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***MANDATORY PROFIT SHARING, ENTREPRENEURIAL
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Abstract

The impact of a mandatory tax on profits which is transferred to workers is analyzed in a general equilibrium entrepreneurial model. In the short run, this distortion reduces the number of firms and the aggregate output. In the long run, if capital and labor are bad substitutes, it fosters capital accumulation and increases the aggregate output. In a small open economy with free movement of capital, it improves the welfare of the economy's average individual. One concludes that the benefits of sharing schemes may go beyond the short run employment-stabilization goal focused by the profit sharing literature.

Keywords: Profit sharing, incentives, capital accumulation, factor substitution

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I - INTRODUCTION

The usual models on profit sharing (PS) study the impact of sharing contracts on the reduction of nominal wage rigidities that cause unemployment. The model presented in this paper does not address the impact of PS on unemployment. It aims at addressing another aspects of sharing schemes, namely: (1) the entrepreneurial incentives that impinge on an individual's decision to set up his own business, thus creating new jobs, instead of being a worker and competing for one of the existing jobs, and; (2) the impact of PS on capital accumulation.

The model differs from the usual models in five ways. First, in stark contrast with the models that focus on the rigidities that generate unemployment, it is assumed that the wage is flexible, which rules out involuntary unemployment. Second, there is no uncertainty. Third, it studies profit sharing schemes that are not freely negotiated between workers and firms, but are imposed from outside by, say, a government decree. Fourth, it takes into account that the number of firms in operation in an economy is not invariant to the compensation scheme adopted. And fifth, it analyzes the impact of profit sharing on capital accumulation.

In short, this paper views PS not from its possible contribution to circumventing nominal rigidities, but from its impact on the allocation of factors of production and capital accumulation. When managerial talent is an "asset" unevenly distributed across individuals, the mandatory introduction of PS works as a tax levied on the rent of this "asset". The proceeds of this tax is transferred to workers as a subsidy. Under this arrangement, is it possible that such a distortion may yet increase the economy's output? Under what circumstances would it happen? In order to make clear the distinction of the focus of this paper and that of the other papers on PS, I provide below a brief description of the literature on this issue.

The work of Martin Weitzman (1983), (1985) and (1987) kindled interest on PS as an alternative compensation system which could be a useful tool to reduce the main nominal rigidity that causes unemployment. Since it emulates a flexible labor market, i.e., one where the nominal wage adjusts instantaneously in order to provide full employment, PS was advocated as a remarkable form of avoiding the dreaded Keynesian unemployment.

In Weitzman's economy, the marginal cost of labor would be below its average cost, creating permanent excess demand for labor, thus eliminating unemployment. Nordhaus (1988) showed that for the Weitzman proposition to be valid, two conditions must hold: the supply price of labor must fall very sharply in recessions, and the marginal cost of labor must be very far below the average cost of labor.

Weitzman's explanation for the fact that profit sharing be rare in most western countries was based on strong externality effects. When *one* wage firm converts to a share contract it will be guaranteeing employment not only to its own (internal) workers, but also serving as the employer of last resort for *all* other (outside) workers. In bad times internal workers would see their compensation falling in order to rescue outsiders. Since most of the benefits accrue not to its own workers, but to the working class as a whole, internal workers face no incentive to accept profit sharing when the workers of other firms do not accept it as well.

This argument is analogous to Keynes' (1936) explanation of why nominal wages are fixed in the short run: no one is willing to be the first to reduce nominal wages. As a result, the economy ends up in an inefficient Nash equilibrium without profit sharing. Weitzman suggests that in order to overcome this perverse coordination failure some sort of government incentive to profit sharing schemes were in order. Brunello (1992) shows that if internal promotion were the only way to climb the rungs of a career within a firm, than internal workers would favor profit schemes since the high rungs would only be attained if outsiders were hired to fill the low ones.

Following Weitzman's emphasis on short run aggregate fluctuations, Cooper (1988) presents a model of monopolistic competition in the presence of multiplier effects in which the introduction of share contracts in one sector changes the response to the adverse shocks and alters the nature of the interaction between the sectors. In his model there is only one very special share contract which Pareto dominates the fixed-wage system. That is, there is only one very special contract which can balance the gains and losses to the various groups of agents in the economy from the introduction of share contracts. Nothing can assure, however, that the real economy has the arcane power to pick out this special contract. Moreover, even if the firms were able to single out the special contract, how could it be coordinated in order to create, say, a (stable) Nash equilibrium? John (1991) casts additional light in the factors that impinge on profit sharing. When a firm's (marginal or total) revenue are very sensitive to employment, greater employment fluctuation may arise in a share firm.

James Mead (1986) not only agrees with Weitzman's view that profit sharing provides greater stability of employment than a wage economy, but also believes that a firm in a share economy will be in excess demand for labor. He also suggests that the share system might increase labor effort. By making worker's income a function of profits, the incentives to shirk are reduced. Moreover, each worker will tend to help with the supervision of fellow workers and might even impose social sanctions to those who shirk. However unobservability of individual effort may undermine this argument when profit

sharing is based on collective output. González (1992) discuss this point. Kandel & Lazear (1992) explores how peer pressure may overcome unobservability of individual effort.

The impact of profit sharing on investment was studied by Wadhvani (1987). He concludes that it increases the cost of capital, thus tending to reduce the level of the capital stock. This point is analyzed in a more complete setting in this paper. It is shown that a necessary condition for cost of capital to increase is that labor and capital be poor substitutes.

The empirical evidence on profit sharing has focused both on productivity as well as on employment. Jones & Svejnar (1985) found evidence that profit sharing had positive productivity effects in Italy. FitzRoy & Kraft (1987) found strong influence of profit sharing on factor productivity in a sample of medium-sized metalworking firms in Germany. Blanchflower & Oswald (1987) found no evidence that profit sharing influenced employment in the United Kingdom. Cable & Wilson (1989) estimated productivity gains of between 3 and 8% due to profit sharing in the UK engineering industry. Kruse (1992) presents evidence that, although in a small magnitude, profit sharing did indeed increase productivity in the USA. Bell & Newmark (1993) using firm-level data for the union sector of the US economy found evidence that profit sharing reduced labor cost growth at firms that adopted these plans.

In short, the literature on PS focuses on unemployment, labor effort and coordination failures that preclude a discentralized economy from adopting PS. In the present paper it will be argued that even in an economy that operates with full employment, PS may have beneficial effects. The basic framework is an adaptation of the model used by Kihlstrom & Laffont (1979) to study firm formation. Kanbur (1979) analyzed the personal distribution of income under uncertainty in a similar model. Lucas (1979) used this basic framework to study the impact of capital accumulation on the average size of firms. In Lucas' paper the source of capital accumulation was exogenous for he was concerned with fitting his model to actual time series. As in Lucas' paper there is no uncertainty in the present model, since capital accumulation is the result of consumption-savings decision of infinite lived individuals.

There are two exogenous characteristics of the economy: (1) the available production technology, which is unique for all potential entrepreneurs, and; (2) the distribution of managerial skills. The production technology is homogeneous of a degree one in management, labor and capital. The most skilled individuals set up firms that hire the least skilled ones. For the former their profits are higher than the wage received by workers. At each instant of time each individual chooses (1) whether to be an entrepreneur or a worker, taking as given the wage and the rental - or interest rate - and; (2) how much

of his income is consumed. There are no price rigidities, so capital and labor markets are continuously in equilibrium.

The mandatory introduction of PS cuts down the net profit received by entrepreneurs, hence leading the least efficient ones to close their firms down. From the aggregate viewpoint, when some entrepreneurs put up the shutters and become workers, there is an increase of the amount of one input (labor) at the expense of another (management). Since the production function exhibits decreasing returns for each input, the short run aggregate output falls.

The impact of the lower (short run) capital/labor ratio on the interest rate is ambiguous: it falls if capital and labor are good substitutes, or if high levels of PS are introduced; it increases if capital and labor are poor substitutes and PS is introduced at a small level. The degree of substitutability between capital and labor plays a key role. It is shown that if the elasticity of substitution between capital and labor is bounded above from one, then the mandatory introduction of PS at a low level increases the interest rate above the rate of time preference and, therefore fosters capital accumulation and hence increases the economy's long run output. In a small open economy, the possibility of attracting foreign capital at a fast pace allows PS to accelerate its long run benefits shortening the spell of time during which the aggregate output falls short of the level it would have without PS.

One concludes that the potential benefits of PS might go beyond the short run employment-stabilization argument focused in the usual literature. A necessary condition for these benefits to obtain, however, is that capital and labor be bad substitutes.

The paper is organized as follows. Section II describes the economic environment. Section III studies the short run equilibrium. Section IV analyzes the long run equilibrium. Section V focuses on the dynamics of the economy from one initial steady state without PS to the final steady state with PS. Section VI studies the welfare consequences of the mandatory introduction of profit sharing. Section VII concludes. For ease of exposition, the proofs of the propositions are presented in the appendix.

II - THE ECONOMIC ENVIRONMENT

The economy is populated by a constant (large) number \bar{N} of individuals dispersed over the interval $[0,1]$ and characterized by a label $\lambda \in [0,1]$. The distribution $G: [0,1] \rightarrow [0,1]$, with continuous and bounded density $g \geq 0$ defines, for each λ , the fraction $G(\lambda)$ of individuals whose labels are lower than λ .

There is only one good in the economy. It can be produced, consumed or saved. The production technology is available to all individuals and they are equally skilled to be

workers but differ in their abilities to manage a firm. The parameter λ is a measure of managerial skills (MS). An individual of type λ who sets up a firm, employing $K_\lambda(t)$ units of capital and $N_\lambda(t)$ units of labor at instant t will get an instantaneous flow of production $Y_\lambda(t)$ given by

$$Y_\lambda(t) = \Phi(\lambda, K_\lambda(t), N_\lambda(t)) \quad (1)$$

where the function $\Phi: \mathbb{R}^3 \rightarrow \mathbb{R}$ is increasing, strictly concave, twice continuously differentiable, homogeneous of degree one and $\Phi(0, K, N) = \Phi(\lambda, 0, N) = \Phi(\lambda, K, 0) = 0$. It is also assumed that the function Φ is homogeneous of degree b in its last two arguments, where $0 < b < 1$. Hence, one can define the function $F: \mathbb{R}^2 \rightarrow \mathbb{R}$ as

$$Y_\lambda = \Phi(\lambda, K, N) = \lambda \Phi(1, K/\lambda, N/\lambda) = \lambda^{1-b} \Phi(1, K, N) \equiv \lambda^{1-b} F(K, N)$$

It follows that $F: \mathbb{R}^2 \rightarrow \mathbb{R}$ is increasing, strictly concave and homogeneous of degree b .

The capital and labor markets are competitive and are continuously in equilibrium at the rental $r(t)$ and the wage $w(t)$ -- which are determined in general equilibrium. Each individual takes $r(t)$ and $w(t)$ as given when he is making his economic choices. The gross profit of an entrepreneur of type λ at instant t , $\Pi_\lambda(t)$, is given by

$$\Pi_\lambda(t) = Y_\lambda(t) - [r(t) + \delta] K_\lambda(t) - w(t) N_\lambda(t) \quad (2)$$

where $\delta > 0$ is the instantaneous rate of capital depreciation.

The profit sharing parameter τ , which is exogenous to the model, defines the net profit of an entrepreneur of type λ as $(1 - \tau)\Pi_\lambda(t)$, and the share $\tau\Pi_\lambda(t)$ which must be transferred to workers. Each worker at a firm of type λ earns the wage $w(t)$ and a profit sharing income $s_\lambda(t)$ which is the workers' share on gross profits equally divided among the $N_\lambda(t)$ employees:

$$s_\lambda(t) = \tau \frac{\Pi_\lambda(t)}{N_\lambda(t)} \quad (3)$$

Equation (3) allows for the possibility that the profit sharing income might vary across firms. As will be proven in equation (14), this will not be the case for the gross profit per employee will be a function of the ratio rental/wage faced by all the firms.

At each instant t the individual λ 's goal is to maximize his life utility as given by

$$\int_t^\infty e^{-\rho x} U_\lambda[c_\lambda(x)] dx \quad (4)$$

where $c_\lambda(x)$ is his flow of consumption at instant $x > t$ and $\rho > 0$ is the rate of time preference. The function U_λ is assumed to be strictly increasing, concave and twice continuously differentiable. The rate ρ does not vary across individuals.

Letting $I_\lambda(t)$ stand for individual λ 's non-asset income at instant t , his net wealth $A_\lambda(t)$, vary across time according to his inter temporal budget constraints

$$\dot{A}(t) = I_\lambda(t) + r(t)A_\lambda(t) - c_\lambda(t) \quad (5a)$$

and

$$\int_t^\infty e^{-\int_t^x r(u) du} (c_\lambda(x) - I_\lambda(x)) dx \leq A_\lambda(t) \quad (5b)$$

where, and the dot on the top of a variable stands for its time derivative.

It is assumed that there is no market for entrepreneurial work, which implies that each firm must be managed by its owner. Since each individual can freely choose to be an entrepreneur or a worker, he decides what kind of agent to be seeking to maximize $I_\lambda(t)$:

$$I_\lambda(t) = \max \{ (1 - \tau)\Pi_\lambda(t); w(t) + s(t) \} \quad (6)$$

Given equations (1)-(6), an individual facing $r(t)$ and $w(t)$ will, at each instant t , maximize his inter temporal utility in the following way:

1. Maximize (2) subject to (1) in order to calculate the gross profit $\Pi_\lambda(t)$ he would receive if he chose to be an employer;
2. Compare the net profit $(1 - \tau)\Pi_\lambda(t)$ with $s(t) + w(t)$ according to (6) and then decide whether to be an entrepreneur or a worker;
3. Once he has chosen what kind of agent to be at instant t , he will determine his flow of consumption $c_\lambda(t)$ seeking to maximize (4) subject to (5), where $I_\lambda(t)$ is given by (6) and $s(t)$ by (3).

The choice above assumes that there is no friction such as a cost to close a firm down or to set one up. It is also assumed that the rental and the wage adjust instantaneously - in general equilibrium - such as to clear the capital and labor markets.

An individual of type λ who has decided to be an entrepreneur, determines his demand for capital $K_\lambda^D(t)$ and for labor $N_\lambda^D(t)$, for given $r(t)$ and $w(t)$, from the maximization of (2) subject to (1). The first order conditions of this maximization are:

$$\lambda^{1-b} F_1(K_\lambda^D, L_\lambda^D) = r + \delta \quad (7)$$

$$\lambda^{1-b} F_2(K_\lambda^D, L_\lambda^D) = w \quad (8)$$

where the time variable t was dropped to avoid cluttering the notation. At the partial equilibrium level, if the size of the share of profits that must be transferred to workers τ is sufficiently low - or if λ is sufficiently high - for the strict inequality $(1 - \tau)\Pi_\lambda(t) > w(t) + s(t)$ to hold with slackness, then the size of τ does not influence the choice of the entrepreneur of type λ . Although the size of τ may not influence the

decision of a highly qualified entrepreneur, for the least qualified ones, it may lead them to close their firms down and become workers as will be studied in the next section.

Lemma 1 recalls some useful properties of homogeneous functions that will facilitate the mathematical development of equations (7) and (8).

LEMMA 1- PROPERTIES OF THE HOMOGENEOUS FUNCTION F

The function $F : \mathcal{R}^2 \rightarrow \mathcal{R}$ is homogeneous of degree b , $0 < b < 1$, increasing, strictly concave, twice differentiable and the following properties hold for any $(K, L) \in \mathcal{R}^2$:

- (P1) $K F_1(K, L) + L F_2(K, L) = b F(K, L);$
- (P2) $F_i(K/L, 1) = L^{1-b} F_i(K, L), \text{ for } i = 1 \text{ or } i = 2;$
- (P3) $(K/L) F_1(K/L, 1) + F_2(K/L, 1) = b F(K/L, 1).$

Applying (P2) to (7) and (8) yields

$$\frac{\lambda^{1-b}}{(L_\lambda^D)^{1-b}} F_1(K_\lambda^D / L_\lambda^D, 1) = r + \delta \quad (9)$$

$$\frac{\lambda^{1-b}}{(L_\lambda^D)^{1-b}} F_2(K_\lambda^D / L_\lambda^D, 1) = w \quad (10)$$

Defining the ratio of inputs at firm λ as $h_\lambda = K_\lambda^D / L_\lambda^D$, equations (9) and (10) imply that $F_1(h_\lambda, 1) / F_2(h_\lambda, 1) = (r + \delta) / w$, which shows that the ratio of inputs does not vary across firms. Hence, h will substitute for h_λ hereafter. Since $F_{11} < 0$ and $F_{21} > 0$, one concludes that h is a decreasing function of the ratio $(r + \delta) / w$, i.e., the ratio of (demanded) inputs is a function of the ratio of input prices. Note that if the function F were homogeneous of degree one, h would be a function of r alone. Another important consequence of working with a homogeneous function of degree $b < 1$ is given by (P1). Substituting (7) and (8) into (P1) one concludes that the gross profit $\Pi_\lambda(t)$ - defined by (2) - is equal to $(1 - b)$ times the output $Y_\lambda(t)$ - defined by (1).

Lemma 2 defines the auxiliary function f - as well as works out its properties - which will greatly facilitate the mathematical development of equations (9) and (10).

LEMMA 2- DEFINITION AND PROPERTIES OF THE AUXILIARY FUNCTION f

Define the function $f : \mathcal{R} \rightarrow \mathcal{R}$ as $f(h) = F(h, 1)$ for all $h > 0$. Then the following properties hold:

- (P4) $f(0) = 0$, $f'(h) = F_1(h, 1) > 0$, and $f''(h) < 0$, for all $h > 0$;

(P5) $F_2(h,1) = b f(h) - h f'(h)$, for all $h > 0$;

(P6) Define the function $\eta_1: \mathcal{R} \rightarrow \mathcal{R}$, $\eta_1(h) = h f'(h)/f(h)$. Then for all $h > 0$, $0 < \eta_1(h) < b$;

(P7) Define the function $\eta_2: \mathcal{R} \rightarrow \mathcal{R}$ as $\eta_2(h) = -h f''(h)/f'(h)$. Then $(1-b) < \eta_2(h)$, for all $h > 0$;

(P8) The elasticity of substitution of F at $h > 0$, $\sigma(h)$, is lower than one if, and only if, $\eta_1(h) + \eta_2(h) > 1$, for all $h > 0$;

(P9) For all $h > 0$, $(1-b)\sigma(h) < \eta_2(h)\sigma(h) < 1$.

(P10) The function $\eta_1: \mathcal{R} \rightarrow \mathcal{R}$ is decreasing at $h > 0$ if, and only if, the elasticity of substitution of F at h , $\sigma(h)$, is lower than one;

From (7), (P2) and the fact that F is homogeneous of degree b , one can interpret the term $\eta_1(h)$ as the capital's share in output:

$$(r + \delta) \frac{K_\lambda}{Y_\lambda} = \frac{\lambda^{1-b} F_1(h,1)}{L^{1-b}} \frac{K_\lambda}{\lambda^{1-b} L^b F(h,1)} = \frac{h f'(h)}{f(h)} = \eta_1(h)$$

Substituting (P5) into (10), the demand for labor of a firm of type λ is written as:

$$L_\lambda^D = \lambda \left[\frac{[b f(h) - h f'(h)]}{w} \right]^{\frac{1}{1-b}} \quad (11)$$

From the definition of h and (11) the demand for capital of a firm of type λ is written as

$$K_\lambda^D = h L_\lambda^D = \lambda h \left[\frac{[b f(h) - h f'(h)]}{w} \right]^{\frac{1}{1-b}} \quad (12)$$

Since h is the same for all firms, (11) and (12) assure that the higher is λ , the higher will be the use of both inputs and hence the scale of production, i.e., the most skilled entrepreneurs manage the largest firms.

Substituting (P1) and (11) into (2) the gross profit of an entrepreneur of type λ is given by

$$\begin{aligned} \Pi_\lambda &= (1-b) \lambda^{1-b} F(K_\lambda^D, L_\lambda^D) = (1-b) \lambda^{1-b} (L_\lambda^D)^b f(h) \\ &= (1-b) \left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{b}{1-b}} f(h) \lambda \end{aligned} \quad (13)$$

Equation (13) asserts that the most qualified entrepreneurs - with higher λ - make the highest profits, for they operate with the largest scales of production. The scale of production and of profits is linearly proportional to the level of managerial skill.

The total compensation $R_\lambda(t)$ of a worker at a firm of type λ , $R_\lambda(t) = w(t) + s_\lambda(t)$, where $s_\lambda(t)$ defined by (3), is given by the substitution of (11) and (13) into (3):

$$R_\lambda = w + \tau \Pi_\lambda / L_\lambda^D = w \left[1 + \tau (1-b) \frac{f(h)}{b f(h) - h f'(h)} \right] = w \left[1 + \frac{\tau (1-b)}{b - \eta_l(h)} \right] \quad (14)$$

The expression above shows that the total compensation of workers, R , does not vary across firms. Therefore the subscript λ will henceforth be dropped from R . Careful examination of (14) shows that workers' total compensation R is proportional to the wage w by a mark-up $\tau(1-b)/(b - \eta_l(h))$. The interpretation of this mark up is immediate. Since b is the sum of the shares of capital and labor payroll - excluding PS - in output, it follows that $\tau(1-b)$ is the ratio of labor's share in profits to output. Likewise, $(b - \eta_l)$ is the share of labor payroll in output.

Examining equations (13) and (14) one concludes that while the workers' total compensation $R(t)$ does not vary across λ , the net profit $(1-\tau)\Pi_\lambda(t)$ is an increasing function of λ with $(1-\tau)\Pi_\lambda(t) = 0$ for $\lambda = 0$. The individual choice of whether to be an entrepreneur or a worker can now be described. Given the wage $w(t)$, those individuals for whom $(1-\tau)\Pi_\lambda(t)$ exceeds $R(t)$ will choose to be entrepreneurs whereas those whom $R(t)$ exceeds $(1-\tau)\Pi_\lambda(t)$ will prefer to be workers. Therefore there must be a watershed $\Lambda(t) \in (0,1)$ such that individuals of type $\lambda \in [0, \Lambda(t)]$ will chose to be workers and individuals of type $\lambda \in (\Lambda(t), 1]$ will prefer to be entrepreneurs. The existence of such $\Lambda(t)$ is assured by the hypothesis of a flexible labor market which continuously adjusts the wage $w(t)$ so that the labor market is cleared.

III - THE SHORT RUN GENERAL EQUILIBRIUM

The concept of short run in this paper is a period of time sufficiently short for the aggregate capital stock to be considered as constant. There are three key general equilibrium conditions: (1) the capital market clearing; (2) the labor market clearing and; (3) the marginal entrepreneur's condition of indifference between being an entrepreneur or a worker. They define a system of three equations and three equilibrium variables: the rental $r(t)$, the wage $w(t)$ and, the watershed $\Lambda(t)$. In order to facilitate the mathematical treatment of the model, the capital/labor ratio $h(t)$, which was shown - in equations (9) and (10) - to be a decreasing function of the ratio $r(t)/w(t)$ of input prices, will substitute for the rental $r(t)$. Hence the general equilibrium variables analyzed hereafter will be: $h(t)$, $w(t)$ and $\Lambda(t)$.

The Capital Market

At each instant t the aggregate supply of capital, $K^S(t)$, is the sum of all individuals' assets. There are $\bar{N}g(\lambda)$ individuals of type λ and each one owns $A_\lambda(t)$ units of assets, so $K^S(t)$ is fixed in the short run and given by:

$$K^S(t) = \bar{N} \int_0^1 A_\lambda(t) g(\lambda) d\lambda \quad (15)$$

The *aggregate demand for capital*, $K^D(t)$, is the sum of all entrepreneurs' demand for capital. There are $\bar{N}g(\lambda)$ entrepreneurs of type λ and each one demands $K_\lambda^D(t)$. From (12), $K^D(t)$ is given by

$$K^D(t) = \bar{N} \int_{\Lambda(t)}^1 K_\lambda^D(t) g(\lambda) d\lambda = \bar{N} \left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-\alpha}} h \int_{\Lambda(t)}^1 \lambda g(\lambda) d\lambda \quad (16)$$

The Labor Market

The aggregate supply of labor at instant t , $N^S(t)$, is equal to the total number of individuals \bar{N} times the fraction of them who choose to be workers:

$$N^S(t) = \bar{N} G(\Lambda(t)) \quad (17)$$

The aggregate demand for labor, $N^D(t)$, is the sum, of all entrepreneurs' demand for labor. There are $\bar{N}g(\lambda)$ entrepreneurs of type λ and each one demands $N_\lambda^D(t)$. From (11), $N^D(t)$ is given by:

$$N^D(t) = \bar{N} \int_{\Lambda(t)}^1 N_\lambda^D(t) g(\lambda) d\lambda = \bar{N} \left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-\alpha}} \int_{\Lambda(t)}^1 \lambda g(\lambda) d\lambda \quad (18)$$

The Marginal Entrepreneur's Indifference Condition

The watershed $\Lambda(t)$ is such that an individual of type $\lambda = \Lambda(t)$ is indifferent between being an entrepreneur or a worker, since for him the net profit he would receive if he set up a firm $(1-\tau)\Pi_{\Lambda(t)}(t)$ would be equal to the workers' compensation $R(t)$. From (13) and (14) this requires:

$$(1-\tau)(1-b) \left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-\alpha}} f(h) \Lambda = w \left[1 + \frac{\tau(1-b)}{b - \eta_l(h)} \right]$$

Rearranging the expression above one gets the *marginal entrepreneur's indifference condition*

$$\left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-\alpha}} \Lambda = \frac{(b - \eta_l(h)) + \tau(1-b)}{(1-b)(1-\tau)} \quad (19)$$

The Short Run General Equilibrium

As already mentioned, it is assumed that the input prices $r(t)$ and $w(t)$ are flexible so that at each instant t , the capital and labor markets clear. It is also assumed that there is no friction that hinders an entrepreneur from closing down his firm and becoming a worker, nor that may hamper a worker from quitting his job and opening up his own firm.

Given the short run aggregate supply of capital $K^S(t)$ defined by (15), equating $K^S(t)$ to the aggregate demand for capital $K^D(t)$ defined by (16) provides the *capital market equilibrium equation*:

$$\frac{K^S(t)}{\bar{N}} = \left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} h \int_{\Lambda}^I \lambda g(\lambda) d\lambda \quad (20)$$

Likewise, equating the aggregate supply of labor $N^S(t)$ given by (17) to the aggregate demand for labor given by (18), one gets the *labor market equilibrium equation*:

$$\frac{G(\Lambda)}{\int_{\Lambda}^I \lambda g(\lambda) d\lambda} = \left[\frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} \quad (21)$$

The market clearing equations (20) and (21) involve three variables: w , h and Λ . The missing equation is the marginal entrepreneur's indifference equation (19). The three equilibrium variables w , h and Λ are, therefore, implicitly determined by the system of equations (19), (20) and (21). Noting that the term on the right hand side of (21) is present in (19), (20) and (21), this system can be simplified into two equations involving the variables h and Λ plus a third equation which determines w given h and Λ . (20) and (21) imply:

$$h = K^S / \bar{N} G(\Lambda) \quad (22)$$

which shows that the capital/labor ratio at each firm h is equal to the ratio of the aggregate stock of capital to the number of workers.

Now, substituting (21) into (19) one gets

$$\frac{G(\Lambda) \Lambda}{\int_{\Lambda}^I \lambda g(\lambda) d\lambda} = \frac{b - \eta_l(h) + \tau(1-b)}{(1-b)(1-\tau)} \quad (23)$$

Define the auxiliary differential function $Z: \mathcal{R}^+ \rightarrow \mathcal{R}^+$ such that $Z(\Lambda)$ is equal to the left hand side of (23). Then, Z has the following properties: $Z(0) = 0$, $dZ/d\Lambda > 0$, and $\lim_{\Lambda \rightarrow I} Z(\Lambda) = +\infty$. Since b is the sum of the shares of capital and labor payroll - excluding PS - in output, it follows that $(b - \eta_l)$ is the share of labor payroll in output. Likewise, since $\tau(1-b)$ is the ratio of labor's share in profits to output, it follows that the numerator

of (23), $(1 - \eta_l) + \tau(1 - b)$, is the total share of labor - including PS - in output. Noticing that the denominator of (23), $(1 - \tau)(1 - b)$, is the entrepreneur's share of net profits in output, one concludes that Z can be interpreted as the ratio, at each firm, of workers' compensation to entrepreneur's net profits. The lower is the number of firms in operation, the higher will be this ratio in each firm. Equation (23) is then rewritten as:

$$\frac{1 - \eta_l(h)}{1 - b} = (1 - \tau)[1 + Z(\Lambda)] \quad (24)$$

The system (22) and (24) determines the equilibrium variables $h(t)$ and $\Lambda(t)$. Substituting (22) into (24) one gets:

$$1 - \eta_l(K(t)/\bar{N}G(\Lambda)) = (1 - b)(1 - \tau)[1 + Z(\Lambda)]$$

This expression determines the watershed $\Lambda(t)$ given the short run aggregate stock of capital $K(t)$ and the PS parameter. This expression embodies the two basic exogenous characteristics of the economy. The first one is the technology as described by the degree of homogeneity b and the auxiliary production function f which defines the function η_l . The second one is the distribution of EP represented by G which defines the auxiliary function Z . As will be seen ahead, the impact of the introduction of PS on most of the endogenous variables of the model will not depend on additional hypothesis about the production F and distribution functions and G . Proposition I presents the short run comparative statics of $h(t)$ and $\Lambda(t)$ with respect to the PS parameter.

PROPOSITION I: *An increase of the PS parameter reduces the number of firms in operation and the capital/labor ratio of these firms.*

The result above shows that when entrepreneurs are compelled to transfer part of their profits to their employees, the least efficient entrepreneurs' net profits will fall short of the prevailing worker's total compensation. As a result, these marginal entrepreneurs close their firms down and become workers. These firms release their previous demand for capital and labor, hence increasing the aggregate supply of capital and of labor. The aggregate capital stock is fixed in the short run, while the new aggregate supply of labor includes the former marginal entrepreneurs. Hence the economy's capital/labor ratio decreases. Since the capital/labor ratio is the same across the firms that remain in operation, it follows that each company's capital labor/ratio falls.

The third equilibrium variable, the wage $w(t)$, is given by the substitution of $h(t)$ and $\Lambda(t)$ into (21):

$$w(t) = [b f(h(t)) - h(t) f'(h(t))] \left(\int_{\Lambda(t)}^1 \lambda g(\lambda) d\lambda / G(\Lambda(t)) \right)^{1-b} \quad (25)$$

PROPOSITION II: *An increase of the PS parameter reduces the wage.*

The increased aggregate supply of labor, for a fixed aggregate supply of capital, cuts down the equilibrium wage. For ease of exposition, the impact of the PS parameter on workers' total compensation $R(t)$ will be analyzed later.

From (P4), (P5), (9), (10) and (25), the rental $r(t)$ is given by

$$r(t) + \delta = w(t) \frac{f'(h(t))}{b f(h(t)) - h(t) f(h(t))} = f'(h(t)) \left(\int_{\Lambda(t)} \lambda g(\lambda) d\lambda / G(\Lambda(t)) \right)^{1-b} \quad (26)$$

Recalling that $\sigma(h)$ stands for the elasticity of substitution F at the capital/labor ratio $h > 0$, let $\sigma_L > 0$ be a lower bound for this elasticity, and $\sigma_U > 0$ be an upper bound for this elasticity, i.e., $\sigma_L \leq \sigma(h) \leq \sigma_U$ for all $h > 0$. Proposition III presents the short run impact of an increase of the sharing parameter on the rental:

PROPOSITION III: (III.1) *If $\tau > (1 - \sigma_L)$, then an increase of the PS parameter reduces the rental⁽¹⁾; (III.2) If $\sigma_U < 1$, then there exists a real number $\tau_r > 0$, $0 < \tau_r < 1$, such that for a PS parameter τ in the range $0 < \tau < \tau_r$, an increase of τ increases the rental.*

In order to interpret the result above it is necessary to recall, from (9) and (10), that each firm's capital/labor ratio h is a decreasing function of the ratio rental/wage, and not of the rental alone as would be the case if the production function F were homogeneous of degree one. From propositions I and II it is known that the introduction of PS reduces the ratio h and the wage, hence the rental may decrease or increase. Proposition III provides sufficient conditions for the rental to fall as well as for it to increase. The introduction of PS reduces number of firms in operation, and the firms that close down release their previous demand for capital and labor, hence increasing the aggregate supply of capital and of labor. Since their previous owners become workers, the increase of the supply of labor is relatively larger than that of the supply capital. The rental falls (rises) if the former entrepreneurs that become workers can (cannot) be easily absorbed by the labor market.

Proposition III.1 assures that if capital and labor are good substitutes, in the sense of $\sigma(h) \geq 1$, then the rental falls regardless of the value of the sharing parameter. This is

⁽¹⁾ Note that if the elasticity of substitution is greater or equal to one for all $h > 0$, then $\sigma_L = 1$. In this case, proposition III.1. asserts that for any positive value of the PS parameter, an increase of this parameter will reduce the rental. If, however, the elasticity of substitution is lower than one for some $h > 0$, then proposition III.1. states that a sufficient condition for an increase of the PS parameter to reduce the rental is that the PS parameter be sufficiently high, i.e., $\tau > (1 - \sigma_L)$;

so because the possibility of substituting labor for capital dampens the demand for capital and hence cuts down the rental. If capital and labor are bad substitutes, in the sense of $\sigma(h) < 1$, then the rental may fall or increase. Proposition III.1. assures that it will fall if the PS parameter is sufficiently high to leave the remaining entrepreneurs with just a small share of their gross profits. The fall of the rental - and not only of the wage-is the means through which gross profits increase sufficiently - the gross profit of a firm is proportional to its output which increases, as will be shown in proposition IV - in order to keep net profits attractive for some entrepreneurs to still prefer to keep their firms in operation. Moreover, the higher is the lower bound of the elasticity of substitution of F - i.e., the closer one is to the case where $\sigma(h) \geq 1$ - the narrower is the range of the PS parameters that assure that the rental falls - this is shown in the proof of the proposition. This is so because when capital and labor are not too bad substitutes, there is some room for substituting capital for labor which dampens the demand for capital, hence cutting down the rental.

The interpretation of III.2. is the following. The increase of the supply of labor relative to the supply of capital, forces the rise of the rental because capital becomes scarce in an environment where labor cannot easily substitute for it. This will happen provided that PS parameter is sufficiently small, i.e., in the range $0 < \tau < \tau_r$. If the PS parameter were high, then, in general equilibrium, gross profits would have to increase by much in order to keep net profits attractive. This would require a fall of the rental. Moreover, the expression of $\tau_r > 0$ - presented in the proof of proposition III.2 - shows that the lowest is the upper bound of the elasticity of substitution of F - i.e., the farthest one is from the case where $\sigma(h) \geq 1$ - the wider is the range of PS parameters that assure that the rental increases. That is, if capital and labor are very bad substitutes, then even high PS parameters can lead to an increase of the rental.

Recalling that (P2) implies that the gross profit of a firm is equal to $(1-b)$ times its output, the output of a firm of type λ that remains in operation after the introduction of PS, $Y_\lambda(t)$, is given by the substitution of (25) into (13):

$$Y_\lambda(t) = \left(G(\Lambda(t)) / \int_{\Lambda(t)} \lambda g(\lambda) d\lambda \right)^b f(h(t)) \lambda \quad (27)$$

PROPOSITION IV: *An increase of the PS parameter increases the output - and hence the gross profit - of each firm that remains in operation.*

The economic intuition of Proposition IV is immediate. From (11) one can see that for a given firm λ_i that remains in operation, its share of the total demand for labor is given by

$\lambda_i / \int_{\Lambda(t)} \lambda g(\lambda) d\lambda$. Likewise, (12) implies that firm λ_i 's share of total capital is also

given by this factor. Hence each firm increases its demand for both factors, which implies that each firm's output must increase.

The aggregate output, $Y(t)$, is the sum of all firm's output. There are $\bar{N}g(\lambda)$ entrepreneurs of type λ , each one producing $Y_\lambda(t)$. From (27),

$$Y(t) = \bar{N} \int_{\Lambda(t)} Y_\lambda(t) g(\lambda) d\lambda = \bar{N} f(h(t)) [G(\Lambda(t))]^b \left(\int_{\Lambda(t)} \lambda g(\lambda) d\lambda \right)^{1-b} \quad (28)$$

PROPOSITION V: *An increase of the PS parameter reduces the aggregate output.*

The introduction of PS has three impacts on the aggregate output. First it reduces the number of firms contributing to the aggregate output. Secondly it augments each firm's scale of production since the firms that close down release inputs that will be used by those that remain in operation. Third it concentrates the available factors of production on the most efficient companies, which increases the average level of MS of the entrepreneurs. The first effect tends to decrease the aggregate output while the last two tend to increase it. Proposition V shows that net effect of PS on $Y(t)$ is negative. The economic intuition of this result is that PS reduces the supply of one input (entrepreneurs) of the aggregate production function, and increases the supply of another (labor); since the technology is concave, this reduction in the diversity of inputs cuts down the aggregate output.

From (27), the net profit of a firm that remains in operation is given by:

$$\begin{aligned} (1-\tau)\Pi_\lambda(t) &= (1-b)(1-\tau)Y_\lambda(t) \\ &= (1-b)(1-\tau) \left(G(\Lambda(t)) / \int_{\Lambda(t)} \lambda g(\lambda) d\lambda \right)^b \lambda f(h(t)) \end{aligned} \quad (29)$$

Proposition IV asserts that an increase of the PS parameter increases the gross profits of each firm that remains in operation. Proposition VI presents the short run impact of an increase of the PS parameter on the net profit of each firm that remains in operation.

PROPOSITION VI: *If $\sigma(h) \leq 1$ for all $h > 0$, then an increase of the PS parameter reduces the net profit of the firms that remain in operation.*

The result above shows that if labor and capital are bad substitutes, then the increase of the gross profit of a firm that remains in operation after the introduction of PS is not sufficiently high to offset the loss of the share that is transferred to workers. If capital and

labor were good substitutes, in the sense of $\sigma(h) \geq 1$, then the fall of both the wage and the rental - as reported in proposition II and III.1 - might in principle increase the gross profit so much that it could offset the loss of the share that has to be transferred to workers.

The impact of PS on workers' total compensation is obtained from (14) and (25):

PROPOSITION VII: *If $\sigma_L > 1/[1 + 2(1 - b)]$, and if the density function g is non increasing, then there exists $\tau_R > 0$ such that for $0 < \tau < \tau_R$, an increase of the sharing parameter increases the workers' total compensation.*

An increase of the PS parameter impinges on the amount of workers' total compensation through four channels. First, it impacts directly on the share of the gross profit that will be transferred to workers. Second, it increases each company's gross profit through the increase of its output as reported in proposition IV. Third, it increases the number of workers among which that share will be divided. And fourth it reduces the wage. The first two effects tend to augment the workers' total compensation, while the last two tend to reduce it.

The result of proposition VII shows that for the workers' total compensation to increase, it is sufficient to make two assumptions: (1°) that labor and capital are not too bad substitutes, and (2°) that EP is a scarce gift, in the sense that the higher the level of EP, the lower is the number of individuals endowed with that level - i.e. g is non-increasing. If labor and capital were very bad substitutes, than - according to proposition III.2. - the rental might increase so much that the gross profit left after paying the rentals could not increase much. As a consequence the first and second effects described above might be outweighed by the last two. When MS is a scarce gift the introduction of PS concentrates among the most gifted entrepreneurs the available inputs of the economy. The average MS of the entrepreneurs increases much and hence the average gross profit as a fraction of output has also a substantial increase. Once again the first and second effects described above would be outweighed by the last two.

IV - THE LONG RUN GENERAL EQUILIBRIUM

Section III described the behavior of the economy immediately after the mandatory introduction of PS, i.e., for a given level of aggregate capital stock. According to Proposition III, the return on accumulated assets $r(t)$ changes after the introduction of PS and this will affect the consumption-savings decisions and hence the evolution of the aggregate stock of capital. This section describes the long run equilibrium that will obtain when the aggregate capital stock reaches its (new) stable level.

The consumption-savings decision of individual λ is determined by the usual Keynes-Ramsey rule.

$$\frac{\dot{c}_\lambda(t)}{c_\lambda(t)} = \varphi_\lambda(c_\lambda(t))[r(t) - \rho] \quad (30)$$

where φ_λ is individual λ 's instantaneous elasticity of substitution of consumption:

$$\varphi_\lambda(c_\lambda(t)) = -U'_\lambda(c_\lambda(t)) / [U''_\lambda(c_\lambda(t)) c_\lambda(t)] > 0$$

The long run equilibrium is defined as the steady state equilibrium at which each individual's savings are zero. This will happen when the net return on accumulated assets r^* is equal to the rate of time preference:

$$r^* = \rho$$

When this last expression holds, (26) implies:

$$\rho + \delta = f'(h^*) \left(\int_{\Lambda^*} \lambda g(\lambda) d\lambda / G(\Lambda^*) \right)^{1-b} \quad (31)$$

where h^* and Λ^* stand for the equilibrium long run capital/labor ratio and watershed.

The *labor market equilibrium equation* and the *marginal entrepreneur's indifference condition* require that (24) must hold in the long run:

$$\frac{1 - \eta_l(h^*)}{1 - b} = (1 - \tau)[1 + Z(\Lambda^*)] \quad (32)$$

The *capital market equilibrium equation* is represented by (22):

$$K^* = \bar{N} h^* G(\Lambda^*) \quad (33)$$

Equations (31) and (32) determine the equilibrium long run capital/labor ratio h^* and watershed Λ^* . Once these two variables are calculated, equation (33) gives the long run capital stock K^* . It is important to compare the determination of the short run equilibrium of section III with the long run equilibrium of this section. The short run variables $h(t)$ and $\Lambda(t)$ were determined by the system of (22) and (24) for a given level of aggregate capital stock; while the long run h^* and Λ^* are determined by (31) and (32) for a given rate of return on the aggregate stock. The long run impact of PS on h^* and Λ^* is given below.

PROPOSITION VIII: *An increase of the PS parameter reduces the capital/labor ratio and the number of firms in operation in the long run.*

The result above shows that the short run reductions of the capital/labor ratio and of the number of firms in operation that follows the introduction of PS - as described in proposition I - can not be reversed by the accumulation or decumulation of capital that takes place thereafter.

The long run wage w^* is obtained by substitution of h^* and Λ^* into (25). The impact of the PS on this variable is given below.

PROPOSITION IX: *An increase of the PS parameter reduces the long run wage.*

Proposition IX shows that the short run impact of PS on the wage - which decreases this variable, as described in proposition II - can not be reversed in the long run, even if there is accumulation of capital after the introduction of PS.

The long run aggregate capital stock K^* is obtained from substitution of h^* and Λ^* into (33). Proposition X shows that the sufficient conditions which assure that K^* is higher (lower) in the long run equilibrium with PS than in the one without PS are the same conditions that, in proposition III, assure that the rental $r(t)$ rises (falls) after the introduction of PS.

PROPOSITION X: (X.1) *If $\tau > (1 - \sigma_L)$, then an increase of the PS parameter reduces the long run aggregate capital stock;* (X.2) *If $\sigma_U < 1$, then there exists a real number $\tau_r > 0$, such that for a PS parameter τ in the range $0 < \tau < \tau_r$, an increase of τ increases the aggregate long run capital stock.*

The interpretation of the result above is straightforward. When the rental falls short of the rate of time preference, as would be the case under the sufficient conditions of X.1., individuals increase their level of consumption and hence deplete part of their accumulated assets. As the aggregate capital stock is gradually whittled down, it becomes scarcer relative to labor and the rental starts to rise until it eventually reaches the rate of time preference in the new long run equilibrium. Likewise, when the rental increases above the rate of time preference, as would be the case under the sufficient conditions of X.2., individuals reduce their level of consumption in order to accumulate assets. As the aggregate capital stock is gradually increased, it becomes abundant relative to labor and the rental starts to fall until it eventually reaches the rate of time preference in the new long run equilibrium.

The long run output of firm λ , Y_λ^* , is given by the substitution of h^* and A^* into (27). The impact of the PS parameter on Y_λ^* is given below.

PROPOSITION XI: *An increase of the PS parameter increases the output - and hence the gross profits - of each firm that remains in operation in the long run.*

Proposition XI shows that the short run upward jump of the output of each firm - as reported in proposition V - following the introduction of PS can not be reversed in the long run, even if there is decumulation of the capital stock thereafter.

The aggregate output Y^* is given by the substitution of h^* and A^* into (28). The impact of the PS parameter on Y^* is given below.

PROPOSITION XII: (XII.1) *If $\tau > b(1 - \sigma_L)$, then an increase of the PS parameter reduces the long run aggregate output¹; (XII.2) If $\sigma_U < 1$, then there exists a real number $\tau_y > 0$, such that for a PS parameter τ in the range $0 < \tau < \tau_y$, an increase of τ increases the long run aggregate output.*

Proposition XII.1 assures that when labor and capital are good substitutes - in the sense of $\sigma_L \geq 1$ - the introduction of PS reduces the long run output regardless of the size of the PS parameter. In order to interpret this result, one must notice that equation (28) states that, the aggregate output is an increasing function of the aggregate capital stock. From proposition X.I, under the hypothesis $\sigma(h) \geq 1$ there is decumulation of capital in the long run, it follows that under this same hypothesis the long run aggregate output must fall.

The range of PS parameters that, in proposition XII.1, assure that the long run output falls is narrower than the range that, in proposition X.2, assure that the aggregate capital stock falls: the former is equal to b times the latter. This means that one can not rule out the possibility that the chosen PS parameter be sufficiently low to lead to an accumulation of the capital stock, but high enough to lead to a reduction of the long run output through its deleterious effects on the incentives that individuals face when they decide to set up a firm.

(1) Note that if the elasticity of substitution is greater or equal to one for all $h > 0$, then $\sigma_L = 1$. In this case, proposition X.1 asserts that for any positive value of the PS parameter, an increase of this parameter will reduce the long run aggregate output. If, however, the elasticity of substitution is lower than one, for some $h > 0$, then X.1 states that a sufficient condition for the long run aggregate to fall is that the PS parameter be sufficiently high, i.e., $\tau > (1 - \sigma_L)$;

Proposition XII.2 assures that when labor and capital are bad substitutes, in the sense of $\sigma_U < 1$, the introduction of PS at a small PS parameter increases the long run output. This result is a consequence of proposition X.2. The range of PS parameters that, in proposition XII.2, assure that the long run output rises is narrower than the range that assure that the aggregate capital stock rises in proposition X.2, i.e., $\tau_Y < \tau_r$ - this is shown in the proof of the proposition.

The long run net profit of a firm λ that remains in operation, $(1 - \tau)\Pi'_\lambda$, is equal to $(1 - b)(1 - \tau)Y_\lambda^*$. The impact of the PS parameter on $(1 - \tau)\Pi'_\lambda$ is given below.

PROPOSITION XIII: *The higher is the PS parameter, the lower will be the net profit of a firm that remains in operation in the long run equilibrium.*

Proposition XII must be compared with proposition VI. In the long run PS necessarily reduces the net profit received by each entrepreneur. The rise of each firm's gross profits - as reported in proposition XI - cannot offset the higher share of profits that entrepreneurs have to transfer to workers, since the long run rental is exogenous and equal to the rate of time preference. In the short run, however, if labor and capital were good substitutes, then the price of both inputs falls - as reported in proposition II and III. Hence one cannot rule out the possibility that the introduction of PS may increase the net profits. This is why, the sufficient hypothesis $\sigma(h) \leq 1$ was required in proposition VI to prove that the short run net profit falls.

The long run workers' total compensation R^* is obtained through the substitution of w^* and h^* into (14).

PROPOSITION XIV: *If the density function g is non increasing, then there exists $\tau_{R^*} > 0$ such that the higher is the PS parameter in the range $0 < \tau < \tau_{R^*}$, the higher will be the workers' total compensation in the long run equilibrium.*

As was seen in the discussion after proposition VII, an increase of the PS parameter impinges on the amount of workers' total compensation through four channels. First, it impacts directly on the share of the gross profit that will be transferred to workers. Second, it increases each company's gross profit through the increase of its output as reported in proposition XI. Third, it increases the number of workers among which that share will be divided. And forth it reduces the wage. The first two effects tend to augment the workers' total compensation, while the last two tend to reduce it.

The main difference between proposition VII and proposition XIV is the fact that the former required the sufficient condition that labor and capital be not too bad substitutes, in the sense of $\sigma_L > 1/[1 + 2(1 - b)]$. This assumption was used because if labor and capital were very bad substitutes, than - according to proposition III.2 - the rental could increase and the gross profits left after paying the rentiers would not increase much, thus reducing the available profits transferable to workers. If that were the case, then in the long run capital would increase up to the level at which its return is equal to the rate of time preference. Hence there is no need to make assumptions on the elasticity of substitution to prove proposition XIV. The assumption on the distribution of MS is still necessary and its interpretation is the same given after proposition VII.

V - THE DYNAMICS

In section III one studied the comparative statics of the endogenous variables of the model immediately after the introduction of PS, when the aggregate capital stock is still unchanged. In section IV one described the comparative statics of the endogenous variables of the model in the long run equilibrium with PS after the aggregate capital stock has already adjusted through accumulation or decumulation. The goal of this section is to analyze the path of all those endogenous variables *during* the transition from an initial long run equilibrium without PS towards a final long run equilibrium with PS.

For ease of exposition in this section it is assumed that at instants $t < 0$ the economy is in a long run equilibrium without PS. At instant $t = 0$ the PS parameter is increased abruptly to a positive value and remains constant at this level thereafter. Therefore the value of the PS parameter is given as: $\tau(t) = 0$ for $t < 0$, and $\tau(t) = \tau > 0$ for $t \geq 0$.

As was seen in proposition III, the rental - and hence the aggregate capital stock - may increase or decrease after the introduction of PS. Since the PS parameter remains constant for $t > 0$, the key variable determining which of the two paths will obtain is the rental. There will be decumulation (accumulation) of capital if the rental falls short of (increases above) the rate of time preference. Once the capital stock starts to move, the other endogenous variables will move accordingly. One concludes that the capital stock is the variable that drives the other endogenous variables. Hence the paths of the other variables can be expressed as a function of the path of the aggregate capital stock. Proposition XV characterizes the paths of the endogenous variables studied in sections III and IV from $t > 0$ onwards:

PROPOSITION XV: *During the transition from the short run equilibrium after the introduction of PS at $t = 0$, to the new long run equilibrium, the aggregate capital stock*

$K(t)$, the watershed $\Lambda(t)$, the capital/labor ratio $h(t)$, the wage $w(t)$, the rental $r(t)$, the output of each firm $Y_\lambda(t)$, the workers' total compensation $R(t)$, and the aggregate output $Y(t)$ move according to:

(XV.1) If $\tau > (1 - \sigma_L) > 0$, then $\dot{K}(t) < 0$; If $\sigma_U < 1$ and $0 < \tau < \tau_r$, then $\dot{K}(t) > 0$;

(XV.2) $\dot{K}(t)\dot{\Lambda}(t) > 0, (=0), (<0)$ as $\sigma(h) < 1, (=1), (>1)$ respectively;

(XV.3) $\dot{K}(t)\dot{h}(t) > 0$; $\dot{K}(t)\dot{w}(t) > 0$; $\dot{K}(t)\dot{r}(t) < 0$; $\dot{K}(t)\dot{Y}_\lambda(t) > 0$; $\dot{K}(t)\dot{R}(t) > 0$;

(XV.4) If $\sigma_L \geq 1$, then $\dot{Y}(t) < 0$; If $\sigma_U < 1$ and $0 < \tau < \tau_r$, then $\dot{Y}(t) > 0$.

Proposition XV.1 asserts that under the hypothesis of proposition III.1 the capital stock decreases; whereas under the hypothesis of proposition III.2, the capital stock increases.

Proposition XV.2 assures that the number of firms $\bar{N}[1 - G(\Lambda)]$ and the capital stock follow the same path if, and only if, capital and labor are good substitutes, i.e. $\sigma(h) \geq 1$ for all $h > 0$. Under this hypothesis, proposition III.1 asserts that the introduction of PS cuts the rental to a level below the rate of time preference, thus leading to a decumulation of the aggregate capital stock. Proposition XV.2 then implies the number of firms decreases as the capital stock depreciates. This happens because the marginal entrepreneurs are lured by the labor market where workers are being demanded to substitute for the falling supply of capital. If $\sigma(h) = 1$ for all $h > 0$, then the number of firms falls abruptly after the introduction of PS at $t = 0^+$ and remains constant thereafter. If $\sigma(h) < 1$ but $\tau > \tau_r$, capital decumulates and the number of firms increases because the dearth of capital dampens the demand for its complementary input labor. When $\sigma(h) < 1$ and $0 < \tau < \tau_r$, there is accumulation of capital and reduction of the number of firms, as the marginal entrepreneurs are lured by the labor market where workers are being demanded to complement the higher supply of capital.

Proposition XV.3 asserts that the capital/labor ratio, the wage, the output of each firm and the workers' total compensation follow the same path of the capital stock, whereas the rental follows the opposite path. These results are quite intuitive. The direct impact of the accumulation/decumulation of capital on the numerator of h cannot be reversed by countervailing changes in the number of individuals supplying labor. Since the aggregate output is shared by all firms, when it increases so do the output of firms. Increases of the aggregate capital stock rise the marginal productivity of labor (the wage) and the workers' total compensation which is closely linked to the wage. On the other hand, increases of the aggregate capital stock dampen the marginal productivity of capital (the rental).

Proposition XV.4.1 must be analyzed together with proposition III.1 and XII.1. If capital and labor are good substitutes, then after the introduction of PS, the rental falls and the capital follows a declining trend and so does the aggregate output. Since from proposition IV the aggregate output falls abruptly after the introduction of PS, one concludes that, under the hypothesis of XV.4.1 it will fall even further in its path towards the new steady state. Proposition XV.4.2 must be analyzed together with proposition III.2 and XII.2. If capital and labor are bad substitutes and the distortion created by PS is low, then the rental rises and there will be accumulation of capital and the aggregate output will rise towards its new long run upper level. From proposition V one knows that the aggregate output falls abruptly after the introduction of PS at $t = 0^+$. From proposition XII.2 one knows that under the hypothesis of XV.4.2 it will reach a long run level with PS above its long run level without it. Hence Proposition XV.4.2 assures that it will follow an increasing trend from $t = 0^+$ onwards.

Figure 1 describes the path followed by the economy from its steady state without PS for $t < 0$ until its steady state with PS at $t = +\infty$ when capital and labor are good substitutes. At $t = 0^+$, according to the results reported in section III, the introduction of PS leads some entrepreneurs to close their firms down and become workers. Hence the supply of capital relative to labor falls. The wage falls because labor becomes more abundant. The output of each firm increases because the firms that close down release inputs that are hired by those that remain in operation and their former owners increase the supply of labor. The aggregate output falls because there is decreasing returns to scale. Since labor substitutes well for capital, the increased supply of labor reduces the demand for capital, and hence the rental falls short of the rate of time preference. From $t > 0$ onwards, the decumulation of capital compounds the short run impacts of PS. As capital is depreciated, more firms close down as there is an upsurge in the demand for labor to substitute for the dwindling capital which lures the least skilled entrepreneurs. As capital becomes relatively scarce and its return starts to increase. In the new long run steady state the rental is equal to the rate of time preference and capital decumulation stops.

Figure 2 describes the path followed by the economy when capital and labor are bad substitutes, but the distortion introduced by PS is high according to the hypothesis of proposition XIII.1. The only path that differs from those of figure 1 is the path of $\lambda(t)$. Since labor is complementary to capital, the fall of the stock of capital releases workers that set up new firms.

Figure 3 describes the path followed by the economy when capital and labor are bad substitutes and the distortion introduced by PS is low according to the hypothesis of proposition XIII.2. The introduction of PS leads the marginal entrepreneurs to close their companies down and become workers. This increases the supply of labor relative to

capital. Since these two inputs are poor substitutes, labor can not easily substitute for capital. Hence the rental rises above the rate of time preference. As a result, savings rise and the aggregate capital stock starts to increase. As the supply of capital increases more labor is needed to complement it, which leads the least skilled workers to close their firms down. The output of each firm increases even further, and the aggregate output rises. As capital increases, labor becomes relatively less scarce and hence the wage rises gradually. In the new steady state the aggregate output with PS will have increased above its initial level without PS.

V - WELFARE ANALYSIS

This welfare analysis will start with a comparison between the long run equilibrium without PS and the one with PS. The transition from one equilibrium to the other will be analyzed later on.

It was shown in proposition XIII that, in the long run, the net profit of entrepreneurs are lower with PS than without it. In proposition XIV it was shown that, in the long run, provided that entrepreneurial talent is a scarce gift, for a wide range of PS parameters, workers' total compensation is larger with PS than without it. This implies that, without any other countervailing distortion, the introduction of PS improves workers' welfare at the expense of entrepreneurs'.

If one could establish a non distortionary transfer from workers to entrepreneurs, one might increase the long run welfare of both, provided that the aggregate output increased. As was shown in Proposition XII, if the elasticity of the production function is bounded above from one, then the introduction of mandatory PS at a sufficiently low PS parameter, increases the long run aggregate output. This fact poses a conundrum: in order to make everybody better off, at least as far as the long run equilibrium is concerned, one would have to use a lump sum transfer from workers to entrepreneurs so as not to change their economic incentives; but this is unfeasible since an individual, when making his choice between being a worker or an entrepreneur, would take into account that he would be lump sum-taxed in the first choice, and would receive a lump sum grant in the second.

One concludes that in this model without uncertainty, nor nominal rigidities, *PS cannot be made Pareto improving in its strict sense*. This is so even when one considers just the long run allocations and disregards the adjustment path. However, if workers and entrepreneurs were lumped together so as to make it possible to consider the *average individual* of the society, than one can fancy a situation in which this individual can be made better off, even when the transition from the short to the long run equilibrium is taken into account. This would happen (1) in an open economy with free movement of

capital; (2) if capital could be brought from abroad sufficiently fast, and (3) if PS were introduced within the conditions of proposition XVI ahead.

The welfare analysis of the average individual consists just in examining what happens to the per capita national income. Let Y_0^* be the economy's (long run) domestic product before the introduction of PS, and NY_0^* be its national product before the introduction. Likewise, let Y_r^* and NY_r^* be their respective (new) values with PS. At instant $t = 0$, PS is introduced within the range defined in proposition XVI. The rental jumps to $r(0^+) > \rho$. Assuming that the international rate of return on capital is equal to ρ , the assumption that capital can come from abroad implies that the aggregate capital increases from K_0^* to K_r^* until the gross rental falls back to ρ . If capital arrives sufficiently fast, there is no change in domestic savings. The jump in the level of the aggregate capital stock is entirely financed by foreign savings.

During the transition, the absorption of foreign capital keeps the rental slightly above ρ . The economy moves from one long run equilibrium with the net rental equal to the rate of time preference plus capital depreciation to another. In the new equilibrium, the net foreign income sent abroad increases by $(K_r^* - K_0^*)\rho > 0$. Hence, the increase of the national product $NY_r^* - NY_0^*$ is equal to the difference between the increase on the domestic product $(Y_r^* - Y_0^*) > 0$ and $(K_r^* - K_0^*)\rho > 0$, as given by:

$$NY_r^* - NY_0^* = (Y_r^* - Y_0^*) - (K_r^* - K_0^*)\rho$$

Since the number of individuals is constant, if the national product increases then so does the per capita national income. Proposition XVI provides the sufficient conditions that assure that the national product increases.

PROPOSITION XVI: *If $\sigma_U < 1$, then there is $\tau_{NY^*} > 0$ such that for the PS parameter in the range $0 < \tau < \tau_{NY^*}$, an increase of τ raises the economy's per capita national income.*

The range of PS parameters that, in proposition XVI, assure that the long run national product rises is narrower than the range that assure that the aggregate domestic output rises in proposition XII.2, i.e., $\tau_{NY^*} < \tau_Y < \tau_r$ - this is shown in the proof of the proposition.

The source of this growth is the increase of the productivity of the national factors through two means. The introduction of PS attracts more capital from abroad, thus increasing the productivity of the workforce formed by entrepreneurs and workers. It also concentrates the factors of production on the best run firms. In this sense PS is a means through which relatively unqualified entrepreneurs are lured away from the entrepreneurial

activity thus increasing the average productivity of the managerial input used in the economy.

It must be noted that the welfare improvement described above is a *national* improvement, not a world improvement. There is no Pareto improvement since the capital that arrives must come from somewhere. The economy from which it flees will miss it somehow. The trick here is that the economy that is been studied generates an adverse externality to other economies and gets away with it. In a closed economy, the savings that generate the accumulation of capital would have to come from a temporary reduction of the consumption level. That would require a sacrifice that is not happening under the hypothesis of a small open economy with free (and fast) movement of capital.

It must be stressed, moreover, that a key assumption for the above result to hold is that there is no free emigration. Under this assumption, proposition XIII would imply that some entrepreneurs would emigrate. Hence the attempt to attract capital through PS would backfire: instead of luring capital in, PS would frighten entrepreneurs away.

VI- CONCLUSION AND EXTENSIONS

The model presented above is an example that mandatory PS schemes distorts the economic incentives that lead individuals to set up their own business, with consequences for the structure of the economy. In an environment where nominal rigidities are absent, and hence full employment obtains, PS reduces the economy's short run output but it may decrease or increase the long run one. The former occurs if capital and labor are good substitutes or if high levels of PS are introduced, while the latter obtains when capital and labor are poor substitutes and PS is introduced at a small level.

The results presented in this paper should not be viewed as a definite case for the mandatory adoption of PS in a real economy. The conditions under which PS improves the welfare of nationals in this paper are somewhat restrictive. A key assumption for the results above was that there was only one good, and that it was produced under a single technology. In a real economy, the degree of substitutability between capital and labor may vary widely across sectors. If, for instance, there are sectors in which capital and labor are good substitutes then the argument for mandatory PS presented above crumbles. Therefore, the model presented here should be viewed as an exploratory theoretical research which does not claim that mandatory PS should be implemented without careful examination of the pre requisites that would bring the benefits of it to bear.

The present model does not address the impact of PS schemes freely negotiated between firms and workers in environments where there are nominal rigidities and uncertainty about demand. This issue has already been studied by the literature surveyed in

section I and its findings suggest that PS should not be mandatory. This literature has shown that PS schemes freely negotiated may indeed reduce risk bearing by firms of very cyclical sectors, thus leading to lower rates of unemployment. It has also shown that PS may create an important incentive to increase labor's effort in sectors where it is observable.

The great contribution of the present model, I think, is to show that even in an environment with fully flexible prices, hence without unemployment, PS may yet be commendable. From the policy maker's viewpoint, the contribution of the paper is to show that the potential negative effect of PS on entrepreneurial initiatives may be offset by its positive effects on capital accumulation and the concentration of the economy's factors of production on the hands of the best entrepreneurs. In short, my view of PS is that it may be good or bad, according to the conditions (nominal rigidities, observability of worker's effort, factor substitutability, etc.). Hence it should not be imposed by government decree, but be freely negotiated between firms and workers. At most it should receive tax incentives in economies where the conditions under which it would be benign are present.

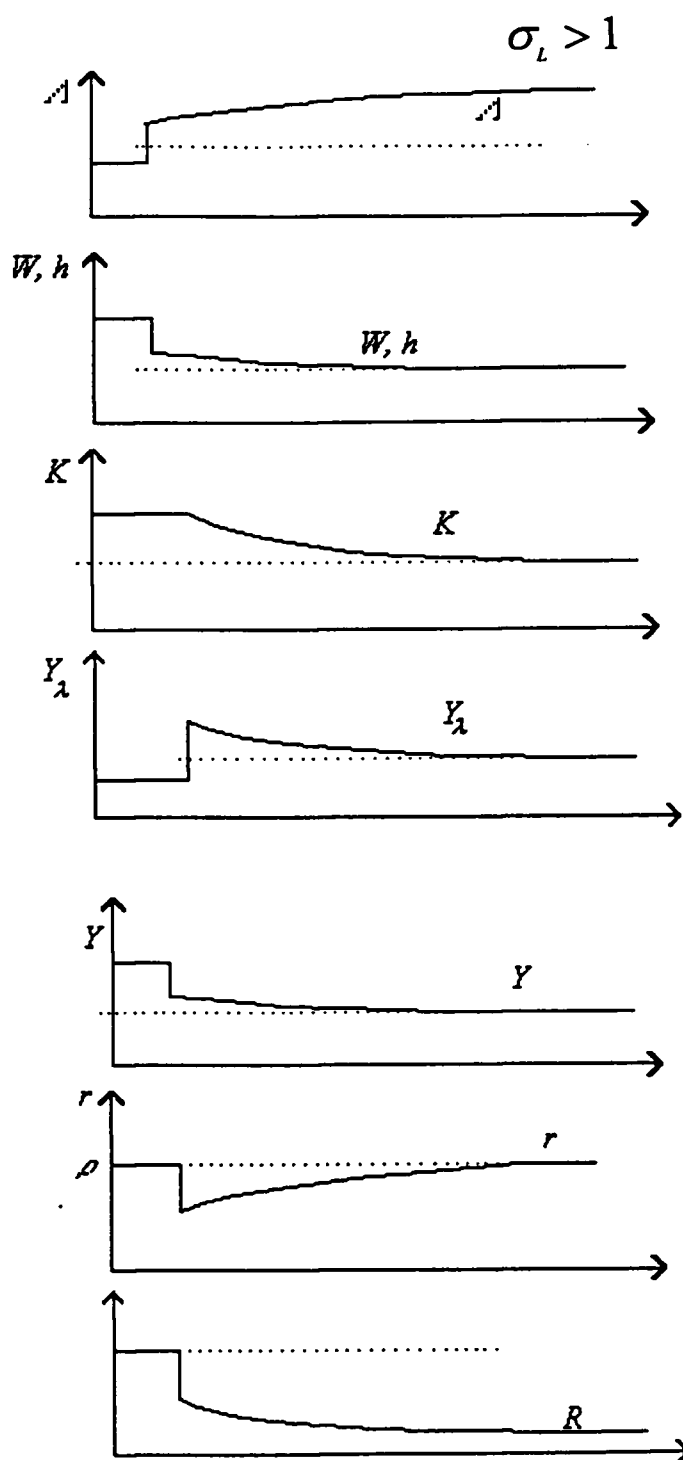
A natural extension of the model is to introduce growth through technical progress. That could be done through knowledge accumulation as a spill over of capital accumulation in a learning by doing process. This is a project for future research.

REFERENCES

- Bell, L. A. & Neumark, D. (1993), "Lump-Sum Payments and Profit-Sharing Plans in the Union Sector of the United States Economy", *The Economic Journal*, Vol. 103, May, 602-619.
- Blanchflower, D. G. & Oswald, A. J., (1987), "Profit Sharing - Can It Work?", *Oxford Economic Papers*, Vol. 39, 1-19.
- Brunello, G., (1992), "Profit Sharing in Internal Labour Markets", *The Economic Journal*, Vol. 102, 570-577.
- Cable, J. & Wilson, N., (1989), "Profit Sharing and Productivity: an Analysis of the UK Engineering Firms", *The Economic Journal*, Vol. 99, June, 366-375.
- Cooper (1988), "Will Share Contracts Increase Economic Welfare?", *American Economic Review*, Vol. 78, No. 1, 138-154
- Fitzroy, F. R. & Kraft, K. (1987), "Cooperation, Productivity, and Profit Sharing", *Quarterly Journal of Economics*, February, 21-35.
- González, P. (1992), "Profit Sharing, Unemployment and effort", *Economics Letters*, Vol. 40, 359-363.
- John, A. (1991), "Employment Fluctuations in a Share Economy", *Oxford Economic Papers*, Vol. 43, 075-084.
- Jones, D. C. & Svejnar, J., (1985), "Participation, Profit Sharing, Worker Ownership and Efficiency in Italian Producer Cooperatives", *Economica*, Vol. 52, 449-465.
- Kanbur S. M., (1979), "Of Risk Taking and the Personal Distribution of Income ", *Journal of Political Economy*, Vol. 87, no.4, 769-797.
- Kandel, E. & Lazear, E. P., (1992), "Peer Pressure and Partnerships", *Journal of Political Economy*, Vol. 100, no.4, 801-817.
- Keynes, J.M. (1936), *The General Theory of Employment, Interest and Money*. London: Macmillan
- Kihlstrom, R. E. & Laffont, J.J., (1979), "A General Equilibrium Entrepreneurial Theory of Firm Formation Based on Risk Aversion ", *Journal of Political Economy*, Vol. 87, no.4, 719-747.
- Kruse, D. L., (1992), "Profit Sharing and Productivity: Microeconomic Evidence From the United States", *The Economic Journal*, Vol. 102, January, 24-36.
- Lucas, R. E. Jr., (1978), "On the size Distribution of Business Firms", *Bell Journal of Economics*, Vol. 9. no. 2, 508-523.
- Mead, J., (1986), *Different Forms of Share Economy*, London: Public Policy Centre.
- Nordhaus, W., (1988), "Can The Share Economy Conquer Stagflation?", *The Quarterly Journal of Economics*, February, 201-217.
- Wadhvani, S. B., (1987), "Profit-Sharing and Mead's Discriminating Labour-Capital Partnerships: a Review Article", *Oxford Economic Papers*, Vol. 39, 421-442.
- Weitzman, M. L. (1983), "Some Macroeconomic Implications of Alternative Compensation Systems" , *Economic Journal*, Vol. VM 93, 763-783.
- (1984), *"The Share Economy"*, Harvard University Press, Cambridge.
- (1985), "The Simple Macroeconomics of Profit-Sharing", *American Economic Review*, Vol. 75, 937-953.
- (1987), "Steady State Unemployment Under Profit Sharing", *The Economic Journal*, Vol. 97, March, 86-105.

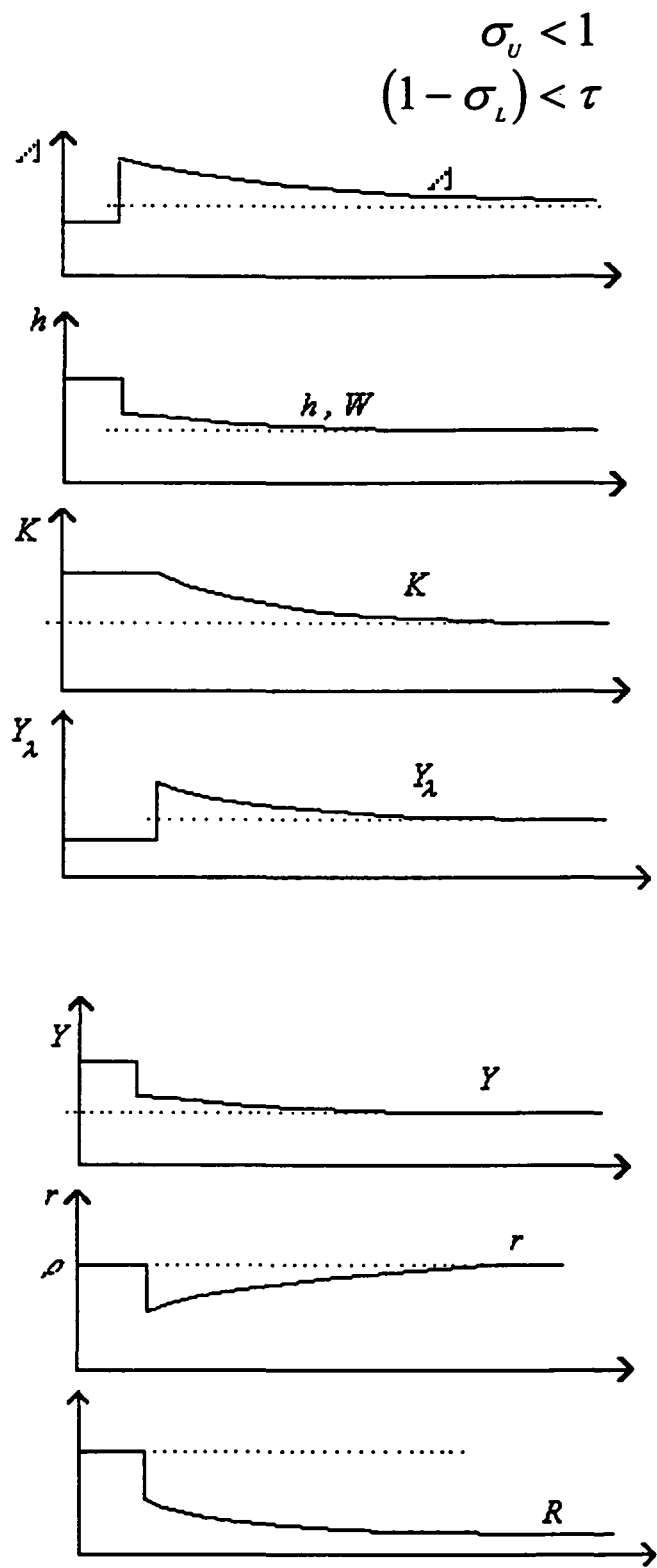
K and L are good substitutes

FIGURE 1



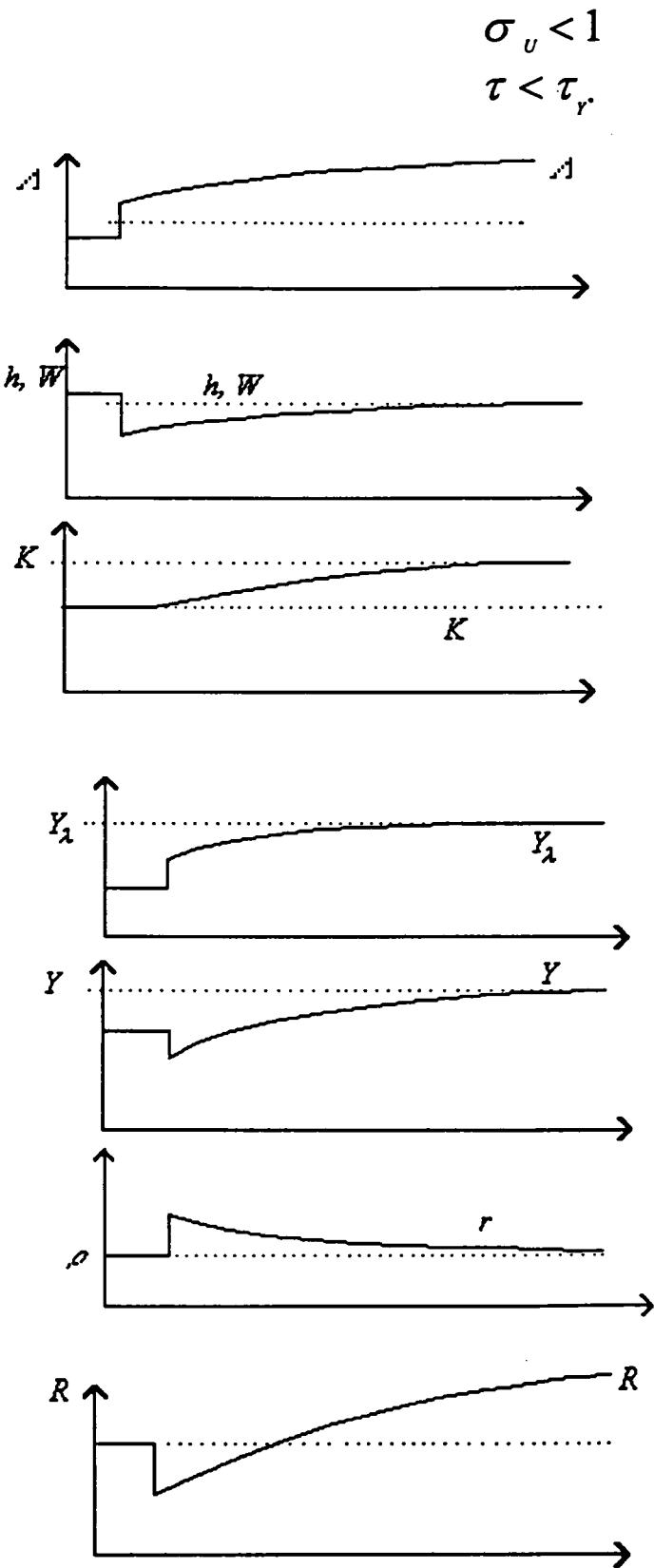
K and L are bad substitutes,
but τ is high

FIGURE 2



K and L are bad substitutes, and τ is sufficiently low

FIGURE 3



APPENDIX

Proof of Lemma I: Since F is homogeneous of degree b , for any $\alpha > 0$,

$$F(\alpha K, \alpha L) = \alpha^b F(K, L) \quad (A1)$$

To obtain (P1), differentiate (A1) with respect to α and then set $\alpha = 1$. To obtain (P2) for $i=1$, differentiate (A1) with respect to K and then set $\alpha = 1/L$. To obtain (P2) for $i=2$, differentiate (A1) with respect to L and then set $\alpha = 1/L$. To obtain (P3), substitute (P2) into (P1) and apply (A1) with $\alpha = 1/L$.

Proof of Lemma II: Property (P4) is straightforward. Property (P5) follows from (P3), (P4) and the definition of f . Property (P6) follows from (P5) and the fact that $F_2 > 0$. To obtain (P7) differentiate (P5) and get: $\eta_2(h) = (1-b) + F_{21}(h,1)/f'(h) > (1-b)$. In order to prove (P8), let $x = F_1(K,L)/F_2(K,L)$. From (P2), (P4) and (P5) one gets: $x = x(h) = F_1(h,1)/F_2(h,1) = f'(h)/[bf(h) - hf'(h)]$. Hence the elasticity of substitution of F , $\sigma(h) = -(x/h)dh/dx$, is given by:

$$\sigma(h) = - \frac{[bf(h) - hf'(h)]}{f''(h)[bf(h) - hf'(h)] - f'(h)^2[\eta_2(h) - (1-b)]} \frac{f'(h)}{h} = \frac{b - \eta_1(h)}{b[\eta_1(h) + \eta_2(h)] - \eta_1(h)} \quad (A2)$$

It follows that $\sigma(h) \leq 1$ if, and only if, $\eta_1(h) + \eta_2(h) \geq 1$. In order to prove (P9), notice that σ can be regarded as a function $\sigma: \mathbb{R}^2 \rightarrow \mathbb{R}$ with $\sigma(\eta_1, \eta_2)$ defined by (A2) which is decreasing in η_1 . It follows that: $\sigma(h) = \sigma(\eta_1, \eta_2) < \sigma(0, \eta_2) = 1/\eta_2$. Hence, $\sigma(h)\eta_2(h) < 1$. From (P7), $(1-b)\sigma(h) < \eta_2(h)\sigma(h) < 1$ which proves (P9). Property (P10) follows from the differentiation η_1 and (P8):

$$d\eta_1/dh = \eta_1(h)[1 - \eta_1(h) - \eta_2(h)]/h \quad (A3)$$

This completes the proof of lemma 2. Expression (A2) implies two useful relations:

$$\eta_1(h) = \frac{b[1 - \eta_2(h)\sigma(h)]}{1 - (1-b)\sigma(h)} \quad (A4)$$

$$\eta_2(h) = \frac{b - \eta_1(h)[1 - (1-b)\sigma(h)]}{b\sigma(h)} \quad (A5)$$

Proof of Proposition I: The derivatives of Λ and h are obtained by the application of the implicit function theorem to (22) and (24). Since $Z(\Lambda)$ is defined as the left hand side of (23), it follows that

$$\frac{dZ}{d\Lambda} = \frac{d}{d\Lambda} \left[G(\Lambda) \Lambda / \int_{\Lambda}^{\infty} \lambda g(\lambda) d\lambda \right] = Z(\Lambda) \left[\frac{1}{\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \right] \quad (A6)$$

Taking the logarithm of (24) and differentiating, one gets:

$$- \frac{1}{1 - \eta_1} \frac{d\eta_1}{dh} \frac{dh}{d\tau} = - \frac{1}{1 - \tau} + \frac{1}{[1 + Z(\Lambda)]} \frac{dZ}{d\Lambda} \frac{d\Lambda}{d\tau} \quad (A7)$$

Likewise, taking the logarithm of (22) and differentiating, one finds that:

$$\frac{1}{h} \frac{dh}{d\tau} = - \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau} \quad (A8)$$

From (A8) one concludes that $dh(t)/d\tau < 0$ if, and only if, $d\Lambda(t)/d\tau > 0$. Hence it suffices to prove this last inequality. Substituting (A3), (A6) and (A8) into (A7) and rearranging yields:

$$\frac{d\Lambda}{d\tau} = \frac{1}{1-\tau} \left\{ \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \frac{1}{\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[Z(\Lambda) - \frac{\eta_1(h) [1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \right] \right\}^{-1} \quad (\text{A9})$$

It is sufficient to show that for all $h > 0$, the term in the bracket which is multiplied by $g(\Lambda)/G(\Lambda)$ is positive. Substituting $Z(\Lambda)$ from (24), this term is given by:

$$\frac{g(\Lambda)}{G(\Lambda)} \left[\right] = \frac{g(\Lambda)}{G(\Lambda)} \frac{[\eta_2(1-b)(1-\tau)\eta_1] + (1-\eta_1)^2 [1-(1-b)(1-\tau)]}{(1-b)(1-\tau)(1-\eta_1)} > 0$$

Proof of Proposition II: Applying logarithms to (25) and differentiating, one gets:

$$\frac{d \ln w}{d\tau} = \frac{f'(h) [\eta_2(h) - (1-b)]}{b f(h) - h f'(h)} \frac{dh}{d\tau} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \frac{d\Lambda}{d\tau}$$

From Lemma 2, $[\eta_2 - (1-b)] > 0$. The derivatives $dh/d\tau < 0$ and $d\Lambda/d\tau > 0$, imply $dw(t)/d\tau < 0$.

Proof of Proposition III: Applying logarithms to (26) and differentiating one gets:

$$\frac{1}{(r+\delta)} \frac{dr}{d\tau} = \frac{f''(h)}{f(h)} \frac{dh}{d\tau} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \frac{d\Lambda}{d\tau} = \frac{[(\eta_1(h) + \eta_2(h) - 1) - \tau \eta_2(h)]}{1-\tau} \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau}$$

where the last equality follows from (A8). From (P8), if $\sigma(h) \geq 1$, $[(\eta_1(h) + \eta_2(h) - 1) - \tau \eta_2(h)] < 0$, and hence $dr/d\tau < 0$. For $\sigma(h) < 1$, if $\tau > (1-\sigma_L)$, then, for all $h > 0$:

$$\tau > (1-\sigma_L) \geq [1-\sigma(h)] > [1-\sigma(h)] \frac{1-(1-b)/\eta_2(h)}{1-(1-b)\sigma(h)} = \frac{\eta_1(h) + \eta_2(h) - 1}{\eta_2(h)}$$

where the second inequality follows from the definition of σ_L , the third inequality from (P9) and the equality from (A4). This completes the proof of III.1.

By the same token, from (A5), $dr/d\tau > 0$ if, and only if, for all $h > 0$:

$$\tau < \frac{\eta_1(h) + \eta_2(h) - 1}{\eta_2(h)} = \frac{[1-\sigma(h)][b-\eta_1(h)]}{b-\eta_1(h)[1-(1-b)\sigma(h)]} \quad (\text{A.10})$$

Define the differentiable function $V: \mathcal{R}^2 \rightarrow \mathcal{R}$, $V(\sigma, \eta_1) = (1-\sigma)(b-\eta_1) / \{b-\eta_1[1-(1-b)\sigma]\}$. This function is decreasing in both its arguments. Recalling from (P10) that, for $\sigma(h) < 1$, the function η_1 is decreasing, it follows that $V(\sigma, h) = V(\sigma, \eta_1(h))$ is decreasing in σ and increasing in h . Define the short run lower bound $h_{\min}(t) \equiv K(t)/\bar{N}$ and let Λ_0 and Λ_τ stand for the watershed when $\tau=0$ and $\tau>0$ respectively. From proposition I, one knows that $1 > G(\Lambda_\tau) > G(\Lambda_0)$. Therefore $h_{\min}(t) = K(t)/\bar{N} \geq K(t)/\bar{N} G(\Lambda_\tau) \geq K(t)/\bar{N} G(\Lambda_0)$. Hence one can write:

$$\frac{[1-\sigma(h)][b-\eta_1(h)]}{b-\eta_1(h)[1-(1-b)\sigma(h)]} \geq \frac{[1-\sigma_U][b-\eta_1(h)]}{b-\eta_1(h)[1-(1-b)\sigma_U]} \geq \frac{[1-\sigma_U][b-\eta_1(h_{\min})]}{b-\eta_1(h_{\min})[1-(1-b)\sigma_U]} \quad (\text{A11})$$

Defining the constant $\tau_r \equiv [1-\sigma_U][b-\eta_1(h_{\min})] / \{b-\eta_1(h_{\min})[1-(1-b)\sigma_U]\} < 1$, one concludes that if $\tau < \tau_r$, then $dr/d\tau > 0$ which completes the proof of III.2.

Proof of Proposition IV: Applying logarithms to (27) and differentiating, yields

$$\frac{d \ln Y_\lambda}{d\tau} = b \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \frac{d\Lambda}{d\tau} + \frac{f'(h)}{f(h)} \frac{dh}{d\tau} = \frac{g(\Lambda)}{G(\Lambda)} [bZ + (b-\eta_1)] \frac{d\Lambda}{d\tau} > 0$$

where the last equality follows from (A8).

Proof of Proposition V: Applying logarithms to (28) and differentiating, one gets:

$$\frac{d \ln Y}{d\tau} = \frac{f'(h)}{f(h)} \frac{dh}{d\tau} + \frac{g(\Lambda)}{G(\Lambda)} [b - (1-b) Z(\Lambda)] \frac{d\Lambda}{d\tau} = - \frac{g(\Lambda)}{G(\Lambda)} [1 - \eta_1(h)] \frac{\tau}{1-\tau} \frac{d\Lambda}{d\tau} < 0$$

Proof of Proposition VI: Substituting (24) into (29) one gets:

$$(1-\tau)\Pi_\lambda = \frac{1-\eta_1(h)}{1+Z(\Lambda)} f(h) \left(G(\Lambda) / \int_\Lambda^\Lambda \lambda g(\lambda) d\lambda \right)^\lambda$$

Taking logarithms and differentiating yields

$$\frac{d \ln[(1-\tau)\Pi_\lambda]}{d\tau} = \left[\frac{f'(h)}{f(h)} - \frac{1}{1-\eta_1} \frac{d\eta_1}{dh} \right] \frac{dh}{d\tau} + \left[b[1+Z(\Lambda)] \frac{g(\Lambda)}{G(\Lambda)} - \frac{1}{[1+Z(\Lambda)]} \frac{dZ}{d\Lambda} \right] \frac{d\Lambda}{d\tau} \quad (\text{A.12})$$

Substituting (A3), (A6) and (A8) into this expression one gets

$$\frac{d \ln[(1-\tau)\Pi_\lambda]}{d\tau} = - \left\{ \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \frac{1}{\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[\frac{\eta_1(h) \eta_2(h)}{1-\eta_1(h)} - b[1+Z(\Lambda)] + Z(\Lambda) \right] \right\} \frac{d\Lambda}{d\tau}$$

From (24), one can write the expression inside the brackets multiplied by $g(\Lambda)/G(\Lambda)$ as:

$$\frac{g(\Lambda)}{G(\Lambda)} \left[\frac{\eta_1(h) \eta_2(h)}{1-\eta_1(h)} - 1 + \frac{1-\eta_1(h)}{1-\tau} \right] \geq \frac{g(\Lambda)}{G(\Lambda)} \frac{\eta_1(h) [\eta_1(h) + \eta_2(h) - 1]}{1-\eta_1(h)} \geq 0$$

where the inequality follows from the hypothesis that $\sigma(h) \leq 1$ and (P8).

Proof of Proposition VII: From (14), (24) and (25), R is given as

$$R = \frac{Z(\Lambda)}{1+Z(\Lambda)} \left(\int_\Lambda^\Lambda \lambda g(\lambda) d\lambda / G(\Lambda) \right)^{1-b} f(h) [1 - \eta_1(h)] \quad (\text{A.13})$$

Taking logarithms, differentiating and substituting (A3), (A6), (A8) and (24) one gets:

$$\frac{d \ln R}{d\tau} = \left\{ \frac{(1-b)(1-\tau)}{[1-\eta_1(h)]\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[1 - \frac{1-\eta_1(h)}{1-\tau} - \eta_1(h) + \frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \right] \right\} \frac{d\Lambda}{d\tau}$$

Eliminating $\eta_2(h)$ with the help of (A5) and noting that since g is assumed non increasing, for any $\lambda \leq \Lambda$

one has $g(\lambda) \geq g(\Lambda)$ and hence $G(\Lambda) = \int_0^\Lambda g(\lambda) d\lambda \geq \int_0^\Lambda g(\Lambda) d\lambda = g(\Lambda) \int_0^\Lambda d\lambda = g(\Lambda) \Lambda$, one needs to show that:

$$\frac{d \ln R}{d\tau} \geq \frac{g(\Lambda)}{G(\Lambda)} \frac{(1-\tau)^2 (1-b)b\sigma + (\eta_1 - \tau)(1-\eta_1)b\sigma - (1-\tau)\eta_1\{b - \eta_1[1 - (1-b)\sigma]\}}{(1-\eta_1)(1-\tau)b\sigma} > 0$$

The numerator above can be written it as a polynomial of τ :

$$P(\tau) = \tau^2(1-b)b\sigma - \tau\{b\sigma[2(1-b) + (1-\eta_1)] - \eta_1\{b - \eta_1[1 - (1-b)\sigma]\}\} + \{b\sigma(1-b) - \eta_1(b - \eta_1)(1-\sigma)\}$$

Since the first term is positive, it is sufficient to show that in the range $0 < \eta_1 < b$ one has:

$$-\tau\{b\sigma[2(1-b) + (1-\eta_1)] - \eta_1\{b - \eta_1[1 - (1-b)\sigma]\}\} + \{b\sigma(1-b) - \eta_1(b - \eta_1)(1-\sigma)\} > 0$$

Defining the polynomials $Q = Q(\eta_1)$ as $Q(\eta_1) = \sigma[b(1-b) + \eta_1(b - \eta_1)] - \eta_1(b - \eta_1)$ and $U = U(\eta_1)$ as $U(\eta_1) = \eta_1^2[1 - \sigma(1-b)] - \eta_1 b(1 + \sigma) + b\sigma[2(1-b) + 1]$, the last inequality can be written as $-\tau U(\eta_1) + Q(\eta_1) > 0$. It will be defined below τ_R such that, for all $0 < \eta_1 < b$, the inequality $-\tau U(\eta_1) + Q(\eta_1) > 0$ holds for any $\tau < \tau_R$. The first polynomial is u-shaped with, $U(0) = \sigma b[1 + 2(1-b)] > 0$, $U(b) = \sigma b(1-b)(3-b) > 0$, and $U'(\eta_1) = 2\eta_1[1 - (1-b)\sigma] - b(1 + \sigma) \leq U'(b) = b\{1 - \sigma[2(1-b) + 1]\}$. Under the assumption $\sigma_L > 1/[2(1-b) + 1]$, one gets $U'(b) < 0$, and hence polynomial is positive in the range $0 < \eta_1 < b$, and is maximum at the vicinity of $\eta_1 = 0$, which implies: $U(\eta_1) \leq \sigma_U b[1 + 2(1-b)] = \sup_{\eta, \sigma} U(\eta_1) > 0$. Under the same assumption $\sigma_L > 1/[2(1-b) + 1]$, one can write: $Q(\eta_1) \geq \frac{[b(1-b) + \eta_1(b - \eta_1)]}{[1 + 2(1-b)]} - \eta_1(b - \eta_1) = \frac{(1-b)[b - 2\eta_1(b - \eta_1)]}{1 + 2(1-b)} \geq \frac{b(1-b)(1-b/2)}{1 + 2(1-b)} = \inf_{\eta, \sigma} Q(\eta_1) > 0$, where the second inequality follows from the fact that $\eta_1(b - \eta_1) \leq b^2/4$ in the range $0 < \eta_1 < b$. Now defining $\tau_R = \inf_{\eta, \sigma} Q(\eta_1) / \sup_{\eta, \sigma} U(\eta_1) > 0$, one has: $\tau_R \equiv \frac{b(1-b)(1-b/2)}{\sigma_U [1 + 2(1-b)]^2}$, for any $\tau < \tau_R$ one gets $\tau < \tau_R \leq \frac{Q(\eta_1)}{U(\eta_1)}$. Hence $\tau < \tau_R$ implies: $-\tau U(\eta_1) + Q(\eta_1) > 0$, then $dR(\tau)/d\tau > 0$.

Proof of Proposition VIII: Applying logarithms to (31) and differentiating one gets,

$$\eta_2(h^*) \frac{dh^*}{h^*} = -(1-b) \frac{g(\Lambda^*)}{G(\Lambda^*)} [1 + Z(\Lambda^*)] d\Lambda^* \quad (A14)$$

Hence, $dh^*/d\tau < 0$ if, and only if, $d\Lambda^*/d\tau > 0$. Applying logarithms to (32) and differentiating yields,

$$\frac{1}{[1 - \eta_1(h^*)]} \frac{d\eta_1}{dh^*} dh^* + \frac{1}{[1 + Z(\Lambda^*)]} \frac{dZ}{d\Lambda^*} d\Lambda^* = \frac{d\tau}{1 - \tau}$$

Substituting (A3), (A6) and (A14), and dropping the * over h and Λ to avoid cluttering one finds:

$$\left\{ \frac{Z(\Lambda)/\Lambda}{[1 + Z(\Lambda)]} + \frac{g(\Lambda)}{G(\Lambda)} \left[Z(\Lambda) - \frac{\eta_1}{\eta_2} \frac{[1 - \eta_1 - \eta_2]}{[1 - \eta_1]} (1-b)[1 + Z(\Lambda)] \right] \right\} d\Lambda^* = \frac{d\tau}{1 - \tau} \quad (A15)$$

Substituting Z from (24), the term in the brackets multiplied by $g(\Lambda)/G(\Lambda)$ can be reduced to:

$$\frac{g(\Lambda)}{G(\Lambda)} \square = \frac{g(\Lambda)}{G(\Lambda)} \left[\frac{(1 - \eta_1)[\eta_2 b - (1-b)\eta_1] + \tau\eta_2(1-b)}{\eta_2(1 - \tau)(1-b)} \right] > 0$$

where the inequality follows from (P6) and (P7). This shows that $d\Lambda^*/d\tau > 0$, which completes the proof.

Proof of Proposition IX: Applying logarithms to (25) and differentiating yields

$$\frac{d \ln W^*}{d\tau} = \eta_1 \frac{\eta_2 - (1-b)}{b - \eta_1} \frac{1}{h^*} \frac{dh^*}{d\tau} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \frac{d\Lambda^*}{d\tau} < 0$$

where the inequality follows from (P7) and the result of proposition VIII.

Proof of Proposition X: Applying logarithms to (33) and differentiating yields

$$\frac{d \ln K^*}{d\tau} = \frac{1}{h^*} \frac{dh^*}{d\tau} + \frac{g(\Lambda^*)}{G(\Lambda^*)} \frac{d\Lambda^*}{d\tau} = \frac{1}{[(1 - \tau)\eta_2]} \frac{g(\Lambda^*)}{G(\Lambda^*)} [(\eta_1 + \eta_2 - 1) - \tau\eta_2] \frac{d\Lambda^*}{d\tau}$$

where the last equality follows from (A14) and eliminating of Z from (24). The expression inside the brackets is the same expression that appeared in the proof of proposition III. Since the hypothesis of X.1 and X.2 are identical to those of III.1. and III.2. respectively, the argument used in that proposition applies.

Proof of Proposition XI: Applying logarithms to (27) and differentiating yields

$$\frac{d \ln Y^*}{d\tau} = \frac{\eta_1}{h^*} \frac{dh^*}{d\tau} + b \frac{g(\Lambda^*)}{G(\Lambda^*)} [1 + Z(\Lambda^*)] \frac{d\Lambda^*}{d\tau} = \frac{g(\Lambda^*)}{G(\Lambda^*)} [1 + Z(\Lambda^*)] \frac{[b\eta_2 - (1-b)\eta_1]}{\eta_2} \frac{d\Lambda^*}{d\tau} > 0$$

where the second equality follows from (A14).

Proof of Proposition XII: Applying logarithms to (28) and differentiating yields

$$\frac{d \ln Y^*}{d\tau} = \eta_1 \frac{1}{h^*} \frac{dh^*}{d\tau} + \frac{g(\Lambda^*)}{G(\Lambda^*)} [b - (1-b)Z(\Lambda^*)] \frac{d\Lambda^*}{d\tau} = \frac{1}{\eta_2(1-\tau)} \frac{g(\Lambda^*)}{G(\Lambda^*)} [\eta_1[\eta_1 + \eta_2 - 1] - \tau \eta_2] \frac{d\Lambda^*}{d\tau}$$

where the second equality follows from (A14) and substitution of Z from (24). From (P8), one concludes that if $\sigma(h) \geq 1$ for all $h > 0$, then $d \ln Y^* / d\tau < 0$. If $\sigma(h) < 1$, then $dY^* / d\tau < 0$ if, and only if,

$$\tau > \eta_1(h^*) \frac{\eta_1(h^*) + \eta_2(h^*) - 1}{\eta_2(h^*)} = \frac{b[1 - \sigma(h)]}{[1 - (1-b)\sigma(h)]^2} [\eta_2(h) - (1-b)][1 / \eta_2(h) - \sigma(h)]$$

where the equality follows from the substitution of η_1 from (A4). For a given σ , the function $X: (1-b, +\infty) \rightarrow \mathbb{R}$, $X(\eta_2) = [\eta_2 - (1-b)][1 / \eta_2 - \sigma]$ has a global maximum at $\eta_2^* = \sqrt{(1-b) / \sigma}$ and $X(\eta_2^*) = (1 - \sqrt{(1-b)\sigma})^2$. Hence, it is sufficient that for all σ ,

$$\tau > b(1-\sigma) \frac{(1 - \sqrt{(1-b)\sigma})^2}{[1 - \sigma(1-b)]^2} = \frac{b(1-\sigma)(1 - \sqrt{(1-b)\sigma})^2}{[(1 - \sqrt{(1-b)\sigma})(1 + \sqrt{(1-b)\sigma})]^2} = \frac{b(1-\sigma)}{(1 + \sqrt{(1-b)\sigma})^2}$$

Since $\sigma(h) \geq \sigma_L$ for all $h > 0$ and the fraction on the right hand side of the inequality above is a decreasing function of σ , one concludes that if $\tau > b(1-\sigma_L) > b(1-\sigma_L) / (1 + \sqrt{(1-b)\sigma_L})^2$, then $d \ln Y / d\tau < 0$, which completes the proof of XII.1.

To prove XII.2, one has to show that there is an interval within which a PS parameter $\tau > 0$ satisfies, for all $h > 0$, the following inequality holds for all $h > 0$:

$$\tau < \eta_1(h) \frac{\eta_1(h) + \eta_2(h) - 1}{\eta_2(h)} = \eta_1(h) \frac{(1 - \sigma(h))(b - \eta_1(h))}{b - \eta_1(h)[1 - (1-b)\sigma(h)]} \quad (\text{A.16})$$

where the equality follows from (A5). The right hand side of (A.16) is equal to $\eta_1(h)$ times the right hand side of (A.10). From $\eta_1(h) < b < 1$, one concludes that any PS parameter that satisfies (A.16), also satisfies (A.10). It follows that if the PS parameter satisfies (A.16), then the time path of aggregate capital stock is increasing. In the proof of proposition XV.1 it will be shown - without any assumption about the elasticity of substitution of F - that the dynamics of capital/labor ratio $h(t)$ and the aggregate capital stock $K(t)$ follow similar trends, i.e., $\dot{h}(t)\dot{K}(t) > 0$. One concludes that if the PS parameter satisfies (A.16), then $K(t)$ and $h(t)$ will follow increasing trends, i.e., $\dot{h}(t) > 0$ and $\dot{K}(t) > 0$. Let h_0^* and K_0^* stand for the long run capita/labor ratio and capital stock without PS. Hence:

$$h_{\min} = \frac{K_0^*}{N} \leq \frac{K(0)}{N G(\Lambda(0^*))} = h_r(0^*) \leq \frac{K(t)}{N G(\Lambda(t))} = h_r(t) \leq \frac{K_r^*}{N G(\Lambda_r^*)} = h_r^* < h_0^* = \frac{K_0^*}{N G(\Lambda_0^*)}$$

The first inequality above follows from the fact that $G(\Lambda) < 1$. The second and third inequalities follow from the fact that $\dot{h}(t) > 0$. The fourth inequality is assured by proposition VIII. From (P10), η_l is a decreasing function of h , hence: $\eta_l(h_{\min}) > \eta_l(h_r(0^*)) \geq \eta_l(h_r(t)) \geq \eta_l(h_r^*) \geq \eta_l(h_0^*)$. By the same argument used in the proof of proposition III.2 to arrive at (A.11), for all $h > 0$ one can write:

$$\frac{[1 - \sigma_U][b - \eta_l(h_{\min})]}{b - \eta_l(h_{\min})[1 - (1-b)\sigma_U]} \leq \frac{[1 - \sigma_U][b - \eta_l(h)]}{b - \eta_l(h)[1 - (1-b)\sigma_U]} \leq \frac{[1 - \sigma(h)][b - \eta_l(h)]}{b - \eta_l(h)[1 - (1-b)\sigma(h)]}$$

Defining $\tau_{\gamma^*} \equiv \eta_l(h_0^*) [1 - \sigma_U] [b - \eta_l(h_{\min})] / \{b - \eta_l(h_{\min})[1 - (1-b)\sigma_U]\}$, if $\tau < \tau_{\gamma^*}$, the inequality above implies that (A16) is satisfied, which completes the proof of XII.2.

Proof of Proposition XIII: Substituting (A.3) into (A.12) with $h = h^*$ and $\Lambda = \Lambda^*$ one gets:

$$\frac{d \ln (1 - \tau) \Pi_{\lambda}^*}{d\tau} = \left[\eta_l - \frac{\eta_l [1 - \eta_l - \eta_z]}{1 - \eta_l} \right] \frac{dh^*}{h^*} + \left[b [1 + Z(\Lambda^*)] \frac{g(\Lambda^*)}{G(\Lambda^*)} - \frac{1}{[1 + Z(\Lambda^*)]} \frac{dZ}{d\Lambda} \right] \frac{d\Lambda^*}{d\tau}$$

Substituting $dh^* / d\tau$ from (A14) and Z from (24), and (A6) one gets:

$$\frac{d \ln (1 - \tau) \Pi_{\lambda}^*}{d\tau} = - \left\{ \frac{1}{\Lambda^*} \frac{Z(\Lambda^*)}{[1 + Z(\Lambda^*)]} + \frac{g(\Lambda^*)}{G(\Lambda^*)} \frac{\tau}{1 - \tau} \right\} \frac{d\Lambda^*}{d\tau} < 0$$

Proof Proposition XIV: R^* is given by (A13) with $h = h^*$ and $\Lambda = \Lambda^*$. Taking logarithms and differentiating yields:

$$\frac{d \ln R^*}{d\tau} = \frac{1}{Z(\Lambda^*)[1 + Z(\Lambda^*)]} \frac{dZ}{d\Lambda} \frac{d\Lambda^*}{d\tau} - (1-b)[1 + Z(\Lambda^*)] \frac{g(\Lambda^*)}{G(\Lambda^*)} \frac{d\Lambda^*}{d\tau} + \left[\frac{f'(h)}{f(h)} - \frac{1}{[1 - \eta_l(h)]} \frac{d\eta_l}{dh} \right] \frac{1}{h^*} \frac{dh^*}{d\tau}$$

Substituting $dh^* / d\tau$ from (A14), Z from (24), $dZ / d\Lambda$ from (A6) one gets:

$$\frac{d \ln R^*}{d\tau} = \left\{ \frac{(1-\tau)(1-b)}{\Lambda^*(1-\eta_l)} - \frac{\tau}{(1-\tau)} \frac{g(\Lambda^*)}{G(\Lambda^*)} \right\} \frac{d\Lambda^*}{d\tau} \geq \frac{g(\Lambda^*)}{G(\Lambda^*)} \frac{(1-\tau)^2(1-b) - \tau(1-\eta_l)}{(1-\eta_l)(1-\tau)} \frac{d\Lambda^*}{d\tau}$$

where the inequality follows from the hypothesis $g' \leq 0$ which implies $1 / \Lambda^* \geq g(\Lambda^*) / g(\Lambda^*)$. Hence:

$$\frac{d \ln R^*}{d\tau} \geq \frac{g(\Lambda^*)}{G(\Lambda^*)} \frac{(1-\tau)^2(1-b) - \tau(1-\eta_l)}{(1-\eta_l)(1-\tau)} \frac{d\Lambda^*}{d\tau}$$

Define the polynomial in τ , $P(\tau)$ as $P(\tau) = (1-\tau)^2(1-b) - \tau(1-\eta_l)$. Then $P(\tau) = \tau^2(1-b) - \tau[2(1-b) + (1-\eta_l)] + (1-b) \geq -\tau[2(1-b) + (1-\eta_l)] + (1-b)$. Hence, defining $\tau_{R^*} \equiv (1-b) / [2(1-b) + 1]$, then for $0 < \tau < \tau_{R^*}$, $\tau < (1-b) / [2(1-b) + (1-\eta_l)]$ and hence $dR^* / d\tau > 0$.

Proof of Proposition XV

Proof of XV.1: The path of the aggregate capital $K(t)$ is increasing if, and only if, the rental rises above the rate of time preference. Since the sufficient conditions of this proposition XV.1 are identical to those of proposition III, the proof of both propositions are identical;

Proof of XV.2: Taking logarithms of (22) and differentiating with respect to time, yields

$$\frac{\dot{K}(t)}{K(t)} = \frac{\dot{h}(t)}{h(t)} + \frac{g(\Lambda)}{G(\Lambda)} \dot{\Lambda}(t)$$

where the dots on the variables stand for the time derivative. Likewise, taking logarithms of (24) and differentiating with respect to time gives:

$$\frac{\eta_1(h)[\eta_1(h) + \eta_2(h) - 1]}{1 - \eta_1(h)} \frac{\dot{h}(t)}{h(t)} = \frac{Z(\Lambda)}{[1 + Z(\Lambda)]} \left[\frac{1}{\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \right] \dot{\Lambda}(t) \quad (\text{A.17})$$

where one has substituted (A3) and (A6). Eliminating \dot{h}/h from the last two expressions one gets

$$\frac{\eta_1[\eta_1 + \eta_2 - 1]}{1 - \eta_1} \frac{\dot{K}(t)}{K(t)} = \left\{ \frac{Z(\Lambda)}{[1 + Z(\Lambda)]} \frac{1}{(1-b)} + \frac{g(\Lambda)}{G(\Lambda)} \left[Z(\Lambda) - \frac{\eta_1[1 - \eta_1 - \eta_2]}{1 - \eta_1} \right] \right\} \dot{\Lambda}(t) = \{+\} \dot{\Lambda}(t) \quad (\text{A.18})$$

The right hand side of (A.18) is equal to $(1 - \tau)$ times the right hand side of (A.9), which was shown to be positive in the proof of proposition I. Since from (P8), $\eta_1(h) + \eta_2(h) \geq 1$ if, and only if, $\sigma(h) \leq 1$, the proof of XV.1 is complete;

Proof of XV.3.1: Eliminating $\dot{\Lambda}(t)$ from (A17) and (A18), and letting $\{+\}$ stand for the positive expression in the braces which is multiplied by $\dot{\Lambda}(t)$ in (A18), one concludes that

$$\frac{\dot{h}(t)}{h(t)} = \frac{Z(\Lambda)}{[1 + Z(\Lambda)]} \left[\frac{1}{\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \right] \{+\}^{-1} \frac{\dot{K}}{K} > 0 \quad (\text{A.19})$$

Proof of XV.3.2: In order to prove $\dot{K}(t)\dot{w}(t) > 0$ one differentiates (25) with respect to time to get:

$$\frac{\dot{w}(t)}{w(t)} = \eta_1(h) \frac{[\eta_2(h) - (1-b)]}{b - \eta_1(h)} \frac{\dot{h}}{h} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \dot{\Lambda}$$

Substituting $\dot{\Lambda}(t)$ and \dot{h}/h from (A.18) and (A.19), and Z from (24) one gets:

$$\frac{\dot{w}}{w} = \eta_1 \{+\}^{-1} \frac{\dot{K}}{K} \left[\frac{[\eta_2 - (1-b)]}{b - \eta_1} \frac{Z/\Lambda}{1+Z} + \frac{g(\Lambda)}{G(\Lambda)} \left(\frac{[\eta_2 - (1-b)]\tau(1-b) + (b - \eta_2)[b\eta_2 - \eta_2(1-b)]}{(b - \eta_1)(1-b)(1-\tau)} \right) \right] > 0$$

Proof of XV.3.3: Applying logarithms to (26) and differentiating one gets:

$$\frac{\dot{r}(t)}{r + \delta} = -\eta_2 \frac{\dot{h}}{h} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1 + Z] \dot{\Lambda} = -\frac{\dot{K}/K}{\{+\}} \left[\frac{\eta_2 Z/\Lambda}{(1+Z)} + \frac{g}{G} \frac{\tau\eta_2(1-b) + (1-\eta_1)[\eta_2 b - (1-b)\eta_1]}{(1-b)(1-\tau)} \right]$$

where the second equality follows from the substitution of $\dot{\Lambda}(t)$, \dot{h}/h and Z from (A.18) (A.19), and (24). From (P6) and (P7), the expression in the brackets is positive, hence $\dot{K}(t)\dot{r}(t) > 0$.

Proof of XV.3.4: In order to prove that $\dot{K}(t)\dot{Y}_\lambda(t) > 0$ one differentiates (27) with respect to time to get:

$$\frac{\dot{Y}_\lambda(t)}{Y_\lambda(t)} = b \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \dot{\Lambda} + \eta_1 \frac{\dot{h}}{h} \frac{\dot{Y}_\lambda}{Y_\lambda} = \eta_1 \frac{\dot{K}/K}{\{+\}} \left[\frac{1-b}{1-\eta_1} \frac{Z/\Lambda}{1+Z} + \frac{g}{G} \frac{[b\eta_2 - (1-b)\eta_1] + \tau(1-b)}{(1-b)(1-\tau)} \right] > 0$$

where the second equality follows from the substitution of $\dot{\Lambda}$, \dot{h}/h and Z from (A.18) (A.19) and (24).

Proof of XV.3.5: In order to prove that $\dot{K}(t)\dot{R}(t) > 0$ one takes logarithms of (A14) and differentiates with respect to time and gets:

$$\frac{\dot{R}(t)}{R(t)} = \left[\frac{1}{Z(\Lambda)[1+Z(\Lambda)]} \frac{dZ}{d\Lambda} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \right] \dot{\Lambda} + \left[\frac{f'(h)}{f(h)} - \frac{1}{[1-\eta_l(\Lambda)]} \frac{d\eta_l}{dh} \right] \dot{h}$$

Replacing $dZ/d\Lambda$ and $d\eta_l/dh$ from (A3) and (A6), substituting of $\dot{\Lambda}(t)$ and \dot{h}/h from (A.18) and (A.19), and Z from (24) one gets:

$$\frac{\dot{R}}{R} = \frac{\{+\}^{-1}}{1-\eta_l} \left\{ \frac{\eta_l(1-b)(1-\tau)}{\Lambda[1+Z(\Lambda)]} + \frac{g(\Lambda)}{G(\Lambda)} [f\eta_l b - (1-b)\eta_l] + \tau(1-b) \right\} \frac{\dot{K}}{K} > 0$$

Proof of XV.4: Applying logarithms to (28) and differentiating with respect to time one gets:

$$\frac{\dot{Y}}{Y} = \eta_l(h) \frac{\dot{h}}{h} + \frac{g(\Lambda)}{G(\Lambda)} [b - (1-b)Z(\Lambda)] \dot{\Lambda} \quad \frac{\dot{Y}}{Y} = \frac{\eta_l \dot{K}}{\{+\}K} \left[\frac{Z/\Lambda}{1+Z} + \frac{g}{G} \left(Z + [b - (1-b)Z] \frac{[\eta_l + \eta_2 - 1]}{1-\eta_l} \right) \right]$$

where the second equality follows from the substitution $\dot{\Lambda}(t)$ and \dot{h}/h from (A.18) and (A.19).

Replacing Z from (24) the expression in the brackets which is multiplied by $g(\Lambda)/G(\Lambda)$ in (A.22) is:

$$\frac{g}{G} \left(\frac{(b-\eta_l)[(b-\eta_l) + (1-b)(1-\eta_l)] + (1-b)[\eta_l[\eta_2 - (1-b)] + \tau[(1-\eta_l) + (1-\eta_l - \eta_2)]]}{(1-\eta_l)(1-b)(1-\tau)} \right) \quad (A20)$$

From (P8), one concludes that under the hypothesis $\sigma_L \geq 1$, then $(1-\eta_l - \eta_2) \geq 0$, and $\dot{K}(t)\dot{Y}(t) > 0$. Proposition III.1 showed that for $\sigma_L \geq 1$, the introduction of PS reduces the rental to $r(t) < \rho$. It follows that $\dot{K}(t) < 0$ under this hypothesis, and therefore $\dot{Y}(t) < 0$, which completes the proof of XV.4.1.

From (A16), if $\tau < \tau_{\gamma^*}$, then $\eta_2(\eta_l - \tau) > \eta_l(1 - \eta_l)$. Substituting this inequality into (A23) one gets:

$$\frac{g}{G} \left(\right) \geq \frac{g}{G} \frac{(b-\eta_l) + 2\tau(1-b)}{(1-b)(1-\tau)} > 0$$

which shows that for $\sigma_U < 1$, if $\tau < \tau_{\gamma^*}$, then $\dot{K}(t)\dot{Y}(t) > 0$. From proposition III.2 one knows that under this hypothesis the introduction of PS increases the rental to $r(t) > \rho$ and hence $\dot{K}(t) > 0$. One concludes that for $\sigma_U < 1$, if $\tau < \tau_{\gamma^*}$, then $\dot{Y}(t) > 0$. This completes the proof of XV.4.2.

Proof of XVI: Dividing through (34) by τ and letting τ go to zero one gets:

$$\frac{dNY_r^*}{d\tau} = \lim_{\tau \rightarrow 0} \frac{(NY_r^* - NY_0^*)}{\tau} = \lim_{\tau \rightarrow 0} \frac{(Y_r^* - Y_0^*)}{\tau} - \rho \lim_{\tau \rightarrow 0} \frac{(K_r^* - K_0^*)}{\tau} = \frac{dY_r^*}{d\tau} - \rho \frac{dK_r^*}{d\tau}$$

Substituting $dY_r^*/d\tau$ and $dK_r^*/d\tau > 0$ as calculated in propositions XII and X, one concludes that:

$$\frac{dNY_r^*}{d\tau} = \frac{(1-\rho)}{(1-\tau)} \frac{g(\Lambda)}{G(\Lambda)} \left\{ \frac{(\eta_l - \rho)}{(1-\rho)} \frac{(\eta_l + \eta_2 - 1)}{\eta_2} - \tau \right\} \frac{d\Lambda}{d\tau}$$

Define $\theta(h) \equiv [1 - \rho/\eta_l(h)]/(1-\rho)$. In the proof of proposition XII.2 it was shown that if $\sigma_U < 1$, then $\eta_l(h_{\min}) > \eta_l(h_r(0^+)) \geq \eta_l(h_r(t)) \geq \eta_l(h_r^*) \geq \eta_l(h_0^*)$. Hence $\theta(h_0^*) \leq \theta(h(t))$ for all $t > 0$. Since τ_{γ^*} is lower than the right hand side of (A16), if $\tau < \tau_U \equiv \tau_{\gamma^*} \theta(h_0^*)$, then:

$$\tau < \tau_{\gamma^*} \theta(h_0^*) \leq \eta_2(h) \frac{(\eta_l(h) + \eta_2(h) - 1)}{\eta_2(h)} \theta(h) = \frac{(\eta_l(h) + \eta_2(h) - 1)}{\eta_2(h)} \frac{(\eta_l(h) - \rho)}{(1-\rho)}$$

which completes the proof.

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