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On Insurance Markets with Endogenous Information Acquisition: A Robust Mechanism Design Approach

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Área de Concentração: Teoria Econômica

Orientador: Carlos Eugênio Ellery Lustosa da Costa

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Resumo

Neste trabalho propomos a aplicação das noções de equilíbrio da recente literatura de desenho de mecanismo robusto com aquisição de informação endógena a um problema de divisão de risco entre dois agentes. Através deste exemplo somos capazes de motivar o uso desta noção de equilíbrio, assim como discutir os efeitos da introdução de uma restrição de participação que seja dependente da informação. A simplicidade do modelo nos permite caracterizar a possibilidade de implementar a alocação Pareto eficiente em termos do custo de aquisição da informação. Além disso, mostramos que a precisão da informação pode ter um efeito negativo sobre a implementação da alocação eficiente. Ao final, são dados dois exemplos específicos de situações nas quais este modelo se aplica.

PALAVRAS CHAVE: *Desenho de Mecanismos. Divisão de Riscos. Aquisição de Informação. Implementação Ex-Post.*

Abstract

We propose an application of the recent robust mechanism design literature within an information acquisition framework to a simple risk sharing example. Through this example we are able to motivate the equilibrium concept usage as well as discuss the introduction of informational dependent participation constraints and its non-standard effects into endogenous information gathering problems. The setup simplicity allows us to characterize Pareto optimality attainability and to find a negative effect of more precise information on implementability. Finally, as to show applicability, we provide specific examples in both macroeconomics and finance terms.
KEYWORDS: *Mechanism Design. Insurance. Information Acquisition. Ex-Post Implementation.*

Contents

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1 Introduction

Since Mirrlees (1971) and Rothschild and Stiglitz (1976) we have learned that both cooperative and competitive behavior may prevent optimal insurance under information frictions. Both adverse selection and moral hazard models have given us the intuition that full insurance prevents the principal from distinguishing bad from good behavior. On the other hand, the extension to which risk sharing may generate information frictions has not received comparable attention by the literature. Two exceptions are Hirshleifer (1971) and Grossman and Stiglitz (1980), as both have emphasized that competitive markets may not be able to generate informationally efficient outcomes. Our aim here is to address this question in a mechanism design framework.

We consider a stylized model of risk sharing between two agents in which each agent may gather information at a cost and then get a better assessment of his actual ex-post payoffs. With this model in hand, we discuss whether or not risk sharing allocations are able to provide incentives for socially optimal information acquisition. In order to make our point clear, we consider the extreme scenario in which information has no social value. Although one may think of a dozen situations in which information is socially useful, even this simple case has not been fully considered so far. Therefore, our problem reduces to consider the incentives which risk sharing generates over information acquisition *per se*. In this case efficiency is equivalent to no information gathering and full insurance. Hence the efficient allocation involves no information frictions (asymmetries).

We follow an approach very much similar to that of Bergemann and Valimaki (2002). However, as in our model agents problem is to trade their endowments in order to achieve better risk sharing, a participation decision is introduced. The rationality constraint introduction on the information gathering - robust mechanism design setup of Bergemann and Valimaki (2002) rises complexities which have not yet been fully considered by the theoretical literature. In fact, the outside option payoff itself may be informational dependent what may generate information value. Thus as a result it may imply inefficient

implementability[1]. Further, we follow Bergemann and Valimaki (2002) robustness equilibrium usage.

Our framework allows a simple characterization of first best implementability in terms of the information gathering cost and also of the signal informativeness. In fact we show that there is a threshold gathering cost level which defines if implementability is possible or not. As intuition suggests, for sufficiently high cost levels full insurance leads to socially optimal information acquisition. However, for low gathering costs, it is impossible to reconcile full insurance and the socially optimal information acquisition. We also consider the case in which only ex-post insurance[2] is provided. In turns out that for low cost levels it is necessary to acquire information in order to achieve ex-post insurance. Further we obtain two important results. First, a necessary condition for ex-post insurance implementability is symmetric information in equilibrium. Second, there are cases in which it is not possible to implement ex-post insurance at all.

Finally, we give two possible economic interpretations of our setup which exemplifies its applicability. The first has a macroeconomics flavor. There is uncertainty with respect to the abilities which will be most valuable in the technology to be used next period. Workers have heterogeneous two-dimensional abilities but in the aggregate both abilities endowments are the same. Finally agent may pool their resources into production or may live in autarky. What would be the effect on equilibrium consumption schedules of letting agents acquire information on the next period technology? We answer numerically this question and show that if we restrict ourselves to pure strategies mechanisms the gathering cost has a non-monotone effect on welfare. On the second application we consider the effect of inside information on an asset market. We interpret states of nature as public information to be released next period and we ask ourselves what is the willingness to pay of agent's for a risk free bond in the ex-ante period if they were at the first best ex-ante (partial equilibrium analysis). We get the numerical result that (for pure strategies

[1]This result shows another direction in which there are limits to Bergemann and Valimaki (2002) optimality theorems.

[2]In the sense that the insured level may vary with the acquired information realization.

equilibrium) these *virtual* interest rates are lower that the discount rate and further are not monotone in the gathering cost.

Review of the Literature

This work is mostly related to the literature on robust mechanism design an on endogenous information gathering in mechanism design. For a survey of both literatures see Bergemann and Valimaki (2005). In particular, we use the framework of Bergemann and Valimaki (2002) and we use an insightful result by Bergemann and Morris (2005) on the conditions under which ex-post implementability is robust. Jehiel, Meyer ter Vehn, Moldovanu, and Zame (2006) contains an important result on the limits of ex-post implementability and also a very well written assessment of this equilibrium concept.

Our risk sharing setup is very much similar to that of Lewis and Sappington (1995). Contrary to our assumptions of imperfectly informative signals and perfectly correlated information and states, they consider information to be perfect[3] and states to be uncorrelated among agents. With imperfectly informative signals the planner is not able to know whether or not agents have reported the true state of the world even after the realization of all uncertainties. On the other hand, as we have a finite number of agents (as opposed to a continuum) risk sharing is definitely dependent on all agents information, so we analyze the easiest case, i.e., information is perfectly correlated among agents. Further, in section 4-B of their work, Lewis and Sappington consider the case of endogenous information acquisition. They show that if the gathering cost is sufficiently low, the government does not even find it optimal to offer partial insurance, this corroborates our inefficiency results for low gathering cost levels.

More generally, our work relates to the economics of information literature. This literature has shown that intuitions of a frictionless economy may be misleading into the analysis of economies with information frictions. One brilliant example is given by Hirshleifer (1971). Would agents be better off if they have had more information? The

[3]The agent observes the state itself, not a signal imperfectly correlated with it.

intuitive, but incorrect, answer is an affirmative. Hirshleifer shows by an insurance market example that all agents would be *ex-ante* better off by agreeing to remain ignorant. The intuition behind his results is that more information breaks down *ex-ante* risk sharing opportunities which are *ex-ante* valuable to the agents. The characterization results of our paper exploit this classical example and its counterintuitive property, which has been known as the *Hirshleifer Effect*.

As concerning the optimal risk sharing framework, we would cite three classical papers. Mirrlees (1971) has considered the trade-off between risk sharing and the induction of optimal effort level in a production economy. Our model is similar to his to the extent a hidden action may also prevent optimal risk sharing. Townsend (1988) has shown that optimal risk sharing implementability is highly dependent on the specifics of information asymmetry one assumes. Finally, Rothschild and Stiglitz (1976) has firstly shown that competitive insurance markets may be unable to provide optimal risk sharing if agents have superior information.

2 Model

In this section we begin to talk specifically about the environment and the equilibrium concept we chose. First we define a stylized model of insurance among two agents and the equilibrium concept used. Then we discuss on the equilibrium concept used and on the relevance of the participation constraint for the value of information.

2.1 Environment

Let $I = \{i_1, i_2\}$ be the set of agents and $S = \{s_1, s_2\}$ be the set of possible states of nature. There is one consumption good and agents are expected utility maximizers with a continuous, strictly concave and strictly increasing bernoulli utility function $u : \mathbb{R}_+ \to \mathbb{R}$ (homogeneous across agents). All uncertainty is resolved with respect to the probability space $(\Omega, \mathcal{A}, \mathbb{P})$.

At an utility cost $c > 0$ each agent may acquire the right to know the realization of a measurable function $\pi : \Omega \to \Pi_0$, which we call information. We assume that $\Pi_0 = \{\pi_1, \pi_2\} \subset (0,1)$; $\pi_1 = 1 - \pi_2 > 1/2$ and $\mathbb{P}(\{\pi = \pi_1\}) = \lambda = 1/2$. States of nature are defined by a variable $s : \Omega \to S$ such that $\mathbb{P}(\{s = s_1\}|\pi) = \pi$. Moreover, we define $p = \lambda\pi_1 + (1-\lambda)\pi_2$ as the ex-ante probability of state s_1. If the agent has not acquired the signal then p is all information he has from the signal. We define $\Pi = \Pi_0 \cup \{p\}$ as the ex-post informational type space. The ex-post informational type of the agent is given by π if he gathered information or p otherwise. We formalize the information gathering decision of agent i as a binary variable $a^i \in \{0,1\} = A$ such that $a^i = 1$ if the agent acquires information and $a^i = 0$ otherwise.

Moreover, each agent has an endowment $e^i = (e^i_1, e^i_2) \in \mathbb{R}^S_+$. We assume that $e^{i_1} = (1+\delta, 1-\delta)$ and $e^{i_2} = (1-\delta, 1+\delta)$ with $\delta \in (0,1)$. Hence, $e^{i_1} + e^{i_2} = (2,2)$ and there is no aggregate risk. It is important to note that the only variables which are private information are the decision to observe the signal and the value of π itself.

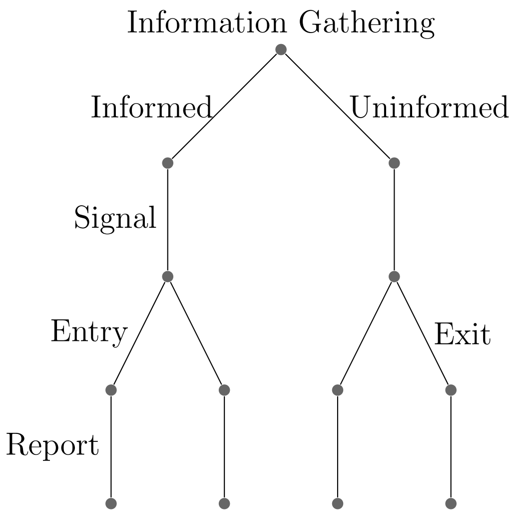

Figure 1: Model Tree

On Figure 1 we may understand the timing of the model. At stage zero the agent decides whether to gather information or not. If he does not acquire information, there is nothing to do at stage 2 and he goes to stage 3. If he acquired information, he observes the realization of π and then goes to stage 3. On stage three, which we also call reporting

stage, agents choose whether to continue and accept the planner's offer or to leave with the endowment. If the agent decided to exit he does nothing at stage four and goes to stage five when the state of nature happens and he finally consumes his endowment. If he decided to participate, he gives up his rights on the endowment and exchanges it with the other agent at stage four according to the planner's offer. Right after the state of nature is observed at stage five, agents consume their resulting state contingent consumption bundles. We assume that before stage zero, the planner is able to send a message for each agent explaining the mechanism that is going to be played next (there is commitment by the planner). Further, the planner's offer is common knowledge.

2.2 Equilibrium

Now we are ready to define social choice and equilibrium. One should note the dynamic nature of our economy. The decision to acquire information or not is relevant on the determination of possible consumption schedules on final stages. Indeed, if none has acquired information, consumption schedules cannot vary with information because it is not available. Hence, we define a social choice function as determining both consumption schedules and information acquisition. Define $\pi : A \times \Omega \to \Pi$ such that $\pi(1, \omega) = \pi(\omega)$ and $\pi(0, \omega) = p$.

Definition 1 (Social Choice) *A* social choice *is a pair of mappings* (f_a, f_x) *such that* $f_a : I \to A$ *is the social choice informational function and the social allocative function is* $f_x : I \times \mathbf{\Pi}_{f_a} \to \mathbb{R}^S_+$*, where* $\sum_{i \in I} f^i_x = \sum_{i \in I} e^i$ *and*

$$\mathbf{\Pi}_{f_a} = \{(\pi^i)_{i \in I} \in \mathbf{\Pi} : \exists\, \omega \in \Omega \text{ such that } \forall\, i \in I\,, \pi^i = \pi(f^i_a, \omega)\}$$

Two remarks need to be done at this point. The first is that although in essence both are very similar, our social choice definition departs from Bergemann and Valimaki (2002). Their *social choice correspondence* does not depend on the information decision stage. As a matter of fact, they assume that all possible information decisions lead to

the same set of possible "signals" (here the π's). Hence, no matter what information you decide to gather there is a full support on all possible signals. Our desire to have an uninformed agent leads us to the no full support case. Therefore, our constraint on the domain of the allocative function is just a natural extension of Bergemann and Valimaki (2002) ideas to this case. The second remark is that, as we will further see in section 2.3, the restriction to social choice allocative functions instead of correspondences allow us to rely on Bergemann and Morris (2005) results and interpret the problem as one of robust mechanism design.

For ease of notation we define a function $v^i : \mathbb{R}_+^S \times \mathbf{A} \times \Pi \to \mathbb{R}$ which gives expected utilities for a state contingent consumption bundle and an aggregate available information. Given the information structure $\mathbf{a} \in \mathbf{A}$ and a realization of the signal $\pi \in \Pi$, we define

$$v(x, \mathbf{a}, \pi) = \begin{cases} \mathbb{E}^{\mathbb{P}}[u(x(s))] & if\ \mathbf{a} = \mathbf{0} \\ \mathbb{E}^{\mathbb{P}}[u(x(s))|\pi] & otherwise \end{cases}$$

for all $x \in \mathbb{R}_+^S$. This function is the relevant objective function of the agent at the time of deciding whether or not to report truthfully if he were to know all available information in the economy.

Definition 2 (Ex-Post Incentive Compatibility) *A social choice allocative function f_x respective to a social choice (f_a, f_x) is said* ex-post incentive compatible *if for all $i \in I$ and all $\boldsymbol{\pi} \in \boldsymbol{\Pi}_{f_a}$*

$$v(f_x^i(\boldsymbol{\pi}), f_a, \pi^i) \geq v(f_x^i(\tilde{\pi}^i, \pi^{-i}), f_a, \pi^i)$$

for all $(\tilde{\pi}, \pi^{-i}) \in \boldsymbol{\Pi}_{f_a}$.

In other words, it is optimal for an agent to reveal its type if others response is truth-telling, no matter what their types are. We restrict reports to the set of available information given f_a. Thus implicitly we assume that the planner is able to severely punish people who report contradictory information or whose report reveals deviation in

information acquisition. Further details on this equilibrium concept will be discussed in section 2.3.

Definition 3 (Ex-Post Individual Rationality) *A social choice* (f_a, f_x) *is said* ex-post individually rational *if for all* $i \in I$

$$v(f_x^i(\boldsymbol{\pi}), f_a, \pi) \geq v(e^i, f_a, \pi)$$

for all $\boldsymbol{\pi} \in \boldsymbol{\Pi}_{f_a}$.

We define individual rationality with the implicit assumption that information gathering and transmission have already happened at the time of the participation decision. Note that, as we emphasized before, the outside option value is information dependent. We call a social choice *ex-post implementable* if it is both ex-post incentive compatible and ex-post individually rational.

In order to consider first stage implementability we must formally define the strategy space of agents. Note that the "single stage - single agent" deviation theorem does not hold in our setup as the strategy set on the reporting stage varies with first stage playing. In fact, the set of possible available information on the reporting stage may change as a result of agent's information gathering decisions. For this reason we should consider all reporting stage strategies when defining first stage implementability. The *set of viable*[4] *strategies* at the ex-post stage by agent $i \in I$, given (f_a, a^i), is defined by $F^i(f_a, a^i)$ such that $(g_1, g_2) \in F^i(f_a, a^i)$ if $g_1 : \Pi \to \Pi$ and $g_2 : \Pi \to \{E, NE\}$ where g_2 is the participation decision and g_1 is a consistent informational type report, i.e., such that

$$(g_1(\pi(a^i, \omega)), (\pi(f_a^{-i}, \omega))_{-i}) \in \Pi_{f_a}$$

for all $\omega \in \Omega$ such that $g_2(\pi(a^i, \omega)) = E$.

[4]In the sense the planner will not actually note a possible deviation.

Definition 4 (Ex-Ante Implementability) *A social choice* (f_x, f_a) *is called* ex-ante implementable *(in pure strategies) if for all* $i \in I$

$$\mathbb{E}^{\mathbb{P}}[v(f_x^i(\pi(f_a^i), \pi^{-i}), f_a, \pi)] - c\mathbf{1}_{[f_a^i=1]} \geq$$
$$\mathbb{E}^{\mathbb{P}}[v(\mathbf{1}_{[g_2(\pi(a'))=E]} f_x^i(g_1(\pi(a')), \pi^{-i}) + \mathbf{1}_{[g_2(\pi(a'))=NE]} e^i, (a', f_a^{-i}), \pi)] - c\mathbf{1}_{[a'=1]}$$

for all $a' \in A$ *and* $(g_1, g_2) \in F^i(f_a, a')$.

Finally a social choice is said *implementable* if it is both ex-ante implementable and ex-post implementable. The implementability problem has both a moral hazard aspect on the decision to acquire information (ex-ante implementability) as well as an adverse selection flavor on the reporting stage (ex-post implementability). Hence we may apply Myerson (1982)'s Revelation Principle for generalized principal-agent problems to this multistage game as proposed in Myerson (1986)[5]. Thus the restriction to truth-telling mechanisms is without loss of generality.

Therefore we consider all mechanisms in which there is ex-post equilibrium at the last stage for every possible information acquisition schedule at the first stage; and, that given these ex-post equilibrium, there is a Nash equilibrium at the first stage. The equilibrium concept we adopt is basically the same of Bergemann and Valimaki (2002). The nonstandard point on their definition, and consequently ours, is the use of ex-post equilibrium at the allocative stage. In the next section we argue that this concept allows what we call non-strategic information transmission.

2.3 Ex-Post Equilibrium

Consider a general game and assume that agents already have information available for themselves about their types at the initial game stage. Define a *payoff environment*[6] $\Gamma = \{I, Y, \boldsymbol{\Theta}, (v^i)_{i \in I}\}$. Where I is the set of agents, Y is a social possibilities set, $\boldsymbol{\Theta} =$

[5] Once you restrict his analysis to ex-post equilibrium at the last stage.
[6] Following Bergemann and Morris (2005) definition.

$\times_{i\in I}\Theta^i$ is the aggregate type set; and $v^i : Y \times \boldsymbol{\Theta} \to \mathbb{R}$ is agent's i payoff function. On our environment we have $\Theta^i = \Pi$, $Y = \{(x^i)_{i\in I} \in (\mathbb{R}^S_+)^I |\sum_{i=1}^2 \frac{1}{2}x^i(s) = 1, \forall\ s \in S\}$ and $v^i(y,\theta) = v(y,\mathbf{a},\theta)$ forall $y \in Y$. A social choice function $f : \boldsymbol{\Theta} \to Y$ is said *ex-post incentive compatible*[7] if for all $i \in I$

$$v^i(f(\theta),\theta) \geq v^i(f(\bar{\theta}^i,\theta^{-i}),\theta) \tag{1}$$

for all $\theta, (\bar{\theta}^i,\theta^{-i}) \in \boldsymbol{\Theta}$. Each agent truth-telling strategy is optimal given that other agents are telling the truth, no matter what their types turn out to be. Although ex-post incentive compatibility is weaker than dominant strategies incentive compatibility[8], both definitions turn out to be equivalent under private values[9]. Indeed, under this assumption and with a notational abuse, equation (1) turns into

$$v^i(f(\theta),\theta^i) \geq v^i(f(\bar{\theta}^i,\theta^{-i}),\theta^i)$$

for all $\theta^{-i} \in \boldsymbol{\Theta}^{-i}$, i.e., dominant strategies incentive compatibility. Jehiel, Meyer ter Vehn, Moldovanu, and Zame (2006) argues that ex-post incentive compatibility is an extension of dominant strategies implementation for common values environments. On these environments agent's allocation payoff ordering is likely to change with other people's types. Hence domination in the usual sense may constraint exactly the most fruitful characteristic of common values. A natural (an easy) solution is to weaken the dominance criterion. That is exactly what ex-post implementation does.

Whenever you think about information asymmetries, whether information may or may not be transmitted is a significant issue. Addressing this point, Bergemann and Morris

[7] Although we consider only ex-post incentive compatibility in this digression, the robustness properties trivially extend to ex-post individual rationality.

[8] In the sense that the last implies the former.

[9] We have private values if and only if, for all $i \in I$, there is a function $\bar{v}^i : Y \times \Theta^i \to \mathbb{R}$ such that

$$v^i(y,\theta) = \bar{v}^i(y,\theta^i)$$

for all $y \in Y$ and $\theta = (\theta^i,\theta^{-i}) \in \Theta$.

(2005) shows that whenever you consider a social choice *function*, ex-post implementability is equivalent to bayesian incentive compatibility on all type spaces[10] (consistent with the payoff environment). But this implies that all possible beliefs are taken into account by the designer, what implies that belief changes due to information transmission (in any degree) are taken into account. Although it is not intuitive, in fact their result has a straightforward proof. If you take expectations over (1), it is easy to see that the inequality is maintained for every distribution you use over θ^{-i} (given θ^{i}). Hence it follows that ex-post incentive compatibility implies Bayesian incentive compatibility for any belief you may fix. On the other hand, assume that you have Bayesian implementability on all type spaces. In particular you have implementability on all spaces with degenerate beliefs over θ_{-i}, but this is equivalent to ex-post incentive compatibility once you note that the social choice cannot depend on variables other than the payoff types (that is where the "function" assumption is useful).

However, there are important restrictions on the implications of Bergemann and Morris (2005) to information transmission on Bergemann and Valimaki (2002) and our setup. No strategic considerations are being made by agents on information transmission at the moment they decide whether to acquire information or not. A model which considers these strategic possibilities should have an explicit game of information transmission. Although we understand the limits of our study, we think it is only a basic model to build upon in the future.

Finally, Jehiel, Meyer ter Vehn, Moldovanu, and Zame (2006) shows that ex-post incentive compatibility is generally[11] impossible to obtain as long as you consider a model with two or more agents; multidimensional signals and quasi-linear preferences. It turns out that only constant social choice functions are generically implementable. Although, this result weakens the appeal of ex-post equilibrium implementability, it does not restrict implementability of constant efficient social choice functions which are our primary object

[10] A type space is a game defined over the payoff environment such that agents have some fixed arbitrary belief about other agents types/higher order beliefs. For a throughout discussion see Bergemann and Morris (2005).

[11] Almost everywhere on a measure space of utility functions.

of study here[12]. Therefore, the impact of this impossibility result on the present paper is limited to the analysis of non ex-ante efficient social choices. However, as the signals are in a finite set Jehiel, Meyer ter Vehn, Moldovanu, and Zame (2006)'s result is not an issue.

2.4 Information Acquisition and Participation

The primary idea in Bergemann and Valimaki (2002) is that information gathering is individually costly but may be socially useful. A particular agent has available a bundle of informational services from which to choose from. Afterwards, he will be able to condition its decisions on the provided signal. So he chooses the service which makes him better off (in expected terms) on the mechanism to be played next, taking into account its present cost. From the mechanism designer point of view, consideration not only of allocative efficiency but also of people's incentives to acquire information optimally is a central problem.

Although the model inherits the main characteristics already in Bergemann and Valimaki (2002), there is one key difference as we allow for outside opportunities. In order to understand the non-triviality of such difference one should first take a look at the agent's problem. Whether considering information acquisition, agents wonder if being able to report information would make them better off in the proposed mechanism. As usual in moral hazard models, information gathering would be induced by letting agents face uncertainty. But, the outside option creates a new source of value for information. In fact, the agent may condition it's participation decision on the piece of information he receives. Therefore, the disparity between the mechanism given payoff and the outside option payoff for differing informational signals turns out to be central in the determination of information demand.

As an example, consider an agent whom is offered a mechanism which provides him

[12] In the conclusion we discuss the case with aggregate risk, i.e., in which Pareto optimal allocations are not constant.

insurance, i.e., information is not relevant for the mechanism given payoff. Without a participation decision, information does not change his payoff on the mechanism to be played next. Hence the agent does not gather costly information. Now, consider the agent has an outside option. If for some signal realization it is better for the agent to stay out, then it may be optimal for him to acquire information if the cost is sufficiently low. In fact, based on the signal he may take a wiser participation decision.

Finally, this source of information value is not considered on Bergemann and Valimaki (2002), but we think it deserves further theoretical analysis. In fact, participation constraints are a feature of some major applications and its full consideration may ease the usage of the (robust version) extension of the basic mechanism design framework.

3 Efficiency

In order to study the allocative properties of this class of economies with the equilibrium concept considered herein, we define efficiency as the natural extension of Bergemann and Valimaki (2002) *ex-ante efficiency* for social choices, i.e., unconstrained Pareto efficiency.

Definition 5 (Pareto Ex-Ante Efficiency) *A social choice (f_a, f_x) is said ex-ante efficient if there is no social choice (g_a, g_x) such that for all $i \in I$*

$$\mathbb{E}^{\mathbb{P}}[v(g_x^i(\boldsymbol{\pi}), g_a, \pi^i)] - c\mathbf{1}_{[g_a^i=1]} \geq \mathbb{E}^{\mathbb{P}}[v(f_x^i(\boldsymbol{\pi}), f_a, \pi^i)] - c\mathbf{1}_{[f_a^i=1]}$$

with strictly inequality for some $k \in I$.

The intuition behind this definition is straightforward. If agents were to choose an enforceable social choice before the information acquisition stage, they would unanimously agree that it must satisfy above property. In our setup there is a simple characterization of ex-ante efficient social choices which we provide through next Lemma (See the Appendix for a proof).

Lemma 1 *A social choice (f_a, f_x) is ex-ante optimal if and only if*

(i) *$f_a = \mathbf{0}$;*

(ii) *f_x is constant in $s \in \mathcal{S}$.*

If a social choice function satisfies (ii) we call it *ex-post efficient*. In words we may say that an ex-ante efficient social choice is a full insurance scheme with no resources spending on information gathering. Therefore, information is socially valueless. Notwithstanding, the mere possibility of information acquisition may imply that private incentives cannot be dealt with without information gathering. Indeed, as we will see below, this sort of inefficiency may generate a positive information value.

3.1 Implementability of Efficient Social Choices

In this section we give necessary and sufficient conditions for implementability of ex-ante efficient social choices. By Lemma 1 the problem reduces to create incentives for no information gathering and at the same time induce participation. There are two cases in which we would be able to do so. First, we do if there is no *Hirshleifer Effect*, i.e., insurance is optimal for all agents; no matter what the information is. Second, we do if the cost of information acquisition is so high that even for an agent who is worse off with insurance it does not worth off to gather information.

For ease of notation we use certainty equivalents instead of state-contingent endowments. As usual, the certainty equivalent of the endowment, $CE^i(\pi)$, is

$$u(CE^i(\pi)) = \mathbb{E}[u(e^i(s))|\pi]$$

for all $\pi \in \Pi$ and all $i \in I$. Next lemma says that if it were possible to give agents their maximal endowments certainty equivalents they would accept the mechanism proposed.

Lemma 2 (No Hirshleifer Effect) *Assume that*

$$1 \geq \sum_{i=1}^{2} \frac{1}{2} \max_{\pi \in \Pi_0} \{CE^i(\pi)\} \tag{2}$$

Then there is an implementable ex-ante efficient social choice.

The intuition behind the result is that information has no value if the planner is able to give agents their maximum outside option payoffs. But this implies valueless information, and hence no information acquisition which is an important element of ex-ante efficient allocations as we see from Lemma 1. The other element, constant consumption across states of nature, may naturally be satisfied under this assumption.

Now it is straightforward to define the key variable in the characterization of all ex-ante efficient implementable social choices: the minimum insurance level such that the agent should be worse off if he acquires information. For agent $i \in I$, MIL^i is defined by

$$MIL^i = \min_x \{x : x \geq CE^i(p) \text{ and } c^i \geq \mathbb{E}[\max\{0, u^i(CE^i(\pi)) - u(x)\}]\}$$

The definition makes clear the relation of insurance implementability and the cost of information. The higher the gathering cost, the lower is going to be the minimum insurance level of a given agent. Since $c^i \geq 0$, we obviously have $\max_{\pi \in \Pi_0}\{CE^i(\pi)\} \geq MIL^i$, what makes last Lemma a corollary of the Proposition below.

Proposition 1 *There is an implementable ex-ante efficient social choice if and only if*

$$2 \geq \sum_{i=1}^{2} MIL^i$$

Hence we have a full characterization of economies in which the first best may be implemented. As for our environment the solution to MIL^{i_1} is given by[13]:

$$MIL^{i_1} = \begin{cases} CE^{i_1}(p) & \textit{if} \;\; c \geq \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(p))] \\ u^{-1}[u(CE^{i_1}(\pi_1)) - 2c] & \textit{otherwise} \end{cases}$$

Therefore $MIL^{i_1}(c)$ is strictly decreasing up to $c = \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(p))]$ and

[13]Once you note that $CE^{i_1}(\pi_1) > CE^{i_1}(p) > CE^{i_1}(\pi_2)$ the solution is trivially obtained.

constant afterwards. Symmetry guarantees that $MIL^{i_1} = MIL^{i_2}$, hence using all our assumptions we have[14]:

Corollary 1 *There is an implementable ex-ante efficient social choice if and only if (i) or (ii) holds where*

(i) $c \geq \frac{1}{2}\left[\left(\pi_1 - \frac{1}{2}\right)u(1+\delta) + \left(\frac{1}{2} - \pi_1\right)u(1-\delta)\right]$

(ii) $c < \frac{1}{2}\left[\left(\pi_1 - \frac{1}{2}\right)u(1+\delta) + \left(\frac{1}{2} - \pi_1\right)u(1-\delta)\right]$ *and*
$1 \geq u^{-1}[\pi_1 u(1+\delta) + (1-\pi_1)u(1-\delta) - 2c]$

Define $c^\star$ as the minimum gathering cost level such that the first-best is implementable for a given vector of the other parameters. It is easy to see that

$$1 = u^{-1}[\pi_1 u(1+\delta) + (1-\pi_1)u(1-\delta) - 2c^\star]$$

thus

$$c^\star = \frac{1}{2}[\pi_1 u(1+\delta) + (1-\pi_1)u(1-\delta) - u(1)] \tag{3}$$

which is exactly the value of information given by the agent's outside option. Therefore implementability of the first best is equivalent to the requirement that the gathering cost be higher than the agent's valuation of information as a participation decision trigger.

With Equation (3) in hand we may ask ourselves how this threshold changes as we vary the informativeness of the signal (an increase in π_1). For $\pi_1 = 1$ we have $c^\star = \frac{1}{2}[u(1+\delta) - u(1)] > 0$ and for $\pi_1 = 1/2$ we have $c^\star = \frac{1}{2}[u(CE^{i_1}(p)) - u(1)] < 0$, hence informativeness has a relevant effect on first best implementability. As for the marginal effect, it also turns out that we have an increase in outside option value of information, i.e., $c^\star$. In fact,

$$\frac{\partial c^\star}{\partial \pi_1} = \frac{1}{2}[u(1+\delta) - u(1-\delta)] > 0$$

If $CE^{i_1}(\pi_1) \leq 1$, there is no Hirshleifer effect and the first best is implementable as we may

[14] Note that $1 > CE^i(p)$ as u is strictly concave and $\delta > 0$.

see from $c^{\star} \leq 0$. But whenever $CE^{i_1}(\pi_1) > 1$, an increase in the signal informativeness leads to a diminution of first best implementability (*ceteris paribus*).

In the next section we will obtain a similar threshold level for ex-post efficient allocations. Furthermore we will be able to compare these particular levels and interpret the set among them as a very special case in which both types of efficiency are not implementable.

3.2 Ex-Post Efficiency and Information Gathering

On this section we analyze implementability of ex-post efficient social choices and the contract properties whenever it is indeed possible to achieve such social choices as incentive compatible and individually rational mechanisms. As in Bergemann and Valimaki (2002) our aim is to evaluate how well these mechanisms perform in terms of information acquisition incentives. We have already considered the case in which information acquisition is efficient, i.e., when there is no gathering. Here we consider the case in which information is acquired. In our current framework there are only two possibilities: one agent acquires information or two agents acquire information. In the next section we introduce mixed strategies and consider a third case. We begin our analysis by the case in which only one agent acquires information.

Lemma 3 *There is no ex-post efficient social choice which is implementable and in which only one agent acquires information (with positive probability).*

Proof: Assume by way of contradiction that there is such social choice. Risk aversion implies that all agents should be ex-post insured, i.e., insured conditional on π. But then the informed agent should be uniformly insured, otherwise this social choice would not be ex-post incentive compatible. However, if this is the case, he has no incentives to acquire costly information. In fact he may not acquire information, report any $\pi \in \Pi_0$ and be strictly better off since $c > 0$. ■

As a standard moral hazard inefficiency result, information acquisition by a single agent must involve a loss in risk sharing opportunities. Note that the result is pretty general as its proof do not relies on particularities of our environment and/or of our equilibrium concept[15], but only on risk aversion and $c > 0$. The analysis of information acquisition by two agents is more fruitful as we will see below.

Proposition 2 *There is an implementable ex-post efficient social choice in which two agents acquire information if and only if one of the following*

(i) $c \le \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$ *and* $1 \ge \frac{1}{2}CE^i_1(\pi_1) + \frac{1}{2}u^{-1}(u(CE^i_1(\pi_2)) + 2c)$

(ii) $c > \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$ *and* $1 \ge u^{-1}\left(c + \frac{1}{2}[u(CE^{i_1}(\pi_1)) + u(CE^{i_1}(\pi_2))]\right)$

holds.

See the Appendix for a proof. In order to understand implementability in this *double* information acquisition case, we are going to show that case (ii) only happens when there is no Hirshleifer Effect and therefore an ex-ante efficient allocation is implementable. In fact (ii) implies

$$1 > u^{-1}(u(CE^{i_1}(\pi_1))) = CE^{i_1}(\pi_1) = \max_{\pi \in \Pi_0} CE^i(\pi) \;\; \forall\, i \in I$$

Therefore, whether considering economies in which there is no implementable ex-ante efficient social choice, it suffices to consider case (i). A null cost of information acquisition implies implementability. In fact u's concavity implies that (i) in Proposition 2 holds for $c = 0$. Since u is strictly increasing, there is an interval $[0, c_{max}]$ such that (i) holds. But from (i) we get

$$c_{max} = \frac{1}{2}[u(2 - CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$$

One could question what the relation between $c^\star$ and c_{max} is, the answer is that such relation strongly depends on the parameters of the problem. In fact, for $\pi_1 = 1$ we have

[15]If you instead use bayesian implementation the result holds.

$c_{max} = 0 < c^\star$ and for $\pi_1 = 1/2$ we have $c_{max} = \frac{1}{2}[u(2 - CE^{i_1}(p)) - u(CE^{i_1}(p)) > 0 > c^\star$. Note that in case $c_{max} \geq c^\star$ it is always possible to implement an ex-post efficient social choice. Therefore, without further restrictive assumptions, it is not possible to guarantee that there is an implementable ex-post efficient social choice for each positive gathering cost.

The marginal effect of signal informativeness on the threshold c_{max} may be assessed by taking the derivative of c_{max} with respect to π_1:

$$\frac{\partial c_{max}}{\partial \pi_1} = \frac{[u(1+\delta) - u(1-\delta)]}{2} \left\{ 1 - \frac{u'(2 - CE^{i_1}(\pi_1))}{u'(CE^{i_1}(\pi_1))} \right\}$$

Hence $\frac{\partial c_{max}}{\partial \pi_1} < 0 \Leftrightarrow CE^{i_1}(\pi_1) > 1$, i.e., whenever there is the Hirshleifer effect an increase in π_1 reduces c_{max}. Now consider again the set $(c_{max}, c^\star)$. In the analysis of $c^\star$, we have shown that $\frac{\partial c^\star}{\partial \pi_1} > 0$. Therefore, together with the possibility that $c^\star > c_{max}$, it leads us to the conclusion that a marginally more informative signal reduces the possibilities of implementing an ex-post efficient social choice in the presence of the Hirshleifer effect.

Proposition 3 *If $CE^{i_1}(\pi_1) > 1$ then*

$$\frac{\partial [c^\star - c_{max}]}{\partial \pi_1} > 0$$

i.e., an increase in the signal informativeness leads to an enlargement of the interval $(c_{max}, c^\star)$ in which it is not possible to implement an ex-post efficient allocation whenever we have $c^\star > c_{max}$.

The intuition is that under the Hirshleifer effect, an increase in the signal informativeness leads to strictly better exit decisions for deviating agents. Note that this results depends not solely on the robustness of our equilibrium concept (which implies that in fact in this region there is no ex-post efficient equilibrium as we will see in next section), but it also strongly depends on our introduction of the information dependent outside

option payoff[16].

3.3 Mixed Strategies On Information Acquisition and Ex-Post Efficiency

Up to now we have characterized necessary and sufficient conditions for ex-post efficiency implementability in pure strategies. In this section we go one step forward and characterize ex-post efficiency implementability in mixed strategies (on information acquisition). First we define a social choice within this context and then we define the extension of incentive compatibility to this new setup.

Definition 6 (Social Choice) *A* social choice *is a pair of mappings* (f_a, f_x) *such that* $f_a : I \to [0,1]$ *is the social choice vector of information gathering probabilities; and the social allocative function,* $f_x : I \times \mathbf{\Pi}_{f_a} \to \mathbb{R}^S_+$, *is such that* $\sum_{i\in I} f^i_x = \sum_{i\in I} e^i$.

The mechanism now is defined in terms of the probability of gathering information for each agent. In our equilibrium definition, we let ex-post incentive compatibility and ex-post individual rationality unchanged. However, incentive compatibility must be changed as in the mixed strategies case $f^i_a \neq a^i$ in general.

Definition 7 (Ex-Ante Implementability) *A social choice* (f_x, f_a) *is said* ex-ante implementable *if*

$$f^i_a \in \arg\max_{\mu\in[0,1]} \left\{ \max_{g\in F^i(f_a)} \left\{ \mathbb{E}^{\mathbb{P}\times\mu\times f_a^{-i}} [v(f^i_x(g(\pi(a^i)), \pi^{-i}), (a^i, a^{-i}), \pi) - c\mathbf{1}_{[a^i=1]}] \right\} \right\}$$

for all $i \in I$.

As we already stated in Lemma 3, it is not implementable for only one agent to acquire information with positive probability in an ex-post efficient mechanism. Again ex-post

[16] On one hand, the result also holds for every $\lambda = \mathcal{P}(\{\pi = \pi_1\}) \in [0,1]$. In fact, the higher λ, the larger is the interval. On the other hand, the hypothesis $\pi_1 = (1-\pi_2) > 1/2$ may be substituted by $\pi_1 > 1/2$, $\pi_1 > \pi_2$ and $\lambda \geq 1/2$.

incentive compatibility implies the agent is completely insured, thus it is not possible to induce him to take the desired effort (information gathering). Next proposition shows that this negative result indeed extends to all types of mixed strategies equilibrium.

Proposition 4 *Assume that a social choice f is ex-post efficient. Then information acquisition is in pure strategies.*

See the Appendix for a proof. The intuition of the proof is that ex-post efficiency in conjunction with ex-post incentive compatibility implies full insurance in the case of both agents using non-trivial mixed strategies. On the other hand, the case in which one agent acquires information with probability one and the other with positive, but lower than one, probability is obtained solely from ex-post incentive compatibility (Lemma 4 on the Appendix). Therefore, the result is a direct consequence of our robustness equilibrium criteria.

Before we go through the applications of our insurance model a remark need to be done. This last result in conjunction with the possibility that it may be not possible to implement an ex-post efficient social choice at all (see the section on Ex-Post Efficiecy) has a close relation to a classical impossibility result. When considering existence of Rational Expectations Equilibrium (REE), Grossman and Stiglitz (1980) has shown that with endogenous information acquisition, it may be the case that there is no REE for low (but strictly positive) information gathering cost levels. Their intuition is that REE may imply information disclosure and that it may lead agent's ex-ante decision to be not to gather information. But as this cost is very low, it may worth for one agent to acquire information if none does. Hence equilibrium fails.

In fact, fully revealing rational expectations equilibrium would be ex-post efficient in their setup. Therefore, it appears to be the case that fully revealing rational expectations equilibrium shares some properties with our robustness criteria on equilibrium definition.

4 Applications

This section has a two-fold objective. First, we give numerical examples which exemplify that it may be the case that there is no ex-post efficient implementable social choice. Second, and more importantly, we give two possible interpretations of our setup in terms of specific economic problems and moreover give a glance on the kind of questions the framework may deal with.

As for both applications we assume $\delta = 1/2$ (what implies $e^{i_1} = (3/2, 1/2)$ and $e^{i_2} = (1/2, 3/2)$) and $\pi_1 = 4/5$. Finally, the bernoulli utility function is assumed to be given by $u(x) = \sqrt{x}$. With these numbers we are able to compute both $c^\star$ and c_{max}. We have $c_{max} = 0.026$ and $c^\star = 0.061$, then for $c \in (0.026, 0.061)$ the social choice should be ex-post inefficient, as we indeed numerically obtain for the planner problem we consider. To show that this relation between c_{max} and $c^\star$ is very sensitive to parameter changes, consider $\pi_1 = 0.68$ and let all remaining parameters unchanged. Then, we obtain $c_{max} = 0.032 > 0.030 = c^\star$, i.e., in this second case there is an implementable ex-post efficient social choice for all $c \geq 0$.

We consider a planner who seeks to maximize the expected level of utilitarian welfare. Furthermore, we restrict the planner choice set to pure strategies implementable mechanisms[17]. The planner may solve the problem separately for the three possible types of information gathering (pure strategies) and then just pick up the optimal method.

4.1 Technological Uncertainty and Social Insurance

Here we apply our model to consider the effects of endogenous information acquisition on technological uncertainty and the resulting worker's optimal consumption schedule. The technological shock we consider is purely redistributive, i.e., it does not change aggregate production possibilities.

[17]With mixed strategies we were not able to guarantee the planner action set is compact. Therefore it may be the case the problem has no solution with mixed strategies. We tried to search numerically for a mixed strategy implementable social choice, but we were unable to find a feasible point.

Each worker is assumed to be endowed with a two dimensional vector of abilities. There are two technologies, one specific to each type of ability. In respect to the previous notation, a particular technology is represented by a state $s \in S$ and the agent's vector of abilities by $e^i = (e^i_{s_1}, e^i_{s_2})$. Each technology s is assumed to be one-to-one with respect to the amount of ability of type s used in the production process. Further, the possible technology distributions are defined by the probabilities $\pi \in \Pi$. Finally, workers maximize expected utility with bernoulli utility function u.

As an example you may think on a rural co-op. There are two possibilities for future technology. One includes intensive use of trucks and the other is more intensive on chemical fertilizers. All workers are able to drive the truck or to apply the fertilizers, but they present heterogeneous abilities in each task. Workers may buy a specialized magazine (from an external source) or exert effort themselves to learn which technology is more probable, but a priori they attribute a fifty-percent chance for each. If a worker has high productivity on using the truck and the truck-technology is more probable, he is in a better position to bargain for more consumption with other individuals.

The planner seeks to maximize ex-ante agent's welfare. He may offer workers a wage schedule and pool their resources into production. Workers may decide to accept this wage contract or not. If they don't, we assume they may decide to be self-employed and that in this case they are restricted to their own production possibilities. Table 1 shows the welfare loss in respect to first best in terms of certain consumption for the varied information gathering types and the gathering cost (bold values indicate the optimal gathering method). We express the cost in terms of first best utility level.

We can see that a marginal increase on the gathering cost has a negative effect on welfare for low cost levels (on the grid of points considered) and a positive welfare effect for high cost levels. This happens because optimal information gathering changes from double acquisition to no acquisition from low to high cost levels. We conclude that if a policymaker considers interventions which may marginally change the gathering cost, this policy should be dependent on the level of the gathering cost. We need to remark that

Cost	Information Gathering* (pure strategies)		
	Double	Single	No Acq
0.0%	**-1.68%**	-3.49%	-6.70%
0.5%	**-2.67%**	-4.11%	-5.29%
1.0%	**-3.65%**	-4.73%	-4.15%
2.0%	-5.61%	-5.99%	**-2.42%**
3.0%	Not Feasible	Not Feasible	**-1.27%**
4.0%	Not Feasible	Not Feasible	**-0.54%**
5.0%	Not Feasible	Not Feasible	**-0.13%**
6.0%	Not Feasible	Not Feasible	**0.00%**
7.0%	Not Feasible	Not Feasible	**0.00%**

* Bold values indicate optimal type of information gathering.

Table 1: Welfare loss (in terms of first best certain consumption) as a function of gathering cost and information gathering type.

it is true given we restrict ourselves to the case of pure strategies mechanisms (we were not able to prove whether or not it is true with the introduction of mixed strategies).

Cost	Agent i_1				Agent i_2			
	s_1, π_1	s_2, π_1	s_1, π_2	s_2, π_2	s_1, π_1	s_2, π_1	s_1, π_2	s_2, π_2
0.0%	1.257	1.257	0.743	0.743	0.743	0.743	1.257	1.257
0.5%	1.257	1.257	0.743	0.743	0.743	0.743	1.257	1.257
1.0%	1.257	1.257	0.743	0.743	0.743	0.743	1.257	1.257
2.0%	1.308	0.692	1.308	0.692	0.692	1.308	0.692	1.308
3.0%	1.224	0.776	1.224	0.776	0.776	1.224	0.776	1.224
4.0%	1.146	0.854	1.146	0.854	0.854	1.146	0.854	1.146
5.0%	1.073	0.927	1.073	0.927	0.927	1.073	0.927	1.073
6.0%	1.004	0.996	1.004	0.996	0.996	1.004	0.996	1.004
7.0%	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Table 2: Optimal contract consumption schedules (pure strategies)

In Table 2 we have computed optimal consumption schedules. We see that for a cost level less than 2% the social choice is ex-post efficient. Further, for costs between 2% and 7% the contract does not depend on information (as it is not acquired). Hence incomplete markets arise endogenously as a consequence of equilibrium. Finally, for a cost level of 7% an ex-ante efficient social choice is implementable.

4.2 Insider Trading and Asset Markets

As a second example, we propose a relabeling of variables in asset market terms. The state of nature is regarded as public information and the information signal is imperfect inside information an agent may acquire at a cost $c \geq 0$ about the public information. There is one risky asset which pays one consumption unit at s_1 and charges one consumption unit at s_2. Each agent is endowed with one consumption unit and an asset position of δ or $-\delta$ in a half-half proportion in the population.

Considering the same planner problem again, what would be the willingness to pay of agents to a risk-free asset? Does this shadow price changes with inside information cost? In order to compute these *virtual* prices, we compare marginal utilities to the first best. One interpretation is that before the ex-ante stage there is another consumption stage in which agents are at the first best. In fact we are doing a partial equilibrium analysis. In Table 3 we see the results. Again we need to remark that only pure strategies equilibrium is being accounted for.

Cost	Shadow Price
0,0%	1.026
0.5%	1.026
1.0%	1.026
2.0%	1.038
3.0%	1.020
4.0%	1.008
5.0%	1.002
6.0%	1.000
7.0%	1.000

Table 3: Implicit Risk Free Bond Price

Results may be understood as fractions of the discount factor, e.g., if β is the discount factor, 1.038β is the risk free bond shadow price at the ex-ante consumption stage for $c = 0.02$. We see that the gathering cost changes the interest rate in a negative direction. Therefore agents are willing to take an investment with a lower rate of interest than in the first best (in which the risk free bond price is β).

5 Conclusion

On the present paper we proposed the usage of the robust mechanism design framework for the analysis of insurance market problems with endogenous information acquisition. Indeed, robustness arises as a natural requirement in these problems as information may be transmitted among agents. Moreover, we show that the introduction of a participation constraint in the theoretical framework may be more challenging than anticipated as another sources of information value arises. The robustness criteria turned it possible to characterize both ex-ante and ex-post efficient social choices. Further, it has implied that a necessary condition for efficiency is that the planner is certain about who is informed. Finally we were able to give two different examples of the sort of economics questions which our model may help to answer.

Our analysis is limited in many dimensions. We considered an example in which the first best is represented by a constant social choice. As we said before, Jehiel, Meyer ter Vehn, Moldovanu, and Zame (2006) has shown that it is impossible to generally implement non-constant social choices if there are two or more agents and signals are multidimensional. If one stick to the assumption of a finite set of possible information signals, this is not an issue. The analysis of more general information structures would require a change in the equilibrium concept used, e.g., one could then use bayesian implementability. In this case a robustness idea could be obtained by building a formal model of information transmission.

In the applications we restricted ourselves to pure strategies on information acquisition. A full treatment of the planner problem (and resulting policymaking prescriptions) should also consider the usage of mixed strategies. Although these social choices are not ex-ante or ex-post efficient, when considering the trade-off between efficiency and the information gathering burden, it could be optimal for the planner to use one of them. The complexities (both numerical an analytical) which this case rises have proven to be of much higher level than we foresaw. Therefore we decided to leave it for further research.

With respect to further research we have a few proposals. On one hand, this work has given a snapshot into what we think may be an interesting theoretical research area: the properties of environments with participation constraints and endogenous information acquisition. In fact our intuition suggests that outside options may be an important source of value for information in the real world. On the other hand, the extension to an infinite horizon dynamic environment would be interesting. Indeed, it would tell us to which extent the inefficiencies which have risen here may vanish as a consequence of a repeated relationship.

Appendix

Proof of Lemma 1: If (i) and (ii) are satisfied, by concavity of u and resources feasibility of the social choice, an improvement for some agent would necessarily imply a loss for other agents (in terms of consumption or information acquisition). Now let's prove the converse by contradiction. Assume by way of contradiction that (g_a, g_x) is an ex-ante efficient social choice but (i) or (ii) are not valid. Define (f_a, f_x) as following

$$f_x^i(s, \boldsymbol{\pi}) = \sum_{\pi' \in \Pi_0} \sum_{s' \in S} \mathbb{P}[\pi(\omega) = \pi'] \mathbb{P}[S = s' | \pi'] g_x(s', \pi')$$

and

$$f_a^i = 0$$

Trivially, f is a social choice. By concavity of u and $c > 0$, we have that (f_a, f_x) is necessarily an ex-ante Pareto improvement over (g_a, g_x), a contradiction. ■

Proof of Lemma 2: We have $\max_{\pi \in \Pi_0} CE^i(\pi) \geq MIL^i$, hence the result follows from Proposition 1. ■

Proof of Proposition 1: If there is an implementable ex-ante efficient social choice,

it must satisfies the properties of Lemma 1. Then it must be the case that the insurance level given to both agents is higher that the respective minimum insurance level, i.e., it cannot be optimal for the agents to acquire information. Hence the inequality follows from consumption resources feasibility of a social choice.

On the other hand, if the inequality is satisfied it is possible to implement $(\mathbf{0}, (MIL^i + 1 - \sum_{i=1}^{2} \frac{1}{2} MIL^i)_{i \in I})$. Let's prove it. Since this social choice is constant, ex-post incentive compatibility is automatically satisfied. Moreover, the definition of MIL implies ex-post participation constraints are also satisfied. Hence it suffices to show that this social choice is ex-ante implementable. The only feasible deviation for the agent is to acquire information and condition its outside option decision on the signal received. However, this is exact what is avoided by giving the agent an insurance level above the MIL. ■

Proof of Proposition 2: An ex-post efficient social choice would be implementable if and only if agents given payoff under the mechanism is higher than the payoff they would obtain from not acquiring information and exiting the game. In fact this is the only possibly profitable deviation for an agent given the other is acquiring information and telling the truth; and given that participation constraints are satisfied for all information signals. It is trivial to note that a deviation does not worth off under (i) or (ii) with the natural social choice allocation g such that for $c \leq \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$

$$g(s, \pi_1) := g(\pi_1) = CE^{i_1}(\pi_1) \text{ and } g(\pi_2) := g(\pi_2) = u^{-1}[u(CE^{i_1}(\pi_2)) + 2c] \;\; \forall\, s \in S$$

and for $c > \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$ and $\forall\, s \in S$

$$g(s, \pi_1) := g(\pi_1) = u^{-1}\left(u(CE_1^i(\pi_1)) + c - \frac{u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))}{2}\right)$$
$$g(s, \pi_2) := g(\pi_2) = u^{-1}\left(u(CE_1^i(\pi_2)) + c + \frac{u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))}{2}\right)$$

As you may arbitrarily give extra resources to a fixed agent, this defines and imple-

mentable social choice (by (i) and (ii) it is indeed a social choice) which is ex-post efficient (it does not vary with s).

Now consider you have an ex-post efficient social choice f which is implementable. It is trivial that g satisfies the participation constraints and that it is incentive compatible with respect to the no information - no participation strategy. Then

$$\begin{aligned} \frac{1}{2}u(f_x^{i_1}(\pi_1,\pi_1)) + \frac{1}{2}u(f_x^{i_1}(\pi_2,\pi_2)) - c &\geq \frac{1}{2}u(CE^{i_1}(\pi_1)) + \frac{1}{2}u(CE^{i_1}(\pi_2)) \\ &= \frac{1}{2}u(g(\pi_1)) + \frac{1}{2}u(g(\pi_2)) - c \end{aligned} \tag{4}$$

Therefore it suffices to show that g spends feasible consumption resources. By concavity of u, we have

$$u(f_x^{i_1}(\pi_1,\pi_1)) - u(g(\pi_1)) \leq u'(g(\pi_1))[f_x^{i_1}(\pi_1,\pi_1) - g(\pi_1)] \tag{5}$$

$$u(f_x^{i_1}(\pi_2,\pi_2)) - u(g(\pi_2)) \leq u'(g(\pi_2))[f_x^{i_1}(\pi_2,\pi_2) - g(\pi_2)] \tag{6}$$

If $c \leq \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$, by u's strict concavity we have $u'(g(\pi_2)) > u'(g(\pi_1))$. This last inequality and the participation constraint ,$f_x^{i_1}(\pi_1,\pi_1) \geq CE^{i_1}(\pi_1) = g(\pi_1)$, implies

$$\frac{\left[u(f_x^{i_1}(\pi_1,\pi_1)) - u(g(\pi_1)) + u(f_x^{i_1}(\pi_2,\pi_2)) - u(g(\pi_2))\right]}{u'(g(\pi_2))} \leq f_x^{i_1}(\pi_1,\pi_1) - g(\pi_1) + f_x^{i_1}(\pi_2,\pi_2) - g(\pi_2)$$

equivalently

$$\frac{u(f_x^{i_1}(\pi_1,\pi_1)) - u(CE^{i_1}(\pi_1))}{u'(g(\pi_2))} + \frac{u(f_x^{i_1}(\pi_2,\pi_2)) - u(u^{-1}[u(CE^{i_1}(\pi_2)) + 2c])}{u'(g(\pi_2))} \leq$$
$$f_x^{i_1}(\pi_1,\pi_1) - g(\pi_1) + f_x^{i_1}(\pi_2,\pi_2) - g(\pi_2)$$

But, by inequality (4), then the left hand side is greater or equal to zero, so we have

$$g(\pi_1) + g(\pi_2) \leq f_x^{i_1}(\pi_1,\pi_1) + f_x^{i_1}(\pi_2,\pi_2)$$

Therefore, g is feasible.

If $c > \frac{1}{2}[u(CE^{i_1}(\pi_1)) - u(CE^{i_1}(\pi_2))]$, it is guaranteed that individual rationality is satisfied. Since now we have $g(\pi_1) = g(\pi_2)$, concavity of u and (4) imply

$$g(\pi_1) + g(\pi_2) \leq f_x^{i_1}(\pi_1, \pi_1) + f_x^{i_1}(\pi_2, \pi_2)$$

and g feasibility is implied by f feasibility. Therefore in any case g is implementable. If g does not uses all resources, we may give it all remaining resources to one agent and maintain ex-post efficiency. ■

Proof of Proposition 4

Before we go to the proof of the proposition itself, an intermediary result will be shown.

Lemma 4 *If (f_x, f_a) is incentive compatible and $f_a^{i_1} = 1$, then we cannot have $f_a^i \in (0,1)$ for some $i \neq i_1$.*

Proof: Assume by way of contradiction that $f_a^{i_2} \in (0,1)$. Then incentive compatibility implies:

$$\mathbb{E}^{\mathbb{P}\times\mu\times f_a^{-i_2}}[v(f_x^{i_2}(\pi(1), \pi^{-i}), (1, a^{-i_2}), \pi)] - c = \mathbb{E}^{\mathbb{P}\times\mu\times f_a^{-i_2}}[v(f_x^{i_2}(\pi(0), \pi^{-i_2}), (0, a^{-i_2}), \pi)]$$

hence by $c > 0$ we get

$$\mathbb{E}^{\mathbb{P}\times\mu\times f_a^{-i_2}}[v(f_x^{i_2}(\pi(1), \pi^{-i}), (1, a^{-i_2}), \pi)] > \mathbb{E}^{\mathbb{P}\times\mu\times f_a^{-i_2}}[v(f_x^{i_2}(\pi(0), \pi^{-i_2}), (0, a^{-i_2}), \pi)]$$

But ex-post incentive compatibility implies

$$\begin{aligned} v(f_x^{i_2}(\pi(0), \pi^{-i_2}), (0, a^{-i_2}), \pi) &\geq v(f_x^{i_2}(\pi(1), \pi^{-i}), (0, a^{-i_2}), \pi) \\ &= v(f_x^{i_2}(\pi(1), \pi^{-i}), (1, a^{-i_2}), \pi) \end{aligned}$$

for all π, since $a^{-i_2} \neq 0$, a contradiction with the strict inequality above. ■

Proof of Proposition 4: We have shown that only one agent acquiring information with positive probability is not feasible. Further, by Lemma 4 it suffices to consider the case in which each agent uses a non-trivial mixed strategy in equilibrium.

Since f is ex-post efficient, we may define $\bar{f}^i_x : \Pi^I \to \mathbb{R}_+$ such that $f_x(s, \boldsymbol{\pi}) = \bar{f}^i_x(\boldsymbol{\pi})$ for all $s \in S$. Then

$$v(f^i_x(\pi(a^i), \pi(a^{-i})), (a^i, a^{-i}), \pi) = u(\bar{f}^i_x(\pi(a^i), \pi(a^{-i})))$$

Assume f is implementable, $f^{i_1}_a \in (0,1)$ and $f^{-i}_a \neq 0$. Then by ex-post incentive compatibility we have

$$\bar{f}^{i_1}_x(\pi_1, p) = \bar{f}^{i_1}_x(\pi_2, p) = \bar{f}^{i_1}_x(p, p)$$

further ex-post incentive compatibility implies

$$\bar{f}^{i_1}_x(p, \pi_1) = \bar{f}^{i_1}_x(\pi_1, \pi_1) \quad \text{and} \quad \bar{f}^{i_1}_x(p, \pi_2) = \bar{f}^{i_1}_x(\pi_2, \pi_2)$$

But by i_2 ex-post incentive compatibility we have $\bar{f}^{i_2}_x(\pi_1, p) = \bar{f}^{i_2}_x(\pi_1, \pi_1)$. Hence feasibility implies

$$2 - \bar{f}^{i_1}_x(\pi_1, p) = 2 - \bar{f}^{i_1}_x(\pi_1, \pi_1)$$

Thus $\bar{f}^{i_1}_x(\pi_1, p) = \bar{f}^{i_1}_x(\pi_1, \pi_1) = \bar{f}^{i_1}_x(p, \pi_1)$. Analogously $\bar{f}^{i_1}_x(\pi_2, p) = \bar{f}^{i_1}_x(\pi_2, \pi_2) = \bar{f}^{i_1}_x(p, \pi_2)$. Finally we have

$$\bar{f}^{i_1}_x(\pi_1, p) = \bar{f}^{i_1}_x(\pi_2, p) = \bar{f}^{i_1}_x(p, p) = \bar{f}^{i_1}_x(\pi_1, \pi_1) = \bar{f}^{i_1}_x(p, \pi_1) = \bar{f}^{i_1}_x(\pi_2, \pi_2) = \bar{f}^{i_1}_x(p, \pi_2)$$

Therefore agent i_1 faces no uncertainty. Hence he may acquire information with zero probability and be strictly better off, a contradiction with f implementability. ■

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