

Nº 76

THE BROWN-VON NEUMANN DIFFERENTIAL

EQUATION FOR BIMATRIX GAMES *

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Chapter II: The Brown-Von Neumann
Differential Equation for Bimatrix Games

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1. Introduction

Von Neumann's classical proof of existence of a Nash equilibrium for a two-person zero-sum game with a finite number of pure strategies used the separating hyperplane theorem. In 1950, G. Brown and J. Von-Neumann gave another proof of this same theorem. Their proof is based on giving a differential equation system on the cartesian product of the sets of mixed strategies and studying the behaviour of the solution path when $t \rightarrow \infty$. This differential equation system is the continuous time equivalent of the method of fictitious play proposed by Brown.

The method of fictitious play is an iterative procedure, it corresponds to each player choosing in turn the best pure strategy against the accumulated mixed strategy of his opponent up to then. Julia Robinson (1951) proved that for two-person zero-sum games the solution path of this process accumulates at the Nash equilibria of the game. Similarly, the BN mechanism is a tâtonement process where each player increases the weight he gives to his i -th pure strategy if, assuming the actions of the other players are fixed, this pure strategy has a payoff greater than the mixed strategy he is taking.

Our aim is to extend the BN differential equation system to 2×2 bimatrix games. We show that the solution of this system approaches a Nash equilibrium when $t \rightarrow \infty$. This should be expected. Indeed, there is a strong analogy between the method of fictitious play and the BN mechanism and in 1961, K. Miyasawa proved that the method of fictitious play

can be used for approximating the solution of 2x2 bimatrix games. Our discussion of the BN mechanism reveals the geometry behind the problem. The convergence of the solution path of the BN mechanism is assured by the Poincaré- Bendixson theorem and by the properties of the best reply correspondences of the players. This type of reasoning is not contained in Miyasawa's result. We are also able to show that the BN mechanism is generically structurally stable. Finally, we give a theorem that may be of some use for testing the convergence of the BN solution path to a Nash equilibrium for any bimatrix game.

2. The BN Differential System for 2x2 Bimatrix Games

Let the payoff matrix of the first player be $A=[a_{ij}]$, while that of the second is $B=[b_{ij}]$, for $i,j=1,2$. The first player picks row i and the second player picks column j . A mixed strategy for player 1 is determined by a number x in $[0,1]$, which is the probability he assigns for picking the first row. For player 2 we also have a number y in $[0,1]$, which is the probability that he picks the first column. Under the pair (x,y) the payoffs of both players are

$$P_1(x,y) = x[a_{11}y + a_{12}(1-y)] + (1-x)[a_{21}y + a_{22}(1-y)]$$

$$P_2(x,y) = x[b_{11}y + b_{12}(1-y)] + (1-x)[b_{21}y + b_{22}(1-y)] .$$

We define the functions

$$Z_{11}(x,y) = [\max(0, P_1(1,y) - P_1(x,y))]^2$$

$$Z_{12}(x,y) = [\max(0, P_1(0,y) - P_1(x,y))]^2$$

$$Z_{21}(x,y) = [\max(0, P_2(x,1) - P_2(x,y))]^2$$

$$Z_{22}(x,y) = [\max(0, P_2(x,0) - P_2(x,y))]^2$$

$$H_1(x,y) = Z_{11}(x,y) + Z_{12}(x,y)$$

$$H_2(x,y) = Z_{21}(x,y) + Z_{22}(x,y) .$$

The BN differential system, from now on called system (A), is formed by the two differential equations

$$dx/dt = Z_{11}(x,y) - xH_1(x,y)$$

$$dy/dt = Z_{21}(x,y) - yH_2(x,y) .$$

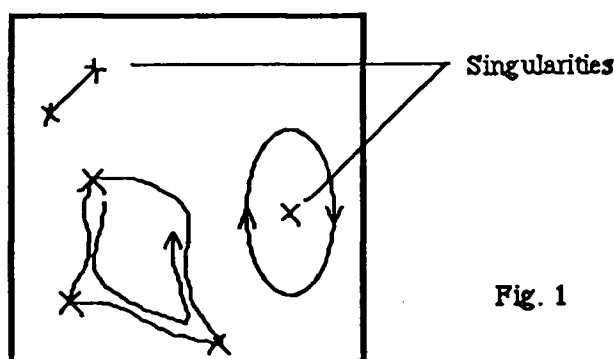
Using Picard's theorem one can solve the system for any given pair of initial conditions (x_0, y_0) . If (x_0, y_0) lies in $[0,1]^2$, then the solution $(x(t), y(t))$ never leaves $[0,1]^2$. Indeed, if $x(t)=0$, then $dx/dt = Z_{11}(x,y) > 0$ and if $x(t)=1$, then $Z_{11}(x,y)=0$ and $dx/dt = -H_1(x,y) < 0$. The same reasoning applies to $y(t)$. We study the behaviour of the maps $x, y: [0, \infty) \rightarrow [0, 1]$ when t goes to infinity. That is, given an initial condition (x_0, y_0) we study the set formed by the accumulation points of $(x(t), y(t))$ when $t \rightarrow \infty$, we call this set the ω -limit set $\omega(x_0, y_0)$.

Notice that $Z_{11}Z_{12}=Z_{21}Z_{22}=0$ for all pairs (x,y) . And this implies that if $H_1 > 0$, then either $H_1 = Z_{11}$ or $H_1 = Z_{12}$. Moreover, all Z_{ij} are simultaneously equal to zero at (x^*, y^*) if and only if this point is a Nash equilibrium. Now, suppose for example that $Z_{11} > 0$, then $dx/dt = (1-x)Z_{11}=0$ only if $x=1$, but this implies $Z_{11}=0$ a contradiction. On the

other hand, if $Z_{12} > 0$, then $dx/dt = -xZ_{12} = 0$ only if $x = 0$ and we again have a contradiction because if $x = 0$ then $Z_{12} = 0$. Repeating this reasoning for the hypothesis $Z_{21} > 0$ and $Z_{22} > 0$ we conclude that the set of singularities of (A) coincides with the set of Nash equilibria.

The dynamical system (A) is planar and autonomous, i.e. the right hand side of the equations in (A) do not depend directly on t . Therefore we can use the Poincare-Bendixson theory for $[0,1]^2$. This says that given any initial condition (x_0, y_0) we shall have that

$\omega(x_0, y_0)$ may consist of singularities or paths leading from one singularity to another or, still, of closed orbits (Figure 1). Moreover, inside the region circumscribed by a closed orbit we always have a singularity.



Closed orbits and orbits that accumulate like those inside the loop S are the problem. They would imply that the solution path of system (A) could accumulate on something other than a singularity of (A). We shall prove that these cases do not arise in system (A). Let us study the functions Z_{ij} . For generic values of the a_{ij} and b_{ij} we have a situation as depicted in Figures 2 and 3. There we assumed without loss of generality that $(a_{11} - a_{21}) - (a_{12} - a_{22}) > 0$ and $(b_{11} - b_{21}) - (b_{12} - b_{22}) > 0$

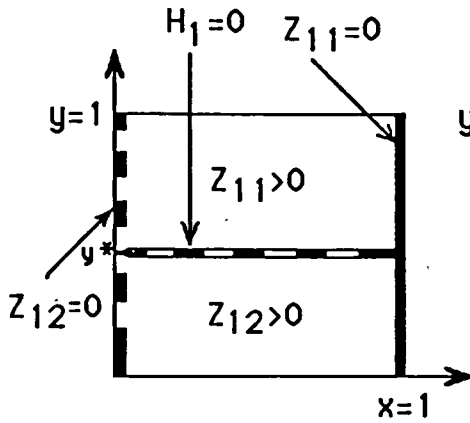


Fig. 2

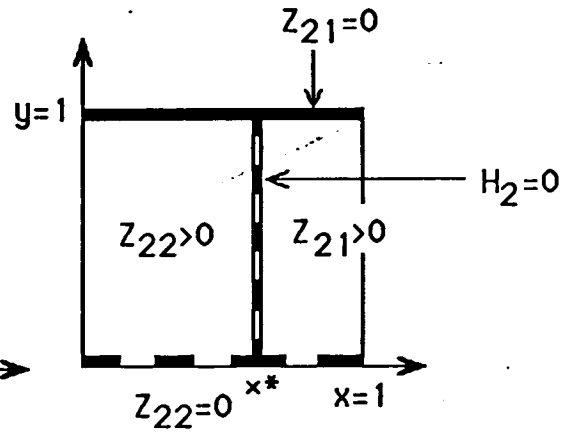


Fig. 3

$$x^* = \frac{-(b_{12} - b_{22})}{(b_{11} - b_{21}) - (b_{12} - b_{22})}$$

$$y^* = \frac{-(a_{12} - a_{22})}{(a_{11} - a_{21}) - (a_{12} - a_{22})}$$

We conclude that if an interior equilibrium exists, that is an equilibrium with (x, y) in $(0, 1)^2$, then it is generically unique. We analyse this case first. In Figure 4 the point P is the unique interior equilibrium. In region I of this Figure we have that $dx/dt = (1-x)Z_{11} > 0$ and that $dy/dt = (1-y)Z_{21} > 0$, this implies that $dx/dy > 0$. However it is clear that along the closed orbit we should have $dx/dy < 0$. This yields a contradiction and therefore no closed orbit is possible. On the other hand, if more than one interior equilibrium exists, then either all the points of $[0, 1]$ are equilibria or we have a situation like that of Figure 5. A closed non-degenerate orbit G of system (A), i.e. a closed orbit that does not degenerate to a point, is not possible in any of these two situations.

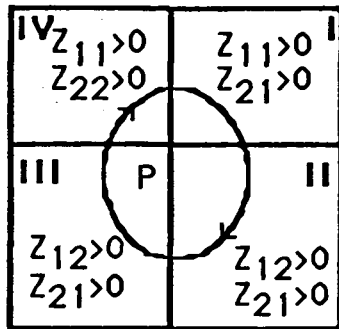


Fig. 4

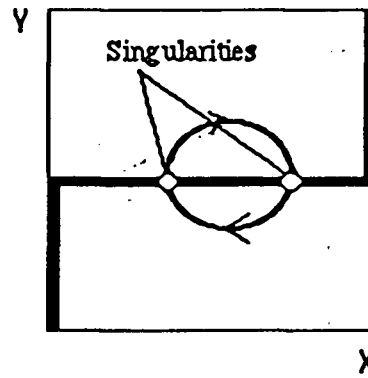


Fig. 5

It is not possible to have a loop like the loop S of Figure 1. Indeed, in order to obtain a loop we need at least 3 equilibria. This is actually the maximum number of equilibria one can have if the number of equilibria is to be finite. Taking a close look at Figure 6 we see that in region I we have two paths along which dy/dx has opposite signs, but in this region we know that the sign of dy/dx is unique. Therefore we have that with a finite number of equilibria the loop is impossible.

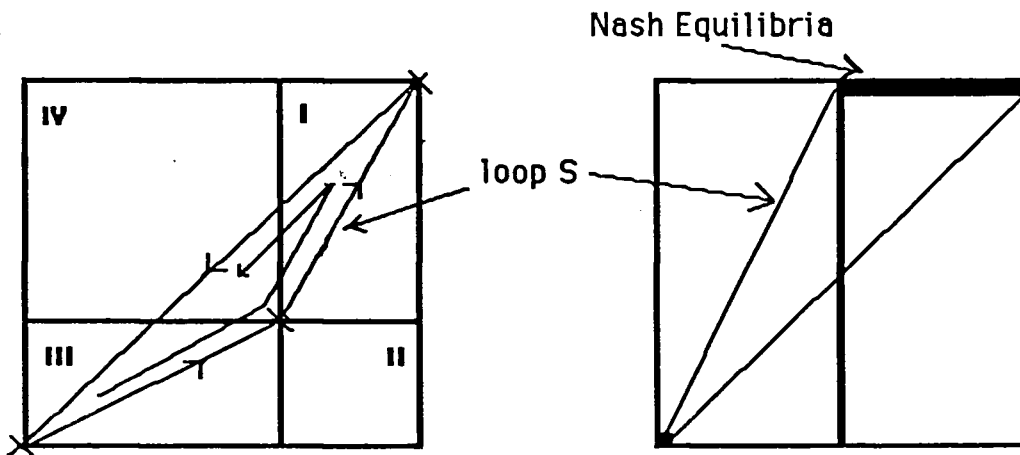


Fig. 6

Fig. 7

Finally, in Figure 7 we consider the case where we try to build a loop for a game with an infinite number of equilibria. The path that cuts the vertical line has $dy/dx < 0$ or $dy/dx > 0$, but not both. But cutting the vertical line means that dy/dx is going to change sign, a contradiction. Therefore, a loop is also impossible. We leave to the reader the analysis of the other similar situations. One has

Theorem A: The BN differential procedure always work in a 2x2 bimatrix game.

3. Some Examples and Structural Stability

It is interesting to look at the dynamics that system (A) yields when we study some well known games. We start with the prisoner's dilemma of A. Tucker.

The payoff matrices are given by

$$A = \begin{bmatrix} 5 & -4 \\ 6 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ -4 & -3 \end{bmatrix}$$

In this case $(a_{11}-a_{21}) - (a_{12}-a_{22}) = 0$. We have that $P_1(x,y) = -x + 9y - 3$, what implies that $Z_{11}(x,y) = (\max(0, -(1-x)))^2 = 0$ and $Z_{12}(x,y) = (\max(0, x))^2 = x^2$. We also have $(b_{11}-b_{21}) - (b_{12}-b_{22}) = 0$ and $P_2(x,y) = 9x - y - 3$. This gives $Z_{21}(x,y) = (\max(0, -(1-y)))^2 = 0$ and $Z_{22}(x,y) = (\max(0, y))^2 = y^2$. The only equilibrium is then the point (0,0) and the phase diagram for this example is given in Figure 8. On the right side of the Figure we have the equations of the path. C is a constant determined by the initial conditions.

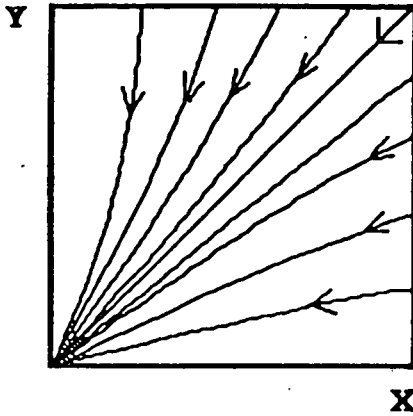


Fig. 8

$$\frac{dy}{dx} = \frac{1}{(1 - Cx^2)^{3/2}}$$

$$\frac{d^2y}{dx^2} = \frac{3Cx}{(1 - Cx^2)^{5/2}}$$

Another interesting example is the "battle of sexes". For this particular game we have the payoff matrices given by

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

It is easy to check that both (0,0) and (1,1) are Nash equilibria. There exists a third equilibrium given by (3/5, 2/5). We have $Z_{11}(x,y) = (\max(0, (1-x)(5y-2)))^2$, $Z_{12}(x,y) = (\max(0, x(2-5y)))^2$, $Z_{21}(x,y) = (\max(0, (1-y)(5x-3)))^2$ and $Z_{22}(x,y) = (\max(0, y(3-5x)))^2$.

The phase diagram is given below.

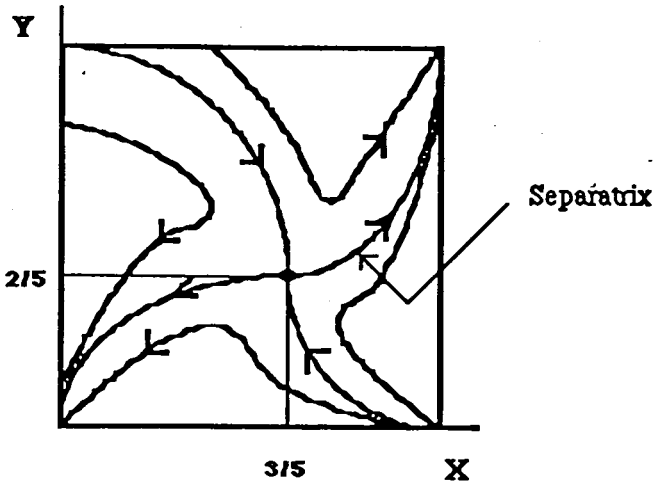


Fig. 9

An interesting feature of the BN mechanism is revealed by the first example. It should be clear to the reader that changing the payoff matrices for rA and rB , where $r > 0$, does not change the dynamics of the game. But if we put $r=0$, then the dynamics change completely, every point of the square will be an equilibrium and given any initial condition system (A) will remain there forever. The passage from $r=0$ to $r > 0$ is not structurally stable.

Another example of lack of structural stability is obtained when we consider the following modification of the battle of sexes,

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 \\ -r & 2r \end{bmatrix}$$

for $r > 0$ the dynamics of this game is similar to that of Figure 9. However, for $r=0$ the dynamics is that of Figure 10. When we pass from $r=0$ to $r > 0$ there is a qualitative modification of the phase diagram.

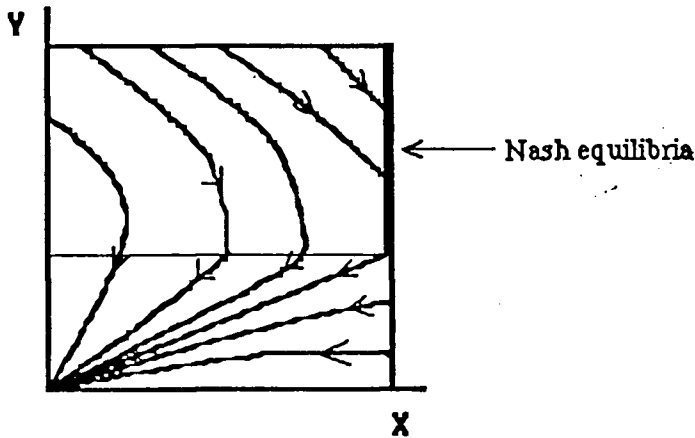


Fig. 10

How often do such jumps appear in this mechanism is a relevant question. Indeed, it is not very difficult to prove that except for a set of Lebesgue measure zero any pair of matrices A, B yield a dynamical system (A) which is structurally stable.

4. A Criterium for Testing the Convergence of the BN -Solution

It is clear that the type of mechanism proposed by Brown and Von Neumann for matrix games can be extended to any m -person game where each player has a finite number of pure strategies. Indeed, as remarked by Kuhn in his Lecture Notes, the differential equation of the BN mechanism can be traced to the proof Nash gave of existence of Nash equilibria. The problem is that Nash's proof gives the existence of an equilibrium, it does not reveal how to calculate it, while the BN mechanism contains a means of computing the solution. Shapley has studied a class of examples of 3×3 bimatrix games where the BN mechanism does not converge. What we propose below is a criterium to check if a certain solution of the BN differential equation converges to an equilibrium or not.

Let $S(n)$ be the standard n -dimensional simplex, that is $S(n) := \{ \xi \in \mathbb{R}^{n+1} : \xi^1 + \dots$

$+ \xi^{n+1} = 1$ and $\xi^j \geq 0$. If player i has (n_i+1) pure strategies then its set of mixed strategies is $S(n_i)$. The BN differential equation is defined over the cartesian product of all the $S(n_i)$ in the following way. Let $P_i: \prod_k S(n_k) \rightarrow \mathfrak{R}$ be the payoff function of player i , i.e. if the mixed strategies of each player is specified in $(\xi_1, \xi_2, \dots, \xi_m)$ then $P_i(\xi_1, \xi_2, \dots, \xi_m)$ is his payoff. Let σ_i^j denote the j -th pure strategy of player i . For each player i we construct a function $b_i: \prod_k S(n_k) \rightarrow S(n_i)$ such that its j -th coordinate is given by $b_i^j(\xi_1, \xi_2, \dots, \xi_m) := [\max(0, P_i(\xi_1, \xi_2, \dots, \xi_{i-1}, \sigma_i^j, \xi_{i+1}, \dots, \xi_m) - P_i(\xi_1, \xi_2, \dots, \xi_m))]^2$. The differential equation is then obtained by putting

$$d\xi_i/dt = b_i(\xi) - \langle v_i, b_i(\xi) \rangle_i \xi_i \quad (*),$$

where $v_i \in \mathfrak{R}^{n_i+1}$ is the vector with all coordinates equal to 1 and \langle, \rangle_i is the internal product in this space. We leave to the reader to do the proof of three facts: 1) with an initial condition in $\prod_j S(n_j)$ the system $(*)$ yields a solution path $\omega(t)$ that never leaves this set; 2) the singularities of this dynamical system coincide with the Nash equilibria of the game; 3) if $\xi(t)$ converges to ξ^* when $t \rightarrow \infty$, then ξ^* is an equilibrium.

We finally come to the result of this section.

Theorem B: Let $\xi(t)$ be solution to $(*)$ with initial condition ξ_0 . Suppose that $b_i(\xi(t))$

converges, then it converges to the zero vector.

Proof: Let b be the unique element of $b(\omega(\xi_0))$ and let $\xi_1 \in \omega(\xi_0)$. From a classical result of Dynamical Systems Theory we have that the solution to (*) with initial condition ξ_1 is contained in $\omega(x_0)$, this implies that this path obeys

$$d\xi_i/dt = b_i - \langle v_i, b_i \rangle \xi_i.$$

This implies that $\omega(\xi_1)$ consists of a single point, namely the solution ξ^* to the equation

$b_i - \langle v_i, b_i \rangle \xi_i = 0$. Suppose some $b_i \neq 0$, then $\xi_i^* = b_i / \langle v_i, b_i \rangle$, since b is a continuous

function and $\omega(\xi_0) \supset \omega(\xi_1)$ we have $\xi_i^* = b_i(\xi^*) / \langle v_i, b_i(\xi^*) \rangle$. But this implies that $\xi_i^{*,j}$

is greater than zero if and only if $b_i^j(\xi^*) > 0$ and this gives that $\langle \xi_i, \xi_i \rangle_i =$

$$(\langle v_i, b_i(\xi_i) \rangle_i)^{-1} \cdot \sum_j (P_i(\xi_1, \xi_2, \dots, \xi_{i-1}, \sigma_i^j, \xi_{i+1}, \dots, \xi_m) - P_i(\xi_1, \xi_2, \dots, \xi_m))^2 \xi_i^j = 0, a$$

contradiction since $\xi_i \in \Sigma(n_i)$. Hence all the $b_i(X^*)$ are equal to zero. QED

Corollary: The BN differential system converges only if for every i $\langle v_i, b_i(\xi(t)) \rangle_i$ converges to zero when t goes to infinity.

With this corollary we may check Shapley's example of non-convergence of the BN mechanism for a 3x3 bimatrix game with payoff matrices

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

A computer simulation of the BN differential system for this game yielded the following results when the initial condition for both players was (1,0,0).

T	$\langle v_1, b_1 \rangle_1$	$\langle v_2, b_2 \rangle_2$
10	0.145055	0.196715
20	0.098662	0.170563
30	0.093496	0.170637
40	0.139185	0.150601
50	0.151957	0.116355
60	0.168024	0.080938
70	0.162864	0.118401
80	0.134780	0.149684
90	0.098187	0.161441
100	0.090857	0.169296

Convergence is failing, when $\langle v_1, b_1 \rangle_1$ decreases $\langle v_2, b_2 \rangle_2$ increases and vice versa.

PROGRAM

(This program was used to make the table)

```

10 REM Brown-Von Neumann for NxN Bimatrix Games
20 REM Setting N
30 N=3
40 DIM A(3,3),B(3,3),X1(3),X2(3)
50 REM Inputting Payoff Matrices
60 OPEN "DATA1.DAT" FOR INPUT AS #1
70 FOR I=1 TO N
80 FOR J=1 TO N
90 READ #1, A(I,J);B(I,J)
100 NEXT J
110 NEXT I
120 REM Asking for initial condition
130 PRINT "INPUT INITIAL CONDITION"
140 FOR I=1 TO N
150 PRINT "X1(";I;") =":INPUT X1(I)
160 PRINT "X2(";I;") =":INPUT X2(I)
170 NEXT I
180 K=0
190 WHILE K<10000
200 FOR I=1 TO N
210 B1(I)=0
220 B2(I)=0
230 NEXT I
240 C1=0
250 C2=0
260 L=0
270 FOR I=1 TO N
280 FOR J=1 TO N
290 FOR K=1 TO N
300 C1=A(I,J)*X1(I)*X2(J) +C1
310 C2=B(I,J)*X1(I)*X2(J) +C2
320 NEXT K
330 NEXT J
340 NEXT I
350 FOR I=1 TO N
360 B1(I)= A(I,I)*X2(I) + B1(I)
370 B2(I)=B(I,I)*X1(I) + B2(I)
380 NEXT I
390 NEXT I
400 FOR I=1 TO N
410 B1(I)=B1(I)-C1
420 IF B1(I)<0 THEN B1(I)=0
430 B2(I)=B2(I)-C2
440 IF B2(I)<0 THEN B2(I)=0
450 NEXT I
460 FOR I=1 TO N

```



```
470 C1=C1 + B1(I)
480 C2=C2 + B2(I)
490 NEXT I
500 FOR I=1 TO N
510 X1(I)=X1(I)*(1-0.01*C1) + 0.01*B1(I)
520 X2(I)=X2(I)*(1-0.01*C2) + 0.01*B2(I)
530 NEXT I
540 L=K MOD 1000
550 IF L=0 THEN 600
560 L=0
570 K=K+1
580 WEND
590 END
600 PRINT K/100
610 PRINT "C1 =";C1;"  C2 =";C2
620 GOTO 560
```

Bibliography

Brown, G. W. and J. Von Neumann, "Differential Equations", Contributions to the Theory of Games, Annals of Mathematics Study No. 24, Princeton University Press, 1950.

Kuhn, H. W. , Lectures on the Theory of Games, Princeton University, 1952.

Nash, J., "Non-Cooperative Games", Annals of Mathematics, vol. 54, No. 2, 1951.

Miyasawa, K., "On the Convergence of the Learning Process in a 2x2 Non-Zero-Sum Two-Person Game", Econometric Research Program, Research Memorandum No. 33, Department of Economics, Princeton University, 1961.

Robinson, J. , "An Iterative Method of Solving a Game", Annals of Mathematics, Vol. 54, pages 296-301, 1951.

ENSAIOS ECONÔMICOS DA EPGE

1. ANÁLISE COMPARATIVA DAS ALTERNATIVAS DE POLÍTICA COMERCIAL DE UM PAÍS EM PROCESSO DE INDUSTRIALIZAÇÃO - Edmar Bacha - 1970 (ESGOTADO)
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