

# Is the New Keynesian IS Curve Forward Looking?

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## **Abstract**

This paper analyzes the solutions of the canonical and hybrid New Keynesian IS curve difference equation. It shows that the usual forward solution is ruled out because it is at odds with the underlying economic theory. This implies that current and expected future real interest rates do not matter for the determination of the current output gap. Indeed, the new Keynesian IS curve has multiple backward solutions and determines not the level of the output gap, as does the traditional IS curve, but its expected rate of change. The paper also shows that the hybrid IS has (multiple) backward solutions regardless the size of the forward coefficient.

*JEL Classification:* E12, E31, E32.

*Keywords:* New Keynesian models; IS curve; Transversality Condition; Forward and Backward Solutions; Unique and Multiple Rational Expectations Equilibrium.

# 1 Introduction

In this paper, I analyze the solutions of the new Keynesian IS curve (NKISC) difference equation. It is usual in the literature to adopt a two-step approach: i) solving each equation independently on the assumption that the forcing variable follows a bounded exogenous process; ii) solving the IS and Phillips curves jointly, together with an interest rate rule. I will not follow the latter approach. I analyze the IS curve on the assumption that the forcing variable follows a bounded process.

The NKISC derives from the consumption Euler equation and assumes that the expected output gap and the current interest rate gap determine the current output gap. Solving forward this IS curve implies that the current output gap is negatively related to the current interest rate gap and to the expected future path of interest rate gaps. Thus, announcing the likely path of future overnight interest rates, i.e., forward guidance, is a powerful monetary policy tool available to the central bank. This is an overwhelmingly held view in the new Keynesian literature.<sup>1</sup>

The Euler equation states that all consumption-smoothing paths have to be such that the expected rate of consumption growth should be proportional to the interest rate. It is common knowledge that current consumption does not depend on the interest rate when the intertemporal elasticity of substitution is equal to one. However, expected future consumption is affected in such a case through the substitution and income effects. Therefore, the usual New Keynesian interpretation's claim that the current interest rate always affects the current output gap is at odds with the underlying consumption theory.<sup>2</sup> Furthermore, the output gap and the real interest rate gap are negatively correlated when the elasticity of substitution is greater than one, but positively correlated when the elasticity of substitution is smaller than one. Thus, the forward solution of the NKISC can be ruled out and provides no theoretical support for forward guidance monetary policy. Indeed, the NKISC has multiple backward solutions and determines not the level of the output gap, as the traditional IS curve does, but its rate of change.

Cochrane (2011) analyzes the new Keynesian model, showing that economic theory cannot rule out multiple nonlocal equilibria. In this paper, I use the same approach as Cochrane, namely to solve a difference equation and analyze it to check whether there are solutions that

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<sup>1</sup>See, e.g. Clarida, Galí and Gertler (1999), McCallum and Nelson (1999), Woodford (2003) Chapter 4, Galí (2008) among others.

<sup>2</sup>This statement supports the claim made by Cochrane [(2011), 604] that “New-Keynesian models and results are often described with old-Keynesian intuition. This is a mistake.”

can be rejected by the economics of the model. I show that economic theory rules out the forward solution of the IS curve but not the multiple backward solutions.

Gertler (2017) uses the canonical new Keynesian model to analyze the forward guidance puzzle (FGP), i.e., the disconnect between the experience and the prediction of strong effects of expected future interest rate changes on the economy. The fact that the IS curve has no forward solution is tantamount to say that there is no FGP in the canonical model.

The new Keynesian model literature has been using the hybrid specification of the IS curve (NKHISC) to account for the empirical evidence of inertial output dynamics. In this hybrid specification, the output gap depends on a weighted average of its expected future value and its lagged value. The coefficient of the expected future value ( $\omega$ ) is the forward coefficient. The hybrid difference equation can be solved forward when the forward coefficient of the output gap is not dominant,  $0 < \omega < 1/2$ , but consumption theory rules out this solution. Therefore, the solutions of the NKHISC are backward looking.

The rest of the paper is organized as follows: Section II shows that the standard interpretation of the NKISC is flawed. Section III discusses the solutions of the NKHISC. Section IV concludes.

## 2 New Keynesian IS Curve

The standard interpretation of the NKISC, derived from the Euler equation, is that expected future output and the current real rate of interest affect current output due to consumption smoothing.<sup>3</sup> Consider the simple canonical model where consumption is the only aggregate demand component. The goods market clearing condition is  $c_{t+i} = y_{t+i}$ . The Euler equation is given by:

$$y_t = E_t y_{t+1} - \sigma (r_t - \rho) \quad (1)$$

When prices are flexible, output is equal to potential ( $\bar{y}_t$ ), or natural, output and the real interest rate is the natural rate. Subtracting the flexible-price Euler equation from (1) gives the NKISC equation:

$$x_t = E_t x_{t+1} - \sigma (r_t - \bar{r}), \quad (2)$$

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<sup>3</sup>See, for example, McCallum and Nelson (1999), Clarida, Galí and Gertler (1999) and Goodfriend and King (1997). According to McCallum and Nelson [(1999), p. 303] “...it represents the typical household’s choice in period  $t$  of  $c_t$  in response to  $r_t$  and expectations concerning  $c_{t+1}$  – *not* the choice of  $c_{t+1}$  in response to lagged values  $c_t$  and  $r_t$ .” The Euler equation determines how the expected growth rate of future consumption depends on the real interest rate. It is not a statement about consumption levels, either present or future.

where  $x_t = y_t - \bar{y}_t$  is the output gap and  $\bar{r}$  is the natural rate of interest. I assume, for simplicity, a constant natural rate of interest. Holding constant the expected future output gap, the current output gap decreases (increases) when the real interest rate gap increases (decreases). Thus, holding constant the expected future output gap, the NKISC looks like the traditional IS curve.<sup>4</sup>

Specification (2) of the NKISC, with the current output gap on the left-hand side and the expected future output gap on the right-hand side, can be misleading because one is tempted to carry out the experiment I have just described, namely that of changing the real interest rate gap and holding constant the expected future output gap.

The intertemporal allocation of consumption that underlies the NKISC states that expected consumption increases (decreases) when the current interest rate increases (decreases). Furthermore, it is common knowledge that current consumption does not depend on the real rate of interest when the intertemporal elasticity of substitution is equal to one.<sup>5</sup> Thus, when this elasticity is equal to one, the interest rate will not affect the current output gap, but will affect the expected future output gap. When the elasticity of substitution is less than one, an increase (decrease) in the interest rate will increase (decrease) both the current output gap and the expected future output gap. When the elasticity of substitution is greater than one, an increase (decrease) in the interest rate will decrease (increase) the current output gap and increase (decrease) the future output gap. Specification (2) can be misleading because it could be interpreted as implying that the interest rate gap is negatively related to the output gap.<sup>6</sup> This statement is true when the elasticity of substitution is greater than one, otherwise the correlation would be either positive or zero.

By iterating forward equation (2), I obtain:

$$x_t = E_t x_{t+T} - \sigma E_t \sum_{i=0}^{T-1} (r_{t+i} - \bar{r}). \quad (3)$$

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<sup>4</sup>See the New IS schedule drawn by King (2000) [Figure 1, p. 52], holding constant next period expected output.

<sup>5</sup>The level of current consumption can decrease, increase or stay constant when the interest rate increases depending on whether the elasticity of substitution is greater than, less than or equal to one. This result is valid for a model with infinite periods and variable interest rates see, Obstfeld and Rogoff [(1996), p. 77, equation (25)]. Most of the new Keynesian calibration uses an elasticity of substitution value equal to one [e.g. Galí (2008), p. 52]. However, Woodford [(2003), Table 5.1, p. 341] assumes  $\sigma = 6.25$ .

<sup>6</sup>Estrella and Fuhrer [(2002), p. 1021] states that “the standard consumption equation with rational expectations implies that the sign of the correlation between the real interest rate and the level of consumption is the opposite of the correlation between the real interest rate and the growth rate of consumption”.

When  $T \rightarrow \infty$  the limit of the expected future output gap is given by

$$\lim_{T \rightarrow \infty} E_t x_{t+T} = x_t + \sigma E_t \sum_{i=0}^{\infty} (r_{t+i} - \bar{r}). \quad (4)$$

I assume, as is usual in the literature, that the infinite sum in this expression has a finite limit. Thus, the current output gap would be proportional to the current interest rate gap and the expected future paths of the interest rate gaps,

$$x_t = -\sigma E_t \sum_{i=0}^{\infty} (r_{t+i} - \bar{r}), \quad (5)$$

if the following limit holds:

$$\lim_{T \rightarrow \infty} E_t x_{t+T} = 0. \quad (6)$$

This assumption became standard in the new Keynesian literature and is accepted without discussion. At first sight this statement – namely that the output gap in the long run is equal to zero – suggests that this is the right assumption to be made in this context, since the economy will not be affected by nominal rigidities in the long run.<sup>7</sup> However, according to the underlying consumption-smoothing Euler equation of the new Keynesian model, this limit is, in general, different from zero because expected future output gaps depend on the path of the real interest rates.<sup>8</sup> The hypothesis is not only *ad hoc* but yields a nonsensical result when the intertemporal elasticity of substitution is equal to one. Bear in mind that, in such a case, current consumption does not depend on the current interest rate and the expected future interest rates. Therefore, in this particular case, current spending does not depend on real interest rates as implied in expression (5) for the current output gap. I conclude, based on the microfoundations of the NKISC, that forward-looking equation (5) is not a solution of (2).

The solutions of equation (2) are indeed backward looking and given by:

$$x_t = x_{t-1} + \sigma (r_{t-1} - \bar{r}) + \epsilon_t, \quad (7)$$

where  $\{\epsilon_t\}$  is any stochastic process such that  $E_{t-1}\epsilon_t = 0$ , and  $x_0$  is arbitrary. To stress the fact that the NKISC determines the expected rate of change of aggregate demand, and not the

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<sup>7</sup>This assumption has been used by Clarida, Galí and Gertler [(1999), p. 1666], Woodford [(2003), p. 244], Galí [(2008), p. 49]. The expectational difference equation  $x_t = aE_t x_{t+1} + u_t$  has a unique solution when  $|a| < 1$ . In the case where  $|a| \geq 1$  there is an uncountable infinite number of solutions [Woodford (2003), p. 636–637]. When  $a = 1$ , the assumption that  $\lim_{T \rightarrow \infty} E_t x_{t+T} = 0$ , which provides a unique solution because it would work as if it were a transversality condition, is at odds with the economic theory that yields the Euler equation.

<sup>8</sup>Let us assume that the real interest rate gap follows an AR (1) process:  $r_{t+i} - \bar{r} = \phi(r_{t+i-1} - \bar{r}) + \xi_{t+i}|\phi| < 1$ . Thus:  $E_t(r_{t+i} - \bar{r}) = \phi^i(r_t - \bar{r})$ , and  $\lim_{T \rightarrow \infty} E_t x_{t+T} - x_t = \frac{\sigma}{1-\phi}(r_t - \bar{r}) \neq 0$ . Thus, the limit of the output gap change is different from zero, unless the real interest rate is equal to the natural rate.

level of aggregate demand, I put the expected change of output gap on the left hand-side:<sup>9</sup>

$$E_t x_{t+1} - x_t = \sigma (r_t - \bar{r}) \quad (8)$$

For instance, when  $\sigma = 1$ , the interest rate will not affect the current output gap, but will affect the expected future output gap. Specification (8) also stresses the fact that it is not possible to misinterpret the monetary policy channel in the new Keynesian model because the real interest rate affects the rate of change of the output gap and not its level, as the traditional Keynesian model would have it. Therefore, the NKISC is not forward, but backward looking. Even so, when the NKISC is inserted into a model with a new Keynesian Phillips curve and a Taylor rule, this model can have a unique locally bounded forward solution.<sup>10</sup>

### 3 NKHIS: Forward and Backward Solutions

When there is habit persistence, or habit formation, in consumption the NKHISC is specified according to:<sup>11</sup>

$$x_t = \omega E_t x_{t+1} + (1 - \omega) x_{t-1} - \sigma (r_t - \bar{r}). \quad (9)$$

This specification when  $0 < \omega < 1$  includes a backward-looking component ( $x_{t-1}$ ). This curve can be written as:

$$E_t \Delta x_{t+1} = \frac{1 - \omega}{\omega} \Delta x_t + \frac{\sigma}{\omega} (r_t - \bar{r}). \quad (10)$$

The solutions of this model depend on the size of the forward coefficient. When ,  $0 < \omega < 1/2, (\omega / (1 - \omega)) < 1$ , there is a unique bounded solution and I iterate forward (10) to obtain:

$$\Delta x_t = -\frac{\sigma}{1 - \omega} \sum_{t=0}^{\infty} \left( \frac{\omega}{1 - \omega} \right)^t E_t (r_{t+1} - \bar{r}). \quad (11)$$

Thus, the current change in the output gap depends on the current short-term interest rate gap as well as on the expected future short-term interest rate gaps. However, for the same reason as the NKISC curve, I rule out this solution because it is at odds with the underlying consumption theory. The solutions, in this case, are given by:

$$\Delta x_t = \frac{1 - \omega}{\omega} \Delta x_{t-1} + \frac{\sigma}{\omega} (r_{t-1} - \bar{r}) + \epsilon_t, \quad (12)$$

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<sup>9</sup>This specification also stresses the fact that both the current output gap and expected future output gap are endogenous variables that can be affected by the real interest rate. When the NKISC is written in continuous time its specification is:  $\dot{x} = \sigma(r - \bar{r})$ . See, for example, Werning (2011).

<sup>10</sup>See, for example, the new Keynesian models presented in Woodford (2003), Galí (2008) and Cochrane (2011).

<sup>11</sup>See, for example, Fuhrer and Rudebusch (2004) and Woodford (2003) habit persistence IS curve [equation (1.25), p. 333].

where  $\{\epsilon_t\}$  is any stochastic process such that  $E_{t-1}\epsilon_t = 0$ , and  $x_0$  is arbitrary.

When  $1 > \omega > 1/2$ ,  $((1 - \omega) / \omega) < 1$ , the NKHISC (10) can be iterated backward and I obtain:

$$\Delta x_t = \sum_{i=0}^{\infty} \left( \frac{1 - \omega}{\omega} \right)^i \left[ \frac{\sigma}{\omega} (r_{t-1-i} - \bar{r}) + \epsilon_{t-i} \right], \quad (13)$$

where  $\{\epsilon_t\}$  is any stochastic process such that  $E_{t-1}\epsilon_t = 0$ , and  $x_0$  is arbitrary. Thus, when the forward-looking coefficient is dominant ( $1 > \omega > 1/2$ ) the current output gap is related to the lagged interest rates and to the current as well as lagged values of an arbitrary stochastic variable, yielding multiple solutions.

## 4 Conclusion

The expectational difference equation of the new Keynesian IS curve has two solutions, one forward and the other backward looking, with multiple solutions. I show in this paper that the underlying economic theory rules out the forward solution.

The new Keynesian IS curve shows how the interest rate gap is related to the rate of change of the output gap and not to the level of the output gap. The correlation between the interest rate and the change in the output gap is positive. However, the correlation between the real interest rate gap and the output gap depends on the intertemporal elasticity of substitution ( $\sigma$ ). This correlation can be positive ( $\sigma < 1$ ), negative ( $\sigma > 1$ ) or zero ( $\sigma = 1$ ).

Based on the analysis of the solutions of the hybrid new Keynesian IS curve I arrive at the conclusion that this curve is backward looking regardless of the size of the forward coefficient.

The backward solution of the new Keynesian IS curve expectational difference equation implies that the current interest rate as well as the expected future real interest rates do not matter for the determination of current output gap. In the canonical new Keynesian Taylor-rule model, the central bank promises to make nominal interest higher when inflation is above its target. When the elasticity of substitution is less than or equal to one ( $\sigma \leq 1$ ) the higher interest rate does not bring about a reduced level of aggregate demand in the current period. Instead, it yields a higher level of aggregate demand in the current period and in the next period. Current inflation would increase due to a positive current output gap and a higher expected inflation, unless inflation jumps to a locally bounded equilibrium.<sup>12</sup>

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<sup>12</sup>When I combine the IS curve with continuous variables,  $\dot{x} = \sigma(r - \bar{r})$ , with the new Keynesian Phillips curve,  $\dot{\pi} = \rho\pi - \kappa x$ , after taking its derivatives with respect to time, I obtain  $\ddot{\pi} = \rho\dot{\pi} - \kappa\dot{x} = \rho\dot{\pi} - \kappa\sigma(r - \bar{r})$ . Thus, the rate of change of inflation is given by:  $\dot{\pi} = \kappa\sigma \int_t^{\infty} e^{-\rho(\tau-t)} (r - \bar{r}) d\tau$ . Due to price stickiness when the central bank raises the nominal rate of interest it increases the real rate

In the canonical new Keynesian model, monetary policy does not operate through the Keynesian interest rate channel. The central bank's promise of a higher interest rate reduces aggregate demand and inflation in the old Keynesian model, but it increases aggregate demand and inflation in the new Keynesian model.

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of interest, and the rate of inflation increases. This model feature is the end result of the fact that in the NKISC the rate of interest gap affects the rate of change of the output gap and not the output gap level, as in the traditional IS curve.



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