m$  shifts down.

right, so that  $k_\mu$  is to the right of  $k_d$ . In both cases there will be no intersection between both curves in the positive quadrant<sup>13</sup>. •

The fact that for large  $\mu$  there will be no monetary equilibrium makes economic sense: as the amount of money creation becomes very large, as does the rate of inflation, individuals will flee from money to alternative assets to avoid losses.

The same reasoning allows us to do a graphic and heuristic analysis of the possible scenarios of the effects of changes in  $\mu$  over capital and money demand. Suppose the economy is initially at the point  $E_0$  in the figure 2 below:

### Figure 2

As  $\mu$  rises, both curves shift down. The economy goes to successively lower levels of money demand and higher levels of capital:  $E_1$ ,  $E_2$ , etc. For a certain value of the rate of money creation, the money phase line will become negatively sloped, or it will be still positively sloped but it will cross the capital axis at the right of  $k_d$ . At this point there will be no monetary equilibrium and the economy will stay forever at  $(k_d, 0)$ . Here the steady state is no longer a balanced growth path but a constant value: because of the flight from money the government can no longer use inflationary financing to improve labor productivity<sup>14</sup>.

If at  $\mu = 0$  the monetary equilibrium is at a point to the left of the maximum of the curve given by equation (16) it is possible that for some levels of money creation both asset demands increase. Here, for a "well behaved" economy (where movements in both curves are approximately of the same size), money creation by the government has a Laffer-effect in money demand and a positive effect on the rate of growth of capital up to a level of  $k = k_d$ . Through money and inflation taxes the government can finance the investments needed to raise labor productivity and consequently to increase the growth rate of income and capital. However, as inflation reaches higher levels, the government capacity to stimulate the real side of the economy declines, as the (inflation) tax base shrinks.

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<sup>13</sup> If  $s/(1-a) \leq 1$ , as said before, the slope of curve 17 is positive for any  $\mu$ , so that as long as curve 17 cuts the capital axis to the left of  $k_d$ , it cuts curve 16 in the positive quadrant and there will be a monetary equilibrium. This assumption is however too strong.

<sup>14</sup> This is only one of the possible scenarios, although for small and large values of the parameter  $\mu$  this will always be the case. We do not know the relative size of the shifts of both curves, unless we make some very unreasonable assumptions. If the shifts in the phase line of capital are, for some levels of  $\mu$ , much larger than the ones in the money phase line, it may be the case that both the demand for money and for capital will decrease for some time.



This graphic reasoning is supported by analytical results presented as a proposition below:

*Proposition 2* : The level of capital per efficiency units of labor in the steady state grows with the rate of money creation, for  $\mu$  sufficiently small ( and very large). Real money balances grow ( fall) with  $\mu$  if  $sw' > I$  (  $sw' < I$ ) and  $\mu$  being either very small or very large.

*Proof* : see appendix C

In the appendix it is shown that the derivative of  $m$  with respect to  $\mathbf{m}$  in the steady state, for small values of the money growth rate is

$$\frac{dm}{d\mathbf{m}} = -\frac{I}{(1+\mathbf{m})^2 f''} [sw'-I], \quad (21a)$$

while for large  $\mathbf{m}$  it is given by

$$\frac{dm}{d\mathbf{m}} = -\frac{1}{f''} [sw'-I]. \quad (21b)$$

In both cases the sign of the derivative will depend on whether the monetary equilibrium is to the left or to the right of the peak of the curve given by equation (17). To the left,  $sw'$  is greater than  $I(.)$ , so that money demand grows with  $\mathbf{m}$ . On the other hand, to the right of the peak of the parabola,  $I(.)$  is larger than  $sw'$  so that individuals are willing to hold less money for larger rates of money creation. However, for intermediate values anything can happen, a result that coincides with what we just saw through graphical analysis.

One interesting result here is that it is possible to have inflation and money demand growing at the same time in the case where  $sw'$  is greater than  $\lambda$ . This is apparently a non intuitive result but can be easily explained because government expenditures are financed by inflation tax, and  $g_t$  has a spillover effect on the variables of the model, so that when it increases ( within certain bounds) national income increases. For this interval, where saving is still growing fast (  $sw' > I$ ), this income effect on the demand for money is stronger than the substitution effect caused by the decrease in the return on money holdings.

On the other hand, the derivative of the steady state capital with respect to  $\mu$  is positive for both large and small rates of money creation. In both cases we can reduce this derivative to

$$\frac{dk}{dm} = -\frac{1}{(1+m)^2 f''}, \quad (22)$$

that is always positive. This is the so called Tobin effect: as inflation increases, the return on money declines boosting the demand for the alternative asset, capital. Money is not super-neutral in this economy. However, in concordance with the graphical analysis, for intermediate values of  $\mu$ , the stock of capital can go to any direction and even be crowded out by government expenditures. For this reason we cannot say that in this model inflationary finance has everywhere a positive and monotone effect on the growth rates of capital and product. It maybe the case that for large intervals of the inflation rate, capital falls with increases in  $\mu$ . In simulations of the model we ran in a previous version of this paper, however, capital always grew with inflation.

#### 4. Optimal Monetary Policy

In this section we study the optimal monetary policy for a government that wants to maximize the utility of its subjects and take their actions as given. In this model, money creation has two opposite effects on the well being of consumers. The first one is that, creating inflation, the government distorts the optimal allocation of the economy. This is a general "by-product" of inflation finance, and it only exists here because the government cannot use lump sum taxes in this economy. The second effect is that money creation finances public investment, and thus increases the growth rate of output and consumption and therefore improves consumer utility.

The government goal is to choose the  $\mu$  that maximizes the flow of utility from consumption in the economy. First, using the fact that along the balanced path all the variables grow at the common rate  $\lambda$ , the consumption of young and old at time  $t$  can be rewritten as  $I^t_{c_1}$  and  $I^t_{c_2}$ , respectively. The government thus solves

$$\max_m \sum_{t=0}^{\infty} b^t I^t U(w - S, RS).$$

where the homotheticity property of the utility function was used when  $I^t$  was taken to the outside. Assume that the term  $(bI)^t$  is always between zero and one (i.e.,  $\lambda$  is bounded by a number smaller than  $1/\beta$ ), the government problem becomes

$$\max_m \frac{1}{1 - bI} U(w - S, RS).$$

We basically want to study the relationship between the optimal  $m^*$  which solves the above problem and the money growth rate that maximizes growth (and seigniorage). It could be expected that a government which does not discount the future very heavily would equalize both rates, maximizing growth while introducing a minimum distortion. The following proposition says that for a large number of economies this may not be the case.

**Proposition 3:** For economies where  $s/(1+s) > a$  ( respectively,  $< a$  ), a government which wants to maximize the welfare of the present and all future generations, should set  $\mu$  at a smaller ( respectively, higher) level than the one which maximizes growth and it should operate on the upward ( respectively, downward ) sloped side of the Laffer curve.

*Proof:* The first order condition for the government problem is:

$$\frac{1}{1-bl} \left[ U_1 \left( \left\{ \frac{f_w}{f_k} - \frac{f_s}{f_k} \right\} \frac{f_k}{f_m} \right) + U_2 \left( \left\{ \frac{f_R}{f_k} S - \frac{f_s}{f_k} R \right\} \frac{f_k}{f_m} \right) \right] + \frac{b}{(1-bl)^2} l \left( \frac{m}{(1+m)^2} + \frac{m}{1+m} \frac{f_m}{f_m} \right) U = 0$$

Using the envelope theorem and collecting terms this expression becomes

$$\left( U_1 \frac{f_w}{f_k} + U_2 \frac{f_R}{f_k} S \right) \frac{f_k}{f_m} + \frac{b}{(1-bl)} \frac{l}{(1+m)} \left( \frac{m}{(1+m)} + m \frac{f_m}{f_m} \right) U = 0$$

Introducing into the above equation the expressions for the derivative of wages and interest rate with respect to capital, we obtain

$$(U_2 S - U_1 k) f'' \frac{f_k}{f_m} + \frac{b}{(1-bl)} \frac{l'}{(1+m)} \left( \frac{m}{(1+m)} + m \frac{f_m}{f_m} \right) U = 0$$

using the fact that  $U_1 = R U_2$ , we have

$$(S - Rk) U_2 f'' \frac{f_k}{f_m} + \frac{b}{(1-bl)} \frac{l'}{(1+m)} \left( \frac{m}{(1+m)} + m \frac{f_m}{f_m} \right) U = 0$$

Finally, introducing in the left most bracket the expression for savings and interest rate, we find that

$$(sf - (1+s)f'k) U_2 f'' \frac{f_k}{f_m} + \frac{b}{(1-bl)} \frac{l'}{(1+m)} \left( \frac{m}{(1+m)} + m \frac{f_m}{f_m} \right) U = 0 \quad (23)$$

Given the concavity assumption of the production function the first expression on the left is negative for  $s/(I+s) > \alpha$ . This implies that in order for equation (23) to be zero, the term in brackets in the second expression has to be positive. But this term is the expression for the slope of the Laffer curve, so that the optimal money growth rate is to the left of the maximum seigniorage and the government is not maximizing revenues from money creation. Given that in this model maximization of seigniorage coincides with maximization of growth, it follows that the optimal policy is not maximization of growth. ●

*Remark :* Note that in this model  $s$  is actually a propensity to save for old age out of wage income and does not correspond to the observed saving rate of the economy. If this was the case the only relevant scenarios would be when  $s/(I+s)$  is smaller than  $\alpha$ , because the observed saving rate is around 0.2 and  $\alpha$  is around one quarter or one third for most economies. The optimal policy would be always excessively distortionary. In this model, however, given the definition of  $s$ ,  $\alpha < s/(I+s)$ <sup>15</sup> looks more plausible, because it implies  $S$  larger than  $Rk$ .

The above result has interesting implications. The first one is that in a model where government expenditures are not "wasted" or lump sum transferred but directly affect the productivity growth of the economy, proposition 3 says that the best policy implies always some inflation and that it is usually optimal to introduce some distortion. Note that if  $\lambda'$  was always equal to zero (government expenditures had no effect on labor productivity) equation (23) would never be equal to zero, and the best policy would be to set  $\mu^*$  equal to zero. This is the standard result: in a world where money introduces only distortion, the best policy is to avoid inflation. For all other cases some inflation is always optimal because the benefits of money creation over accumulation are larger than the distortion costs, up to a certain level. Endemic inflation episodes can thus be explained, because if a government is unable to enlarge tax collection (for social or political reasons), the importance of government expenses for the economy and its external effect over capital, force the authorities to resort to money creation.

The second implication is that, under the more plausible scenario ( $s/(I+s) > \alpha$ ), it is never optimal for the authorities to maximize growth and they should keep inflation below the level that maximizes seigniorage. As the rate of money creation rises, the increase in seigniorage becomes progressively small, and so do the welfare gains from

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<sup>15</sup> It may be more intuitive to think of a specialization of this problem where technology is Cobb-Douglas and utility logarithm. In this case both the capital share and the propensity to save  $s$  are constants and do not change with capital. We illustrate this argument further in the end of this section.

economic growth. At a certain point, the loss due to the distortionary effect of inflation tax overcome the gains from growth.

In a previous version of this paper, we replicated this last result through simulation for a large number of combinations of parameters and usual functional forms (it was assumed  $U(c_1, c_2) = b \ln(c_1) + (1-b) \ln(c_2)$ , Cobb-Douglas production function, and  $\lambda(g_t) = 2 - \exp(-g_t/\mathbf{f})$ ). For instance, in the case where  $\alpha$  is one quarter and beta 0,35, the optimal inflation rate is 10 points below the maximand of growth when the discount rate is 0.9.

If, however, the opposite case ever holds and the ratio  $s/(1+s)$  is smaller than  $\alpha$ , we have an intriguing result: a benevolent government should always introduce excessive distortion in the economy. The rate of growth, however, will not be maximized in both cases.

## 5. Conclusions

An overlapping generations model where government expenses positively affects the growth rate of human capital and consequently the rate of growth of the economy was developed in this article. Endemic inflation is thus explained because of, in one hand, the inability of the government to enlarge the tax base (which we did not model here) and, in the other hand, the essential role that infrastructure and education, mostly public financed, play in the economy. The lack of alternative financial sources forces the government to use money to pay for its projects, which are instrumental for economic growth.

It was proved that monetary equilibria exist in this economy for large intervals of parameters and also that steady state capital (per efficiency units of labor) increases with the rate of money creation. However, for large enough rates of inflation, the authorities cannot affect real variables and we are there are only nominal effects. It was also proved that for reasonable values of parameters, the optimal rate of money creation is never above the one that maximizes growth and seigniorage.

There are some extensions of the present framework that we think are worth pursuing. The first and more immediate one is to include taxes in the model. Although there are plenty of historical experiences to justify the present model, such additions would add more realism to it. A second extension would be to endow agents with a more sophisticated human capital investment decision, in such a way that human capital growth would depend not only on public investment but also on the number of hours

individuals spend on training. This would give individuals a more important role in the determination of the rate of growth of the economy.

## Appendix A

In this appendix the necessary conditions for equation (17) in section 2 to be positively sloped and to cut equation (16) to the left of  $k_d$  are shown. Equation (17) is given by

$$f' \left( \frac{sw(k^*) - m^*}{I \left( \frac{m}{1+m} m^* \right)} \right) = \frac{I \left( \frac{m}{1+m} m^* \right)}{1+m}$$

First let us rewrite it taking  $m_t$  as an implicit function of  $k_t$  and  $\mu$ , in which we will take  $\mu$  as a given parameter:

$$m_t = H(k_t, \mu)$$

To obtain the slope of H, we implicitly differentiate  $m_t$  with respect to  $k_t$ , which gives us, after some simplifications and substitutions

$$H_k = \frac{dm}{dk} = \frac{sw'}{1 + I' k \frac{\mu}{1+\mu} \left[ 1 - \frac{s}{1-a} \right]}, \quad (A1)$$

where  $\sigma$  is the elasticity of substitution and  $\alpha$  the capital share. It is easily verified that when the parameter  $\mu$  is zero the slope of this curve is equal to  $sw'$  for any value of  $s/(1-a)$ . This slope will still be positive (as it will always be the case for production functions with small elasticity of substitution) for values of  $\mu$  close to zero or not large enough<sup>16</sup>.

For large values of  $\mu$ , the denominator becomes  $\{1 + I' k [1 - s/(1-a)]\}$ . Clearly it can be negative if the expression  $I' k [.]$  is greater than one in absolute value as, for many production functions, the Cobb-Douglas among them, the expression in brackets is negative. However, for small and large values of  $m_t$  and  $k_t$  we can suppose without problems that the numerator is positive. For large values, as  $\lambda$  is a bounded function we only have to assume that  $I'$  approaches zero at a faster pace than that the capital goes to infinity. On the other hand, for small

<sup>16</sup> For "not large enough" it is meant  $\mu$  such that  $1/\mu > I' k [s/(1-a) - 1] - I$ . Note that for values of  $s/(1-a)$  lower than one this inequality always holds.

values of  $m_t$  and  $k_t$ , we only need that the derivative  $I'(0)$  be bounded<sup>17</sup>. In both cases the denominator is reduced to one and the slope of the money phase line is again  $sw'$ .

For intermediate values of  $m$  everything depends on the value of the ratio  $s/(1-a)$ . Again, it can be the case that the value of the slope is negative everywhere except for very large and for very small values of capital and money holdings, if this ratio is greater than one. On the other hand, it will cross the capital line (if it crosses at all) at  $k_\mu$  defined by

$$f'(sw(k_m)) = \frac{1}{(1+m)} \quad (A2)$$

while its intercept with the money axis will be negative. This can be easily seen because when capital is zero, the money phase line becomes

$$f' \frac{-m}{I\left(\frac{m}{1+m}m\right)} = \frac{I\left(\frac{m}{1+m}m\right)}{1+m} \quad (A3)$$

Note that the derivative of the production function is only defined in  $R_+$ , so that  $m$  has to be negative for  $\{-m/I\}$  be positive.

## Appendix B

This section deals with the dynamic properties of the system composed by equations (16)-(17). Figure 1 in section 2 depicts the phase diagram of this system. We will briefly comment on the determination of the arrows of motion of the system. Let us first study the capital phase line, equation 16:

$$\text{If } k_{t+1} > k_t, \Rightarrow m_t = sw(k_t) - I(g_t)k_{t+1} < sw(k_t) - I(g_t)k_t$$

so that capital is increasing below the phase line.

For the money phase line, suppose that  $m_{t+1} > m_t$ , so that

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<sup>17</sup> This assumption makes economic sense. It says that when the government starts its investments, the marginal gain in labor productivity is not unbounded. This maybe be the case because there are some scale problems with respect to the size of schools, hospitals, etc. However, in order to avoid jumps in the  $\lambda$  function we assumed a slow but smooth increase in the labor productivity for small values of  $g_t$ .

$$f'(k_{t+1}) = \frac{I\left(\frac{m}{1+m}m_t\right)}{1+m} \frac{m_{t+1}}{m_t} > f'\left(\frac{sw(k_t) - m_t}{I\left(\frac{m}{1+m}m_t\right)}\right) = \frac{I\left(\frac{m}{1+m}m_t\right)}{1+m}$$

which implies that

$$k_{t+1} < \frac{sw(k_t) - m_t}{I\left(\frac{m}{1+m}m_t\right)} = k_t$$

This result, in turn, implies that the arrows of motion are upward to the left of the money phase line. By visual inspection, the tree steady states of the system,  $(k_t, m_t) = (0,0)$ ,  $(k_d, 0)$  and  $(k^*, m^*)$ , seen to be a source point, an attractor point and a saddle point, respectively. In what follows, while examining the Jacobian of the linearized system, this properties will be proved under mild assumptions.

For ease in computing the stability conditions it will be useful to rewrite the system as

$$h(k_t, m_t) = k_{t+1} \equiv \frac{sw(k_t) - m_t}{I\left(\frac{m}{1+m}m_t\right)} \quad (B1)$$

$$g(k_t, m_t) = m_{t+1} \equiv \frac{(1+m)m_t}{I\left(\frac{m}{1+m}m_t\right)} f'(h(k_t, m_t)) \quad (B2)$$

We can now state the following proposition:

**Preposition 4:** If  $(sw'/\lambda) < 1$ , the monetary steady state is saddle path so that this equilibrium is determinate. Otherwise, a sufficient condition for a saddle is given by expression (B3) below. The trivial steady state is a source point, while the Diamond equilibrium is an attractor if the sufficient condition  $sw'f' < (1+m)$  holds.

Proof: The Jacobian of the system (B1)-(B2) at the steady state is given by

$$J = \begin{bmatrix} h_k & h_m \\ \frac{1+m}{I} m f'' h_k & \frac{1+m}{I} \left[ m f'' h_m + f' \left( 1 - \frac{I'}{I} \frac{m m}{1+m} \right) \right] \end{bmatrix}$$

where  $h_k = (sw')/I \neq 0$

$$h_m = -(1/I) \cdot [1 + (I' m / (1+m))] \neq 0$$



We can see immediately that the trivial steady state  $(k, m) = (0, 0)$  is a source point. The trace of the Jacobian evaluated at this point reduces to  $sw' + (1+m)f'$  which is positive and greater than one for any  $\mu$ . The determinant becomes  $sw'(1+m)f'$  which is also positive and greater than one. We can conclude that both eigenvalues are positive (it is also easily checked that they are real). Thus, we need only to check the characteristic polynomial at one. It is given by:

$$P(I) = (1 - sw')(1 - f'(1+m))$$

As  $P(1)$  is positive and the determinant is greater than one, both eigenvalues have to be greater than one, so that the trivial steady state is in fact a source.

For the Diamond steady state  $(k, m) = (k_d, 0)$ , both the trace and the determinant have the same expression as the trivial case only now the  $f'$  and  $sw'$  evaluated at  $k_d$  are relatively small. The trace and the determinant are still positive, but the determinant may be less than one depending on the relative magnitude of  $fsw'$  vis-a-vis  $(1+m)$ . If this is the case, the Diamond equilibrium is an attractor point: the characteristic polynomial evaluated at one for  $(k_d, 0)$  is positive, which, together with a determinant between zero and one, implies that both eigenvalues are positive and smaller than one.

For the monetary steady state  $(k, m) = (k^*, m^*)$  the Jacobian of the system can be rewritten as

$$J = \begin{bmatrix} h_k & h_m \\ m \frac{f'}{f} h_k & 1 - h + m \frac{f'}{f} h_m \end{bmatrix}$$

where  $\eta$  is the elasticity of the government expenditure function with respect to  $g_t$ .

It is easily checked that the eigenvalues are always real. The characteristic polynomials evaluated at one and minus one are, respectively, after collecting terms

$$\begin{aligned} p(-1) &= (2-h)(1+(sw')/1) + m.h_m(f''/f'), \\ p(1) &= h(1-(sw'/1)) - m.h_m(f''/f'). \end{aligned}$$

The condition for a stationary equilibrium to be a saddle point is that the value of these characteristic polynomials have opposite signs. The first expression is positive if we assume  $\eta$  smaller than (or close to) two. The second polynomial is always negative if  $(k^*, m^*)$  are at the upward part of equation (B1) where  $(sw'/1)$  is greater than one. Otherwise a sufficient condition for negativity is

$$1 - \frac{sw'}{1} < \frac{1-s}{s} \left[ 1 + \frac{1+m}{m} \frac{1}{kl'} \right] \quad (B3)$$

We see no problem with condition (B3) except perhaps for large values of  $k$ . In this case, the ratio  $sw'/I$  is very small. But when capital is very large, money demand is close to zero in this economy, so that the derivative  $I'$  is also near zero. This fact would imply that  $kI'$  is very small if  $I'$  goes faster to zero, so that the condition would hold. For a small stock of capital, the ratio  $sw'/I$  is greater than one so that the left hand side of the inequality is negative, while the right hand side is always positive.

The condition that the elasticity of government expenditures be less than two, which is only sufficient, is not a strong one. It places a reasonable bound on the magnitude of the relative response of labor productivity to increases in the government investments in infra-structure. In summary, if condition B3 (together with  $\eta$  not much bigger than two) holds, the monetary stationary equilibrium is a saddle point.

## Appendix C

This section studies the derivative of the steady state money demand and capital with respect to the rate of money creation. It is proved that for  $\mu$  small and large these derivatives can be reduced to equations (21) and (22), respectively, of section II. To simplify calculations and notation rewrite the system as

$$m_t + I\left(\frac{m}{1+m}m_t\right)k_t - sw(k_t) = 0 \quad (C1)$$

$$m_t + I\left(\frac{m}{1+m}m_t\right)\Omega(k_t, m) - sw(k_t) = 0 \quad (C2)$$

Where the function  $\Omega(k_t, \mu)$  solves for  $k_{t+1}$  implicit as the solution of

$$f'(k_{t+1}) = \frac{I\left(\frac{m}{1+m}m_t\right)}{1+m}$$

Note that this time the system was constructed plugging equation (14) into equation (15) to define the system in the steady state. In the paper we worked in the reversed order. This does not change the results, but only simplify things.

The derivative of money demand with respect to  $\mu$  is given by

$$\frac{dm}{d\mathbf{m}} = \frac{\Omega_m[sw'-I] + \frac{I' m \Omega_m}{1+m}}{\Omega_m[sw'-I] - \left[1 + \frac{I' m \Omega_m}{1+m}\right]}, \quad (C3)$$

where  $W_m$  is given by  $\{(I' m)/(1+m)^2 f''\}$ , which is non-positive, while  $\Omega_m$  is given by  $\{[I/(1+m)^2 f''][(h/m) - I]\}$ , which can be either positive or negative, depending on  $h/m$  being smaller or greater than one.

As  $\mu$  becomes larger it can be immediately seen, given previous hypothesis, that  $W_m$  approach zero because  $\lambda'$  goes to zero and  $m'(1+m)^2$  becomes small. On the other hand,  $I'W$  also approaches zero, as  $\Omega$  is bounded by  $k_d$ , and the denominator goes to minus one. This all results that  $dm/d\mathbf{m}$  is reduced to

$$\frac{dm}{d\mathbf{m}} = \Omega_m[sw'-I] \quad .$$

This expression can be reduced immediately to equation (21a).

For  $\mu$  small we have that  $W_m$  goes to zero, because  $\lambda' \mu$  goes to zero, while the second term in the numerator reduces to minus one, again because of the behavior of  $I'm$ . Given that  $h(0)$  is zero,  $W_m$  is reduced to  $(-I/(1+m)f'')$ . Collecting terms we get

$$\frac{dm}{d\mathbf{m}} = \frac{I'}{(1+m)f''}[sw'-I],$$

which is equation (21b).

Differentiating the system (C1)-(C2) with respect to capital we obtain

$$\frac{dk}{d\mathbf{m}} = \frac{-\Omega_m \left[1 + \frac{I' m \Omega_m}{1+m}\right] + \frac{I' I m \Omega_m \Omega_m}{(1+m)^2}}{\Omega_m[sw'-I] - \left[1 + \frac{I' m \Omega_m}{1+m}\right]}$$

Once again there is no definite sign for this expression, unless we make some unreasonable assumptions. However is not difficult to define its sign for limit rates of money supply.

For  $\mu$  close to zero, the second term in the numerator goes to zero (as both  $\Omega_m$  and  $\lambda'$  are zero), while the denominator reduces to minus one, as we have seen in the previous case. The whole derivative reduces to  $W_m$  which approaches equation (22) close to zero. For large  $\mu$ , we already know that the

denominator goes to minus one. As for the numerator, the second term goes to zero as  $W_m$ ,  $I'$  and  $1/(1+m)^2$  go to zero, while  $\lambda$  is bounded. The second term reduces again to  $\Omega_\mu$ , and the proof that the derivative of capital stock in the steady state can be reduced to equation 22 for both small and large rates of money creation is completed.

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