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# Vintage Capital, Distortions and Development

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## Abstract

This paper asks to what extent distortions to the adoption of new technology cause income inequality across nations. We work in the framework of embodied technological progress with an individual, C.E.S. production function. We estimate the parameters of this production function from international data and calibrate the model, using U.S. National Income statistics. Our analysis suggests that distortions account for a bigger portion of income inequality than hitherto has been assessed.

JEL Classification numbers: D24, D33, E25, O11, O47, O49

Key words: Vintage Capital, Embodied Technological Progress, Putty-Clay, Total Factor Productivity, Elasticity of Substitution.

## 1 Introduction

In this paper we assess to what extent distortions to the purchase of capital-goods account for the huge per-capita income differences among countries of the world. We approach this question by considering a world in which there is exogenous technological progress and where the decision to adopt new technology depends on the price of capital that embodies this technology. We construct a model of this adoption decision and use it to determine

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whether the observed variation in the price of adopting technology accounts for the observed variation in per-capita incomes.

In analyzing this question, we hypothesize that technological progress is embodied, rather than disembodied. In other words, we hypothesize that technological progress is effective only if new capital-goods that embody the new technology are installed. In such a world, the effect of distortions is strong for the simple reason that when no new capital-goods are installed (due to distortions), there is no growth in income. By contrast, if technological progress were disembodied, it would apply to old capital-goods and, hence, would raise incomes even without the installation of new capital-goods. Our main objective is to quantify this reasoning; i.e., contrast the quantitative effects of distortions in a disembodied versus an embodied model.

This approach was first suggested in a paper by Jovanovic and Rob (1997) and continued in papers by Parante (2000) and Mateus-Planas (2001). Compared with those papers, we offer the following improvements. First, instead of picking a production function in advance (for example, the Leontieff production function), we estimate a production function from empirical data. More specifically, we consider the C.E.S. family of production functions, and pin down one member of this family based on international data on investments and prices. This makes the model more amenable to empirical and quantitative exercises for two reasons. First, because the production function is derived from data, the results pertaining to it are more empirically relevant. Second, because the production function we use allows firms to choose the quantity of capital on top of the timing of adoption, the model mimics more accurately the way firms adjust their capital in the real world.

A second improvement compared to previous models with embodied technological progress is that our theory cranks out a price-profile for capital of different vintages. This price-profile is then used to evaluate the inventory of capital goods in the economy, which is the common practice in National Income statistics. As a result of this procedure, we make a clear and operational distinction between three types of depreciation: physical, technological (or obsolescence) and economic (or scrapping). By contrast, previous models construct an ad-hoc measure of the value of the inventory of capital goods and, as a result, lump together different types of depreciation.

As a consequence of these improvements, we show that distortions account for a larger fraction of income inequality than hitherto has been assessed. This is reflected first by the fact that our simulated model economy (where “simulated” means we allow only prices to

vary across countries, holding other parameters constant) can easily account for the observed income inequality when distortions are allowed to vary over a reasonable range. This is to be contrasted with the aforementioned models of embodied technological progress where that is not the case. It is also to be contrasted with models of disembodied technological progress, which require an unrealistic capital share of income, namely  $2/3$ , instead of the “traditional”  $1/3$  in order to have the same explanatory power (see Chari et al. (1997)). Related to this, a version of our model is able to account for growth miracles based purely on factor accumulation, as documented by Young (1995). A second indicator of the success of our theory is that, even if we accommodate other, non-economic factors, distortions remain an important explanatory variable. This can be contrasted with Hall and Jones (1999) who argue that these factors play an overwhelming role by comparison with economic factors. The reason we are able to improve upon the Hall and Jones (1999) exercise is that our embodied model generates an endogenous Total Factor Productivity (TFP) term, which accounts for some of the variation in income. By contrast, when technological progress is disembodied, there is no such thing as endogenous TFP and, thus, the entire unexplained variation in incomes is absorbed into the exogenously specified TFP term.

We proceed as follows. In the following subsection we describe verbally the mechanics of our model. In Section 2 we formalize the model. In section 3 we solve the firm’s maximization program. In section 4 we solve for general equilibrium. In section 5 we aggregate various variables and derive various National Income statistics. In Section 6 we calibrate the model. In Section 7 we estimate the individual production function. In section 8 we simulate the model, showing what per-capita incomes it predicts as distortions vary from country to country. In Section 9 we report various other predictions of our model, including the capital-output ratio and the price-earning ratio. In Section 10 we perform a development decomposition exercise, showing how much inequality is explained by distortions when we take a more comprehensive view of the sources of inequality. Section 11 concludes.

## 1.1 Verbal description of the Model and its Mechanics

The unit of analysis in the model is the individual firm and its production function. A firm in the model faces exogenous technological progress and has to decide how frequently to adopt new technology. A firm adopts new technology by scrapping its old capital and buying new capital that embodies the new technology. Thereby, a firm’s decision is twofold: it chooses

the timing of adoption and, at each date of adoption, it chooses how much capital to install.

The economy is populated by a continuum of firms, which upgrade their technology in sequence. As a result of these sequential upgradings, we have, at any moment in time, a window of capital goods in the economy. The width of this window corresponds to the waiting period between upgrades and the height of the window corresponds to how much capital is being installed by each firm. In addition, at any moment in time, we have a (general equilibrium) price-profile of capital-goods; i.e., a function relating the vintage of a capital-good to its price. The essence of the model is to determine how the window and prices of capital-goods are affected by distortions and other parameters. Once we determine these effects, we proceed to determine the effect of distortions on various National Income statistics. In particular, we determine the effect of distortions on the holding period of capital-goods, the per-capita income, the investment-capital ratio, the investment-output ratio, wage-rates, and the price-earning ratio. These effects can then be contrasted both with micro and macro-data.

## 2 The Model

We consider an infinite horizon, continuous-time economy.

**Agents and Goods.** The economy is populated by a continuum of identical, infinitely-lived individuals of measure 1. Each individual consumes output, and supplies inelastically one unit of labor. The productivity of individuals' labor is determined by their educational level.

In addition to individuals, there is a continuum of infinitely-lived firms. Each firm sells output, hires individuals, and buys capital. Firms are owned by individuals, as specified below. Individuals and firms take prices as given. Since production in each firm is according to a constant returns to scale technology, the measure (and size) of firms is indeterminate. For convenience, we normalize this measure to be 1.

There is one output in the economy, which is used both for consumption and capital accumulation. When a firm buys capital, output is withdrawn from consumption on a one-to-one basis.

**Technological Progress, Vintage Capital and embodiment.** The economy enjoys exogenous technological progress. The frontier technology at date  $s$  is of quality  $A(s)$ , where

the meaning of “quality” is specified immediately below. We assume:

$$A(s) = A_0 e^{gs},$$

where  $g$  is the rate of technological progress.

Firms periodically upgrade their technology by buying new capital. Capital installed at date  $s$  reflects the date- $s$  frontier technology and is said to be vintage- $s$  capital. A firm that last upgraded its technology at date  $s$  is said to be a vintage- $s$  firm.<sup>1</sup>

A single firm at a given point in time. A firm is characterized, at a given point in time  $t$ , by its technology  $s$  ( $s \leq t$ ), and its capital-worker input-combination, denoted  $(K, L)$ , where  $K$  is the capital-worker ratio and  $L$  is the number of workers. Assume all workers have the same educational level  $h$ , and assume  $h$  is constant through time. Then, an  $(s, K, L)$ -firm is using  $LA(s)e^{\phi(h)}$  efficiency units of labor.  $h$  is measured by years of schooling and  $e^{\phi(h)}$  is a Mincerian function which translates years of schooling into labor productivity. According to this specification, a newer-vintage technology is of higher quality in that it translates the time of a worker into more efficiency units of labor. A higher educational level has the same effect.

Production in a single firm. A firm produces output according to a C.E.S. production function. Namely, the date- $t$  per-worker flow of output of an  $(s, K, L)$ -firm is:

$$Y(s, t) = e^{-\delta(t-s)} B \left[ (1 - \alpha)(A(s)e^{\phi(h)})^{\frac{\sigma-1}{\sigma}} + \alpha K^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \quad (1)$$

where  $\sigma$  is the elasticity of substitution and  $\alpha$  is the distribution parameter.  $B$  is total factor productivity (TFP), i.e., it measures the efficiency with which a firm converts inputs into output, and  $\delta$  is the physical depreciation factor.<sup>2</sup> The values of these parameters are the same for all firms within one country, and remain constant through time.<sup>3</sup> On the other hand, as we go across countries (which we do later) the values of parameters change. Hence differences across countries are captured by differences in parameter values.

To make the language less cumbersome we refer sometimes to capital per-worker simply

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<sup>1</sup>To be precise one should distinguish between a firm and a plant. A firm is an infinitely-lived entity that upgrades its plant every once in a while. A plant consists of a certain vintage capital and ceases to exist once this capital is upgraded. For simplicity we suppress this distinction.

<sup>2</sup>Note that depreciation affects output, not capital; we comment on this below.

<sup>3</sup>Parameter values satisfy:  $B > 0$ ,  $\alpha \in (0, 1)$ ,  $\delta, \sigma \geq 0$ .

as “capital” and to output per-worker as “output.”

**Upgrading.** Consider a point in time, say  $s$ , at which a firm upgrades its technology. At that point the firm also chooses how much capital of that vintage to install and how many workers to employ. Once the firm makes that determination it is committed to the vintage- $s$  technology and to the  $(K, L)$  input-combination - until its next technological upgrade. Therefore, a firm has full flexibility ex-ante, and full rigidity ex-post.<sup>4</sup> Between upgrades the firm makes no economic decisions; it merely collects the output flow specified by (1), and pays wages.

The cost of upgrading to a new technology is  $p$  units of output per-unit capital of the new vintage. Thus, if a firm upgrades at date  $s$  and installs  $K$  units of capital per-worker, it pays  $pKL$ .  $pKL$  is paid up-front (at  $s$ ), and  $p$  is independent of  $s$ .

When a firm upgrades to a new technology it dumps the capital of its old technology; there is no such thing as a second-hand market in capital-goods,<sup>5</sup> and there is no such thing as combining capital of different vintages. Also, although capital is “made of” output and output is the same through time, there is no way of converting capital-goods of an old vintage into a newer vintage. There is also no way of converting capital-goods back into output that can be consumed. The decision to create capital goods is irrevocable.

**The Firm’s Maximization Problem.** The objective of a firm is to maximize the discounted value of revenues net of wages and capital upgrading costs. Discounting is with respect to the constant, instantaneous interest rate  $r$ , which the firm takes as given.

The economic decision facing a firm is as follows. At a given point in time, say  $t$ , the firm operates with a certain combination  $(s, K, L)$ . Then it has to decide on its next upgrade date say  $s + T$ , where  $T$  is labelled the “waiting period” (and  $\frac{1}{T}$  is labelled the “frequency of upgrades.”) The firm has also to decide on how much capital to install and how many workers to employ at  $s + T$ .

The trade-off governing these decisions is as follows. The capital stock the firm has on its hands is already paid for so it “comes for free.” On the other hand, as time goes on the technological frontier keeps moving out so this capital becomes more and more obsolete. On top of that, the firm had pre-committed itself to employ a certain number of workers whose equilibrium wage keeps increasing (see below).<sup>6</sup> Therefore, a point comes where it no longer

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<sup>4</sup>Known in the 1960s literature as the “putty-clay” formulation.

<sup>5</sup>We introduce this market later.

<sup>6</sup>Obviously, this cost is lessened if labor is divisible and perfectly mobile across firms/vintages. Nonethe-



pays to keep the old technology, i.e., it pays to upgrade. We determine below the optimal upgrade date, or, equivalently, the optimal waiting period,  $T$ .

Considering output used for consumption as the numeraire, the dynamic programming formulation of the firm's problem is as follows:

$$V(s, K, t) = \max_{T, K'} \left\{ \int_t^{T+s} e^{-r(\tau-t)} [Y(s, \tau) - w(\tau)] d\tau + e^{-r(T+s-t)} [V(T+s, K', T+s) - pK'] \right\}, \quad (2)$$

where  $w(\tau)$  is the wage rate at date  $\tau$ .  $V$  is the per-worker value function. The firm takes  $p, r$ , and  $w(\cdot)$  as given.

**Distortions.** As stated earlier there is, at a fixed point in time, a one-to-one (social) rate of transformation between consumption and investment. Nonetheless, a firm (privately) pays  $p$  for capital, rather than paying 1. The reason is that there are distortions (or subsidies) to the adoption of technology, which equal  $p - 1$ . At the level of the theory, distortions are broadly interpreted; distortions could be taxes or tariffs on the acquisition of machines, bureaucratic hurdles on the construction of new structures, regulatory delays in implementing new discoveries, corruption, etc. When we (later) quantify the theory, distortions are more narrowly interpreted as taxes and tariffs. At that point we also verify that this narrow interpretation does not constrain the empirical usefulness of our quantitative exercises.

Several comments on our modeling choices are in order.<sup>7</sup>

1. Production in our model is according to the C.E.S. production function, which is in contrast with the “usual” production function in the growth and development literature, namely Cobb-Douglas. Our idea is to allow a parametric family of production functions and, then, pin down a member of this family, based on empirical data; in particular, we are not ruling out the Cobb-Douglas case, which is a special case of a C.E.S. production function when  $\sigma = 1$ .<sup>8</sup> As we show below a wealth of empirical

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less, upgrading still occurs as long as  $\sigma < 1$  (even if labor is divisible and perfectly mobile). Thus, our results do not necessarily hinge on the ex-post rigidity of labor demand.

<sup>7</sup>Our model is closely related to a class of models popularized in the 1960, in particular, Phelps (1963), Solow et al (1966), Bliss (1968) and Bardhan (1969). Our working paper explains in what ways we extend that class of models.

<sup>8</sup>We are also not ruling out the Leontief case, which is a special case of a C.E.S. production function when  $\sigma = 0$ . Compared to the Leontief case our theory allows the firm to vary the quantity of capital, which is impossible under Leontief.

implications hinges on the production function at hand. Thus, we are able to assess the plausibility of the production function we pin down by comparing the implications it yields to those yielded by the Cobb-Douglas (or some other) production function.

2. Technological progress in our model is embodied in capital, but has an effect on labor. Intuitively, what the new-vintage capital does is to enable workers to use better equipment, thereby raising their productivity. This is known in the literature as “labor-augmenting” (or “labor-saving”) technological progress. Two major reasons for considering the labor-augmenting, as opposed to the capital-augmenting, case are: (i) Since we focus on a balanced growth-path (see below), since such path does not exist when technological progress is capital-augmenting (unless the production function is Cobb-Douglas), and since it does exist when technological progress is labor-augmenting, the choice of which technological progress to consider is pinned down by what we focus on. (ii) If technological progress were capital-augmenting and labor were mobile between vintages the model would be simpler in that there would be an aggregate quantity of capital and an aggregate production function. See Fisher (1965) for a general result, and Greenwood et al. (1997) who use such model to study growth features of the U.S. economy. The advantage of using our more complex model is that it parallels the treatment of capital goods in U.S. National Accounting, NIPA (whereas the simpler model does not). In particular, NIPA reaches an aggregate value of capital by distinguishing between capital goods of different vintages, pricing them differentially and adding up their values, which is exactly what we do here.<sup>9</sup>
3. As time progresses a firm is able to produce less and less (unless it upgrades) due to what we call “physical depreciation.” More conventionally, physical depreciation affects the capital stock, not the output that results from it.<sup>10</sup> The primary reason for our specification is analytical convenience. As we shall see, we are able to solve

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<sup>9</sup> Another obvious advantage is that the model with mobile labor has been extensively studied, whereas the model with immobile labor has not.

<sup>10</sup> A plausible story to tell about our way of treating depreciation (and, at the same time, about the assumption that labor is immobile) is the following. Old machines break down or require maintenance more frequently than new machines. Hence, more labor has to be shifted away from actual production into maintenance, and the assumption is that the time required for maintenance is proportional to cumulative depreciation. Therefore, the ratio of machines to labor employed in actual production remains constant, which, under constant returns to scale, allows us to move the depreciation factor outside the production function.

the model analytically under this assumption, whereas under the more conventional assumption it can only be solved numerically. As it turns out, it makes little difference whether output or capital depreciates. We have solved (numerically) the model under the alternative assumption that capital depreciates, and found the quantitative results to be very close; we explain the reason for this later.

To complete the model's description one needs to specify the economy's initial conditions, and individuals' maximization problems (in their capacity as consumers). We do that after we state the solution to the firm's problem.

### 3 Solution to the Firm's Maximization Problem

We seek a balanced growth-path, along which all real variables grow (exponentially) at the constant rate,  $g$ . With this goal in mind, we consider, from this point onwards, only constant-growth wage paths,  $w(\tau) = w_0 e^{g\tau}$ , where  $w_0$  is endogenous and yet to be determined.

The firm's maximization program, (2), reflects two sub-programs which have a long tradition in economic theory. Holding the quantity of capital constant, the problem of when to upgrade the technology is a stopping-time problem (e.g., when to cut a tree). Holding the upgrade dates constant, the problem of how much capital to install is a capital accumulation problem. Although the solutions to these problems - in isolation - are well understood, when we combine them, use a C.E.S. production function, and assume ex-post input rigidity, several issues come up. The first issue is the existence of an optimal solution to the sequence program. To prove existence, one ordinarily exploits the assumption of a fixed (and less than 1) discount factor to show compactness and continuity over some relevant domain. Here, however, we have an endogenous upgrading period and, hence, an endogenous discount factor, which is not necessarily bounded away from 1. Thus, the ordinary method is not directly applicable. A second issue is the uniqueness of an optimum. Ordinarily, one uses (strict) concavity to establish that. Here, again because of the timing of upgrades decision, we don't have concavity. The third issue is that we may have qualitatively different types of optima, depending on parameter values. Indeed as  $\sigma$  increases, it becomes more profitable to upgrade the technology more frequently, and install less capital at each upgrade date. And, in the limit, the firm may upgrade continuously and install no capital at all. Thus, technological progress may become "disembodied" in that the firm uses labor only. Our working paper

deals with these issues and characterizes the solution across different parameter values. The following Proposition states the net result for a range of parameter values which includes the empirically relevant case. In stating the Proposition we use the following notation:

$$f(x) \equiv \left[ 1 - \alpha + \alpha x^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where  $f(x) \equiv \frac{Y(0)}{A_0 B e^{\phi(h)}}$  and  $x \equiv \frac{K}{A_0 e^{\phi(h)}}$ .

For simplicity we set, from this point onwards,  $A_0 = 1$ .

**Proposition 1** Assume  $0 \leq \sigma < 2$  and  $w(s) = f(k)e^{-(g+\delta)T}e^{\phi(h)+gs}$ , where  $k$  and  $T$  are specified immediately below. (i) Assume also that  $p \leq \bar{p} \equiv \frac{B}{r+\delta}\alpha^{\frac{\sigma}{\sigma-1}}$  for  $0 \leq \sigma < 1$ , or that  $p \geq \bar{p}$  for  $1 < \sigma < 2$ . Then there exist unique optimal waiting periods between upgrades, which are constant, strictly positive and finite. The optimal waiting period  $T$  is the solution to the following equation:

$$0 = e^{-\delta T} B \left[ \frac{p}{\alpha B D(r+\delta, T)} \right]^{\sigma} - e^{gT} \frac{1}{D(r-g, T)} \left[ D(r+\delta, T) \left( \frac{p}{\alpha B D(r+\delta, T)} \right)^{\sigma} B - p \right], \quad (3)$$

where  $D(x, T) \equiv \frac{1-e^{-xT}}{x}$ .

(ii) Consider a date  $s$  at which the firm upgrades. Then the optimal capital-worker ratio is:

$$K(s) = k e^{\phi(h)+gs}, \quad (4)$$

where  $k \equiv \left[ \frac{Z^{\sigma-1}-\alpha}{1-\alpha} \right]^{\frac{\sigma}{1-\sigma}}$  and  $Z \equiv \frac{p}{\alpha B D(r+\delta, T)}$ . The optimal labor demand is indeterminate,  $L(s) \in [0, \infty)$ .

The maximized output at  $s$  is:

$$Y(s, s) = B f(k) e^{\phi(h)+gs}. \quad (5)$$

(iii) If the hypotheses underlying (i) are not satisfied, the firm never upgrades its capital or demands an infinite quantity of capital.<sup>11</sup>

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<sup>11</sup>We are not discussing these possibilities in greater detail because we are interested in a balanced-growth path, and these possibilities are ruled out by it.

As the Proposition states, the scale of production is indeterminate whenever  $w(s) = f(k)e^{-(g+\delta)T}e^{\phi(h)+gs}$ . This is a standard feature of models with price-taking firms and with constant returns to scale. If the average cost in such model is equal to the product price - and  $w(s) = f(k)e^{-(g+\delta)T}e^{\phi(h)+gs}$  is the analogue of this condition in our model - the firm is indifferent between all scales of production.<sup>12</sup> Another standard feature is that although the scale of production is not determinate the capital-worker ratio  $K(s)$  is.

Other features of the solution are as follows: (i) The firm's optimum choices are determined sequentially. We first solve for the optimal waiting period  $T$  using equation (3). Then, we plug this  $T$  into (4) and solve for the optimal quantity of capital. Therefore,  $T$  is scale-independent so it becomes an "intensive" variable, like  $K$ . (ii) The optimal quantity of capital  $K(s)$  grows at the constant rate  $g$ ; the firm is on a balanced growth-path. (iii) The optimal choice of  $T$  and  $K$  depend only on the ratio  $\frac{p}{B}$ , not on each one separately. (iv)  $h$  affects the choice of  $K$ , but not the choice of  $T$ .<sup>13</sup>

Regarding the comparative statics of the optimum, and maintaining the assumptions underlying Proposition 1, we have the following result:

**Proposition 2** (i)  $K(s)$  is decreasing in  $p$ . (ii) If  $\sigma < 1$ ,  $T$  increases in  $p$  and if  $\sigma > 1$ ,  $T$  decreases in  $p$ . If  $\sigma = 1$ ,  $T$  is independent of  $p$ .

The specific mechanism via which  $p$  affects  $T$  relates to capital and labor being substitutes or complements. In particular, when  $\sigma > 1$  labor and capital are substitutes. Hence, when  $p$  goes up the firm uses less capital, and, because capital and labor are substitutes, it uses more labor. In turn, one way of using more labor is to upgrade more frequently (because then the "effective" quantity of labor increases).

Proposition 2(ii) suggests a first glimpse at why the Cobb-Douglas case or, more generally,  $\sigma \geq 1$  might not be the most realistic production function in our context. Indeed it would

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<sup>12</sup>If  $w(s) > f(k)e^{-(g+\delta)T}e^{\phi(h)+gs}$ , the firm produces zero. If  $w(s) < f(k)e^{-(g+\delta)T}e^{\phi(h)+gs}$ , the firm produces infinite quantity.

<sup>13</sup>The reason for this is that  $e^{\phi(h)}$  multiplies  $L$ , rather than  $K$  or the output  $Y$ . This provides a testable implication of the way we incorporate  $h$  into our production function, the implication being that as one goes across countries with different educational levels one should observe no variation - on account of  $h$  alone - in the frequency of technological upgrades. This implication can be used to contrast our way of incorporating  $h$  into the production function with alternative ways.

Recent empirical findings show, at the macro level, that the return to education is consistent with the way we incorporate  $h$  into the production function; see Topel (1999) and Krueger and Lindhl (2000). On top of that, the cross-section study by Psacharopoulos (1994), using micro data, suggests the equality between the private and the social return to education, validating thereby our approach at the macro level.

seem that as  $p$  goes up the firm would want to spread the higher cost of buying capital over a longer holding period. However, as Proposition 2 states, that holds true only if  $\sigma < 1$ . Later in the paper we provide empirical evidence (and further theoretical arguments) corroborating this intuition, i.e., supporting  $\sigma < 1$ .

## 4 General Equilibrium

We consider now the whole economy and determine equilibrium prices and the mechanics of consumption and investment over time and across firms.

### 4.1 The household sector

We start with the problem facing a typical individual in her capacity as a consumer. We assume such individual has a C.E.S. flow utility function,  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ , and a constant rate of time-preference parameter,  $\rho$ . The life-time utility of such consumer from a consumption stream,  $c(\cdot)$ , is:

$$\int_0^{\infty} e^{-\rho t} \frac{c(t)^{1-\gamma}}{1-\gamma} dt.$$

The consumer has some initial wealth, call it  $\omega$ , which comes from owning equities, as we specify in the next two subsections. On top of that wealth, the individual receives a stream of wages  $w(\cdot)$ . The consumer chooses a consumption stream that maximizes her life-time utility subject to the life-time budget constraint:

$$0 = \int_0^{\infty} e^{-rt} [c(t) - w(t)] dt - \omega. \quad (6)$$

As is well-known, the solution to the household's problem is:

$$c(t) = c_0 e^{\frac{r-\rho}{\gamma} t}, \quad (7)$$

where  $c_0$  is determined in the next section.

## 4.2 Pricing of Capital Goods

When a firm upgrades its capital it finances this activity by going to the bond market and borrowing at the constant interest rate  $r$ . Equivalently, a firm can go to the equity market and issue shares. Let the unit of measurement be such that one share is backed by one unit of capital. Then a share yields a stream of dividends which equals the marginal productivity of this capital. Denote the date- $\tau$  marginal productivity of vintage- $s$  capital by  $R(s, \tau)$ . Then, considering a share of a vintage- $s$  firm, its price must be the discounted value of the dividends it yields:<sup>14</sup>

$$p = \int_s^{s+T} R(s, \tau) e^{-r(\tau-s)} d\tau.$$

By the same token, if we consider a later date, say  $t$ , where  $s < t < s + T$ , the date- $t$  price of a share issued at date- $s$ ,  $p(s, t)$ , is the discounted value of the remaining dividends it yields:

$$p(s, t) = \int_t^{s+T} R(s, \tau) e^{-r(\tau-t)} d\tau. \quad (8)$$

We interpret  $p(s, t)$  either as the share price of a vintage- $s$  firm, or, equivalently, as the price of the capital backing this share. In the second interpretation we consider  $p(s, \cdot)$ , for a fixed  $s$ , as tracking the price of vintage- $s$  capital over its life-time.<sup>15</sup> We consider  $p(\cdot, t)$ , for a fixed  $t$ , as a “price-profile” of capital of different vintages at date  $t$ . A familiar example of such price profile is the blue book, which lists the prices of cars of different “models,” i.e., it shows how the price of a car relates to its vintage (or, equivalently, its age).

## 4.3 Initial Conditions

To focus on a balanced growth-path we assume that firms are initially staggered across vintages, so their upgrading decisions come in sequence. Specifically, we assume that at  $t = 0$  firms are uniformly distributed across vintages  $[-T, 0)$  with a constant density  $= \frac{1}{T}$ .

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<sup>14</sup>The price of a share is understood as its *shadow* price since in our economy we don’t need a separate equity market to decentralize feasible allocations.

<sup>15</sup>Fixing  $s$ ,  $p(s, \cdot)$  is the function  $p(s, t)$  as  $t$  varies over  $[s, s + T]$ . For  $t > T + s$ ,  $p(s, t) = 0$ . Analogous remarks apply to  $p(\cdot, t)$ .

The amount of capital per worker of a vintage- $s$  firm, for  $-T \leq s < 0$ , is the  $K(s)$  specified in equation (4). This pins down the economy's initial conditions.

Since firms upgrade in sequence, a density  $\frac{1}{T}$  of firms upgrades and a density  $\frac{1}{T}$  of workers gets re-assigned to new vintage capital - at each point in time. Thus, a uniform distribution over vintages (or, equivalently, over capital of different ages) is preserved for the indefinite future.

## 4.4 Equilibrium Prices

We find now equilibrium wage rates, interest rates and shadow prices of capital and dividends.

**Labor Market.** At each instant, say  $s$ , a density  $\frac{1}{T}$  of workers seek new employment as their employment with firms that dump their old capital is terminated. These workers give rise to a perfectly inelastic supply curve of workers, with a quantity intercept of  $\frac{1}{T}$ . The equilibrium wage rate at  $s$  must be such that the set of firms that are upgrading their technology (the same firms that used to employ these workers), demands this quantity of labor. Under the labor demand we derived in Section 3 this happens if and only if:

$$w(s) = f(k)e^{-(g+\delta)T}e^{\phi(h)+gs}. \quad (9)$$

A noteworthy property of (9) is that the equilibrium wage rate at date  $s$  equals the per-worker output of an  $s - T$ -vintage firm. Thus, such firm dumps its capital at  $s$  even if it does not have the option to upgrade.

**Bond Market.** Since output net of investment grows at the rate  $g$ , and since consumption grows at the rate  $\frac{r-\rho}{\gamma}$ , see equation (7), market clearing dictates we have  $g = \frac{r-\rho}{\gamma}$ . Thus,  $r = \gamma g + \rho$ .

**Dividends.** The dividend corresponding to vintage- $s$  capital at time  $t$  is the marginal product of that capital. Given constant returns to scale, the total dividend distributed by a vintage- $s$  firm is the output net of wages:

$$\begin{aligned} R(s, t)K(s) &= e^{-\delta(t-s)}B \left[ (1-\alpha)(e^{\phi(h)}A(s))^{\frac{\sigma-1}{\sigma}} + \alpha K(s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - w(t) \\ &= e^{-\delta(t-s)}B f(k)[1 - e^{-(g+\delta)(s-(t-T))}]K(s), \end{aligned} \quad (10)$$

after we substitute in from (4) and (9).



Price Profile. Substituting (10) into (8) we get:

$$p(s, t) = p e^{-\delta(t-s)} \frac{D(r + \delta, T - (t - s))}{D(r + \delta, T)} \frac{1 - H(T - (t - s))}{1 - H(T)}, \quad (11)$$

where:

$$H(T) \equiv \frac{D(r - g, T)}{D(r + \delta, T)} e^{-(g+\delta)T}. \quad (12)$$

Consider a point in time  $t$  and a vintage  $s$ ,  $t - T < s < t$ . Then (10) and (11) show that dividends and capital prices are independent of  $t$ . Rather, they depend on the age of capital only,  $t - s$ . Consequently, from this point onwards we write  $p(t - s)$ , instead of  $p(s, t)$  and  $R(t - s)$ , instead of  $R(s, t)$ . At times it will be convenient to work with the variable  $a$ , which denotes the age of capital, e.g.,  $p(a)$ .

Figure 1 below shows two  $p(a)$  curves, one for a high- $p$  country ( $p = 3$ ), the other for a low- $p$  country ( $p = 1$ ). The parameter values underlying these curve are those we report in Sections 6 and 7.

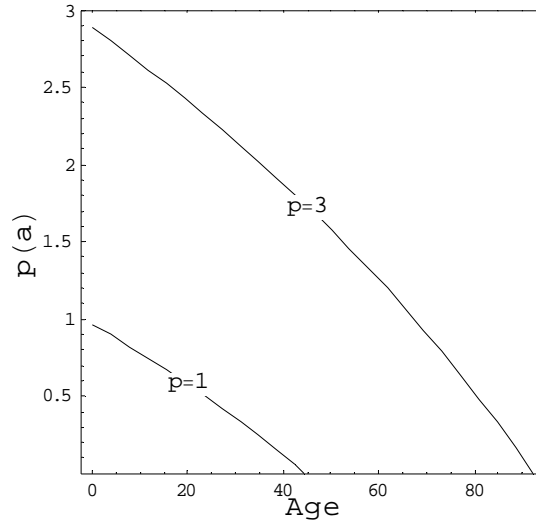


Figure 1: Price Profiles. Parameters values are as calibrated for the U.S.

Both curves decrease, intersect the vertical axis at  $p$ , and intersect the horizontal axis at  $a = T$ , i.e., at the age at which capital is retired.<sup>16</sup> Note that the high- $p$  curve lies everywhere

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<sup>16</sup>One can show that:

$$\lim_{T \rightarrow \infty} p(a) = e^{-(g+\delta)T},$$

above and declines at a slower rate than the low- $p$  curve. Moreover, the high- $p$  curve declines at a slower rate even if we control for the higher price-intercept by dividing by  $p$  (in which case the two curves start at the same point on the vertical axis).

By contrast, if we consider  $\sigma > 1$  we get counter-factual shapes of price-profiles. A high- $p$  price-profile starts (obviously) higher than a low- $p$  price-profile. However, the high- $p$  price-profile declines faster and, as a result, intersects the low- $p$  price-profile. Furthermore, it intersects the horizontal axis at a lower  $a$ , which is due to the fact that the life-time of capital is shorter the higher  $p$  is; see Proposition 2. This contrast between the shapes of price profiles, depending on  $\sigma$  being below or above 1, provides further evidence in favor of  $\sigma < 1$ .

Given equity prices, as given by (11),  $\omega$  is determined as the value of initial equity holdings:

$$\omega = \int_{-T}^0 p(s, 0) K(s) ds.$$

In turn,  $c_0$  is determined so that the consumer exhausts her budget.

## 5 Aggregation

Given the above equilibrium, we construct now the theoretical counterparts of various National-Income Statistics.

**Per-capita output.** At  $t$  firms in the economy operate with vintage- $s$  technology, where  $s \in [t - T, t]$ . A vintage- $s$  firm employs  $\frac{1}{T}$  workers and produces a flow of output  $Y(s, t)$ . Thus, using (5), the aggregate per-capita output flow at  $t$  is:

$$\begin{aligned} y(t) &= \frac{1}{T} \int_{t-T}^t Y(s, t) ds \\ &= \frac{1}{T} \int_{t-T}^t e^{-\delta(t-s)} B f(k) e^{\phi(h) + gs} ds \\ &= y e^{\phi(h) + gt}, \end{aligned} \tag{13}$$

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which is the price profile when capital-goods from different vintages are perfect substitutes instead of perfect complements, as is the case in our model.

where:

$$y \equiv Bf(k)D(g + \delta, T).$$

**Investment.** At  $t$  a density  $\frac{1}{T}$  of firms upgrades its capital, and each such firm installs the quantity  $K(t)$  of new capital. Using (4), this implies that the per-capita investment at  $t$  is:

$$i(t) = \frac{k}{T} e^{\phi(h) + gt}. \quad (14)$$

**Investment-Output ratio.** Using (13) and (14) it follows that the investment-output ratio is:

$$\begin{aligned} i &\equiv \frac{i(t)}{y(t)} = \frac{\frac{k}{T}}{\frac{B}{T}f(k)D(g + \delta, T)} \\ &= \frac{1}{B\psi(k)D(g + \delta, T)}, \end{aligned} \quad (15)$$

where

$$\psi(x) \equiv \frac{f(x)}{x} = [(1 - \alpha)x^{-\frac{\sigma-1}{\sigma}} + \alpha]^{\frac{\sigma}{\sigma-1}}.$$

**Factor-Shares of Income.** Capital-income is the sum of dividends on capital employed by all firms. Since there is a density  $\frac{1}{T}$  of vintage- $s$  firms at time  $t$ , where  $s \in [t - T, t]$ , capital-income at  $t$  is:

$$\frac{1}{T} \int_{t-T}^t R(t-s)K(s)ds.$$

After we substitute in from (4) and (10) we get:

$$\text{Capital Income} = \frac{1}{T} Bf(k) \left[ \frac{1 - e^{-(g+\delta)T}}{g + \delta} - T e^{-(g+\delta)T} \right] e^{\phi(h) + gt}.$$

And dividing this by output it follows that:

$$\alpha_K = 1 - \frac{(g + \delta)T e^{-(g+\delta)T}}{1 - e^{-(g+\delta)T}}, \quad (16)$$

where  $\alpha_K$  stands for the capital-share of income.

By working out a similar computation, the labor-share of income is:<sup>17</sup>

$$1 - \alpha_K = \frac{(g + \delta)T e^{-(g+\delta)T}}{1 - e^{-(g+\delta)T}}. \quad (17)$$

**The Value of the Economy's Capital Stock at Market Prices.** Given the market prices for installed capital, (11), and the optimal quantity of installed capital, (4), the economy-wide market value of capital at  $t$  is:

$$\begin{aligned} k_M(t) &= \frac{1}{T} \int_{t-T}^t p(t-s)K(s)ds \\ &= i_M(t) \int_0^T e^{-(g+\delta)\tau} \frac{D(r+\delta, T-\tau)}{D(r+\delta, T)} \frac{1 - H(T-\tau)}{1 - H(T)} d\tau, \end{aligned}$$

where:

$$i_M(t) = \frac{pk}{T} e^{\phi(h)+gt}.$$

Given the above formula, the capital-investment ratio is:

$$\frac{k_M(t)}{i_M(t)} = \int_0^T e^{-(g+\delta)\tau} \frac{D(r+\delta, T-\tau)}{D(r+\delta, T)} \frac{1 - H(T-\tau)}{1 - H(T)} d\tau \equiv \frac{1}{\delta_{EF}}. \quad (18)$$

The analogue of the investment-capital ratio in a model with disembodied technological progress is  $\delta + g$ , the rate of physical depreciation plus the rate of technological depreciation or “obsolescence.” Here the investment-capital ratio, which we denote by  $\delta_{EF}$  (where EF stands for “effective”), includes, in addition to  $\delta + g$ , a third term, which we call economic depreciation (we later show that  $\delta_{EF} \geq \delta + g$ ). Economic depreciation is due to the fact that capital is scrapped (or “retired” or “dumped”), which does not happen in a model with disembodied technological progress.<sup>18</sup> One of the points we make later is that this distinction is quantitatively significant. In particular, for the U.S. economy, economic depreciation is

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<sup>17</sup>Although it is “natural” that labor and capital shares add up to one this hinges on computing these shares at market prices. If we compute them at factor costs they add up to less than one. The difference is the “distortion-share of income.”

<sup>18</sup>As a consequence, what is true for the disembodied model is also true in the embodied model when economic depreciation is of no relevance, i.e., when  $T \rightarrow \infty$ . Formally, one can show that:

$$\lim_{T \rightarrow \infty} \delta_{EF} = g + \delta.$$

almost 3 times bigger than the sum of physical and technological depreciations.

Our concept of capital, which is to evaluate different vintages at their market prices, is in accordance with NIPA practice and is not found in previous models of vintage capital. Instead, previous models use an ad-hoc measure of aggregate capital, which accommodate physical and technological depreciation, but not economic depreciation.

**The Price-Earning Ratio.** A commonly used statistic of the stock market is the price-earning ratio. Here it is convenient to work with its inverse, the earning-price ratio or, in short, the profit-rate. In our environment, the profit-rate of an age- $a$  firm (at any point in time) is:

$$\Pi(a) \equiv \frac{R(a)}{p(a)}.$$

After substituting in from (10) and (11) we get:

$$\Pi(a) = (g + \delta) \frac{D(g + \delta, T - a)}{D(r + \delta, T - a) - e^{-(g+\delta)(T-a)} D(r - g, T - a)}.$$

It can be readily verified that this profit rate satisfies the no-arbitrage condition:

$$r = \Pi(a) + \frac{p'(a)}{p(a)}. \quad (19)$$

Consider now the cross-section of all firms in the economy, and call the collection of equities associated with them the “market portfolio.” At any point in time, say  $t$ , this portfolio consists of  $K(s)$  shares of vintage- $s$  firms, where  $s \in [t-T, t]$ . Since  $K(s)$  grows exponentially at the rate  $g$ , this is equivalent to a portfolio consisting of age- $a$  firms, with  $a \in [0, T]$ , and where the weight assigned to an age- $a$  firm is  $\frac{e^{-ga}}{T}$ . The average profit-rate of this portfolio,  $\Pi$ , is:

$$\begin{aligned} \Pi &\equiv \frac{\int_0^T \frac{e^{-ga}}{T} \Pi(a) da}{\int_0^T \frac{e^{-ga}}{T} da} \\ &= g(g + \delta) \frac{\int_{-T}^0 e^{-ga} \frac{D(g+\delta, -a)}{D(r+\delta, -a) - e^{(g+\delta)a} D(r-g, -a)} da}{e^{gT} - 1}. \end{aligned}$$

## 6 Calibration

We calibrate the model now, assigning numerical values to its parameters. To do this we consider the National Income statistics we derived in the last section along with the optimality conditions of Section 3 as mapping parameters of the economy into endogenous variables. Taking the values of these endogenous variables, as observed in actual data, inserting them into this mapping and “inverting” we recover the values of unobserved parameters.<sup>19</sup> The parameter values we recover in this way are:  $\sigma, \alpha, B, \delta, p$  and  $T$ .

We proceed in two steps. In the first step we fix  $\sigma$  and calibrate the remaining five parameters as a function of  $\sigma$ , using U.S. data. In the second step, we estimate the value of  $\sigma$ , using the Summers-Heston (1991) (SH) international data set. This approach is dictated by the fact that  $\sigma$  is a curvature parameter of the production function, and the U.S. economy is at a single point along this production function (along a balanced growth-path). This makes it impossible to identify the value of  $\sigma$  from U.S. data alone. On the other hand, if we had data points which vary along the production function, we could identify  $\sigma$ , and this is where the international data comes in handy.

In the remainder of this section we work out the first step of the calibration.

### 6.1 System of Equilibrium Equations

The system that maps parameters into endogenous variables consists of the following equations: The per-capita output, (13), the investment-output ratio, (15), the capital-share of income, (16), the first-order condition for the timing of upgrades, (3), and the capital-investment ratio, (18). We consider these equations at a fixed point in time, say 0. After we flesh out these equations, by substituting in for  $k, Z, f$  and  $\psi$ , we obtain the following

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<sup>19</sup>As will be (or has been) noted, the dichotomy parameters-endogenous variables is not the same as the dichotomy observables-unobservables. Nonetheless, to make the language less cumbersome, we ignore this distinction and identify observables with endogenous variables and unobservables with parameters.

What we call parameters according to this convention is:  $\sigma, \alpha, B, \delta, p$  and  $T$ . What we call endogenous variables is:  $\alpha_K, i, r, g, y, h, \delta_{EF}$ , and  $k$ .

system:

$$\begin{aligned}
y_0 &= B e^{\phi(h)} \left[ \frac{1-\alpha}{1-\alpha \left( \frac{p}{\alpha B D(r+\delta, T)} \right)^{1-\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \frac{D(g+\delta, T)}{T} \\
i &= \frac{1}{B} \left( \frac{\alpha B D(r+\delta, T)}{p} \right)^{\sigma} \frac{1}{D(g+\delta, T)} \\
\alpha_K &= 1 - \frac{(g+\delta) T e^{-(g+\delta)T}}{1-e^{-(g+\delta)T}} \\
1 &= e^{-(g+\delta)T} \left( \frac{p}{\alpha B D(r+\delta, T)} \right)^{\sigma} \frac{B}{p} D(r-g, T) - \left( \frac{p}{\alpha B D(r+\delta, T)} \right)^{\sigma} \frac{B}{p} D(r+\delta, T) \\
\frac{1}{\delta_{EF}} &= \int_0^T e^{-(g+\delta)\tau} \frac{D(r+\delta, T-\tau)}{D(r+\delta, T)} \frac{1-H(T-\tau)}{1-H(T)} d\tau.
\end{aligned} \tag{20}$$

## 6.2 The values of Observables

In this sub-section we specify the value of observables and where they come from. Since we focus on a balanced growth-path we computed long-run averages of certain observables.

- The U.S. growth-rate,  $g$ , is 1.36% per year, which is obtained by estimating a trend line for the variable RGDPW of SH for the period 1960-1992.
- The annual riskless interest-rate,  $r$ , is 4.5%, which corresponds to a long-run average of 30-year T-bills.
- The investment-output ratio is 0.22, which is the average value for the U.S. variable  $i$  in SH for the period 1960-1985 (what constitutes investment is spelled out immediately below).
- The Mincerian factor,  $e^{\phi(h)}$ , for the U.S. is normalized to 1.
- The per-capita output for the U.S.,  $y_0$ , is normalized to 1.

The reason we normalize  $e^{\phi(h)}$  and  $y_0$  is that we later focus on the ratio of other countries' per-worker output to the U.S., and those ratios are independent of scale.

- The income-share of capital takes its “traditional” value,  $1/3$ .

The last observable, the investment-capital ratio,  $\delta_{EF}$ , comes from NIPA, using the following procedure. NIPA reports the market value of various assets, which we consider to be the empirical counterparts of “capital” in the model. These assets consist of: Private Equipment, Private Commercial Structures (e.g., plants), Private Residential Structures (e.g., houses),

and Government Fixed Assets. We exclude Household Durable Goods.<sup>20</sup> We have chosen these assets so that the concept of capital in the U.S. is consistent with the concept of capital in other countries, as reported by SH.

The value of these assets is calculated by NIPA,<sup>21</sup> using the “perpetual inventory method,”<sup>22</sup> which corresponds to the way we have theoretically priced capital in the model - our price profile,  $p(\cdot, t)$ .<sup>23</sup> We took the market values of investment and capital reported by NIPA over a 30-year window, 1970-1999, and computed the long-run average of the investment-to-capital ratio:

$$\delta_{\text{EF}} = \frac{1}{30} \sum_{t=1970}^{1999} \frac{\frac{1}{40} \sum_{j=t-41}^{t-1} i_j}{K_t} = 0.066,$$

where  $K_t \equiv \frac{k_M(t)}{y(t)}$ .

Altogether we have the following values of observables:

$y_0$	$e^{\phi(h)}$	$\alpha_K$	$g$	$r$	$i$	$\delta_{\text{EF}}$
1	1	1/3	0.0136	$\log(1.045)$	0.22	0.066

We plug these values into the system (20), which gives us 5 equations in 5 unknowns,  $p, T, \alpha, B$  and  $\delta$ . We consider  $\sigma$  as a free parameter.

### 6.3 Solving the System

Our working paper shows that system (20) is uniquely solvable. Furthermore, although we have not assigned a numerical value to  $\sigma$  as yet, we are able to pin down numerical values for  $p, T$  and  $\delta$ . The net result is as follows:

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<sup>20</sup>Jovanovic and Rob (1997) consider a narrower definition of capital, which includes equipment but not structures. Our concept of capital is broadened to include structures because we hypothesize that structures undergo technological change as well (possibly at a lower rate than the rate of technological progress applying to equipment). Evidence supporting our hypothesis is found in Gort et al. (1999). As a result of considering a broader concept of capital our model explains much more income inequality across nations than the Jovanovic-Rob (1997) model.

<sup>21</sup>It is found on the BEA web page <http://www.bea.doc.gov/bea/an1.htm>. See “Fixed and Reproducible Tangible Wealth in the United States 1925-94,” BEA, August 1999.

<sup>22</sup>On the depreciation methodology see Fraumeni (1997).

<sup>23</sup>There is slight discrepancy between our theoretical concept of capital and NIPA practice. What NIPA does is to “fit” an exponential curve to the empirical price-profile. For obvious reasons, we prefer to work with the empirical price-profile itself. The difference between the two is negligible, and when it is not negligible, NIPA uses the empirical price-profile, rather than an exponential fit.



$p$	$T$	$\delta$
1.04	44	0.0035

As regards  $\alpha$  and  $B$ , we are only able to express them as functions of  $\sigma$ :

$$\alpha = \frac{1}{1 + \frac{H(T)}{1-H(T)} (iT)^{\frac{\sigma-1}{\sigma}}}, \quad (21)$$

$$B = \frac{1}{i} \left\{ 1 - H(T) \left[ 1 - (iT)^{\frac{\sigma-1}{\sigma}} \right] \right\}^{\frac{\sigma}{\sigma-1}} \frac{1}{D(g + \delta, T)}. \quad (22)$$

Once we have a value for  $\sigma$  (which we obtain in the next section) we can, obviously, pin down numerical values for  $\alpha$  and  $B$  as well.

Remarks on calibrated values.

1. We are able to solve numerically for  $\delta$  and  $T$  because we get them by solving the third and fifth equations of system (20), and these equations are independent of  $\sigma$ . Hence, the implied values of  $\delta$  and  $T$  depend only on  $\alpha_K, \delta_{EF}, g$  and  $r$ .

2. As noted earlier, there is no empirical counterpart to what we call  $p$  in the model. To see this more clearly, generalize the production function (1) as follows:

$$Y(s, t) = e^{\delta(t-s)} B \left[ (1 - \alpha)(e^{\phi(h)} A(s))^{\frac{\sigma-1}{\sigma}} + \alpha (CK(s))^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where  $C$  is a constant which translates forgone output into capital goods, and capital goods into capital services. Re-doing all the calculations one arrives at:

$$p = C \frac{1 - H(T)}{i} \frac{D(r + \delta, T)}{D(g + \delta, T)}.$$

Thus, one can either calibrate  $p$  so that  $C = 1$ , which is what we do; or, if  $p$  is an observable, one can recover the value of  $C$ . Since our model does not distinguish between consumption and capital-goods or between capital-goods and capital-services (which implies that  $C$  is not necessarily 1), there is no natural candidate to which  $p$  should be equated.

3. Likewise, there is no empirical counterpart to  $T$ , since capital in the real world consists of many items. So it is not obvious the lifetime of which particular item one should equate with  $T$ . Nonetheless, we can think about the calibrated value of  $T$  in the following manner. Let us consider capital goods as belonging to two broad categories, “equipment”

and “structures.” According to Coen (1980)<sup>24</sup> the average lifetime of equipment is 12 years, and according to Gort et al. (1999) the average lifetime of structures is 50 years. In order for these two lifetimes to average to 44 years (our calibrated value) the share of equipment in total capital stock should be 14%. According to NIPA data, the share of equipment in total private capital is 16%, which is not far off the “required” 14%. Consequently, we interpret  $T$  as the average lifetime of capital goods belonging to these two categories. An explicit treatment of the multiple capital-goods case is, naturally, the next step in this line of research.

4. The calibrated value of  $\delta + g$  is 1.7%, whereas  $\delta_{\text{EF}}$  equals 6.6%, which is quite a bit bigger. The difference is economic depreciation. Therefore, economic depreciation in the U.S. is almost three times bigger than  $\delta + g$ .

5. The calibrated value of  $\delta$  may seem small by comparison with other estimates. Note, however, that some of the effective depreciation (6.6%) is accounted for by what we call economic depreciation, which others have lumped together with physical depreciation. Furthermore, our empirical concept of output is gross of maintenance costs.<sup>25</sup>

## 7 Estimating the Elasticity of Substitution

### 7.1 Methodology

In this Section we estimate  $\sigma$ , using the SH international data-set. Our starting point is equation (15), which, for convenience, we reproduce here:

$$i = \frac{1}{B} \left( \alpha \frac{B}{p} \right)^\sigma \frac{\left( \frac{1 - e^{-(r+\delta)T}}{r+\delta} \right)^\sigma}{\frac{1 - e^{-(g+\delta)T}}{g+\delta}}. \quad (23)$$

We consider this equation as the long-run demand for investment. Assuming all countries have the same technology (apart from the TFP term  $B$ ), this demand is the same for all countries. As regards the long-run supply of investment it is derived as a by-product of the maximization of the individual consumer’s program (stated in Section 4.1). This supply curve is infinitely elastic and its price intercept is monotonically related to  $p$ . Thus, since

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<sup>24</sup>Apud in Parente (2000), page 691.

<sup>25</sup>Also, our model suppresses learning by doing, which has the effect of “negative depreciation.”

different countries face different  $p$ 's, as one goes across countries, one is tracing points along the demand for investment, which enables the identification of  $\sigma$ . This is the approach we pursue here.

A potential problem with this approach is that different countries differ not only with respect to  $p$ , but also with respect to total factor productivity  $B$ , which is an unobservable. We shall control for that by constructing a panel data set, which accommodate both variations. Then we shall estimate  $\sigma$  from that panel.

To make this approach operational we take the logarithms of both sides of (23). This gives us the econometric specification:

$$\log i_{jt} = \log FE_j - \beta \log p_{jt} + \epsilon_{jt}, \quad (24)$$

where  $FE_j$  is “fixed effect” for country  $j$ ,  $p_{jt}$  is the relative price of capital in country  $j$  in year  $t$ , and  $\epsilon_{jt}$  is noise term.  $\beta$ , as we explain immediately below, is monotonically related to  $\sigma$ , so once we get an estimate for  $\beta$  we can infer an estimate for  $\sigma$ . A plot of the monotonic relationship between  $\sigma$  and  $\beta$  is shown in Figure 2 below.<sup>26</sup>

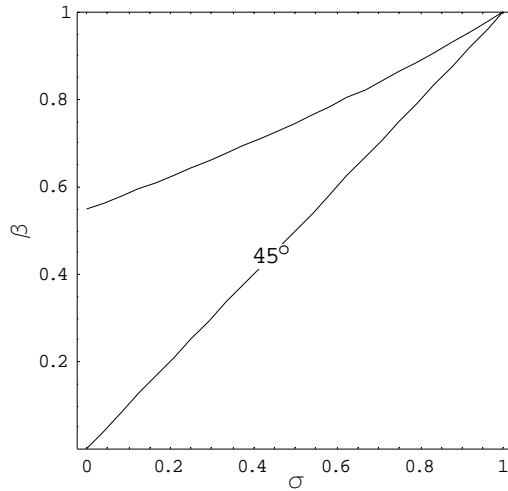


Figure 2: Theoretical Mapping Connecting  $\sigma$  and  $\beta$

The economic relationship between  $\sigma$  and  $\beta$  is that  $\sigma$  is the elasticity of demand (for investment) when the timing of upgrades is fixed, whereas  $\beta$  is the elasticity of demand

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<sup>26</sup>Although there is no simple, closed-form expression linking  $\sigma$  and  $\beta$  this functional dependence is numerically approximated with a great degree of precision.

when the timing of upgrades is a choice variable.<sup>27</sup> Because of the extra flexibility, the demand with timing is more elastic than the demand without timing, i.e.,  $\beta$  is bigger than  $\sigma$ , which is seen by inspecting Figure 2.<sup>28</sup>

A word of caution is in order before we report our estimation results. When (23) is transformed into (24) we don't actually get a log-linear relationship (the reason is that  $p$  affects  $T$  in a non log-linear way). Nonetheless, if we fix a value for  $\sigma$  and plot the true relationship between  $\log i$  and  $\log p$ , the curve we get is 'nearly' linear, and its slope is in one-to-one correspondence with  $\sigma$ . Thus, we proceed with the log-linear approximation (24) as our econometric specification.

## 7.2 Data and Estimation

Using SH, we constructed a panel data-set for the period 1960 - 1985. The relative price of capital in this panel is the ratio between the price level of investment, SH variable PI, and the price level of consumption, SH variable PC. The investment in this panel is real gross domestic investment at 1985 international prices. This is the same panel that Restuccia and Urrutia (2001) constructed. For details, see the Statistical Appendix of their paper.

Regressing (24) by OLS, we got 0.57 for  $\beta$ . On the other hand, regressing (24) from the same panel - on the assumption that the fixed effect is constant across countries,  $FE_j = FE$  - we got 1.09 for  $\beta$ . The reason for this difference is that  $p$  is negatively correlated with  $B$ . Because of that, a cross-section estimation of (23), without controlling for variation in  $B$ , biases  $\beta$  upwards, making it appear as though the production function is Cobb-Douglas,  $\beta = \sigma = 1$ . The rest of this section reports the results of further econometric exercises, which confirm that  $\beta$  is indeed significantly  $< 1$ .<sup>29</sup>

To confirm that  $\beta < 1$  we first ran various other regressions. We tried OLS with random effects, weighted least-squares, instrumental-variable with lagged-price as the instrument, and instrumental-variable, weighted least-squares with lagged-price as the instrument. The estimated values of  $\beta$  for these regressions were 0.64, 0.45, 0.63, and 0.45, respectively.

Second, we dealt with the following measurement problem. It has been suggested (see

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<sup>27</sup> $\beta$  is the same as  $\sigma$  in a model without embodiment and, hence, without timing decisions.  $\beta$  is also the same as  $\sigma$  when  $\sigma = 1$  (and embodiment) because then  $T$  is independent of  $p$ .

<sup>28</sup>As an example, consider the Leontief production function,  $\sigma = 0$ . Then, the elasticity of demand without timing decisions is 0. On the other hand, the elasticity of demand with timing decisions is 0.55.

<sup>29</sup>The fact that  $\beta < 1$  implies that  $\sigma < 1$ ; see Figure 2.

Pritchett (2000)) that the investment data reported for the poorest/high- $p$  economies are way above their true investments.<sup>30</sup> If this suggestion is true, that would bias the estimated value of  $\beta$  downwards. To check for that possibility, we constructed another panel by omitting all African economies. The unweighted instrumental-variable estimation of the new panel produced 0.73 for  $\beta$ . The other estimates, OLS, weighted OLS, and weighted IV estimation produced 0.56, 0.40, and 0.47, respectively. Thus, the estimated  $\beta$  from the restricted panel did not reverse the conclusion that  $\beta$  is less than 1.<sup>31</sup>

A third problem we addressed relates to using panel-data as opposed to cross-section data. A potential problem with panel-data is that it is difficult to distinguish between short-run (business-cycle type fluctuations) and long-run movements. Since what we are trying to estimate is the long-run demand for investment, we smoothed the data by constructing Hodrick-Prescott (HP) filtered data. The resulting OLS and weighted OLS estimates were 0.73 and 0.63. The fact that the estimated  $\beta$ , which reflects price elasticity, increases when we go from the original data to the smoothed data is no surprise: as we smooth data, we go from short-run to long-run demand, so the price elasticity is expected to increase.

An important by-product of this smoothing exercise is that it shows we have enough variability in the data - in the time dimension - in order to estimate  $\beta$ . Indeed, if there were not enough variability we would have gotten smaller, not bigger, estimates for  $\beta$ . Overall, our results suggest that  $\beta$  is somewhere in the range  $[0.45, 0.75]$ ; it is definitely less than 1.

Given these results and using the monotonic transformation between  $\beta$  and  $\sigma$ , see Figure 2, we find that  $\sigma$  is somewhere in the range  $[0, 0.4]$ . For the rest of this paper we pick  $\sigma = 0.4$  which correspond to the OLS HP filtered-data estimate of  $\beta$ .<sup>32</sup>

For this value of  $\sigma$  the calibrated value of  $\alpha$  is 0.95, and of  $B$ , 0.27.

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<sup>30</sup>Because the share of public-sector investments in total investment is high, and because data on public-sector investments is exaggerated.

<sup>31</sup>We experimented with alternative ways of working with the data, including running different regressions for different regions of the world. These experimentations did not alter our finding. See our working paper.

<sup>32</sup>Further evidence corroborating our findings is found in Yuhn (1991). That paper surveys works that estimate the elasticity of substitution for the U.S., and reports values between 0.078 and 0.763. See his discussion on page 343 and thereafter.

On the other hand, Lucas (1969), working with industry data, showed that cross-section estimation of the elasticity of substitution is consistent with the Cobb-Douglas production function, while time series estimation yields values between 0.3 and 0.5, which is consistent with our findings here.

## 8 Quantitative Exercises

In this section, we examine to what extent distortions account for income inequality across nations. The way we approach this issue here is that we “shut down” TFP and educational differences and ask how much income inequality can be explained by differences in distortions alone. The thought experiment therefore is to consider the whole world as having access to the same technology, and quantify the impact of bad economic policies (or other institutional and cultural features) that raise the cost of adopting new technology.<sup>33</sup> We refer to this exercise as “model simulation.”

We also examine how the answer to this quantitative exercise hinges on particular features of our model, which distinguish it from other models. In particular, we examine the sensitivity of our findings to the embodiment hypothesis.

### 8.1 Simulating the model

We feed varying values of  $p$  into the (U.S.) calibrated version of the model and read off what effect this has on per-capita income. In doing so we hold the educational level  $h$  and the total factor productivity  $B$  constant, so as to isolate the effect of distortions.

We use the following procedure. The first equation of system (20) gives us per-capita income (at  $t = 0$ ) as a function of all parameters:

$$Be^{\phi(h)} \left[ \frac{1 - \alpha}{1 - \alpha \left( \frac{p}{\alpha BD(r + \delta, T)} \right)^{1-\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \frac{D(g + \delta, T)}{T}.$$

Plugging calibrated and observed values, other than  $p$ , into this expression we get a function, call it  $y(p)$ , showing how per-capita income depends on distortions. We plot this function as the lower curve of Figure 3.

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<sup>33</sup>A recent paper by Djankov et al. (2000) conducts an empirical comparison between countries regarding the ease of entry. In doing so that paper provides direct micro evidence on how countries differ with respect to distortionary policies.

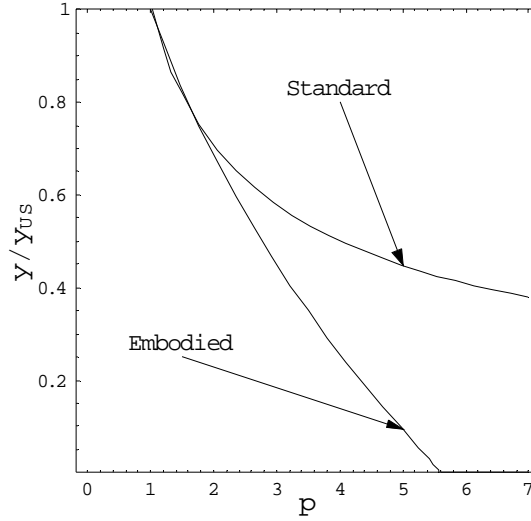


Figure 3

The most apparent feature of Figure 3 is that the lower curve spans a large range of incomes as  $p$  varies over a “reasonable domain.” In particular, if  $1 \leq p \leq 5$ , which is the observed domain of prices in the SH data-set,<sup>34</sup> then  $0 < y < 1$ . Since  $y$  is income relative to U.S. income, the theory explains the full range of observed income ratios. This is, of course, a very limited explanation because we are ignoring the roles of  $B$  and  $h$  (which we discuss in Section 10). Nonetheless, the theory we put forward here already does much better than the “standard” model with disembodied technological progress and an aggregate, Cobb-Douglas production function; consult Lucas (1988) and Mankiw (1995) for how the standard model is used to assess development facts, and Chari et al. (1997) for a recent application. Indeed, if we do a similar calibration-estimation exercise for the latter (see next subsection) we get the higher curve in Figure 3.

Visual inspection reveals that the higher curve is flatter and, consequently, spans a much smaller range of incomes than the lower curve. The reason for that is that, under embodiment and  $\sigma < 1$ , the capital-elasticity of output increases when  $k$  decreases (and hence when  $p$  increases), which magnifies the effect of distortions, compared with disembodiment and  $\sigma = 1$ .

An extreme form of this is the possibility of a “poverty-trap;” i.e., that per-capita income

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<sup>34</sup>From SH we computed  $5.1 = \frac{\sum \bar{p}_i}{\sum \underline{p}_i}$ , where the numerator is the 20 lowest- $p$  countries and the denominator is the 20 highest- $p$  countries.

is zero. A poverty-trap might occur with  $\sigma = 0.4$  but not with  $\sigma = 1$ . Indeed, whenever  $\sigma < 1$ , the marginal product of capital at  $K = 0$  is finite, whereas when  $\sigma = 1$ , the same marginal product is infinite (the celebrated Inada condition). Thus, as  $p$  goes up, a point comes where an economy with  $\sigma < 1$  stops upgrading its capital altogether, and becomes poverty-trapped. On the other hand, an economy with  $\sigma = 1$  (or  $\sigma > 1$ ) always upgrades its capital - no matter how high  $p$  is; such an economy is never poverty-trapped.

## 8.2 Embodied versus Disembodied Technological Progress

Next let us compare - in greater detail - our model of embodied technological progress to the standard model of disembodied technological progress. Our point of departure from the standard model is two-fold. First, we consider **embodied** technological progress. Second, we consider individual production functions with  $\sigma = 0.4$ , whereas the standard model considers an aggregate, Cobb-Douglas production function with  $\sigma = 1$ . This raises the natural question why our model performs better. Is it the embodiment hypothesis? Or, is it the elasticity of substitution?

To address that question we do the following exercise. We construct the balanced growth-path of the disembodied model with a C.E.S. aggregate production function, whose elasticity of substitution is  $\beta$ . Then, we calibrate this model to U.S. data and estimate  $\beta$ , using the SH data-set. The estimated  $\beta$  is  $\beta = 0.7$ .<sup>35</sup> These steps are perfectly analogous to those we performed above; details are found in our working paper. Once we have these calibration and estimation results we construct the same  $y(p)$  functions for the disembodied model with  $\beta = 0.7$  and with  $\beta = 1$ , and superimpose them on our, embodied  $y(p)$  function with  $\sigma = 0.4$ . The result is shown in Figure 4.

As Figure 4 shows, going from  $\beta = 1$  to  $\beta = 0.7$  (i.e., keeping disembodiment, but using a different elasticity of substitution) already represents an improvement. However, going from disembodiment (with  $\beta = 0.7$ ) to embodiment (with  $\sigma = 0.4$ ) produces a far bigger improvement. Quantitatively, therefore, embodiment plays a more significant role in explaining income inequality.

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<sup>35</sup>Same regression as in Section 7.



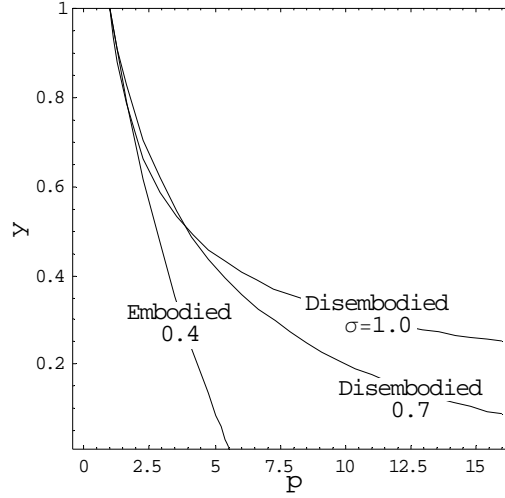


Figure 4

Another aspect of the model that this exercise points to is the following. Our model with individual production functions and embodied technological progress accommodate two features. On the one hand, the model gives rise to an aggregate demand for investment, which, when we estimate it, gives an elasticity of demand with respect to price of 0.73. On the other hand, the model features an individual production function with elasticity of substitution of 0.4, which, when we simulate it, produces poverty-traps with reasonable distortions, e.g.,  $p = 5$ . By contrast, if one were to work with a disembodied model, one can either assume  $\beta = 0.4$ , get poverty-traps, but work with the wrong elasticity of demand for investment, or, one can assume  $\beta = 0.73$ , which corresponds to the right elasticity of demand for investments, but does not generate poverty-traps under reasonable distortions. Under disembodiment, it is impossible to get both a realistic elasticity of demand and poverty-traps for reasonable distortions. The advantage of our model is that it disentangles the price elasticity of investment from the elasticity of substitution (of the individual production function) and is thereby able to capture both features.

### 8.3 Growth Miracles

Using our simulated  $y(p)$  curve, we relate Young's (1995) findings to the theory we put forward here. Young (1995) documents the fact that much of the growth miracles of the "East-Asian tigers" are attributed to the most traditional economic factors - physical and

human capital accumulation, as opposed to changes in total factor productivity.<sup>36</sup> Young's findings are easier to reconcile with our model than with the standard model, as the following computation suggests. Consider the  $y(p)$  function that our theory cranks out evaluated at the price  $p = 4$ . This hypothetical price is taken to represent a highly distorted economy, which the East-Asian tigers are said to have been in the 50's. Consider a change in economic policy that removes these distortions, resulting in  $p = 1$ . Then, as Figure 3 shows, we encounter a phenomenal growth experience - per-capita income increases five-fold, which, if we consider the time period 1960-1990, amounts to a compounded 9% annual growth-rate. On the other hand, if we consider the standard model, as calibrated in Section 8.2, per-capita income merely doubles, amounting to 3% per-year growth rate. So if  $B$  remained constant over the period 1960-1990, then our theory reproduces the actual growth experience of the East-Asian tigers much better than the standard model.

Although suggestive, one should be a bit cautious with this exercise. This is because the  $y(p)$  curve connects steady states rather than showing a transition path between steady states. The next natural step in this line of research, which goes beyond the scope of the present paper, is to investigate explicitly transitory dynamic and what it implies about growth-rates when  $p$  goes down.

## 8.4 Contrasting the Simulated Model with Data

Another way of contrasting our model with (cross section) data is the following. We can ask how actual data compares to our simulated  $y(p)$  curve. That curve is derived on the assumption that  $h$  and  $B$  are constant, and equal to their U.S. levels, whereas, in the real world,  $h$  and  $B$  are (obviously) not constant. To partially account for cross-country variation in  $(h, B)$ , we use the following procedure. Using Hall and Jones (1999) (HJ) data we construct the ratios (country  $j$ 's income)/(country  $j$ 's educational factor)\*(U.S. income), where per-capita incomes  $y_j$ , which are net of the mineral sector output, and educational factors  $e^{\phi(h_j)}$  come from HJ. This accounts for the  $h$ , but not for the  $B$ , variation, which we are not able to observe. Then, we pair up these ratios with the  $p_j$ 's reported by SH, and put these pairs on a two-dimensional diagram. On the same diagram we superimpose the  $y(p)$  function calculated above. The result is shown below as Figure 5.

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<sup>36</sup>As far as total factor productivity,  $B$ , if anything, the  $B$  of the East-Asian tigers is below the average  $B$  of countries with similar per-capita income.

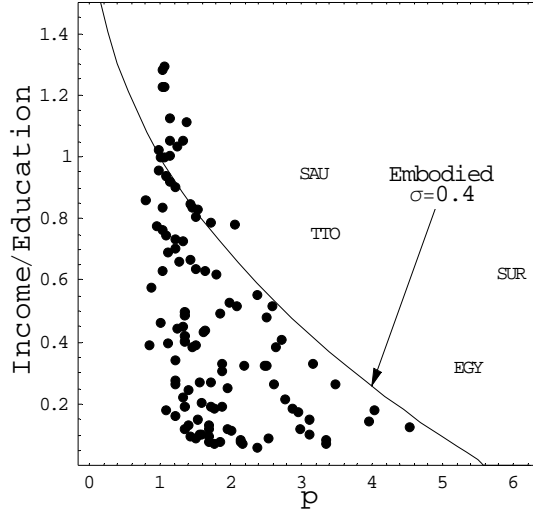


Figure 5

As this Figure shows, most scatter points lie below the  $y(p)$  curve. We take this fact to imply that the U.S. economy operates at a high efficiency; i.e., has a higher TFP than most countries. Indeed, the scatter points in this diagram are created without correcting for different countries having different  $B$ 's. Hence, whenever a country has a smaller  $B$  than the U.S., its scatter point lies below  $y(p)$ , which is what the Figure shows.<sup>37</sup>

## 9 Model Predictions

In Section 8 we examined how the model works as a theory explaining income inequality. In this section we discuss various other predictions along which the model can be tested. The methodology here is similar to that of Section 8. We fix  $h$  and  $B$  while varying  $p$ , and ask what effect this has on various economic statistics. We also compare the predictions we get in this way to those we get when the elasticity of substitution is 1 (instead of 0.4).<sup>38</sup>

<sup>37</sup>We tried to account for the  $B$ 's by taking the implied  $B$  from the regressions in Section 7. However, the  $B$  for some African countries is excessively high because the model does not work well for large  $p$ 's. As a result of this the scatter diagram became very dispersed.

A natural solution to this is to throw out some countries whose  $p$  and, hence, implied  $B$  is high. However, it is not obvious which criterion to use when deciding which observations are to be thrown out. Because of that we treat  $B$  as a residual in this exercise.

<sup>38</sup>This comparison is closely related to a comparison with the standard model, since, when  $\sigma = 1$ , the qualitative implications of a model with embodied technological progress are the same as the implications of a model with disembodied technological progress.

Many of our model’s predictions hinge on the mechanics of upgrading and how it relates to distortions. In particular, each firm periodically upgrades its technology. The length of the upgrading period varies positively with  $p$ ; namely, a higher  $p$  implies a firm will hold on to its technology for a longer time (see Proposition 2). This creates a “window effect” where more distorted economies hold more antiquated and, hence, lower-quality capital. Related to this, the price of old capital decays more slowly the more distorted the economy is (see Figure 1). These two effects create a “paradoxical” valuation of capital stocks, where poor countries appear to have a large aggregate capital stock.<sup>39</sup> Many of the patterns we report below are due to these features of the model.

- The Investment-Capital ratio. This ratio is decreasing in  $p$ , and quantitatively the decrease is quite steep; see Figure 6.1. As suggested, the logic behind this is that capital goods are held for a longer time and their prices decline at a lower rate as  $p$  increases. This increases the market value of the capital stock, which decreases the investment-capital ratio. On the other hand, for  $\sigma = 1$  this ratio is constant in  $p$  (because the holding period is independent of  $p$ ).

Another prediction that this exercise brings out is that economic depreciation is large relative to the sum of physical and technological depreciations. Indeed, as  $p$  increases the height of the curve converges to  $\delta + g$ , which is the sum of physical and technological depreciations. Everywhere else the curve is higher and, for “reasonable” values of  $p$ , it is significantly higher.

- The Capital-Output Ratio. To make this ratio comparable across countries, we divide it by  $p$ . For  $\sigma = 1$ , this curve is downward sloping, with unit elasticity (see Figure 6.2). For  $\sigma < 1$ , the curve declines at a lower rate and may even bend upwards and increase. For our  $\sigma = 0.4$ , the curve is, in fact, U-shaped. The reason for this is that as  $p$  increases, output always decreases. However, the market value of capital is subject to two opposing effects. The quantity of capital installed always decreases (see Proposition 2), which makes the capital stock smaller. On the other hand, capital is held for a longer time and its price declines more slowly. This makes the market value of capital bigger. Which of these effects dominates depends on the value of  $\sigma$ .

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<sup>39</sup>This is due to the fact that, at a given point in time, more capital goods are “floating around” and that they are evaluated at higher prices, the higher is  $p$ .

This gives us a testable implication of our theory. If we isolate the effect of  $p$ , then, under  $\sigma = 0.4$ , the capital-output ratio should be (approximately) constant across countries. On the other hand, under  $\sigma = 1$ , the same ratio is significantly decreasing in  $p$ . Related to this is the prediction about the capital-output ratio among the very poorest (high- $p$ ) countries. Under  $\sigma = 0.4$ , this ratio is positive, whereas under  $\sigma = 1$ , it is asymptotically zero.

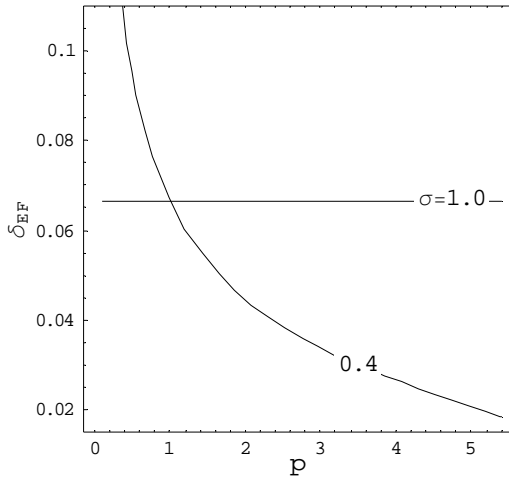


Figure 6.1

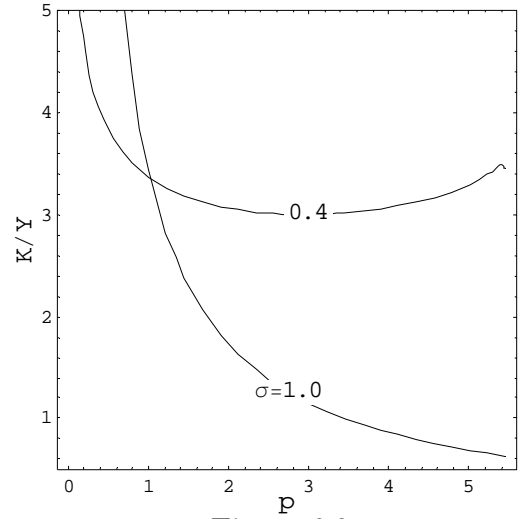


Figure 6.2

- The Economy-wide Profit-Rate. Under  $\sigma = 0.4$ , this rate is decreasing in  $p$  or, equivalently, more distorted economies exhibit a higher price-earning ratio (see Figure 7.1). The logic here is that an increase in  $p$  reduces economic depreciation (i.e.,  $T$  is bigger) and, thereby, reduces the profit rate of the economy. This follows from the no-arbitrage condition (19). On the other hand, under  $\sigma = 1$ , the profit rate is constant. This follows from the fact that economic depreciation is unaffected by  $p$  (i.e.,  $T$  is constant) in the  $\sigma = 1$  case.
- The wage rate. In general, the effect of distortions on wages is negative. Quantitatively, the effect is stronger when  $\sigma = 0.4$  as compared to  $\sigma = 1$ ; see Figure 7.2. The logic behind this is that the wage rate equals the output of the marginal plant. At the same time, the age of the marginal plant under  $\sigma = 1$  is unaffected by  $p$ , whereas the marginal plant under  $\sigma = 0.4$  is older when  $p$  increases. Thus, under  $\sigma = 0.4$  labor works with older (and hence lower quality) capital as  $p$  increases, whereas under  $\sigma = 1$

the age of capital is independent of  $p$  (the wage decreases in  $p$  even when  $\sigma = 1$  because less capital is being installed) .

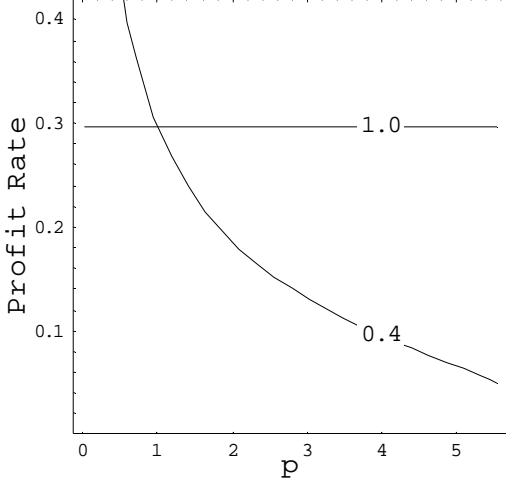


Figure 7.1

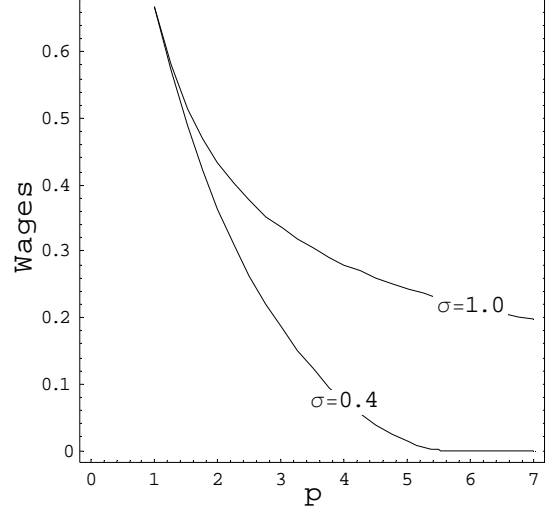


Figure 7.2

## 10 Development Decomposition Exercise

In this section, we take a more comprehensive look at the sources of income inequality across nations. In particular, we consider income inequality as stemming from two sources: differences between countries in distortions and differences in TFP; differences in educational levels are accounted for by considering income per efficiency unit of a year of schooling. Given these two sources, we do a decomposition exercise for our model as well as for the standard model, and compare the results. This allows us to determine which model explains more income inequality by means of differences in distortions.

Our procedure is as follows: we take the panel of 125 countries we analyzed in Section 7 and consider it at a fixed point in time - 1988. For each country  $j$  we take the first and fourth equations of system (20):

$$\frac{y_j}{e^{\phi(h_j)}} = A_j B \left[ \frac{1-\alpha}{1-\alpha \left( \frac{p_j}{\alpha A_j B D(r+\delta, T_j)} \right)^{1-\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \frac{D(g+\delta, T_j)}{T_j} \quad (25)$$

$$1 = e^{-(g+\delta)T_j} \frac{D(r-g, T_j)}{D(r+\delta, T_j)} + \alpha \left( \frac{p_j}{\alpha A_j B D(r+\delta, T_j)} \right)^{1-\sigma},$$

where  $B$  is the TFP in the U.S., as calibrated in Section 6, and  $A_j B$  is the TFP of country

$j$  ( $A_j$  is yet to be determined). We substitute into (25) values for  $\alpha, \sigma, g, \delta$ , and  $r$  from Sections 6 and 7. In addition we substitute into equations (25) the observed values of per-capita income (net of the mineral sector)  $y_j$  and the educational factors  $e^{\phi(h_j)}$ , as reported in HJ for the year 1988.<sup>40</sup> We also substitute in the price of capital  $p_j$ , which we define as the SH average price of capital for the period 1965-1985. We exclude 7 countries from this exercise. Ethiopia, Iraq, Liberia and Nepal are excluded because they are not in the HJ data-set. Madagascar, Mozambique, and Chad are excluded because their reported  $p$  is bigger than  $\bar{p}^{\text{US}}$  (which means they are poverty-trapped). After this exclusion we get, for each remaining country  $j$ , a system of two equations in two unknowns,  $T_j$  and  $A_j$ . We solve the system (it is uniquely solvable), and plot  $A_j$  against  $y_j$ . The plot is shown in Figure 8.1.

As this figure makes clear, TFP, which has the status of a residual, is correlated with per-capita income and, hence, has an explanatory power even after the role of distortions is accounted for. However, as we show briefly it has less explanatory power than the analogous TFP (and correspondingly  $p$  has more explanatory power than) in the standard model.

To do an analogous decomposition exercise for the standard model we follow a similar procedure. We take the same panel of 121 countries at the same point in time. The balanced growth path for country  $j$  is characterized by one equation in one unknown,  $A_j^{\text{S}}$ :

$$y_j = (A_j^{\text{S}} B^{\text{S}})^{\frac{1}{1-\alpha}} e^{\phi(h_j)} \left[ \frac{\alpha}{p_j (r + \delta^{\text{S}})} \right]^{\frac{\alpha}{1-\alpha}}, \quad (26)$$

where  $B^{\text{S}}$  is the U.S. calibrated TFP and  $\delta^{\text{S}}$  is the calibrated depreciation factor in the standard model (which are the same across economies). We solve equation (26) and plot  $A_j^{\text{S}}$  against  $y_j$ , as shown in Figure 8.2.

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<sup>40</sup>This data set can be downloaded from Jones web page (<http://www-econ.stanford.edu/faculty/jones.html>).

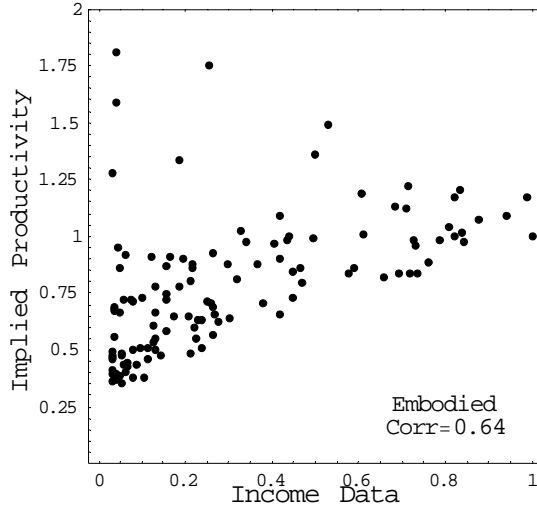


Figure 8.1

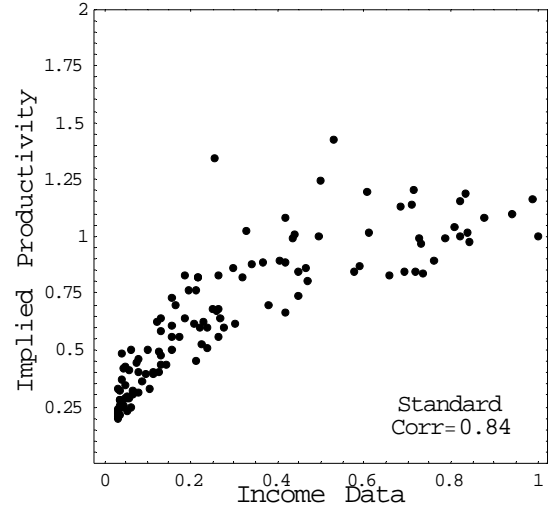


Figure 8.2

As visual comparison of Figures 8.1 and 8.2 reveals, there is more correlation between  $A_j$  and  $y_j$  in the standard model than in the embodied model. More systematically, we have computed the correlation between  $A_j$  and  $y_j$  and found it be 0.64 in our model and 0.84 in the standard model.<sup>41</sup> We also computed the mean TFP for the ten countries where it is highest and the ten where it is lowest and divided the former by the latter. For our model, we got the ratio of 3.4, whereas for the standard model, we got the ratio 5.4. On all three counts, our model does better as far as accounting for income inequality via prices.

Given the implied TFPs for the embodied and the standard models,  $A_j$  and  $A_j^S$ , we performed a further exercise to compare the explanatory role of  $p$  in the two models. We computed the coefficient of variation of per-capita incomes in the data and got the value 0.91. Then, we “shut down” the variation in distortions by assuming that all countries have a common  $p$ ,  $p = 2$ , which is the average  $p$  in our data-set. Then, inserting this value into  $y(p)$  we determined implied per-capita incomes, and computed the coefficient of variation for them - under the embodied and the standard models. The numbers we got were 0.60 for the embodied model and 0.81 for the standard model.<sup>42</sup> This suggests that the role of  $p$  in the embodied model is significantly bigger than in the standard model.

<sup>41</sup>We got 0.54 instead of 0.64 when we consider the full sample, including Madagascar, Mozambique, and Chad.

<sup>42</sup>When we use  $p = 1$  instead of  $p = 2$ , we get a coefficient of variation of 0.45 for our model and 0.81 for the standard model. So using  $p = 1$  strengthens the conclusion that  $p$  has more explanatory power in the embodied model than in the standard model.



To understand why  $p$  explains more income inequality in the embodied model we re-write the first equation of (25) as follows:

$$\frac{y_j}{e^{\phi(h_j)}} = A_j B f(k_j) \frac{D(g + \delta, T_j)}{T_j}.$$

This expression suggests that per-capita income can be decomposed into three components: the residual  $A_j B$ , the endogenous TFP  $\frac{D(g + \delta, T_j)}{T_j}$ , and the ‘capitalization effect’  $f(k_j)$ . Figure 9.1 displays the embodied model’s prediction for the endogenous TFP. It shows that  $p$  affects significantly the endogenous TFP. For example, take a country whose per-capita income is 1/10 of the U.S. per-capita income. Then, as figure 9.1 shows, its endogenous TFP is 1/2 of the U.S. endogenous TFP. This is because when  $p$  is high, capital is held for a longer duration, labor works with more antiquated capital and, consequently, labor productivity is lower. This effect on capital quality/labor productivity is non-existent in the standard model. Thus,  $p$  has an extra effect in the embodied model, which explains why distortions are more powerful in that model.

To complete this explanation one has to make sure that effect of  $p$  on capitalization does not wipe out its effect on endogenous TFP. To that end, Figure 9.2 shows that the value for  $f(k)$  implied by the embodied model is very close to the one implied by the standard model. Thus, the extra TFP effect is still decisive even after we take into account the capitalization effect.<sup>43</sup>

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<sup>43</sup>In the embodied model  $k$  is a function of  $p$  and  $T$ , and  $T$  is a function of  $p$ . When  $\sigma < 1$ , the effect of  $p$  on  $k$  via  $T$  offsets the direct effect of  $p$  on  $k$  (this is because when  $p$  is higher capital is kept for a longer time and, hence, more capital is purchased). Therefore the effect of  $p$  (on  $k$ ) in the embodied model is weaker than the effect of  $p$  in the disembodied model. This makes most scatter points in Figure 9.2 lie above the 45° line. The same Figure shows, however, that this difference is minute and, consequently, the effect on the endogenous TFP term dominates the capitalization effect.

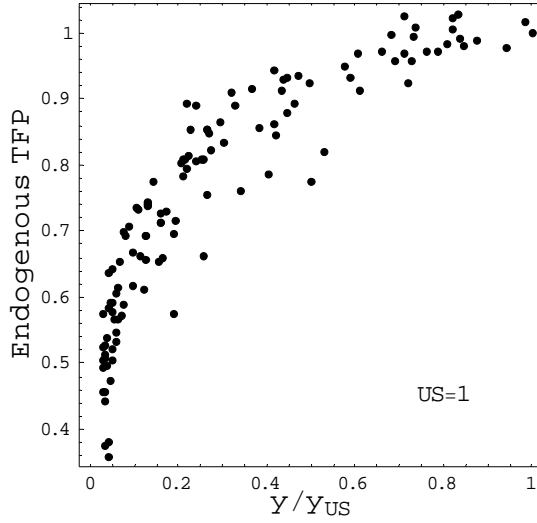


Figure 9.1

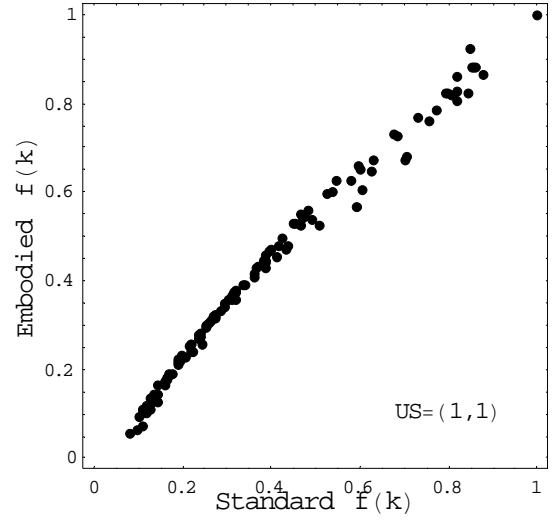


Figure 9.2

## 11 Conclusion

This paper constructs a theory of embodied technological progress, which strikes a middle ground between generality and empirical applicability. The theory is more general than previous theories in that we accommodate a whole class of production functions, and each of these production functions allows a firm to upgrade its capital stock by choosing the timing of upgrades and how much capital to install at each point of upgrade. Yet our theory is sufficiently tractable in that we are able to take it to data and do various quantitative exercises with it. In particular, we apply our theory to the question whether distortions to the adoption of new technology, such as taxes and tariffs, account for the huge per-capita income gaps among nations of the world. Our main conclusion is that distortions account for a greater portion of the observed income inequality in our model than in competing models. Nonetheless, much remains to be explained even after the role of distortions is taken into account.

The theory can be, naturally, extended in several directions. In closing, let us point out a shortcoming of our theory and suggest how this shortcoming may be remedied by one such extension. Our main Proposition (Proposition 1) states that economies with a high value of the distortion parameter  $p$  become poverty-trapped; i.e., the model predicts zero per-capita income for such economies. This raises several problems when we take the model to data because, obviously, observed per-capita income is not zero for any economy. One problem is

that the model predicts that the labor-share of income vanishes as  $p$  increases, which the data does not corroborate. A second problem is that the implied productivity  $B$ , when treated as a residual, is biased upwards for the high- $p$  economies (since observed per-capita income is not zero even for high- $p$  economies, the model attributes it to a high value of  $B$ ). As a consequence of this,  $p$  and the implied  $B$  are positively, instead of negatively, correlated.

A natural solution to this problem is as follows. When distortions are sizable, one would expect resources to flow from more distorted sectors to less distorted sectors. This cannot happen in our model since we have only one sector. Imagine, however, a two-sector extension of our model. In particular, assume there was a “traditional” sector, which used capital less intensively (or, in the extreme, used no capital at all), and were therefore less affected by distortions. Then, we would expect resources to flow towards the traditional sector. This would imply that per-capita income is never zero; it equals at least the output of the traditional sector when all resources are employed in that sector. It would also imply that the labor-share of income is not as small when  $p$  increases as it is in our model. Finally such formulation would imply that the implied residuals are not as big as they are in our model. In this way, a two-sector economy can remedy the above three problems that the present model leaves open. An explicit treatment of this extension remains, however, the subject of future work.

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