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Integrability and The Demand for Monetary Assets: An Alternative Approach to an Old Problem¹.

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Abstract

This note provides necessary and sufficient conditions for some specific multidimensional consumer's surplus welfare measures to be well posed (path independent). We motivate the problem by investigating partial-equilibrium measures of the welfare costs of inflation. The results can also be used for checking path independence of alternative definitions of Divisia indexes of monetary services. Consumer theory classically approaches the integrability problem by considering compensated demands, homothetic preferences or quasi-linear utility functions. Here, instead, we consider demands of monetary assets generated from a shopping-time perspective. Paralleling the above mentioned procedure, of finding special classes of utility functions that satisfy the integrability conditions, we try to infer what particular properties of the transacting technology could assure path independence of multidimensional welfare measures. We show that the integrability conditions are satisfied if and only if the transacting technology is blockwise weakly separable. We use two examples to clarify the point.

1 Introduction

The answer to the question of under what conditions the Marshallian consumer's surplus integral furnishes acceptable measures of welfare change has been given, among others, by Chipman and Moore (1976). The two well known cases are those of homothetic preferences and of parallel preferences with respect to the numeraire.

The attempt to introduce money into the analysis of integrability, with money in the utility function, raised several difficulties. In the words of Samuelson and Sato (1984, pp. 591): "once money enters into the model in an essential way, demand theory not only loses its crown jewels (of testable well behaved curvature and perhaps reciprocity relations), but worse than that, in a sense it loses its *raison d'être* as a theory." These conclusions are a consequence of contrasting homogeneities of the demands for goods and for money. In Samuelson and Sato's analysis, money is deduced to be homogeneous of degree one - not zero - in prices and income. Samuelson and Sato showed that the problem can be fixed by considering utility functions that are weakly separable in goods and money. By assuming this hypothesis, integrability with money in the utility function can be treated in the same way as when only goods and services are considered, and we are back to the classical goods-and-services approach.

In this note, we consider specifically the integrability of the demand for monetary assets, and approach the problem with a different framework. We derive the demand for monetary assets under the shopping-time perspective popularized by the work of McCallum-Goodfriend (1983). Monetary assets are demanded because they save agents time which can be allocated to the production of the consumption good. Money does not enter into the utility function directly.

We do not rely on compensated demands and we are not directly concerned with the type of problem raised by Samuelson and Sato. Indeed, we do not consider monetary assets in the utility function.

Our procedure can be understood as paralleling the search, relatively to classical consumer theory, of special configurations of the utility function that could assure integrability of non-compensated demands. We try to infer the same with respect to demands of monetary assets and the underlying transacting technology. We conclude that integrability conditions are satisfied if and only if the transacting function is blockwise weakly separable with respect to the monetary variables and shopping time. Under this hypothesis, integrability is assured independently of income variations. Of course, integrability of non-compensated demands is a desirable property, since empirical data are based on such demands.

We motivate our study of integrability by considering partial-equilibrium measures of the welfare costs of inflation in economies where more than one asset performs monetary functions. Some examples of works in the literature that use multidimensional consumer's surplus measures with this intent are Marty and Chaloupka (1988), Marty (1994, 1999) and Baltensperger and Jordan (1997). In such works, which in a certain sense extend Bailey's (1956) original contribution, the measurement of welfare variations is made by calculating the areas under the inverse demand of each asset and then adding the results. These authors, however, concentrated on other issues, not on integrability. The results here derived support the pioneering contributions of Marty and Chaloupka (1988) and Marty (1984, 1999), when the transacting technology is separable.

Besides the application in the study of the welfare costs of inflation, our results can also be useful when checking for path independence of alternative definitions of Divisia indexes. Cysne (2000) investigates this issue.

The controversy about the exactness of consumer's surpluses as welfare measures or, relatedly¹, of analyzing the problem of path dependence when performing an integration in the n -dimensional space, is by no means a new issue in economics. An example that dates from the mid-nineteenth century is given by Walras' (1874 [1954] ; p:443) critique of Dupuit's (1844) ingenious observation that demand schedules can be used to infer welfare cost of price changes. As Hines Jr. (1999) reminds us, Walras, commenting on Dupuit's work, felt compelled to "call attention to an egregious error which Dupuit committed in a matter of capital importance." Walras was referring to the questionability of assuming the marginal utility of income to be constant as prices changed. Hotelling (1938) provided a practical defense of Dupuit's method, by analyzing its implications in a situation in which more than one price changes simultaneously (which is going to be the same type of question that we investigate in this work, regarding the opportunity cost of holding different monetary assets).

Hotelling showed that, provided that the vector field generated by the simultaneous consumer's surplus calculations is conservative, Dupuit's analysis could be applied to each commodity separately, for a given path in the n -dimensional Euclidean space, and the results could then be added. Empirically, Hotelling did not consider income effects to be large enough to make ordinary demand curves inappropriate for Dupuit's purposes. Hicks (1939, 1942, 1946) took another approach, showing that the conditions associated with the conservativeness of the respective vector field, also called integrability conditions, are trivially satisfied by using compensated demand curves.

¹ Both cases demand that the integrand represents an exact differential of utility.

Beyond these achievements, many works have emphasized, after the 1970s, the importance of using line integrals in the discussion of consumer's surplus. This happens when one wants to take into consideration the influences of the market for one good in the market for another good, or service. Two examples are Silberberg (1972, 1990), and Chipman and Moore (1980).

The integrability problem can be described as follows. Suppose that one makes two computations, c_1 and c_2 ; of the areas under the demand curves of the assets. In computation c_1 , one first calculates the area under the inverse demand curve of the first asset, holding the price of the second asset constant, and allowing the demand of the second asset to shift to a new position. Then one calculates the area under the demand curve of the second asset and adds it to the previous result. Computation c_2 is made by means of a symmetric procedure. One starts by changing the price of the second asset and calculating the area under its demand, while allowing the demand of the first asset to shift. Then, one calculates the area under the demand of the first asset, and adds it to the previous area. If the integrability conditions are not satisfied, we can have $c_1 \neq c_2$:

We use Simonsen and Cysne's (1994, 1999) model, which, in turn, draws on Lucas' (1993, 2000) previous analysis of welfare costs of inflation in shopping-time economies. To simplify, we present a model using currency and one kind of alternative monetary asset, which we call deposits. Cysne (2000) uses a version of this model with several different types of deposits.

The work is structured as follows: In section 2, we present the basic model. Households are assumed to maximize discounted utility subject to their budget constraint and to the constraint that the total time spent producing the consumption good and shopping must sum to one. A transacting technology specifies how currency and deposits permit agents to economize on the amount of time spent on transactions in the goods market. Utility-maximizing behavior generates the shopping-time curve as well as standard assets' demand curves. Both are functions of the nominal interest rate on bonds, which is the opportunity cost of holding currency, and of the opportunity cost of holding deposits.

Section 3 presents the partial-equilibrium measure of consumer's surplus, here equivalent to the welfare costs of inflation, and investigates its path independence. Two examples presented at the end of the Section aim at making the main point of the paper clearer. In the first example, technology is not separable and we get two different measures of the welfare costs of inflation, for the same initial and final values of interest rates. In the second example technology is separable and we show that such a problem does not happen. Section 4 offers the conclusions of the note.

2 The Model

Household preferences are determined by:

$$\int_0^{\infty} e^{-\rho t} U(c) dt \quad (1)$$

where $U : X_0 \rightarrow \mathbb{R}$; $X_0 \subset \mathbb{R}_+$; is a strictly concave function of the consumption at instant t and $\rho > 0$. The household is endowed with one unit of time that can be used to transact or to produce the consumption good with constant returns to scale:

$$y + s = 1 \quad (2)$$

Here, y stands for the production of the consumption good and s for the fraction of the initial endowment spent as transacting time. Households can accumulate three assets: currency (M); bonds (B) and deposits (X). To simplify, we assume that all assets are issued by the government.

In their maximization, households take as given the nominal interest rate on bonds, i ; and the opportunity cost of holding deposits, $j = i - i_x$, where i_x is the interest rate paid by X and $0 \leq j \leq i$. Letting $P = P(t)$ be the price of the consumption good, the household faces the budget constraint:

$$\dot{M} + \dot{B} + \dot{X} = iB + i_x X + P(y - c) + H$$

H indicates the (exogenous) flow of currency transferred to the household by the government and the dot over the variable its time derivative. Making $\bar{m} = \dot{M}/P$ (inflation rate), $\bar{b} = \dot{B}/P$; $\bar{x} = \dot{X}/P$ and $\bar{h} = \dot{H}/P$; and taking (2) into account, the budget constraint reads:

$$\bar{m} + \bar{b} + \bar{x} = 1 - (c + s) + \bar{h} + (i - \bar{m})\bar{b} + (i_x - \bar{m})\bar{x} - \bar{m} \quad (3)$$

Compared to currency, or to deposits, bonds are obviously preferable as a reserve of value. However, currency, as well as deposits, are useful because they save transaction time, as the transacting function describes:

$$c = F(m; x; s) \quad (4)$$

Here, F is supposed to be differentiable and strictly increasing in each of its variables, with decreasing marginal returns.

The household maximizes (1) subject to the budget constraint (3) and subject to the time-transacting technology (4). We are interested in steady-state solutions where m ; x and b converge to constant values. In this case, Euler's equations lead to the equilibrium relations:

$$\begin{aligned} i &= \frac{1}{\tau} + g \\ F_m &= i F_s \end{aligned} \quad (5)$$

$$F_x = j F_s \quad (6)$$

In equilibrium, since the consumption good is non-storable and since all households are equal, $y = c$: Using (2) and (4), we get the fourth equation that completes the description of household behavior:

$$1 - i - s = F(m; x; s) \quad (7)$$

We use equations (5), (6) and (7) to determine, locally, i ; j and s ; as functions of the asset holdings of m and x : Later we will be interested in assuring that these functions are globally integrable, and that the necessary and sufficient conditions of the Potential Function Theorem (presented in the next Section) are satisfied. Hence, we shall assume that $i(m; x)$ and $j(m; x)$ are defined over an open and connected set, U ; which is also simply connected.

3 The Multidimensional Consumer's Surplus Measure

In what follows we will be interested in the evaluation of $i(m; x)$ and $j(m; x)$ along paths $C(t) = (m(t); x(t))$; $C(t) \in U$; with $a \leq t \leq b$: With currency and interest-bearing deposits, the partial-equilibrium measure of the welfare costs of inflation is given by the line integral:

$$PE = \int_C i dm + j dx \quad (8)$$

PE can be thought a generalization of the area under a demand curve, although it is a different object from the mathematical point of view³.

²The same necessary and sufficient conditions for integrability are achieved if one works, when it is possible, with m ; x and s determined as functions of i and j ; and considers PE along paths $t \mapsto (i, j)(t)$:

³Simonsen and Cysne (2000), in the particular case when j is constant, or Cysne (2000), in the more general case, show that PE can be regarded as an approximation to the welfare measure (variable s) which emerges from the model that we present here.

4 Path Independence

Our objective here - paralleling the derivation of special conditions of the utility function that make the marginal utility of income constant in consumer theory - will be investigating if the same type of procedure can be fruitful in regards to making particular assumptions concerning the transacting technology.

It is natural to hope that the measure PE takes a unique value for different paths of m and x ; when the initial and final points are the same. In order to assure necessary and sufficient conditions for this result to hold, we need a well-known result from calculus generally called the "Potential Function Theorem".

Theorem 1 (Potential Function Theorem [PFT]): Let $F = (A; D)$ be a C^1 vector field in an open connected set L ; which is also simply connected. Then F is conservative if and only if

$$\frac{\partial A(i;j)}{\partial j} = \frac{\partial D(i;j)}{\partial i}$$

Proof. See any good textbook on Calculus. ■

Given (8), the conditions of the PFT for the path independence of the welfare measure PE in our problem are met if:

$$\frac{\partial i}{\partial x}(m; x) = \frac{\partial j}{\partial m}(i; j) \quad (9)$$

is verified for all $i; j$ considered in $C(m(t); x(t))$:

The formal definition and an encompassing analysis of separability can be found in Leontief (1947). For our purposes, the function $F(m; x; s)$ is said to be blockwise weakly separable when there are functions G and H such that $F(m; x; s) = H(G(m; x); s)$: It also follows from the analysis made by Leontief that this condition is equivalent to having the marginal rate of substitution between m and x independent of s :

$$\frac{\partial}{\partial s}(F_m = F_x) = 0$$

Proposition 3 uses this definition to establish our main result:

Proposition 2 Suppose that both $m(i; j)$ and $x(i; j)$ satisfy the conditions required for the application of the PFT: Then, it is a necessary and sufficient condition, for the welfare measure PE to be well defined (path independent), that the transacting technology $F(m; x; s)$ is blockwise weakly separable with respect to the monetary aggregator and the shopping time variable, s :

Proof. Considering (4), taking the partial derivatives of the first order and equilibrium conditions (5), (6), and (7) and making:

$$\Phi = \det \begin{pmatrix} F_s & 0 & iF_{ss} & iF_{ms} \\ 0 & F_s & jF_{ss} & jF_{xs} \\ 0 & 0 & 1 & F_s \end{pmatrix} = F_s^2(1 + s) > 0$$

We have

$$\begin{aligned} \Phi \frac{\partial i}{\partial x} &= -F_s [(iF_{sx} + jF_{mx})(1 + F_s) + (iF_{ss} + jF_{ms})F_x] \\ \Phi \frac{\partial j}{\partial m} &= -F_s [(jF_{sm} + iF_{xm})(1 + F_s) + (jF_{ss} + iF_{xs})F_m] \end{aligned}$$

Therefore,

$$F_s(1 + F_s) \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial m} \right) = (iF_{sx} + jF_{sm})(1 + F_s) + F_{ss}(iF_x + jF_m) + (F_mF_{xs} + F_xF_{ms})$$

Using the first order conditions the above equation reduces to:

$$F_s(1 + F_s) \left(\frac{\partial i}{\partial x} + \frac{\partial j}{\partial m} \right) = (iF_{sx} + jF_{sm}) + F_s(iF_{xs} + jF_{ms})$$

It follows that $\frac{\partial i}{\partial x} = \frac{\partial j}{\partial m}$ for any values of m and x if and only if $F_{ms}F_x = F_mF_{xs}$; , $\frac{\partial}{\partial s}(F_m=F_x) = 0$.⁴ Proposition 3 then follows from Lemma 2. ■

When m and x can be determined as functions of i and j ; this result has a dual economic interpretation. Under the blockwise weak separability of the transacting technology, the representative consumer's intertemporal problem is formally equivalent to a two-stage problem, in which the shopping time and total amount of monetary services are decided in a first stage, and the relative quantities of m and x are decided in a second stage. In the second stage the consumer takes the opportunity cost vector $(i;j)$ as given and performs a minimization of the cost of holding monetary assets, which is given by $R(m; x) = im + jx$; subject to the constraint that the total quantity of monetary services equals the one decided in stage one.

The set $\{(m; x) | G(m; x) \leq G\}$ is closed and non-empty, and the expenditure function R is precisely its support function. Since $(m; x) = \text{grad}$

⁴Once one knows that the functions $i(m; x)$ and $j(m; x)$ can be defined, the condition $F_m=F_x = F_{ms}=F_{xs}$ can be directly achieved by deriving $i = F_s=F_m$ with respect to x , $j = F_s=F_x$ with respect to m ; and equaling the results.

$R(i; j)$, the reciprocity condition $\partial_m \partial_j i(i; j) = \partial_j \partial_i i(i; j)$ is a direct consequence of the differentiability of R :

In the following two examples, we consider the functions $i(m; x)$ and $j(m; x)$ defined in R_{++}^2 :

Example 3 Here we consider an economy with a non-weakly separable transacting technology, and show that the multidimensional consumers' surplus measure is dependent on the path followed by the monetary assets. The transacting technology is given by:

$$F(m; x; s) = A(m^{1/2} + s)x^{1/2}, \quad A > 0:$$

First notice that this technology is not weakly separable, since $F_m = F_x \neq F_{ms} = F_{xs}$. Given the first order conditions, (5), (6) and (7), we have the functions:

$$\begin{aligned} i(m; x) &= \frac{1}{2m^{1/2}} \\ j(x; m) &= \frac{1 + m^{1/2}}{2x(1 + Ax^{1/2})} \end{aligned}$$

and the partial-equilibrium welfare measure:

$$PE = \int_C \frac{1}{2m^{1/2}} dm + \frac{1 + m^{1/2}}{2x(1 + Ax^{1/2})} dx \quad (10)$$

Since in this example we are interested only in comparing the values of PE along different paths, we make $A = 1$ to simplify the calculations. Let us then suppose that this economy presents an initial value of $(m; x)$ given by $(0.04; 0.04)$, and a final value of $(0.01; 0.01)$. We consider two different paths. In the first, C_1 ; the economy moves from $(0.04; 0.04)$ to $(0.01; 0.01)$ along the straight line $x = m = t$. The second path is a sum of two intermediate paths. In the first, C_{21} ; in which m is always kept constant, the economy moves from $(0.04; 0.04)$ to $(0.04; 0.01)$: In the second, C_{22} ; in which x is always kept constant in its new level, 0.01; the economy moves from $(0.04; 0.01)$ to $(0.01; 0.01)$: By first calculating the value of PE for C_1 ; making $x = m = t$ and $dx = dm = dt$ in (10), we get $PE_1 = 0.1 + \log 2$: On the other hand, along C_{22} ; $PE_{22} = 0.1$; whereas along C_{21} we have:

$$PE_{21} = 0.6 \int_{0.01}^{0.04} \frac{1}{x(1 + x^{1/2})} dx$$

Since $PE_2 = PE_{21} + PE_{22}$; $PE_1 = PE_2$, $PE_{21} = \log 2$: However, this does not occur, as the inequality below shows:

$$PE_{21} = \int_{0.01}^{0.04} \frac{1}{x(1+x^{1/2})} dx > \int_{0.01}^{0.04} \frac{1}{1.2x} dx = 0.5 \int_{0.01}^{0.04} \frac{1}{x} dx = \log 2$$

Any evaluation of PE for this economy would therefore have to consider the path followed by the variables.

Example 4 Here we consider the technology $F(m; x; s) = G(B; m; x)s = Bm^\alpha x^{1-\alpha} s$; $B > 0$: This technology is separable, since $F_m = F_x = F_{ms} = F_{xs} = (\alpha/(1-\alpha))(x/m)$: Along a path C ; this technology leads to the following measure of PE:

$$PE = \int_C idm + jdx = \int_C \frac{\alpha}{m} \frac{1}{m^\alpha x^{1-\alpha} + 1} dm + \frac{1-\alpha}{x} \frac{1}{m^\alpha x^{1-\alpha} + 1} dx \quad (11)$$

We prove that this integral is path independent in two different ways. In the first, we show that for all closed path C_c its value is equal to zero. This is equivalent to path independence (Lang, 1997, p. 399). We consider the generic closed path C_c . If K is the region in R^2 that is bounded by C_c ; it follows from Green's theorem that :

$$\int_{C_c} idm + jdx = \int_K \left(\frac{\partial j}{\partial x} - \frac{\partial i}{\partial m} \right) dmdx$$

Checking the partial cross derivatives in (11), one easily concludes that $\frac{\partial j}{\partial x} - \frac{\partial i}{\partial m} = 0$: Hence, PE is always equal to zero for closed paths, and therefore path independent.

As a second way to prove the path independence, we show that this problem admits a potential function. By making $G(m; x) = Bm^\alpha x^{1-\alpha}$ and solving the partial differential equations:

$$\begin{aligned} \frac{\partial i(m; x)}{\partial m} &= \frac{\alpha}{m} \frac{1}{1 + G} \\ \frac{\partial j(m; x)}{\partial x} &= \frac{1-\alpha}{x} \frac{1}{1 + G} \end{aligned}$$

one easily finds the potential function to be $i(m; x) = \log \frac{G}{1+G}$: Hence, for initial and final values of m and x given by, respectively, $(m_1; x_1)$ and $(m_2; x_2)$:

$$PE = i(m_2; x_2) - i(m_1; x_1)$$

whatever the path taken by $(m; x)$ between these two points.

5 Concluding Remarks

Traditional demand theory approaches the integrability problem by considering compensated demand functions or special features of the utility function. Here we follow this second type of approach with respect to a transacting technology. The demands for assets are derived from a shopping-time perspective, in an intertemporal utility maximization. We show that the necessary and sufficient conditions for integrability of non-compensated demands for monetary assets are given by the blockwise weak separability of the transacting technology. This outcome, derived in this note, establishes under what conditions P E is well defined. Elsewhere (Simonsen and Cysne (2000) and (2000)) we also show (assuming weak separability) that P E can be regarded as an approximation to the welfare measure (variable s) which endogenously emerges from the model that we present here. Added together, these results establish the grounds for, like suggested by the pioneering contributions of Marty and Chaloupka (1988) and Marty (1984, 1999), using P E in order to assess the welfare costs of inflation.

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