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COMMON KNOWLEDGE

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1. Knowledge and Common Knowledge

The concepts of knowledge and belief are central in economics, and in many other sciences, whose object of study is the human being, like psychology, linguistics, and artificial intelligence. Let us assume that the meaning of knowledge about a fact, or about the truth of a proposition is well understood. We will sometimes also refer to the word belief. If one person believes something is true, this does not imply that this something is true. People are allowed to hold false beliefs.

Since the knowledge by one person of a fact is itself a fact, we may speak of the knowledge by an individual, say i, about knowledge of another individual, say j, about a fact. In general, we may repeat this process as many times as we want.

Hence, if we have several agents, we say that a fact is known up to level m by these agents if: everyone knows that (everyone knows that)$^{m-1}$ the fact is true. That is, the words "everyone knows that" appear side by side m times, before "the fact is true". In the same fashion we define common knowledge of a fact if: the fact is known up to level m for all

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$m > 1$. In other words, a fact is common knowledge if everyone knows it, everyone knows that everyone knows it, everyone knows that everyone knows that everyone knows it, ... ad infinitum.

A question immediately comes to mind when we see these definitions. If a fact is know by everyone, why should higher levels of knowledge matter?

In order to illustrate the importance of all levels of knowledge, let us refer to a well known puzzle. (For a more formal analysis of such puzzles, see Moses, Dolev and Halpern (1986), Halpern and Moses(1986), and Barwise(1981)).

Long time ago a king decided to give amnisty to a large group of prisoners. This prison is a very special one, and the prisoners are not allowed to speak to each other. To communicate his decision, the king summons the prisoners, and each one is put on a hat, which they are told they cannot see under any circunstance (the penalty for looking at the top their that is death). All but two hats have a white top. Those two hats are red topped. All the prisoners see the hats of the others, but not its own.

The king delivers the following speech: "As you noticed, most of you have a white hat. But some have a red hat. From now on every day you will be brought to this room. The day you find out the color of your hat you are free to go. If any of you guess the wrong color, this person is going to be behéaded."

How many days did it take to the prisoners with the red hat to find out the color of their hats? And the ones with white hat?

Let us reason day by day. On the first day, after the speech, the prisoners with white hat see two with red hat, and the prisoners with red hat see only one other with red hat. Hence there is nothing they can conclude about the color of their own hat.

As a result, in the second day every prisoner is back in the main room. Again, the ones with red hat see one red hat only, and the others two. But there is additional information that one day has passed. Hence, a prisoner with a red hat knows that the other prisoner with the red hat did not leave, which means that someone else must be wearing a red hat. This someone has to be himself, since he sees no one else except one with the red hat.

So the prisoners with red hat guess correctly on the second day. On the third day the others will guess, because they will see no one with red hats.

What happens if we had three red hats instead of two? The same reasoning apllies and the prisoners with red hat would leave on the third day and the others on the fourth. It is clear that this reasoning can go on for any number of red hats, provided the white hats are in majority.

This puzzle and its solution serve the purpose to illustrate that all layers of knowledge are important. In fact, in the case of two hats, a person with red hat sees at least one red hat, but does not know whether the other person with red hat sees at least one red hat. So, in the case of two hats, everyone sees a red hat, but it is not the case that everyone knows everyone sees a red hat.

In the case of three red hats, the person with a red hat sees two red hats, and knows that everyone sees at least one red hat. So that now everyone sees at least one red hat, and everyone knows everyone sees at least one red hat.

We could go inductively showing that all levels of knowledge of the fact "everyone sees a red hat" are attained exactly for versions of the puzzle with different numbers of red hats. As these puzzles have a different solution for different numbers of red hats, it is clear that all levels of knowledge are important in a general problem, in the sense that they affect the outcome.

We will see later, what are the effects of knowledge, higher orders of knowledge, and common knowledge of rationality in a game.

2. Historics

The notions of knowledge and belief have been discussed for a long time in philosophy. The idea of common knowledge (and of the importance of higher layers of knowledge) was first defined by Lewis(1966). In order to study social conventions, how they arise, how they are stable, Lewis uses coordination games and defines common knowledge.

Harsanyi(1967-68) independently, notices the importance for incomplete information games of guesses about the unknown parameters, guesses about guesses, and so on. His argument was roughly as follows: if what any player thinks about the unknown parameters influence how this player plays, then the same should occur with other players, so that any player should try to guess

what other players are guessing, and so on. Unfortunately, Harsanyi dismisses the problem as being mathematicaly very hard. This mathematical problem was solved by Böge(1974), Armbruster and Böge (1979), Böge and Eisele(1979) and Mertens and Zamir(1985). They developed the so-called model of infinite hierarchies of beliefs.

The first formal definition of common knowledge was given by Aumann(1976). Since the publication of his paper the field has had an enourmous developement. We will see this in the next sections.

3. <u>Formalising Common Knowledge</u>

There are several ways of formalising the notion of common knowledge. In all the cases we will be dealing with a basic set Ω of states of the world. Aumann(1976) considers given a priori $\Pi_1, \ldots, \Pi_n$ partitions of this set of states of the world, each of them representing the information structure of an agent. Implicitly the space Ω and the partitions $\Pi_1, \ldots, \Pi_n$, are to be common knowledge. Aumann's concern is solely with the definition of common knowledge of an event A Ω, when the true state of the world is an $\omega \in \Omega$. If $\Pi_1 \wedge \ldots \wedge \Pi_n$ is the meet of the partitions, i.e., the finest common coarsening, we say that <u>A is common knowledge at ω</u> if $\exists$ B $\in \Pi_1 \wedge \ldots \wedge \Pi_n$ with $\omega \in$ B A. It is also possible to define knowledge, and finite iterations of knowledge of an event A using the same framework. For this see the appendix of Tan and Werlang(1988). An alternative formulation of this model is given by Milgrom(1981). Aumann treats only the case the state space is finite. The extension to infinite state spaces is not

immediate. It is done in two alternative ways, which are closely related, by Nielsen(1984) and Brandenburger and Dekel(1987).

The second model to formalise common knowledge is based on infite hierarchies of beliefs. This framework was used by Tan and Werlang(1984, 1988, 1988b), Werlang(1986) and Brandenburger and Dekel(1985). Tan and Werlang(1988) use this explicitly to define common knowledge of the partitions. Then They show that given the partitions are common knowledge, an event is common knowledge in the infinite hierarchies sense if and only if it is in Aumann's sense. Brandenburger and Dekel are interested in findind an uncertainty space which is large enough to encompass all possible information, including the implicit description of the partitions.

Tan and Werlang(1984, 1988b) and Werlang(1986) use this formalism to derive implication for a game of the knowledge and common knowledge of facts like rationality. We will see this in more detail later on the text.

We will discuss some of the implications of Aumann's and the infinite hierarchies of belief's approaches later on the text.

The third way to model common knowledge is through the logic-theoretic framework. The most important source of reference for this approach is Fagin, Halpern and Vardi(1984). The logic formulation introduces new characters called knowledge operators in the usual propositional logic. The authors show that this is equivalent to the model of Kripke(1963) of modal logic (they are also known as epistemic logics) as far as common knowledge is concerned. Recently, many authors used the logic model: Bacharach (1985), Kaneko(1987), Kaneko and Nagashima(1988), Shin(1987), Barwise(1988), and Gilboa(1988). The logic - theoretic models

of knowledge have taught us three main results. The first is that it is possible to model the states of the world as including a complete description of the knowledge status of the agents, particularly the knowledge about the information partitions. The second is that this set of states of the world has to be infinite. Third, it clarifies the relation between properties of the knowledge operator and the information partitions. As a further interesting remark, Gilboa(1988) conjectures (but does not prove) that it is possible to obtain a logical model in which it makes sense to talk about knowledge and common knowledge of the model itself.

4. Agreeing to Disagree

One of the most striking results of the theory of common knowledge is proved in Aumann(1976). He shows that if the agents have a common prior which is common knowledge, their information partitions are common knowledge and their posteriors are common knowledge, then their posteriors have to coincide. This result is very important to show how strong of a requirement is common knowledge. It can be thought of as saying that "it is impossible to agree to disagree, whenever this fact and a common prior are common knowledge."

This claim was extended by Cave(1983) and Bacharach (1985) to more general properties, not only posteriors.

These results all seem extremely strong. One question that comes to mind is how it is possible to achieve common knowledge about posteriors. This question was answered in three related

ways by Geanakoplos and Polemarchakis(1982),Sebenius and Geanakoplos (1983) and McKelvey and Page(1986). The first makes use of an iterative procedure where each of two agents announce to each other their posteriors, based on the common prior, the partition they have, and the value of the posterior that the other announced. This process converges in finite time (if the state space, Ω is finite), to numbers which are equal, and common knowledge in the final partitions.

On the other hand, Sebenius and Geanakoplos(1983) use the system of bets being offered iteratively, until they are rejected. This way of transfering information is equivalent to the previous one.

The third approach is given by McKelvey and Page (1986). They showed how common knowledge can arise through the publication of aggregate statistics iteratively.

In the case of n > 2 agentes if one tries to have only private announcements between two agents, the process will not converge in general, except for very restrictive cases (see Parikh and Krasucki(1987)).

It is important to notice that the possibilitie of lies, and of strategic aunouncements have not yet been studied in these information transmission procedures.

One might conjecture that Aumann's result on the impossibility of agreeing to disagree is dependent on the fact that the information structure is given by partitions of the set of states of the world. It is well known that this is very restrictive. In particular, if we think of the knowledge operator, it is possible to show that, in the logic context, this follows from requiring that an agent knows what she does not know (the

well known modal logic S5 see Fagin, Halpern and Vardi(1984)).

It has been shown that whenever knowledge is finitely generated, i.e., generated by finitely many first facts, Aumann's result goes through even when one relaxes the requirement of one knowing what one does not know. Shin(1987) does it in terms of the weaker S4 modal logic. More striking even is the result of Samet(1987). He shows that independently of the logic properties of the knowledge operator, Aumann's result will follow, in the case of finitely generated knowledge. He has also some results for the case of beliefs.

5. Incomplete Information Games

An incomplete information games is a game where there is an unknown parameter.

As remarked above, Harsanyi was the first person to be concerned with iterated guesses about the value of this unknown parameter. In particular, through informal arguments Harsanyi(1967-68) reduces a general game of incomplete information to a game of complete information which is common knowledge among the players. His reasoning was formalised by Mertens and Zamir(1985). By means of the framework of infinite hierarchies of beliefs, they construct a complete information game which is common knowledge and which is equivalente to the incomplete information one. Of course, the game is extremely complicated, with very complex strategy spaces. They show, however, that it is possible to approximate this game as well as one pleases by games where players can be of finitely many types. On this see also Myerson(1985).

6. Foundations of Solution Concepts of Games - Normal Form

There is a plethora of notions of equilibrium in games. The theory of knowledge and common knowledge applied to games helps us to understand the implicit assumptions on the players which underliesome of the solution concepts.

In this section we will concentrate on the study of normal form noncooperative games. The interested reader should refer to Tan and Werlang(1984, 1988a, 1988b), and to the papers cited below. The programme of this literature is simply stated: given a solution concept, find the implicit assumptions about the knowledge and common knowledge of the players which will lead to this solution concept. One should be aware of another trend to find foundations of solution concepts, the evelutionary view. On this see Samuelson(1988) and Maynard Smith(1982).The following normal form solution concepts will be analysed: iterative elimination of strictly dominated strategies, rationalisble strategic behaviour, correlated equilibrium, Nash equilibrium, perfect (trembling - hand perfect) equilibrium and proper equilibrium Bernheim(1987) has an analysis which is related.

(i) Iterative elimination of strictly dominated strategies.

We say that a player is Bayesian rational when this player chooses an action which maximises the expected utility given her beliefs about the actions of the other players. Thus, in the game of figure I,

		II ℓ	r
I	u	(1000, 1000)	(-1000, 999)
	d	(900, 1000)	(900, 999)

Figure I

player II has a strictly dominant strategy, which is ℓ. Hence, whatever player II thinks player I will do (i.e. given any beliefs about player I's actions), player II will always play ℓ, if she is rational. If player I knows that player II is rational, then player I knows player II will play ℓ, so that player I, being himself rational, will play u. We can see clearly in this example the links between layers of knowledge of rationality and iterations in the elimination of strictly dominated strategies.

This can be formally and very generaly shown as in Tan and Werlang(1988b): if Bayesian rationality is known up to level m, then the players will play strategies which survive m + 1 iterations of elimination of strictly dominated strategies. Carrying this argument to the limit, if it is common knowledge that players are Bayesian rational, then the outcome of the game has to survive iterative elimination of strictly dominated strategies. Converses of the results above are also true: in a game with n players, any n - tuple which survives m + 1 rounds of eliminations of strictly dominated strategies is played by some n - tuple of players for whom Bayesian rationality in known up to level m. The same goes for m = ∞ (i.e., common knowledge).

(ii) Rationalisable Strategic Behaviour.

Bernheim(1984) and Pearce(1984) argue that one should eliminate iteratively not only the strictly dominated strategies, but also the strategies which never are a best response. In two person games these two concepts coincide (see Pearce(1984)). For more than two players they are different if it is required that the players play independently, as it is usual in non cooperative game theory. They define a solution concept, known as rationalisable strategic behaviour, which is the set of all actions (strategies) of the players which survive iterative elimination of actions which never are best responses.

It turns out that it is common knowledge that players are Bayesian rational and that they act independently if, and only if, they play a rationalisable action (i.e., an action which survivers iterative elimination of actions which never are best responses).

(iii) Correlated Equilibrium.

Aumann(1987) shows that if Bayesian rationality is common knowledge, and a common prior is common knowledge, then the players play a correlated equilibrium (as defined in Aumann(1974)). See also Tan and Werlang(1988b) for a converse. Combining this result with previous ones, we see that the solution concepts of rationalisability and correlated equilibrium are closely related. This analysis is made in Brandenburger and Dekel(1987).

(iv) Nash Equilibrium.

There are several ways to derive Nash behaviour in games, none of then entirely satisfactory. For example, as in Bernheim(1987), we could require common knowledge of rationality, of

independence, and of a common prior. Or, in two-person games, as in Armbruster and Böge(1979) we could require common knowledge of rationality and of each other's beliefs. Also Werlang(1986) and Tan and Werlang(1988b), notice that if one requires a theory to be single valued, then the only theory which is consistent with common knowledge of itself and of rationality is a Nash equilibrium. Bacharach(1987) gets to the same conclusion. This last explanation for Nash behaviour, shows the strong coordination requirements behind this solution concept. Not only rationality has to be common knowledge, but also the actions taken, before they are taken. Nevertheless, in the realm of Bayesian foundations of solution concepts there is nothing much better that can be done.

(v) Perfect and Proper Equilibria.

The concept of (trembling-hand) perfect equilibrium was defined by Selten(1975). The concept of proper equilibrium by Myerson(1978). Those are refinements of Nash equilibrium which require the equilibrium to be robust to some small mistakes by all the players. Clearly, to have a notion of equilibrium in games justified by means of mistakes is somewhat disturbing. This problem was overcome by the results of Blume(1986) and Brandenburger and Dekel(1986). They use a different theory of Bayesian rationality. Savage's(1954) theory of decision under uncertainty implies the Bayesian view: there exists a subjective probability distribution over the space of uncertainty such that the agent maximises expected utility. Blume and Brandenburger and Dekel use a different of decision under uncertainty, known as subjective expected utility under lexicographic beliefs. They define two types of rationality using this theory of expected utility a weak and a strong sence. If rationality is common knowledge and the actions also, as in (iv),

then with the weak reationality concept we get perfect equilibrium. With the strong rationality concept, proper equilibrium.

7. Foundations of Solution Concepts - Extensive Form

In the extensive form things get much more complicated. Reny(1988) and Basu(1985) notice that the requirement of common knowledge of rationality at all modes of a tree may be inconsistent. It is very easy to see why this is so. Consider a part of a game tree which is attained only if some player chooses a strictly dominated alternative at some previous information set. It is clear that, once this part of the tree is reached, it makes no sense to require common knowledge of rationality. Binmore(1988) suggests that the solution to the problem passes through introducing automata. He argues that to ask from human beings that they go into the infinite regress of common knowledge is too much. Bicchieri (1988) shows that in finite perfect information games, finite levels of rationality at the first node will indice backward induction. By far the deeper results in this area were found by Reny(1988). He found a theory with the property that the common knowledge of this theory is consistent at all modes of a tree. One can think that this is a minimally consistent theory, taht any other equilibrium theory should be a subset of. It should be notice that Reny's this theory is neither a subset nor a superset of Nash equilibria.

8. Miscellanea of Applications in Economics

Gilboa and Schmeidler(1988) define a class of games for which the knowledge about the outcome affects the payoff. Theses games encompass several phenomena and paradoxes of philosophy. An illustration of one such game is given by gossip. It is conceivable that one likes to gossip about some person, but that gossiper likes much more when the person (object of the gossip) finds out about the fact the gossip is going on. Gilboa and Schmeidler show that these games may be incompatible with common knowledge of rationality.

Rubinstein(1988) exibits a game which has an unknown parameter, for which information transmission in possible, and it may go on forever but common knowledge of the value the parameter may never arise. This is a result akin to Halpern and Fagin(1985).

For the case of economies with asymmetric information, Milgrom and Stokey(1982) show that if it is common knowledge ex-post that everyone benefits from a feasible trade, then ex-ante the initial allocation cannot be Pareto efficient. In a certain sense this is a no-trade result: after the uncertainty is realised and the agents see their private information, there cannot be a mutually advantageous trade which is common knowledge, if the initial allocation were Pareto efficient ex-ante.

9. Conclusion - Applications Outside Economics

The theory of common knowledge has been applied to the theory of social convention Lewis(1966, 1969), Gilbert (1981,1983). The theory of communication also has been

developed based on common knowledge ideas, as in Parikh(1988).

In parallel processing it is important, sometimes, that common knowledge between two (ou more) programs be attained. This study is reviewed in Halpern and Fagin(1985).

We tried to give a broad overview of the literature in common knowledge, with enphasis in its application in economics, and in particular in game theory. The more interested reader can have a more detailed view in the surveys by Binmore and Brandenburger (1988), Brandenburger(1986) and Tan and Werlang(1988a).

References

Armbruster, W. and W. Böge, "Bayesian Game Theory", in O. Moeschlin and D. Pallaschke ed., Game Theory and Related Topics, North-Holland, Amsterdam.

Aumann, Robert(1974), "Subjectivity and Correlation in Randomized Strategies", Journal of Mathematical Economics 1, 67-96.

__________(1976), "Agreeing to Disagree", The Annals of Statistics 4, 1236-1239.

__________(1987), "Correlated Equilibrium as an Expression of Bayesian Rationality", Econometrica 55, 1-18.

Bacharach, Michael(1985), "Some Extensions to a Claim of Aumann in an Axiomatic Model of Knowledge", Journal of Economic Theory 37, 167-190.

__________(1987), "A Theory of Rational Decision in Games", Erkenntnis 27, 17-55.

Barwise, Jon(1981), "Scenes and Other Situations", Journal of Philosophy LXXVIII, 369-397.

__________(1988), "Three Views of Common Knowledge", in M. Vardi ed. Theoretical Aspects of Reasoning About Knowledge II, Annals of the 1988 conference, held in Asilomar, KA, Morgan Kaufman.

Basu Kaushik(1985), "Strategic Irrationality in Extensive Games", Institute of Advanced Studies, Princeton, mimeo.

Bernheim, B. Douglas(1984), "Rationalizable Strategic Behavior", Econometrica 52, 1007-1028.

_____________(1987), "Axiomatic Characterizations of Rational Choice in Strategic Environments", Scandinavian Journal of Economics 88, 473-488.

Bicchieri, Cristina(1988), "Common Knowledge and Backward Induction: A Solution to the Paradox", in M.Vardi ed. Theoretical Aspects of Reasoning About Knowledge II, Conferece held in Asilomar, CA. Morgan Kaufman.

Binmore, Ken(1988), "Modelling Rational Players I and II", Journal of Economics and Philosophy, forthcoming.

Binmore, Ken and Adam Brandenburger(1988), "Common Knowledge and Game Theory", mimeo, London School of Economics.

Blume, L.(1986), "Lexicographic Refinements of Nash Equilibrium", Department of Economics, University of Michigan, mimeo.

Böge, W.(1974), "Gedanken Über die Angewandte Mathematik", in Mathematiker Uber die Mathematik, M. Otte (ed.) Heidelberg: Springer.

Böge, W. and Th. Eisele(1979), "On Solutions of Bayesian Games", International Journal of Game Theory 8, 193-215.

Brandenburger, Adam(1987), "The Role of Common Knowledge Assumptions in Game Theory", to appear in F. Hanh ed., The Economics of Information, Games, and Missing Markets.

Brandenburger, Adam and Eddie Dekel(1985), "Hierarchies of Beliefs and Common Knowledge", Research Paper No. 841, Graduate School of Business, Stanford University.

Brandenburger, Adam and Eddie Dekel(1986), "On an Axiomatic Approach to Refinements of Nash Equilibrium", Department of Economics, University of California at Berkeley, mimeo.

__________(1987), "Common Knowledge With Probability 1", Journal of Mathematical Economics 16, 237-245.

__________(1987), "Rationalizability and Correlated Equilibria", Econometrica 55, 1391-1402.

Cave, Jonathan, "Learning to Agree", Economic Letters 12, 147-152.

Fagin, Ronald, Joseph Y. Halpern and Moshe Vardi(1984), "A Model-Theoretic Analysis of Knowledge: Preliminary Report", Proc. 25th IEEE Symposium on Foundations of Computer Science, West Palm Beach, Florida.

Geanokoplos, John D. and Heraklis M. Polemarchakis(1982), "We Can't Disagree Forever", Journal of Economic Theory 28, 192-200.

Gilbert, Margaret(1981), "Game Theory and Convention", Synthese 46, 41-93.

__________(1983), "Agreements, Conventions and Language", Synthese 54, 375-404.

Gilboa, Itzahk(1988), "Information and Meta-Information", in M. Vardi ed. Theoretical Aspects os Reasoning About Knowledge II, annals of the 1988 conference, held in Asilomar, CA, Morgan Kaufman.

__________ and David Schmeidler(1988), "Information-dependent Games: Can Common Sense be Common Knowledge?" in M. Vardi ed. Theoretical Aspects of Reasoning About Knowledge II, annals of the conference held in Asilomar, CA, Morgan Kaufman.

Halpern, Joseph Y.(1986), "Proceeedings of the 1986 Conference on Theoretical of Reasoning About Knowledge", Asilomar, CA, Morgan Kaufman.

Halpern, Joseph Y. and Ronald Fagin(1985), "A Formal Model of Knowledge, Action and Communication in Distributed Systems: Preliminary Report", Proc. ACM Symposium on Principles of Distributed Computation.

__________ and Yoram Moses(1986), "Knowledge and Common Knowledge in a Distributed Environment", IBM Research Report RJ 4421, January.

Harsanyi, John(1967-68), "Games With Incomplete Information Played by 'Bayesian' Players", Management Science 14, 159-182, 320-334, 486-502.

Kaneko, Mamoru(1987), "Structural Common Knowledge and Factual Common Knowledge", RUEE Working Paper No. 87-27, Hitotsubashi University.

Kaneko, Mamoru and Takashi Nagashima(1988), "Player's Deductions and Deductive Knowledge and Common Knowledge on Theorems", Working Paper no. E88-02-01, Department of Economics, Virginia Polytechnic Institute and State University.

Kripke, Saul(1963), "Semantical Analysis of Modal Logic", Zeitschrift für Mathematische Logik und Grundlagen der Mathematik 9, 67-96.

Lewis, David(1966), "Conventions of Language", Ph.D. Dissertation, Harvard University.

__________(1969), Convention: A Philosophical Study, Cambridge, Massachusetts, Harvard University Press.

Maynard Smith, John(1982), Evolution and the Theory of Games, Cambridge University Press.

McKelvey, R. and T. Page(1986), "Common Knowledge, Consensus, and Aggregate Information", Econometrica 54, 109-127.

Mertens, Jean François and Shmuel Zamir(1985), "Formulation of Bayesian Analysis for Games with Incomplete Information", International Journal of Game Theory 14, 1-29.

Milgrom, Paul(1981), "An Axiomatic Characterization of Common Knowledge", Journal of Economic Theory 49, 219-222.

Milgrom, Paul and Nancy Stokey(1982), "Information, Trade and Common Knowledge", Journal of Economic Theory 26, 17-29.

Moses, Yoram, Danny Dolev and Joseph Y. Halpern(1986), "Cheating Husbands and Other Stories: A Case Study of Knowledge, Action, and Communication", Distributed Computing 1, 167-176.

Myerson, Roger(1978), "Refinements of the Nash Equilibrium Concepts", International Journal of Game Theory 7, 73-80.

______________(1985), "Bayesian Equilibrium and Incentive - Compatibility: An Introduction", in L. Hurwicz, D. Schmeidler and H. Sonnenschein eds., Social Gools and Social Organization: Essays in Memory of Elisha Pazner, Cambridge University Press.

Nielsen, L.(1984), "Common Knowledge, Communication, and Convergence of Beliefs", Mathematical Social Sciences 8, 1-14.

Parikh, Prashant(1988), "Language and Strategic Inference", Ph.D. Dissertation, Stanford University.

Parikh, Rohit and Paul Krasucki(1987), "Communication, Consensus and Knowledge", mimeo, Department of Mathematics, CUNY Graduate Center, New York, NY.

Pearce, David(1984), "Rationalizable Strategic Behavior and the Problem of Perfection", Econometrica 52, 1029-1050.

Reny, Philip(1988), "Rationality, Common Knowledge and the Theory of Games", Ph.D. Thesis Princeton University, Chapter 1.

Rubinstein, Ariel(1988), "A Game with Almost Common Knowledge: An Example", mimeo, London School of Economics.

Samet, Dov(1987), "Ignoring Ignorance and Agreeing to Disagree", MEDS Discussion Paper, Nortwestern University.

Samuelson Larry(1988), "Evolutionary Foundations of Solution Concepts for Finite, Two Players, Normal - form Games", in M. Vardi ed., Theoretical Aspects of Reasoning About Knowledge II, annals of the conference held in Asilomar, CA, Morgan Kaufman.

Savage, Leonard(1954), "The Foundations of Statistics". New York, Wiley.

Sebenius, James K. and John D. Geanakoplos(1983), "Don't Bet On It: Contingent Agreements With Asymmetric Information", Journal of the American Statistical Association 78, 424-426.

Selten, Reinhardt(1975), "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games", International Journal of Game Theory 4, 25-55.

Shin, Hyun Song(1987), "Logical Structure of Common Knowledge, I and II", Nuffied College, mimeo.

Tan , Tommy Chin-Chiu and Sérgio Ribeiro da Costa Werlang(1984),"The Bayesian Foundations of Rationalisable Strategic Behaviour and Nash Equilibrium Behaviour", mimeo, Princeton University.

______________(1988), "On Aumann's Notion of Common Knowledge - An Alternative Approach", CARES Working paper, University of Pennsylvania.

______________(1988a), "A Guide to Knowledge in Games", in M. Vardi ed. Theoretical Aspects of Reasoning About Knowledge II, annals of the conference held on Asilomar, CA, Morgan Kaufman.

______________(1988b), "The Bayesian Foundations of Solution Concepts of Games", forthcoming in Journal of Economic Theory.

Vardi, Moshe(ed.)(1988), "Proceedings of the Second Conference on Theoretical Aspects of Reasoning About Knowledge", Asilomar, CA, Morgan Kaufman.

Werlang, Sérgio Ribeiro da Costa(1986), "Common Knowledge and Game Theory", Ph.D. Thesis, Princeton University.

ENSAIOS ECONÔMICOS DA EPGE

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