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SIGNALLING AND ARBITRAGE*

by

Vicente Madrigal
Department of Economics
Princeton University

Tommy C. Tan
Graduate Schoo of Business
University of Chicago

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Vicente Madrigal
Department of Economics
Princeton University

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Tommy C. Tan
Graduate School of Business
University of Chicago

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Abstract

Given a competitive signalling game, we study a transformed game which includes the natural enrichment of contractual opportunities in a market. We show that subgame perfect equilibrium outcomes in the new game which satisfy certain competitive hypotheses induce sequential equilibrium outcomes in the original game which satisfy the Cho-Kreps extended intuitive criterion. Arbitrage opportunities in the transformed game achieve results similar to refinements of sequential equilibria in the original game.

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1. INTRODUCTION

Much recent research has been devoted to narrowing down the set of equilibria in games with asymmetric information.[1] The aim of this program has been to isolate the "reasonable" equilibria, thus allowing the models to have a stronger set of testable hypotheses. In this paper, we want to provide a framework in which one could analyze the claim that for certain problems, in particular those involving "perfect competition," a good model should include the correct mechanisms so that less restrictive equilibrium concepts would still select the "reasonable" outcomes. For the case of perfect competition,[2] the mechanism we suspect to be important is arbitrage and the claim to be investigated is that if arbitrage opportunities are modelled properly then a solution which involves optimizing behavior at every information set should bring out the expected outcomes.

We concentrate on competitive signalling games, although our results are applicable to a broader class of market models with adverse selection. In competitive signalling games, one informed agent sends a signal, two uninformed agents receive the signal and simultaneously choose actions, and the informed agent then selects one uninformed agent to "trade" with. The problem is that along certain sequential equilibra, ("unreasonable") off the equilibrium path beliefs have the effect of not allowing trades contingent on the observable event, the message sent, to occur. This restricts the opportunity to arbitrage across types. We want to study the implication of returning these arbitrage opportunities to market models. In doing so, we also hope to get a better idea of how refinements of Nash equilibrium work.

Spence (1973) brought the phenomenon of signalling to the attention of economists. The canonical signalling model consists of informed agents who first observe some private information (this determines their "type") and then send a

costly signal to the uninformed agents. Uninformed agents must infer the information or type of the informed agents from the signals they receive, and then choose an action which affects the payoffs of both agents. The cost of sending different signals must be systematically related to the type of the informed agent for there to be non-trivial equilibria.

The analysis of Spence (1974) demonstrated the crucial role of beliefs -- the posteriors of the uninformed agents on the type of the informed agents, conditioned upon the signal they receive -- in determining the nature of the equilibrium. There is a severe multiplicity of equilibria in the Spence model of signalling. Some of these involve pooling (different types of the informed agent sending the same signal), some involve separating (different types send different signals), and still others exhibit hybrid behavior. In the separating equilibria, the uninformed are able to perfectly infer the type of the informed agents from observing the signals in equilibrium, whereas in the pooling equilibria, the signals reveal nothing.

Each of these equilibria are supported by posteriors which are Bayes-rule consistent with the equilibrium behavior. The multiplicity of equilibria is caused by the large degree of freedom in specifying posteriors at zero probability events. The uninformed may be specified to believe almost anything when they observe a signal which should not have been sent in an equilibrium since that occurs with probability zero. Consequently, equilibrium conditions place few restrictions on the behavior of the uninformed off the equilibrium path, and therefore many types of equilibrium behavior may be supported by appropriate specifications of the off-equilibrium beliefs and actions.

Beliefs gained the attention of game theorists as a result of their essential role in sequential equilibrium (Kreps and Wilson, 1982). Prior to this, the standard equilibrium concept -- that of Nash -- had no explicit mention

of posteriors at information sets in a game. Sequential equilibrium basically requires beliefs to be Bayes-rule consistent with the equilibrium strategies at every information set, and equilibrium strategies to remain optimal at every information set when expected payoffs are computed relative to the posteriors. Hence, the notion of a competitive signalling equilibrium of Spence is very close in spirit to sequential equilibrium.

More recently, Kreps (1984) made this relationship exact by providing an extensive form game which captures the Spence paradigm precisely. In that thought-provoking paper, Kreps suggested further restrictions on the beliefs of players beyond the relatively weak consistency requirements of sequential equilibrium. His "intuitive criterion" was very effective in narrowing the set of equilibria. In the Spence-Kreps signalling market, only one survived elimination. The equilibrium singled out by the criterion was the pareto optimum among the separating equilibria.

Other important papers which study refinements of sequential equilibrium include McClennan (1985), which preceded Kreps, Cho and Kreps (1985)[3], Cho (1985), Banks and Sobel (1985), and Grossman and Perry (1986). Kohlberg and Mertens (1985) are concerned with developing an axiomatic basis for solutions of games.

The trend in the literature appears to be to take the Spence-Kreps extensive form as a given and then to apply refinement theory to select the "reasonable" outcomes. Instead of studying a new solution concept, this paper analyzes a transformation of a class of competitive signalling games (in which the Spence-Kreps game belongs) into a new extensive form. In the new game, the uninformed move first and commit themselves to a signal to action function before the informed agent chooses an action. The informed agent, after observing the possibilities offered by the market (the profile of signal to action functions)

chooses a signal and an uninformed agent to carry out the action corresponding to the signal chosen.

One way of comparing the two extensive forms is in terms of contracts. The signals sent by the informed agents and the actions taken by the uninformed agents are public information (in contrast to the private information of the informed agent). We therefore allow the agents to enter into contracts which are contingent upon public information. These contracts can be signed before the informed agent chooses a signal. This allows actions of the uninformed agent to depend on the commonly observed signal and since payoff functions and the distribution of types are common knowledge, uninformed agents must arbitrage across types when they compete.[4]

The transformed game thus opens up more arbitrage opportunities. Moreover, in contrast to the original game which has no subgames, the transformed game has many -- a subgame is characterized by the functions chosen by the firms and a type for the informed player, who acts thereafter with complete and perfect information. Consequently, beliefs play a minor role in any sequential equilibrium of the transformed game.

In fact, the subgame perfect equilibrium (Selten, 1965) outcomes of the transformed game correspond in a natural way to outcomes in the original game which satisfy what we call an "arbitrage" condition. In turn, outcomes of equilibria in the transformed game which satisfy certain competitive hypotheses induce equilibria in the original games which also satisfy these hypotheses <u>and</u> the Cho-Kreps intuitive criterion.

The transformed game that we study is very similar in spirit to the Rothschild and Stiglitz (1976) model of the insurance market. With minor changes in the transformed game from signals to insurance contracts, the relationship is exact. The competitive market equilibrium of Rothschild and Stiglitz is

precisely the subgame perfect equilibrium of the transformed game. This brings out the relationship between the Spence model and the Rothschild and Stiglitz model. At the same time, it provides an explanation for the seemingly different conclusions of the two classic models of incomplete information markets. Studying the transformed game tells us that it is the additional arbitrage opportunities in Rothschild-Stiglitz which breaks all pooling equilibria. At this point we want to bring to attention the paper of Riley (1979) which studies an asymmetric information model where uninformed agents move first and to Stiglitz and Weiss (1986) which anticipates one of our results by noting that in the context of their model "the set of outcomes when the uninformed move first is a subset of the outcomes when the informed move first."

There are several ways to interpret our results. The first is that many recently studied solutions of games which are refinements of Nash equilibria are naturally related to an arbitrage argument. Moreover, there exists an extensive form which supports the equilibrium outcome selected by these refinements. As a result, one can be more comfortable with this method of singling out equilibria. See, however, Section 2, where we discuss some properties of the Cho-Kreps criterion, the Grossman-Perry criterion and our arbitrage condition in the context of the education model.

Another interpretation is that for certain economic environments, by a judicious specification of the model, subgame perfect equilibria achieve the same results as more complicated refinements of Nash equilibrium and that in some situations, it achieves even more. Since these refinements are not thoroughly understood (see for instance Madrigal, Tan and Werlang (1985), Reny (1985), Kreps and Ramey (1985), Tan and Werlang (1986)), it may be more profitable to specify the economic model more carefully for each situation than to adhere strictly to a canonical extensive form.

In Section 2, we demonstrate the issues discussed above in the context of the education signalling market as rendered by Kreps (1984) and Cho and Kreps (forthcoming). We analyze the similarities and differences between Cho-Kreps, PSE, Rothschild-Stiglitz and the arbitrage condition. Section 3 provides our general results.

We want to emphasize that this paper intends to be exploratory. Rather than claiming conclusions about a particular approach to the analysis of games, we only want to suggest a line of inquiry -- some "food for thought" -- which may help us in understanding solutions of games.

2. THE SPENCE-KREPS EDUCATION MARKET

Before analyzing the general case, it will be constructive to consider our arguments in the context of the education model of Spence (1974) as rendered by Kreps (1984) and Cho and Kreps (forthcoming).

The worker A receives private information about his own ability. He can be a low ability type $(t = 1)$ with probability λ, or a high ability type $(t = 2)$ with probability $(1 - \lambda)$. He then chooses an education level $e \in [0, \infty]$.

The marginal products of a type 1 and a type 2 who obtain an education level e are e and $2e$ respectively.

There are two uninformed firms $B1$ and $B2$ who move after the informed worker turns up with an education level e. Firms do not know the ability of the worker and must infer it from the level of education.

The firms simultaneously choose wages y_1 and y_2 from $[0, \infty)$ after observing e, and the worker goes to the firm with the higher bid. Hence the strategy of firm i is $w_i\colon [0, \infty) \to [0, \infty)$. Where $w_i(e)$ is the wage it will pay if e is sent. Let $y = \max\{y_1, y_2\}$.

The payoff for the worker of type t who chooses education level e is $y - e^2/t$. Firms are risk neutral and they bid to maximize the probability that they land the employee multiplied by the value $te - y_i$ if they win.

It should be clear that the firms earn zero expected profits in equilibrium. Hence the "Bertrand" specification of labor demand exactly captures the Spencian competitive labor market.

We refer to Kreps (1984) for a more detailed analysis of this model. Here, we demonstrate the effects of enriching the space of contracts between workers and firms by discussing two types of equilibria in the education game. We show that when we enrich the space of contracts, market arbitrage by the "Bertrand" firms will destabilize the pooling equilibria which exist for the Spence-Kreps game. In the transformed signalling game that we consider, pooling cannot occur in <u>any</u> subgame perfect equilibrium.

Consider the pooling equilibrium (e^*, w^*) in which both types of workers take the same education e^* and both firms offer wage $w^* = (2 - \lambda)e^*$ in equilibrium. w^* is the expected marginal product of a worker given that both types are taking the same education level e^*. This is an equilibrium as long as Type 1's prefer (e^*, w^*) to 0: $w^* - (e^*)^2 \geq 0$. This equilibrium is supported by out of equilibrium wages below the indifference curves of both types through (e^*, w^*). See figure 1 [which is figure 3 in Kreps (1984)].

Insert Figure 1 here

Some of these pooling equilibria can be made into sequential equilibria of the game by appropriately high posterior probability on a low ability type off the equilibrium path. The Kreps intuitive criterion (see Section 3 for a precise definition) operates on these "unreasonable" beliefs and removes sequential

equilibrium which are supported by "unreasonable" beliefs. The criterion removes all but one separating equilibrium, regardless of the value of λ.

Our approach focuses on the arbitrage opportunities in this market rather than on restricting beliefs. We feel that pooling equilibria exist in the Spence-Kreps model because of insufficient contractural arrangements.

At (e^*, w^*), the high ability type would rather move to $(\hat{e}, \hat{w})$, above the equilibrium indifference curve. In verifying the Nash or sequential equilibrium of the Spence-Kreps game, such a deviation from (e^*, w^*) to $(\hat{e}, \hat{w})$, say, is not relevant because it requires a coordinated deviation by two players -- the worker and a firm.

The Kreps criterion facilitates such a coordinated deviation because the <u>unilateral</u> deviation by the worker to $\hat{e}$ forces the two firms to introspect and to re-evaluation their posteriors to a more "reasonable" one and hence to offer a wage such as $\hat{w}$. The reasoning of the Kreps criterion is as follows. Beginning from the pooling equilibrium (e^*, w^*) if the firms observe an out-of-equilibrium education level $\hat{e}$, the firms should realize that the low ability type would never benefit from such a deviation as long as the firms' responses were optimal relative to <u>any</u> posterior. A non-empty set of types will therefore not send $\hat{e}$ and rather stay with e^*. Hence, the firm should place zero probability on the deviant being of a low ability type. On the other hand, when the firms now believe that high ability type sent $\hat{e}$, then the high ability type strictly prefers to send $\hat{e}$ than to send e^* as long as the firms play to a Nash equilibrium given that their posterior is concentrated on type 2. Notice that this line of argument does <u>not</u> consider the desirability of such a deviation from the point of view of the firms. Moreover, it takes advantage of giving the informed agent the first move to force the uninformed to introspect.

Let us reconsider $(\hat{e}, \hat{w})$. Firm B1 earns zero expected profits at $(e^*,$

w^*) the pooling equilibrium. B1, however, knows the model and knows that if it commits itself to $(\hat{e}, \hat{w})$ (keeping the rest of his strategy as at the pooling equilibrium), before the worker chooses an education level, then any rational worker who is a high ability type, would choose $\hat{e}$ and work for B1. Whereas the low ability types will choose e^* and work for B2. Hence B1 will earn strictly positive profits in contrast to the pooling equilibrium (e^*, w^*).

The coordinated deviation from (e^*, w^*) to $(\hat{e}, \hat{w})$ is something which both the worker and the firm B1 would like to see happen and they would be happy to sign a contract to commit to such a coordination. The Spence-Kreps extensive form restricts the set of permissible contracts and prevents this arbitrage from occuring.

The transformed game we consider permits players to engage in these types of arbitrage. In the transformed game, the uninformed firms choose a wage schedule first, before the worker chooses an education level. They are bound to it by contract (or by truth-in-advertising regulations). The worker, after observing the wage schedules available in the market, chooses an education level e and works for the firm with the highest wage rate for that level of education.

This game, unlike the Spence-Kreps game, has many subgames. Also, since the uninformed move first, beliefs play a very minor role -- they simply have to be consistent with the prior distribution λ. The subgames arise after the firms have chosen their wage schedules. The informed player faces no uncertainty and simply chooses an education level and firm with all the information in his hand. Hence, a subgame is defined by the wage schedules offered by firms and a type for the worker.

Consider now the pooling equilibrium (e^*, w^*) of the original game, as well as the wage schedule $w(e)$ supporting it. This cannot be a subgame perfect equilibrium in the transformed game. Suppose both firms offer this wage schedule

$w(e)$ to begin with. Both would be earning zero profits. It pays firm B1 to now offer

$$\hat{w}(e) = \begin{cases} w(e) & \text{if } e \neq \hat{e} \\ \hat{w} & \text{if } e = \hat{e} \end{cases}$$

instead. Consider the subgame in which firm B1 offers $\hat{w}(e)$ and B2 stays with $w(e)$, and the worker is type 2. Since

$$\hat{w} - \frac{\hat{e}^2}{2} > w^* - \frac{e^{*2}}{2},$$

this worker will choose B1 with probability one in any subgame perfect strategy of the worker. Hence the firm B1 now earns strictly positive profits, upsetting (e^*, w^*) and $w(e)$.

This is precisely the coordinated deviation from (e^*, w^*) to $(\hat{e}, \hat{w})$ by a firm and a high ability worker. In the transformed game, standard techniques like subgame perfection, achieve what required the Kreps criterion on top of sequential equilibrium in the original game.

Subgame perfection is required here because firm B1, when it offers $\hat{w}(e)$, must be assured that the workers do not have "crazy" suboptimal strategies off the equilibrium path. (e^*, w^*) can be implemented as a non-subgame optimal equilibrium of the transformed game. Just have the <u>low</u> ability type 1's take $(\hat{e}, \hat{w})$ if it is offered even if it is suboptimal compared to (e^*, w^*).

The essential feature of the transformed game is that a firm and a worker can coordinate to take advantage of an arbitrage. The transformed game in which the firm moves before the workers is one particular extensive form which permits this coordination via subgame perfection. There is little reason to prefer this over the Spence formulation as long as the contractual space is enriched there. Notice that these contracts do not require more information than that already present in the Spence-Kreps extensive form. We simply require that a court can

verify the education level of the worker and the wage he actually receives from the firm. Observe that the above arbitrage argument breaks all pooling equilibrium regardless of λ, the proportion of the low ability types, just as the Kreps criterion. The Kreps criterion discards most of the pooling hybrid (mixed strategy), and even separating equilibria.

The only equilibrium which the Kreps criterion does not discard is the one which maximizes the utility of both types <u>subject</u> to it being a separating equilibrium. This is regardless of the value of λ. See Figure 2.

Insert Figure 2 here

$(\underline{e}, \underline{w})$ is the equilibrium outcome for the low ability types and $(\bar{e}, \bar{w})$ for the high ability types. The low ability type is at the full information first best outcome, and the high ability type is at the constrained optimal outcome subject to incentive compatibility.

It can be shown using straightforward arbitrage arguments that in our transformed signalling game, only this separating equilibrium picked out by the Kreps criterion, is a candidate outcome for <u>any</u> subgame perfect equilibrium in <u>pure</u> strategies. Hence, the pooling, hybrid, and constrained inefficient separating equilibria <u>cannot</u> be equilibrium outcomes in the transformed game.

Section 3 shows that the outcome path of any sequential equilibrium in the original game which corresponds to an outcome path of a subgame perfect equilibrium in the transformed game, satisfies the Cho-Kreps extended intuitive criterion.

This, however, does not mean that $(\underline{e}, \underline{w})$ and $(\bar{e}, \bar{w})$ is a subgame perfect pure strategy outcome of the transformed game! There are values of λ such that $(\underline{e}, \underline{w})$ and $(\bar{e}, \bar{w})$ cannot be supported as subgame perfect outcomes of the

transformed game. Consider figure 3.

Insert Figure 3 here

This picture represents a low value of λ, i.e., very few low ability types. Consider $(\hat{e}, \hat{w})$. Both low ability and high ability workers prefer that point to $(\underline{e}, \underline{w})$ and $(\bar{e}, \bar{w})$ respectively. Moreover, $(\hat{e}, \hat{w})$ lies below the break even line for pooling. Hence there is an arbitrage opportunity in the transformed game. As a result, $(\bar{e}, \bar{w})$ and $(\underline{e}, \underline{w})$ will not be equilibrium outcomes by themselves.

Consequently, for λ low enough, the only candidate pure strategy subgame perfect equilibrium is also not robust to arbitrage. As a result, for these values of λ, the only subgame perfect equilibria of the transformed game are in mixed strategies.

Let us examine this final, seemingly peculiar, property of the transformed game. When a pure strategy subgame perfect equilibrium exists, then it is the same one as that picked out by Kreps in the original game. Yet it may fail to exist.

We argue that the Cho-Kreps criterion is useful in eliminating "unsound" equilibria but it does not really support the one it picks out. The $(\underline{e}, \underline{w})$, $(\bar{e}, \bar{w})$ equilibrium is one in which all parties can be better off provided they can coordinate their deviation. When such an arbitrage opportunity appears in a market, we believe that contractual arrangements will arise to eliminate them. Hence, unless there are economic reasons to restrict the class of permissable contracts, we believe that this separating equilibrium is also "unreasonable" and it would be destabilized by market forces. We therefore view the lack of a pure strategy subgame perfect equilibrium of the transformed game when λ is low, as

a virtue rather than a weakness when we are studying markets. It should be noted that the Grossman-Perry (1985) perfect sequential equilibrium also rejects this separating equilibrium when λ is small.

It is useful to review the Grossman-Perry requirement in this example (see Section 3 for a precise definition). Consider the pooling equilibrium (e^*, w^*) of figure 1 again. Grossman and Perry require that if a deviation such as $\hat{e}$ in figure 1 is observed, if a subset of types K can be found such that if the posterior of each firm were the prior restricted to the set K and if the firms then played to a Nash equilibrium given that posterior, every type in K prefers the deviation than to the pooling e^*, and every type not in K prefers the pooling outcome to the deviation, <u>then</u> each firm's posterior on observing $\hat{e}$ should be the prior restricted to the set K. The complement of K <u>may</u> be empty.

This requirement differs from Kreps' in two essential ways in the signalling game. First, Kreps requires there to be a non-empty set of types who would <u>not</u> want to deviate from the proposed equilibrium. Grossman and Perry allow that set to be empty -- that is, every type may wish to deviate if the response is appropriate. The Kreps condition does not reject the separating equilibrium in figure 3, when the arbitrage condition fails, because <u>both</u> types want to deviate. Grossman-Perry, on the other hand, allows the reasoning to proceed despite this.

The second main difference is that Grossman-Perry requires the posterior on the set of deviants to be the prior restricted to the set. Kreps permits the posterior to be any probability measure on the set of deviants. In both considerations, the Grossman-Perry concept places more discipline on the equilibrium.

The arbitrage condition is very similar to Grossman-Perry in that it permits the set of deviants to be the universal set. Moreover, it assesses the posterior to be the prior restricted to the set of deviants. It differs from both refine-

ment concepts in that it requires at least one uninformed agent to benefit from the deviation as well.

The reader familiar with Rothschild and Stiglitz (1976) would recognize the similarity between the model of the insurance market there and the transformed game here. The transformed game (with appropriate changes from signals to insurance contracts) is precisely the extensive form game underlying their insurance model. Hence, the Rothschild-Stiglitz equilibrium is the subgame perfect equilibrium of the transformed game. Much of the intuition about the various types of equilibria for the transformed game can be gotten from the Rothschild-Stiglitz model.

Section 3 demonstrates the relationship between equilibria in the transformed game and the Cho-Kreps equilibria in the original game. Hence, the economic justification for eliminating pooling equilibria are those which are well known from the pioneering work of Rothschild-Stiglitz.

3. THE MODEL AND RESULTS

A (3 agent) competitive signalling game[5] is defined by the tuple $S = (M, A, T, p, U, V^1, V^2)$ and involves the following "moves." Nature chooses a type t of the informed player from a finite set T according to the probability distribution p, where $p(t) > 0 \quad \forall t \in T$. The informed player, knowing his type, then chooses a message m from a finite set M. There are two uninformed players. Each learns the message m but do not know the type of the informed player. Uninformed player i then chooses an action a_i from the finite set A. (Uninformed players move simultaneously.) The informed player then chooses $j \in \{1,2\}$ and receives a payoff $U(t, m, a_j)$. Uninformed player $k \neq j$ receives a payoff of 0 while uninformed player j receives a payoff $V^j(t, m, a_j)$.[6] We assume that $V_1 = V_2 = V$. If x is a topological space endowed with

its Borel σ-algebra, then let $F(x)$ be the set of regular probability measures on x.

A behavior strategy for the informed player is then a map $\sigma = (\sigma_1, \sigma_2)$ where σ_1: $T \to F(M)$ and σ_2: $T \times M \times A \times A \to F\{1,2\}$. A behavior strategy for uninformed player i is a map τ_i: $M \to F(A)$. Pure strategies are special cases of behavior strategies. It is straightforward to show that at any Nash equilibrium, σ_2 can always be chosen not to be random. (That is if at a Nash equilibrium σ_2 is random, then there exists another Nash equilibrium with exactly the same specification of strategies except that σ_2 is not random.)

For any signalling game S we can define a transformed game $\hat{S}$ as follows. Let $A(M)$ denote the set of all maps from M to A. After nature chooses $t \in T$, the uninformed players (simultaneously) move, each choosing a random function $\hat{\alpha}_i = (\hat{\alpha}_i^1, \hat{\alpha}_i^2) \in F(A(M)) \times F(A(M))$, $i = 1, 2$. (Thus, in $\hat{S}$ $F(A(M)) \times F(A(M))$ is the set of <u>actions</u> of the uninformed player.) The informed player then chooses a message $m \in M$ and a firm $j \in (1, 2)$. Values of the random functions $(\hat{\alpha}_1, \hat{\alpha}_2)$ are realized and observed by all the players. Let $(\alpha_j^1, \alpha_j^2) \in A(M) \times A(M)$ denote the realized pair of functions of the chosen firm j. The informed player then chooses $k \in \{1, 2\}$ and receives a payoff of $U(t, m, \alpha_j^k(m))$. Uninformed player j receives a payoff $V^j(t, m, \alpha_j^k(m))$ while uninformed player $i \neq j$ receives a payoff of 0.

The main difference between S and $\hat{S}$ is that in $\hat{S}$ the uninformed player moves first. His pure strategy is therefore only an element of his set of actions $F(A(M)) \times F(A(M))$. Let $F^4 \equiv F(A(M)) \times F(A(M)) \times F(A(M)) \times F(A(M))$. A pure strategy of the informed player is a map $\hat{\sigma} = (\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3)$ where $\hat{\sigma}_1$: $T \times F^4 \to M$ and $\hat{\sigma}_2$: $T \times F^4 \to \{1,2\}$ and $\hat{\sigma}_3$: $T \times F^4 \times M \times A(M) \times A(M) \times \{1, 2\} \to \{1, 2\}$. Notice, however, that since the uninformed player moves first, $\hat{S}$ now has many proper subgames, unlike S which has none. We can then

study the subgame perfect equilibria of $\hat{S}$. Our main result below establishes that the subgame perfect equilibria in $\hat{S}$ induces outcome paths in S which must belong to equilibria in S which satisfy certain strong criteria formulated in attempts to "solve" signalling games. We interpret this result as meaning that the transformed game $\hat{S}$ makes clear the arbitrage opportunities inherent in situations which competitive signalling games are modelling. Before we proceed we need some definitions.

Definition 1. Let $(\mu, \sigma, \tau) = \left((\mu_1, \tau_1), (\mu_2, \tau_2), (\mu_0, \sigma)\right)$ be a sequential equilibrium in S, where the μ's are beliefs. Let U_t denote the payoff to an informed player of type t under (μ, σ, τ) and V_i the payoff to uninformed player i under (μ, σ, τ). We will say that (μ, σ, τ) fails to satisfy the arbitrage condition if there exists $m' \in M$ unsent in equilibrium (that is, $m' \notin \text{supp } \sigma_1(t)$ $\forall t \in T$, where supp x denotes the support of the random variable x), and there exists a non-empty $K \subset T$, $f \in F(A)$ such that for some $i \in \{1,2\}$

(i) $\forall\ t \in T - K,\quad U_t \geq E^f U(t, m', a)$

(ii) $\forall\ t \in K,\quad U_t < E^f U(t, m', a)$

(iii) $E_{k,f} V(t, m', a) > V_i$

where $E^f U(t, m', a)$ denotes the expected payoff to the informed player of type t, after sending message m', and where the uninformed agent randomizes action according to f; $E_{k,f} V(t, m', a)$ denotes the expected payoff to uninformed agent i, who receives message m' and randomizes according to f, taking the distribution over types to be the prior p restricted to k. (Types in T - K choose firm $j \neq i$.) [7]

For a given $m \in M$ and $\phi \in F(T)$, consider the following simple game $G(m, \phi)$: nature moves first choosing $t \in T$ with probability $\phi(t)$; 2 "uninformed" players, not knowing nature's move, then simultaneously choose actions $a_i \in A$, $i = 1, 2$; if $V(m, a_i, t) > V(m, a_j, t)$ $i, j \in \{1, 2\}$ $i \neq j$; then player i receives $V(m, a_i, t)$ and player j receives 0; otherwise, players receive $V(m, a_k, t)$ $k = 1, 2$ with equal probability. For $K \subset T$, denote by $EQ(K, m)$ the union of the sets of all Nash equilibria of $G(m, \phi)$ as ϕ varies through all measures (probability assessments) with support contained in K.

The following is a definition of the Cho-Kreps criterion specialized to the context of competitive signalling games. It is the definition used for the Spence-Kreps game (see section 2 above and Cho-Kreps (forthcoming)). It is also the notion of equilibrium implied by Cho's (1985) concept, which is an extension of the Cho-Kreps criterion to arbitrary finite extensive form games.

<u>Definition 2</u>: Let $(\mu, \sigma, \tau) = \big((\mu_1, \tau_1), (\mu_2, \tau_2), (\mu_0, \sigma)\big)$ be a sequential equilibrium in S . Then (μ, σ, τ) fails to satisfy the Cho-Kreps criterion if there exists a nonempty $J \subset T$ and there exists $m' \in M$ unsent by the informed agent in equilibrium such that

(i) $\forall t \in J$, $\forall g = (g_1, g_2) \in EQ(T, m')$

$$E^g U(t, m', a) < U_t \tag{1}$$

(ii) $\forall g \in EQ(T - J, m')$, $\exists t \in T - J$ s.t.

$$E^g U(t, m', a) > U_t$$

where $E^g U(t, m', a) = E \max_{x_1, x_2} \{U(t, m', x_1), U(t, m', x_2)\}$ where x_1, x_2 are independent random variables and x_i has probability distribution given by g_i $i = 1, 2$.

The interpretation is that when m' is sent both uninformed agents will introspect and reject all $t \in J$ as candidate senders of m', hence reaching some $(g_1, g_2) \in EQ(T - J, m')$ in the following subform.

<u>Definition 4</u>: Let $\hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{b}_2, \in F(A(M))$. Let $(\hat{a}_1, \hat{a}_2, \hat{\sigma}), (\hat{b}_1, \hat{b}_2, \hat{\sigma})$ be strategy profiles in $\hat{S}$. We will say that $\hat{\sigma}$ is <u>non-capricious</u> if whenever $\hat{\sigma}(t, \hat{a}_1, \hat{a}_2) = (f, i)$ and $\hat{\sigma}(t, \hat{b}_1, \hat{b}_2) = (f, k)$ (where $f \in F(M)$) and for all m in supp f, $\hat{a}_1(m) = \hat{a}_2(m) = \hat{b}_1(m) = \hat{b}_2(m)$, <u>then</u> $i = k$.

A non-capricious strategy $\hat{\sigma}$ will therefore not change the choice of uninformed agents if an uninformed agent changes his strategy in such a way that under $\hat{\sigma}$ there will be no change in the final message and action (and if the informed player could have chosen the other firm and induced the same outcome of messages and actions in the original profile).

<u>Definition 5</u>: A strategy profile (τ_1, τ_2, σ) in S induces an (possibly stochastic) outcome path

$$\tilde{x} = (t, \tilde{m}, \tilde{a}_j, j)_{t \in T} .$$

The probability that $\tilde{x}$ takes on a particular volume is calculated from (τ_1, τ_2, σ).[8]

<u>Definition 6</u>: A strategy profile $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$ in $\hat{S}$ induces an outcome path

$$\hat{x} = (t, \tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{m}, \tilde{j}, \tilde{k})_{t \in T} .$$

where $\tilde{\alpha}_i = (\tilde{\alpha}_i^1, \tilde{\alpha}_i^2)$. Note that since an action in $\hat{S}$ is an element of $F(A(M)) \times F(A(M))$, $\tilde{\alpha}_1$ is a pair of random variables over $A(M)$. Again, the probability that $\hat{x}$ is a particular value is derived from $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$.[9]

Definition 7: Given any outcome path $\hat{x}$ in $\hat{S}$, we can associate an outcome path $\phi(\hat{x})$ in S by $\phi(\hat{x}) = \phi((t, \tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{m}, \tilde{j}, \tilde{k})_{t \in T}) = (t, \tilde{m}, \tilde{\beta}, \tilde{j})_{t \in T}$ where $\tilde{\beta}$ is the random variable over A with probability distribution determined from $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$.[10]

Theorem 3.1: Let $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$ be a subgame perfect equilibrium of $\hat{S}$ with equilibrium outcome path $\hat{x}$. Let $\hat{\sigma}$ be non-capricious.[11] Then any sequential equilibrium in S with outcome path $\phi(\hat{x})$ does not fail to satisfy the arbitrage condition.

Proof: Let $(\mu, \sigma, \tau) = [(\mu_1, \tau_1), (\mu_2, \tau_2), (\mu_0, \sigma)]$ be a sequential equilibrium with outcome path $\phi(\hat{x})$. Suppose that (μ, σ, τ) fails to satisfy the arbitrage condition. Then there exists m' unsent in equilibrium and there exists $K \subset T$, $f \in F(A)$ such that $U_t < E^f U(m', a, t)$ for all $t \in K$ and $E_{K,f} V(m', a, t) > V_i$ for some i. Consider the alternative strategy in $\hat{S}$ for i defined by

$$\hat{\xi}_i(m) = \hat{\tau}_i(m) \quad \text{if} \quad m \neq m'$$

and $\hat{\xi}_i(m') = (f,f)$ (Each coordinate denotes the random action where $\hat{\xi}_i(m') = a$ with probability $f(a)$.)

By subgame perfection, $\hat{\sigma}_2(t, \hat{\tau}_{-i}, \hat{\xi}_i) = i$ for $t \in K$ and since $\hat{\sigma}$ is non-capricious $\sigma_2(t, \hat{\tau}_{-i}, \hat{\xi}_i) = \sigma_2(t, \hat{\tau}_1, \hat{\tau}_2)$ for $t \in T - K$. Thus, the expected payoff to i is strictly greater under $\hat{\xi}_i$ which contradicts the hypothesis that $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$ is a subgame perfect equilibrium in $\hat{S}$. Q.E.D.

We will now look at three hypotheses (in the context of signalling games) which are often associated with models of perfect competition. We will show that equilibria in S which have the same outcome paths as equilibria in $\hat{S}$ and

which satisfy these hypotheses never fail to satisfy the Cho-Kreps criterion. This result is therefore important for models (such as the Spence-Kreps game discussed in section 2) in which all equilibria satisfy these hypotheses.[12]

<u>Assumption A</u> (Free exit): For each $m \in M$, there exists $a(m) \in A$ such that for all $t \in T$ $V(t, m, a(m)) \geq 0$.

Of course, a sufficient condition for assumption A to hold is that there exists an $\bar{a} \in A$ such that for all $m \in M$ and all $t \in T$ $V(t, m, \bar{a}) \geq 0$. In the Spence-Kreps model, for example, this $\bar{a}$ is a zero wage offer. Actually, to obtain the result of Theorem 3.2 below we only need the substantially weaker assumption that given a pair of strategies of the informed player and uninformed player j there exists an action for uninformed player $i \neq j$ which yields i at least zero profits. However, assumption A seems simpler and more transparent so we will retain it.

Let y be an outcome path in S or in $\hat{S}$ and let $(V_1, V_2; U_1, \ldots, U_t)$ be the profile of expected payoffs to uninformed and informed players along y. We will say that y is a <u>perfectly competitive outcome path</u> if:

<u>Assumption B</u> (zero profits): $V_1 = V_2 = 0$

<u>Assumption C</u> (monopoly profits are not incompatible with welfare losses): For all $D \subset T$ and $m \in M$ there exists a $(D, m) \in A$ such that

$$\sum_{t \in D} V(t, m, a(D, m))p(t) > 0 \quad \text{and} \quad U(t, m, a(D, m)) < U_t \quad \text{for all} \quad t \in T - D.$$

In many economic applications, Assumption C holds for natural reasons. Assumption C is implied by the following stronger hypothesis: there exists $a' \in A$ such that for all $m \in M$ and all $t \in T$ $V(t, m, a') > 0$ and $U(t, m, a') < U_t$. This hypothesis states that if one uninformed player has <u>monopoly</u> power and the informed agent is forced to trade with him then he can <u>find</u> an

action (not necessarily an optimal action) which will yield him nonzero payoff and which makes the informed player worse off than in the original (competitive) outcome path. In the context of the Spence-Kreps model, for example, a' is any sufficiently low wage. Note that $\hat{x}$ is a perfectly competitive outcome path of $\hat{S}$ if and only if $\phi(\hat{x})$ is a perfectly competitive outcome path of S.

We will say that a sequential equilibrium in S (or a subgame perfect equilibrium in $\hat{S}$) is perfectly competitive if its outcome path satisfies Assumptions B and C above. We will say that S is a perfectly competitive signalling game if all sequential equilibria in S are perfectly competitive and S satisfies Assumption A. The Spence-Kreps game is an example of a perfectly competitive signalling game.

Theorem 3.2 Let S satisfy Assumption A. Let $\hat{x}$ be a perfectly competitive outcome path of a subgame perfect equilibrium of $\hat{S}$. Let $(\mu, \sigma, \tau) = [(\mu_1, \tau_1), (\mu_2, \tau_2), (\mu_0, \sigma)]$ be a sequential equilibrium in S with outcome path $\phi(\hat{x})$. Then (μ, σ, τ) does not fail to satisfy the Cho-Kreps criterion.

Corollary 3.3 Let S be a perfectly competitive signalling game. Let $\hat{x}$ be the outcome path of a subgame perfect equilibrium of $\hat{S}$. Then any sequential equilibrium in S with outcome path $\phi(\hat{x})$ does not fail to satisfy the Cho-Kreps criterion.

Proof of Theorem 3.2 (part of the proof uses an idea from Grossman-Perry (1986), section 4): Suppose that (μ, σ, τ) fails to satisfy the Cho-Kreps criterion. Then there exists m' unsent in equilibrium and $J \subset T$ such that (1) holds.

For every $\varepsilon > 0$. Consider the game S^{ε} defined as follows. Informed players can be of type $t \in T - J$ with prior probability $\dfrac{p_t}{\sum_{t \in T-J} p_t}$ The informed players move first choosing $m \in \{m'\} \cup \{m\colon \exists t \in T \;\; m \in \operatorname{supp} \sigma_1(t)\}$.

If $m \neq m'$ is sent by the informed player of type t the game ends and the player receives U_t -- his equilibrium payoff in S. Similarly, uninformed agent $i \in \{1, 2\}$ receives V_i, his equilibrium payoff in S. If m' is sent then the game proceeds as in the corresponding subform in S: The two uninformed players simultaneously choose actions $a_1, a_2 \in A$. Then the informed player chooses a firm $j \in \{1, 2\}$. The informed player of type t receives a payoff $U(t, m', a_j) - \varepsilon$; uninformed player j receives a payoff $V(t, m', a_j)$ and uninformed player $k \neq j$ receives 0.

Choose a sequential equilibrium of S^ε. Let $(f_1^\varepsilon, f_2^\varepsilon) \in F(A) \times F(A)$ denote the behavioral strategy profile defined by this sequential equilibrium at the information set given by m' and let U_t^ε, V_1^ε, V_2^ε denote the equilibrium payoffs. For ε sufficiently small, by (1), m' is observed along the equilibrium path with positive probability. Let $K \subset T$ be the types who send m'. By Bayes' rule, when m' is sent, the uninformed agents' posterior probability that the sender is of type t is given by $(\sum_{t \in K} p(t))/(\sum_{t \in T-J} p(t))$.

For $t \in K$ $U_t^\varepsilon \geq U_t$ (so that $U_t^\varepsilon + \varepsilon > U_t$). By Assumption A, $V_i^\varepsilon \geq 0$ $i = 1, 2$. Note that only types who do strictly better (under the payoff functions of S) than in the original equilibrium (μ, σ, τ) send m'.

By Assumption C, there exists $a^* \in A$ such that $\sum_{t \in K} V(t, m', a^*)p(t) > 0$ and $U(t, m', a^*) < U_t$ for all $t \in T - K$. Let $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$ be a subgame perfect equilibrium of $\hat{S}$ with outcome path $\hat{x}$. Consider the alternative strategy $\hat{\xi}_1$ for firm 1 in $\hat{S}$ defined by: $\hat{\xi}_1(m) = \hat{\tau}_1(m)$ $m \neq m'$, $\hat{\xi}_1(m') = (\rho a^* + (1 - \rho)f_1^\varepsilon, \rho a^* + (1 - \rho)f_2^\varepsilon) \in F(A) \times F(A)$ where $\rho a^* + (1 - \rho)f_K^\varepsilon$ denotes the probability measure over A which gives probability $\rho + (1 - \rho)f_1^\varepsilon(a^*)$ to a^* and probability $(1 - \rho)f_2^\varepsilon(a)$ to $a \in A$ $a \neq a^*$.

For ρ and ε sufficiently small, $\hat{\sigma}_1(t, \hat{\xi}_1, \hat{\tau}_2) = m'$ and $\hat{\sigma}_2(t, \hat{\xi}_1, \hat{\tau}_2) = 1$ if and only if $t \in K$, by the hypothesis that $(\hat{\tau}_1, \hat{\tau}_2, \hat{\sigma})$ is subgame perfect. But the strategy profile $(\hat{\xi}_1, \hat{\tau}_2, \hat{\sigma})$ yields uninformed player 1 a payoff greater than or equal to $\rho \sum_{t \in K} V(t, m', a^*)p(t) + (1 - \rho) \min (V_1^\varepsilon, V_2^\varepsilon) > 0$, a contradiction. Q.E.D.

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FOOTNOTES

[1] See Cho-Kreps (forthcoming), Grossman and Perry (1986), McClennan (1985), Banks and Sobel (1985), Kohlberg and Mertens (1985).

[2] Of course, it is not exactly clear what "perfect competition" means for a game. Implicit here is that a modeller will decide a priori if the phenomenon he is modeling will involve a competitive outcome. In the context of signalling games, we make precise exactly what we mean by competition.

[3] Kreps (1984) and Cho-Kreps (1985) have since been fused together into Cho and Kreps (forthcoming).

[4] This contract interpretation of the transformed game should not be taken literally. However, previous research (see for example Hart (1975)) has taught us how the set of equilibria changes with the set of markets of contingent claims contracts open in the economy. We are studying only one market so the contract interpretation merely highlights the implicit restrictions placed on trade by the alternative extensive forms used.

[5] From now on we will just say "signalling game" instead of 3 agent competitive signalling game.

[6] Thus, one can think of 0 as the (normalized) payoff level at which an uninformed agent is indifferent between being active in or existing the market.

[7] Theorem 3.1 below is also true if we use the weaker arbitrage condition which requires a strong inequality in (i).

[8] Thus, for a given t, $\tilde{x} = (t, \tilde{m}, \tilde{a}_j, \tilde{j})$ with probability

$\sigma_1(t)\tau_j(a_j)\tau_i(a_i)\sigma_2(t, m, a_1, a_2)(j)$ $i, j = 1,2$ $i \neq j$.

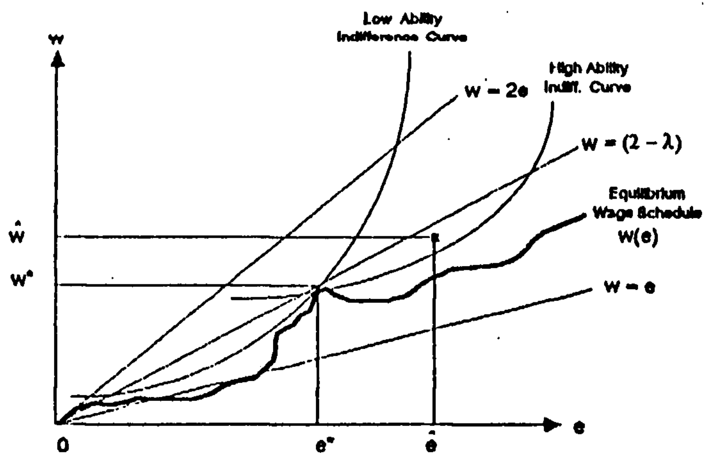
[9] For a given t, $\tilde{x} = (t, \tilde{\alpha}_1, \tilde{\alpha}_2, \tilde{m}, \tilde{j}, \tilde{k}) = (t, \hat{\alpha}_1, \hat{\alpha}_2, \tilde{m}, \tilde{j}, \tilde{k})$ with probability $\tau_1(\hat{\alpha}_1)\tau_2(\hat{\alpha}_2)\sigma_1(t, \hat{\alpha}_1, \hat{\alpha}_2)(m)\sigma_2(t, \hat{\alpha}_1, \hat{\alpha}_2)(j)\hat{\alpha}_1(\alpha_1)\hat{\alpha}_2(\alpha_2)$

$\sigma_3(t, \hat{\alpha}_1, \hat{\alpha}_2, \alpha_1, \alpha_2)(k)$. Recall that $\hat{\alpha}_i \in F(A(M)) \times F(A(M))$ and α_i is a realization of $\hat{\alpha}_i$, so that $\alpha_i \in A(M) \times A(M)$ $i = 1,2$.

[10] For a given t, $\phi(\hat{x}) = (t, \tilde{m}, \tilde{\beta}, \tilde{j}) = (t, m, \beta, j)$ with probability $\sum_{a \in A} [\hat{\sigma}_1(t, \hat{\alpha}_1, \hat{\alpha}_2)(m)\hat{\tau}_1(\hat{\alpha}_1)\hat{\tau}_2(\hat{\alpha}_2)\hat{\alpha}_3^1(m)(\beta)\hat{\alpha}_j^2(m)(a)\hat{\sigma}_2(t, \hat{\alpha}_1, \hat{\alpha}_2)(j)$ $\hat{\sigma}_3(t, \hat{\alpha}_1, \hat{\alpha}_2, m, \beta, a)(1) + \hat{\sigma}_1(t, \hat{\alpha}_1, \hat{\alpha}_2)(m)\hat{\tau}_1(\hat{\alpha}_1)\hat{\tau}_2(\hat{\alpha}_2)\hat{\alpha}_j^2(m)(\beta)\alpha_j^1(m)(a)\sigma_2(t, \alpha_1, \alpha_2)(j)$ $\hat{\sigma}_3(t, \hat{\alpha}_1, \hat{\alpha}_2, a, \beta)(2)$. Again, recall that $\hat{\tau}_i$ may be a mixed strategy over the action space $F(A(M)) \times F(A(M))$ so that its realization $\hat{\alpha}_i$ is an element of $F(A(M)) \times F(A(M))$.

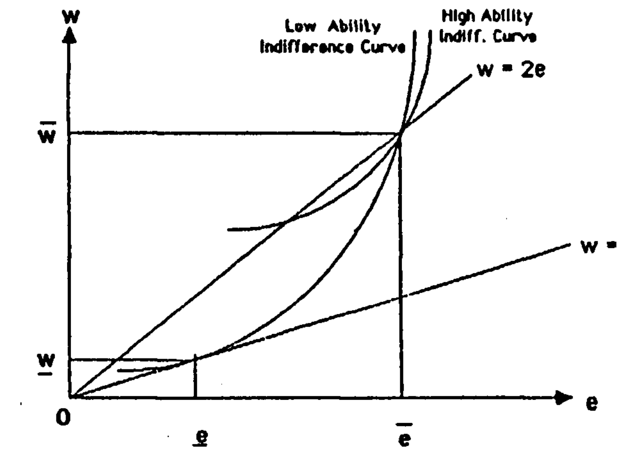
[11] We would not need to assume that $\hat{\sigma}$ is non-capricious, if we assume Assumption C below. Thus, Theorem 3.1 holds without non-capriciousness for S or $\hat{S}$ perfectly competitive (see below).

[12] Note that these assumptions involve only "primitives" of the game and the values of payoffs along a particular outcome path. Thus, satisfaction of these hypotheses can be checked either in the context of S or of $\hat{S}$.



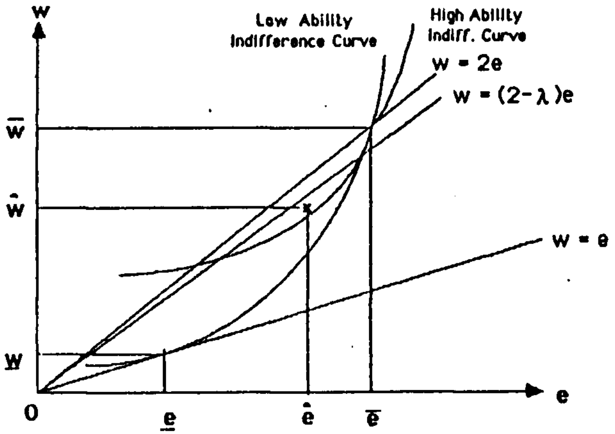
(e*, w*) = Pooling Equilibrium
(ẽ, ŵ) = Arbitrage Oppurtunity

Figure 1



Separating Equilibrium
(ē, w̄) = equilibrium outcome for high ability types
(e̲, w̲) = equilibrium outcome for low ability types
w(e) is any wage function below the indifference curves

Figure 2



Separating Equilibrium with Arbitrage Oppurtunity
$(\bar{e}, \bar{w})$ = equilibrium outcome for high ability types
$(\underline{e}, \underline{w})$ = equilibrium outcome for low ability types

$(\hat{e}, \hat{w})$ is preferred by both types over the equilibrium and makes positive profits

Figure 3

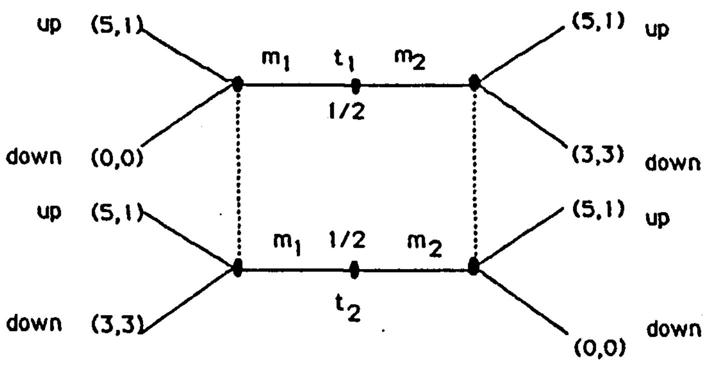


Figure 4

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