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TEA Model Documentation

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Abstract

The TEA model is a multi-regional and multi-sectorial Computable General Equilibrium (CGE) model that tracks the production and distribution of goods in a dynamic recursive setup for the global economy. The model is built in GAMS and departs from the framework of the GTAPinGAMS model [1]. The dynamic structure and parameters are based in other CGEs and Integrated Assessment Models (AIM) as the MIT EPPA model [2, 3] and the COFFEE model [4], considering the evolution of primary factors and technologies in 18 regions and 21 economic sectors. The TEA model was built to perform economic analysis of future greenhouse gas emissions scenarios, considering technological and structural changes in the global economy, under different climate policies. The TEA model can work on a stand-alone basis but also soft-linked to other tools and models (such as the COFFEE model), in order to increase the capacity of Brazilian research groups to contribute to the scientific and policy debate on climate change and its related topics. This document describes the TEA model structure and its functionalities.

TEA Model Documentation

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1 Introduction

Computable general equilibrium (CGE) modeling combines economic theory with data. CGE models represent the the market choices of agents under resource allocation constraints. In addition, CGE models capture industry-to-industry linkages (indirect effects), highlighting effects that are not usually intuitive. Such models can support policies, evaluating their costs and benefits. CGE modeling brings a greater level of detail and complexity that can be exploited when compared to simple analytical models, due to the advances in microcomputer technology (Shoven and Whalley, 1984). These models are a standard tool used in the analysis of aggregate welfare and measurement of policy impacts through multiple markets, including taxes, subsidies, quotas or transfer instruments (Wing, 2004).

Figure 1 shows the representation of the circular flow of the economy (Walrasian box). The main agents are households and firms. Households supply production factors (labor, capital and land) to the productive sectors. Firms demand these factors of production to produce the goods and services of that economy. Firms pay rent, salaries, and interests for households in exchange for production factors. In turn, households spend income on the purchase of goods and services, including investment. These exchanges occur in the factors of production and goods and services markets, respectively. Prices adjust so that supply meets demand, reaching market equilibrium.

General equilibrium (all markets) is achieved through *market clearance*, under perfect competition and *zero profit* conditions. Governments collect taxes and spend their revenue on consumption and direct transfers to households and firms. Economic agents have preferences over resource allocation. The optimal choice of agents occurs under constraints, such as disposable income, physical and technological availability, and social and political institutions rules. Resource availability affects the choices of agents.

Firms with constant returns of scale aim to maximize profits, while households aim to maximize the utility of their *income balance*. Thus, at each period, households exhaust their income in the purchase of goods and services, even for the purpose of saving (saving equals investment). These three conditions – market clearance, zero profit and income balance – are solved simultaneously to reach the general equilibrium.

TEA (Total Economy Assessment) is a multi-regional and multi-sectorial CGE model that tracks the production and distribution of goods in a dynamic recursive setup for the global economy. The core equations and the static part of the model is based on GTAPinGAMS [1], while the recursive dynamic setup is based on the EPPA model [2, 3]. The model is formulated as mixed complementary problem (MCP) and is solved through Mathematical Programming System for General Equilibrium – MPSGE [5] within GAMS. It assumes total market clearance (through commodity price equilibrium), zero profit condition for producers (with constant-returns-to-scale) and perfect competition to reach general equilibrium.

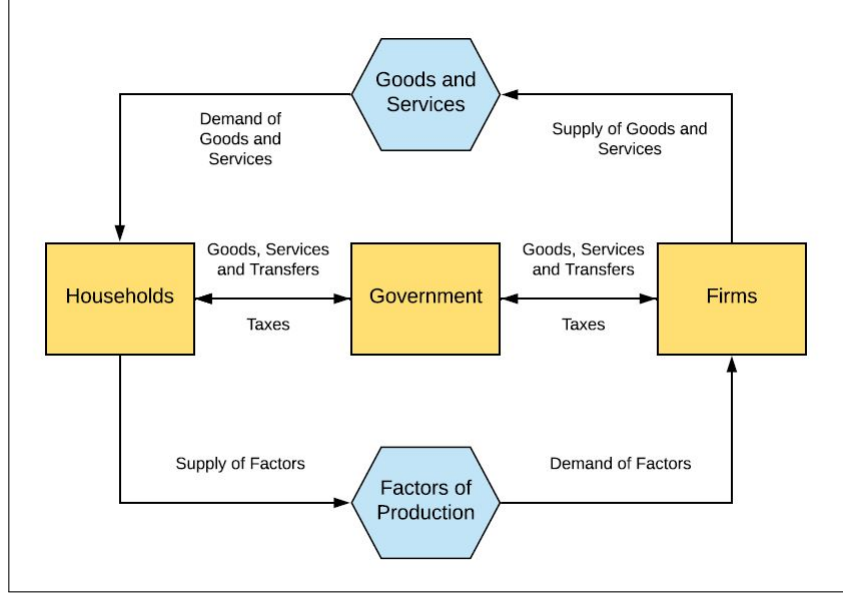


Figure 1: Circular flow of the economy.
Source: own elaboration

1.1 Data sources, regional and sectoral disaggregation

TEA is designed to assess the evolution of the global economy from 2011 (base year) until 2100, at five-year intervals, although it can run on a yearly basis. The major data sources are the GTAP9 database [6], the International Energy Agency (IEA) reports of energy statistics [7] and the World Bank indicators [8], also a source for the GTAP database. The Gross domestic product (GDP) and the population follow exogenous growth rates. In most applications, these rates are based on the Shared Socio-Economic Pathway 2 (SSP2) – Middle of the Road [9, 10].

Regarding the regional breakdown, the world is divided in 18 regions (Figure 2). Regions were selected based on their long-term relevance on energy and environmental matters, such as the China, United States, European Union and Japan, as well as developing countries with growth potential for growth, such as Brazil, India, Russia and South Africa. In addition, the regional representation is compatible with the COFFEE bottom-up energy and land-use systems model [4] to enhance their integration.

The default sectoral disaggregation of the TEA model comprises 16 sectors that were selected among the 57 productive goods / sectors of the GTAP database [6]. 1 shows the 16 sectors grouped in 5 macro sectors: Agriculture (represented by 4 goods / sectors); Industry (represented by 6 goods / sectors); Energy (represented by 5 goods / sectors); Transportation (represented by 3 sectors); and Services. The macro sectors representation is also compatible with the COFFEE model [4] to enhance their integration.

2 Static model

The core equations of the static part of the model, including the variables and the parameters names, are fully described in Section 2 of the document *GTAP6inGAMS: The Dataset and Static Model* by Thomas Rutherford [11]. The static model is defined by three conditions that are solved simultaneously to reach the general equilibrium of the economy: (i) normal economic profit or zero (cost of production equals revenue); (ii) market clearance (supply equals demand for all

¹AFR (Africa), AUS (Australia and New Zealand), BRA (Brazil), CAM (Central America), CAN (Canada), CAS (Caspian Region), CHN (China), EEU (Eastern Europe), IND (India), JPN (Japan), KOR (South Korea), MEA (Middle East), RAS (Rest of Asia and Oceania), RUS (Russia), SAF (South Africa), SAM (South America), USA (United States) and WEU (Western Europe).

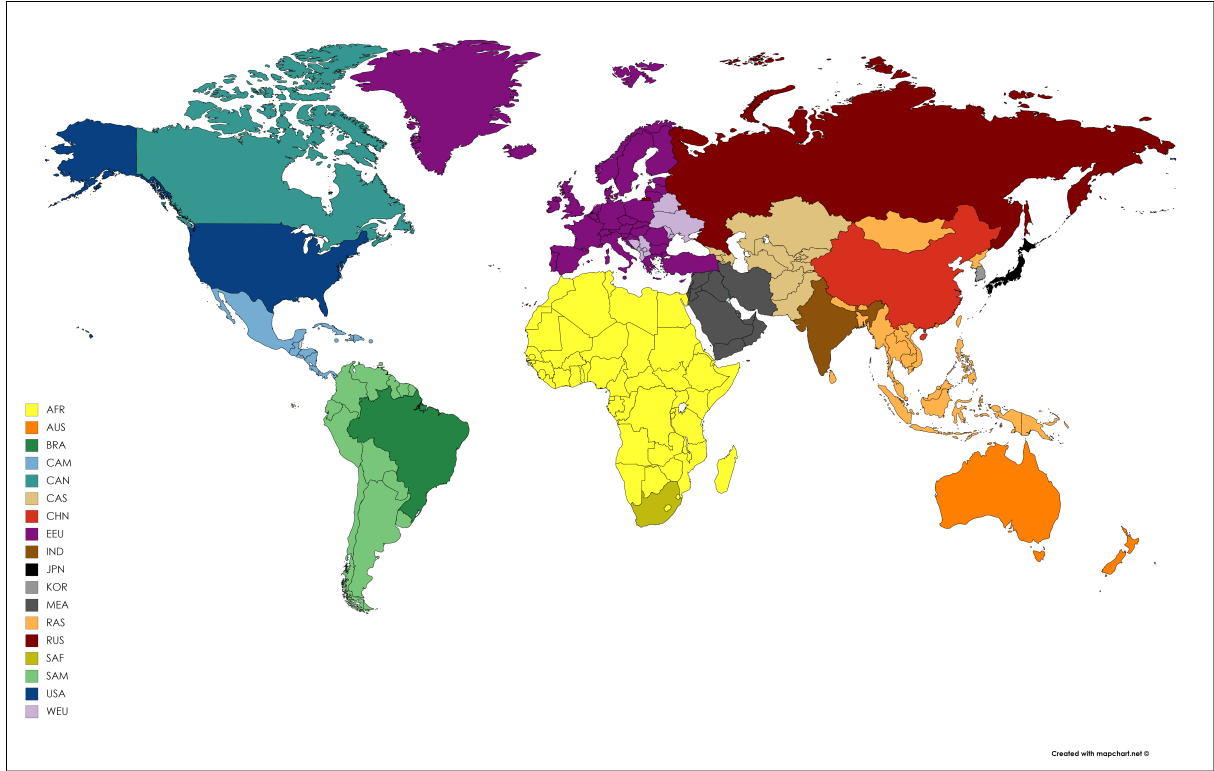


Figure 2: TEA regional breakdown¹
Source: own elaboration.

goods and factors of production) and (iii) income balance (net income equals net expenditure). The equilibrium conditions are consistent with the GTAP database across the national Social Accounting Matrix (SAM) for each region of the model.

2.1 General structure and equilibrium conditions

The equilibrium conditions rely on microeconomic theory, where consumers (representative agent) maximize their welfare subject to budget constraints, and producers minimize the total cost of inputs for a given technology. The productive sectors use intermediate inputs and primary factors for the production of goods and services. Table 1 shows that the factors of production are classified as fixed or free mobility. Free mobility factors are represented by labor and capital, which have free movement only within each region. Fixed factors of production are represented by land and other natural resources.

Final demand comprises public and private expenditure on goods and services. Economic agents are represented by firms / sectors, households (*HH*) or private agents, and the government or public sector of each region r . Table 3 defines the primary variables (activity levels) that express the accounting identities of the regional social account matrices (SAM).

The Equations 1-8 represent the accounting identities of the economic structure for domestic supply (1), imported goods (2), factor market equilibrium (3), international market clearance (4), international transportation services (5), bilateral trade flows (6), tax revenue transfers (7) and household budget constraints (8), respectively. These equations represent the conditions for: (i) market clearance, in which supply equals demand in all markets, and (ii) income balance, in which the net income equals the net expenditure for all agents.

Table 1: TEA sectoral breakdown

Sector	Code	Description
Agriculture	AGR	Agriculture crops and vegetables
	LIV	Livestocks
Energy	COL	Coal
	CRU	Crude Oil
	ELE	Electricity
	GAS	Natural Gas
	OIL	Petroleum coal products
Industry	I_S	Iron and steel
	CRP	Chemical rubber and plastic
	NMM	Manufacture of non-metallic mineral products
	MAN	Others manufacture
Transport	OTP	Transport nec
	WTP	Water transport
	ATP	Air transport
Services	SER	Services
	DWE	Dwellings

Table 2: Factors of production.

Productive Factors	Code	Description
Capital	K	Free intra-regional mobility
Labor	L	
Land	T	Fixed factors
Natural Resources	R	

Market Clearance:

$$vom_{(i,r)} = vdp_{m(i,r)} + \sum_j vdf_{m(i,r)} + vdg_{m(i,r)} + vdim_{(i,r)} + \sum_s vxmd_{(i,s,r)} + vst_{(i,r)} \quad (1)$$

$$vim_{(i,r)} = vip_{m(i,r)} + \sum_j vif_{m(i,r)} + vigm_{(i,r)} \quad (2)$$

$$\sum_i vfm_{(f,i,r)} = evom_{(f,r)} \quad (3)$$

$$vxm_{(i,r)} = \sum_s vxmd_{(i,s,r)} \quad (4)$$

$$vt_{(r)} = \sum_j vst_{(j,r)} \quad (5)$$

$$vt_{(j)} = \sum_{i,r,s} vtwr_{(i,j,s,r)} \quad (6)$$

Income Balance:

$$vgm_{(r)} = \sum_i R_{(i,r)}^Y + R_{(r)}^C + R_{(r)}^G + \sum_i R_{(i,r)}^M + R_{(r)}^{HH} + vb_{(r)} \quad (7)$$

Table 3: Activity levels.

Variables	Code	Description
Y	$vom_{(i,r)}$	Total production or supply of goods and services at market prices
C	$vpm_{(r)}$	Private consumption or household demand
G	$vgm_{(r)}$	Public consumption or government demand
I	$vdim_{(i,r)}$	Demand for investment
M	$vim_{(i,r)}$	Imports
X	$vxml_{(i,r)}$	Bilateral trade flows
YT	$vt_{(i,r)}$	International shipping services
FT	$evom_{(f,r)}$	Specific factor transformation

where i, j are sectors and r, s are regions based on the aggregation of the GTAP database; and f are the factors of production, as described in Table 2.

$$vpm_{(r)} + vim_{(r)} = \sum_f evom_{(f,r)} - R_{(r)}^{HH} \quad (8)$$

where $R_{(i,r)}^Y$, $R_{(r)}^C$, $R_{(r)}^G$ and $R_{(r)}^M$ are the indirect taxes on production/exports, consumption, public demand and imports, respectively; and $R_{(r)}^{HH}$ is the revenue from direct taxes on households.

In addition, the condition of perfect competition with constant returns to scale applies to all productive sectors. Thus, the cost of inputs including indirect taxes equals the value of outputs, and there are no excess profits (zero profit condition) for each of the production sectors, as represented by the Equations 9-15:

Zero Profit:

$$Y_{(i,r)} : \sum_f vfm_{(f,i,r)} + \sum_j \left(vifm_{(j,i,r)} + vigm_{(j,i,r)} \right) + R_{(i,r)}^Y = vom_{(i,r)} \quad (9)$$

$$C_{(r)} : \sum_i \left(vdp_{(j,i,r)} + vip_{(j,i,r)} \right) + R_{(r)}^C = vpm_{(r)} \quad (10)$$

$$G_{(r)} : \sum_i \left(vdg_{(j,i,r)} + vig_{(j,i,r)} \right) + R_{(r)}^G = vgm_{(r)} \quad (11)$$

$$I_{(r)} : \sum_i vdim_{(i,r)} = vim_{(r)} \quad (12)$$

$$M_{(i,r)} : \sum_s \left(vxmd_{(i,s,r)} + \sum_j vtwr_{(j,i,s,r)} \right) + R_{(i,r)}^M = vim_{(i,r)} \quad (13)$$

$$FT_{(f,r)} : evom_{(f,r)} = \sum_i vfm_{(f,i,r)} \quad (14)$$

$$YT_{(j)} : \sum_r vst_{(j,r)} = vt_{(j)} = \sum_{i,s,r} vtwr_{(j,i,s,r)} \quad (15)$$

where $Y_{(i,r)}$, $C_{(r)}$, $G_{(r)}$, $I_{(r)}$, $M_{(i,r)}$, $FT_{(f,r)}$ and $YT_{(j)}$ represent total production, private consumption, public consumption, demand for investment, imports, specific factor transformation and international shipping services, respectively.

2.2 Functional forms

The model solves a Mixed Complementarity Problem (MCP) to find equilibrium solutions. The MCP is given by a system of equations, where three conditions must be satisfied: zero profit, market equilibrium and budget balance. Thus, the MCP approach involves three sets of non-negative variables: prices, quantities and income levels. Moreover, the equilibrium solution indirectly incorporates the efficient and the optimizing behavior of agents (both firms and consumers).

As described in Section 4 of the document *GTAP6inGAMS: The Dataset and Static Model* by Thomas Rutherford [11], the Mathematical Programming System for General Equilibrium (MPSGE) is a model/tool within GAMS that provides a simplified representation of complex systems of nonlinear inequalities, including functions and Jacobian evaluation routines, which eases the formulation of the MCP in general equilibrium models. MPSGE is based on nested structures of functions with CES, Cobb-Douglas and/or Leontief functions, representing blocks of equations to calibrate them according to input data. Hence, the use of MPSGE makes the economic structure of the model more transparent, allowing flexibility in the choice of input data (parameters and elasticities).

The values for elasticities are crucial for the model calibration, influencing the degree of substitution and transformation between goods, services and factors of productions. In the absence of elasticity data or sufficient information for estimation, data published in the literature is assumed, to reflect the structures of production and consumption and the flexibility of sectors to changes in relative prices. Table A1 in the Appendix describes the parameters and the values for the elasticities in the model.

The market equilibrium conditions are achieved through activity levels and relative prices. Table 4 describes the activity levels and the relative prices of each economic activity in the model.

Table 4: Parameters description.

Parameter	Description	Price	Description
$w_{(i,r)}$	Welfare - including investments	$pw_{(i,r)}$	Domestic supply price of goods and services (gross basis)
$y_{(i,r)}$	Total supply	$py_{(i,r)}$	Private consumption price index
$c_{(r)}$	Private demand	$pc_{(r)}$	Government provision price index
$g_{(r)}$	Government Demand	$pg_{(r)}$	Investment price index
$inv_{(r)}$	Investments	$pinv_{(r)}$	Import price (gross basis)
$m_{(i,r)}$	Imports	$pm_{(i,r)}$	Armington Aggregation Price
$A_{(r)}$	Armington Aggregation	$pa_{(r)}$	Prices to identify bilateral exports
$vxmdb_{(i,s,r)}$	Bilateral trade flows	$py_{(j,r)}\$vxmd_{(j,r)}$	Import price for transport services
$vtwrb_{(j,i,s,r)}$	Bilateral Shipping Costs	$ptw_{(i,s,r)}$	Price of specific productive factor (fixed) in the sector
$ft_{(ind,r)}$	Land specific Factor Transformation	$pf_{(ind,r)}$	Marginal cost of international transport service
$yt_{(i)}$	Transportation services	$pt_{(i)}$	Domestic price of Coal power generation
$col_e_{(r)}$	Coal power generation	$pcol_e_{(r)}$	Domestic price of Natural gas power generation
$gas_e_{(r)}$	Natural gas power generation	$pgas_e_{(r)}$	Domestic price of Oil products power generation
$oil_e_{(r)}$	Oil products power generation	$poil_e_{(r)}$	Domestic price of Nuclear power generation
$nuc_e_{(r)}$	Nuclear power generation	$pnuc_e_{(r)}$	Domestic price of Hydroelectric power generation
$hyd_e_{(r)}$	Hydroelectric power generation	$phyd_e_{(r)}$	Domestic price of Wind power generation
$wnd_e_{(r)}$	Wind power generation	$pwnd_e_{(r)}$	Domestic price of Solar power generation
$sol_e_{(r)}$	Solar power generation	$psol_e_{(r)}$	Domestic price of Biomass power generation
$bio_e_{(r)}$	Biomass power generation	$pbio_e_{(r)}$	Domestic price of Biofuel production
$bfu_e_{(r)}$	Biofuel production	$pbfu_e_{(r)}$	Private household transport price (aggregate)
$htrn_{(r)}$	Private household transport	$phtrn_{(r)}$	Price of private transport (ICE)
$htrn_ice_{(r)}$	Private household transport (internal combustion engine)	$phtrn_ice_{(r)}$	Price of private transport (electric/hybrid vehicles)
$htrn_ele_{(r)}$	Private household transport (electric/hybrid vehicles)	$phtrn_ele_{(r)}$	
$carb_land_{(r)}$	Land expansion	$carb_land_{(r)}$	

In addition, Table 5 describes the parameters for taxes, subsidies and tariffs, including the range for the values found in the GTAP database for the selected regions in the model.

Table 5: Parameters for taxes, subsidies and tariffs.

Parameter	Description		Value range
$rto_{(i)}$	Output taxes (gross basis)		-0.2959 – 0.5406
$rtf_{(f,i,r)}$	Factor taxes		-0.3368 – 0.3972
$rtpd_{(i,r)}$	Private demand taxes	Domestic	-0.1603 – 1.2895
$rtpi_{(i,r)}$		Imported	-0.2836 – 1.033
$rtgd_{(i,r)}$	Public demand taxes	Domestic	-0.1025 – 0.2562
$rtgi_{(i,r)}$		Imported	-0.095 – 1.9031
$rtfd_{(i,j,r)}$	Intermediate input taxes	Domestic	-0.2020 – 1.2895
$rtfi_{(i,j,r)}$		Imported	-0.2836 – 1.9031
$rtxs_{(i,r,s)}$	Export subsidies		-4.4309 – 0.1588
$rtms_{(i,r,s)}$	Import tariffs		0.000 – 4.067

The functional forms for the consumer utility, the demand, the production and the constraint functions are part of the static model and provide a base year equilibrium based on the reference GTAP database. Indeed, the model is solved from balanced data for its parameter values, not the opposite [12], and the equilibrium is reached by solving a system of equations, in which optimization problems are converted into equilibrium problems of the economy.

Representative consumers maximize welfare subject to budget constraint in each region. Such choices are determined by the parameters of substitution and transformation elasticities in the utility and production functions. Preferences and technologies are based on nested Constant Elasticity of Substitution (CES) and nested Constant Elasticity of Transformation (CET) functions, respectively².

Private consumption is represented by a nested structure that includes goods and services from different sectors, such as transport (otp, atp, wtp, aut), energy (ele, oil, gas, col), among others. The values in red in Figure 3 represent the elasticities of substitution between goods and services consumed by the representative agent.

The consumer problem of maximizing welfare subject to budget constraint is, therefore, equivalent to the expenditure minimization problem of a given aggregate consumption level. Thus, household consumption is characterized by the choice of goods and services subject to the minimization of expenses, as represented by Equation 17.

$$\min \sum_i py_{i,r} (1 + rtpd_{i,r}) ddp_{i,r} + pm_{i,r} (1 + rtpi_{i,r}) dip_{i,r} \quad (17)$$

$$s.t. \quad H_r(ddpm, dipm) = C_{i,r}$$

²The nested CES structure that describes consumer preferences or the substitution possibilities between factors of production and intermediate inputs in the productive process (CET) can be generically expressed by the following Equation 16:

$$Y = A \cdot \left(\sum_{i=1}^n \beta_i X_i^\rho \right)^{\frac{1}{\rho}}, \quad (16)$$

where Y represents the consumer utility (or the output), A is a constant that represents the propensity to consume (or the productivity), β is the share of goods and services (or input) i , and X is the good and service (or the factor of production). The elasticity of substitution is a positive constant $s = \frac{1}{(1-\rho)}$.

The CES and CET production functions lead to the linear production functions as $\rho \rightarrow 1$, to the Cobb - Douglas production functions as $\rho \rightarrow 0$, and to the production functions with fixed proportions (Leontief) as $\rho \rightarrow \infty$.

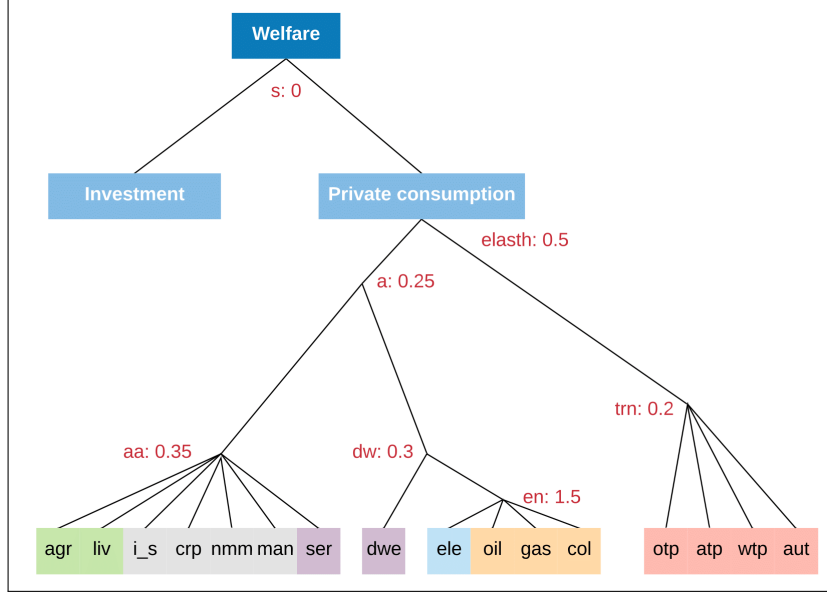


Figure 3: Welfare nested structure.
Source: own elaboration.

where $py_{i,r}$ and $pm_{i,r}$ are the domestic output and import prices; $rtpd_{i,r}$ and $rtpi_{i,r}$ are consumption taxes on domestic and imported goods and services; and $ddpm_{i,r}$ and $dipm_{i,r}$ are the demand functions for domestic and imported goods and services, respectively.

Production functions also follow a nested (technological) structure. For instance, the energy production bundle uses resources as a fixed factor of production, but with a certain degree of substitution among other factors, like capital (K) and labor (L). In Figure 4, electricity (ele) and inputs from the other sectors are represented by colored boxes, while the text in red shows the values for the elasticities of transformation (CET) among the inputs used in the production of different forms of energy.

The producer problem of maximizing profits with constant returns to scale is, therefore, equivalent to minimizing costs subject to technical constraints. So the production of any given sector $Y_{i,r}$ is characterized by the choice of inputs subject to the minimization of unit costs, as represented by the Equation 18.

$$\begin{aligned}
 \min_{ddfm, difm, dfm} \quad & c_{i,r}^D + c_{i,r}^M + c_{i,r}^F \\
 s.t. \quad & c_{i,r}^D = \sum_j py_{j,r} (1 + rtf_{j,i,r}) ddfm_{j,i,r} \\
 & c_{i,r}^M = \sum_j pm_{j,r} (1 + rtf_{j,i,r}) difm_{j,i,r} \\
 & c_{i,r}^F = \sum_f pff_{f,r} (1 + rtf_{f,i,r}) dfm_{f,r} \\
 & F_{i,r}(ddfm, difm, dfm) = Y_{i,r}
 \end{aligned} \tag{18}$$

where $py_{i,r}$, $pm_{i,r}$ and $pff_{i,r}$ are the domestic output, import and factor prices, respectively; $rtpd_{i,r}$, $rtpi_{i,r}$ and $rtpf_{i,r}$ are intermediate consumption taxes on domestic, imported goods and services, and factor taxes, respectively; and $ddfm_{i,r}$, $difm_{i,r}$ and $dfm_{i,r}$ are the demand functions for domestic, imported goods and services, and the demand function for factors, respectively.

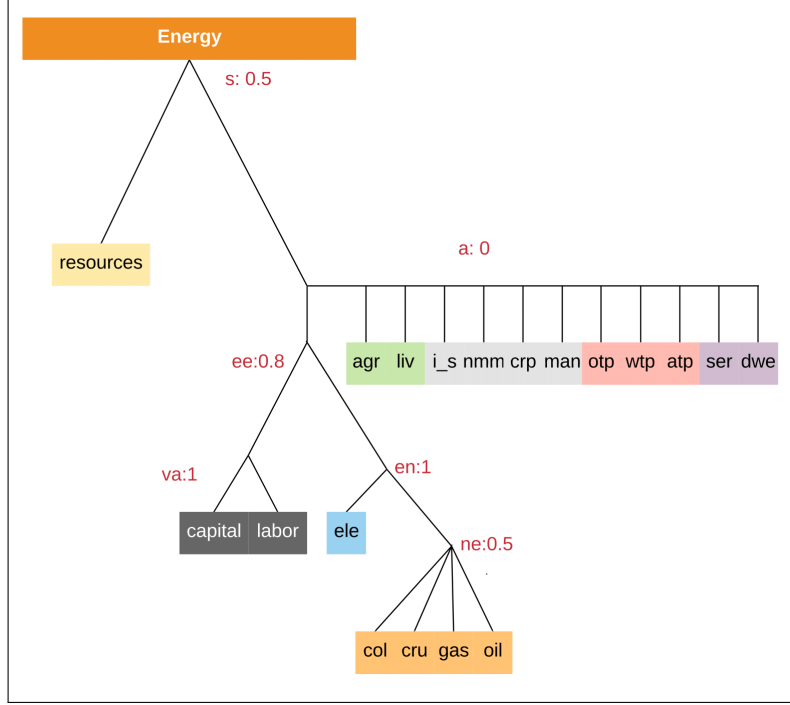


Figure 4: Electricity technological structure – fuel and emissions
Source: own elaboration.

Produced goods and services are traded internationally and follow the Armington formulation [13], that allows an explicit representation of bilateral trade flows. The Armington function $A_{i,r}$ assumes imperfect substitution between the same good and service from regions abroad, so that each imported good is actually an aggregate of that good imported from different regions of the model. Substitution between imports from different sources is driven by the substitution elasticity ($esubm_i$) and bilateral imports follow the cost minimization problem:

$$\min_{dxmd, dtwr} \sum_s (1 + rtmd_{i,s,r}) \left(py_{i,s} (1 - rtxd_{i,s,r}) dxmd_{i,s,r} + \sum_j pt_j dtwr_{j,i,s,r} \right) \quad (19)$$

$$s.t. \quad A_{i,r}(dxmd, dtwr) = M_{j,r}$$

where $M_{i,r}$ is the aggregation function for billateral imports, in which the international shipping services are added proportionately to the value of imports from different regions, thus reflecting marginal cost differences between countries for these transport services.

The international shipping services are an aggregation of shipping services offered by different regions in the model, under an unitary elasticity of substitution (Cobb-Douglas function) between services from different sources. The choice can be represented by a cost minimization problem:

$$\min_{dst} \sum_r py_{i,r} dst_{i,r} \quad (20)$$

$$s.t. \quad T_i(dst) = YT_i$$

where T_i is the aggregation function for international shipping services.

Trade flows are subject to export taxes, import tariffs and international transport margins. The governments might transfer (subsidies) or collect (taxes) revenues on trade. Subsequently,

the aggregate of imports is combined with domestic production of the same good, under elasticity $esubd_{i,r}$, creating a basket of goods to be offered within that region.

Government consumption (public agent) is represented by a CES function, between goods composed of domestic and imported parcels. The elasticity $elast_{gsg}$ drives the substitution between energy and other goods, while imported and domestic goods substitution is driven by the elasticity $esubd_i$.

2.3 Macroeconomic closure

The macroeconomic closure assumes full employment of the factors of production. On the demand side, the marginal propensity to save is constant and specific to each region according to its share of total consumption and savings in the initial database. Thus, savings equals investment in the general equilibrium, but regionally the imbalances are closed by a surplus (or deficit) in the current account.

International trade follows an Armington's aggregation [13], in which a composite CES function differentiate consumer's preferences between imported and domestic goods. An endogenous real exchange rate clears the current accounts and the regional capital accounts converge exogenously in the long-run.

The macroeconomic closure is crucial to capture the policy effects on general equilibrium models. Since the TEA model does not aim to address tax or public budget issues, the demand blocks of the public agent (government) and the representative private agent (families) are unified, reducing computational processing time. Families consume goods and services and, in turn, receive income for supplying factors of production. Thus, aggregate private demand (vpm) equals the factor income ($evom$) less the demand for investment ($vdim$). The government receives international transfers and tax revenues, and spends on goods and services to provide public services. Thus, aggregate public demand (vgm) equals the income from international transfers (vb) plus tax revenues (R).

The macroeconomic closure of the demand from private and public agents, therefore, implies that all income received by households is completely spent on private consumption of goods and services and household savings, which is assumed to be equivalent to the demand for investments. As such, all income received by the public agent must be equal to government spending. Thus, private agent spending (vpm), plus government spending (vgm), plus investments ($vdim$) equals the income for factor income ($evom$), plus international transfers (vb), plus taxes (R). Since international transfers are given by the negative current account balance ($X-M$), the macroeconomic closure reveals the identity between aggregate demand of the economy and income (Equations 21 and 22).

$$vpm + vdim + vgm - vb = evom + R \quad (21)$$

or, equivalently:

$$C + I + G - (M - X) = FT + T \quad (22)$$

The endowment of factors of production is exogenous and fixed over a given period. Factors are assumed to be fully unemployed, perfectly mobile between sectors and within a region (capital and labor), but without mobility between regions (for capital, labor, land and other natural resources). Finally, government spending may adjust to changes in prices and tax revenues. Government spending is assumed to be proportional to the change in household spending for each period, avoiding mismatches between public and private activity.

3 Dynamic process

The benchmark solution of the static model provides the equilibrium for the base year – the starting point for a next equilibrium solution within the analysis horizon. Thus, based on

the changes in the model parameters the solution for the subsequent periods will capture the dynamics of the economy over time. Modelling such dynamics is a sensible task, due to the treatment of the expectations of economic agents in an intertemporal process (PALTSEV, 1999).

The dynamics is modelled through a recursive algorithm that simulates the trajectory of the economy as a succession of medium term equilibria. Given the research focus on prioritizing the nexus between economy, energy and climate change, the model considers a long-term simulation horizon, starting from 2011 (base year) and simulating the production and allocation of goods and services in the global economy at 5 year interval until the year 2100, although most of the applied work runs the model until 2050. The recursive dynamic setup of the TEa model is based on the EPPA model [2, 3].

3.1 Baseline calibration

Regarding data inputs and requirements, the values of the elasticities are kept constant. The main exogenous variables include the rate of economic and population growth; the growth of the workforce; the total factor productivity (TFP); structural changes in demand; formation of new capital; availability of backstop technologies; energy efficiency and energy intensity.

In the baseline, Total-Factor Productivity (TFP) is endogenously target to meet the GDP. Thus, both labor and capital productivity are endogenously defined, assuming that the evolution of total factor productivity will reproduce GDP levels. Labor supply follows population growth rates, such as the trend provided by the SSP2 [14], and the workforce is derived (exogenously) from the population growth. The land resource is specific to agricultural sectors and to biofuel production. Land productivity is assumed to increase exogenously by 1% a.a., but no increase is assumed for labor and capital productivity.

Demand for goods and services grows as income and yield increase. Baseline additional adjustments (structural changes) in long-term demand were made when necessary, usually, based on estimated curves relating GDP per capita and demand share. Hence, the shares of food and agriculture goods in final demand can be adjusted to capture long-term structural changes in preferences. This approach is adopted to overcome the limits of constant returns to scale in consumption (CES consumption function is first order homogeneous), thus making the shares of consumption of goods and services dependent on the change in income between periods.

For instance, long-term structural changes in demand have been made to characterize this reduction in food participation in total household spending as the average per capita income of the population increases in a given region. This prevents food production from growing at the same rate as GDP growth. Correlating GDP per capita and share of demand for agricultural products and food industry by region, curves are estimated to guide household demand for food as a function of changes in per capita income between periods of the model.

3.2 Investment and capital flows

In dynamic recursive models, savings and investment are based only on current period variables. In the TEA model, an MPSGE production block produces an aggregate level of investments equal to the level of savings determined by the household welfare / utility function (representative private agent). Capital accumulation is therefore accounted from the generation of new capital based on the depreciation rate of the investment made in the previous period. The marginal propensity to save is kept constant over time, thus avoiding shocks related to economic cycles. Capital stock evolves at each period with the formation of new capital that depends on the investment level in that period and the capital depreciation rate, as described in Equation 23.

$$K_{r,t} = I_{r,t} + (1 - \delta_r)K_{r,t-1} \quad (23)$$

where, $K_{r,t}$ is the capital stock in region r and time t ; $I_{r,t}$ is the investment in new capital goods in region r and time t ; and δ_r is the depreciation rate of capital in region r .

In addition, the model allows for tracking foreign ownership of capital and investment behavior. The model can assume, in its macro closure, international capital allocation according to changes in regional rates of return [5]. Thus, the impacts of endogenous capital accumulation and the movement of investment between countries in response to differing expected rates of return can be tested.

3.3 Technological change and efficiency improvements

The availability of backstop technologies might be emulated by the penetration of alternative technologies from the COFFEE model, based on the parameter e_trend .

The model assumes that there are technological changes in energy demand, given a reduction in the amount of energy per unit of GDP as each region's GDP per capita increases. Reductions in energy use per unit of product over time are characterized by technological changes, currently represented by the parameter AEEI (Autonomous Energy Efficiency Improvement), which is exogenously assumed to grow at the standard rate of 1%a.a.

3.4 Energy system

TEA includes a detailed representation of the energy sector. This representation is based on the COFFEE (COMputable Framework For Energy and the Environment) model [4], a partial equilibrium (PE) bottom-up model, that provides detailed technological information for the energy system. The soft-link with COFFEE (3.7) improves energy system analysis, achieving a more comprehensive representation of the energy system. This feature is particularly interesting because COFFEE describes energy conversion technologies based on discrete techniques with pre-defined technological (size, lead time, efficiency, availability, etc.) and economic (overnight costs, fixed and variable O&M costs, contingency factors etc.) variables, thus capturing technological deployment over time in a least cost approach. The linking procedure between the models relies on base year data harmonization that includes:

- energy production and consumption (for energy and non-energy uses);
- explicit technological representation of nuclear, hydro, wind, solar and biomass sources;
- implementation of autonomous energy efficiency improvement (AEEI);
- share of power generation and energy trends; and
- GHG emissions (CO_2 , CH_4 and N_2O).

Data for electricity generation (in energy physical units) and the shares of production factors (capital, labor, services, resources, fuel and land) are inputted into TEA in order to explicitly represent nuclear, hydro, wind, solar and biomass technologies. The production functions of these technologies were changed from CES to typical Leontief structures in order to facilitate that results from COFFEE could be completely embedded by the TEA model. Thus, the substitution elasticity between the different energy inputs is set to equal zero so that there is no substitutability between factors. The power generation branch has fixed input proportions and the penetration of different technologies carriers is determined by the COFFEE model.

Figure 5 presents a generic mapping of the base year energy balance performed for each region of the model. The mapping shows the main data sources that were used to calibrate the base year of the model in terms of energy use.

Table 6 describes the equations used to calculate the amount of energy consumed by each region (r) during the time horizon. First, monetary values in the TEA model are converted into energy units. Then, the energy and non-energy use is aggregated from different sources (oil products, electricity) to calculate the total amount of energy consumed by each sector (j) and by each region (r).

	COL	CRU	GAS	OIL	BIO	ELE	
Primary Energy Production	1				1		
COL						3	
GAS							
OIL				2			
BIO							
Hydro							
Wind						4	
Solar							
Nuclear							
Secondary Energy Production				4		4	
COL	5		5			5	
GAS							
OIL							
AGR	6		6				
CTL							
OAP							
FSH							
I_S							
CRP							
NMM							
MAN							
OFD							
OMT							
ATP							
WTP							
OTP							
SER							
DWE							
NEU (non-energy use)	7		7				
Final Consumption	8		8				
Note: COFFEE TEA IEA-WEO							

Figure 5: Mapping of the energy data for base year calibration.
Source: own elaboration.

Table 6: Equations and parameters definition for the mapping of the energy data – base year calibration.

N	Equation	Parameter	Description
1	$p_enecoe(r, ene) = \frac{p_energy(r, ene)}{vom(ene, r)}$	p_enecoe	Converts the monetary value of the production (vom) for each of the energy sectors (ene).
	$p_eneout(r, ene) = enecoe(r, ene) \cdot vom_l(ene, r)$	p_eneout	Primary energy production in energy units based on the evolution of the production in time (vom_l).
2-3	$s_enecoe(r, ene) = \frac{s_energy(r, ene)}{vom(ene, r)}$	s_enecoe	Converts (vom) of oil products oil or electricity production ee into energy units for the base year.
		s_energy	Data for the supply of secondary energy (oil or ee).
4	$s_eneout(ene, r, t) = enecoe(r, ene) \cdot vom_l(ene, r)$	s_eneout	Supply of oil products oil or electricity ee in energy units based on the evolution of the sectoral production in time (vom_l).
5-7	$c_enecoe(r, j, ene) = \frac{c_energy(r, j, ene)}{vafm(j, ene, r)}$	c_enecoe	Converts the monetary value of consumption $vafm$ for each sector (j) into energy units for the base year, including energy (ene) and non-energy (eit) use sectors.
		c_energy	Data for the final energy consumption.
5-6	$enecons_{en}(r, tot, ene, t) = \sum_{ene} enecons_en(r, ene_ , ene, t)$	$enecons_en$	Evolution of the total amount of energy consumption for each energy sector (ene).
7	$enecons_{neu}(r, tot, fos, t) = \sum_{neu} enecons_neu(r, eit, fos, t)$	$enecons_neu$	Evolution of the total amount of energy consumption for non-energy end uses (eit).
8	$enecons(r, tot, ene, t) = enecons_{en}(r, tot, ene, t) + enecons_{neu}(r, tot, fos, t)$	$enecons$	Total amount of energy consumed (in energy units) for all sectors (j), including energy and non-energy uses.

Note: $ene_$ is an alias for ene , a set that comprises (col, gas, oil, cru, ele). $fos(ene)$ is a subset of ene and comprises (col, gas, oil) only.

Source: own elaboration.

Table 7 shows the share of the total production factors (capital, labor, services, resources, fuel and land) for non-fossil fueled power generation technologies. The production factors are an input in the production function. Fixed factors (land and resources) are specific and are based on penetration trends for each technology. The observed penetration rates for these technologies generally show gradual penetration (JACOBY et al., 2004). The TEA model replicates this trend by allocating the fixed factors according to the regional trends of COFFEE expansion capacity [4]. In turn, the trend is affected by three factors: (i) the amount of resource; (ii) the substitutability by other inputs; and (iii) the learning curves for each technology.

Table 7: Share of the production factors inputs by backstop technology

Input / Technology	Wind	Solar	Biomass	Hydro	Nuclear
Capital	66%	89%	52%	40-60%	44-55%
Labor	9%	4%	13%	5-20%	10-30%
Resources	-	-	-	30%	10-15%
Services	-	-	-	3-9%	2-10%
Others	20%	7%	16%	2-7%	2-10%

Source: data based on [3]

As output increases over time, the fixed factor endowment grows, reducing the constraint on the expansion of the installed capacity of the power generation technologies. New industries are bounded by scarce engineering resources, expertise, and specific or customized materials to build alternative power plants. Thus, as long as demand for capacity expansion grows, firms will face higher returns and gain know-how (learning curve process), so that they can expand the fixed factor endowment of such technologies in future periods.

3.5 Land-use system

The amount of the production factor land is endogenously calculated by a land supply curve, that also adding biophysical information on land productivity, for each region. The supply of agricultural land generally depends on its biophysical availability (potential arable land area), institutional factors (agricultural and urban policy, price). According to EICKHOUT et al. (2008), land income and its income are related, so that rising incomes result in lower land incomes and vice versa. From an economic point of view, agricultural land supply is a function of land prices.

In the TEA model, a reciprocal relationship represents the average price of land and its expansion, as shown in Equation 24 and Figure 6, where $lexp$ is the land use and pf is the price of land.

$$lexp_r = \phi_r + \theta_r \cdot \frac{1}{pf_r} \quad (24)$$

The reciprocal relationship indicates that, when the price of land increases indefinitely, the term $\theta_r \cdot \frac{1}{pf_r}$ tends to zero and the expansion of land ($lexp$) reaches a limit (ϕ). Thus, if agricultural land is available, an increase in demand for land will lead to the conversion of potentially available land into productive land, as well as a modest increase in its price to offset the cost of making it productive. On the other hand, since most agricultural land is already in use (REF), an increase in demand for land will lead to large increases in its price, so land conversion will be limited and, therefore, the price elasticity of land supply will become lower.

The value of θ is estimated for all regions of the model, based on the initial point $(x, y) = (pf, lexp) = (1, 0)$ for any region r in the model benchmark. The value of ϕ is estimated from the difference between the amount of land available in the TEA model benchmark and the maximum land expansion by 2050, according to literature data, that projects land expansion in different regions of the world between 8% and 25% by 2050 (SMITH et al., 2010; ALEXANDRATOS and

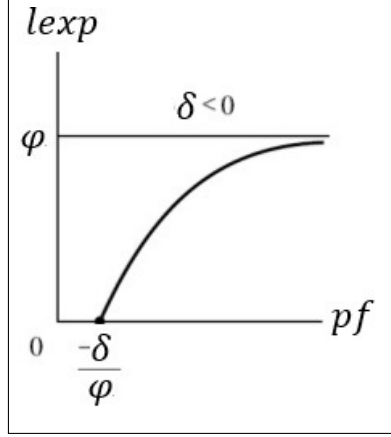


Figure 6: Land supply curve.
Source: own elaboration

BRUINSMA, 2012; LOBELL et al., 2013; Schmitz et al., 2014). The calibration of the regional curves of the TEA model also relies on FAO land use projections and regional area per hectare (ha) data from the COFFEE model [4], as shown in Table 8.

Table 8: Land expansion limits for each region of the TEA model.

Region	$lexp_{max}$	Region	$lexp_{max}$
AFR	20%	JPN	1%
AUS	3%	KOR	3%
BRA	25%	MEA	3%
CAM	6%	RAS	25%
CAN	3%	RUS	10%
CAS	6%	SAF	6%
CHN	3%	SAM	25%
EEU	1%	USA	1%
IND	3%	WEU	3%
Global avg.			8%

Source: own elaboration based on []

The Equation 25 was added to the model to limit the land expansion for each region (8), according to the land supply curve and the data on 8.

$$lsup_r = \max \left[1, \left(1 + \frac{lexp(r)}{evom("lnd", r)} \right) \right] \quad (25)$$

where $lsup(r)$ is the supply of land and $evom("lnd", r)$, the initial endowment of land for each region r in the model benchmark.

3.6 Greenhouse gas emissions

In the TEA model, emissions of carbon dioxide (CO₂), methane (CH₄) and nitrous oxide (N₂O) are represented. The GHG emissions are classified according to its sources: (i) emissions related to the burning of fossil fuels, which are proportional to energy consumption, such as in the transport sector and industries; and (ii) emissions related to the activity level, such as emissions from transformations processes in industry, energy and agriculture sectors.

Parameters represent the average CO₂ emission factors for each of the energy sources according to IPCC (2006; 2014). Regional emission factors were estimated based on data from the COFFEE model to represent GHG emission factors from land use change and from industrial

processes, Non-CO2 emissions are converted into CO2 equivalent according to the Global Warming Potential for 100-year horizon (GWP100). The method does not consider climate change feedbacks into the model.

In addition, GHG emission control policies can be explicitly represented in the model. Taxes and subsidies might be applied on production and consumption to incorporate the social cost of carbon of emitting activities. The model also allows to simulate quantitative emission restrictions, such as carbon budgets, and to simulate the design of different cap-and-trade policies, whether sectoral, regional or global.

3.7 Linking procedure (under development)

Figure 7 shows the procedure for linking the CGE model TEA with the energy model COFFEE. Both models rely on the same exogenous population and GDP projections (e.g., SSP2 – Middle of the Road or any other narrative/projection). After its first run, TEA key outputs on sectoral production and private consumption are processed by a demand generator and serve as key inputs to COFFEE – particularly, energy service demands. In the second step, COFFEE runs and sends TEA information of the power generation mix and energy supply, which becomes an exogenous trend on energy efficiency and technical progress for the TEA model in its next run.

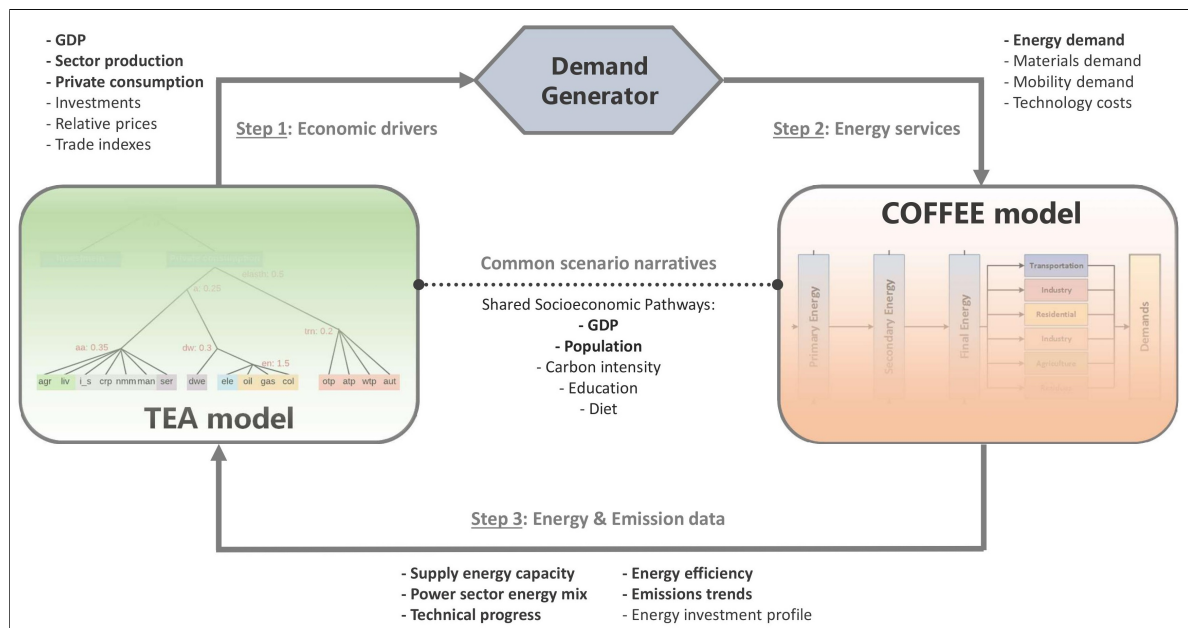


Figure 7: COFFEE - TEA linking procedure

Source: own elaboration

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Appendix

This is the the set and sub-set declaration for elasticities:

```
\textbf{set} nst elasticities nests /en, ee, ne, so, vo, ik, toh,
                                htoh, gesh, gsh, doh, eh, foh, smh, gg/,
nsty(nst) elasticities in production nests /en, ee, ne, so, vo, ik/,
nsth(nst) elasticities in consumption nests /toh, htoh, gesh, gsh, doh, eh, foh, smh/,
nstg(nst) elasticities in government consumption nests /gg/;
```

Table A1 shows the parameters and the values (range) for the elasticities.

Table A1: Parameters and values for elasticities).

Parameter	Elasticity of substitution among...	Value range
$esubva_{(i)}$	Primary factors	0.26 – 1.68
$esubd_{(i)}$	Domestic and Imported (goods and services)	1.25 – 12.88
$esubm_{(i)}$	Imports from different sources	0.50 – 15.00
Inputs:		
$elast^{("so",r,j)}$	Sluggish factors and Others	0.30 – 0.60
$elast^{("vo",r,j)}$	Value added and Others	0.10 – 0.70
$elast^{("ee",r,j)}$	Intermediate inputs and Energy mix	0.00 – 1.00
$elast^{("ele",r,j)}$	Electricity and Other energy sources	1.00 – 1.50
$elast^{("ik",r,j)}$	Intermediate inputs and Capital goods	1.00
Power generation (non-fossil based):		
$neta_{(r)}$	Nuclear supply elasticity	0.25 – 1.00
$heta_{(r)}$	Hydro supply elasticity	0.25 – 0.95
$sigma_{(r)}$	Nuclear resources and other resources	0.04 – 0.17
$hsigma_{(r)}$	Hydro resources and other resources	0.20 – 0.32
$wsigma_{(r)}$	Wind resources and other resources	0.30
$ssigma_{(r)}$	Solar resources and other resources	0.30
$bsigma_{(r)}$	Biomass resources and other resources	0.30
$bfsigma_{(r)}$	Biomass resources - land - and other resources	0.30
$bfelast_{(r)}$	Biofuels and oil	1.00 – 2.00
HH consumption:		
$elasth^{("gsh",r)}$	Goods and services	0.35
$elasth^{("gseh",r)}$	Goods and services and Energy	0.25
$elasth^{("food",r)}$	Food items	0.50
$elasth^{("doh",r)}$	Housing (dwellings) and Other goods and services	0.30
$elasth^{("eh",r)}$	Energy sources	1.50
$elasth^{("toh",r)}$	Household transport services and Other goods and services	0.50
$elasth^{("htoh",r)}$	Household transport services and Other transport services	0.20
$elasth^{("foh",r)}$	Fuels and Other goods and services	1.00
$elasth^{("smh",r)}$	Services and Manufactured goods	1.00
$elastg^{("gg",i)}$	Public demand for goods and services	0.50
$eta_{(i,r)}$	Income elasticity of demand	0.44 – 1.38
$epsilon_{(i,r)}$	Own-price elasticity of demand	-0.93 – -0.20
$etrae_{(f)}$	Elasticity of transformation	0.001 – +inf