

***Nº 265***

**ISSN 0104-8910**

***PROFIT SHARING WITH HETEROGENEOUS  
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***Julho de 1995***

# **PROFIT SHARING WITH HETEROGENEOUS ENTREPRENEURIAL PROWESS**

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## **Abstract**

This paper analyzes the impact of profit sharing on the incentives that individuals face to set up their own business. It presents a model of capital accumulation in which individuals are equally skilled to be workers but differ in their abilities to manage a firm. Given an exogenous profit sharing parameter, an individual chooses whether to be an employer or an employee in order to maximize his net income. It is shown that profit sharing can inhibit entrepreneurial initiatives, reducing the number of firms in operation, the aggregate output and the economy's long run capital stock.

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## I - INTRODUCTION

The work of Martin Weitzman (1983), (1985) and (1987) kindled interest on profit sharing as an alternative compensation system which could be a useful tool to reduce the main nominal rigidity that causes unemployment. Since it emulates a flexible labor market, i.e., one where the nominal wage adjusts instantaneously in order to provide full employment, profit sharing was advocated as a remarkable form of avoiding the dreaded Keynesian unemployment.

In a profit sharing economy, the marginal cost of labor would be below the average cost of labor, creating permanent excess demand for labor, thus eliminating unemployment. Nordhaus (1988) showed that for the Weitzman proposition to be valid, two conditions must hold: the supply price of labor must fall very sharply in recessions, and the marginal cost of labor must be very far below the average cost of labor.

Weitzman's explanation for the fact that profit sharing be rare in most western countries was based on strong externality effects. When *one* wage firm converts to a share contract it will be guaranteeing employment not only to its own (internal) workers, but also serving as the employer of last resort for *all* other (outside) workers. In bad times internal workers would see their compensation falling in order to rescue outsiders. Since most of the benefits accrue not to its own workers, but to the working class as a whole, internal workers face no incentive to accept profit sharing when the workers of other firms do not accept it as well.

This argument is analogous to Keynes' explanation of why nominal wages are fixed in the short run: no one is willing to be the first to reduce nominal wages. As a result, the economy ends up in an inefficient Nash equilibrium without profit sharing. Weitzman suggests that in order to overcome this perverse coordination failure some sort of government incentive to profit sharing schemes were in order. Brunello (1992) shows that if internal promotion were the only way to climb the rungs of a career within a firm, than internal workers would favor profit schemes since the high rungs would only be attained if outsiders were hired to fill the low ones.

Following Weitzman's emphasis on short run aggregate fluctuations, Cooper (1988) presents a model of monopolistic competition in the presence of multiplier effects in which the introduction of share contracts in one sector changes the response to the adverse shocks and alters the nature of the interaction between the sectors. In his model there is only one very

special share contract which Pareto dominates the fixed-wage system. That is, there is only one very special contract which can balance the gains and losses to the various groups of agents in the economy from the introduction of share contracts. Nothing can assure, however, that the real economy has the arcane power to pick out that special contract. Moreover, even if the firms were able to single out the special contract, how could it be coordinated in order to create, say, a (stable) Nash equilibrium?

John (1991) casts additional light in the factors that impinge on profit sharing. When a firm's (marginal or total) revenue are very sensitive to employment, greater employment fluctuation may arise in a share firm. While Cooper's skepticism about profit sharing was based on general equilibrium effects, John's was based on fundamentals at the firm's level.

James Mead (1986) not only agrees with Weitzman's view that profit sharing provides greater stability of employment than a wage economy, but also believes that a firm in a share economy will be in excess demand for labor. He also suggests that the share system might increase labor effort. By making worker's income a function of profits, the incentives to shirk are reduced. Moreover, each worker will tend to help with the supervision of fellow workers and might even impose social sanctions to those who shirk. However unobservability of individual effort may undermine this argument when profit sharing is based on collective output. González (1992) discuss this point. Kandel & Lazear (1992) explores how peer pressure may overcome unobservability of individual effort.

The impact of profit sharing on investment was studied by Wadhwani (1987). He concludes that it increases the cost of capital, thus tending to reduce the level of the capital stock. This point is analyzed in a more complete setting in this paper. The conclusions are at one with Wadhwani's.

The empirical evidence on profit sharing has focused both on productivity as well as on employment. Jones & Svejnar (1985) found evidence that profit sharing had positive productivity effects in Italy. FitzRoy & Kraft (1987) found strong influence of profit sharing on factor productivity in a sample of medium-sized metalworking firms in Germany. Blanchflower & Oswald (1987) found no evidence that profit sharing influenced employment in the United Kingdom. Cable & Wilson (1989) estimated productivity gains of between 3 and 8% due to profit sharing in the UK engineering industry. Kruse (1992) presents evidence that, although in a small magnitude, profit sharing did indeed increase productivity in the USA. Bell & Newmark (1993) using

firm-level data for the union sector of the US economy found evidence that profit sharing reduced labor cost growth at firms that adopted these plans.

In short, there are three main cases for more income sharing: (i) the morale and productivity argument, i.e., profit sharing may incentive workers to increase their effort; (ii) the wage flexibility argument, i.e., profit sharing may reduce the incentives of firms to sack workers in recessions, and; (iii) Weitzman's macro economic argument, i.e., profit sharing may eliminate unemployment through the creation of permanent excess demand for labor. Argument (i) seems to be a compelling one when profit sharing is adopted in a small firm where observability of individual effort is possible; (ii) also seems to be true, specially in highly cyclical sectors; argument (ii), however, has cogent theoretical, as well as empirical evidence, against it.

The model presented in this paper does not address the first two arguments summarized above, but is an additional case against the third argument. The main conclusion is that the gains from sharing schemes can only be found at the very micro level, not at the macro one. A new case against tax incentives to encourage profit sharing is provided. The model differs from the above discussed literature in two ways.

First, is analyzes is the impact of sharing schemes on the individual incentives to set up a firm. Although neoclassical macro economists have been obsessed with micro foundations for their macro models, very often they take for granted who is employing whom. Why should the number of firms operating in an economy be invariant to the compensation scheme adopted? The present paper takes into account this first choice of individuals before turning to the following choices of how many workers a firm will hire.

The second way this paper differs from the usual literature on profit sharing is on its departure from the short run analysis. Weitzman's claim that all compensation systems have the same long run equilibrium - proposition I in his (1993) paper - does not hold if the micro choices are more carefully studied. Actually the model shows that Weitzman's claim does not hold even in the short run.

The paper is organized as follows. Section II describes the economic environment. In section III the model is solved. This section is divided into three subsections in order to simplify the exposition. III.1 works out the short run equilibrium, III.2 the long run equilibrium, III.3 analyses the dynamics following the introduction of profit sharing. Section IV concludes.

## II - THE ECONOMIC ENVIRONMENT

The economy is populated by a constant (large) number  $\bar{N}$  of individuals dispersed over the interval  $[0,1]$  and characterized by a label  $\lambda \in [0,1]$ . The distribution  $G: [0,1] \rightarrow [0,1]$ , with bounded density  $g$  defines, for each  $\lambda$ , the fraction  $G(\lambda)$  of individuals whose labels are lower than  $\lambda$ .

There is only one good in the economy. It can be produced, consumed or saved. All individuals know the production technology and are equally skilled to be workers but differ in their abilities to manage a firm. An individual of type  $\lambda$  who sets up a firm, employing  $K_\lambda(t)$  units of capital and  $N_\lambda(t)$  units of labor at instant  $t$  will get a flow of production  $Y_\lambda(t)$  given by

$$Y_\lambda(t) = \lambda F(K_\lambda(t), N_\lambda(t)) \quad (1)$$

where the production function  $F: \mathcal{R}^2 \rightarrow \mathcal{R}^+$  is homogeneous of degree  $b$ ,  $0 < b < 1$ , and satisfies the usual hypothesis of increasingness, strict concavity, twice differentiability and  $F(0, N) = F(K, 0) = F(0, 0) = 0$ . The hypothesis of decreasing returns to scale is necessary to generate pure profits in a competitive equilibrium. The parameter  $\lambda$  is a measure of entrepreneurial prowess.

The capital and labor markets are competitive and are continuously in equilibrium at the rental  $r(t)$  and wage  $w(t)$ . The gross profit of an entrepreneur of type  $\lambda$  at instant  $t$ ,  $\Pi_\lambda(t)$ , is given by

$$\Pi_\lambda(t) = Y_\lambda(t) - [r(t) + \delta]K_\lambda(t) - w(t)N_\lambda(t) \quad (2)$$

where  $\delta > 0$  is the instantaneous rate of capital depreciation.

The profit sharing parameter  $\tau$ , which is exogenous to the model, defines the net profit of an entrepreneur of type  $\lambda$  as  $(1 - \tau)\Pi_\lambda(t)$ . Each worker at a firm of type  $\lambda$  earns the wage  $w(t)$  and a profit sharing income  $s_\lambda(t)$  which is the workers' share on gross profits equally divided among the  $N_\lambda(t)$  employees:

$$s_\lambda(t) = \tau \frac{\Pi_\lambda(t)}{N_\lambda(t)} \quad (3)$$

Equation (3) allows for the possibility that the profit sharing income may vary across firms. As will be proven on equation (14), this will

not be the case for the gross profit per employee will be a function of the ratio rental/wage faced by the firms.

At each instant  $t$  the individual  $\lambda$ 's goal is to maximize his life utility as given by

$$\int_t^{\infty} e^{\epsilon - \rho x} U_{\lambda}[c_{\lambda}(x)] dx \quad (4)$$

where  $c_{\lambda}(x)$  is his flow of consumption at instant  $x$  and  $\rho > 0$  is the rate of time preference. The function  $U_{\lambda}$  is strictly increasing concave and continuously differentiable. The rate  $\rho$  does not vary across individuals.

Individual  $\lambda$ 's assets at instant  $t$ ,  $A_{\lambda}(t)$ , vary across time according to the intertemporal budget constraints

$$dA_{\lambda}(t)/dt = I_{\lambda}(t) + r(t)A_{\lambda}(t) - c_{\lambda}(t) \quad (5a)$$

and

$$\int_t^{\infty} e^{-\int_t^x r(u) du} (c_{\lambda}(x) - I_{\lambda}(x)) dx \leq A_{\lambda}(t) \quad (5b)$$

where  $I_{\lambda}(t)$  is his non-interest income. Since the individual can freely choose to be an entrepreneur or a worker, he decides what kind of agent to be seeking to maximize  $I_{\lambda}(t)$ :

$$I_{\lambda}(t) = \max \{ (1 - \tau)\Pi_{\lambda}(t); w(t) + s(t) \} \quad (6)$$

Given equations (1)-(6), an individual facing  $r(t)$  and  $w(t)$  will, at each instant  $t$ , maximize his intertemporal utility in the following way:

1. Maximize (1) subject to (2) in order to calculate the gross profit  $\Pi_{\lambda}(t)$  he would receive if he chose to be an employer;
2. Compare the net profit  $(1 - \tau)\Pi_{\lambda}(t)$  with  $s(t) + w(t)$  according to (6) and then decide whether to be an entrepreneur or a worker;
3. Once he has chosen what kind of agent to be at instant  $t$ , he will determine his flow of consumption  $c_{\lambda}(t)$  seeking to maximize (4) subject to (5), where  $I_{\lambda}(t)$  is given by (6).

The choice above assumes that there are no frictions such as a cost to close a firm down or to set one up. It is also assumed that the rental

and the wage adjust instantaneously - in general equilibrium - such as to clear the capital and labor markets.

### III - SOLVING THE MODEL

An individual of type  $\lambda$  who is an entrepreneur determines his demand for capital  $K_\lambda^D(t)$  and for labor  $N_\lambda^D(t)$  in order to maximize (2) subject to (1). The first order conditions of this maximization are:

$$\lambda F_1(K_\lambda^D, L_\lambda^D) = r + \delta \quad (7)$$

$$\lambda F_2(K_\lambda^D, L_\lambda^D) = w \quad (8)$$

where the time variable  $t$  was dropped to avoid cluttering the notation.

Lemma I recalls some useful properties of homogeneous functions which will facilitate the mathematical development of equations (7) and (8).

#### **Lemma 1 - Properties of the Homogeneous Production Function $F$**

If the function  $F: \mathcal{H}^2 \rightarrow \mathcal{H}$  is increasing, strictly concave, twice differentiable and homogeneous of degree  $b$ ,  $0 < b < 1$ , then for any point  $(K, L) \in \mathcal{H}^2$  the following properties hold:

$$(P1) \quad K F_1(K, L) + L F_2(K, L) = b F(K, L);$$

$$(P2) \quad F_i(K/L, 1) = L^{1-b} F_i(K, L), \text{ for } i = 1 \text{ or } i = 2;$$

$$(P3) \quad (K/L) F_1(K/L, 1) + F_2(K/L, 1) = b F(K/L, 1).$$

Proof: See the appendix.

Applying (P2) to (7) and (8) yields

$$\frac{\lambda}{(L_\lambda^D)^{1-b}} F_1(K_\lambda^D / L_\lambda^D, 1) = r + \delta \quad (9)$$

$$\frac{\lambda}{(L_\lambda^D)^{1-b}} F_2(K_\lambda^D / L_\lambda^D, 1) = w \quad (10)$$



Equations (9) and (10) show that the ratio  $h = K_\lambda^D / L_\lambda^D$  of inputs is a decreasing function of the ratio  $(r + \delta)/w$  of their respective costs and therefore does not vary across firms.

Another important consequence of working with a homogeneous function of degree  $b < 1$  is given by (P1). Substituting (7) and (8) into (P1) one concludes that the gross profit  $\Pi_\lambda(t)$  - defined by (2) - is equal to  $(1-b)$  times the out  $Y_\lambda(t)$  - defined by (1).

Lemma 2 defines the auxiliary function  $f$  - as well as works out its properties - which will greatly facilitate the mathematical development of equations (9) and (10).

### **Lemma 2 - Definition and Properties of the Auxiliary Function $f$**

Define the function  $f: \mathcal{R} \rightarrow \mathcal{R}$  as  $f(h) = F(h, 1)$  than the following properties hold for all  $h > 0$ :

(P4)  $f(0) = 0$ ,  $f'(h) = F_1(h, 1) > 0$ , and  $f''(h) = F_{11}(h, 1) < 0$ ;

(P5)  $F_2(h, 1) = b f(h) - h f'(h)$ ;

(P6) Define the function  $\eta_1: \mathcal{R} \rightarrow \mathcal{R}$ ,  $\eta_1(h) = h f'(h) / f(h)$ . Then  $0 < \eta_1(h) < b$  for all  $h > 0$ ;

(P7) Define the function  $\eta_2: \mathcal{R} \rightarrow \mathcal{R}$  as  $\eta_2(h) = -h f''(h) / f'(h)$ . Then  $(1 - b) < \eta_2(h)$  for all  $h > 0$ ;

(P8) The elasticity of substitution of  $F$  at  $h$ ,  $\sigma(h)$  is lower than one if, and only if,  $\eta_1(h) + \eta_2(h) > 1$ ;

(P9) For all  $h > 0$ ,  $\sigma(h) < 1 / \eta_2(h)$ .

Proof: See Appendix

Substituting (P5) into (10), the demand for labor of a firm of type  $\lambda$  is written as:

$$L_\lambda^D = \lambda^{\frac{1}{1-b}} \left[ \frac{[b f(h) - h f'(h)]}{w} \right]^{\frac{1}{1-b}} \quad (11)$$

From (9) and (11) the demand for capital of a firm of type  $\lambda$  can be written as

$$K_\lambda^D = h L_\lambda^D = \lambda^{\frac{1}{1-b}} h \left[ \frac{[b f(h) - h f'(h)]}{w} \right]^{\frac{1}{1-b}} \quad (12)$$

Substituting (P2) into (2) the gross profit of an entrepreneur of type  $\lambda$  is given by

$$\begin{aligned} \Pi_\lambda &= (1-b) \lambda F(K_\lambda^D, L_\lambda^D) = (1-b) \lambda (L_\lambda^D)^b f(h) \\ &= (1-b) \left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} f(h) \lambda^{\frac{1}{1-b}} \end{aligned} \quad (13)$$

where the last equality follows from (11).

The total compensation  $R_\lambda(t)$  of a worker at a firm of type  $\lambda$ ,  $R_\lambda(t) = w(t) + s_\lambda(t)$ , where  $s_\lambda(t)$  defined by (3) is given by the substitution of (11) and (13) into (3):

$$\begin{aligned} R_\lambda &= w + \tau \Pi_\lambda / L_\lambda^D \\ R &= w \left[ 1 + \tau(1-b) \frac{f(h)}{b f(h) - h f'(h)} \right] \end{aligned} \quad (14)$$

The expression above shows that the total compensation of workers,  $R$ , does not vary across firms. Therefore the subscript  $\lambda$  is dropped from  $R$  in (14).

Examining equations (13) and (14) one concludes that while the workers' total compensation  $R(t)$  does not vary across  $\lambda$ , the net profit  $(1-\tau)\Pi_\lambda(t)$  is an increasing function of  $\lambda$  with  $(1-\tau)\Pi_\lambda(t) = 0$  for  $\lambda = 0$ . The individual choice of whether be an entrepreneur or a worker can now be described. Given the wage  $w(t)$ , those individuals for whom  $(1-\tau)\Pi_\lambda(t)$  exceeds  $R(t)$  will choose to be entrepreneurs whereas those whom  $R(t)$  exceeds  $(1-\tau)\Pi_\lambda(t)$  will be workers. Therefore there must be a watershed  $\Lambda(t) \in (0,1)$  such that individuals of type  $\lambda \in [0, \Lambda(t)]$  will be workers and individuals of type  $\lambda \in (\Lambda(t), 1]$  will be entrepreneurs. The existence of such  $\Lambda(t)$  is assured by the hypothesis of a flexible labor market which adjusts the wage  $w(t)$  so that there is  $\Lambda \in [0,1]$  such that  $R(t)$  exceeds  $(1-\tau)\Pi_\lambda(t)$ .

The hypothesis that the wage is flexible rules out involuntary unemployment. Most models on profit sharing focus on the frictions that generate unemployment. They study the consequences of sharing contracts freely negotiated between employers and employees in order to appraise their impact on unemployment. The model of this paper aims at addressing another aspect of sharing schemes, namely, the incentives to set up a business, thus creating new jobs instead of being a worker and competing for one of the existing jobs. In this paper sharing schemes are not negotiated between workers and firms but imposed from outside by, say, a government decree.

### III.1 - THE SHORT RUN GENERAL EQUILIBRIUM

#### The Capital Market

At each instant  $t$  the aggregate supply of capital,  $K^S(t)$ , is the sum of all individuals' assets. There are  $\bar{N}g(\lambda)$  individuals of type  $\lambda$ , each owning  $A_\lambda(t)$  units of assets, so  $K^S(t)$  is fixed in the short run and given by:

$$K^S(t) = \bar{N} \int_0^1 A_\lambda(t) g(\lambda) d\lambda \quad (15)$$

The aggregate demand for capital  $K^D(t)$  is the sum over all entrepreneurs' demand for capital. From (12),  $K^D(t)$  is given by

$$\begin{aligned} K^D(t) &= \bar{N} \int_{\Lambda(t)} K_\lambda(t) g(\lambda) d\lambda \\ &= \bar{N} \left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-\alpha}} h \int_{\Lambda(t)} \lambda^{\frac{1}{1-\alpha}} g(\lambda) d\lambda \end{aligned} \quad (16)$$

#### The Labor Market

The aggregate supply of labor at instant  $t$ ,  $N^S(t)$ , is just the total number of individuals  $\bar{N}$  times the fraction of them who choose to be workers:

$$N^S(t) = \bar{N} G(\Lambda(t)) \quad (17)$$

The aggregate demand for labor,  $N^D(t)$ , is the sum, over all entrepreneurs' demand for labor. From (11),

$$N^D(t) = \bar{N} \int_{\Lambda(t)} N_\lambda(t) g(\lambda) d\lambda$$

$$N^D(t) = \bar{N} \left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} \int_{\Lambda(t)}^1 \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda \quad (18)$$

### The Individual Choice

The watershed  $\Lambda(t)$  is such that individuals of type  $\lambda = \Lambda(t)$  are indifferent between being an entrepreneur or a worker, i.e., for them the net profit they would receive if they set up a firm  $(1-\tau)\Pi_{\Lambda(t)}(t)$  would be equal to the workers' compensation  $R(t)$ . From (13) and (14) this requires:

$$(1-\tau)(1-b) \left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} f(h) \Lambda^{\frac{1}{1-b}} = w \left[ 1 + \frac{\tau(1-b) f(h)}{b f(h) - h f'(h)} \right]$$

Rearranging the expression above one gets the *marginal entrepreneur's indifference condition*

$$\left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} \Lambda^{\frac{1}{1-b}} = \frac{b f(h) - h f'(h) + \tau(1-b) f(h)}{(1-b)(1-\tau) f(h)} \quad (19)$$

### Short Run General Equilibrium

It is assumed that the input prices  $r(t)$  and  $w(t)$  are flexible so that at each instant  $t$ , the capital and labor markets clear. It is also assumed there are no frictions that hinder an entrepreneur from closing down his firm and becoming a worker, as well as that may impede a worker from quitting his job and opening up his own firm.

Given the short run aggregate supply of capital  $K^S(t)$  defined by (15), equating  $K^S(t)$  to the aggregate demand for capital  $K^D(t)$  in (16) provides the *capital market equilibrium equation*:

$$\frac{K^S(t)}{\bar{N}} = \left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} h \int_{\Lambda}^1 \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda \quad (20)$$

Likewise, equating the aggregate supply of labor  $N^S(t)$  given by (17) to the aggregate demand for labor given by (18), one gets the *labor market equilibrium equation*:

$$\frac{G(\Lambda)}{\int_{\Lambda}^1 \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda} = \left[ \frac{b f(h) - h f'(h)}{w} \right]^{\frac{1}{1-b}} \quad (21)$$

The market clearing equations (20) and (21) involve three variables:  $w$ ,  $h$  and  $\Lambda$ . The missing equation is the marginal entrepreneur's indifference equation (19). The three equilibrium variables  $w$ ,  $h$  and  $\Lambda$  are, therefore, implicitly determined by the system of equations (19), (20) and (21).

Noting that the term in the right hand side of (21) is present in (19), (20) and (21), the system can be simplified into two equations involving the variables  $h$  and  $\Lambda$  plus a third equation which determines  $w$  given  $h$  and  $\Lambda$ . (20) and (21) imply:

$$h = K^s / \bar{N} G(\Lambda) \quad (22)$$

which shows that the capital/labor ratio at each firm  $h$  is equal to the ratio of the aggregate stock of capital to the number of workers.

Now, substituting (21) into (19) one gets

$$\frac{G(\Lambda) \Lambda^{\frac{1}{1-b}}}{\int_{\Lambda}^1 \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda} = \frac{b f(h) - h f'(h) + \tau(1-b) f(h)}{(1-b)(1-\tau) f(h)} \quad (23)$$

Define the auxiliary function  $Z: \mathcal{R}^+ \rightarrow \mathcal{R}^+$  such that  $Z(\Lambda)$  is equal to the left hand side of (23). Then,  $Z$  has the following properties:  $Z(0) = 0$ ,  $dZ/d\Lambda > 0$ , and  $\lim_{\Lambda \rightarrow 1} Z(\Lambda) = +\infty$ . Equation (23) is then rewritten as:

$$(1-b) f(h) [1 + Z(\Lambda)] - [f(h) - h f'(h)] = \tau(1-b) f(h) [1 + Z(\Lambda)]$$

From (P6),  $\eta_t(h) = h f'(h) / f(h) < b$ . This yields:

$$\frac{1 - \eta_t(h)}{1-b} = (1-\tau) [1 + Z(\Lambda)] \quad (24)$$

The system (22) and (24) determines the equilibrium variables  $h(t)$  and  $\Lambda(t)$ . The third equilibrium variable  $w(t)$  is given by the substitution of  $h(t)$  and  $\Lambda(t)$  into (21):

$$w(t) = [b f(h(t)) - h(t) f'(h(t))] \left( \int_{\Lambda(t)}^1 \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda / G(\Lambda(t)) \right)^{\frac{1}{1-b}} \quad (25)$$

From (P4), (P5), (9) and (10), the rental  $r(t)$  is given by

$$r(t) + \delta = w(t) \frac{f'(h(t))}{b f(h(t)) - h(t) f'(h(t))}$$

$$= f'(h(t)) \left( \int_{\Lambda(t)}^I \lambda^{\frac{1}{1-b}} \sigma(\lambda) d\lambda / G(\Lambda(t)) \right)^{\frac{1}{1-b}} \quad (26)$$

where the last equality follows from (25).

Recalling that (P2) implies that the gross profit of a firm is equal to  $(1-b)$  times its output, the output  $Y_\lambda(t)$  of a firm of type  $\lambda$  is given by the substitution of (25) into (13):

$$Y_\lambda(t) = \left( G(\Lambda(t)) / \int_{\Lambda(t)}^I \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda \right)^b f(h(t)) \lambda^{\frac{1}{1-b}} \quad (27)$$

From (27) the aggregate output,  $Y(t)$ , is given by

$$\begin{aligned} Y(t) &= \bar{N} \int_{\Lambda(t)}^I Y_\lambda(t) g(\lambda) d\lambda \\ &= \bar{N} f(h(t)) [G(\Lambda(t))]^b \left( \int_{\Lambda(t)}^I \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda \right)^{1-b} \end{aligned} \quad (28)$$

Once the complete determination of all the variables of the model is accomplished, the short run impact of a sudden increase of the profit sharing parameter can be described. It must be stressed that the concept of "short run" here means, a spell sufficiently short for the state variables  $\Lambda_\lambda(t)$ , and hence the aggregate supply of capital  $K^s(t)$ , to be considered as constant.

### PROPOSITION I - Short Run Impact of Mandatory Profit Sharing

Given the supply of capital at instant  $t$ ,  $K^s(t)$ , the short run endogenous variables  $\Lambda(t)$ ,  $h(t)$ ,  $w(t)$ ,  $Y_\lambda(t)$ ,  $\Pi_\lambda(t)$ ,  $P^L(t)$ ,  $P^K(t)$ ,  $Y(t)$ ,  $r(t)$ , and  $R(t)$  are differentiable functions of the sharing parameter,  $\tau$ , with the following derivatives:

$$\text{I.1 - } d\Lambda(t)/d\tau > 0;$$

$$\text{I.2 - } dh(t)/d\tau > 0;$$

$$\text{I.3 - } dw(t)/d\tau > 0;$$

$$\text{I.4 - } dY_\lambda(t)/d\tau > 0;$$

- I.5 -  $d\Pi_\lambda(t)/d\tau > 0$ ;
- I.6 -  $d \ln P^L(t)/d\tau < d \ln P^K(t)/d\tau < 0$ ;
- I.7 -  $dY(t)/d\tau < 0$ ;
- I.8 - If  $\sigma(h) \leq 1$  for all  $h > 0$ , then  $\frac{d}{d\tau}[(1-\tau)\Pi_\lambda(t)] < 0$ ;
- I.9 - Let  $\bar{\sigma} \in \mathcal{H}^*$  be such that  $\bar{\sigma} \leq \sigma(h) \leq 1$  for all  $h > 0$ . If  $\tau \geq (1-\bar{\sigma})$ , then  $dR(t)/d\tau < 0$ ;
- I.10 - There exists  $\tau_1 \in (0,1)$  such that for  $\tau \in (\tau_1,1]$ ,  $dR(t)/d\tau < 0$ . If  $1/2 \leq \sigma(h) \leq 1$  for all  $h > 0$  and  $g' \leq 0$ , then there exists  $\tau_2^* \in (0, \tau_1)$  such that for  $\tau \in [0, \tau_1)$ ,  $dR(t)/d\tau > 0$ .

In order to interpret the results summarized in the proposition I one has to divide the impact of an increase of the profit sharing parameter into its partial as well as its general equilibrium components. The partial equilibrium impact of an increase of the share of profits that must be transferred to workers is to reduce the net profits received by the employers and to increase the total compensation paid to workers. As a result of this, the least efficient entrepreneurs close their companies down, thus dismissing their employees and releasing capital, and then join the labor market.

The increased supply of labor and the release of capital, together with a reduced number of firms demanding these factors, tend to reduce the wage and rental. At the new lower factor prices, the firms that remain in operation hire more workers and rent more capital. When the new equilibrium is achieved, each firm is using both more capital and labor, thus producing a higher level of output. Although the output of each firm increases, the aggregate output decreases for there are fewer companies in operation.

In the new equilibrium the capital/labor ratio is higher than before because the aggregate supply of capital is fixed in the short run, while the aggregate supply of labor was increased by the number of (previous) entrepreneurs that became workers. The higher capital/labor ratio does not necessarily mean a higher marginal productivity of capital - see equation (9) - when the production function is not homogeneous of degree one. In fact, equations (9) and (10) state that the higher the capital/labor ratio, the higher the relative factor prices. Since the wage falls, the rental may well fall too, provided that its relative fall be lower than the rental's.

One surprising result of the model is that the net profit of a firm that remains in operation does not necessarily fall. On the one hand, since gross profits are equal to  $(1-b)$  times the firm's output, a higher profit sharing parameter tends to reduce its net profit, lost on the other hand, a higher output tends to increase it. If capital and labor are not close substitutes - i.e., if  $\sigma \leq 1$  - then the net profit falls.

### III.2 - THE LONG RUN GENERAL EQUILIBRIUM

Section III.1 described what happens to the economy immediately after the introduction of profit sharing, i.e., for a given level of aggregate capital stock. According to Proposition I, the return on accumulated savings  $r(t)$  tends to change and this will affect the consumption-savings decisions and hence the evolution of the aggregate stock of capital. This section describes the (new) long run equilibrium that will obtain with profit sharing.

The consumption-savings decision of individual  $\lambda$  is determined by the usual Keynes-Ramsey rule.

$$\frac{\dot{c}_\lambda(t)}{c_\lambda(t)} = \sigma_\lambda(c_\lambda(t))[r(t) - \rho] \quad (30)$$

where  $\sigma$  is the instantaneous elasticity of substitution of consumption:

$$\sigma_\lambda(c_\lambda(t)) = -U'_\lambda(c_\lambda(t))/[U''_\lambda(c_\lambda(t))c_\lambda(t)]$$

The long run equilibrium is defined as the steady state equilibrium at which each individual's savings are zero. This will happen when the return on accumulated assets  $r^*$  is equal to the rate of time depreciation

$$r^* = \rho$$

When this last expression holds, (26) implies:

$$\rho + \delta = f'(h^*) \left( \int_{\Lambda^*}^1 \lambda^{\frac{1}{1-\sigma}} g(\lambda) d\lambda / G(\Lambda^*) \right)^{1-\sigma} \quad (31)$$

where  $h^*$  and  $\Lambda^*$  are the long run equilibrium variables.

The labor market equilibrium equation and the marginal entrepreneur's indifference condition require that (24) must hold in the long run:



$$\frac{1 - \eta_h(h^*)}{1 - b} = (1 - \tau)[1 + Z(\Lambda^*)] \quad (32)$$

The capital market equilibrium equation is represented by (22):

$$K^* = \bar{N}h^*G(\Lambda^*) \quad (33)$$

Equations (31) and (32) determine the equilibrium long run capital/labor ratio  $h^*$  and watershed  $\Lambda^*$ . The long run aggregate capital stock  $K^*$  is obtained from substitution of these equilibrium variables into (33). The long run wage  $w^*$  is obtained by substitution of  $h^*$  and  $\Lambda^*$  into (25). The long run workers' total compensation  $R^*$  is obtained through the substitution of  $w^*$  and  $h^*$  into (14). The long run output of firm  $\lambda Y_\lambda^*$  and the aggregate output  $Y^*$  are similarly given by the substitution of  $\Lambda^*$  and  $h^*$  into (27) and (28).

After having completed the determination of the long run equilibrium, one can turn to the analysis of the long run effects of an increase of the profit sharing parameter. It must be stressed that the concept of "long run" here means a spell sufficiently long for the control variables  $c_\lambda(t)$  to reach a constant level,  $dc(t)/dt = 0$ , and the stock of capital to be constant.

## PROPOSITION II Long Run Impact of Mandatory Profit Sharing

The long run equilibrium variables  $h^*$ ,  $\Lambda^*$ ,  $w^*$ ,  $Y_\lambda^*$ ,  $\Pi_\lambda^*$ ,  $P_L^*$ ,  $P_K^*$ ,  $Y^*$ ,  $(1 - \tau)\Pi_\lambda^*$ ,  $K^*$  and  $R^*$  are differentiable functions of the profit sharing parameter  $\tau$  with the following derivatives:

$$\text{II.1 - } d\Lambda^*/d\tau > 0 ;$$

$$\text{II.2 - } dh^*/d\tau < 0 ;$$

$$\text{II.3 - } dw^*/d\tau < 0 ;$$

$$\text{II.4 - } dY_\lambda^*/d\tau > 0 ;$$

$$\text{II.5 - } d\Pi_\lambda^*/d\lambda > 0 ;$$

$$\text{II.6 - } d \ln P_L^*/d\tau < d \ln P_K^*/d\tau \text{ and if } \sigma(h) < 1 \text{ for } h > 0, \text{ then } d \ln P_L^*/d\tau < 0 ;$$

II.7 - Let  $\bar{\sigma} \in (0,1)$  be such that  $\bar{\sigma} \leq \sigma(h) \leq 1$  for all  $h > 0$ . If  $\tau \geq b(1-\bar{\sigma}) / (1 + \sqrt{(1-b)\bar{\sigma}})^2$ , then  $dY^* / d\tau < 0$ ;

II.8 -  $d((1-\tau)II_\lambda^*) / d\tau < 0$ ;

II.9 - Let  $\bar{\sigma} \in (0,1)$  be such that  $\bar{\sigma} \leq \sigma(h) \leq 1$  for all  $h > 0$ . If  $\tau \geq b(1-\bar{\sigma})$ , then  $dK^* / d\tau < 0$ ;

II.10 - There exist  $\tau_1^*, \tau_2^* \in (0,1)$ ,  $\tau_1^* < \tau_2^*$ , such that for all  $\tau \in [0, \tau_1)$ ,  $dR^* / d\tau > 0$  and for all  $\tau \in (\tau_1, 1]$ ,  $dR^* / d\tau < 0$ .

## VI- CONCLUSION AND EXTENSIONS

The model presented above is an example that mandatory profit sharing schemes may harm the economic incentives that lead individuals to set up their own business. Moreover, since it can increase worker's gross compensation at the expense of their employer's pure profits, populist governments may feel tempted to meddle with labor-firms relations, introducing compulsory profit sharing systems. Weitzman's case for public policy measures taken to induce firms to adopt share-type compensation schemes might be used as the populist's theoretical underpinnings to go on bullying the work of the invisible hand.

The model does not deny that profit sharing schemes freely negotiated between firms and workers may indeed reduce risk bearing by firms of very cyclical sectors, thus leading to higher rates of employment. Nor does it deny that the impact of sharing schemes on workers' effort may also be of importance in some industries. In short, my view on profit sharing is that it can be of great help in some industries and of great harm in others. While the micro conditions in one sector of the economy may recommend sharing schemes, they may not recommend it other sectors may. Aggregate arguments such as Weitzman's cannot justify any public intervention to foster profit sharing. The choice of whether to adopt sharing schemes or not must be left to the agents directly involved: workers and firms.

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## APPENDIX

### **Proof of Lemma I**

Since  $F$  is homogeneous of degree  $b$ , for any  $\alpha \in \mathbb{R}^+$ ,

$$F(\alpha K, \alpha L) = \alpha^b F(K, L) \quad (A1)$$

to obtain (P1), differentiate (A1) with respect to  $\alpha$  and then set  $\alpha=1$ . To obtain (P3) for  $i=1$ , differentiate (A1) with respect to  $K$  and then set  $\alpha=1/L$ . To obtain (P2) for  $i=2$ , differentiate (A1) with respect to  $L$  and then set  $\alpha=1/L$ . To obtain (P3), substitute (P2) into (P1) and apply (A1) with  $\alpha=1/L$ .

### **Proof of Lemma II**

Property (P4) is straightforward. To obtain (P5) substitute (P4) into (P3). Property (P6) follows from (P5) and the fact that  $F_2 > 0$ . To obtain (P7) differentiate (P5) and note that:

$$-\frac{h f''(h)}{f'(h)} = (1-b) + \frac{F_{21}(h,1)}{f'(h)} > (1-b)$$

In order to prove (P8), let  $x = F_1(K, L)/F_2(K, L)$  and write the elasticity of substitution  $\sigma(h)$  of  $F$  as

$$\sigma(h) = -\frac{d(K/L)}{dx} \frac{x}{K/L} = -\frac{dh}{dx} \frac{x}{h}$$

From (P4) and (P5),

$$x = \frac{f'(h)}{b f(h) - h f'(h)} \quad (A2)$$

The derivative  $dh/dx$  is

$$\frac{dh}{dx} = \frac{[b f(h) - h f'(h)]^2}{f''(h)[b f(h) - h f'(h)] - f'(h)^2 [\eta_2(h) - (1-b)]} =$$

Substituting the expression above and (A2) into the definition of  $\sigma(h)$  yields:

$$\begin{aligned}
\sigma(h) &= - \frac{[b f(h) - h f'(h)]}{f''(h) [b f(h) - h f'(h)] - f'(h)^2 [\eta_2(h) - (1-b)]} \frac{f'(h)}{h} = \\
&= \frac{b - \eta_1(h)}{b[\eta_1(h) + \eta_2(h)] - \eta_1(h)}
\end{aligned}
\tag{A3}$$

From the expression above, it follows that  $\sigma(h) \leq 1$  if, and only if,  $\eta_1(h) + \eta_2(h) \geq 1$ . For a given  $\eta_2$ , (A3) implies that  $\sigma$  is a decreasing function of  $\eta_1$ . Since  $\eta_1 \in (0, b)$ , it follows that:

$$\sigma(h) = \frac{b - \eta_1(h)}{b\eta_2(h) - (1-b)\eta_1(h)} \leq \frac{b - 0}{b\eta_2 - (1-b)0} = \frac{1}{\eta_2} < \frac{1}{1-b}$$

(A3) implies the useful relation:

$$\eta_1(h) = \frac{b[1 - \eta_2(h)\sigma(h)]}{1 - (1-b)\sigma(h)}$$

(A4)

## Proof of Proposition I

### Proof of I.1 and I.2

The derivatives of  $\Lambda$  and  $h$  are obtained by applying the implicit function theorem to (22) and (24). Since  $Z(\Lambda)$  is defined as the left hand side of (23), it follows that

$$\frac{dZ}{d\Lambda} = \frac{d}{d\Lambda} \left[ G(\Lambda) \Lambda^{\frac{1}{1-b}} / \int_{\Lambda}^1 \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda \right] = Z(\Lambda) \left[ \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \right]$$

(A5)

The definitions of  $\eta_1(h)$  and  $\eta_2(h)$  from Lemma 2 imply that:

$$\frac{d\eta_1}{dh} = \frac{d}{dh} \left[ \frac{h f'(h)}{f(h)} \right] = \frac{f'(h)}{f(h)} \left[ 1 - \frac{h f'(h)}{f'(h)} + \frac{h f''(h)}{f'(h)} \right] = \frac{1}{h} \eta_1(h) [1 - \eta_1(h) - \eta_2(h)]$$

(A6)

Taking the logarithm of (24) and differentiating, one gets:

$$- \frac{1}{1 - \eta_1} \frac{d\eta_1}{dh} \frac{dh}{d\tau} = - \frac{1}{1 - \tau} + \frac{1}{[1 + Z(\Lambda)]} \frac{dZ}{d\Lambda} \frac{d\Lambda}{d\tau}$$

(A7)

Likewise, taking the logarithm of (22) and differentiating, one finds that:

$$(A8) \quad \frac{1}{h} \frac{dh}{d\tau} = - \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau}$$

Substituting (A5), (A6) and (A8) into (A7) yields

$$\frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau} = - \frac{1}{1-\tau} + \frac{Z(\Lambda)}{1+Z(\Lambda)} \left[ \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \right] \frac{d\Lambda}{d\tau}$$

Rearrangig one gets

$$(A9) \quad \frac{d\Lambda}{d\tau} = \frac{1}{1-\tau} \left\{ \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[ Z(\Lambda) - \frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \right] \right\}^{-1}$$

In order to prove (I.1), it is sufficient to show that for all  $h > 0$  and  $\Lambda$ ,

$$Z(\Lambda) = \frac{1-\eta_1(h)}{(1-b)(1-\tau)} - 1 \geq \frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)}$$

where the equality follows from (24). From (P6) of Lemma 2, it is sufficient to prove that the polinomial  $P(\eta_1)$  defined below is not negative for  $0 < \eta_1 \leq b$ :

$$\begin{aligned} P(\eta_1) &\equiv (1-\eta_1)^2 - (1-\eta_1)(1-b)(1-\tau) - (1-b)(1-\tau)\eta_1[1-\eta_1-\eta_2] \\ &= \eta_1^2 [1+(1-b)(1-\tau)] - \eta_1 [2-\eta_2(1-b)(1-\tau)] + [1-(1-b)(1-\tau)] \end{aligned}$$

From (P7) of Lemma 2,  $\eta_2 > 1-b$ , thefore for all  $\eta_1$ , the inequality below holds

$$P(\eta_1) \geq \eta_1^2 [1+(1-b)(1-\tau)] - \eta_1 [2-(1-b)^2(1-\tau)] + [1-(1-b)(1-\tau)]$$

Let  $Q(\eta_1)$  stand for the polinomial on the right hand side of the inequality above. It is sufficient to show that  $Q(\eta_1) > 0$  for all  $\eta_1 \in (0, b)$ .  $Q$  is u-shaped and if it has real roots, both are positive and the derivative of  $Q$  is negative for all  $\eta_1$  lower than its lowest root.

Since the derivative of the polynomial  $Q$  at  $\eta_1 = b$ ,  $Q'(b) = -(1-b)[2 - (1+b)(1-\tau)] < 0$ , and  $Q(b) = \tau(1-b)^2 \geq 0$ , it follows that for all  $\eta_1 \in (0, b)$ ,  $Q(\eta_1) \geq Q(b) \geq 0$ . This completes the proof of (I.1).

The proof of (I.2) is immediate for (A8) assures that  $d\Lambda(t)/d\tau > 0$  implies  $dh(t)/d\tau < 0$ .

### Proof of I.3

Applying logarithms to (25) and differentiating, one gets:

$$\frac{d \ln w}{d\tau} = \frac{f'(h)[\eta_2(h) - (1-b)]}{b f(h) - h f'(h)} \frac{dh}{d\tau} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \frac{d\Lambda}{d\tau}$$

From Lemma 2,  $[\eta_2 - (1-b)] > 0$ . The derivatives  $dh/d\tau < 0$  and  $d\Lambda/d\tau > 0$ , imply  $dw(t)/d\tau < 0$ .

### Proof of I.4 and I.5

Applying logarithms to (27) and differentiating, yields

$$\begin{aligned} \frac{d \ln Y_\lambda}{d\tau} &= b \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \frac{d\Lambda}{d\tau} + \frac{f'(h)}{f(h)} \frac{dh}{d\tau} \\ &= \frac{g(\Lambda)}{G(\Lambda)} \left[ b [1 + Z(\Lambda)] - \frac{h f'(h)}{f(h)} \right] \frac{d\Lambda}{d\tau} \end{aligned}$$

where the last equality follows from (A8). Rearranging terms, one gets

$$\frac{d \ln Y_\lambda}{d\tau} = \frac{g(\Lambda)}{G(\Lambda)} [b Z + (b - \eta_1)] \frac{d\Lambda}{d\tau} > 0 \quad (\text{A10})$$

where the inequality follows from Lemma 2, (P6) and  $d\Lambda/d\tau > 0$ . The proof of (I.5) is immediate since  $\Pi_\lambda = (1-b)Y_\lambda$ .

## Proof of I.6

From (14), (25) and  $\Pi_\lambda = (1-b)Y_\lambda$ ,

$$P_L = \frac{Y_\lambda}{N_\lambda} = \frac{\Pi_\lambda}{(1-b)N_\lambda} = \frac{w f(h)}{b f(h) - h f'(h)} = f(h) \left( \int_\lambda^1 \lambda^{-\frac{1}{1+b}} g(\lambda) d\lambda / G(\Lambda) \right)^{1-b}$$

Taking logarithms, differentiating and substanting (A8) yields

$$\frac{d \ln P_L}{d\tau} = -[\eta_l(h) + (1-b)(1+Z)] \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau} < 0$$

The average productivity of capital is given by  $P_K = Y_\lambda / K_\lambda = (Y_\lambda / N_\lambda)(N_\lambda / K_\lambda) = P_L h$ . Hence, from (A8) and (24) one gets

$$\frac{d \ln P_K}{d\tau} = -(1-\eta_l) \frac{\tau}{(1-\tau)} \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau} < 0.$$

Since  $h$  falls, it follows that the fall of  $h$  is relatively lower than that of  $P_L$ .

## Proof of I.7

Applying logarithms to (28) and differentiating, one gets:

$$\frac{d \ln Y}{d\tau} = \frac{f'(h)}{f(h)} \frac{dh}{d\tau} + \frac{g(\Lambda)}{G(\Lambda)} [b - (1-b)Z(\Lambda)] \frac{d\Lambda}{d\tau}$$

Substituting (A9) and (24) yields:

$$\begin{aligned} \frac{d \ln Y}{d\tau} &= \frac{g(\Lambda)}{G(\Lambda)} \left[ [b - \eta_l(h)] - (1-b) \left[ \frac{1 - \eta_l(h)}{(1-\tau)(1-b)} - 1 \right] \right] \frac{d\Lambda}{d\tau} \\ &= -\frac{g(\Lambda)}{G(\Lambda)} [1 - \eta_l(h)] \frac{\tau}{1-\tau} \frac{d\Lambda}{d\tau} < 0 \end{aligned}$$

## Proof of I.8

From (13), (14) and (25),

$$(1-\tau)\Pi_\lambda = \frac{1-\eta_l(h)}{1+Z(\Lambda)} f(h) \left( G(\Lambda) / \int_\lambda^1 \lambda^{-\frac{1}{1+b}} g(\lambda) d\lambda \right)^b \lambda^{\frac{1}{1+b}}$$

Taking logarithms and differentiating yields



$$\frac{d \ln[(1-\tau)\Pi_\lambda]}{d\tau} = \left[ \frac{f'(h)}{f(h)} - \frac{1}{1-\eta_1} \frac{d\eta_1}{dh} \right] \frac{dh}{d\tau} + \left[ b[1+Z(\Lambda)] \frac{g(\Lambda)}{G(\Lambda)} - \frac{1}{[1+Z(\Lambda)]} \frac{dZ}{d\Lambda} \right] \frac{d\Lambda}{d\tau}$$

Substituting (A5), (A6) and (A8) into this expression one gets

$$\frac{d \ln[(1-\tau)\Pi_\lambda]}{d\tau} = \left\{ \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[ \frac{\eta_1(h) \eta_2(h)}{1-\eta_1(h)} - b[1+Z(\Lambda)] + Z(\Lambda) \right] \right\} \frac{d\Lambda}{d\tau}$$

It is sufficient to show that the expression inside the brackets multiplied by  $g(\Lambda)/G(\Lambda)$  is positive. From (24),

$$\frac{g(\Lambda)}{G(\Lambda)} \left[ \frac{\eta_1(h) \eta_2(h)}{1-\eta_1(h)} - 1 + \frac{1-\eta_1(h)}{1-\tau} \right] \geq \frac{g(\Lambda)}{G(\Lambda)} \frac{\eta_1(h) [\eta_1(h) + \eta_2(h) - 1]}{1-\eta_1(h)} \geq 0 \quad (\text{A11})$$

where the inequality follows from the hypothesis that  $\sigma(h) \leq 1$  and (P8).

### Proof of I.9

Applying logarithms to (26) and differentiating onw gets:

$$\frac{1}{(r+\delta)} \frac{dr}{d\tau} = \frac{f''(h)}{f(h)} \frac{dh}{d\tau} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \frac{d\Lambda}{d\tau}$$

Substituting (A8) and (24) into this expression yields:

$$\frac{1}{(r+\delta)} \frac{dr}{d\tau} = \frac{g(\Lambda)}{G(\Lambda)} \left[ \eta_2(h) - \frac{1-\eta_1(h)}{1-\tau} \right] \frac{d\Lambda}{d\tau} = \frac{[(\eta_1(h) + \eta_2(h) - 1) - \tau \eta_2(h)]}{1-\tau} \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau}$$

From (P8), it follows that for  $\sigma(h) \geq 1$ ,  $[(\eta_1(h) + \eta_2(h) - 1) - \tau \eta_2(h)] < 0$ , and hence  $dr/d\tau < 0$ . For  $\sigma(h) < 1$ , (A4) gives  $\eta_1(h) = b(1 - \sigma \eta_2(h)) / [1 - (1-b)\sigma(h)]$ , and  $dr/d\tau < 0$  if and only if, for all  $h > 0$ :

$$\tau > \frac{\eta_1(h) + \eta_2(h) - 1}{\eta_2(h)} = [1 - \sigma(h)] \frac{1 - (1-b)/\eta_2(h)}{1 - (1-b)\sigma(h)}$$

From (P9),

$$\sigma < \frac{1}{\eta_2(h)} \Rightarrow -(1-b) \sigma(h) > -\frac{1-b}{\eta_2(h)} \Rightarrow 1 > \frac{1-(1-b)/\eta_2(h)}{1-(1-b)\sigma(h)}$$

Therefore, if  $\tau > (1-\bar{\sigma})$ ,  $dr/d\tau < 0$ .

### Proof of I.10

From (14) and (25),  $R$  is given as

$$R = f(h) [b - \eta_1(h)] \left( \int_{\Lambda} \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda / G(\Lambda) \right)^{1-b} \left[ 1 + \frac{\tau(1-b)}{b - \eta_1(h)} \right]$$

Substituting (24) into the expression in the last bracket yields:

$$R = \frac{Z(\Lambda)}{1+Z(\Lambda)} \left( \int_{\Lambda} \lambda^{\frac{1}{1-b}} g(\lambda) d\lambda / G(\Lambda) \right)^{1-b} f(h) [1 - \eta_1(h)]$$

Taking logarithms and differentiating one gets:

$$\frac{d \ln R}{d\tau} = \left[ \frac{1}{Z(\Lambda)[1+Z(\Lambda)]} \frac{dZ}{d\Lambda} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \right] \frac{d\Lambda}{d\tau} + \left[ \frac{f'(h)}{f(h)} - \frac{1}{[1-\eta_1(\Lambda)]} \frac{d\eta_1}{dh} \right] \frac{dh}{d\tau}$$

Substituting (A5), (A6) and (A8) into expression gives

$$\frac{d \ln R}{d\tau} = \left\{ \frac{1}{(1-b)\Lambda[1+Z(\Lambda)]} + \frac{g(\Lambda)}{G(\Lambda)} \left[ 1 - \frac{1-\eta_1(h)}{1-\tau} - \eta_1(h) + \frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \right] \right\} \frac{d\Lambda}{d\tau}$$

The expression inside the brackets which is multiplied by  $g(\Lambda)/G(\Lambda)$  is negative for  $\sigma \leq 1$  - see (A11). The first term is a decreasing function of  $\tau$  and tends to zero as  $\Lambda$  tends to 1, i.e., as  $\tau$  tends to one. Hence, for a bounded density  $g$ , one concludes that  $dR(t)/d\tau < 0$  for  $\tau$  close to 1.

For  $\tau$  close to zero,  $dR(t)/d\tau$  is given as

$$\left. \frac{d \ln R(t)}{d\tau} \right|_{\tau=0} \cong \left\{ \frac{1}{(1-b)\Lambda[1+Z(\Lambda)]} + \frac{g(\Lambda)}{G(\Lambda)} \frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \right\} \frac{d\Lambda}{d\tau} \quad (A12)$$

Two alternative conditions are sufficient to assure that (A12) is positive. The first is the hypothesis that  $\sigma(h) \geq 1$  for all  $h > 0$ , which from (P9) implies  $(1 - \eta_1 - \eta_2) \geq 0$ .

If, however  $\sigma(h) < 1$  for some  $h > 0$ , than a second alternative sufficient condition is that  $g' \leq 0$  together with  $\frac{1}{5} \leq \sigma(h)$  for all  $h > 0$ . In fact, for  $g' \leq 0$ ,

$$G(\Lambda) = \int_0^\Lambda g(\lambda) d\lambda \geq \int_0^\Lambda G(\Lambda) d\Lambda = g(\Lambda) \Lambda$$

Hence

$$\frac{d \ln R(t)}{d\tau} \geq \frac{g(\Lambda)}{G(\Lambda)} \left[ \frac{1}{(1-b)[1+Z(\Lambda)]} + \frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \right] \frac{d\Lambda}{d\tau}$$

From (24), the expression above can be written as:

$$\frac{d \ln R(t)}{d\tau} \geq \frac{g(\Lambda)}{G(\Lambda)} \frac{1 + \eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \frac{d\Lambda}{d\tau}$$

From (A4),

$$1 + \eta_1(h)[1-\eta_1(h)-\eta_2(h)] = 1 - \frac{b[1-\sigma(h)]}{[1-(1-b)\sigma(h)]^2} [1-\sigma\eta_2(h)][\eta_2(h)-(1-b)]$$

For a given  $\sigma$ , the function  $V: (1-b, +\infty) \rightarrow \mathcal{R}$ ,  $V(\eta_2) = [1-\sigma\eta_2][\eta_2-(1-b)]$  has a global maximum at  $\eta_2^* = [\sigma(1-b)+1]/2\sigma$ , and  $V(\eta_2^*) = [1-\sigma(1-b)]^2/4\sigma$ . Hence,

$$1 + \eta_1(h)[1-\eta_1(h)-\eta_2(h)] \geq 1 - \frac{b(1-\sigma)}{4\sigma} = \frac{4+b}{4\sigma} \left( \sigma - \frac{b}{4+b} \right)$$

Since,  $b < 1$ ,  $\frac{b}{4+b} < \frac{1}{5} \leq \sigma$ , which completes the proof.

## Proof of Proposition II

### Proof of (II.1) and (II.2)

Applying logarithms to (31) and differentiating one gets,

$$\eta_2(h) \frac{dh}{h} = -(1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] d\Lambda \quad (\text{A14})$$

Applying logarithms to (32) and differentiating yields,

$$\frac{1}{[1-\eta_1(h)]} \frac{d\eta_1}{dh} dh + \frac{1}{[1+Z(\Lambda)]} \frac{dZ}{d\Lambda} d\Lambda = \frac{d\tau}{1-\tau}$$

Substituting (A5) and (A6) into this last expression one finds:

$$\frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \frac{dh}{h} + \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \left[ \frac{a}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \right] d\Lambda = \frac{d\tau}{1-\tau}$$

Substituting (A14) into this expression gives:

$$\left\{ \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[ Z(\Lambda) - \frac{\eta_1(h)}{\eta_2(h)} \frac{[1-\eta_1(h)-\eta_2(h)]}{[1-\eta_1(h)]} (1-b)[1+Z(\Lambda)] \right] \right\} d\Lambda = \frac{d\tau}{1-\tau} \quad (A15)$$

Substituting Z from (24), the term in the brackets multiplied by  $g(\Lambda)/G(\Lambda)$  can be reduced to:

$$\begin{aligned} \frac{g(\Lambda)}{G(\Lambda)} [ ] &= \frac{g(\Lambda)}{G(\Lambda)} \left[ \frac{\eta_1(h)}{\eta_2(h)} \frac{[\eta_1(h)+\eta_2(h)-1]}{(1-\tau)} + \frac{1-\eta_1(h)}{(1-\tau)(1-b)} - 1 \right] \\ &= \frac{\eta_2(h)\{b[1-\eta_1(h)] + \tau(1-b)\} - (1-b)\eta_1(h)[1-\eta_1(h)]}{(1-\tau)(1-b)\eta_2(h)} \end{aligned}$$

It is sufficient to show that the numerator of this fraction is positive. Since  $\eta_2 > (1-b)$ , letting N stand for this numerator, one has:

$$N > (1-b)[b(1-\eta_1) + \tau(1-b) - \eta_1(1-\eta_1)] = (1-b)[(b-\eta_1)(1-\eta_1) + \tau(1-b)] > 0$$

This shows that  $d\Lambda^*/d\tau > 0$ . From (A14), it follows that  $dh^*/d\tau < 0$

### Proof of II.3

Applying logarithms to (25) and differentiating and substituting (A3) yields

$$\frac{d \ln W}{d\tau} = \eta_1(h) \frac{\eta_2(h) - (1-b)}{b - \eta_1(h)} \frac{1}{h} \frac{dh}{d\tau} - (1-b) \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \frac{d\Lambda}{d\tau} < 0$$

where the equality follows from propositions (II.1) and (II.2).

### Proof of II.4 and II.5

Applying logarithms to (27) and differentiating yields

$$\frac{d \ln Y_\lambda}{d\tau} = \frac{\eta_1(h)}{h} \frac{dh}{d\tau} + b \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \frac{d\Lambda}{d\tau}$$

Substituting (A14) into this expression one gets

$$\frac{d \ln Y_\lambda}{d\tau} = \frac{g(\Lambda)}{G(\Lambda)} [1 + Z(\Lambda)] \frac{[b\eta_2(h) - (1-b)\eta_1(h)]}{\eta_2(h)} \frac{d\Lambda}{d\tau} > 0 \quad (\text{A17})$$

where the inequality follows from (P6), (P7) and proposition (II.1). The proof of (II.5) is immediate since  $\Pi_\lambda = (1-b)Y_\lambda$

### Proof of II.6

From (14), (25) and  $\Pi_\lambda = (1-b)Y_\lambda$ ,

$$P_L = \frac{\Pi_\lambda}{(1-b)N_\lambda} = f(h) \left( \int_\Lambda^\Lambda \lambda^{-\frac{1}{\sigma}} g(\lambda) d\lambda / G(\Lambda) \right)^{1-b}$$

Taking logarithms and differentiating

$$\frac{d \ln P_L}{d\tau} = \frac{f'(h)}{f(h)} \frac{dh}{d\tau} - (1-b) [1 + Z(\Lambda)] \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau}$$

Substituting (A14) into the expression gives

$$\frac{d \ln P_L}{d\tau} = -(1-b) [1 + Z(\Lambda)] \frac{\eta_1(h) + \eta_2(h)}{\eta_2(h)} \frac{d\Lambda}{d\tau} < 0$$

The average productivity of capital is given by  $P_K = P_L / h$

$$\frac{d \ln P_K}{d\tau} = \frac{d \ln P_L}{d\tau} - \frac{d \ln h}{d\tau}$$

Substituting (A14) and (A15) into the last expression yields

$$\frac{d \ln P_K}{d\tau} = (1-b) [1 + Z(\Lambda)] \frac{1 - \eta_1(h) - \eta_2(h)}{\eta_2(h)} \frac{d\Lambda}{d\tau} \leq 0$$

Where the inequality follows from the hypothesis that  $\sigma(h) \leq 1$  for all  $h > 0$ .

### Proof of II.7

Applying logarithms to (28) and differentiating yields

$$\frac{d \ln Y}{d \tau} = \eta_1(h) \frac{1}{h} \frac{dh}{d \tau} + \frac{g(\Lambda)}{G(\Lambda)} [b - (1-b)Z(\Lambda)] \frac{d \Lambda}{d \tau}$$

Substituting (A14) into this expression gets

$$\frac{d \ln Y}{d \tau} = \frac{g}{G(\Lambda)} \left[ b - (1-b)Z(\Lambda) - \frac{(1-b)\eta_1(h)[1+Z(\Lambda)]}{\eta_2(h)} \frac{d \Lambda}{d \tau} \right]$$

Substituting Z from (24)

$$\begin{aligned} \frac{d \ln Y}{d \tau} &= \frac{1}{\eta_2(h)} \frac{g(\Lambda)}{G(\Lambda)} \left[ \eta_2(h) \left[ b - (1-b) \left[ \frac{1 - \eta_1(h)}{(1-b)(1-\tau)} - 1 \right] \right] - \eta_1(h) \frac{1 - \eta_1(h)}{1-\tau} \right] \frac{d \Lambda}{d \tau} \\ &= \frac{1}{\eta_2(h)(1-\tau)} \frac{g(\Lambda)}{G(\Lambda)} [\eta_1(h)[\eta_1(h) + \eta_2(h) - 1] - \tau \eta_2(h)] \frac{d \Lambda}{d \tau} \end{aligned}$$

From (P8), one concludes that if  $\sigma(h) \geq 1$  for all  $h > 0$ , then  $d \ln Y / d \tau > 0$ . If  $\sigma(h) < 1$  for some  $h > 0$ , then  $dY^* / d \tau < 0$  if, and only if,

$$\tau > \eta_1(h) \frac{\eta_1(h) + \eta_2(h) - 1}{\eta_2(h)}$$

Substituting  $\eta_1$  from (A4) into this expression, then  $dY^* / d \tau < 0$  if, and only if,

$$\tau > \frac{b[1 - \sigma(h)]}{[1 - (1-b)\sigma(h)]^2} [\eta_2(h) - (1-b)][1 / \eta_2(h) - \sigma(h)]$$

For a given  $\sigma$ , the function  $X: (1-b, +\infty) \rightarrow \mathcal{R}$ ,  $X(\eta_2) = [\eta_2 - (1-b)][1 / \eta_2 - \sigma]$  has a global maximum at  $\eta_2^* = \sqrt{(1-b) / \sigma}$  and  $X(\eta_2^*) = (1 - \sqrt{(1-b)\sigma})^2$ . Hence, it is sufficient that

$$\tau > b(1-\sigma) \frac{(1 - \sqrt{(1-b)\sigma})^2}{[1 - \sigma(1-b)]^2} = \frac{b(1-\sigma)(1 - \sqrt{(1-b)\sigma})^2}{[(1 - \sqrt{(1-b)\sigma})(1 + \sqrt{(1-b)\sigma})]^2} = \frac{b(1-\sigma)}{(1 + \sqrt{(1-b)\sigma})^2}$$

Since  $\sigma(h) \geq \bar{\sigma}$  for all  $h > 0$  and the fraction on the right hand side of the inequality above is a decreasing function of  $\sigma$ , it follows that if

$$\tau > b(1 - \bar{\sigma}) / (1 + \sqrt{(1 - b)\bar{\sigma}})^2$$

than  $d \ln Y / d\tau < 0$ .

### Proof of II.8

Taking logarithms of (A11), differentiating and substituting (A6) yields

$$\frac{d \ln}{d\tau} (1 - \tau) \Pi_\lambda = \left[ \eta_1(h) - \frac{\eta_1(h)[1 - \eta_1(h) - \eta_2(h)]}{1 - \eta_1(h)} \right] \frac{dh}{h} + \left[ b[1 + Z(\Lambda)] \frac{g(\Lambda)}{G(\Lambda)} - \frac{1}{[1 + Z(\Lambda)]} \frac{dZ}{d\Lambda} \right] \frac{d\Lambda}{d\tau}$$

Substituting (A7) and (A11) one gets:

$$\frac{d \ln}{d\tau} (1 - \tau) \Pi_\lambda = - \left\{ \frac{Z(\Lambda)}{(1 - b)\Lambda[1 + Z(\Lambda)]} + \frac{g(\Lambda)}{G(\Lambda)} \left[ Z(\Lambda) - b[1 + Z(\Lambda)] + \frac{\eta_1(h)}{1 - \eta_1(h)} (1 - b)[1 + Z(\Lambda)] \right] \right\} \frac{d\Lambda}{d\tau}$$

Substituting (24) one obtains

$$\frac{d \ln}{d\tau} (1 - \tau) \Pi_\lambda = - \left\{ \frac{1}{\Lambda(1 - b)} \frac{Z(\Lambda)}{[1 + Z(\Lambda)]} + \frac{\tau}{1 - \tau} \right\} \frac{d\Lambda}{d\tau} < 0$$

### Proof of II.9

Applying logarithms to (33) and differentiating yields

$$\frac{d \ln K^*}{d\tau} = \frac{1}{h} \frac{dh}{d\tau} + \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau}$$

Substituting (A14) into this expression one obtains

$$\frac{d \ln K}{d\tau} = \frac{g(\Lambda)}{G(\Lambda)} \left[ 1 - \frac{(1 - b)[1 + Z(\Lambda)]}{\eta_2(h)} \right] \frac{d\Lambda}{d\tau}$$

Substituting Z from (24) one gets

$$\frac{d \ln K}{d\tau} = \frac{1}{[(1 - \tau)\eta_2(h)]} \frac{g(\Lambda)}{G(\Lambda)} [(\eta_1(h) + \eta_2(h) - 1) - \tau \eta_2(h)] \frac{d\Lambda}{d\tau}$$

The expression inside the brackets is the same expression that appeared in the proof of (I.9) which was shown to be negative if  $\tau > 1 - \bar{\sigma}$ .

## Proof II.10

$R^*$  is given by (A13). Taking logarithms and differentiating yields

$$\frac{d \ln R}{d\tau} = \frac{1}{Z(\Lambda)[1-Z(\Lambda)]} \frac{dZ}{d\Lambda} \frac{d\Lambda}{d\tau} - (1-b)[1+Z(\Lambda)] \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau} + \left[ \frac{f'(h)}{f(h)} - \frac{1}{[1-\eta_1(h)]} \frac{d\eta_1}{dh} \right] \frac{1}{h} \frac{dh}{d\tau}$$

Substituting  $dZ/d\Lambda$  from (A5),  $d\eta_1/dh$  from (A6) and  $dh/d\tau$  from (A14) one gets

$$\begin{aligned} \frac{d \ln R}{d\tau} &= \left\{ \frac{1}{[1+Z(\Lambda)]} \left[ \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \right] - (1-b)[1+Z(\Lambda)] \frac{g(\Lambda)}{G(\Lambda)} \right\} \frac{d\Lambda}{d\tau} \\ &\quad - \eta_1(h) \left[ 1 - \frac{1-\eta_1(h)-\eta_2(h)}{1-\eta_1(h)} \right] \frac{(1-b)[1+Z(\Lambda)]}{\eta_2(h)} \frac{g(\Lambda)}{G(\Lambda)} \frac{d\Lambda}{d\tau} \\ &= \left\{ \frac{1}{(1+Z)(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} \left[ 1 - (1-b)[1+Z(\Lambda)] \left[ 1 + \frac{\eta_1(h)}{1-\eta_1(h)} \right] \right] \right\} \frac{d\Lambda}{d\tau} \end{aligned}$$

Substituting  $Z$  from (24) one obtains

$$= \left\{ \frac{1}{(1-b)\Lambda[1+Z(\Lambda)]} - \frac{\tau}{1-\tau} \frac{g(\Lambda)}{G(\Lambda)} \right\} \frac{d\Lambda}{d\tau}$$

the first term above tends to zero as  $\tau$  tends to one, hence for a bounded density  $dR^*/d\tau < 0$  for  $\tau$  close to 1. For  $\tau=0$ ,  $dR^*/d\tau > 0$ .

## Proof of Proposition III

Taking logarithms of (22) and differentiating with respect to time, yields

$$\frac{\dot{K}(t)}{K(t)} = \frac{\dot{h}(t)}{h(t)} + \frac{g(\Lambda)}{G(\Lambda)} \dot{\Lambda}(t)$$

where the dot on the variables stands for the time derivate. Likewise, taking logarithms of (24) and differentiating with respect to time gives:

$$-\frac{\eta_1(h)[1-\eta_1(h)-\eta_2(h)]}{1-\eta_1(h)} \frac{\dot{h}(t)}{h(t)} = \frac{Z(\Lambda)}{[1+Z(\Lambda)]} \left[ \frac{1}{(1-b)\Lambda} + \frac{g(\Lambda)}{G(\Lambda)} [1+Z(\Lambda)] \right] \dot{\Lambda}(t) \quad (\text{A.18})$$



Eliminating  $\dot{h}/h$  from the last two expressions one gets

$$\begin{aligned} & \frac{\eta_1(h)[\eta_1(h) + \eta_2(h) - 1]}{1 - \eta_1(h)} \frac{\dot{K}(t)}{K(t)} = \\ & = \left\{ \frac{Z(\Lambda)}{[1 + Z(\Lambda)]} \frac{1}{(1-b)} + \frac{g(\Lambda)}{G(\Lambda)} \left[ Z(\Lambda) - \frac{\eta_1(h)[1 - \eta_1(h) - \eta_2(h)]}{1 - \eta_1(h)} \right] \right\} \dot{\Lambda}(t) \end{aligned}$$

The expression in the brackets which is multiplied by  $g(\Lambda)/G(\Lambda)$  was shown to be positive - see proof of proposition I.1. Therefore, according to (P8), if  $\sigma(h) \leq 1$  for all  $h > 0$ , then  $\dot{K}(t)\dot{\Lambda}(t) \geq 0$  and if  $\sigma(h) > 1$  for all  $h > 0$  then  $\dot{K}(t)\dot{\Lambda}(t) < 0$ . By (A.18), if  $\sigma(h) \leq 1$  for all  $h > 0$ , then  $\dot{K}(t)\dot{h}(t) \geq 0$  and if  $\sigma(h) > 1$  for all  $h > 0$  then  $\dot{K}(t)\dot{h}(t) < 0$ .

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