Escola de Pós-Graduação em Economia - EPGE
Fundação Getulio Vargas

Current Account and Capital Mobility Hypothesis: evidence from the G-7

Dissertação submetida à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas como requisito para obtenção do Título de Mestre em Economia

Aluno: Wagner Piazza Gaglianone

Orientador: João Victor Issler

Rio de Janeiro
2004

Escola de Pós-Graduação em Economia - EPGE
Fundação Getulio Vargas

Current Account and Capital Mobility Hypothesis: evidence from the G-7

Dissertação submetida à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas como requisito para obtenção do Título de Mestre em Economia

Aluno: Wagner Piazza Gaglianone

Banca Examinadora:
João Victor Issler (Orientador, EPGE/FGV)
Maria Cristina Terra (EPGE/FGV)
Luiz Renato Lima (EPGE/FGV)
Fabiana Fontes Rocha (FEA-USP)

Rio de Janeiro
2004

Abstract

This paper investigates an intertemporal optimization model in order to analyze the current account of the G-7 countries, measured as the present value of the future changes in net output. The study compares observed and forecasted series, generated by the model, using Campbell & Shiller's (1987) methodology. In the estimation process, the countries are considered separately (with OLS technique) as well as jointly (SURE approach), to capture contemporaneous correlations of the shocks in net output. The paper also proposes a note on Granger causality and its implications to the optimal current account. The empirical results are sensitive to the technique adopted in the estimation process and suggest a rejection of the model in the G-7 countries, except for the USA and Japan, according to some papers presented in the literature.

Agradecimentos

A Deus, sempre.

Aos meus pais Paulo e Sandra, e minha irmã Karina, que sempre se dedicaram, com muito amor, ao meu crescimento pessoal e profissional.

À Carla, pelo seu amor, carinho e incentivo constante.

Aos amigos Eduardo, Fábio e Otávio, pelo eterno companheirismo.

À Angela Rangel, Zachary Lee e Colin Jolly, pelo auxílio na revisão do texto.

Ao meu orientador no Banco Central do Brasil, Octávio M. Bessada Lion, que sempre me apoiou nesta jornada.

Ao Professor João Victor Issler, pela orientação e oportunidade de pesquisa conjunta, sempre com muita paciência e dedicação.

Aos membros da banca examinadora, Prof.ª Fabiana Fontes Rocha, Prof.ª Maria Cristina Terra e Prof. Luiz Renato Lima, pelas valiosas críticas e contribuições a este trabalho.

Aos colegas de turma, pela amizade e companheirismo durante estes anos de convivência.

Aos funcionários da EPGE e da Biblioteca Mario Henrique Simonsen, pelo apoio vital à conclusão deste trabalho.

Aos Professores da EPGE, pelo compromisso com a excelência acadêmica.

Finalmente, ao Banco Central do Brasil, pelo patrocínio e pela oportunidade de uma formação continuada.

Contents

1 Introduction

In spite of growing international trade and the evolution of global capital markets, in recent decades, there is still no consensus in the literature about the perfect capital mobility hypothesis (CMH). The studies on international capital mobility use several measurement criteria and can be divided into three branches. The first approach, initially proposed by Feldstein & Horioka (1980), tests CMH by regressing the investment ratio on the saving ratio, where a positive close-to-one coefficient suggests imperfect capital mobility. The second strand measures capital mobility in terms of deviations from international parity conditions (covered, uncovered and real interest parities - CIP, UIP and RIP), since the rate of return should be equal across countries, in a CMH framework. Finally, a more recent approach assesses capital mobility in terms of consumption smoothing, in response to shocks in expenditure variables.

Concerning this topic, several authors have analyzed the open economy model, initially proposed by Sachs (1982) and later detailed by Obstfeld & Rogoff (1994), that incorporates the Permanent Income Hypothesis (PIH). In this context, the current account is used as a 'buffer' to smooth consumption, which is proportional to permanent income ('people save for a rainy day'). The current account presents deficits, in this theoretical outline, whenever there are expectations of a larger future net output (also known as national cash flow), defined as the gross domestic product excluding investment and the government's expenses.

This theoretical model defines the optimal current account from the agents' intertemporal optimization problem, supposing that agents can freely smooth consumption in the presence of shocks. The comparison of this optimal value with the observed current account allows us to test the capital mobility hypothesis. Additionally, the estimation of the volatility of these two series permits the identification of speculative capitals in the current account, not related to the smoothing consumption component. The econometric methodology adopted in this paper, to analyze the theoretical model, was proposed by Campbell & Shiller (1987) and consists of estimating an unrestricted VAR, whose parameters are used in the construction of the optimal current account.

Although the literature based on Campbell & Shiller's approach is relatively extensive, the following papers should be mentioned: Sheffrin and Woo (1990) test the theoretical model for Canada, Belgium, Denmark and the United Kingdom, and do not reject it for Belgium, suggesting that agents can freely smooth consumption in this country. Otto (1992) rejects the model for Canada and the USA, and Ghosh (1995) rejects the model for the USA (except for one period), Canada, Japan, the United Kingdom, and Germany. On the other hand, Ghosh and Ostry (1995) test the model for developing countries and do not reject it in about 2/3 of the sample, indicating a low degree of capital control for these countries.

Several recent papers have presented some extensions of the theoretical framework, in order to justify the rejection of the model in previous works: Ghosh & Ostry (1997) consider precautionary saving, Gruber (2000) studies habit formation, İşcan (2002) allows for durables and non-traded goods, and Kano (2003) introduces a stochastic world interest rate (see Literature Review).

Unlike this recent literature, the objective of this paper is to provide a simple econometric approach to the standard model, in order to improve the efficiency of the estimation process. The SURE estimation provides a gain in efficiency, when compared to the OLS method, and could lead to a rejection of the model previously accepted in the OLS framework, widely used in the literature.

Hence, the contribution of this paper is twofold: to analyze the model for the G-7 countries and verify the impact of the estimation method of the VAR (accomplished with OLS and SURE techniques),[1] and to provide a note on Granger causality, suggesting that this implication of the theoretical model is a necessary condition to perform a Wald test for the VAR and verify the validation of the model.

This study is structured in the following manner. Section 2 provides an overview of the theoretical model and the econometric techniques used to test it. Section 3 presents a brief literature review. Section 4 describes the datasets, the statistical tests and the results, and Section 5 concludes it.

2 Methodology

2.1 Theoretical Model

The Present Value Model (PVM) adopted to analyze the intertemporal optimization problem of a representative agent is based on Sachs (1982), considering the perfect capital mobility hypothesis across countries. In this context, countries save through flows of capital in their current accounts, according to their expectations of future changes in net output. Thus, the current account is used as an instrument of consumption smoothing against possible shocks to the economy, and can be expressed by

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - I_t - G_t - C_t \tag{1}$$

where B_t represents foreign assets, Y_t gross domestic product (GDP), r the world interest rate, I_t total investment, G_t the government's expenses and C_t aggregated consumption.

The consumption path, related to the dynamics of the current account, can be divided into two components: the trend term, generated by the difference between the world interest rate and the rate of time

[1] Following the methodology adopted in Issler-Teixeira (2003).

preference, and the smoothing component, related to the expectations of changes in permanent income. This paper only studies the second component effect, by isolating from the current account, the trend component in consumption. Thus, the optimal current account (only associated with the consumption smoothing term) is given by

$$CA_t^* = Y_t + rB_t - I_t - G_t - \theta C_t \tag{2}$$

where θ is a parameter that removes the trend component in consumption.[2] The net output Z_t , also known in the literature as national cash flow, is defined by

$$Z_t \equiv Y_t - I_t - G_t \tag{3}$$

Substituting the optimal consumption expression in equation (2), it can be shown that the present value relationship between the current account and the future changes in net output is given by (see Ghosh and Ostry, 1995):

$$CA_t^* = -\sum_{j=1}^{\infty}(\frac{1}{1+r})^j E_t(\Delta Z_{t+j} \mid R_t) \tag{4}$$

where R_t is the agent's information set. It should be mentioned that the main assumptions of the model are time-separable preferences, zero depreciation of capital, and complete asset markets. A quadratic form is also adopted for the utility function, without precautionary saving effects (see Ghosh & Ostry, 1997).

According to equation (4), the optimal current account is equal to minus the present value of the expected changes in net output. For instance, the representative agent will increase its current account, accumulating foreign assets, if a future decrease in income is expected, and vice-versa.

2.2 Econometric Model

The econometric model is based on the methodology developed by Campbell & Shiller (1987), which consists of an alternative method to test the Permanent Income Hypothesis. According to the authors, several difficulties might arise when testing a PVM. In this manner, Campbell & Shiller suggest an alternative way to verify a PVM, when the involved variables are stationary.

The idea is to test a set of restrictions imposed to a Vector Auto Regression (VAR), used to forecast the current account through equation (4). The advantage of this approach is that, although the econometrician does not observe the agent's information set, this framework allows us to summarize all the relevant information through the variables used in the construction of the VAR.

[2] The tilt parameter (θ) is not equal to one whenever the rate of time preference differs from the world interest rate.

However, to apply this methodology, the VAR must be stationary. Hence, the first empirical proposition is to verify whether ΔZ_t is a stationary variable (or in other words, is an integrated of order zero I(0) variable). The current account (in level) must also be a stationary variable, since it can be written as a lineal combination of stationary variables (via equation (4)). The stationarity of these variables will be checked later by unit root tests. Campbell & Shiller argue that series represented by a Vector Error Correction Model (VECM) can be rewritten as an unrestricted VAR. Thus, consider the following VAR representation:

$$\begin{bmatrix} \Delta Z_t^i \\ CA_t^i \end{bmatrix} = \begin{bmatrix} a^i(L) & b^i(L) \\ c^i(L) & d^i(L) \end{bmatrix} \times \begin{bmatrix} \Delta Z_{t-1}^i \\ CA_{t-1}^i \end{bmatrix} + \begin{bmatrix} \mu_1^{i*} \\ \mu_2^{i*} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t}^i \\ \varepsilon_{2t}^i \end{bmatrix} \tag{5}$$

where the index i represents the analyzed country and $a^i(L)$, $b^i(L)$, $c^i(L)$ and $d^i(L)$ are polynomials of order p. It should be mentioned that, hereafter, CA_t will be constructed considering the parameter θ, to remove the trend component in consumption, as it follows (see appendix for further details):

$$CA_t = Y_t + rB_t - I_t - G_t - \theta C_t \tag{6}$$

Hence, the estimation of the VAR must be preceded by the estimation of θ, which occurs in the cointegration analysis between C_t and $(Y_t + rB_t - I_t - G_t)$. The model VAR(p) can be described as a VAR(1), in the following way:

$$\begin{bmatrix} \Delta Z_t^i \\ \vdots \\ \vdots \\ \Delta Z_{t-p+1}^i \\ CA_t^i \\ \vdots \\ \vdots \\ CA_{t-p+1}^i \end{bmatrix} = \begin{bmatrix} a_1^i & \cdots & a_p^i & b_1^i & \cdots & b_p^i \\ 1 & & & & & \\ & \ddots & & & 0 & \\ & & 1 & & & \\ c_1^i & \cdots & c_p^i & d_1^i & \cdots & d_p^i \\ & & & 1 & & \\ & 0 & & & \ddots & \\ & & & & & 1 \end{bmatrix} \begin{bmatrix} \Delta Z_{t-1}^i \\ \vdots \\ \vdots \\ \Delta Z_{t-p}^i \\ CA_{t-1}^i \\ \vdots \\ \vdots \\ CA_{t-p}^i \end{bmatrix} + \begin{bmatrix} \mu_1^{i*} \\ 0 \\ \vdots \\ 0 \\ \mu_2^{i*} \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t}^i \\ 0 \\ \vdots \\ 0 \\ \varepsilon_{2t}^i \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{7}$$

or, in a compact form:

$$X_t = AX_{t-1} + \mu^* + \varepsilon_t \tag{8}$$

where $X_t \equiv \begin{bmatrix} \Delta Z_t^i & \cdots & \Delta Z_{t-p+1}^i & CA_t^i & \cdots & CA_{t-p+1}^i \end{bmatrix}'$, A is the companion matrix, μ^* represents a vector of intercepts, and ε_t is a vector that contains the residuals. The VAR(1) is stationary by assumption, and the equation (8) can be rewritten removing the vector of means μ:

$$(X_t - \mu) = A(X_{t-1} - \mu) + \varepsilon_t \tag{9}$$

where $\mu^* = (I - A)\mu$. The forecast of the model j periods ahead is given by

$$E[(X_{t+j} - \mu \mid H_t) = A^j(X_t - \mu) \tag{10}$$

where H_t is the econometrician's information set (composed of current and past values of CA and ΔZ), contained in the agent's information set R_t. Define h' as a vector with $2p$ null elements, except the first: $h' = \begin{bmatrix} 1 & 0 & \dots & 0 \end{bmatrix}$. Then, one can select ΔZ_t in the vector X_t, in the following way:

$$\Delta Z_t = h'X_t \therefore \Delta Z_{t+j} = h'X_{t+j} \therefore (\Delta Z_{t+j} - \mu_{\Delta Z}) = h'(X_{t+j} - \mu) \tag{11}$$

where the vector μ contains the means $\mu_{\Delta Z}$ and μ_{CA^*}. Thus, applying the conditional expectation in the previous expression, it follows that:

$$E[(\Delta Z_{t+j} - \mu_{\Delta Z}) \mid H_t] = E[h'(X_{t+j} - \mu) \mid H_t] = h'E[(X_{t+j} - \mu) \mid H_t] = h'A^j(X_t - \mu) \tag{12}$$

where the last equality comes from equation (10). In order to calculate the optimal current account CA_t^*, one can take expectations of equation (4):

$$E(CA_t^* \mid H_t) = CA_t^* = -\sum_{j=1}^{\infty}(\frac{1}{1+r})^j E(\Delta Z_{t+j} \mid H_t) \tag{13}$$

The first equality comes from the fact that CA_t^* is contained in H_t, and the second is given by the law of iterated expectations ($H_t \subseteq R_t$). Applying the unconditional expectation in the previous expression:

$$E(CA_t^*) = -\sum_{j=1}^{\infty}(\frac{1}{1+r})^j E(\Delta Z_{t+j}) \therefore \mu_{CA^*} = -\sum_{j=1}^{\infty}(\frac{1}{1+r})^j \mu_{\Delta Z} \tag{14}$$

Combining equation (13) with equation (14), it follows that:

$$(CA_t^* - \mu_{CA^*}) = -\sum_{j=1}^{\infty}(\frac{1}{1+r})^j E(\Delta Z_{t+j} - \mu_{\Delta Z} \mid H_t) \tag{15}$$

Applying the expression (12) in the equation above:

$$(CA_t^* - \mu_{CA^*}) = -\sum_{j=1}^{\infty}(\frac{1}{1+r})^j h'A^j(X_t - \mu) = -h'(\frac{A}{1+r})(I - \frac{A}{1+r})^{-1}(X_t - \mu) \tag{16}$$

where the last equality is due to the convergence of an infinite sum, since the variables ΔZ_t and CA_t are stationary. Rewriting the previous equation in a simplified form:

$$(CA_t^* - \mu_{CA^*}) = K(X_t - \mu) \tag{17}$$

$$K = -h'(\frac{A}{1+r})(I - \frac{A}{1+r})^{-1} \tag{18}$$

where the vector K is derived from the world interest rate r and the matrix A. To formally test the model, one can analyze the null hypothesis $(CA_t^* - \mu_{CA^*}) = (CA_t - \mu_{CA})$. Define g' as a vector with $2p$ null elements, except the $(p+1)th$ element, that assumes a unit value. Thus, under the null hypothesis, it follows that:

$$(CA_t^* - \mu_{CA^*}) = (CA_t - \mu_{CA}) = g'(X_t - \mu) \tag{19}$$

Combining equations (17) and (19), the model can be formally tested through a set of restrictions imposed to the coefficients of the VAR:

$$g'(X_t - \mu) = -h'(\frac{A}{1+r})(I - \frac{A}{1+r})^{-1}(X_t - \mu) \therefore g'(I - \frac{A}{1+r}) = -h'(\frac{A}{1+r}) \tag{20}$$

Applying the structure of matrix A into equation (20), the following restrictions[3] can be derived:

$$\begin{aligned} a_i &= c_i \ \ ; i = 1...p \\ b_i &= d_i \ \ ; i = 2...p \\ b_1 &= d_1 - (1+r) \end{aligned} \tag{21}$$

Another important implication of the model is that the current account Granger-cause changes in net output, or in other words, CA_t helps to forecast ΔZ_t. This causality can be tested by means of the statistical significance of the $b(L)$ coefficients. Therefore, the implications of the intertemporal optimization model,[4] according to Otto (1992), can be summarized by:

1. Verifying the stationarity of CA_t and ΔZ_t, through unit root tests;
2. Checking if CA_t Granger-cause ΔZ_t;
3. Analyzing the cointegration between C_t and $(Y_t + rB_t - I_t - G_t)$, and calculating the parameter θ;
4. Formally investigating, by means of a Wald test, the equality of the optimal and observed current accounts, given by restrictions (21).

It should be mentioned that the previous list constitutes a set of testable propositions of the PVM. Therefore, the statistical acceptance of the model occurs only if all of these implications could be verified.

[3] These restrictions can be verified by a Wald test.

[4] Examined in details in the following sections.

2.3 Estimation Method

The VAR is usually estimated in the literature, equation-by-equation, using Ordinary Least Squares (OLS). However, the Seemingly Unrelated Regression Equations (SURE) method can also be adopted. The SURE technique is a Generalized Least Squares (GLS) estimation applied to a system of equations as a whole, in which the data for several countries is examined simultaneously. As well as GLS, the SURE estimation is more efficient than OLS, leading to smaller standard errors for the estimated coefficients, since it considers the contemporaneous covariance between residuals in different countries. It should be noted that if the regressors of all equations are the same, then the SURE technique yields estimates identical to those obtained by OLS equation-by-equation.

The joint estimation is given by stacking the system of equations that compose the VAR (for each country $i = 1, \ldots, N$) in the following way:

$$\begin{bmatrix} \Delta Z_t^i \\ CA_t^i \end{bmatrix} = \begin{bmatrix} a^i(L) & b^i(L) \\ c^i(L) & d^i(L) \end{bmatrix} \times \begin{bmatrix} \Delta Z_{t-1}^i \\ CA_{t-1}^i \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t}^i \\ \varepsilon_{2t}^i \end{bmatrix} \tag{22}$$

Then, define $Y_t^i = \begin{pmatrix} \Delta Z_t^i \\ CA_t^i \end{pmatrix}_{(2\times 1)}$, $X_t^i = \begin{pmatrix} \Delta Z_{t-1}^i \\ \vdots \\ \Delta Z_{t-p_i}^i \\ CA_{t-1}^i \\ \vdots \\ CA_{t-p_i}^i \end{pmatrix}_{(2p_i\times 1)}$, $\varepsilon_t^i = \begin{pmatrix} \varepsilon_{1t}^i \\ \varepsilon_{2t}^i \end{pmatrix}_{(2\times 1)}$

and $\beta_i' = \begin{pmatrix} a_1^i & \ldots & a_{p_i}^i & b_1^i & \ldots & b_{p_i}^i & c_1^i & \ldots & c_{p_i}^i & d_1^i & \ldots & d_{p_i}^i \end{pmatrix}_{(1\times 4p_i)}$

Thus, the system of equations can be expressed by

$$\begin{pmatrix} Y_t^1 \\ \vdots \\ Y_t^N \end{pmatrix}_{(\text{2N x 1})} = \begin{pmatrix} X_t^{1\prime} & 0 & \ldots & & 0 \\ 0 & X_t^{1\prime} & & & \\ \vdots & & \ddots & 0 & \vdots \\ & & 0 & X_t^{N\prime} & 0 \\ 0 & & \ldots & 0 & X_t^{N\prime} \end{pmatrix}_{(\text{2N x 4P})} \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_N \end{pmatrix}_{(\text{4P x 1})} + \begin{pmatrix} \varepsilon_t^1 \\ \vdots \\ \varepsilon_t^N \end{pmatrix}_{(\text{2N x 1})} \tag{23}$$

or in a compact form $Y_t = X_t\beta + \varepsilon_t$. It should be noted that $\sum_{i=1}^{N} p_i = P$, where p_i is the number of lags of the VAR, for a country i. The residuals ε_t have mean zero and are serially uncorrelated, with covariance matrix given by $E(\varepsilon_t \varepsilon_t') = \sigma^2\Omega$. Hence, the GLS estimator of β and its variance-covariance matrix are given by

$$\widetilde{\beta} = \left(X_t'\Omega^{-1}X_t\right)^{-1} X_t'\Omega^{-1}Y_t \tag{24}$$

$$E(\widetilde{\beta} - \beta)(\widetilde{\beta} - \beta)' = \sigma^2 \left(X_t'\Omega^{-1}X_t\right)^{-1} \tag{25}$$

In general, Ω is unknown and the last expression cannot be applied directly. However $\widetilde{\beta}$ can be calculated by an estimate of the ijth element of Ω, given by

$$\widehat{w}_{ij} = \frac{e_i' e_j}{T}, \qquad \text{where } i, j = 1, \ldots, N \tag{26}$$

where e_i is a $(T \times 1)$ vector containing the residuals of the ith equation estimated by OLS. In this case, a feasible SURE estimator of β is obtained as

$$\widetilde{\beta}^* = \left(X_t'\widehat{\Omega}^{-1}X_t\right)^{-1} X_t'\widehat{\Omega}^{-1}Y_t \tag{27}$$

$$E(\widetilde{\beta}^* - \beta)(\widetilde{\beta}^* - \beta)' = \sigma^2 \left(X_t'\widehat{\Omega}^{-1}X_t\right)^{-1} \tag{28}$$

The OLS estimator, on the other hand, is given by

$$\widehat{\beta} = \left(X_t'X_t\right)^{-1} X_t'Y_t \tag{29}$$

$$E(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)' = \sigma^2 \left(X_t'X_t\right)^{-1} X_t'\Omega X_t \left(X_t'X_t\right)^{-1} \tag{30}$$

The difference between their variance-covariance matrices is a positive semidefinite matrix, and can be expressed by

$$E(\widehat{\beta} - \beta)(\widehat{\beta} - \beta)' - E(\widetilde{\beta} - \beta)(\widetilde{\beta} - \beta)' = \sigma^2 \eta \Omega \eta' \tag{31}$$

where $\eta = \left(X_t'X_t\right)^{-1} X_t' - \left(X_t'\Omega^{-1}X_t\right)^{-1} X_t'\Omega^{-1}$, indicating the gain in efficiency of SURE estimators in comparison to the OLS counterpart.

2.4 A note on Granger Causality and Wald Tests

The optimal current account is generated from the vector K (see expressions (17) and (18)), which depends on matrix A and the world interest rate r. However, it should be noted that an estimated coefficient for matrix A could not be statistically significant, as suggested by some t-statistics presented in this work. These results could seriously compromise the subsequent optimal current account analysis, as it follows.

The Granger causality between the current account and net output (CA_t Granger-cause ΔZ_t) is a primordial implication of the theoretical model, and as argued before, can be alternatively tested through the significance of the $b(L)$ coefficients.[5] Moreover, if this implication is not empirically observed, the model should be rejected irrespective of any other results, since equation (4) is the theoretical foundation of the whole study. In this case, the current account could not help to predict variations in net output, suggesting that the agents are badly described by the model. Thus, one should not construct the optimal current account and perform a comparison with the observed series. Unfortunately, this is done in several papers presented in the literature.

To study this topic more carefully, a simple VAR(1) is initially presented. The Granger causality between CA_t and ΔZ_t, in this case, will be determined by the statistical significance of the b_1 coefficient.

$$\begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix} = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \times \begin{bmatrix} \Delta Z_{t-1} \\ CA_{t-1} \end{bmatrix} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{32}$$

In this case, the VAR is represented in a compact form by $X_t = AX_{t-1} + \mu + \varepsilon_t$ and, after some algebraic manipulations, the vector K takes the form:

$$K = -h'(\frac{A}{1+r})(I_2 - \frac{A}{1+r})^{-1} = \begin{bmatrix} \alpha & \beta \end{bmatrix} \tag{33}$$

where

$$\alpha = \frac{-a_1(1+r-d_1) - b_1c_1}{(1+2r-d_1+r^2-rd_1-a_1-a_1r+a_1d_1-b_1c_1)} \tag{34}$$

$$\beta = \frac{-a_1b_1 - b_1(1+r-a_1)}{(1+2r-d_1+r^2-rd_1-a_1-a_1r+a_1d_1-b_1c_1)} \tag{35}$$

If the Granger causality is rejected by the data (e.g., b_1 is not significant), then equation (35) indicates that $\beta = 0$, or in other words, CA_t^* is not a function of CA_t. In this case, the optimal current account would be given by

$$(CA_t^* - \mu_{CA^*}) = K(X_t - \mu) = \begin{bmatrix} \alpha & 0 \end{bmatrix} \begin{bmatrix} \Delta Z_t - \mu_{\Delta Z} \\ CA_t - \mu_{CA} \end{bmatrix} = \alpha\,(\Delta Z_t - \mu_{\Delta Z}) \tag{36}$$

[5] Presented in equation (5).

Hence, if $\beta = 0$ the null hypothesis $(CA_t^* - \mu_{CA}^*) = (CA_t - \mu_{CA})$ is always rejected, since under Ho β should be equal to one (and α should be zero).

A further analysis of the vector K for a VAR(2) is presented in the appendix, in a similar way. The generalization of this cautionary note for a VAR(p) is straightforward, and can be summarized by the Theorem below. According to Hamilton (1994), in the context of a bivariate VAR(p), if one of the two variables does not Granger-cause the other, then the companion matrix is lower triangular (e.g., $b(L) = 0$). Thus, the β_i coefficients of the vector K ($i = 1, \ldots, p$) are always zero, because of the algebraic structure of the vector, as also detailed in the appendix.

Theorem 1 *The Granger causality from the current account* (CA_t) *to the first difference of the net output* (ΔZ_t) *is a necessary condition to perform the Wald test and verify the validation of the model. This way, if the* $b(L)$ *coefficients of the VAR(p) model are not statistically significant, then the Wald test is not applicable and the model should be rejected.*[6]

Therefore, if the Granger causality could not be confirmed by the dataset, neither a Wald test should be performed nor the optimal current account should be generated, since the basic assumption of the model is not verified, as summarized in table 1.

Table 1 - A note on Granger causality and Wald tests

Result of Granger causality	Wald test	Model	Conclusion
CA_t not Granger-cause ΔZ_t	not applicable (β=0)	rejected	model cannot generate CA_t^*(*)
CA_t Granger-cause ΔZ_t	rejects Ho ($\beta \neq 1$)	rejected	$CA_t^* \neq CA_t$ (**)
	does not reject Ho (β=1)	not rejected	$CA_t^* = CA_t$ (***)

Notes: (*) indicates that CA_t^* only depends on ΔZ, instead of CA_t

(**) suggests that agents do not smooth consumption

(***) means that agents perfectly smooth consumption

[6] Proof is presented in the appendix A.7.

3 Literature Review

In this section, several papers describing the empirical results of the standard PVM, as well as some extensions for the theoretical model, are briefly presented in a chronological order.

Sheffrin & Woo (1990) perform a study of the current account of Belgium, Canada, Denmark, and the UK (1955-85), selecting countries with a low degree of capital control. The paper indicates non-stationarity for the current account and assumes implicitly $\theta = 1$. The paper suggests a rejection of the model for Denmark (at 1% level), and for Canada and the UK (at 5% level). The model is not rejected for Belgium, but the Granger causality is not verified for none of the countries. The authors do not perform the Granger causality test, but it can be noticed that in all countries this implication is not verified, given that the $b(L)$ coefficients are not statistically significant.

Otto (1992) studies the current account for the USA and Canada (1950-88), and rejects the model in both countries (at 2% level). The author finds evidence of stationarity of CA_t and ΔZ_t. The paper also rejects the null hypothesis (CA_t does not Granger-cause ΔZ_t) for the USA, but does not reject it for Canada.

Glick & Rogoff (1995) study the current account response to different productivity shocks in the G-7 countries (1961-90). An initial regression of ΔCA_t onto ΔI_t yields significant coefficients (ranging from -0.16 to -0.55), providing some evidence for capital mobility. These findings motivate the study of a structural model including global and country-specific shocks.[7] The results suggest that investment responds positively to both shocks, whereas the current account does not usually respond to global shocks, but reacts negatively to country-specific ones. The empirical evidences also indicate that CA_t responds by much less than I_t does to these country-specific ones.[8]

Ghosh (1995) analyzes 5 major industrialized countries: the USA, Japan, Germany, the UK, and Canada (1960-88). The paper suggests stationarity for CA_t and ΔZ_t in all countries, but rejects the cointegration hypothesis for Germany.[9] The Wald test indicates rejection of the model in all countries, except for the USA, the only country in which the Granger causality is also verified.

Ghosh & Ostry (1995) test the model for 45 developing countries. The results support the perfect capital mobility hypothesis for about 2/3 of the sample (29 countries). However, a careful analysis of the tests reveals that only 25 countries, in the whole sample, support the Granger causality implication (at 5% level). This

[7] In a world of complete markets, CA_t and I_t should move one for one in response to country-specific shocks.

[8] The very slow mean reversion feature of these shocks could apparently explain this evidence.

[9] This evidence could be explained by the fast growth of CA_t in the decade of 80, given that in small periods of time consumption and income are not necessarily cointegrated series.

way, the paper should conclude that in only 18 countries (instead of 29) the model could not be rejected. In this case, the use of a more efficient method (SURE) to estimate the coefficients of the VAR could reduce still further this number of countries, casting doubts to the main result of the paper, supporting the capital mobility hypothesis among developing countries.

Ghosh & Ostry (1997) extend the standard model and test it for Japan and the UK (1955-90), and for the USA (1919-90). The uncertainty in ΔZ_t is represented by the innovations ξ_t.[10] The paper assumes a CARA[11] utility function (with coefficient α), instead of the usual quadratic form, and postulates a process for consumption considering the process of σ_{ξ_t}.[12] The optimal current account consists of two components: the standard term and the precautionary saving component, which depends on α, r, ρ and σ_{ξ_t}.

$$CA_t^* = -\sum_{i=1}^{\infty}(\frac{1}{1+r})^i E_t(\Delta Z_{t+i} \mid \Omega_t) + \frac{\alpha\rho\sigma_{\xi_t}^2}{2\,(r+(1-\rho))}$$

Assuming $\alpha = 3$, the precautionary motives respond for 2-7% of observed current account surpluses. On the other hand, using the estimates of ρ and α, the precautionary term can account for 1% of the observed CA_t of Japan and about 5-19% of the American surplus.

Agénor et alli (1999) focus on the current account of France (1970-96), concluding that the data are consistent with the theory, and the analyzed country was perfectly able to smooth consumption. The main results are a p-value of 0.314 in the Wald test and a high correlation between CA_t and CA_t^* (optimal current account tracks the observed one quite closely). However, it should be noted that the paper suggests a p-value of 0.10 for the Granger causality test. In other words, at 1% and 5% significance levels, the null hypothesis that CA_t does not Granger-cause ΔZ_t should be accepted. Therefore, the results are doubtful, given that one of the premises of the model is not verified.

Hussein & Mello (1999) test the standard PVM to developing countries (Chile, Greece, Ireland, Israel, Malaysia, Mexico, South Africa, South Korea and Venezuela, 1955-92), and find evidences to support the capital mobility hypothesis. The null hypothesis of a unit root for ΔZ_t is rejected in all countries, and for CA_t is rejected in the full sample, except in Ireland. The exogeneity test shows that all components of ΔZ_t are endogenous with respect to CA_t, except in Mexico, and in Malaysia. Overall, the hypothesis that capital flows do not respond to consumption-smoothing behavior is strongly rejected in all cases (at 1% level). Moreover, the null hypothesis that the ratio of observed and benchmark variances of CA_t equals unity cannot be rejected (at 5% level) in any country.

[10] Which are supposed to be normally distributed with mean zero and variance σ_{ξ_t}.

[11] Constant Absolute Risk Aversion.

[12] Empirically a constant or an AR(1) with parameter ρ.

Bergin & Sheffrin (2000) extend the standard PVM to allow for a time-varying world interest rate (r_t) and exchange rate, and perform the analysis for Australia, Canada and the UK (1961-96). The model also considers tradable and non-tradable goods (with relative price p_t), through a Cobb-Douglas consumption index (with parameter a) and a CRRA[13] utility function (with parameter σ)[14]. The solution of the model, around the steady state, is given by

$$CA_t^* = -\sum_{i=1}^{\infty} \beta^i E_t(\Delta Z_{t+i} - \gamma r_{t+i}^* \mid \Omega_t)$$

where r_t^* is a function of r_t, γ, a, p_t.[15] This equation is tested through an augmented VAR, considering the additional variable r_t^*. The benchmark model (not considering r_t^*) rejects the theoretical restrictions in all countries, but the extended version only rejects the model for the UK[16]. Therefore, including r_t^* in the model improves its performance. However, experiments with r_t (instead of r_t^*) show that it is relative price (p_t) and not interest rate that really improves the fit of the model.

Gruber (2000) includes habit formation into the standard PVM and analyzes the model for 10 countries, with post-war data. In this framework, utility is quadratic in its argument, given by $C_t^* \equiv (C_t - \gamma C_{t-1})$.[17] Supposing that the interest rate is equal to the rate of time preference, the paper shows that C_t^* is expected to remain constant across time.[18] When output rises agents only gradually raise consumption. Thus, more of this output increase is saved, and thereby is passed to the current account (which becomes more volatile). The paper performs a test of the PVM based on an orthogonality restriction, and does not reject it for all countries (except Canada). Therefore, the main result of the paper is that a significant habit formation implies increased current account volatility.[19]

İşcan (2002) analyzes Canadian data (1926-97) and modifies the basic model introducing durables and nontraded goods. The paper supposes that consumption can be divided into durables D (traded goods) and nondurables C (traded C^G, or nontraded goods C^S), and postulates a quadratic form to utility function. The author also assumes that the relative prices of durables (P^D) and the real interest rate (r) are constant, and adopts a depreciation rate $\delta = 0.18$. In this framework, the optimal current account is given by

$$CA_t^* = -\sum_{i=1}^{\infty} (\frac{1}{1+r})^i E_t(\Delta Z_{t+i} \mid \Omega_t) - P^D \left(\frac{1-\delta}{1+r}\right) \Delta D_t$$

[13] Constant Relative Risk Aversion.

[14] Here the coefficient of relative risk aversion (σ) is equal to the inverse of the intertemporal elasticity of substitution (γ).

[15] The real exchange rate is used as a proxy for relative price of goods (p_t).

[16] The coefficient associating the optimal current account with the observed one (theoretically equal to one) goes from 0.41 (benchmark model) to 0.93 (extended model) in Australia; from 0.08 to 0.64 (in Canada) and from 0.07 to 0.55 (in the UK).

[17] Where γ is the strength of consumption habits. If $\gamma = 0$ the model collapses to the usual case.

[18] In the model with habit formation it is C_t^* that the agents smooth (rather than C_t).

[19] The empirical results also suggest that γ is statistically significant in eight out of ten samples, ranging from 0.80 to 0.90.

The paper also suggests a more pertinent measure of income, decreasing ΔZ_t by nontraded production. The estimated VAR is augmented to a three-variable framework.[20] Introducing only durables or nontraded goods alone seems not to be enough to improve the model upon the standard approach. However, extending it by both assumptions in annual data (r=14%) increases its performance[21], and Canadian data do not fail the econometric tests. However, the results depend on the sample period, frequency of the data, and the real interest rate assumption. Furthermore, in the sensitive analysis, following Ghosh & Ostry (1997), the author finds no evidence for significant precautionary saving motives in the data.

Nason & Rogers (2003) analyze Canada (1961-98) in a real business cycle (RBC) model, which nests the PVM and serves as a benchmark model. In addition, the paper extends the RBC model to consider (one at a time): non-separable preferences, shocks to fiscal policy and world interest rate, and imperfect capital mobility.[22] These experiments fail to achieve complete success in matching the data, but reveal that the world interest rate shocks move the RBC model closest to the data.

Kasa (2003) presents a cautionary note about the PVM and warns that it might not be testable with a VAR, following the Campbell & Shiller methodology.[23] In the standard framework, the observed decisions of agents can be sufficient statistics for any omitted information. However, the author argues that robustness to omitted information could be illusory and postulates a decomposition of ΔZ_t into permanent and transitory components. The paper shows that if ΔZ_t is trend stationary or its components are perfectly correlated, then the Campbell & Shiller methodology is applicable, but if ΔZ_t is driven by two orthogonal shocks, the methodology might indicate erroneous conclusions about the current account dynamics. In these cases, the author suggests dealing directly with the MA representation.

Kano (2003) proposes an extension for the PVM, introducing a stochastic world interest rate, and tests the model for Canada and the UK (post-war data). The paper considers 3 different shocks in ΔZ_t: global, country-specific permanent and country-specific transitory shocks, which are identified by a structural VAR, with two restrictions: a world interest rate (r) orthogonal to any country-specific shock,[24] and transitory shocks with no long-run effects on ΔZ_t.[25] Hence, the structural VAR contains 3 endogenous variables: $r_t, \Delta \ln(Z_t)$ and CA_t/Z_t. The results indicate that CA_t does not respond to global or country-specific permanent shocks, but responds positively to country-specific transitory shocks, according to the traditional PVM approach.[26] However, the result for global shocks is sensitive to identification.

[20] CA_t, ΔZ_t and $\alpha \Delta D_t$,where $\alpha = (1-\delta)/(1+r)$.

[21] Comparing correlations of CA_t and CA_t^* as well as standard deviation ratios of different models.

[22] Explanations broadly presented in the literature for the rejection of the PVM to Canada.

[23] Since the theoretical moving average (MA) representation could not be invertible.

[24] Allowing r to vary stochastically makes it possible to identify global and country-specific shocks.

[25] Which divides the country-specific shock into permanent and transitory components.

[26] In the standard framework a global shock would provide no effect in CA_t, given that all economies are assumed to be

4 Empirical Results

4.1 Data

All data are from the national accounts of IFS – International Financial Statistics (IMF). The CA_t and ΔZ_t series are constructed from annual and seasonally adjusted quarterly data (at annual rates), and are expressed in 1995 local currency (see the appendix A.1 for further details). In addition, all data are converted in per capita real terms, by dividing it by the implicit GDP deflator and the population. It is worth mentioning that the current account data are not directly obtained from the balance of payments datasets, since these series are not available for all of the countries for an extensive period of time, and it would lead to an arbitrary allocation of 'net errors and omissions' in the current account.

Sample 1: USA, Canada, Japan, United Kingdom, Germany, Italy, and France.
Period: 1960–1998, annual data (39 observations).

Sample 2: USA, Canada, Japan, United Kingdom, and Germany.
Period: 1960–1998, quarterly data (156 observations).

4.2 Unit Root

The first step of the empirical analysis is the study of the stochastic properties of the observed series. The Augmented Dickey Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests of unit root will be conducted to verify whether the current account and the first difference of the net output are, in fact, stationary series.

The plots of these series (shown in appendix A.2) indicate that stationarity does not always occur in the data, according to the ADF results summarized in table 2 (see also appendix A.3 for Phillips-Perron and KPSS results). The null hypothesis of one unit root for ΔZ_t is rejected at 1% level by the ADF and PP tests (except for France),[27] in both samples, and the stationarity hypothesis is rejected, at 5% level, by the KPSS test for the UK (in both samples), and for the USA (quarterly data).

Hereafter, CA_t^+ represents the observed current account including the trend and the smoothing components in consumption, while CA_t refers to the observed current account without the trend component, as already presented in equation (6).[28] Regarding CA_t^+, for several countries (in both samples) the tests do

homogeneous and react symmetrically to the shock. The standard PVM also predicts that the response of CA_t to a country-specific shock depends on the persistence of the shock.

[27] In France, the PP test rejects Ho at 1% level, but the ADF test does not reject it for the usual levels.

[28] $CA_t^+ = Y_t + rB_t - I_t - G_t - C_t$ and $CA_t = Y_t + rB_t - I_t - G_t - \theta C_t$

not reject Ho at usual levels, indicating non-stationary series. This empirical finding could be explained by the tilt component in consumption.

On the other hand, CA_t series are expected to be stationary, since they are constructed exactly to purge this tendency effect in consumption through the parameter θ. However, the results of the ADF and PP tests do not reject Ho (of a unit root) for Italy and France (annual data) and for the United Kingdom (in both samples), suggesting that CA_t is non-stationary in these countries. Furthermore, the KPSS test indicates that CA_t is non-stationary (at 5% level) for the UK (in both samples), for France (annual data), and for the USA (quarterly data).

Table 2 also presents the ADF tests for C_t and $(Y_t{+}rB_t - I_t - G_t)$.[29] The results, in general, suggest the existence of one unit root in the level series, but usually reject the null hypothesis for the first differenced series. This way, if C_t and $(Y_t{+}rB_t - I_t - G_t)$ are integrated of order one, I(1), then these series might cointegrate, since the theoretical results indicate that a lineal combination of them, presented in equation (1), is stationary.

Table 2 - ADF Unit Root Test

	USA	CAN	JPN	UK	GER	ITA	FRA	USA q	CAN q	JPN q	UK q	GER q
C_t	-1.16	-3.19	-2.27	-1.14	-2.03	-3.30 (+)	-1.47	-0.30	-2.34	-2.39	-0.86	-2.22
ΔC_t	-3.95 (**)	-4.03 (**)	-5.69 (**)	-3.54 (*)	-5.25 (**)	-5.29 (**)	-5.54 (**)	-6.23 (**)	-13.10 (**)	-15.15 (**)	-5.41 (**)	-13.41 (**)
GNI_t^*	-0.52	-2.98	-3.30 (+)	0.90	-1.58	-2.21	-2.04	-0.47	-3.38 (+)	-3.01	0.28	-2.11
ΔGNI_t^*	-4.30 (**)	-4.83 (**)	-5.64 (**)	-3.98 (**)	-4.86 (**)	-5.42 (**)	-2.46	-5.15 (**)	-15.63 (**)	-13.45 (**)	-17.63 (**)	-19.55 (**)
CA_t^+	-1.57	-2.41	-0.42	-2.54	-2.36	-1.84	-0.69	-1.92	-2.74 (+)	-1.11	-2.27	-2.78 (+)
CA_t	-2.84 (+)	-2.68 (+)	-3.67 (**)	-0.18	-2.90 (+)	-2.36	-2.26	-3.19 (*)	-3.01 (*)	-3.44 (*)	-0.76	-3.26 (*)
ΔZ_t	-4.53 (**)	-5.25 (**)	-5.63 (**)	-4.47 (**)	-4.69 (**)	-4.70 (**)	-2.44	-12.59 (**)	-14.84 (**)	-13.59 (**)	-16.78 (**)	-18.61 (**)

Notes: a) $GNI_t^* \equiv Y_t{+}rB_t - I_t - G_t$ and USAq indicates quarterly data (sample 2)

b) (**) indicates rejection of the null hypothesis at 1% level; (*) at 5% level and (+) at 10% level

c) $CA_t^+ = Y_t + rB_t - I_t - G_t - C_t$ and $CA_t = Y_t + rB_t - I_t - G_t - \theta C_t$

d) See appendix A.3 for truncation lag, Phillips-Perron and KPSS results.

[29] Unit root tests of C_t and $(Y_t{+}rB_t{-}I_t{-}G_t)$ include constant and lineal trend. In other cases, just a constant term is considered.

4.3 Cointegration

The purpose of this section is to identify long-term relationships between C_t and $(Y_t{+}rB_t - I_t - G_t)$. The technique adopted is the cointegration method, proposed by Johansen, using the information criteria SCI and HQ to select the optimal order of the VAR for each country, which includes an intercept but not trend. The diagnosis tests (not reported) lead to the following results: Jarque-Bera rejects the normality hypothesis (at 1% level) for Germany (in both samples), Italy (annual data), Japan and Canada (quarterly data). The non-autocorrelation hypothesis is rejected at 1% level for France (annual data), and in sample 2 (quarterly data) for the USA and Japan (at 1% level), and for Canada and the United Kingdom (at 5% level). Concerning heteroskedasticity, the null hypothesis is rejected in sample 2 for Japan (at 1% level) and for Germany and the United Kingdom (at 5% level).

Table 3 - Johansen´s Cointegration Test

The cointegration vector between $(Y_t{+}rB_t - I_t - G_t)$ and C_t is given by $(1, -\theta)$

		Ho:	p=0	Ho:	p≤1
Country	θ	trace stat.	$\lambda_{máx.}$ stat.	trace stat.	$\lambda_{máx.}$ stat.
USA	0.955 (0.012)	16.91 (*)	14.81 (*)	2.10	2.10
CAN	0.927 (0.026)	16.67 (*)	16.61 (*)	0.06	0.06
JPN	1.076 (0.015)	11.76	11.53	0.23	0.23
UK	0.806 (0.037)	21.06 (**)	17.44 (*)	3.62	3.62
GER	1.029 (0.031)	8.51	7.34	1.17	1.17
ITA	1.088 (0.049)	8.61	8.54	0.07	0.07
FRA	1.334 (0.124)	8.48	8.45	0.03	0.03
USA q	0.945 (0.013)	20.16 (**)	17.29 (*)	2.87	2.87
CAN q	0.923 (0.030)	14.97	14.76 (*)	0.22	0.22
JPN q	1.079 (0.015)	15.16	14.86 (*)	0.30	0.30
UK q	0.826 (0.039)	19.94 (*)	15.68 (*)	4.26 (*)	4.26 (*)
GER q	1.043 (0.032)	11.44	10.68	0.76	0.76

Notes: a) p is the number of cointegrating relations and USAq means quarterly data (sample 2).

b) (*) indicates rejection of Ho at 5% level and (**) at 1% level.

c) In column θ the standard deviation is presented in parentheses.

Values of θ different from one, obtained in the cointegration analysis, suggest that the world interest rate is not equal to the country rate of time preference ($\beta \neq 1/(1+r)$) and that there is a trend in the consumption path. When this parameter is smaller than one, agents anticipate the future consumption for the present, since the country consumes more than its permanent net output, suggesting a degree of "impatience". Where ($\theta > 1$), the inverse argument is applicable and the country increases its stock of foreign assets. The results indicate $\theta < 1$ for the USA, Canada and the UK, and $\theta > 1$ for the other countries, in accordance with Ghosh (1995).[30]

The null hypothesis that the number of cointegration vectors (p) is smaller than (or equal to) one is not rejected in any case (except for the United Kingdom, quarterly data),[31] where the cointegration vector between $(Y_t + rB_t - I_t - G_t)$ and C_t is given by $(1, -\theta)$. It should be mentioned that the tests do not reject the null hypothesis of no cointegration for Japan, Germany, Italy and France (annual data), and for Germany (quarterly data).

4.4 OLS estimation

4.4.1 Unrestricted VAR

After the estimation of the tilt parameter θ, one can construct the unrestricted VAR in CA_t and ΔZ_t. The HQ and SCI information criteria are used to choose the lag order of the VAR in each country. It should be noted that the current account is constructed according to equation (6), purging the trend component in consumption. If this procedure is not adopted, one cannot guarantee the stationarity for the current account, because of this trend component.

The unrestricted VAR is constructed with intercept, since the series clearly do not have null mean.[32] In fact, the results suggest a statistically significant intercept for the equations of the VAR, unlike several studies, which analyze demeaned series and a VAR without intercept. Furthermore, according to equation (14), in steady state, it follows that $\mu_{CA} = -(1/r)\mu_{\Delta Z}$. Plugging the observed mean values of the series into this expression, then, according to Otto (1992), the world interest rate implied by the model would be extremely high (about 32% per quarter). Hence, this theoretical restriction is clearly not supported by the data and justifies the use of non-demeaned series into an unrestricted VAR with intercept.

[30] See table 18 (in appendix A.6) for further details.

[31] The results suggest no cointegration for the UK (quarterly data) probably because CA_t is non-stationary.

[32] The omission of the intercept in the econometric model, with demeaned series, could result in a spurious rejection of the Granger-causality hypothesis, since the effect of the constant term would be transferred to the other variables of the model.

The coefficients estimated for the VAR are presented in tables 16 and 17 (in the appendix A.4). The diagnosis tests indicate the rejection of the normality hypothesis in sample 1 (annual data) for Canada (at 1% level) and Japan (at 5% level), and at 1% level in all sample 2 (except for the USA). Concerning the non-autocorrelation hypothesis, the tests reject Ho (at 5% level) for Canada (in both samples) and for the United Kingdom (quarterly data). Regarding heteroskedasticity, the null hypothesis is rejected (at 1% level) in quarterly data for Japan, Germany and the UK.

4.4.2 Granger Causality

One of the four implications of the theoretical model, listed by Otto (1992), is that CA_t helps to forecast ΔZ_t. According to the empirical results, presented in table 18,[33] one can verify that the null hypothesis (CA_t does not Granger-cause ΔZ_t) is only rejected for the USA (at 1% level in sample 1, and at 5% level in sample 2) and for Japan (at 8% level in sample 1, and at 1% level in sample 2).

These results are in accordance with the literature. Otto (1992) rejects Ho for the USA (at 1% level), but does not reject it for Canada. Ghosh (1995) also rejects Ho for the USA (at 1% level) and does not reject it for Canada, Japan[34], the UK and Germany. Agénor et alli (1999) present a p-value of 0.10 for France, and do not reject Ho for the usual levels of significance.

Thus, with the exception of the USA and Japan, the current account does not help to forecast the net output of the G-7 countries, indicating that the agents possibly do not have any additional information to predict ΔZ_t, other than those contained in the past of their own series. Table 18 also presents the results for Granger causality with the SURE estimation, leading to the same previous conclusions.[35]

4.4.3 Optimal Current Account

The optimal current account $(CA_t^* - \mu_{CA}^*)$ is generated by equations (17) and (18), and depends on the coefficients of the VAR (matrix A) and the world interest rate (supposed 2% per year) [36]. In a VAR(2), the optimal current account is given by

$$(CA_t^* - \mu_{CA^*}) = K(X_t - \mu) = \begin{bmatrix} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} \Delta Z_t - \mu_{\Delta Z} \\ \Delta Z_{t-1} - \mu_{\Delta Z} \\ CA_t - \mu_{CA} \\ CA_{t-1} - \mu_{CA} \end{bmatrix} \tag{37}$$

[33] See the Current Study (CS) column, for OLS estimation.

[34] Against the results of this paper.

[35] Excepting the UK (sample 1), that rejects (at 5% level) the null hypothesis (CA_t does not GC ΔZ_t) with SURE.

[36] Other values for r, ranging from 1% to 6%, were also used and do not modify the magnitude of the results.

where $K = -h'(\frac{A}{1+r})(I - \frac{A}{1+r})^{-1}$. The vector K is presented in the following tables.[37]

Table 4 - Vector K

(sample 1 - annual data)

	CAN	UK	GER	ITA		USA	FRA	JPN
α	-0.10	0.67	-0.37	0.10	α_1	0.16	-0.91	0.14
β	-0.07	-7.89	-1.22	-0.22	α_2	0.35	-0.97	0.06
					β_1	2.10	0.19	-0.10
					β_2	-0.92	-0.30	0.49

Note: A VAR(1) is estimated for CAN, UK, GER, ITA and a VAR(2) for the USA, FRA and JPN.

Table 5 - Vector K

(sample 2 - quarterly data)

	USA	CAN	UK	GER		JPN
α	-0.07	0.13	0.29	0.31	α_1	0.31
β	0.64	0.28	-0.43	-0.23	α_2	0.15
					α_3	-0.09
					β_1	-0.56
					β_2	0.36
					β_3	0.33

Note: A VAR(1) is estimated for the USA, CAN, UK, GER, and a VAR(3) for JPN.

A formal comparison between $(CA_t^* - \mu_{CA}^*)$ and $(CA_t - \mu_{CA})$, to measure the fit of the model with the data, is provided by the restrictions imposed to the coefficients of the VAR, through a Wald test, which asymptotically follows a χ^2 distribution (with the degrees of freedom equal to the number of restrictions). The acceptance of those restrictions in the Wald test means that both series of current account (optimal and observed) are statistically the same. A less formal and more intuitive approach, also presented in the literature, can be addressed by table 6 in a graphical analysis (see also the appendix A.5).

Table 6 - Variances and Correlations

	USA	CAN	JPN	UK	GER	ITA	FRA	USA q	CAN q	JPN q	UK q	GER q
Corr (.)	0.84	-0.79	0.63	-0,99	-0.95	-0.97	-0.69	0.99	0.97	0.36	-0.97	-0.81
V R	0.55	42.34	4.22	0.02	0.52	23.23	14.80	2.41	11.43	10.34	5.81	15.45

Notes: a) Corr (.) means Correlation $(CA_t;CA_t^*)$

b) VR means variance ratio of $\sigma^2(CA_t)/\sigma^2(CA_t^*)$

[37] Where the α_i coefficients are associated with the ΔZ terms, and β_i with the CA terms.

Imperfect capital mobility might occur in a country where the optimal current account is more volatile than the actual current account, or in other words, where restrictions hinder consumption smoothing. The opposite case, where the optimal current account is less volatile than the observed current account, suggests that the presence of short term larger capital flows can be due to speculative movements. In those cases, the correlation coefficient between the optimal and actual current accounts can be analyzed, indicating whether these two series move, in the presence of shocks, in the same direction.

It should be reminded that if the Granger causality implication is not verified, then the optimal current account should not be generated, since it would lead to spurious results. In the present work, the Granger causality is only verified for Japan and the USA (see table 18). Thus, for the other countries, the model should be rejected and the optimal current account should not be generated. However, following the literature, the results concerning the optimal current account are presented in this paper for all countries, in both samples.

The Germany case (sample 1) can be analyzed as example of spurious results. The VAR(1) estimated for this country is given by

VAR coefficients for **Germany**

$$\begin{bmatrix} \Delta Z_t \\ CA_t \end{bmatrix} = \begin{bmatrix} a_1 = 0.17 \ \ {\scriptstyle (1.03)} & b_1 = 0.26 \ \ {\scriptstyle (1.33)} \\ c_1 = 0.11 \ \ {\scriptstyle (1.09)} & d_1 = 0.73 \ \ {\scriptstyle (6.11)} \end{bmatrix} \times \begin{bmatrix} \Delta Z_{t-1} \\ CA_{t-1} \end{bmatrix} + \begin{bmatrix} 366 \ \ {\scriptstyle (3.37)} \\ -9 \ \ {\scriptstyle (-0.14)} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

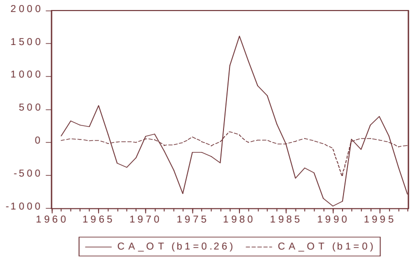
Note: t-statistics in parentheses

where d_1 is statistically significant, but this is not the case for b_1. As described in table 7, the vector K is extremely sensitive to variations in b_1 and could generate completely different CA_t^* series (as shown in Figure 1). Assuming $b_1 = 0.26$ (instead of zero, since b_1 is not statistically significant), the model indicates that $\beta = -1.221$ (unlike the correct value of $\beta = 0$).

Table 7 - Vector $K = [\alpha; \beta]$ (Germany)

	b_1= 0.26	b_1= 0
α	-0.366	-0.205
β	-1.221	0.000

Figure 1- CA_t^* (Germany)



Note: The graphic above presents two optimal current accounts generated from different K vectors.

4.4.4 Wald Test

The Wald test can be implemented for several values of the world interest rate (r). The results, however, are almost the same for values of r ranging from 1% to 6%, as presented in the following table:

Table 8 - Wald Test (**Canada -** sample 1 - annual data)

world interest rate (r)	$\chi^2_{(2)}$	p-value
1%	3.45	0.178
2%	3.62	0.164
4%	3.97	0.137
6%	4.35	0.114

Note: Ho: $(CA_t^* - \mu_{CA}^*) = (CA_t - \mu_{CA})$

As described in the table above, the null hypothesis $(CA_t^* - \mu_{CA}^*) = (CA_t - \mu_{CA})$ cannot be rejected at the usual levels of significance, irrespective of the interest rate adopted. Hence, the presented result, obtained by the OLS technique, suggests that Canada uses its current account to smooth consumption in the presence of shocks to net output.

However, it should be noted that this result is not robust when a more efficient estimation technique is adopted. According to Ghosh and Ostry (1995), the non-rejection of the model can occur because of the magnitude of the standard deviations in the coefficients of the VAR. High values for the standard errors could lead to a statistical equality between the optimal and observed current accounts, even if these series are graphically different. The results of the Wald test, for the two samples, are described in the following subsection, where a comparison between the two estimation techniques (OLS and SURE) is also presented.

4.5 SURE estimation

An empirical starting point to justify the SURE technique for the estimation of the VAR is presented in table 9. Proceeding with the Granger causality tests for the CA_t and ΔZ_t series across the G-7 countries, it could be verified that, in several cases, the null hypothesis is rejected. Thus, the dataset indicates the existence of Granger causality among countries, and the joint estimation of the VAR, through the SURE technique, could make use of this information, resulting in a more efficient estimation.

Table 9 - Granger causality (sample 1 - annual data)

null hypothesis	p-value
USA_CA not Granger-cause UK_DZ	0.003
JPN_CA not Granger-cause USA_DZ	0.049
UK_CA not Granger-cause CAN_CA	0.034
FRA_DZ not Granger-cause GER_CA	0.005

Note: This table only presents a few cross-country cases that reject Ho at 5% level

Moreover, the residual correlation matrix obtained in the joint estimation[38] could also justify the SURE technique, since the contemporaneous correlation across the G-7 countries should not be ignored.

Table 10 - Residual Correlation Matrix (sample 1 - annual data)

	USA_DZ	USA_CA	CAN_DZ	CAN_CA	JPN_DZ	JPN_CA	UK_DZ	UK_CA	GER_DZ	GER_CA	ITA_DZ	ITA_CA	FRA_DZ	FRA_CA
USA_DZ	**1.00**	-0.06												
USA_CA	-0.06	**1.00**												
CAN_DZ	0.46	-0.21	**1.00**	0.60										
CAN_CA	0.04	0.08	0.60	**1.00**										
JPN_DZ	0.16	0.02	-0.24	-0.34	**1.00**	0.27								
JPN_CA	0.14	-0.52	-0.17	-0.38	0.27	**1.00**								
UK_DZ	0.31	-0.03	0.08	-0.18	0.48	0.10	**1.00**	0.64						
UK_CA	0.14	-0.02	-0.18	-0.18	0.32	0.30	0.64	**1.00**						
GER_DZ	-0.10	0.02	-0.56	-0.19	0.24	0.33	0.01	0.27	**1.00**	0.03				
GER_CA	0.16	-0.38	0.36	0.02	0.00	0.24	-0.05	-0.37	0.03	**1.00**				
ITA_DZ	0.34	-0.06	-0.12	-0.50	0.37	0.19	0.43	0.22	-0.07	-0.02	**1.00**	0.46		
ITA_CA	0.24	-0.08	0.09	0.03	0.25	0.15	0.51	0.25	-0.18	0.14	0.46	**1.00**		
FRA_DZ	0.32	-0.12	-0.01	-0.16	0.35	0.37	0.19	0.12	0.09	0.33	0.13	0.37	**1.00**	0.38
FRA_CA	0.46	-0.06	0.08	-0.15	0.11	0.37	0.35	0.38	-0.18	0.07	0.44	0.60	0.38	**1.00**

Furthermore, a Likelihood Ratio (LR) test is able to provide a formal argument to adopt the SURE approach, instead of the OLS technique. Under the null hypothesis, the residuals covariance matrix (Ω) is a diagonal (band)[39] matrix, suggesting the OLS method. On the other hand, the alternative specification (H_1) supposes that Ω is a non-diagonal (band) matrix, recommending the SURE approach. This way, Ho imposes a set of restrictions on the residuals covariance matrix, since all elements out of the diagonal (band) are set to zero. In this case, according to Hamilton (1994), twice the log likelihood ratio for a Gaussian VAR is given by

$$2\left(L_1^* - L_0^*\right) = T\left(\ln\left|\widehat{\Omega}_0\right| - \ln\left|\widehat{\Omega}_1\right|\right) \tag{38}$$

where L_0^* is the maximum value for the log likelihood under Ho (and L_1^* under the alternative hypothesis), T is the number of effective observations, $\left|\widehat{\Omega}_0\right|$ is the determinant of the residuals covariance matrix estimated

[38] Future research could analyze a dynamic panel structure with a long-run correlation matrix (see Phillips & Moon, 1999, "Linear Regression Limit Theory for Nonstationary Panel Data", Econometrica, vol. 67).

[39] The residuals covariance matrix is diagonal-band, in OLS estimation, because of the structure of the VAR, since each country has two equations (CA_t and ΔZ_t).

by OLS, and $\left|\widehat{\Omega}_1\right|$ is the determinant of the same matrix estimated by SURE. Under the null hypothesis, the difference between L_1^* and L_0^* is statistically zero, and the LR statistic asymptotically follows a χ^2 distribution, with degrees of freedom equal to the number of restrictions imposed under Ho.

Table 11 - Results of the LR test

	Sample 1 - annual data	Sample 2 - quarterly data
T	36	152
$\left\|\widehat{\Omega}_0\right\|_{OLS}$	3.22e+86	3.56e+45
$\left\|\widehat{\Omega}_1\right\|_{SURE}$	5.65e+83	2.50e+45
ζ_{LR}	228.50	53.75
ζ_{LR}^*	196.76	-
10% critical value	-	$\chi^2_{(40)}$=51.81
5% critical value	$\chi^2_{(80)}$=101.88 ; $\chi^2_{(90)}$=113.15	$\chi^2_{(40)}$=55.76
1% critical value	$\chi^2_{(80)}$=112.33 ; $\chi^2_{(90)}$=124.12	-

Notes: a) T is the number of effective observations, and $\zeta_{LR} = T\left(\ln\left|\widehat{\Omega}_0\right| - \ln\left|\widehat{\Omega}_1\right|\right)$ is the LR statistic.

b) ζ_{LR}^* is a modification to the LR test to take into account small-sample bias, replacing T by $(T-k)$, where k is the number of parameters estimated per equation.

c) In sample 1, the degrees of freedom (dof)= 84, and in sample 2, dof=40.

Hence, the null hypothesis could be rejected in both samples,[40] since the LR statistics are larger than the critical values. Therefore, the residuals covariance matrices are non-diagonal (band), and the SURE approach is better recommended than the OLS method.

A brief mention, involving both the theoretical and econometric models, could be made using the Glick & Rogoff (1995) approach of global and country-specific shocks. A global shock, in the standard theoretical model, should provide no effect in CA_t, given that all economies are homogeneous and react symmetrically to the shock, leading to a diagonal (band) residual correlation matrix, considered in the OLS estimation.

However, the previous tables show a non-diagonal case, in which cross-country effects are presented, due to idiosyncratic shocks which are only considered in the SURE estimation. Therefore, in this analysis, the OLS method incorporates the global shock to net output, whereas the SURE technique could also consider the country-specific one.

[40] Considering a 1% level of significance in sample 1, and a 10% level in sample 2.

The SURE estimation, as already mentioned, improves the efficiency when compared to the OLS method. Thus, the covariance matrix of a system estimated by SURE could result in a rejection of the model previously accepted in the OLS estimation framework. A comparison of results for the Wald test, using the two methods of estimation (OLS and SURE), is presented in table 12 and summarized in table 13.

Table 12 - Wald Test

Country	p-value (OLS)	p-value (SURE)
USA	0.0327 (*)	0.0295 (*)
CAN	0.1637	0.0001 (**)
JPN	0.1915	0.1191
UK	0.1575	0.0000 (**)
GER	0.0377 (*)	0.0037 (**)
ITA	0.0589	0.0000 (**)
FRA	0.1123	0.0000 (**)
USA q	0.2553	0.0861
CAN q	0.3065	0.1956
JPN q	0.0004 (**)	0.0000 (**)
UK q	0.0757	0.0009 (**)
GER q	0.0008 (**)	0.0001 (**)

Notes: a) Ho: $(CA_t^* - \mu_{CA}^*) = (CA_t - \mu_{CA})$

b) (*) means rejection at 5% level and (**) at 1% level

c) USAq indicates quarterly data (sample 2)

Table 13 - Comparison of Results (OLS *versus* SURE)

	Number of countries that reject Ho at 5% level (OLS)	Number of countries that reject Ho at 5% level (SURE)
Sample 1 (7 countries - annual data)	2	6
Sample 2 (5 countries - quarterly data)	2	3

Note: Ho: $(CA_t^* - \mu_{CA}^*) = (CA_t - \mu_{CA})$

4.6 Summary of Results

The four testable implications of the econometric model are summarized in table 14, bearing in mind that all of these propositions should be verified at the same time, in order to fully accept the model. Furthermore, as mentioned by Ghosh (1995), the theoretical framework adopts several joint assumptions, such as quadratic utility function, a small open economy and a single consumption good. Therefore, a rejection of the model cannot necessarily be attributed to any of these hypotheses individually.

Table 14 - Summary of the 4 testable implications of the model

	(1a) Unit Root for CA_t	(1b) Unit Root for ΔZ_t	(2) CA_t does not GC ΔZ_t	(3) Cointegration (p=0)	(4a) Wald (OLS) $CA_t^* = CA_t$	(4b) Wald (SURE) $CA_t^* = CA_t$
USA	(+)	(**)	(**)	(*)	(*)	(*)
CAN	(+)	(**)	-	(*)	-	(**)
JPN	(**)	(**)	(+)	-	-	-
UK	-	(**)	(-)	(*)	-	(**)
GER	(+)	(**)	-	-	(*)	(**)
ITA	-	(**)	-	-	-	(**)
FRA	-	-	-	-	-	(**)
USA q	(*)	(**)	(*)	(*)	-	-
CAN q	(*)	(**)	-	(*)	-	-
JPN q	(*)	(**)	(**)	(*)	(**)	(**)
UK q	-	(**)	-	(*)	-	(**)
GER q	(*)	(**)	-	-	(**)	(**)

Notes: a) GC means Granger-Cause, (**) means rejection at 1% level, (*) at 5% level, and (+) at 10% level.

b) (-) means rejection at 5% level (only with SURE estimation) and USAq indicates quarterly data (sample 2).

c) The null hypothesis for Cointegration is that the number of cointegrating relations (p) is zero.

d) The Wald test verifies whether the demeaned series CA_t^* and CA_t are statistically the same.

e) The statistical acceptance of the model occurs only if all of these 4 implications could be verified: the Wald test must not reject Ho and all the other three implications must reject Ho.

A rigorous analysis of the previous table suggests that the model should be rejected in all countries. However, as the results are also sensitive to the periodicity of the data, the perfect capital mobility should not be rejected for the USA and Japan. A comparison of these results with the empirical evidence of the literature is presented in table 15: Otto (1992) rejects the model for the USA and Canada, Ghosh (1995) rejects the model for Canada, Japan, the UK and Germany, but does not reject it for the USA, and Agénor et alli (1999) do not reject it for France (see also table 18, in the appendix A.6, for further details).

Table 15 - Comparison with the Literature

Country	**Wald Test** (p-value)			
	Current Study (SURE)	Otto (92)	Ghosh (95)	Agénor (99)
USA	0.0295 (*)	0.0041 (**)	1.19	-
CAN	0.0001 (**)	0.0020 (**)	95 (**)	-
JPN	0.1191	-	75 (*)	-
UK	0.0000 (**)	-	464 (**)	-
GER	0.0037 (**)	-	90 (**)	-
ITA	0.0000 (**)	-	-	-
FRA	0.0000 (**)	-	-	0.314
USA q	0.0861			
CAN q	0.1956			
JPN q	0.0000 (**)			
UK q	0.0009 (**)			
GER q	0.0001 (**)			

Notes: a) Ghosh (1995) presents qui-squared values.

b) USAq indicates quarterly data (sample 2).

c) (**) means rejection at 1% level, and (*) at 5% level.

5 Conclusion

The standard intertemporal optimization model of the current account is adopted to analyze the G-7 countries. In this framework, the perfect capital mobility allows the agents to smooth consumption via current account. The econometric approach of the model, developed by Campbell & Shiller (1987), consists of estimating an unrestricted VAR to verify the adherence of the theoretical framework onto the data.

This paper reveals that the empirical results are extremely sensitive to the method adopted for the estimation of the VAR. The SURE method leads to a rejection of the model for 6 countries,[41] while the OLS technique, which is widely used in the literature, in spite of being less efficient, only indicates a rejection of the model for 2 countries. This study also proposes a note on Granger causality and Wald tests: the Granger causality [42] is addressed as a 'sine qua non' condition for the entire validation of the model, since the construction of the optimal current account leads to spurious results when this condition is not verified.

Furthermore, the analysis of the four implications of the theoretical model suggests that there is no empirical evidence, in the analyzed framework, to support the capital mobility hypothesis. The standard intertemporal optimization model is not able to statistically explain the dynamic behavior of the current account of the G-7 countries. However, as the results are sensitive to the periodicity of the data, the perfect capital mobility should not be rejected for the USA and Japan.

Future research could extend the standard model, as outlined in recent papers described in the Literature Review, considering a more efficient estimation technique, such as the SURE approach, to further verify the empirical results about the capital mobility hypothesis.

[41] Sample 1 - annual data.

[42] CA_t Granger-cause ΔZ_t.

References

[1] AGÉNOR, P.R. et alli, 1999, "Consumption smoothing and the current account: evidence for France, 1970-1996", *Journal of International Money and Finance*, 18, 1-12.

[2] BERGIN, P. R. & SHEFFRIN, S. M., 2000, "Interest rates, exchange rates and present value models of the current account", *The Economic Journal*, 110, 535-558.

[3] CAMPBELL, J.Y., 1987, "Does saving anticipate declining labor income? An alternative test of the permanent income hypothesis", *Econometrica*, 55, 1249-1273.

[4] CAMPBELL, J.Y. & DEATON, A.S., 1989, "Why is consumption so smooth?", *Review of Economic Studies*, 56, 357-374.

[5] CAMPBELL, J.Y. & SHILLER, R., 1987, "Cointegration and tests of present value models", *Journal of Political Economy*, 95, 1062-1088.

[6] FELDSTEIN, M. & HORIOKA, C., 1980, "Domestic saving and international capital flows", *Economic Journal*, 90, 314-329.

[7] GHOSH, A. R., 1995, "International capital mobility amongst the major industrialised countries: too little or too much", *The Economic Journal*, 105, 107-128.

[8] GHOSH, A.R. & OSTRY, J.D., 1995, "The current account in developing countries: a perspective from the consumption smoothing approach", *World Bank Economic Review*, 9, 305-333.

[9] GHOSH, A.R. & OSTRY, J.D., 1997, "Macroeconomic uncertainty, precautionary saving, and the current account", *Journal of Monetary Economics*, 40, 121-139.

[10] GLICK, R. & ROGOFF, K., 1995, "Global versus country-specific productivity shocks and the current account", *Journal of Monetary Economics*, 35, 159-192.

[11] GRUBER, J. W., 2000, "A present value test of habits and the current account", *manuscript, Johns Hopkins University.*

[12] HAMILTON, J.D., 1994, "Time series analysis", *Princeton University Press*, 296-298, 302-305.

[13] HARVEY, A., 1993, "The econometric analysis of time series", *MIT Press.*

[14] HUSSEIN, K. A. & MELLO, L. R., 1999, "International capital mobility in developing countries: theory and evidence.", *Journal of International Money and Finance*, 18, 367-381.

[15] İŞCAN, T. B., 2002, "Present value tests of the current account with durables consumption", *Journal of International Money and Finance*, 21, 385-412.

[16] ISSLER, J.V. & SENNA, F.A., 2000, "Mobilidade de capitais e movimentos da conta corrente do Brasil: 1947-1997", *Estudos Econômicos*, 30, USP.

[17] ISSLER, J.V. & TEIXEIRA, H.C.F., 2003, "Consumption smoothing in latin america: an empirical assessment of present value models", *dissertation of master's degree in economics*, EPGE-FGV.

[18] KANO, T., 2003, "A structural VAR approach to the intertemporal model of the current account", *Bank of Canada, Working Paper*, 42.

[19] KASA, K., 2003, "Testing present value models of the current account: a cautionary note", *Journal of International Money and Finance*, 22, 557-569.

[20] NASON, J. M. & ROGERS, J.H., 2003, "The present-value model of the current account has been rejected: round up the usual suspects", *Board of Governors of the Federal Reserve System, International Finance Discussion Papers*, 760.

[21] OBSTFELD, M. & ROGOFF, K., 1994, "The intertemporal approach to the current account", *National Bureau of Economic Research, Working Paper,* 4893.

[22] OTTO, G., 1992, "Testing a present-value model of the current account: evidence from the U.S. and Canadian time series", *Journal of International Money and Finance*, 11, 414-430.

[23] SACHS, J., 1982, "The current account in the macroeconomic adjustment process.", *Scandinavian Journal of Economics*, 84, 147-159.

[24] SHEFFRIN, S. M. & WOO, W. T., 1990, "Present value tests of an intertemporal model of the current account", *Journal of International Economics*, 29, 237-253.

[25] SIMON, C. P. & BLUME, L., 1994, "Mathematics for economists", *W.W. Norton & Company Inc.*, 180-182.

Appendix

A.1 Data

The CA_t and ΔZ_t series are constructed in the following way, adjusted by the implicit deflator of GDP, in per-capita terms, and are expressed in local currency at constant 1995 prices.

$$CA_t^+ = (GNI_t - I_t - G_t - C_t) * 100/(GDP\_Defl_t \times \mathit{Population})$$

$$CA_t = (GNI_t - I_t - G_t - \theta C_t) * 100/(GDP\_Defl_t \times \mathit{Population})$$

$$Z_t = (GDP_t - I_t - G_t) * 100/(GDP\_Defl_t \times \mathit{Population})$$

where

$$GDP_t = C_t + I_t + G_t + NX_t$$

$$GNI_t = Y_t + rB_t = GDP_t + NIA_t$$

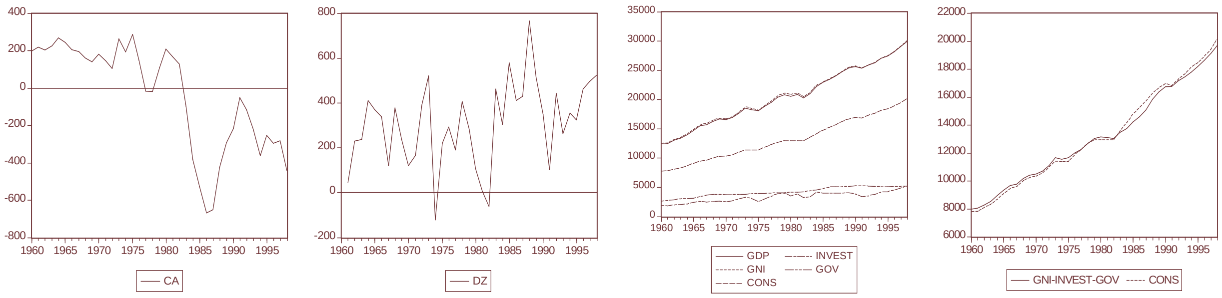
and

GDP: Gross Domestic Product (line 99b of IFS)
GNI: Gross National Income (99a)
C: Household Consumption Expenditures (96f)
I : Investment = Gross Fixed Capital Formation (93e) + Change in Inventories (93i)
G: Government Consumption Expenditures (91f)
NX: Exports (90c) - Imports (98c)
NIA: Net Primary Income From Abroad (98n)
GDP_Deflator (99bip): Deflator of GDP (1995=100)
Population: Midyear estimates (99z), quarterly data are constructed by interpolation of annual data

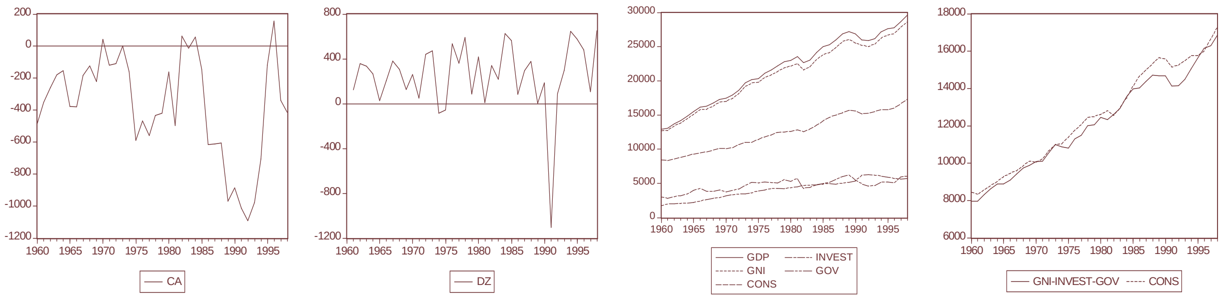
A.2 Time Series

Sample 1 (annual data)

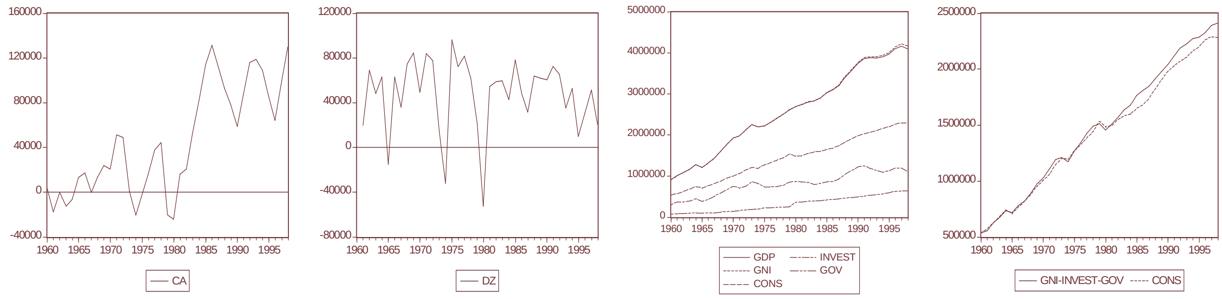
USA



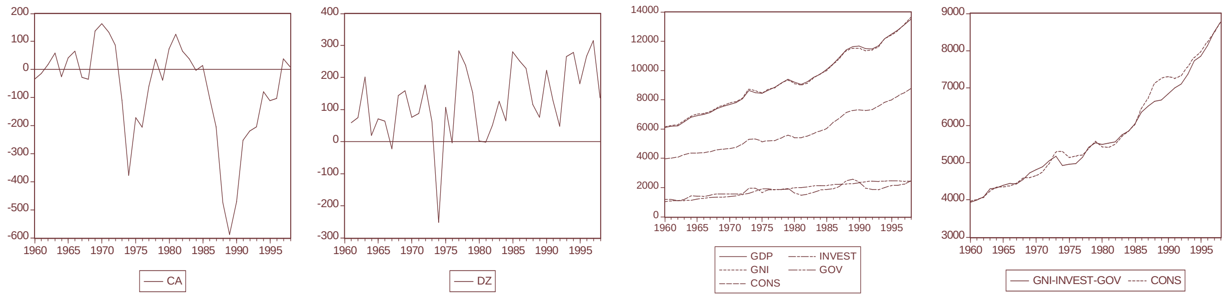
CAN



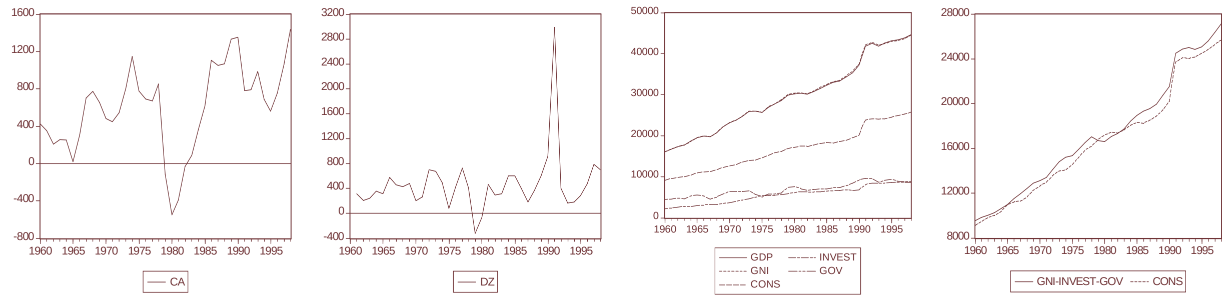
JPN



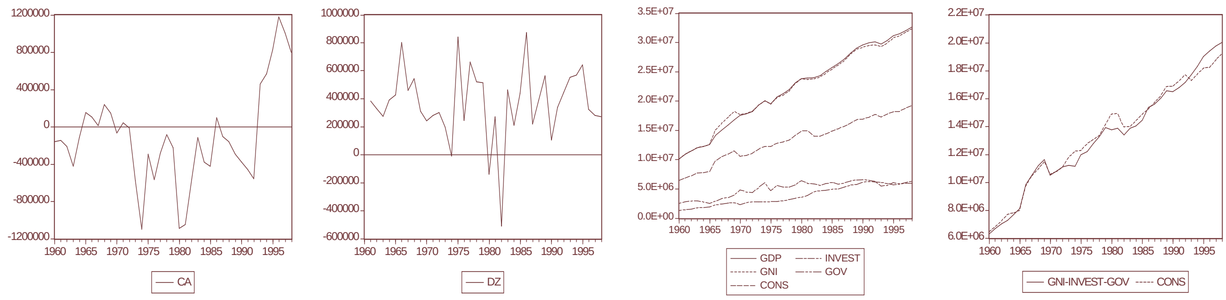
UK



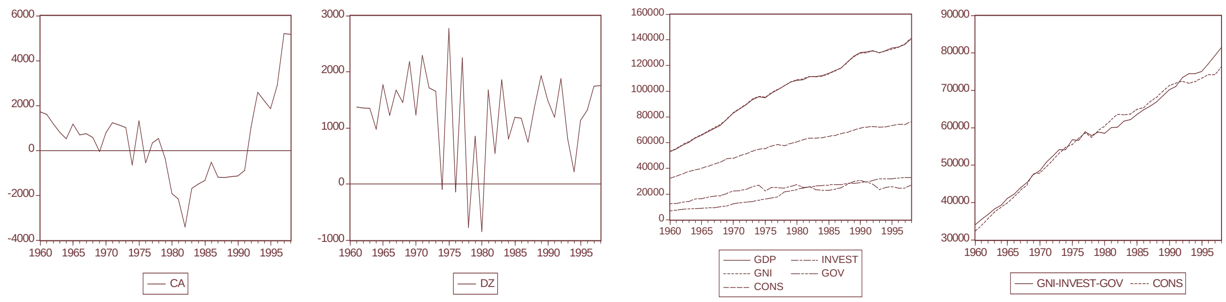
GER



ITA

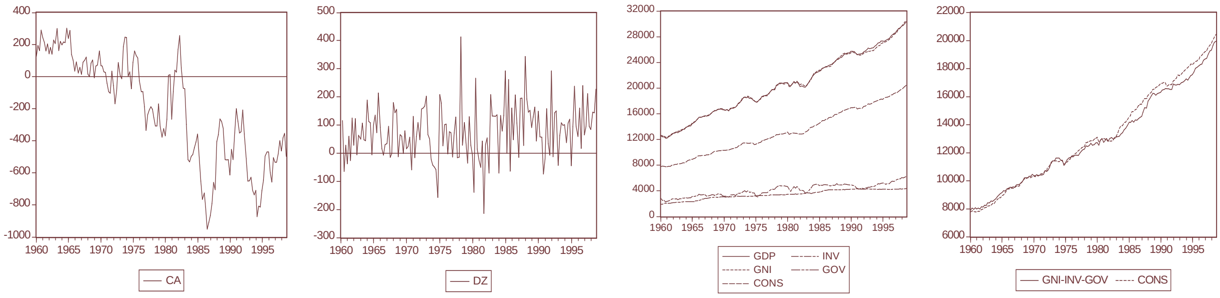


FRA

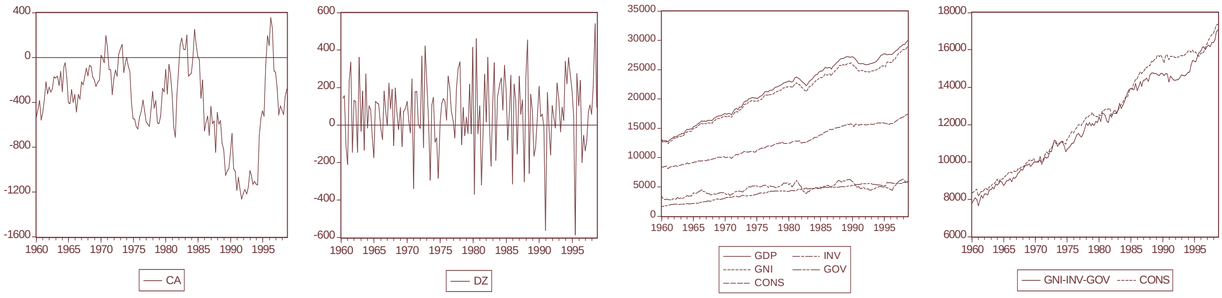


Sample 2 (quarterly data)

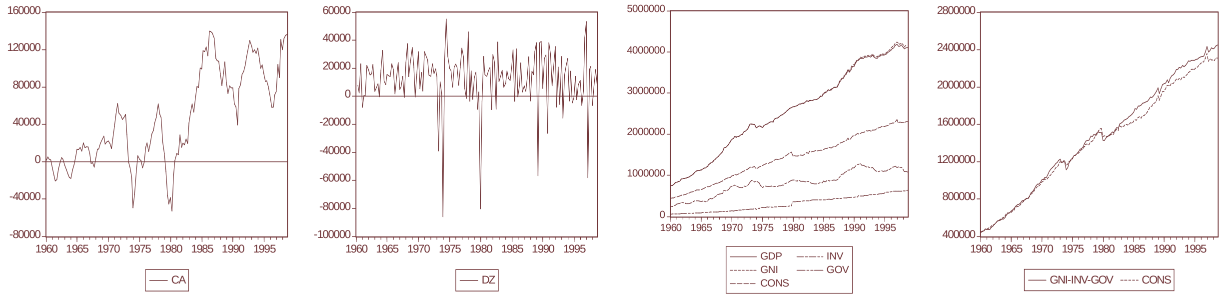
USA



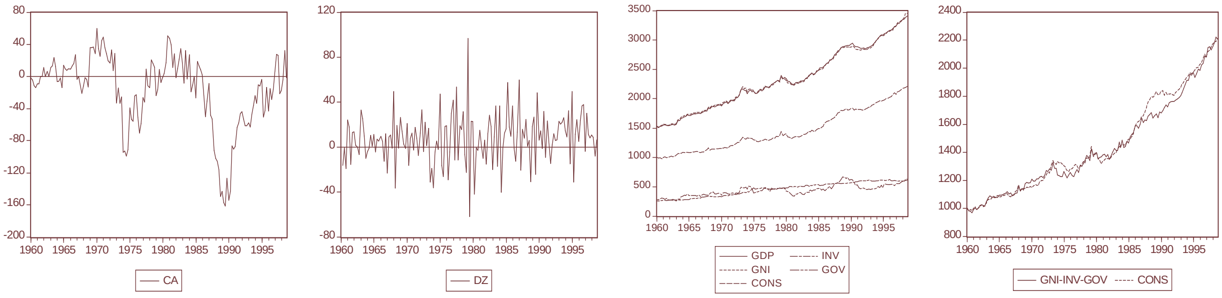
CAN



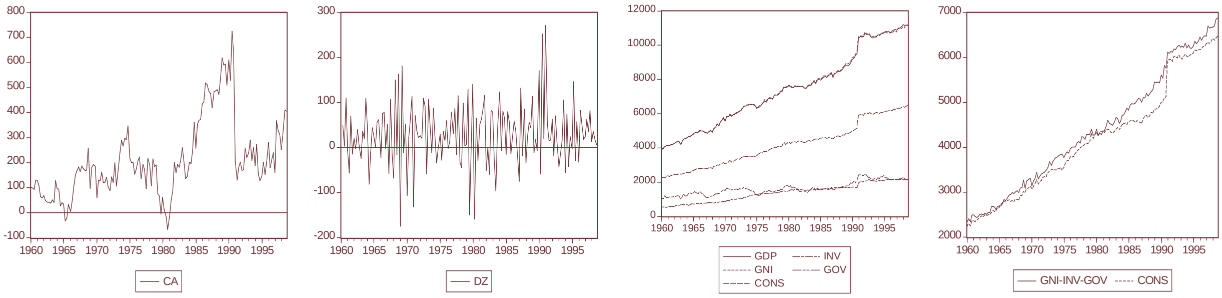
JPN



UK



GER



A.3 Unit Root tests

The results of ADF, Phillips-Perron (PP) and KPSS tests are presented in the following tables. The column 'lags' represents the number of lags in ADF test, selected by Schwartz Information Criterion (SIC). In ADF and PP columns, p-values are presented in parentheses. In KPSS column, the LM statistic is presented (which is followed by the 5% critical value).

The unit root tests of C_t and $(Y_t+rB_t-I_t-G_t)$ include constant and lineal trend. In other cases, just a constant term is considered. It should also be mentioned that the KPSS test differs from the other unit root tests, since under the null hypothesis the series are assumed to be stationary. Furthermore, in the following tables $CA_t^+ = Y_t + rB_t - I_t - G_t - C_t$ and $CA_t = Y_t + rB_t - I_t - G_t - \theta C_t$.

Sample 1 (annual data)

USA

Variable	lags	ADF	PP	KPSS
C_t	1	-1.157195 (0.9047)	-0.963902 (0.9373)	0.199923 0.146
ΔC_t	0	-3.945069 (0.0043)	-3.669281 (0.0088)	0.352971 0.463
$(Y_t+rB_t-I_t-G_t)$	0	-0.521898 (0.9780)	-0.824123 (0.9542)	0.191192 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	0	-4.302770 (0.0016)	-4.254064 (0.0019)	0.515044 0.463
CA_t^+	1	-1.573543 (0.4858)	-0.921909 (0.7703)	0.593808 0.463
CA_t	1	-2.840038 (0.0625)	-2.100991 (0.2454)	0.221408 0.463
ΔZ_t	0	-4.532315 (0.0009)	-4.532315 (0.0009)	0.432923 0.463

CAN

Variable	lags	ADF	PP	KPSS
C_t	1	-3.192046 (0.1016)	-2.554813 (0.3019)	0.067965 0.146
ΔC_t	0	-4.025179 (0.0035)	-4.039189 (0.0033)	0.126816 0.463
$(Y_t+rB_t-I_t-G_t)$	1	-2.985415 (0.1497)	-2.610936 (0.2779)	0.048019 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	0	-4.826465 (0.0004)	-4.690303 (0.0005)	0.083414 0.463
CA_t^+	0	-2.409067 (0.1461)	-2.478414 (0.1285)	0.261955 0.463
CA_t	0	-2.679784 (0.0868)	-2.750387 (0.0751)	0.132264 0.463
ΔZ_t	0	-5.245342 (0.0001)	-5.168390 (0.0001)	0.059161 0.463

JPN

Variable	lags	ADF	PP	KPSS
C_t	0	-2.266674 (0.4410)	-2.387441 (0.3799)	0.067472 0.146
ΔC_t	0	-5.693834 (0.0000)	-5.693834 (0.0000)	0.06517 0.463
$(Y_t + rB_t - I_t - G_t)$	1	-3.298768 (0.0822)	-2.870817 (0.1828)	0.068201 0.146
$\Delta(Y_t + rB_t - I_t - G_t)$	0	-5.639680 (0.0000)	-6.929501 (0.0000)	0.167163 0.463
CA_t^+	4	-0.415276 (0.8954)	-0.845669 (0.7943)	0.636305 0.463
CA_t	1	-3.666683 (0.0089)	-2.664380 (0.0896)	0.095422 0.463
ΔZ_t	0	-5.627289 (0.0000)	-5.996121 (0.0000)	0.142124 0.463

UK

Variable	lags	ADF	PP	KPSS
C_t	1	-1.137342 (0.9086)	-0.723393 (0.9639)	0.183367 0.146
ΔC_t	0	-3.536980 (0.0124)	-3.382338 (0.0181)	0.467566 0.463
$(Y_t + rB_t - I_t - G_t)$	0	0.903876 (0.9997)	2.171536 (1.0000)	0.194052 0.146
$\Delta(Y_t + rB_t - I_t - G_t)$	0	-3.981742 (0.0039)	-3.954622 (0.0042)	0.591701 0.463
CA_t^+	1	-2.537704 (0.1151)	-2.199868 (0.2096)	0.281427 0.463
CA_t	0	-0.178655 (0.9327)	-0.364623 (0.9052)	0.669806 0.463
ΔZ_t	0	-4.467796 (0.0010)	-4.467796 (0.0010)	0.561099 0.463

GER

Variable	lags	ADF	PP	KPSS
C_t	0	-2.027416 (0.5681)	-2.179185 (0.4870)	0.130342 0.146
ΔC_t	0	-5.245994 (0.0001)	-5.186801 (0.0001)	0.113339 0.463
$(Y_t + rB_t - I_t - G_t)$	0	-1.579443 (0.7824)	-1.633699 (0.7604)	0.161035 0.146
$\Delta(Y_t + rB_t - I_t - G_t)$	0	-4.859054 (0.0003)	-4.724019 (0.0005)	0.25115 0.463
CA_t^+	1	-2.361575 (0.1592)	-1.920103 (0.3199)	0.288818 0.463
CA_t	1	-2.896646 (0.0554)	-2.370089 (0.1567)	0.083758 0.463
ΔZ_t	0	-4.693462 (0.0005)	-4.536736 (0.0008)	0.237372 0.463

ITA

Variable	lags	ADF	PP	KPSS
C_t	1	-3.301770 (0.0817)	-2.579694 (0.2911)	0.138605 0.146
ΔC_t	0	-5.289426 (0.0001)	-5.316586 (0.0001)	0.149426 0.463
$(Y_t + rB_t - I_t - G_t)$	0	-2.206942 (0.4723)	-2.347726 (0.3996)	0.086242 0.146
$\Delta(Y_t + rB_t - I_t - G_t)$	0	-5.420334 (0.0001)	-5.420334 (0.0001)	0.077799 0.463
CA_t^+	0	-1.841187 (0.3556)	-1.841187 (0.3556)	0.260649 0.463
CA_t	0	-2.357381 (0.1603)	-2.423078 (0.1424)	0.277123 0.463
ΔZ_t	4	-4.701019 (0.0006)	-6.050856 (0.0000)	0.069826 0.463

FRA

Variable	lags	ADF	PP	KPSS
C_t	0	-1.467058 (0.8234)	-1.404951 (0.8434)	0.19581 0.146
ΔC_t	0	-5.545581 (0.0000)	-5.591024 (0.0000)	0.524792 0.463
$(Y_t + rB_t - I_t - G_t)$	2	-2.035030 (0.5630)	-1.844454 (0.6631)	0.11634 0.146
$\Delta(Y_t + rB_t - I_t - G_t)$	1	-2.456154 (0.1344)	-7.390173 (0.0000)	0.15884 0.463
CA_t^+	0	-0.686168 (0.8382)	-0.686168 (0.8382)	0.185811 0.463
CA_t	0	-2.260467 (0.1895)	-2.356604 (0.1605)	0.648143 0.463
ΔZ_t	1	-2.435291 (0.1396)	-7.762947 (0.0000)	0.15011 0.463

Sample 2 (quarterly data)

USA

Variable	lags	ADF	PP	KPSS
C_t	0	-0.295220 (0.9902)	-0.990336 (0.9414)	0.302198 0.146
ΔC_t	1	-6.233189 (0.0000)	-10.56560 (0.0000)	0.412463 0.463
$(Y_t+rB_t-I_t-G_t)$	0	-0.473954 (0.9838)	-0.540192 (0.9807)	0.308329 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	2	-5.151329 (0.0000)	-13.44103 (0.0000)	0.431295 0.463
CA_t^+	0	-1.921537 (0.3217)	-1.962137 (0.3034)	1.270898 0.463
CA_t	0	-3.197535 (0.0220)	-3.387662 (0.0129)	0.568082 0.463
ΔZ_t	0	-12.58527 (0.0000)	-12.74298 (0.0000)	0.518509 0.463

CAN

Variable	lags	ADF	PP	KPSS
C_t	0	-2.339153 (0.4101)	-2.656870 (0.2562)	0.095883 0.146
ΔC_t	0	-13.10451 (0.0000)	-13.11749 (0.0000)	0.092766 0.463
$(Y_t+rB_t-I_t-G_t)$	0	-3.381035 (0.0577)	-3.187706 (0.0907)	0.066399 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	0	-15.62751 (0.0000)	-15.83949 (0.0000)	0.085856 0.463
CA_t^+	0	-2.743747 (0.0691)	-2.775200 (0.0642)	0.428733 0.463
CA_t	0	-3.007405 (0.0364)	-3.028077 (0.0345)	0.145866 0.463
ΔZ_t	0	-14.84436 (0.0000)	-14.94291 (0.0000)	0.030964 0.463

JPN

Variable	lags	ADF	PP	KPSS
C_t	0	-2.390898 (0.3828)	-2.314982 (0.4230)	0.153853 0.146
ΔC_t	0	-15.14648 (0.0000)	-15.15494 (0.0000)	0.069081 0.463
$(Y_t+rB_t-I_t-G_t)$	0	-3.012862 (0.1321)	-3.058727 (0.1201)	0.108213 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	0	-13.45073 (0.0000)	-13.54160 (0.0000)	0.037083 0.463
CA_t^+	0	-1.111693 (0.7106)	-1.524293 (0.5188)	1.12758 0.463
CA_t	2	-3.438944 (0.0111)	-2.986649 (0.0384)	0.160007 0.463
ΔZ_t	0	-13.59015 (0.0000)	-13.60027 (0.0000)	0.053106 0.463

UK

Variable	lags	ADF	PP	KPSS
C_t	0	-0.855194 (0.9573)	-0.916368 (0.9506)	0.319188 0.146
ΔC_t	2	-5.413380 (0.0000)	-13.80443 (0.0000)	0.484394 0.463
$(Y_t+rB_t-I_t-G_t)$	1	0.280990 (0.9984)	-0.241521 (0.9916)	0.340128 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	0	-17.62711 (0.0000)	-17.26982 (0.0000)	0.421844 0.463
CA_t^+	1	-2.273837 (0.1818)	-2.729779 (0.0713)	0.387352 0.463
CA_t	1	-0.757300 (0.8277)	-1.656364 (0.4514)	0.898279 0.463
ΔZ_t	0	-16.77832 (0.0000)	-16.64067 (0.0000)	0.535831 0.463

GER

Variable	lags	ADF	PP	KPSS
C_t	0	-2.220133 (0.4747)	-2.268694 (0.4481)	0.181267 0.146
ΔC_t	0	-13.41021 (0.0000)	-13.39538 (0.0000)	0.098447 0.463
$(Y_t+rB_t-I_t-G_t)$	1	-2.108709 (0.5365)	-2.873701 (0.1740)	0.250588 0.146
$\Delta(Y_t+rB_t-I_t-G_t)$	0	-19.54841 (0.0000)	-20.17804 (0.0000)	0.186134 0.463
CA_t^+	0	-2.782748 (0.0631)	-2.482801 (0.1216)	0.6885 0.463
CA_t	0	-3.260318 (0.0185)	-3.082903 (0.0299)	0.233715 0.463
ΔZ_t	0	-18.60645 (0.0000)	-18.54294 (0.0000)	0.174557 0.463

A.4 VAR estimation

The coefficients of the unrestricted VAR, estimated by OLS, as well as the t-statistics (in parentheses), are shown in the following tables.

Table 16 - Coefficients of the VAR (**sample 1** - annual data)

Country	USA		CAN		JPN		UK		GER		ITA		FRA	
equation	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)
ΔZ_(t-1)	-0,08	-0,12	0,09	0,14	-0,09	-0,08	0,13	-0,09	0,17	0,11	-0,07	-0,15	0,00	0,23
	(-0,43)	(-1,09)	(0,43)	(1,04)	(-0,50)	(-0,60)	(0,64)	(-0,48)	(1,03)	(1,09)	(-0,40)	(-0,61)	(-0,01)	(0,78)
ΔZ_(t-2)	-0,17	0,10	-	-	-0,07	0,06	-	-	-	-	-	-	0,57	0,49
	(-1,00)	(1,01)			(-0,42)	(0,51)							(3,67)	(1,93)
CA_(t-1)	-0,55	1,23	0,03	0,61	0,57	1,06	0,13	1,01	0,26	0,73	0,06	0,76	-0,19	0,73
	(-2,20)	(7,98)	(0,13)	(4,28)	(2,20)	(5,95)	(1,22)	(10,18)	(1,33)	(6,11)	(0,69)	(6,12)	(-1,55)	(3,69)
CA_(t-2)	-0,15	-0,51	-	-	-0,52	-0,53	-	-	-	-	-	-	0,17	0,15
	(-0,52)	(-2,81)			(-2,10)	(-3,06)							(1,59)	(0,82)
c	786	156	222	193	59 302	-29 220	-22	22	366	-9	479 985	-255 805	277	-3 555
	(4,44)	(1,44)	(2,02)	(2,53)	(2,58)	(-1,85)	(-0,22)	(0,23)	(3,37)	(-0,14)	(2,89)	(-1,14)	(0,36)	(-2,81)
Adj. R-squared	0,29	0,75	0,01	0,47	0,06	0,54	0,06	0,81	0,04	0,54	0,01	0,55	0,32	0,93

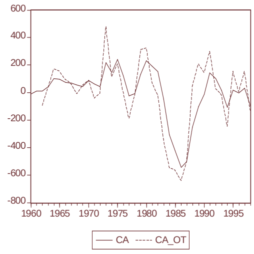
Table 17 - Coefficients of the VAR (**sample 2** - quarterly data)

Country	USA		CAN		JPN		UK		GER	
equation	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)	ΔZ_(t)	CA_(t)
ΔZ_(t-1)	-0,05	-0,20	-0,17	-0,08	-0,22	-0,02	-0,31	-0,16	-0,41	-0,15
	(-0,58)	(-2,40)	(-2,13)	(-1,26)	(-2,64)	(-0,33)	(-3,97)	(-2,03)	(-5,36)	(-1,85)
ΔZ_(t-2)	-	-	-	-	-0,22	-0,15	-	-	-	-
					(-2,66)	(-2,89)				
ΔZ_(t-3)	-	-	-	-	0,15	-0,03	-	-	-	-
					(1,89)	(-0,58)				
CA_(t-1)	-0,09	0,86	-0,04	0,90	0,53	1,01	0,03	0,96	0,05	0,88
	(-2,39)	(22,13)	(-0,82)	(23,59)	(4,13)	(12,39)	(1,07)	(32,06)	(1,24)	(21,66)
CA_(t-2)	-	-	-	-	-0,27	0,27	-	-	-	-
					(-1,51)	(2,40)				
CA_(t-3)	-	-	-	-	-0,20	-0,36	-	-	-	-
					(-1,54)	(-4,39)				
c	129	86	98	71	20 463	-3 033	2	11	39	9
	(5,62)	(3,70)	(3,18)	(2,84)	(4,24)	(-0,99)	(0,32)	(1,53)	(7,04)	(1,47)
Adj. R-squared	0,02	0,77	0,03	0,79	0,13	0,88	0,08	0,87	0,15	0,75

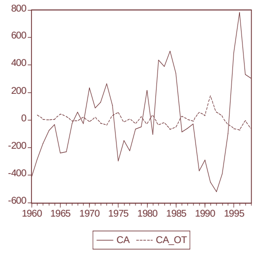
A.5 Optimal *versus* Observed current accounts

Sample 1 (annual data)

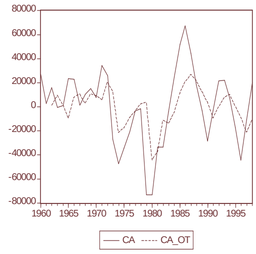
USA



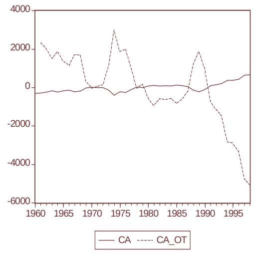
CAN



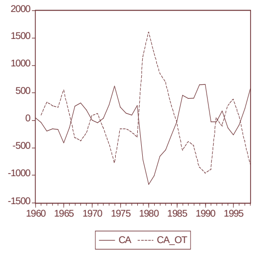
JPN



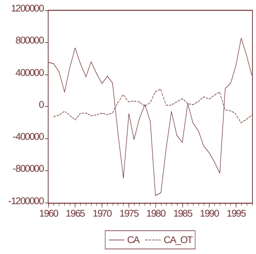
UK



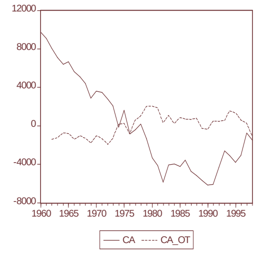
GER



ITA

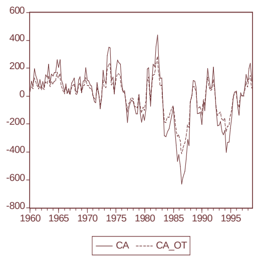


FRA

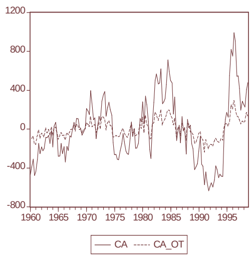


Sample 2 (quarterly data)

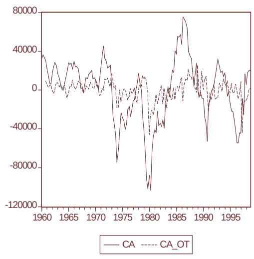
USA



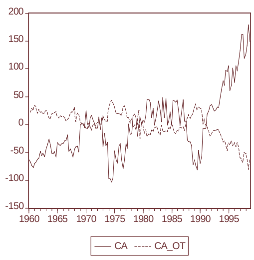
CAN



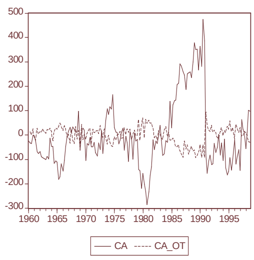
JPN



UK



GER



A.6 Comparison of results

Table 18 - **Main Results**

	θ			lags (VAR)				Granger causality (p-value)					Wald Test				Correlation(.)		
Author	**CS**	**G**	**A**	**CS**	**O**	**G**	**A**	**CS**	**CS2**	**O**	**G**	**A**	**CS2**	**O**	**G**	**A**	**CS**	**O**	**G**
USA	0.955	0.994	-	2	5	1	-	0.002	0.000	0.0001	0.0004	-	0.029	0.0041	1.19	-	0.84	0.932	0.998
CAN	0.928	0.96	-	1	4	1	-	0.899	0.668	0.21	0.40	-	0.0001	0.002	95	-	-0.79	0.735	0.98
JPN	1.076	1.04	-	2	-	2	-	0.079	0.057	-	0.62	-	0.119	-	75	-	0.63	-	0.996
UK	0.806	0.98	-	1	-	2	-	0.231	0.034	-	0.68	-	0.000	-	464	-	-0.99	-	0.70
GER	1.029	1.08	-	1	-	3	-	0.194	0.209	-	0.76	-	0.004	-	90	-	-0.95	-	0.810
ITA	1.088	-	-	1	-	-	-	0.495	0.369	-	-	-	0.000	-	-	-	-0.97	-	-
FRA	1.334	-	0.982	2	-	-	1	0.295	0.109	-	-	0.10	0.000	-	-	0.314	-0.69	-	-
USA q	0.945			1				0.018	0.013				0.0861				0.99		
CAN q	0.923			1				0.415	0.162				0.1956				0.97		
JPN q	1.079			3				0.001	0.000				0.0000				0.36		
UK q	0.826			1				0.285	0.167				0.0009				-0.97		
GER q	1.043			1				0.216	0.189				0.0001				-0.81		

Notes: a) Correlation (.) refers to the observed and optimal current accounts (CA_t;CA_t^*).

b) CS means Current Study (OLS estimation), CS2 means Current Study (SURE estimation),

O refers to Otto (92), G indicates Ghosh (95), A refers to Agénor (1999).

c) Granger causality refers to the null hypothesis (CA_t does not Granger-cause ΔZ_t).

d) P-values are shown in the Wald test column, except for Ghosh (1995),

that presents qui-squared values and rejects the model to CAN, UK and GER (at 1% level),

and to JPN (at 5% level), but does not reject it to the USA.

A.7 A note on Granger Causality and Wald Tests

A VAR(2) model can be rewritten as a VAR(1) in the following way:

$$\begin{bmatrix} \Delta Z_t \\ \Delta Z_{t-1} \\ CA_t \\ CA_{t-1} \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & b_1 & b_2 \\ 1 & 0 & 0 & 0 \\ c_1 & c_2 & d_1 & d_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} \Delta Z_{t-1} \\ \Delta Z_{t-2} \\ CA_{t-1} \\ CA_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ 0 \\ \varepsilon_{2t} \\ 0 \end{bmatrix}$$

or in a compact form $X_t = AX_{t-1} + \varepsilon_t$. Hence, the vector K is given by

$$K = \begin{bmatrix} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 \end{bmatrix} = -h'(\frac{A}{1+r})(I_4 - \frac{A}{1+r})^{-1}$$

$$\therefore K = -\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}(\frac{1}{1+r}\begin{bmatrix} a_1 & a_2 & b_1 & b_2 \\ 1 & 0 & 0 & 0 \\ c_1 & c_2 & d_1 & d_2 \\ 0 & 0 & 1 & 0 \end{bmatrix})(\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \frac{1}{1+r}\begin{bmatrix} a_1 & a_2 & b_1 & b_2 \\ 1 & 0 & 0 & 0 \\ c_1 & c_2 & d_1 & d_2 \\ 0 & 0 & 1 & 0 \end{bmatrix})^{-1}$$

Thus, after some algebraic manipulations, the β_1 coefficient of the vector K is given by:

$$\begin{aligned} \beta_1 &= -\frac{a_1(1+r)(b1+b1r+b2)}{\phi} - \frac{a_2(b1+b1r+b2)}{\phi} \\ &\quad -\frac{b1(1+r)(1+2r+r^2-a1-a1r-a2)}{\phi} - \frac{b2(1+2r+r^2-a1-a1r-a2)}{\phi} \end{aligned}$$

$$\begin{aligned} \text{where} \quad \phi &= 1+2a_1d_1r+4r+a_1d_1r^2-c_1b_2r-2c_1b_1r+a_2d_1r-d_1-d_2-a_2-a_1-c_2b_1r+6r^2 \\ &\quad -3d_1r-2a_2r-a_2r^2+a_2d_1+a_2d_2-c_2b_1-c_2b_2+a_1d_2r-c_1b_1r^2+4r^3+r^4 \\ &\quad -3d_1r^2-2rd_2-3a_1r-3a_1r^2+a_1d_1+a_1d_2-d_1r^3-r^2d_2-a_1r^3-c_1b_1-c_1b_2 \end{aligned}$$

In this case, the optimal current account is given by

$$(CA_t^* - \mu_{CA^*}) = K(X_t - \mu) = \begin{bmatrix} \alpha_1 & \alpha_2 & \beta_1 & \beta_2 \end{bmatrix} \begin{bmatrix} \Delta Z_t - \mu_{\Delta Z} \\ \Delta Z_{t-1} - \mu_{\Delta Z} \\ CA_t - \mu_{CA} \\ CA_{t-1} - \mu_{CA} \end{bmatrix}$$

and the Wald test analyzes the joint restrictions: $\alpha_1 = \alpha_2 = \beta_2 = 0$ and $\beta_1 = 1$. Again, one should note that if the Granger causality is not verified (e.g., $b_1 = b_2 = 0$) then $\beta_1 = 0$. In this manner, the implication of Granger causality becomes a necessary condition for the validation of the VAR(2) model. The generalization of this result to a VAR(p) framework is straightforward, as it follows.

Proof of Theorem 1. The VAR(p) can be rewritten as a VAR(1), in the following way

$$\begin{bmatrix} \Delta Z_t \\ \vdots \\ \vdots \\ \Delta Z_{t-p+1} \\ CA_t \\ \vdots \\ \vdots \\ CA_{t-p+1} \end{bmatrix} = \begin{bmatrix} a_1 & \cdots & \cdots & a_p & b_1 & \cdots & \cdots & b_p \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ c_1 & \cdots & \cdots & c_p & d_1 & \cdots & \cdots & d_p \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta Z_{t-1} \\ \vdots \\ \vdots \\ \Delta Z_{t-p} \\ CA_{t-1} \\ \vdots \\ \vdots \\ CA_{t-p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ 0 \\ \vdots \\ 0 \\ \varepsilon_{2t} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or in the same compact form $X_t = AX_{t-1} + \varepsilon_t$. Supposing that the Granger causality is not provided by the dataset, it follows that $b(L) = 0$, and the companion matrix A becomes:

$$A = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$$

where $A_{11} = \begin{bmatrix} a_1 & \cdots & \cdots & a_p \\ 1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$; $A_{21} = \begin{bmatrix} c_1 & \cdots & \cdots & c_p \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$; $A_{22} = \begin{bmatrix} d_1 & \cdots & \cdots & d_p \\ 1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

In this case, vector K can be written as $K = -h'BC^{-1}$, where the matrices B and C are defined as $B = (\frac{A}{1+r})$ and $C = (I_{2p} - \frac{A}{1+r})$. It should be noted that matrices B and C are also partitioned matrices with a null upper-right block (NURB). According to Simon & Blume (1994), in this case, the inverse of the partitioned matrix C will also result in a NURB matrix (see Lemma reproduced below).

Lemma 1 *Let C be a square matrix partitioned as $C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$ where C_{11} and C_{22} are square sub-matrices. If both C_{22} and $D \equiv C_{11} - C_{12}C_{22}^{-1}C_{21}$ are nonsingular, then C is nonsingular and*

$$C^{-1} = \begin{bmatrix} D^{-1} & -D^{-1}C_{12}C_{22}^{-1} \\ -C_{22}^{-1}C_{21}D^{-1} & C_{22}^{-1}\left(I + C_{21}D^{-1}C_{12}C_{22}^{-1}\right) \end{bmatrix}$$

■

Thus, in our case, $C_{12} = 0$ and the term $-D^{-1}C_{12}C_{22}^{-1}$ becomes a null submatrix, suggesting that C^{-1} and the following product BC^{-1} are also NURB matrices.

Finally, the vector $K = \begin{bmatrix} \alpha_1 & \ldots & \alpha_p & \beta_1 & \ldots & \beta_p \end{bmatrix}$ is given by selecting (through the vector $-h'$) the first line from the matrix (BC^{-1}), suggesting that all β_i $(i = 1, \ldots, p)$ coefficients are zero.