# Nonparametric Assessment of Hedge Fund Performance*

Caio Almeida [†] Kym Ardison [‡] René Garcia [§]

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**Abstract**

We propose a new class of performance measures for Hedge Fund (HF) returns based on a family of empirically identifiable stochastic discount factors (SDFs). These SDF-based measures incorporate no-arbitrage pricing restrictions and naturally embed information about higher-order mixed moments between HF and benchmark factors returns. We provide full asymptotic theory for our SDF estimators that allows us to test for the statistical significance of each fund's performance and for the relevance of individual benchmark factors in identifying each proposed measure. Empirically, we apply our methodology to a large panel of individual hedge fund returns, revealing sizable differences across performance measures implied by different exposures to higher-order mixed moments. Moreover, when we compare SDF-based measures to the traditional linear regression approach (Jensen's alpha), our measures identify a significantly smaller fraction of funds in the cross-section of HFs with statistically significant performances.

**Keywords**: Hedge Fund Performance, Non Parametric Estimation, Higher order Moments.

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[†] Email: calmeida@fgv.br, EPGE/FGV, Rio de Janeiro, Brazil.

[‡] Email: kym.ardison@fgvmail.br, EPGE/FGV, Rio de Janeiro, Brazil.

[§] Email: rene.garcia@umontreal.ca, Université de Montréal, Canada, and Toulouse School of Economics, France.

# 1 Introduction

Investment management and portfolio performance evaluation go hand in hand. Professional portfolio managers propose services to investors in the form of funds. Pension committees, endowments or retail investors need to assess whether the proposed funds achieve a superior performance to the portfolios they could form given their available information. In their seminal paper, Chen and Knez (1996) propose a general framework for evaluating the performance of a managed portfolio and specify a minimum set of conditions that any performance measure must satisfy[1]. While these conditions appear reasonable and intuitive and are obeyed by many well-known performance measures such as the Jensen's alpha or the Sharpe ratio, they are not restrictive enough to rank funds uniformly. Adding the condition that each admissible performance measure must be positive[2], this arbitrariness in the ranking disappears. Their article provides a framework to conduct performance evaluations independently of asset pricing models and applies it to the performance evaluation of mutual funds.

We propose a novel family of admissible positive measures that are particularly suited to evaluate the performance of hedge funds. Hedge fund returns distributions are distinctly non-normal and measures based on the mean and variance are not able to capture the risks reflected in their higher moments. Therefore we extend the nonparametric performance evaluation procedures proposed by Chen and Knez (1996) for mutual funds to address the specificities of hedge fund investing strategies. First and foremost, hedge funds can short assets and use derivatives to manage their portfolios, creating highly nonlinear payoffs. Our family of measures includes the Hansen and Jagannathan (1991) measures as a special case but adds a continuum of measures that incorporate information about moments of returns higher than the variance. From an investor perspective, our measures will give more weight to positive returns in states of nature where marginal utility is high (bad times), a feature that traditional performance measures fail to capture Kosowski (2011).

We minimize general convex functions of stochastic discount factors (SDF) called

[1]If the managed return, $R_i$, lies in the set of basis assets $R_K$, then it is given a zero performance ($\alpha_i = 0$). Furthermore, the performance function is linear, continuous and nontrivial (i.e., it is not the case that $\alpha_i = 0$ for all funds).

[2]Such a performance measure will exist if and only if the securities market offers no-arbitrage opportunities.

Minimum Discrepancy (MD) measures (Corcoran, 1998) in order to obtain a projected nonlinear SDF that prices exactly a set of selected reference assets (Almeida and Garcia, 2016). A well-known example of such discrepancy measures is the Kullback-Leibler information criteria (KLIC) or entropy[3]. Our family of discrepancy functions offers other information criteria that have different implications for assessing performance. Entropy considers a specific combination of SDF moments that gives similar weights to pairs of odd and even moments in the space of SDFs.[4] Therefore, allowing for more distinct weights across these two sets of moments (odd versus even) might be helpful to better identify and separate the effects of skewness (odd moments) from kurtosis (even moments) on pricing kernel dispersion.

The solutions for the nonlinear SDFs are obtained through dual portfolio problems that are easier to solve than the primal problems. The first-order conditions for these portfolio optimization problems provide solutions for the optimal weights of the chosen reference assets. We provide a complete estimation and inference theory for these weights and the resulting performance alpha, which measures the expected discounted value of the fund return under the nonlinear SDF. Given the small number of returns often found in individual hedge fund returns, we complement these asymptotic tests with bootstrap tests.

The implementation of the performance measurement tests involves the selection of the reference assets. We choose four sets of reference assets or risk factors that have been used in the literature on alpha measurement in hedge funds. The most popular set is the ten-factor model from Fung and Hsieh (2001), where risk factors including equities, bonds, credit, currencies, and commodities are considered together with several trend following strategies. We also include in our analysis a nine-factor model where we replace the Fung and Hsieh (2001) trend following strategies by the options strategies from Agarwal and Naik (2004). The Fama, French and Carhart four-factor model and the CRSP value-weighted market portfolio are added for comparison purposes and completeness.

We evaluate the performance of hedge funds reporting to the Lipper-Tass database. It is well-known in the finance literature that commercial hedge fund data suffers from severe biases (e.g. Fung and Hsieh (2001), Aggarwal and Jorion (2010), Patton et al. (2015),

[3]Stutzer (1996) used the same information theoretical approach based on the entropy measure to extract canonical probabilities, that is, risk-neutral probabilities that price consistently a set of options using as basis assets the underlying asset or adding to it other traded options.

[4]By a pair of moments we define two neighbors, like for instance, the third and fourth moments.

Bollen and Pool (2009) and Aiken et al. (2013) among several others). Traditionally, the literature focuses on two primary sources of bias: survivorship and backfill (e.g. Fung and Hsieh (2001) and Aggarwal and Jorion (2010) respectively). Recent articles have raised some additional issues. Straumann (2009) notes that several commercial databases (including the one we used in this paper) suffer from serious problems such as an alarming number of hedge funds with excessive zero returns, consecutive equal returns, and repeated "blocks" of returns[5]. To conduct our evaluation of performance, we apply the strictest filters that have been put forward to eliminate patterns that could bias our findings. As a by-product of our analysis we complement previous studies and show how neglecting these filters might affect one conclusion about the cross-sectional distribution of hedge fund performance.

In our empirical analysis, we will contrast our approach to the Jensen's alpha, which is the performance measure used in most of the literature. The second benchmark will be the Hansen and Jagannathan (1991) measure used in Chen and Knez (1996). Our family of measures will be indexed by a parameter $\gamma$ that will capture the curvature of the functions optimized in our dual portfolio problems. A Taylor expansion of our performance measure shows the intrinsic relationship between the fund's performance and the higher order mixed moments with respect to the benchmark assets. In particular, depending on the value of $\gamma$ more or less weight is given to, say, co-skewness and co-kurtosis. For values close to one (Hansen and Jagannathan case) the weights assigned to higher-order moments are negligible, as is the case of the Jensen's alpha. On the contrary, moving to the negative values ($\gamma < 0$), positive weights are assigned to co-skewness (consistent with Kraus and Litzenberger (1976) skewness preference) and negative weights are assigned to co-kurtosis (kurtosis aversion, Dittmar (2002)). Moreover, the relation between the performance measure and the higher-order moments is intrinsic to the method. Thus, our method is neither restricted to a single asset nor does it rely on imprecise estimates of higher-order moments.

We will summarize in broad strokes the main features of our empirical findings, leaving the rich and detailed analysis for the core of the text. A first important finding concerns the statistical significance of the reference assets in the sets we selected. A reassuring

[5] Bollen and Pool (2012) investigated these suspicious patterns in returns and showed that several of them are indicative of a heightened risk of fraud.

fact is that all investors[6] will want to hold a statistically significant positive share of the market, regardless of the set of factors we consider. With the Carhart (1997) set of factors, both the high-minus-low and the momentum factors are significant and all investors are long in these two factors in the dual portfolio. When we consider the Fung and Hsieh (2001) set of factors, the credit spread appears as an additional significant asset held positively in all portfolios, while the only significant trend following strategy is to short the stock lookback straddle. The results for the Agarwal and Naik (2004) model, where we switch the trend-following factors for the option portfolios, are somewhat consistent. The weights assigned to the S&P 500 increase substantially when the options are introduced in the estimation of the dual portfolios, and very interestingly all investors short out-of-the-money puts and buy the at-the-money ones.

Looking next at the categories of strategies traditionally used to group the funds according to their own reporting, we arrive at the following conclusions for our measures of alpha performance. First we look at the benchmarks used in Chen and Knez (1996), the Jensen's alpha (ordinary least squares regression on the factors) and the Hansen and Jagannathan alpha (based on a positive linear SDF that prices exactly the reference assets or factors). Even though the alphas are computed differently, their estimated values are practically the same for all categories of hedge funds. Alphas exhibit a much smaller variance than the mean return of the indexes (their range is from 0.20 to 0.45 as opposed to 0.47 to 0.93 for the mean return). There are more notable differences when we consider the other values of the curvature parameter $\gamma$, both between the various sets of reference assets and with respect to the linear benchmarks. For example, in the sets of Fung and Hsieh and Agawal and Naik reference assets, all estimates for the Jensen's alpha are positive and statistically significant. The estimated alphas with the different values of $\gamma$ reveal quite a different picture. For several categories such as CTA, fixed income arbitrage, global macro, managed futures and funds of funds, the estimate for very negative values of $\gamma$ is much lower than in the linear case and becomes often not statistically different from zero. These important differences at the aggregate level of fund categories are indicative of a strong heterogeneity in performance at the individual fund level.

A first important statistic is the percentage of individual funds with a positive and

[6] We use as light abuse of language by saying all for all the values of $\gamma$ over the large spread considered. Coefficients and t-stats exhibit monotonic patterns.

statistically different from zero performance[7]. While for the linear model the percentage is around 20 % for the various sets of factors, it falls considerably for all values of $\gamma$ to around 13%[8]. In terms of cross-sectional distributions, the estimated alphas differ considerably between the $\gamma$s and across the sets of reference assets. It means that, depending on the higher-order mixed moments of the individual hedge fund returns with the basis assets, significant differences in the alpha estimates may arise. We analyze the cross-section of alphas with respect to several variables in a linear regression setting. The most important explanatory variables beyond the hedge fund mean are the co-skewness and co-kurtosis with the market return, as well as the idiosyncratic volatility, the skewness and the kurtosis of the hedge fund returns.

We have already connected our paper to several strands of the literature on performance evaluation and on hedge funds data biases. We will conclude this introduction by mentioning several papers on the performance evaluation of hedge funds and how we do relate to them. Kosowski et al. (2007) use a robust bootstrap procedure to evaluate the performance of hedge funds. The bootstrap ensures that the performance measure is not produced by luck and accounts, together with the Bayesian estimation method, for the short-sample problem inherent in hedge fund returns. We also double our asymptotic tests with bootstrap tests that often increase the significance of the alpha tests. Capocci and Hubner (2004) investigate hedge funds performance using various asset pricing models, including the Carhart (1997) and the Agarwal and Naik (2004) models. We include these sets of factors in our analysis and extend their linear specification to our family of nonlinear positive SDFs. Hubner et al. (2015) put forward the key roles of US equity skewness and kurtosis in the hedge fund return generating process. We show that the cross-section of hedge fund alphas indeed strongly depends on the co-skewness and co-kurtosis with the market returns. Billio et al. (2013) stress that most measures of performance depend only on the first two moments of the hedge fund return distribution and propose utility-based measures that consider moments up to the fourth-order to measure performance. Our nonparametric approach does not depend on specifying utility models and endogenously captures all moments of the joint distribution of the benchmark factors.

Our paper is also intimately related to the SDF performance measurement literature

[7]We set the level of the test at 10%.

[8]This value considers, for both cases, Newey and West standard errors to compute the t-statistics.

(e.g. Beja (1971))[9]. More specifically, Farnsworth et al. (2002), Ferson and Mo (2016) and Li et al. (2016) propose several SDF-based performance measures. Our approach differs from theirs in four main aspects. First, our non-parametric SDFs naturally embed non-arbitrage features, providing a consistent measure for hedge fund performance evaluation. Second, our novel family of performance measurements expands the traditional linear SDF approach. In particular, our family includes the Hansen and Jagannathan estimator as a particular case, but also provides a continuum of possibilities with different degrees of non-linearities. Third, theoretically and empirically, our non-parametric SDFs have zero pricing errors with respect to the benchmark assets. Without this feature, hedge fund alphas may be driven purely by the mispricing of the underlying factors. Finally, our paper provides a full set of theoretical results linking the hedge fund performance with its higher-order mixed moments with the benchmarks.

The rest of the paper is organized as follows. In Section 2, we describe the theoretical properties of our measure of performance. Section 3 offers an interpretation and comparison of our performance measures with previously proposed alpha estimators, such as the Jensen's and the Hansen and Jagannathan estimators. Section 4 describes our estimation method and the asymptotic properties of our estimators. Section 5 describes our data set with a particular attention to the multiple filters we apply to the data to construct reliable measures of performance. Section 6 contains all our empirical findings. First, we describe the estimated measures for the various sets of reference assets. Then we report the performance measures and their asymptotic and bootstrap t-statistics both for aggregate categories of funds and individual funds. Section 7 concludes. For space considerations, we have placed all theorem and lemma proofs as well as many tables and figures (with numbers preceded by an A) in an online Appendix.

# 2 A Family of Nonparametric Performance Measures

In their seminal paper about performance measurement, Chen and Knez (1996) establish desirable conditions that performance measures should obey. Our approach builds on these conditions and expands them into two general sets of properties.

Throughout the paper, $\alpha_i$ will denote any generic performance measurement, $R_i$ the

[9] For a careful review of this literature see Ferson (2013)

time series of returns of an individual hedge fund $i$, and $R_k$ the time series of of returns of a benchmark portfolio $k$, with $k \in K$ ($K$ integer). We refer to $R_K$ as the set of benchmark portfolios, also called basis assets. Denote by $R_s = \{R_i \in L^2 : \exists \theta_k(s) \in \mathbb{R} \text{ s.t. } \sum_{k \in K} \theta_k(s) = 1 \text{ and } \sum_{k \in K} \theta_k(s) R_k = R_i\}$ the set of all achievable managed returns, where $S$ denotes information set available to investment managers and $s \in S$ one signal about the state of nature. We denote by $R_p$ the set of investments available to general public, that is $R_p$ is a subset of $R_s$ where $\theta_k(s)$ does not depend on $s$ (the general public does not have access to signals).

**Condition 1.** If the managed return, $R_i$, lies in the span of the basis assets $R_K$ ($R_i \in R_p$), then it is given a zero performance ($\alpha_i = 0$). Furthermore, the performance function is linear, continuous and nontrivial (i.e., it is not the case that $\alpha_i = 0$ for all funds).

The above condition summarizes conditions (1) to (4) in Chen and Knez (1996). Although reasonable, condition 1 is too flexible. Without restricting it any further, performance measurements might have arbitrary values and undesirable properties. Both Chen and Knez (1996) and Goetzmann et al. (2006) discuss issues with the traditional Jensen's alpha (which obeys condition 1) and several other performance measures (e.g. Sharpe Ratio) that satisfy the above properties. Hence, we need a more restrictive condition, also introduced in Chen and Knez (1996).

**Condition 2.** The performance function is positive, i.e. $\alpha_i \geq 0$ whenever $R_i \geq 0$ and $\alpha_i > 0$ whenever $R_i > 0$.

Condition 2 introduces a fairly restrictive property that rules out several performance measurements (e.g., Chen and Knez show that this condition is not satisfied by the Jensen's Alpha). Nevertheless, even this restriction can be shown to be too loose. For instance, Theorem 4 in Chen and Knez (1996) states that if $R_i$ is not in the span of $R_K$ then for any value $\upsilon$ there is a performance measure, in the SDF representation, such that $\alpha_i = \upsilon$.

Notwithstanding these negative results, Chen and Knez (1996) showed that any performance measure that satisfies the above conditions can be cast in the stochastic discount factor framework. That is, for any performance measure that satisfies these conditions there exists an SDF, say $m$, such that $\alpha_i = E[mR_i - 1]$. Thus, Chen and Knez (1996)

provide some guidance for future attempts to develop new measures of performance. The challenge is then to restrict the set of admissible SDFs ($m$) in a meaningful way.

We approach the problem of developing a new class of performance measurements by introducing two restrictions on the admissible SDFs. First, we propose to fix the mean of the SDF (to 1 for simplicity and without loss of generality[10]). Second, we focus on SDFs that satisfy non-arbitrage restrictions. By fixing the SDF mean, we can restrict the range of possible outcomes for the non-parametric alpha performance. The non-arbitrage restrictions allow us to compute performance measures that are more meaningful for hedge funds, which typically trade derivative contracts (see Li et al. (2010)).

**Theorem 1.** Let $R_K$ denote the matrix of basis assets and $\Pi = \{m : E[mR_k] = 1 \text{ for all } k \in K, E[m] = 1 \text{ and } m \geq 0\}$. Here $\Pi$ defines the set of all admissible non-negative SDFs in this economy. For all hedge funds with return $R_i$ we have that:

$$inf(r_i) \leq \alpha_i^m \leq sup(r_i) \tag{1}$$

where $r_i = R_i - 1$ is the net return of the hedge fund and $\alpha_i^m$ the performance measure associated with the stochastic discount factor $m$ .

See proof in the Appendix.

Theoretically, our results are quite similar to Chen and Knez (1996) but empirically they imply more positive results: for all non-negative SDFs with mean equal to one, the alpha performance is bounded below by $min(\tilde{R}_{i,t})$ and above by $max(\tilde{R}_{i,t})$. We must highlight that to prove the preceding theorem we rely on non-arbitrage restrictions ($m \geq 0$). This restriction also buy us several good properties: (i) If $R_i >> R_k$ for some $k \in K$ then $\alpha_i^m > 0$ for all $m$. The reciprocal is true for $R_i << R_k$. (ii) $\alpha_i^m$ is strictly increasing in $R_i$. Thus, if one fund return dominates another, any two SDFs will rank both funds in the same way (proofs are in Chen and Knez (1996)). Furthermore, using (i), we note that the derived bounds can be tightened. In particular, for non-arbitrage SDFs we know that if $R_i >> R_k$ then $\alpha_i^m > 0$ for all $m$ and the converse is also true.

A final point with respect to Chen and Knez (1996) is worth highlighting. Empirically, they focus their applications on the Hansen and Jagannathan estimator for a subset

[10]Note that the result holds with a minor change if the mean of the SDF does not equal one. In this case, the bounds are given by the discounted returns. I.e. equation (2) takes to following form: $E[m\tilde{R}_i] = E^*[\tilde{R}_i]R_f$.

of mutual fund returns. Below we expand this setting and provide a broader set of SDFs. We show that both theoretically and empirically how the choice of the Hansen and Jagannathan measure affects performance measurement. In particular, given the high nonlinearities of both hedge fund returns and standard benchmarks (e.g. Fung and Hsieh (2001)), we show that the additional flexibility of our novel family of measures embed several desired properties.

We now formally introduce our family of performance measures. We are looking for admissible risk-adjustment weights, also known as stochastic discount factors (SDFs), that make the average weighted excess returns on the factors equal to zero:

$$\frac{1}{T}\sum_{t=1}^{T} m_t \left(R_t - 1_K\right) = 0_K. \tag{2}$$

where $0_K$ is a K-dimensional vector of zeros and $R_t$ is a vector of realizations of $R_K$, the basis assets.

Of course in an incomplete market setting where the law of one price is satisfied, there are many such weighting functions (see Cochrane (2001)). Therefore, we need to restrict the set of such admissible SDFs. Hansen and Jagannathan (1991) find an admissible linear SDF with minimum variance, obtained by minimizing a quadratic function in the space of admissible SDFs. This approach will be similar to the usual regression approach to compute the alpha of hedge funds, except that it imposes a zero pricing error on the factors. Instead, assuming absence of arbitrage on the market defined by the primitive risk-factors[11], we consider a convex discrepancy function $\phi$ and we search for a Minimum Discrepancy (MD) SDF that solves the following minimization problem in the more restricted space of admissible strictly positive SDFs:

$$\begin{gathered}\hat{m}_{MD} = \underset{\{m_1,\dots m_T\}}{arg\ min}\ \tfrac{1}{T}\textstyle\sum_{t=1}^{T}\phi(m_t),\\ \text{subject to } \tfrac{1}{T}\textstyle\sum_{t=1}^{T} m_t\left(R_t - 1_K\right) = 0_K,\ \tfrac{1}{T}\textstyle\sum_{t=1}^{T} m_t = 1, m_t > 0\ \forall t.\end{gathered} \tag{3}$$

In this optimization problem, restrictions to the space of admissible SDFs come directly from the general discrepancy function $\phi$. The conditions $\sum_{t=1}^{T} m_i\left(R_t - 1_K\right) = 0_K$

[11]In fact, absence of arbitrage on the market defined by the primitive risk-factors guarantees the existence of at least one admissible SDF that is strictly positive (see Cochrane (2001)).

and $\frac{1}{T}\sum_{t=1}^{T} m_t = 1$ must be obeyed by any admissible SDF $m$ with mean 1. In addition, we explicitly impose a positivity constraint to guarantee that the implied MD SDF is compatible with absence of arbitrage in an extended economy containing any derivatives of primitive risk factors (Chen and Knez, 1996).

This minimization problem is based on the space of discrete SDFs with dimension T (the dimension of the sample of data), which can become impractical. According to Borwein and Lewis (1991), the minimization problem can be solved in a generally much smaller dimensional space by using the following dual problem:

$$\hat{\lambda} = \underset{\alpha\in\Re,\lambda\in\Lambda}{arg\ sup}\ \alpha - \sum_{t=1}^{T} \frac{1}{T}\phi^{*,+}\left(\alpha + \lambda'\left(R_t - 1_K\right)\right), \tag{4}$$

where $\Lambda \subseteq R^K$ and $\phi^{*,+}$ denotes the convex conjugate of $\phi$ restricted to the positive real line:

$$\phi^{*+}(z) = \underset{w>0}{sup}\ zw - \phi(w). \tag{5}$$

Note that any convex discrepancy function can be chosen to arrive at empirical estimates of these minimum discrepancy SDFs. Following Almeida and Garcia (2016), we adopt the Cressie-Read (1984) family of discrepancies defined as:

$$\phi^{\gamma}(m) = \frac{(m)^{\gamma+1} - 1}{\gamma(\gamma+1)}, \gamma \in \Re, \tag{6}$$

where each fixed value of $\gamma$ implies one specific discrepancy function. Almeida and Garcia (2016) shows how to solve the dual problem and recover the admissible SDF from the optimization problem above. Here we synthesize their results in Corollary 1, which will be needed for us to develop the full asymptotic framework for our performance measure.

**Corollary 1.** Let $\phi(m)$ be as in 6, and assume that there is no-arbitrage in the economy (such that there exists at least one strictly positive admissible SDF)[12]. Let $\Lambda(R) = \{\lambda \in \mathbb{R}^K : (1 + \gamma\lambda'(R-1)) > 0\}$:

(i) If $\gamma > 0$ the dual problem specializes to:

$$\lambda^* = \underset{\lambda\in\mathbb{R}^K}{\arg\sup}\ E\left[-\frac{1}{\gamma+1}\left(1+\gamma\lambda'(R-1)\right)^{\frac{\gamma+1}{\gamma}} I_{\Lambda(R)}(R)\right] \tag{7}$$

[12] To ease the notation we specify Almeida and Garcia (2016) corollary for the case where the SDF mean equals one.

where $I_A(x) = 1$ if $x \in A$ and 0 otherwise.

(ii) If $\gamma < 0$ the dual problem specializes to:

$$\lambda^* = \arg\sup_{\lambda \in \mathbb{R}^K} E\left[-\frac{1}{\gamma+1}\left(1+\gamma\lambda'(R-1)\right)^{\frac{\gamma+1}{\gamma}} - \delta(\lambda|\Lambda(R))\right] \tag{8}$$

where $\delta(x|A) = 0$ if $x \in A$ and $+\infty$ otherwise.

(iii) If $\gamma = 0$ the maximization is unconstrained:

$$\lambda^* = \arg\sup_{\lambda \in \mathbb{R}^K} E\left[exp\left(\lambda'(R-1)\right)\right] \tag{9}$$

Note that for each choice of $\gamma$ we obtain a distinct set of estimates for $\lambda$ ($\hat{\lambda}_\gamma$) that will lead to a different MD stochastic discount factor ($\hat{m}^\gamma$). The MD SDF $\hat{m}^\gamma$ is recovered by solving the first-order conditions of the problem and is know in close form:

**Corollary 2.** Let $\hat{\lambda}_\gamma$ be the solution of the dual problem above. Give the above mentioned regularity conditions the MD implied SDF will be given by:

(i) If $\gamma > 0$:

$$\hat{m}_t^\gamma = \left(1+\gamma\hat{\lambda}_\gamma'\left(R_t - 1_K\right)\right)^{\frac{1}{\gamma}} I_{\Lambda(R)}(\hat{\lambda}_\gamma) \tag{10}$$

(ii) If $\gamma < 0$:

$$\hat{m}_t^\gamma = \left(1+\gamma\hat{\lambda}_\gamma'\left(R_t - 1_K\right)\right)^{\frac{1}{\gamma}} \tag{11}$$

(iii) If $\gamma = 0$:

$$\hat{m}_t^\gamma = exp\left(\hat{\lambda}_0(R-1)\right) \tag{12}$$

Concluding this section, following the Chen and Knez framework, we have that the sample estimate of the performance of a particular fund $i$, $\alpha_i^\gamma$, is then obtained by:

$$\alpha_i^\gamma = \frac{1}{T}\sum_{t=1}^{T}[m_t^\gamma R_{it} - 1] \tag{13}$$

In the next section we explain how to estimate the vector of parameters $\lambda^{\gamma}$ and derive the asymptotic properties of the estimator. We also derive the asymptotic variance of the $\alpha^{\gamma}$ to test whether the performance is significantly different from zero.

# 3 Interpretation and Comparison of our Performance Measures

We have insisted on the fact that our family of nonlinear, nonparametric SDFs is better able to risk-adjust the risk embedded in hedge fund returns. In this section, we characterize the theoretical properties of the alphas obtained with the various members of the family and compare them with the other available performance measures. Our approach relies on a Taylor expansion that links the performance measure with the higher-order mixed moments between the hedge fund returns and a particular linear combination of the benchmark assets.

To simplify the notation, from this point onwards, we denote by $R$ a $K$-dimensional vector in the space of basis assets returns. Whenever necessary we will use the notation $R_k$ to indicate a specific column vector of $R$ (i.e. time series of returns for the benchmark asset $k$) and $R_t$ to denote a column vector containing the realizations, at time $t$, of all $K$ assets returns.

## 3.1 Linking our Performance Measures with Higher-Order Moments

We start by defining an auxiliary function $g_{m,i,t}$, which gives the risk-neutralized returns at each date $t$ as follows:

$$g_{m,i,t}(\lambda' R_t) = (1 + \gamma\lambda' R_t)^{1/\gamma} R_{i,t} \tag{14}$$

To see how the alpha performance will vary with the parameter $\gamma$ and the co-moments of the benchmark returns with the fund returns, we Taylor expand $g_{m,i,t}(\lambda' R_t)$ around $E[\lambda' R_t]$. Note that $E[-\lambda' R_t]$ can be interpreted as the unconditional return of the optimal portfolio in the dual representation problem[13]. Taylor expanding the auxiliary function

[13]Almeida and Garcia (2012) and Almeida and Garcia (2016) showed how to link out non-parametric

and taking expectations we have that:

$$
\begin{aligned}
E[g_{m,i,t}(\lambda' R_t)] = {} & E[g_{m,i}(E[\lambda' R_t])] \\
& - (1+\gamma E[\lambda' R_t])^{\frac{1-\gamma}{\gamma}} E[R_{i,t}(-\lambda' R_t + E[\lambda' R_t])] \\
& + \frac{1}{2}(1-\gamma)(1+\gamma E[\lambda' R_t])^{\frac{1-2\gamma}{\gamma}} E[R_{i,t}(-\lambda' R_t + E[\lambda' R_t])^2] \\
& - \frac{1}{6}(1-\gamma)(1-2\gamma)(1+\gamma E[\lambda' R_t])^{\frac{1-3\gamma}{\gamma}} E[R_{i,t}(-\lambda' R_t + E[\lambda' R_t])^3] + \dots
\end{aligned} \tag{15}
$$

The above expansion reveals several noteworthy features. First, the alpha of a fund is shown to explicitly depend on the relationship between its return and the higher-order mixed moments with the benchmark assets. In particular, the expansion indicates that the alpha is naturally linked to the co-skewness and co-kurtosis between the fund returns and the optimal portfolio $-\lambda' R_t$ [14].

Ever since Kraus and Litzenberger (1976) a significant strand of the finance literature focused on understanding the implications of higher-order moments for asset pricing. In particular, Kimball (1993), Dittmar (2002), among others, link investors' utility with their preferences for skewness and kurtosis. The all-important paper by Harvey and Siddique (2000) analyzes empirically the implications of systematic skewness and kurtosis for asset pricing[15]. In the hedge fund literature, Ranaldo and Favre (2003) show that higher-order moments are relevant for characterizing the returns of some funds on top to the market returns. In a related work, Billio et al. (2013) proposes a new performance measure that takes into account the hedge fund skewness and kurtosis.

In contrast to this literature, our performance measure embeds information from all higher-order **mixed moments** with respect to the benchmark assets. In particular, two features are worth highlighting. First, our performance measure naturally embeds information about higher-order risks without estimating the risks themselves. This is a key feature since we avoid two main statistical problems – introducing too many parameters

approach with the portfolio optimization problem from a representative agent with HARA utility function. A key feature of our analysis here is that the portfolio holdings of the representative agent are proportional to $-\lambda$.

[14] To be more precise, to calculate the co-skewness and co-kurtosis, we would need to re-parameterize the relations to recover the correct formulas for these objects.

[15] A number of papers further investigated the properties of skewness, kurtosis, and tail events in different settings (e.g. Chang et al. (2013) focus on market skewness and the cross-section of stock returns).

and facing the imprecise estimation of higher-order moments. Second, our measure contains information about higher-order risks related to a meaningful portfolio: the linear portfolio formed using all relevant benchmark assets. Note that the typical Harvey and Siddique (2000) approach becomes rapidly unfeasible in the hedge fund context since too many co-skewness and co-kurtosis statistics will have to be estimated for the numerous risk factors used to characterize returns[16].

Further exploring each of the terms multiplying the mixed moments in (15) and noting that $E[g_{m,i}(E[\lambda' R])]$ can be rewritten as $(1+\gamma E[\lambda' R_t^B])^{\frac{1}{\gamma}} E[R_i]$, we conclude that the hedge fund alpha is linearly related to its average return and that this average is re-scaled by the SDF evaluated at the mean value of the endogenous portfolio return. From the Jensen's inequality applied to the function $(\cdot)^{\frac{1}{\gamma}}$, we note that for all $\gamma < 1$ (convex function), $1 = E[m] \geq (1+\gamma E[\lambda' R_t^B])^{\frac{1}{\gamma}}$, which in turn implies that the hedge fund average $E[R_i]$ is pinned down in the alpha computation. The opposite effect happens for $\gamma > 1$. The linearity of the Hansen-Jagannathan SDF implies that the corresponding weight is exactly equal to one.

Looking at the weights assigned to the higher-order mixed moments, we note that they are linked both to the Cressie-Read parameter $\gamma$ as well as to the optimal $\lambda$. From the SDF estimation, we have that $1 + \gamma\lambda' R_t$ is nonnegative. This implies that, $1 + \gamma E[\lambda' R_t]$ is also non-negative. Therefore, the signs of the weights are determined only by $\gamma$. In particular, $\gamma > 1$ ($\gamma < 1$) implies a negative (positive) weight for co-skewness. As usual, when $\gamma = 1$ the estimator reduces to the Hansen and Jagannathan case, and all higher-order mixed moments are disregarded. Furthermore, the signs associated with weights on co-kurtosis are provided by a deterministic quadratic term on $\gamma$. Therefore, except for the region where $\gamma \in \{0.5, 1\}$ the weights on co-kurtosis are all negative.

To give an idea of the behavior of these weights under the different settings we adopt in the paper, Figure 1 plots the estimated weights for five different sets of basis assets: (i) the CRSP value-weighted market portfolio, (ii) the Fama, French and Carhart four-factor model, (iii) the Fung and Hsieh underlying factors (the five factors that generate the trend-following strategies), (iv) the Fung and Hsieh ten factor model, and (v) the five underlying factors from Fung and Hsieh with the addition of the option portfolios from Agarwal and Naik. To compute this figure we use the full sample for the underlying

[16]In the empirical section we replicate Ranaldo and Favre (2003) results and further discuss issues related to the linear approach of Harvey and Siddique (2000) in the hedge fund performance setting.

factors and a grid of $\gamma = \{-4, -3.5, \ldots, 4\}$ with 0.5 increments. The figure contains four panels, each indicating the weights for the hedge-fund average return, covariance, co-skewness, and co-kurtosis respectively.

Starting with the weights on the hedge-fund average return, Figure 1 exemplifies the effects of Jensen's inequality. Note that for all $\gamma's \neq 1$ we have a significant difference in weights depending on the basis assets. In particular, when $\gamma < 0$ we note that the introduction of the trend-following factors/option factors significantly reduces the weights on the average hedge fund return. In aggregate, this reduction can be as high as 15%. Additionally, the plot is steepest between $-2 < \gamma < 2$, revealing that the biggest gains of the non-linearities occur in this region[17]. The same analysis holds when $\gamma > 1$. However, in this case, the Jensen's inequality works the other way around: the alphas are augmented in comparison with the average hedge fund return.

The relationship between the hedge-fund alpha and the covariance with the basis assets is also interesting. Here we note that, across all values of $\gamma$, the weights assigned to this moment are monotonic. In particular, for all negative values this weight is higher than minus one and somewhat stable across different sets of basis assets. The picture changes radically for positive values: in this case, the introduction of the trend following/option factors significantly amplify the weights.

Finally, the weights on co-skewness and co-kurtosis behave as expected: (i) both are zero for the Hansen and Jagannathan case, (ii) for co-skewness (co-kurtosis) they are a decreasing (parabolic) function, and (iii) introducing the trend-following/option factors significantly distort these weights for all values of $\gamma$, with a higher effect for values greater than 1. More importantly, for $\gamma < 1$ the signs of the weights agree with the theory: investors like skewness and dislike kurtosis.

## 3.2 The Relationship with the Jensen's Alpha

The Taylor expansion above revealed that the Cressie-Read alphas load both on the hedge fund mean and covariance with the benchmark assets, as the Jensen's alpha, but also on the higher-order mixed moments. In this section, we test statistically the contribution of these higher-order moments to the alpha determination. As a by-product of our tests, we are also able to provide some guidance on whether there is evidence of

[17] In other words, the effects of the non-linearities are similar for the values of $\gamma$ smaller than two.

disagreement between investors.

From the definition of the Cressie-Read alpha, we have that $\alpha^\gamma = E[m^\gamma(R_i - 1)]$. Now suppose we model $R_i - 1$ linearly on the benchmark assets, as in the OLS framework. We have that $R_i - 1 = \alpha + \sum_{k=1}^{K} \beta_k R_k + \epsilon_i$. Note that by construction $E[\epsilon_i] = 0$ and $\epsilon_i$ is uncorrelated with $R_k$ for all $k \in K$. Therefore we can decompose the Cressie-Read alpha into two terms, the regular OLS alpha plus the covariance between the OLS residuals and the estimated stochastic discount factor, that is $\alpha^\gamma = \alpha + E[m_\gamma \epsilon_i]$. Thus, any difference between the two estimates is due only to the $E[m^\gamma \epsilon_i]$, i.e., the correlation between the OLS residuals and the estimated Cressie-Read SDF.

Clearly, if the SDF is linear in the underlying factors, as it is the case for the strictly positive Hansen-Jagannathan case, this term will be zero. Thus, theoretically, the Hansen-Jagannathan estimator will only differ from the OLS one if the indicator function in corollary 1 is "active". As it will be clear in the empirical section, even in these cases it turns out that the contribution of the $E[m^{\gamma=1}\epsilon_i]$ is small.

By construction $E[\epsilon_i] = 0$ and $\epsilon_i$ is uncorrelated with $R_k$ for all $k \in K$. Therefore, the first two terms in the Taylor expansion for $E[m^\gamma \epsilon_i]$ are equal to zero. This is a direct consequence of $E[\epsilon_i] = 0$ for the first term and the orthogonality condition for the second one. This feature implies that by calculating $\alpha^\gamma - \alpha$ we can isolate, for each value of $\gamma$, the contribution of the higher-order moments for the Cressie-Read alpha in comparison to the OLS counterpart[18].

Nonetheless, by testing whether the OLS alpha equals the Cressie-Read alpha we are able to: (i) provide direct statistical evidence on investor disagreement, and (ii) evaluate if there is any information in $\epsilon_i$ captured by the Cressie-Read alpha and neglected by the OLS counterpart.

## 3.3 A Comparison with the Hansen-Jagannathan Estimator

In the previous sections, we discussed the theoretical properties of the Cressie-Read alpha and their differences with the conventional OLS counterpart. Chen and Knez (1996) proposes an empirical application relying on the Hansen and Jagannathan (1991)

[18]Note that this is a comparison between the two alphas, and does not reflect the true contribution of the higher-order moments for the Cressie-Read alpha determination. This is a consequence of the fact that the OLS alpha does not equal the first two terms of the Taylor expansion for $E[m^\gamma R_i]$. More precisely, the higher-order mixed moments of $\epsilon_i$ with $\lambda' R_k$ are not equal to the higher-order mixed moments of $R_i$ and $\lambda' R_k$.

stochastic discount factor to evaluate mutual fund performance. In this section, we illustrate empirically the differences between the Cressie-Read risk neutralization and that implied by Hansen and Jagannathan (1991) estimator. We tailor our examples to illustrate the effects of higher-order moments on both the stochastic discount factor estimation and the performance measurement.

To this end, we borrow from Almeida et al. (2017) and replicate their analysis here using the full sample of Fung and Hsieh (2001) factors to estimate the Cressie-Read implied SDFs. Figure 2 plots the estimated risk neutral densities for three values of $\gamma$, $-3.5, -0.5, 1$. These choices of parameters parallel Almeida et al. (2017) ones. Basically, $\gamma = 1$ collapse to the Hansen and Jagannathan estimator and $\gamma = -0.5$ is the Hellinger estimator. We also select $\gamma = -3.5$ because this estimator gives substantial weight to skewness and kurtosis as seen in the Taylor expansion. Overall, the results in this picture are quite instructive: while the Hansen and Jagannathan estimator is linear in the returns both Hellinger and $\gamma = -3.5$ generates non-trivial hyperbolic nonlinearities [19]. Aditionally, the risk neutral density for the negative gammas puts significant more weight on adverse outcomes of the portfolio. This property translates to an important feature when measuring performance: for negative values of gamma, hedge funds performance can be significantly increased (penalized) if the fund generate returns that are positively (negatively) correlated with the SDF.

We now address the effect on the non-linearities implied by the different Cressie-Read estimators on the hedge fund alphas. To keep the illustration as simple as possible, we consider a one-factor model where the only factor is the market return (CAPM), simulated from a Student-t distribution. We simulate two hedge fund returns from different distributions (Normal and Student-t), but we maintain the same mean, variance and covariance with the market return for both funds. Therefore, by construction, each hedge fund is assigned the same OLS alpha. Therefore, it is quite clear that the linear model is not able to capture the higher-order moments of the hedge fund distributions. Moreover, the addition of non-linear terms, e.g. $R_M^2$, only partially mitigate this caveat (this will become quite clear in the empirical section 6, where we explicitly consider this possibility). In general, as long as the correlation structure of two hedge fund returns and the factor in the model remain equal, the estimated alphas for the hedge funds are identical[20].

[19]Note that when we refer to returns, we mean the endogenous portfolio returns, i.e., $\lambda' R$

[20]Note that, even in more general linear factor models, if the correlation structure between the two

We apply the non-parametric methodology of the previous sections for a grid of $\gamma's$ from $[-5, 3]$ with 0.5 increments tot the simulated data. The point-wise estimated alphas and the bootstrapped confidence intervals (discussed further in section 4.4) are plotted in Figure 3. The solid line and the dashed line feature the point-wise alphas when hedge fund returns follow a normal and a Student-t distribution respectively. The alpha estimates for negative values of gamma differ substantially between the two hedge funds. For extreme negative values of $\gamma$ the higher-order moments from the Student-t distribution play a significant role. For example, when $\gamma = -5$ the difference in estimated alphas is almost 0.4, considering that the hedge funds were simulated with a mean equal to 0.2. Furthermore, the estimated alphas vary considerably across the values of $\gamma$ indicating the difference in risk compensation across SDFs. As expected, when $\gamma = 1$, the Hansen and Jagannathan case, both hedge funds are assigned the same performance. This is consistent with the fact that, if non-arbitrage conditions hold in-sample, then the Hansen and Jagannathan estimator collapse to a linear combination of the basis assets (and produces the same alpha as the OLS estimator). Note that this is not the case if this condition is violated. In fact, as Chen and Knez (1996) pointed out, this feature results from the imposed positivity constraint in the stochastic discount factor.

# 4 Estimation and Tests of the Performance Measures

Our measures are based on the computation of the Lagrange multipliers $\lambda_\gamma$ associated with the dual optimization problem in Corollary 1. In this section, we develop a consistent estimator for these parameters and derive their asymptotic distribution. We then infer the distribution of the implied performance measures $\alpha_i^\gamma$ in (13) based on the SDFs defined in Corollary 2 and computed with the estimated $\lambda_\gamma$.

## 4.1 Definitions

In order to derive the estimator of the Lagrange multipliers $\hat{\lambda}_T$ and its asymptotic properties, we need to define functions that will enter in the objective function to be maximized (18), as well as their first and second derivatives. For each fixed $\gamma$ in the

hedge funds and the underlying factors are the same then it must be the case that they have identical alphas (considering that the hedge funds have the same mean and standard deviation).

family of Cressie-Read discrepancies, let us define the following functions:

$$f^{\gamma}(\lambda, R) = \frac{(1+\gamma\lambda' R)^{\frac{\gamma+1}{\gamma}}}{\gamma+1} \tag{16}$$

$$\mathfrak{M}^{\gamma}(\lambda, R) = E[f^{\gamma}(\lambda, R)] \tag{17}$$

with its in-sample version, $\mathfrak{M}_T^{\gamma}(\lambda, R)$ :

$$\mathfrak{M}_T^{\gamma}(\lambda, R) = \sum_{t=1}^{T} \frac{1}{T} \left[ \frac{(1+\gamma\lambda' R_t)^{\frac{\gamma+1}{\gamma}}}{\gamma+1} \right] \tag{18}$$

To prove the asymptotic normality of the estimator we will need the Hessian and the Fisher Information matrix (outer product of the scores), that we denote $H_{\lambda\lambda}(\lambda, R)$ and $S_{\lambda}(\lambda, R)$ respectively, and which are based on the first and second derivatives of the above-defined functions. The first derivative of the $f^{\gamma}(\lambda, R)$ function is given by:

$$\begin{aligned} h^{\gamma}(\lambda, R) &= \frac{\partial}{\partial \lambda'} f^{\gamma}(\lambda, R) \\ &= (1+\gamma\lambda' R)^{\frac{1}{\gamma}} R, \end{aligned} \tag{19}$$

from which we can calculate the score vector:

$$\begin{aligned} \frac{\partial \mathfrak{M}_T^{\gamma}(\lambda, R)}{\partial \lambda'} &= \frac{1}{T} \sum_{t=1}^{T} h^{\gamma}(\lambda, R_t) \\ &= \frac{1}{T} \sum_{t=1}^{T} (1+\gamma\lambda' R_t)^{\frac{1}{\gamma}} R_t \end{aligned} \tag{20}$$

The expectation of the cross-product of the score vector by its transpose will provide the matrix $S_{\lambda}$. The elements of the $H_{\lambda\lambda,T}$ are given by:

$$\begin{aligned} H_{\lambda\lambda,T} &= \frac{\partial^2 \mathfrak{M}_T^{\gamma}(\lambda, R)}{\partial \lambda' \partial \lambda'} \\ &= \frac{1}{T} \sum_{t=1}^{T} R_t R_t' (1+\gamma\lambda' R_t)^{\frac{1-\gamma}{\gamma}} \end{aligned} \tag{21}$$

To characterize the estimators with the non-negativity restriction on the SDF, we will modify the notation as follows. Let $f^{\gamma+}(\lambda, R)$ denote the restricted version of $f^{\gamma}(\lambda, R)$.

$$f^{\gamma+}(\lambda, R) = \left[ \frac{(1+\gamma\lambda' R)^{\frac{\gamma+1}{\gamma}}}{\gamma+1} \right] I_{\{(1+\lambda'(R-1)\geq 0\}} \tag{22}$$

Similarly, we redefine $\mathfrak{M}^{\gamma}(\lambda, R)$ for the modified $f$ as:

$$\mathfrak{M}^{\gamma+}(\lambda, R) = E[f^{\gamma+}(\lambda, R)] \tag{23}$$

From equation 23, we can define its in-sample version, $\mathfrak{M}_T^{\gamma+}(\lambda, R)$ as follows:

$$\mathfrak{M}_T^{\gamma+}(\lambda, R) = \sum_{t=1}^{T} \frac{1}{T} \left\{ \left[ \frac{(1+\gamma\lambda' R_t)^{\frac{\gamma+1}{\gamma}}}{\gamma+1} \right] I_{\{(1+\lambda'(R_t - 1) \geq 0\}} \right\} \tag{24}$$

Let $H_{\lambda\lambda}^{+}(\lambda, R)$ and $S_{\lambda}^{+}(\lambda, R)$ denote the restricted Hessian and Fisher Information matrices respectively[21].

## 4.2 Estimator of the Lagrange Multiplier Vector and its Asymptotic Properties

Almeida and Garcia (2012) use Cressie-Read discrepancies to estimate parametric models under the assumption of misspecification. Their procedure comprised the joint estimation of the parameters of the models and of the Lagrange multipliers associated with the dual problem. The proved the consistency of the estimator for the whole parameter vector and derived its asymptotic distribution. Our problem is simpler since we have to estimate only the Lagrange multiplier vector. Therefore we simplify their approach to address the more specific estimation problem.

First we need to state two assumptions in order to establish the convergence in probability of our estimator (Theorem 2) and derive its asymptotic distribution (Theorem 3).

**Assumption 1.** We assume that $\lambda^*$ is the unique solution to the following problem:

$$\lambda^* = \underset{\lambda \in \mathbb{R}^K}{\operatorname{argmin}}\ \mathfrak{M}^{\gamma}(\lambda, R) \tag{25}$$

Assumption 1 guarantees that the problem is well specified. This hypothesis parallels Assumption (1.A) in Newey and Smith (2004). Under the assumption that $E[RR']$ is non-singular, the Hessian of $-\mathfrak{M}^{\gamma}$ is non-singular and negative definite. Thus, $-\mathfrak{M}^{\gamma}$ is

[21]Note that for the points where the derivative is defined, the Fisher Information and Hessian matrices will coincide with the above-defined unconstrained ones.

strictly concave in $\lambda$. This, coupled with the fact that $\lambda \in \mathbb{R}^K$ is sufficient to guarantee the uniqueness of $\lambda^*$ [22].

Using the notation defined in the previous section, the sample version of the dual parameters $\hat{\lambda}_T$ can be estimated as follows:

$$\hat{\lambda}_T = \underset{\lambda \in \mathbb{R}^K}{\operatorname{argmin}} \, \mathfrak{M}_T^{\gamma}(\lambda, R) \tag{26}$$

**Assumption 2.**

(A) The process $R_t$ is stationary and ergodic.

(B) The process $R_t$ is strongly mixing with mixing coefficients $\alpha_t$ satisfying $\sum_{t=1}^{\infty} \alpha_n^{1-1/b} < \infty$ and $b > 1$.

(C) $E[RR^T]$ is non-singular.

(D) $Var\left(\sqrt{T}\left[\frac{\partial}{\partial\lambda}\mathfrak{M}_T^{\gamma}(\lambda^*, R)\right]\right) \to_p S_\lambda > 0$ when $T \to \infty$.

(E) For a sufficient small $\delta > 0$, $-E[\sup_{\lambda \in \Gamma(\lambda^*, \delta)} \|h^{\gamma}(\lambda, R)\|] < \infty$ in a neighbor of $\lambda^*$, where $\Gamma$ denotes a open sphere in $\mathbb{R}^K$ with radius $\delta$.

(F) $H_{\lambda\lambda}$ is of full rank.

(G) $H_{\lambda\lambda}$ is continuous for any $\lambda \in \mathbb{R}^K$ almost surely in $R$.

(H) $\exists \delta, \epsilon' > 0$ such that for all $\epsilon \in (0, \epsilon')$:

$$E\left[sup_{\lambda \in \Gamma(\lambda^*, \delta)} \|h^{\gamma}(\lambda, R)\|^{2+\epsilon}\right] < \infty \tag{27}$$

where $\Gamma$ denotes a open sphere in $\mathbb{R}^K$ with radius $\delta$.

Assumptions (A) and (B) guarantee that the basis assets returns are well-behaved. In particular assumption (B) ensures that given a sufficient "amount of time" the realizations of $R_t$ are independent. Assumption (C) guarantees the absence of multicollinearity. Assumptions (D) to (H) are based on Kitamura and Stutzer (1997) and allow us to apply a weak version of the law of large numbers and the Central Limit theorem (CLT). In

[22] We focus on the case $E[m] = 1$, which is the value considered in our empirical applications, to concentrate out the constants in the dual space problem. All the results generalize to the case where $E[m] = a, a \in \mathbb{R}$.

particular, (D) guarantees that the variance in the CLT is well-defined (covariance stationarity condition), (E) can be relaxed with additional algebraic costs (see comments in Kitamura and Stutzer (1997)), (F) and (G) allow us to prove the asymptotic convergence of $H_{\lambda\lambda}$ by relying on the continuous mapping theorem. Assumption (H) is a standard smoothness condition of $h^{\lambda}$ (see Andrews (1993)).

**Theorem 2.** Under assumption 2, $\hat{\lambda}_T$ converges in probability to $\lambda$ (defined in assumption 1).

The proof of this theorem is available in Almeida and Garcia (2012) and relies on an application of Theorem 2.7 of Newey and McFadden (1994).

**Theorem 3.** Under Assumption 2 we have that:

$$\sqrt{T}(\hat{\lambda} - \lambda^{*}) \rightarrow_{d} N(0, V_{\lambda}) \tag{28}$$

where $V_{\lambda}$ is given by $H_{\lambda\lambda}^{-1} S_{\lambda} H_{\lambda\lambda}^{-1}$.

See proof in the Appendix.

## 4.3 Imposing The Non-Negativity Constraint on the SDF

From Almeida and Garcia (2016) we know that imposing non-negativity restrictions has different implications depending on three regions for the parameter $\gamma$ that indexes the Cressie-Read family: (i) $\gamma > 0$. (ii) $\gamma < 0$ and (iii) $\gamma = 0$ (see corollary 1).

Starting with $\gamma = 0$, we have an unconstrained maximization problem. This comes directly from the fact that when $\gamma = 0$ the Cressie-Read family converges to the exponential tilting case, which is positive by construction. Thus, in this particular case, the consistency of the estimator, as well as the asymptotic normality, follows directly from the results in the previous section.

The second case that deserves attention is when $\gamma < 0$. In this case, the optimization function for $\lambda$ needs to be modified when restricting the SDF to be non-negative. In particular, corollary 1 reveals that this restriction introduces a $\delta$ set-indicator function that takes value zero if $(1 + \lambda'(R - 1)) > 0$ or $\infty$ otherwise. To prove consistency and asymptotic normality we must guarantee the stability of the objective function near the solution. Therefore, it must be the case that $\mathfrak{M}^{\gamma}(\lambda^{*}, R) < \infty$ at the solution. From

the continuity of $\mathfrak{M}^{\gamma}(\cdot, R)$ we have that in the neighborhood of $\lambda^*$, $\delta(x|A) = 0$. Thus, asymptotically, the central arguments of the previous section hold.[23]

Finally, the last case that must be dealt with is when $\gamma > 0$. Note that here we lose the pointwise differentiability of the objective function. Precisely, when $\gamma > 0$ the objective function is affected by an indicator function for the set $(1+\lambda'(R-1)) \geq 0$ which introduces a "kink" when the equality holds. As this case is the more complex one, below we modify our previous framework so we can still prove the asymptotic properties. For the consistency result, the main problem is to establish the validity of (iv) in Newey and McFadden (1994) theorem. The basic idea for the normality condition is to find a different approximation (different than the Taylor expansion) that holds even though we lose the differentiability. To do so, we expand the results in Li et al. (2010) for the more general Cressie-Read family [24].

We restate here the assumptions needed to prove the consistency and asymptotic normality of the Lagrange multipliers in the positivity constrained case.

**Assumption 3.**

(A) We assume that $\lambda^*$ is the unique solution to the following problem:

$$\lambda^* = \underset{\lambda \in \Lambda}{\operatorname{argmin}}\ \mathfrak{M}^{\gamma+}(\lambda, R) \tag{29}$$

where $\Lambda \in \mathbb{R}^K$ is a compact subset of $\mathbb{R}^K$

(B) The process $R_t$ is stationary and ergodic.

(C) The process $R_t$ is strongly mixing with mixing coefficients $\alpha_t$ satisfying $\sum_{t=1}^{\infty} \alpha_n^{1-1/b} < \infty$ and $b > 1$.

(D) $E[RR^T]$ is non-singular and $E[\|R\|^2] < \infty$.

(E) The set $\{(1 + \lambda'(R - 1)) = 0\}$ has probability zero.

(F) The first-order derivatives $h^{\gamma+}(\lambda, R)$ form a Donsker class for $\lambda$ in a neighborhood of $\lambda^*$.

[23] Technically, we can approximate the objective function by dropping the Delta term. This approximation dominates the true objective function and is both continuous and differentiable.

[24] Li et al. (2010) expanded the results in Lars Peter Hansen and Luttmer (1995) and proved the asymptotic convergence of the Hansen and Jagannathan estimator

(G) The matrix $E[H^{+}_{\lambda\lambda}(\lambda^*)]$ is non singular and positive definite. The second derivatives are well specified except on a set with zero probability. $H^{+}_{\lambda\lambda}$ is continuous for any $\lambda \in \mathbb{R}^K$ almost surely in $R$.

(H) $Var\left(\sqrt{T}\left[\frac{\partial}{\partial\lambda}\mathfrak{M}^{\gamma+}_T(\lambda^*, R)\right]\right) \to_p S_\lambda > 0$ when $T \to \infty$.

(I) For a sufficient small $\delta > 0$, $-E[\sup_{\lambda\in\Gamma(\lambda^*,\delta)} \| h^{\gamma+}(\lambda, R)\|] < \infty$ in a neighbor of $\lambda^*$, where $\Gamma$ denotes a open sphere in $\mathbb{R}^K$ with radius $\delta$.

(J) $\exists \delta, \epsilon' > 0$ such that for all $\epsilon \in (0, \epsilon')$:

$$E\left[sup_{\lambda\in\Gamma(\lambda^*,\delta)} \| h^{\gamma+}(\lambda, R)\|^{2+\epsilon}\right] < \infty \tag{30}$$

where $\Gamma$ denotes an open sphere in $\mathbb{R}^K$ with radius $\delta$.

As in the previous section, the above assumptions are standard. Assumption (A) can be relaxed to non-compactness with some additional complications. In particular, the compactness will allow us to easily prove the consistency of the estimator. Assumptions (B) and (C) guarantee that the stochastic processes are well behaved and, "given sufficient time", are independent. This allows us to make inference on the population moments using the time series of the variables. Assumption (D) requires that the random variables $R$ are square integrable, which is needed to compute the asymptotic covariance matrix. Assumption (E) guarantees that the set of non-differentiable points in the optimization function is not too big. This is required for the validity of our differentiability in quadratic mean approximation. Assumption (F) is used to guarantee the first-order approximation of the stochastic term in the quadratic approximation (See the Lemmas below). Assumption (G) ensures that the optimization problem is well defined. The remainder assumptions parallel the ones in assumption 2 and are used to prove the asymptotic normality of the score vector. Among the above assumptions, assumption (F) is the less standard one and is discussed further in the Appendix.

Below we start by establishing the consistency of the Lagrange multipliers and proceed with the proof of the asymptotic normality.

**Theorem 4.** Under assumption 3, $\hat{\lambda}_T$ converges in probability to $\lambda$ (defined in assumption 3).

See proof in the Appendix.

For the theorem on the asymptotic normality of the estimators, we need two lemmas that are stated below and proven in the Appendix.

**Lemma 1.** If assumption 3 holds, we can write the following relation:

$$\begin{aligned} E_T[f^{\gamma+}(\lambda, R)] &= E[f^{\gamma+}(\lambda^*, R)] + (E_T - E)[f^{\gamma+}(\lambda^*, R)] + \tilde{S}_\lambda^{+'}(\lambda - \lambda^*) \\ &+ \frac{1}{2}(\lambda - \lambda^*)' H_{\lambda\lambda}^+(\lambda - \lambda^*) + o(\|\lambda - \lambda^*\|) + o_p(\|\lambda - \lambda^*\| T^{-1/2}) \end{aligned} \tag{31}$$

where $\tilde{S}_\lambda^+ = (E_T - E)\left[\frac{\partial f^{\gamma+}(\lambda,R)}{\partial\lambda}\right]$ and $H_{\lambda\lambda}^+ = E\left[\frac{\partial^2 f^{\gamma+}(\lambda,R)}{\partial\lambda\partial\lambda'}\right]$.

**Lemma 2.** If assumption 3 holds, the following asymptotic approximation is valid:

$$\begin{aligned} \max_{\lambda\in\mathbb{R}^K} E_T[f^{\gamma+}(\lambda, R)] &= E[f^{\gamma+}(\lambda, R)] + (E_T - E)[f^{\gamma+}(\lambda, R)] \\ &- \frac{1}{2}\tilde{S}_\lambda^{+'} H_{\lambda\lambda}^+ \tilde{S}_\lambda^+ + o_p(T^{-1}) \end{aligned} \tag{32}$$

where $\tilde{S}_\lambda^+$ and $H_{\lambda\lambda}^+$ where defined in Lemma 1.

Now, using these two lemmas, we can establish the asymptotic convergence of the estimator $\hat{\lambda}$.

**Theorem 5.** If assumption 3 holds, the estimator for the Lagrange multipliers, $\hat{\lambda}$, has the following asymptotic distribution:

$$\sqrt{T}(\hat{\lambda} - \lambda^*) \to_d N(0, V_\lambda) \tag{33}$$

where $V_\lambda$ is given by $(H_{\lambda\lambda}^+)^{-1} S_\lambda^+ (H_{\lambda\lambda}^+)^{-1}$.

The proof is given in the Appendix.

## 4.4 Alpha Performance Estimation and Testing

In section 2, we have underlined the fact that there might exist multiple SDFs that correctly prices the $K$ assets chosen as benchmark assets to evaluate performance. Moreover, in the inherent incomplete-market setting that we described, the SDF might be investor dependent. That is, even though each investor prices the assets equally, the

internal pricing structure might vary across investors (see Mas-Colell et al. (1995) for a formal treatment). In the hedge fund performance framework this means that if the hedge fund returns $R_{i,t}$ is not spanned by the basis assets in the economy each investor might differ in the fund valuation. In particular, let $m^{\gamma}$ be the SDF associated that corresponds to a particular investor with parameter $\gamma$. For this investor, the hedge fund performance is measured as $\alpha_i^{\gamma} = E[m^{\gamma} R_i - 1]$.

Note that if $R_i \in R_p$, i.e. the hedge fund return is accessible to the general public trough a combination of the benchmark assets, we have that $\alpha_i^{\gamma} = 0$ since the SDF correctly price all the basis assets, and their linear combinations (given the linearity of the expectation operator), by construction. Nonetheless, hedge funds might have superior information about $s \in S$. Using this superior information they might construct state-dependent strategies such that $R_i$ is not spanned by the basis assets. Therefore, in this case, $\alpha_i^{\gamma} \neq 0$.

The question is then: how to test whether $\alpha_i^{\gamma} = 0$? One simple way to proceed is to define the following statistic (Chen and Knez, 1996):

$$\mathcal{H}_T = T\left[\frac{1}{T}\sum_{t=1}^{T}\alpha_{i,t}^{\gamma}(\lambda^*)\right] W_T \left[\frac{1}{T}\sum_{t=1}^{T}\alpha_{i,t}^{\gamma}(\lambda^*)\right] \tag{34}$$

where $W_T$ is the inverse of the variance of $\alpha_{i,t}^{\gamma}(\lambda^*)$. Additionally, note that we explicitly write $\alpha_{i,t}^{\gamma}(\lambda^*)$ as dependent on the first-step estimate of $\lambda^*$. Therefore $W_T$ should account for the uncertainty attached to the estimates of the vector of parameters $\lambda$. Rewriting the sample estimate of $\alpha_i^{\gamma}$ taking into account the SDF estimation we have that [25]

$$\alpha_i^{\gamma} = \frac{1}{T}\sum_{t=1}^{T}\left[\left(1+\gamma\lambda_{\gamma}'(R_{K,t}-1_K)\right)^{\frac{1}{\gamma}} R_{i,t} - 1\right] \tag{35}$$

Using the above representation of the Cressie-Read performance measures, we can write $\alpha_i^{\gamma}$ as a function of $\{R, R_i, \lambda\}$. Hence, by applying the delta method we can compute the matrix $W_T$ as follows:

$$(W_T)^{-1} = Var(\alpha_i^{\gamma}) = \frac{\partial \alpha_i^{\gamma\prime}}{\partial\lambda} Var(\lambda) \frac{\partial \alpha_i^{\gamma}}{\partial\lambda} \tag{36}$$

In the online appendix we provide closed-form formulas for $W_T$ for the three cases

[25] To ease the notation we are loosely defining $\alpha_i^{\gamma}$ without taking into account the limiting cases for $\gamma$.

corresponding to corollaries 1 and 2. Finally, under the null of zero alpha, $\mathcal{H}_T$ converges asymptotically to a $\chi^2$ distribution with one degree of freedom.

Despite our asymptotic results, when using hedge fund data, another problem is of first order: typically we only have a handful of observations for the hedge fund return. Therefore, we also approach the problem by relying on bootstrap techniques (see Harvey and Harvey (2016) and Davidson and MacKinnon (2010)) that are particularly suitable for the small sample issue. Specifically, we apply a nonparametric bootstrap re-sampling both the basis assets and the hedge fund returns to calculate the bootstrapped standard errors (we follow the methodology in MacKinnon (2006)). Given that most hedge fund time series do not overlap with each other we perform the bootstrap on a fund-by-fund basis. While this is costly in terms of computation time, our sample characteristics significantly restrict other methods (joint resampling of the full panel for instance). In the internet appendix, we provide a detailed comparison between alternative standard-error estimates.

# 5 Hedge Fund Data and Filters

It is well-known in the finance literature that commercial hedge fund data bases suffer from severe biases (see Fung and Hsieh (2001), Aggarwal and Jorion (2010), Patton et al. (2015), Bollen and Pool (2009) and Aiken et al. (2013) among others). Over the years, researchers have applied stricter filters as it became obvious that some reporting issues were severely biasing the performance results. Even though we use the Lipper-TASS data set from Thomson Reuters, the problems we identified are in line with the former literature.

Traditionally, the literature focused on two primary sources of bias: survivorship and backfill (e.g. Fung and Hsieh (2001) and Aggarwal and Jorion (2010) respectively). Given the lack of regulation, fund managers not only can choose when to report their returns but also to which commercial database to report it. In particular, fund managers also have the discretion to change previously reported returns (Patton et al., 2015). All these problems are carefully investigated in previous papers. The general conclusion is three-fold: the main consensus consists of using data post-1994 when most commercial databases started to include defunct funds. This potentially alleviates the survivorship bias. To deal with

the backfill bias two possibilities are proposed: to exclude the first twelve months (or 24 months) of return for all funds (e.g. Bali et al. (2011)) or, as proposed by Aggarwal and Jorion (2010), to detect back-fillers according to the difference between the hedge fund inception date and the data it started to report to TASS (or any data provider).

In addition to the usual biases addressed in the literature, there is a growing effort to improve data quality in empirical applications. This is reflected in the number of papers investigating data biases and data problems in commercial databases (e.g. Patton et al. (2015) and Bollen and Pool (2009)). Despite this effort, recent articles raised additional concerns. Straumann (2009) found several problematic idiosyncrasies in many databases (including the one used in this paper). Specifically, the author found an alarming number of hedge funds with excessive zero returns, consecutive equal returns, and repeated "blocks" of returns. Building on this, Bollen and Pool (2012) investigated these data problems further and showed that several of them are indicative of future sanctions from the Security Exchange Commission (SEC).

In our online appendix, we carefully describe the filters we apply to the original data and explore the implications of the corrected data biases for performance measurement. Following and expanding Straumann (2009), we consider eight filters (in addition to the traditional ones): (1) Funds with an abnormal number of zero returns. (2) Funds with repeated blocks of returns (e.g. repetitions of the block A,B,C with $A, B, C \in \mathbb{R}$) (3) funds with an unusual amount of repeated returns (e.g. A, A, A). (4) Funds whose returns calculated from reported net asset values do not coincide with reported net returns. (5) Funds with rounded returns. (6) Funds that do not report the assets under management. (7) Funds that share blocks of returns in the time series (e.g. two funds share the same return for, say, Jan - 2000 to Dec - 2000). (8) Funds where returns calculated using the net asset value (NAV) are different from the reported returns. not report them here).

## 5.1 Summary for the Final Sample of Hedge Funds

Table 1 provides these statistics for each year from 1994 till 2015. We report the number of hedge funds in the sample at the beginning of the year, the number of entrants, the number of funds dissolved as well as the number of hedge funds at year-end and their total assets under management (in billions of dollars). Using all active hedge funds active within a given year, we construct an equally-weighted portfolio and compute its mean,

median, standard deviation as well as minimum and maximum returns[26]. Notably the number of hedge funds peaked around 2007 and fell sharply after the financial crisis. The same pattern applies to the assets under management. In fact, by the end of 2014, the AUM figure was less than half its peak level observed in 2007. Overall, the mean and median returns are positive throughout the sample. The exceptions are, as expected, 2008 and 2011, two crisis years where high numbers of hedge funds were dissolved.

Table 2 complements this analysis with information regarding the cross-section of hedge funds. Here we present the average return across funds, the average assets under management, fund age, management fee and performance fee. In contrast with the equally-weighted index, we note that the mean is lower for the individual hedge fund (0.57 versus a historical mean of 0.67 for the equally-weighted index). The same goes for the individual standard deviation. Nonetheless, the minimum (maximum) returns are significantly lower (higher) than those reported for the historical data. The fund age reveals an important piece of information for our empirical application: the median fund has only 71 months of observable returns. Therefore, dealing with sample sizes is a first-order problem for inference. The median management and performance fees are close to the industry 2/20 standard. Finally, we note that the median hedge fund manages about 30 million dollars. When compared with the mean value of 130 million dollars this fact reveals that a small number of funds manage most of the money. Overall our results are very close to Bali et al. (2011) who use a similar data set.

A significant portion of our paper focuses on the higher-order properties of hedge fund returns and its implications for performance measurement. Table 3 reports the average across funds, segregated by the fund primary strategy, mean, standard deviation, skewness, and kurtosis. Overall, we note a significant difference in terms of average return across categories. While equity neutral hedge funds have an average return of 0.44% per month, long-short funds produce about 0.76%. The difference in standard deviation is even higher, ranging from 2.61 to 6.12. Consistent with previous papers we also document a negative skewness for most of the primary-strategy groups (similarly to Kosowski et al. (2007)). Regarding kurtosis, all the groups we analyze have distributions that are platykurtic. Table 4 complements this with the respective values for an equally-weighted index for each category. In general, the results are somewhat similar. Nonetheless, the

[26]Given the high number of hedge funds that do not declare their assets under management on a consistent basis an AUM-weighted portfolio is unpractical.

magnitudes of the higher-order moments change substantially for the index data. Thus, our empirical analysis focuses both on individual and index-level performance measurement.

## 5.2 Summary of the Factors

In this paper, we stress two sources of nonlinearities and its implications for performance measurement: the hedge fund returns and the benchmark factor returns. Previous studies on hedge fund returns have typically used both linear and nonlinear factors. To this end, we include two sets of linear factors (Fama and French (1993) and Carhart (1997)) and two sets of factors explicitly designed to capture non-linear hedge fund strategies. First, we adopt the workhorse model of Fung and Hsieh (2001). The updated model consists of ten factors (see Fung's website). Five of these factors are based on primitive trend following strategies for the bonds (PTFSBD), currency (PTFSFX), commodities (PTFSCOM), interest rate (PTFSIR) and stocks (PTFSSTK) markets. The complementary factors are the underlying portfolios used to construct the trend-following ones: the monthly return on the S&P 500, the spread between the Russell 2000 index total monthly return and the S&P 500 total monthly return, the monthly total return on the MSCI Emerging Markets index, the change in constant maturity yield 10-year Treasury bond, and the change in the spread between Moody's Baa and the 10-year Treasury[27].

Agarwal and Naik (2004) propose a new set of nonlinear factors constructed using option prices. We follow the authors and construct four portfolios of at and out of the money put and calls using raw data from Option Metrics. In our empirical applications when considering the option portfolios, we also add the same linear factors from Fung and Hsieh (2001).[28]

In Table 5 we report the summary statistics for all of the above-mentioned factors. Overall, the sample properties are fairly close to the original factors by Fama and French (1993), Carhart (1997), Fung and Hsieh (2001), and Agarwal and Naik (2004). The statistics about primary equity and size risk factors are without surprise - a mean close to 10% for the S&P 500 index and a spread of about to 4% with the Russell index. The

[27]The underlying data are acquired trough Datastream using the "RI" code, from the Federal Board of Governors H15 forms and from St. Louis FRED dataset.

[28]The SAS code to generate these factors is available upon request. Note that a version of the code is available through WARDS. This version does not take into account recent changes in the expiration schedule (see CBOE website).

bond and the credit factors exhibit much lower means (between -2% and 4%). Most indexes exhibit little skewness and excess kurtosis. This is also the case for the Fama and French (1993) and Carhart (1997) factors. The picture is radically different for the trend-following factors constructed by Fung and Hsieh (2001). Means are sizable and typically negative. Additionally, the volatilities are substantially higher than those of the underlying factors. The picture for the option factors is even stronger: all means are extremely negative with substantial standard deviations. Common to both option and trend-following factors is the high skewness of the returns. In contrast, on average, the kurtosis is lower for these factors than for the underlying. The only exception is the interest rate trend-following strategy, which has an extremely high kurtosis.

# 6 Empirical Findings

In this section we apply the nonparametric measures defined in the previous sections to evaluate the performance of hedge fund indexes and individual hedge funds over the 1994 to 2015 period. Thanks to our estimation and testing methodologies developed in the previous sections, we will be able to identify the actual factors that are significant for evaluating performance depending on investors' preferences (proxied by our gamma parameter in the Cressie-Read family of measures). We then proceed to assess alpha performance based on the estimated SDFs. We pay particular attention to testing the hypothesis that the alphas are different from zero and to measuring their heterogeneity across sets of benchmark factors and strategies reported by the hedge funds. While conducting this analysis we pay attention to the alpha relevance of nonlinear exposures of hedge funds to risks. In particular we compare the effect of adding linearly higher-order moments of factor returns to our nonparametric approach that accounts functionally for the exposure to higher moments.

## 6.1 SDF Estimation Results

The first step to compute our performance measure involves the computation of the stochastic discount factor. To this end, we need to choose the basis assets (factors) to be used in the estimation as well as a range of $\gamma$s. For the benchmark factor models, we select two types of models. First we use linear models that are workhorse models in the

empirical finance literature (CAPM and Fama, French and Carhart four-factor models), in order to isolate the effects of the nonlinear benchmarks designed specifically to capture hedge fund dynamics, that is Fung and Hsieh (2001) and Agarwal and Naik (2004) sets of factors. Overall, these choices provide us with a broad set of possibilities that capture the most common models used in the literature.

For the Cressie-Read gammas, we consider the following $\gamma$ values: $\{-3.5, -2, -1, -0.5, 0, 0.5, 1\}$. This set includes the HJ linear SDF with positivity constraint ($\gamma = 1$), a set of SDFs that give mild weights to skewness and kurtosis ($\gamma = -1, -0.5, 0, 0.5$), and two SDFs that give larger absolute weights to skewness and kurtosis ($\gamma = -2 \text{and} - 3.5$). This range of $\gamma$ values are reasonably representative of the Cressie-Read family and generate substantial variability in both the estimated SDF as well as the hedge fund alphas. In particular, for $\gamma << -3.5$, the implied SDF concentrates most of the probability density on a few states of nature. On the contrary, for values of $\gamma >> 1$ the implied SDF assigns zero probabilities for several states of nature in order to correctly price the basis assets. Both these features are uninteresting for performance measurement, and thus we are comforted in the representativeness of investors' weighting of risks by the chosen $\gamma$ values.

Starting with full-sample estimation results, from January 1994 to June 2015, we look at the estimated $\lambda$s and discuss the time-series dynamics of the estimated SDF. The sign of the lambda indicates whether the investor corresponding to a particular $\gamma$ will be positively loaded or long (a negative sign for lambda) or negatively loaded or short (a positive sign for lambda) in the corresponding factor. The results for the CAPM and Fama, French and Carhart models are reported in Tables A6 and A7. For both specifications, the market portfolio emerges as a statistically significant factor with a positive load. Second, the addition of the long-short portfolios, in the Fama, French, and Carhart model, slightly increases the absolute value assigned to the market portfolio across all $\gamma$s. In addition, the momentum portfolio also appears as a long and statistically significant factor.

The estimated $\lambda$s for the Fung and Hsieh model are reported in Table A8. First, two of the five underlying portfolios, i.e. the S&P 500 and the credit spread, are held long and are statistically significant. For the trend following strategies, only the S&P 500 lookback straddle appears statistically significant. The results for the Agarwal and Naik model (in Table A9), which replaces the trend-following factors by option portfolios, are

somewhat consistent. The portfolio with the highest $\lambda$ is precisely the S&P 500 (the one with the highest mean), but all other underlying assets are insignificant or only marginally significant across all $\gamma$s. For the option portfolios, the positions in the put options are particularly striking. Across all $\gamma$s the investor shorts out-of-the-money puts and hedge out the position by buying at-the-money ones. This is somewhat consistent with the typical results that selling insurance is profitable, on average. Even more interestingly, the $\lambda$s associated with more negative $\gamma$s are smaller in magnitude. This finding indicates that even though selling insurance is profitable on average, more risk averse agents reduce their exposure to this strategy given that rare disastrous outcomes might happen.

A fundamental tenet of the methodology is that the estimated SDF should price exactly the risk factors. Empirically, we note that the estimated SDFs are reasonably precise across all models. In all settings, the pricing errors are smaller than 0.1 basis points for all $\gamma$s. In fact, for most of the assets and $\gamma$s combinations, the pricing errors are lower than the small tolerance level we set in our computational procedure. This feature should reassure the reader that a zero alpha performance will rightfully be attributed to a hedge fund with static strategies on the factors.

Figure 4 features the time series of the estimated SDFs for three selected values of $\gamma$ (1, 0 and -3.5) and for the four sets of basis assets. A first obvious remark is that the SDFs without trend-following and/or options factors are much less volatile across all $\gamma$s. In particular, the SDFs with non-linear factors contain a significant number of spikes in comparison with the linear CAPM and Fama, French and Carhart ones. Within the same set of basis assets, Figure 4 also reveals that more negative gammas are associated with a higher SDF volatility. Interestingly, the introduction of non-linear assets "forces" the Hansen and Jagannathan SDF to assign zero probability to some states of nature (see the figure for the Fung and Hsieh SDF). Apart from a higher volatility, the SDFs with trend following/options portfolios exhibit distinctively large peaks associated with events that had a large impact on wealth. They amplify considerably the events that were already visible in the SDFs without these portfolios. The same peaks appear for the three values of $\gamma$ in 1998, 2002 and 2006 and during the recent financial crisis.

## 6.2 Analyzing Hedge Fund Indexes

Hedge fund indexes are constructed according to their self-declared primary strategy. Although pooling hedge fund returns hide some significant heterogeneity among the funds, we gain in two dimensions. First, the hedge fund indexes allow us to construct portfolios with observed returns for the whole available sample. This is in clear contrast with the median 71 observations for the individual hedge funds. Second, by pooling the data, we are better able to contrast the performance across combinations of $\gamma$ and basis assets since aggregate results are easier to present.

We construct eleven indexes based on the reported primary strategy: Convertible Arbitrage, CTA, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Fund of Funds, Global Macro, Long-Short Equity Hedge, Managed Futures and Multi-Strategy. The summary statistics for these indexes are presented in Table 4. Overall, we note a significant difference in the average return across categories. While equity neutral hedge funds have an average return of 0.60% per month, long-short funds produce a monthly return of about 0.93%. The difference in standard deviation is even higher, ranging from 0.88 for the market-neutral index to 4.21 for the emerging market one. Consistent with previous papers we also document a negative skewness for most of the primary strategies groups (similar to Kosowski et al. (2007)). Regarding kurtosis, all the groups we analyze have distributions that are platykurtic.

### 6.2.1 Alpha Performance

For each hedge fund index, we calculate the full sample alpha as well as the associated asymptotic and bootstrapped t-statistics. For completeness, to compare our estimates with the standard in the literature, we also include the results for the Jensen's alpha[29].

We start with a summary of the results for the CAPM and Fama, French, and Carhart models. In Tables A10 and A11 we report the alpha point estimates for these models. First we note that the Hansen and Jagannathan and the OLS alphas are extremely close to each other (not detectable at two decimals) even though they are not based on the same methodology. This is in line with the Taylor expansion we perform, which revealed that the Hansen and Jagannathan estimator loads only on the mean and variance of the

[29] In this case, the bootstrapped t-statistics are calculated using the residual bootstrap proposed by Kosowski et al. (2006).

hedge fund returns. Focusing on the Jensen's alpha, we also observe that the introduction of a few factors drives down the mean return from $[0.47\%, 0.93\%]$ range to a risk-adjusted alpha from 0.06% to 0.45% per month. Thus, even in models with few factors, we see that most of the hedge funds returns do not translate into $\alpha$ performance.

Focusing on the individual indexes for the CAPM model, we note a considerable heterogeneity in the convertible arbitrage strategy, even though the alphas are insignificant across all gammas. For the -3.5 gamma case we have a point-wise estimate of 0.05 in comparison to the corresponding 0.21 estimate in the Hansen and Jagannathan case. The same type of effect is present for the emerging markets index. For fixed income arbitrage, statistical significance varies hugely across gammas. While for $\gamma = -1$ the alphas are statistically significant, it becomes only marginally so for $\gamma = -3.5$ case. On the opposite direction, for managed futures, we compute a higher alpha for $\gamma = -3.5$ in comparison with the Hansen and Jagannathan case. Comparing these results with those for the Fama, French and Carhart model we note that the introduction of the extra factors typically increases the heterogeneity of the alpha estimates for certain strategies (e.g. convertible arbitrage). In particular, these differences are more noticeable for the Cressie-Read alphas associated with negative gammas, while the Jensen's estimates remain similar to the CAPM.

The introduction of the Fung and Hsieh trend-following factors generates additional heterogeneity (see Table 6). **All** estimates for the Jensen's alpha are positive and statistically significant[30], but the Cressie-Read alphas offer a different picture. For example, for the CTA group, we note a substantial increase in the pointwise alpha across all estimates, but especially for the more negative $\gamma$s. This indicates that CTA's returns are strongly counter-cyclical, providing investors with positive returns in bad times when their marginal utility is high.

The emerging markets category provides an excellent example of how non-linearities introduced by the Cressie-Read estimation might alter the investors' evaluation of performance. The Jensen's alpha obtained the Fung and Hsieh factors is positive, statistically significant and much higher than Fama, French and Carhart one. However, the Cressie-Read estimates reveal a more mixed behavior. While alphas associated with positive gammas also increase substantially and remain marginally significant, the alphas asso-

[30]The OLS $R^2$s, not reported in the table, increase substantially with the introduction of the Fung and Hsieh non-linear factor in comparison to the CAPM model (54% versus 30%).

ciated with the negative gammas are smaller in comparison with the Fama, French and Carhart case and are not significantly different from zero.

Another interesting category is Funds of Funds. In the Fama, French, and Carhart setting, all alphas are close to zero and insignificant (see Table A11), but they almost double with the Fung and Hsieh factors (around 0.20 across the $\gamma$s) and gain significance with the bootstrap t-statistics. A similar phenomenon is also noted for the managed futures index, where all alphas also raise substantially.

The introduction of Agarwal and Naik option factors, in place of Fung and Hsieh trend-following ones, adds some very interesting facts to the evaluation of hedge fund performance. Table 7 compiles these results. For convertible arbitrage, the estimated alphas are statistically significant across all $\gamma$s based on the bootstrapped t-statistics and for the more negative $\gamma$s with the asymptotic t-statistics. It should be noted that among all four risk-factor settings this is the only one where the convertible arbitrage produces statistically significant Cressie-Read alphas.

Perhaps the more interesting cases are the Funds of Funds, global macro and managed futures indexes. First, for all three indexes, the estimates for the Jensen's alpha remain virtually unchanged when substituting the trend-following factors by the option portfolios. In clear contrast, for the more negative $\gamma$s we note a significant reduction of almost half of the previous value and the loss of statistical significance for several categories. when option factors are introduced. For CTA funds, we note that "monotonicity" of the alphas across $\gamma$s reverts with options compared with trend-following factors.

To sum up, it is clear that the choice of benchmark factors matters substantially when assessing hedge fund performance. The heterogeneity in performance evaluation appears much stronger when considering the investors diversity through the various Cressie-Read measures alphas than in the usual linear case, whether it is measure by the Jensen's alpha or the Hansen and Jagannathan one. These two linear measures provide a very similar performance even though the second one imposes absence of arbitrage. It reinforces the fact that the nonlinear exposures in the hedge funds strategies are evaluated differently by investors with different attitudes towards asymmetry or tail risks. Therefore, investors' heterogeneity appears to matter more for some indexes than for others based on the dispersion of alphas across $\gamma$ measures. Finally, our results also emphasize the importance of testing properly for the significance of the estimates. These points are analyzed in more

detail in the next section.

### 6.2.2 Implications of Higher-Order Moments and SDF Non-Linearities

We start by analyzing how the degrees of non-linearities affect the alpha estimates. We do it in the simplest factor setting that is the CAPM alpha[31]. We proceed by adding polynomial terms to the market factor in the linear regression, as follows[32]:

$$R^e_{i,t} = \alpha_i + \beta_{i,R^m}(R^m_t - Rf_t) + \sum_{p=1}^{P} \beta_{i,p}(R^m_t - Rf_t)^{p+1} + \epsilon_{i,t} \tag{37}$$

$R^e_{i,t}$ denotes the excess return for fund $i$ and $R^m_t - Rf_t$ the return of the market portfolio in excess of the risk-free rate. The addition of the polynomial terms is similar to the co-skewness methodology proposed by Harvey and Siddique (2000). In particular, these authors explicitly model the stochastic discount factors as $m_{t+1} = a_t + b_t R_{m,t+1} + c_t R^2_{m,t+1}$, naturally introducing non-linearities. Furthermore, the cubic term can be viewed as the co-kurtosis, as introduced in Ang et al. (2006)[33]. We want to contrast the performance evaluation based on this linear addition of non-linear terms to the performance implied by our SDF approach where the non-linearities are weighted differently by diverse investors.

Table 8 illustrates perfectly the important similarities and differences between the two approaches. It is informative to compare the fourth-order linear model, which incorporates all higher-order moments up to the fourth power, to the $-3.5$ value for $\gamma$ in our nonparametric approach. The two corresponding alphas are similar, positive and significantly different from zero for strategies that are immune from market risk, Equity market neutral, Long-short equity hedge, Event driven and Multi-strategy. In fact for these categories all specifications give about the same performance assessment. It means that the strategies are risk neutral to many moments (see Patton (2009). The two measures also agree in attributing zero performance to the Convertible arbitrage and Funds of funds categories. Note that all the other measures are more optimistic about the per-

[31]In the hedge fund performance measurement literature, the typical approach relies on multiple factors, often the Fung and Hsieh 10 factors). Introducing polynomial terms with these more elaborate models is unfeasible. In contrast, as seen in our Taylor expansion, our methodology provides a parsimonious higher-order exposure to an efficient linear combination of all factors in the model. So the number of factors is not a problem in our approach.

[32]This is similar to Agarwal et al. (2008), who investigate if higher moments of equity risk explain hedge fund returns.

[33]Ranaldo and Favre (2003) incorporate these higher-order moments to evaluate the performance of hedge funds and show that they are relevant for some funds.

formance of these indexes since they do not penalize the high kurtosis associated with these strategies.

The divergences between the two measures are also striking. For CTA, Global Macro and Managed Futures, the fourth-order specification attributes a zero-performance while the $-3.5$ gamma estimates a positive and significant alpha. The latter measure likes the positive skewness and the low kurtosis of these strategies. Introducing the power terms in the linear regression reduces to zero the positive performance implied by the CAPM. Finally for the Emerging markets, the Cressie-Read measure inverts the positive alpha estimated with the fourth-order specification and puts it to a value of zero both numerically and statistically.

A few remarks are in order to conclude on the nonlinearities. A significant difference between the two approaches is that, for all indexes, we note a monotonic relationship between the alphas and gammas in the SDF approach. On the contrary, the results of the polynomial approach are more erratic. Another important point is that the introduction of the polynomial terms in the linear framework suppresses the typical "excess return" interpretation of the OLS alpha. Our stochastic discount factor methodology naturally embeds different degrees of nonlinearities and preserves the interpretation of the alphas as the risk-adjusted hedge fund return. Furthermore, cross-correlation terms are already taken into account in the $\lambda's$ estimation, allowing us to expand the number of benchmarks used to assess performance significantly.

## 6.3 Analyzing Individual Hedge Funds

In this section we further analyze the implications of our non-parametric methodology for the performance of individual hedge funds. To estimate the hedge fund alphas, we re-estimate the SDF using the basis assets sample that matches the fund time series, which implies multiple optimization procedures. The solution consisting in estimating the SDF for the full sample and taking the corresponding range for each fund is not appropriate. The main reason is that we would use information that might not have been available during the period the fund existed. This will translate into downward weighting all states of nature for which the hedge fund existed. As an example, let us consider the trend-following and option-based factors introduced by Fung and Hsieh (2001) and Agarwal and Naik (2004) respectively. The full-sample estimate of the SDF including these factors will

be very different before and after the 2008 crisis. So for hedge funds that died before the crisis, the weights attributes to these factors are not the proper ones to evaluate the performance of the funds[34].

### 6.3.1 Individual Hedge Fund Performance

There are many ways to analyze the whole set of individual alphas. We provide summary statistics, draw kernel densities of alphas and their t-statistics, test for the difference of performance between the linear and nonlinear estimators, and conclude by analyzing the performance across strategies.

**6.3.1.1 Summary statistics** Table 9 presents, for all four sets of basis assets, the summary statistics of the Jensen's and Cressie-Read alpha estimates for all funds. We compute the mean, standard deviation, skewness, kurtosis and the percentage of alphas that are positive and statistically significant at 10%. Similarly to hedge fund indexes, the average alpha estimate for the CAPM model is quite similar across $\gamma$s, but the differences are bigger for higher moments. Note also the difference in the percentage of statistically significant alphas. The Jensen's estimator generates a percentage of 21% while it is 17% for the Hansen and Jagannathan estimator. This difference is explained by the non-arbitrage restrictions imposed by the Hansen and Jagannathan estimator, which may assign zero probabilities for some states of nature.

By adding the Fama, French and Carhart factors, we observe an increase in the kurtosis as well as a decrease in the number of statistically significant funds. The introduction of trend-following factors in the Fung and Hsieh model significantly increases the dispersion of the alpha estimates. Surprisingly, 22% of the estimated alphas are statistically significant for the OLS framework. This picture is in contrast with the Cressie-Read results for which the percentage of significant alphas are lower or equal those of Fama, French and Carhart model. Furthermore, across all estimators, the point estimates for the alphas are superior to those of the previous two models. Finally, the Agarwal and Naik model increases substantially the variance of the cross-sectional alphas. The inclusion of option strategies in the factor model generates alpha distributions with positive skewness

[34] Using the full-sample estimate of the SDF to compute the $\alpha$s will also have implications for bootstrapping the t-statistics. If we use the full-sample $\lambda$s without re-estimating them in the bootstrap, this would be equivalent to bootstrapping the hedge fund returns and a fixed estimated SDF, implying that our re-sampling would not be entirely independent.

and higher kurtosis. Nonetheless, the percentages of significant alphas are similar to that of the Fung and Hsieh model.

**6.3.1.2 Alpha Comparisons between Estimators** In Figure 5, we plot the kernel densities of the cross-sectional distributions of alphas for three values of gamma (-3.5,-1 and 1) and for the Jensen measure. The densities reflect our previous results. They are similar for the CAPM model, but start to differ with the introduction of long-short portfolios in the Fama, French and Carhart model, with more mass in the tails for the negative $\gamma$s. The biggest differences arise when we include the non-linear, option-like, portfolios. The two bottom panels share three features: (i) for all $\gamma$s, as well as the Jensen's alpha, the tails of the cross-sectional distribution are fatter (note the difference in the height of the distribution); (ii) The difference in the distributions between Jensen and Cressie-Read alphas is stronger for $\gamma = -3.5$; (iii) There is a slightly perceptible difference between the distribution of the Hansen and Jagannathan alpha and the Jensen one, with the former more concentrated towards zero. This feature is explained by the theoretical properties of this estimator[35].

The similarity between the cross-sectional alpha distributions between estimators does not mean that the individual fund alphas are similar to each other. To illustrate the potential differences, we report in Table 10 the estimated alphas of the funds located at selected quantiles of the Hansen and Jagannathan distribution ($\gamma = 1$) for the Fung and Hsieh model. For the other values of $\gamma$, we report for each quantile the performance of the corresponding fund. The estimated alphas for the median fund vary substantially, from 0.23% for the Hansen and Jagannathan case to 0.05% when $\gamma = -3.5$. Moving towards the right of the distribution (75% and 90% quantiles), the alphas are more similar across $\gamma$s. An interesting result occurs in the 95% quantile. While all alphas are positive, the alphas associated with more negative gammas are statistically insignificant, while for the Hansen and Jagannathan case the estimates are significant at a 5% level. In the appendix, we also report the results of the other three models, which we synthesize here. The results are similar, but the heterogeneity across $\gamma$s for each percentile increases in the

[35]When $\gamma > 0$ our theoretical results shows that the SDF might be zero for some states of nature. This is notably the case with the non-linear, option-like, portfolios that force the SDF to behave in this way to price all basis assets correctly. Thus, depending on the degree of non-linearity of the basis assets, positive $\gamma$ SDFs might contain this arguably undesirable property (assign zero probability to some states).

Agarwal and Naik model, with switches from negative to positive values. It emphasizes how a fund can be evaluated vry differently by different investors.

To further investigate the issue, Figure 8 present the estimated Kernel Density of the difference between the Cressie-Read and Jensen's alpha for each fund for all $\gamma$s except the CAPM and the Hansen and Jagannathan estimators [36]. Note that as highlighted in section 3.2 this difference allows us to capture the disagreement about fund performance between investors as well as test for the contributions of the higher-order moments in the Cressie-Read alpha on top of the OLS one. Overall, results behave as expected: (i) when we include the non-linear portfolios the heterogeneity increases substantially, in comparison with the Fama, French and Carhart model. (ii) Moving away from the Hansen and Jagannathan case further increases this heterogeneity. In particular, for negative $\gamma s$ it is not rare to encounter funds with a difference of 0.25% on the estimated alpha. Note that is a meaningful difference compared to the median Jensen's alpha of about 0.20% per month.

To formally test whether this difference in performance is statistically meaningful we implement a paired mean test. Figure 9 plots the cross-sectional kernel density of the estimated t-statistics. This figure contains four panels, one for each set of basis assets we consider throughout the paper. Notably, for both the CAPM and Fama, French and Carhart models most estimated alpha differences are insignificant. This feature indicates that for both models we have no evidence of disagreement in performance across investors and that the additional alpha contribution of the higher-order moments is zero. This picture changes when we include the nonlinear factors in both Fung and Hsieh or Agarwal and Naik models. In both cases, the variance of the cross-sectional distribution of the t-statistics is much higher. In fact, for the later model, almost 10% of the funds have Cressie-Read alphas that are statistically significant than their OLS counterparts when $\gamma = -3.5$. As before, when the Cressie-Read gammas approach the Hansen and Jagannathan case the difference in alphas become less significant.

**6.3.1.3 T-statistics Comparisons between Estimators** Figure 6 plots the cross-sectional kernel densities of the asymptotic t-statistics for the four sets of factors and for the three same values of gamma and the Jensen's alpha. The Cressie-Read cross-sectional

[36] Given that in both cases the heterogeneity is quite small, this made it harder to visualize the densities for these two cases.

t-statistics are much more conservative than the OLS ones, but are very similar to one another. The only significant difference is for the Agarwal and Naik estimates when $gamma = -3.5$. In this case, the cross-sectional distribution is slightly more positive skewed tahn for the other gamma values.

Figure 7 plots the same set of figures for the bootstrapped t-statistics. Note that to bootstrap the OLS alpha we adopt the procedure of Kosowski et al. (2006), while for our procedure we rely on a non-parametric bootstrap. One advantage of our method, in comparison with Kosowski et al. (2006), is that our approach re-samples both the hedge fund returns as well as the underlying factors. Thus, we partially accommodate the critiques in Fama and French (2010) regarding the joint sample of fund's returns and factors[37]. Interestingly, the bootstrap distribution of the OLS alpha t-statistics is much closer to the Cressie-Read alphas. This reveals that the asymptotic approximation in the OLS case bias the t-statistics estimate upward (as we see in table 9).

Focusing on the bottom two panels of Figure 7 we note that the Hansen and Jagannathan bootstrapped t-statistics distribution is much more concentrated around zero. Again, this arises from the fact that when $\gamma > 0$ the SDF may assume zero probability. In the bootstrapped distribution, due to the inherent "repetition" of the re-sampling method, this feature is exacerbated. That is, for the Hansen and Jagannathan to correctly price the underlying assets it must assume zero probability in several states of nature. This means that the alpha estimates are significantly biased towards zero, implying a much more concentrated distribution. Once more, this feature reveals that departing from the quadratic case can not only provide theoretical interesting properties (higher order moments exposures) but also avoid empirical shortcomings.

Complementing this analysis, figure 10, compares the asymptotic and bootstrapped cross-section distributions for selected $\gamma s$ as well as for the OLS case. In the top panels, we plot the empirical densities, while the bottom panels plot the cumulative distribution functions. Several interesting features arise from this analysis. First, note that the cross-sectional and asymptotic distributions are very similar to each other for the negative gammas (as highlighted before). Again, for the positive range of the Cressie-Read family, the bootstrapped distribution is much more concentrated around zero. As discussed,

[37] To fully incorporate the issues raised by Fama and French we should jointly re-sample all funds, instead of performing the fund-by-fund bootstrap as we do. Unfortunately, due to the small samples we have for most hedge funds (the median hedge fund contains 71 observations), this procedure is not practically implementable.

this feature results from the fact that in the bootstrap samples positive members of the Cressie-Read family need to assign a considerable number of zero probability states to price the basis assets correctly. More interestingly, however, is the comparison between the CDF's across negative gammas and the OLS case. Contrary to the OLS, when evaluating funds using the bootstrapped t-statics for the negative gammas, we note that more funds get positive statistically significant alphas. This is indicated by the blue lines below the red ones (for the negative gammas) and the inverse relationship for the OLS alpha. Thus, when comparing the bootstrapped t-statistics we note a convergence on the cross-sectional distribution across estimators (compared to the asymptotic one). Again, we must stress that this does not imply a convergence in a fund by fund basis.

**6.3.1.4 Performance across strategies** Table 11 collects for all categories of funds the median, 10% and 90% quantile alphas when using the Fung and Hsieh basis assets. Not surprisingly, across all categories, there is a substantial variability across all estimators on the individual hedge fund alphas. It is interesting to compare the median fund alphas with the average for the corresponding index. For some strategies (e.g. Convertible Arbitrage, Emerging Markets, etc) there is a substantial difference in the median alphas and that of the constructed index. Second, while in the individual fund analysis only a handful of funds presented a statistically significant alpha (see the summary statistics table), the results for the indexes are significantly better due to the diversification across funds. This can also be seen in the smaller interquartile for the distribution of the individual funds in the Funds of Funds category.

### 6.3.2 The Cross Section of Hedge Fund Alphas

Our empirical findings revealed a significant heterogeneity in the alpha estimates both across models (CAPM, Fama, French, and Carhart, etc) and estimators (different gammas and Jensen). Theoretically, we showed that under the Cressie-Read framework each fund alpha loads on its higher-order mixed moments. Moreover, from the OLS results, we know that the Jensen's alpha is by definition given by covariances with the chosen factors. In this section we provide a different perspective by analyzing the cross-section of hedge fund alphas and t-statistics. First we measure in a regression framework the contribution of several potential drivers of the alphas cross-sectionally. Second we propose a measure

of investors' divergence about the performance of a fund.

**6.3.2.1 Alpha Drivers** For the 4815 funds, we regress their individual alphas on several key variables. Based on our theoretical results, we construct the co-skewness and co-kurtosis of each hedge fund with the market return (CRSP value-weighted portfolio). Our findings about the SDF $\lambda s$ indicate that for all gammas and basis assets, the market factor is uniformly significant. Thus, these two measures capture potentially the two most important higher-order mixed moments that drive each hedge fund alpha. In addition, we also include a proxy for the hedge fund idiosyncratic volatility, measured according to the Fama, French, and Carhart factors[38]. To control for the hedge fund own higher-order moments we also include as explanatory variables their skewness and kurtosis. Furthermore, given the known persistence in hedge fund returns (see Jagannathan et al. (2010)), we add the first autocorrelation of the returns as an additional control. Finally, size captures the average assets under management throughout the hedge fund life span and age the number of observable returns.

Table 12 presents the regression results with the Fung and Hsieh alphas for all possible gammas. Not surprisingly, the average hedge fund return is by far the more relevant explanatory variable, but the coefficient decreases with $\gamma$. More interestingly, for all estimators, the co-skewness is not statistically significant. In contrast, funds with high co-kurtosis are penalized in terms of alpha. Intuitively, this is expected, since co-kurtosis captures the likelihood of joint extreme events. Furthermore, the latter effect is stronger for the Hansen and Jagannathan estimator (-0.23) in comparison with the more negative gammas. Idiosyncratic volatility affects negatively and significantly hedge fund alphas for the negative gammas. Concerning the hedge fund own higher-order moments, skewness increases significantly the estimated alpha, but a fund's kurtosis is not significantly related to its alpha.

Considering the additional control variables, for most gammas, size and age are statistically significant with positive coefficients. Size is less important for more negative gammas and the age effect seems to be insensitive to nonlinearities. As expected, funds with smoother returns have, on average, higher alphas. This effect is mechanically related

[38]Our results also hold for both the Fung and Hsieh factors as well as the Agarwal and Naik factors. Given that the standard in the literature is to calculate idiosyncratic volatility using the Fama, French and Carhart factors we present the main results using this framework.

to the way we compute our alphas[39].

**6.3.2.2 Investors' Divergence about fund performance** To capture the substantial variation in the fund-by-fund alpha estimates across estimators, we propose a divergence measure defined as the logarithm of the alpha variance for each fund across $\gamma s$. Table 13 relates this measure to the above-discussed drivers of the alphas for the four sets of basis assets. Except for the CAPM, we note that the higher the hedge fund mean, the lower is the disagreement between investors. As expected, both co-skewness and co-kurtosis affect the disagreement positively. Theoretically, this is explained by the different loads on these variables depending on the gammas used in the estimation (see figure 1). Moreover, across all basis assets, a higher idiosyncratic volatility also implies a higher disagreement.

The effects of the hedge fund own higher order moments on disagreement are also intuitive. First, given the investor's known skewness preference, higher skewness is associated with lower disagreement. Second, higher kurtosis indicates that the hedge fund return distribution is more concentrated around its mean. Thus, higher kurtosis gives less room for disagreement (the empirical bounds on the alpha estimates will be tighter). Finally, the last interesting result is for the age variable. Since it captures the number of monthly observations available for each fund, increasing the number of observations reduces the disagreement in valuation (i.e. the more information one have about the fund, the more the investors agree about its performance).

# 7 Conclusion

In this article, we build on and expand the performance measure methodology proposed by Chen and Knez (1996). Under this general framework, we propose a novel class of performance measures based on Cressie-Read discrepancies that are easy to estimate, satisfy non-arbitrage restrictions, and contains both theoretical and empirical desirable properties. These performance measures encompass the known Hansen and Jagannathan estimator explored in Chen and Knez (1996) and Li et al. (2010). Additionally, we demonstrate that departing from the quadratic case (HJ) allows our performance measure to

[39] One possible way to by-pass this is to follow Jagannathan et al. (2010) and estimate the "relative" alphas (i.e. include, say, the hedge fund primary strategy index as an additional factor on top of the ones we already consider).

embed information about the higher-order mixed moments of the hedge fund return with respect to the basis assets used in the stochastic discount factor estimation.

Concerning hypothesis testing, we extend Almeida and Garcia (2012) and provide a full set of asymptotic results for both the Lagrange multipliers, associated with the stochastic discount factor estimation, as well as the performance measure. Given the known non-normality of both hedge fund returns as well as the traditional hedge fund benchmarks (e.g. Fung and Hsieh (2001)) we propose a nonparametric bootstrap in the lines of Kosowski et al. (2006) to test for the statistical significance of the individual hedge fund alpha estimates.

Empirically, we illustrate our methodology using a large set of individual hedge fund data. We pay particular attention to the filtering of the raw data by combining a large set of filters. We show that neglecting these filters might severely bias upwards the evaluation of performance. Our results reveal a sizable heterogeneity in the funds' valuation depending on the loads on the higher-order moments. In particular, we often find hedge funds with statistically significant OLS alphas but insignificant, and even negative alphas for our non-parametric estimators. Moreover, we find meaningful economic differences between performance measures both in absolute terms as well as in relative terms (the cross-section ranking of the hedge funds).

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Table 1: **Summary Statistics: Hedge Funds Final Sample - Time Series**

| Year | Year Start | Entries | Dissolved | Year End | AUM | Mean | Median | Sd. Dev. | Min | Max |
|---|---|---|---|---|---|---|---|---|---|---|
| 1994 | 529 | 139 | 0 | 668 | 41.71 | 0.17 | 0.18 | 1.03 | -1.86 | 2.23 |
| 1995 | 668 | 166 | 0 | 863 | 51.05 | 1.30 | 1.57 | 1.32 | -1.19 | 3.62 |
| 1996 | 863 | 157 | 67 | 1002 | 63.75 | 1.35 | 1.22 | 1.82 | -1.74 | 4.16 |
| 1997 | 1002 | 192 | 58 | 1183 | 94.77 | 1.28 | 1.82 | 2.10 | -1.72 | 4.87 |
| 1998 | 1183 | 188 | 77 | 1335 | 98.97 | 0.22 | 0.06 | 1.88 | -4.27 | 2.84 |
| 1999 | 1335 | 184 | 95 | 1453 | 113.10 | 1.77 | 0.71 | 2.04 | -0.11 | 5.81 |
| 2000 | 1453 | 204 | 124 | 1589 | 115.01 | 0.71 | 0.23 | 2.13 | -2.26 | 4.47 |
| 2001 | 1589 | 214 | 120 | 1710 | 141.39 | 0.47 | 0.39 | 1.13 | -1.51 | 2.47 |
| 2002 | 1710 | 228 | 107 | 1841 | 163.93 | 0.30 | 0.45 | 0.87 | -1.26 | 1.55 |
| 2003 | 1841 | 235 | 118 | 2020 | 238.63 | 1.37 | 1.20 | 1.14 | -0.57 | 3.84 |
| 2004 | 2020 | 260 | 125 | 2205 | 323.82 | 0.67 | 0.82 | 1.33 | -1.53 | 3.09 |
| 2005 | 2205 | 280 | 194 | 2319 | 358.23 | 0.70 | 1.33 | 1.41 | -1.69 | 2.09 |
| 2006 | 2319 | 260 | 221 | 2374 | 419.88 | 0.98 | 1.30 | 1.44 | -1.57 | 3.32 |
| 2007 | 2374 | 186 | 282 | 2286 | 486.48 | 0.93 | 0.87 | 1.48 | -1.75 | 3.01 |
| 2008 | 2286 | 168 | 417 | 2034 | 376.31 | -1.52 | -1.84 | 2.54 | -5.75 | 1.92 |
| 2009 | 2034 | 152 | 220 | 1948 | 343.50 | 1.43 | 1.33 | 1.67 | -1.10 | 5.02 |
| 2010 | 1948 | 102 | 229 | 1798 | 315.79 | 0.85 | 0.98 | 1.79 | -2.77 | 3.31 |
| 2011 | 1798 | 79 | 271 | 1586 | 278.70 | -0.43 | -0.36 | 1.69 | -3.52 | 2.04 |
| 2012 | 1586 | 51 | 222 | 1388 | 270.08 | 0.55 | 0.65 | 1.22 | -1.93 | 2.46 |
| 2013 | 1388 | 22 | 179 | 1198 | 223.44 | 0.85 | 0.95 | 1.25 | -1.74 | 3.23 |
| 2014 | 1198 | - | - | - | 234.54 | 0.27 | -0.28 | 0.97 | -0.75 | 1.90 |
| 2015 | 1013 | - | - | - | - | 0.46 | 0.70 | 1.12 | -1.65 | 1.68 |

This table presents the summary statistics for the Hedge Fund sample that survive all the filters described in the Appendix. There are a total of 4815 funds in this sample. In each column we report (in order), for a given year, the number of hedge funds in the beginning of the year, the number of entrants, the number of dissolved, the number of hedge funds in the year end, and the total assets under management in billions of dollars. The subsequent columns report, for a equally weighted index, the mean, median, standard deviation, minimum and maximum return.

Table 2: **Summary Statistics: Hedge Funds Final Sample - Cross Section**

| | N | Mean | Median | S.d. | Min | Max |
|---|---|---|---|---|---|---|
| Avg. Ret | 4815 | 0.57 | 0.53 | 0.83 | -8.76 | 18.30 |
| Avg. AUM | 4815 | 0.13 | 0.03 | 0.41 | 0.00 | 17.44 |
| Age | 4815 | 85.43 | 71.00 | 53.52 | 24.00 | 258.00 |
| Man. Fee | 4806 | 1.50 | 1.50 | 0.72 | 0.00 | 8.00 |
| Perf. Fee | 4789 | 16.02 | 20.00 | 7.47 | 0.00 | 50.00 |

This table presents the cross sectional summary statistics for the Hedge Fund sample that survive all the filters described in the Appendix. There are a total of 4815 funds in this sample. In each column we report (in order), the mean, median, standard deviation, minimum and maximum return for the average hedge fund return (first we calculate the average return for each hedge fund. Second we compute the statistics).

Table 3: **Summary Statistics: Hedge Funds Higher Order Moments**

| Category | N | Mean | SD | Skew | Kurtosis |
|---|---|---|---|---|---|
| Convertible Arbitrage | 103 | 0.53 | 2.61 | -0.85 | 10.59 |
| CTA | 412 | 0.65 | 5.70 | 0.37 | 5.03 |
| Emerging Markets | 354 | 0.60 | 6.12 | -0.41 | 7.79 |
| Equity Market Neutral | 176 | 0.44 | 2.43 | -0.22 | 6.47 |
| Event Driven | 310 | 0.73 | 3.00 | -0.46 | 7.44 |
| Fixed Income Arbitrage | 113 | 0.53 | 2.47 | -1.40 | 14.45 |
| Fund of Funds | 1222 | 0.36 | 2.67 | -0.75 | 7.25 |
| Global Macro | 187 | 0.58 | 4.26 | 0.24 | 6.29 |
| Long/Short Equity Hedge | 1162 | 0.76 | 4.98 | -0.03 | 5.56 |
| Managed Futures | 351 | 0.53 | 5.44 | 0.13 | 5.61 |
| Multi-Strategy | 244 | 0.52 | 3.24 | -0.42 | 8.18 |

This table presents the summary statistics for the higher order moments of the Hedge Fund sample that survive all the filters described in the Appendix. We segregate Hedge Funds by their primary category. There are a total of 4815 funds in this sample. In each column we report (in order), the number of hedge funds in each category, the mean, standard deviation, skewness and kurtosis. All statistics are calculated at the fund level and then averaged across funds.

Table 4: **Summary Statistics: Hedge Funds Index Higher Order Moments**

| Category | N | Mean | SD | Skew | Kurtosis |
|---|---|---|---|---|---|
| Convertible Arbitrage | 103 | 0.61 | 2.20 | -2.40 | 22.54 |
| CTA | 412 | 0.62 | 2.07 | 0.45 | 3.25 |
| Emerging Markets | 354 | 0.81 | 4.21 | -1.08 | 7.93 |
| Equity Market Neutral | 176 | 0.60 | 0.88 | -0.37 | 5.66 |
| Event Driven | 310 | 0.77 | 1.74 | -1.51 | 8.61 |
| Fixed Income Arbitrage | 113 | 0.61 | 1.38 | -2.87 | 19.27 |
| Fund of Funds | 1222 | 0.47 | 1.61 | -0.48 | 5.61 |
| Global Macro | 187 | 0.65 | 1.72 | 0.76 | 4.48 |
| Long/Short Equity Hedge | 1162 | 0.93 | 2.71 | -0.26 | 4.41 |
| Managed Futures | 351 | 0.64 | 2.89 | 0.28 | 2.80 |
| Multi-Strategy | 244 | 0.72 | 1.38 | -0.82 | 5.43 |

This table presents the summary statistics for the higher order moments of the Hedge Fund equally weighted indexes, by category, for the sample that survive all the filters described in the Appendix. In each column we report (in order), the number of hedge funds in each category, the mean, standard deviation, skewness and kurtosis for each category of primary strategy.

Table 5: **Summary Statistics: Hedge Funds Factors**

| | Mean | Median | S.d. | Min | Max | Skew | Kurt |
|---|---|---|---|---|---|---|---|
| MKT | 0.63 | 1.33 | 4.42 | -17.23 | 11.35 | -0.74 | 4.19 |
| SMB | 0.17 | 0.01 | 3.40 | -17.17 | 22.08 | 0.81 | 11.76 |
| HML | 0.20 | -0.01 | 3.11 | -11.25 | 12.91 | 0.09 | 5.80 |
| MOM | 0.45 | 0.58 | 5.16 | -34.58 | 18.38 | -1.59 | 13.62 |
| Bond | -0.16 | -0.71 | 6.07 | -31.44 | 19.52 | -0.23 | 6.52 |
| Credit Spread | 0.34 | -0.36 | 6.41 | -20.75 | 40.06 | 1.41 | 9.53 |
| S&P 500 | 0.83 | 1.35 | 4.29 | -16.80 | 10.93 | -0.70 | 4.17 |
| Size Spread | 0.05 | -0.01 | 3.29 | -16.38 | 18.41 | 0.26 | 7.89 |
| Emerging MKT | 0.67 | 0.85 | 6.71 | -28.91 | 17.14 | -0.70 | 4.97 |
| PTFSBD | -1.33 | -3.70 | 15.31 | -26.63 | 68.86 | 1.34 | 5.37 |
| PTFSFX | -0.61 | -4.86 | 19.58 | -30.13 | 90.27 | 1.38 | 5.60 |
| PTFSCOM | -0.28 | -2.96 | 14.28 | -24.65 | 64.75 | 1.09 | 4.65 |
| PTFSIR | -0.66 | -5.75 | 25.84 | -35.13 | 221.92 | 4.28 | 30.75 |
| PTFSSTK | -4.99 | -6.89 | 13.38 | -30.19 | 60.48 | 1.35 | 6.46 |
| ATM PUT | -16.74 | -49.48 | 86.39 | -96.63 | 333.20 | 1.43 | 4.78 |
| OTM PUT | -18.62 | -56.83 | 90.00 | -97.09 | 345.90 | 1.54 | 4.98 |
| ATM CALL | -6.15 | -27.32 | 82.55 | -99.55 | 241.70 | 0.70 | 2.44 |
| OTM CALL | -7.45 | -32.38 | 87.23 | -99.50 | 300.00 | 0.86 | 2.93 |

This table presents the summary statistics for the Hedge Fund factors we use throughout our empirical applications. MKT, SMB, HML and MOM indicate the CRSP value weighted portfolio, the Small minus Big factor, the High minus Low factor and the Momentum factor from Kenneth French website. The Bond Market factor and Credit Spread factor are constructed using the monthly change in the 10-Year constant maturity yield and Moody's Baa yield minus the 10-Year constant maturity yield following Fung and Hsieh (2001) (available at the H15 forms, Federal Board of Governors and at St. Louis FRED respectively). The S&P 500 return is constructed using the total returns from Datastream (RI code) as in Fung and Hsieh (2001). The size spread is constructed as in Fung and Hsieh (2001) using the difference between the Russel 2000 and the S&P 500 total return from Datastream. The emerging market factor is constructed using the MSCI Emerging Market index monthly total return from Datastream. The trend following strategies (PTFS) are available in Fung's web-site. The four option strategies are constructed following Agarwal and Naik (2004) using raw data from Option Metrics. In each column we report (in order), the mean, median, standard deviation, minimum, maximum, skewness and kurtosis for each factor.

Table 6: **Fung and Hsieh (2001) Alpha for Hedge Fund Indexes**

| | -3.5 | -2 | -1 | -0.5 | 0 | 0.5 | 1 | Lin |
|---|---|---|---|---|---|---|---|---|
| Convertible Arbitrage | 0.31 | 0.31 | 0.31 | 0.30 | 0.30 | 0.29 | 0.29 | 0.29 |
| T (Asym) | (1.02) | (1.09) | (1.07) | (1.03) | (1.01) | (1) | (0.98) | (1.74) |
| T (Boot) | (2.37) | (2.55) | (2.61) | (2.64) | (2.49) | (2.4) | (2.36) | (1.74) |
| CTA | 0.70 | 0.67 | 0.63 | 0.61 | 0.60 | 0.59 | 0.58 | 0.58 |
| T (Asym) | (3.01) | (3.1) | (3.35) | (3.56) | (3.73) | (3.84) | (3.88) | (4.81) |
| T (Boot) | (5.33) | (5.41) | (5.54) | (5.36) | (5.19) | (5.21) | (5.04) | (4.85) |
| Emerging Markets | 0.07 | 0.16 | 0.22 | 0.26 | 0.30 | 0.35 | 0.38 | 0.38 |
| T (Asym) | (0.08) | (0.22) | (0.34) | (0.41) | (0.53) | (0.66) | (0.77) | (2.28) |
| T (Boot) | (0.32) | (0.8) | (1.24) | (1.49) | (1.86) | (2.3) | (2.56) | (2.26) |
| Equity Market Neutral | 0.35 | 0.35 | 0.35 | 0.34 | 0.34 | 0.34 | 0.35 | 0.35 |
| T (Asym) | (4.13) | (4.56) | (4.57) | (4.49) | (4.48) | (4.47) | (4.47) | (5.69) |
| T (Boot) | (4.9) | (5.46) | (5.72) | (6.14) | (6.39) | (6.11) | (6.69) | (5.58) |
| Event Driven | 0.33 | 0.35 | 0.37 | 0.37 | 0.39 | 0.40 | 0.41 | 0.41 |
| T (Asym) | (0.97) | (1.16) | (1.32) | (1.42) | (1.55) | (1.66) | (1.74) | (5.06) |
| T (Boot) | (3.79) | (4.47) | (4.85) | (5.25) | (5.68) | (5.96) | (6.38) | (5.15) |
| Fixed Income Arbitrage | 0.44 | 0.45 | 0.43 | 0.42 | 0.42 | 0.41 | 0.42 | 0.42 |
| T (Asym) | (2.91) | (3.11) | (3.04) | (2.91) | (2.82) | (2.76) | (2.76) | (4.39) |
| T (Boot) | (4.92) | (5.39) | (5.46) | (5.58) | (5.22) | (5.29) | (5.54) | (4.64) |
| Fund of Funds | 0.20 | 0.21 | 0.20 | 0.20 | 0.20 | 0.21 | 0.21 | 0.21 |
| T (Asym) | (0.83) | (0.98) | (1.02) | (1.04) | (1.1) | (1.16) | (1.21) | (3.03) |
| T (Boot) | (2.62) | (2.93) | (2.83) | (2.94) | (2.95) | (3.14) | (3.18) | (2.95) |
| Global Macro | 0.43 | 0.42 | 0.43 | 0.44 | 0.45 | 0.45 | 0.45 | 0.45 |
| T (Asym) | (2.85) | (3.18) | (3.41) | (3.52) | (3.59) | (3.64) | (3.67) | (4.97) |
| T (Boot) | (3.74) | (3.9) | (4.47) | (4.6) | (4.88) | (4.89) | (4.79) | (4.71) |
| Long/Short Equity Hedge | 0.42 | 0.44 | 0.43 | 0.42 | 0.42 | 0.42 | 0.42 | 0.42 |
| T (Asym) | (1.02) | (1.17) | (1.25) | (1.28) | (1.33) | (1.36) | (1.39) | (4.58) |
| T (Boot) | (4.8) | (5.48) | (5.72) | (5.62) | (5.65) | (5.51) | (5.53) | (4.57) |
| Managed Futures | 0.93 | 0.84 | 0.76 | 0.73 | 0.71 | 0.69 | 0.69 | 0.69 |
| T (Asym) | (2.61) | (2.61) | (2.74) | (2.89) | (3.05) | (3.19) | (3.27) | (3.91) |
| T (Boot) | (3.94) | (3.83) | (3.81) | (3.84) | (3.7) | (3.88) | (3.83) | (3.68) |
| Multi-Strategy | 0.43 | 0.43 | 0.42 | 0.41 | 0.41 | 0.41 | 0.41 | 0.41 |
| T (Asym) | (2.13) | (2.4) | (2.47) | (2.49) | (2.54) | (2.57) | (2.57) | (5.85) |
| T (Boot) | (7.09) | (7.69) | (7.54) | (7.79) | (7.52) | (7.26) | (7.35) | (5.95) |

This table presents the estimated non-parametric $\alpha$ measure for Hedge Fund indexes associated with the Cressie Read stochastic discount factor when we use the Fung and Hsieh (2001) factors as the benchmark assets. In the last column we also add the OLS Jensen's alpha for comparison. Additional columns indicate the Cressie Read $\gamma$ used in the estimation. We report the estimated $\alpha$, the asymptotic $t-statistics$ as well as the bootstrapped t-statistics (calculated using bootstrapped standard errors from 1000 re-samples). Bootstrapped t-statistics for the linear model follow the residual bootstrap suggested by Kosowski et al. (2006). Asymptotic T-statistics are calculated using Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors with the optimal number of lags given by $\lfloor 4(T/100)^{2/9} \rfloor$. Hedge Fund indexes are equally weighted based on self reported primary strategy using all available data, after applying our filters, for each month. The estimation use the full sample from January 1994 to June 2015.

Table 7: **Agarwal and Naik (2004) Alpha for Hedge Fund Indexes**

| | -3.5 | -2 | -1 | -0.5 | 0 | 0.5 | 1 | Lin |
|---|---|---|---|---|---|---|---|---|
| Convertible Arbitrage | 0.33 | 0.33 | 0.30 | 0.27 | 0.26 | 0.25 | 0.25 | 0.25 |
| T (Asym) | (1.58) | (1.47) | (1.21) | (1.07) | (0.96) | (0.88) | (0.85) | (1.49) |
| T (Boot) | (2.38) | (2.48) | (2.42) | (2.22) | (2.02) | (1.95) | (1.89) | (1.43) |
| CTA | 0.43 | 0.52 | 0.58 | 0.60 | 0.61 | 0.61 | 0.61 | 0.61 |
| T (Asym) | (3.35) | (3.82) | (4) | (4.13) | (4.23) | (4.29) | (4.32) | (4.92) |
| T (Boot) | (2.21) | (3) | (3.85) | (4.34) | (4.43) | (4.52) | (4.38) | (4.83) |
| Emerging Markets | 0.27 | 0.23 | 0.21 | 0.22 | 0.25 | 0.28 | 0.31 | 0.33 |
| T (Asym) | (0.63) | (0.46) | (0.39) | (0.42) | (0.5) | (0.58) | (0.64) | (2.26) |
| T (Boot) | (1.33) | (1.27) | (1.22) | (1.42) | (1.58) | (1.96) | (2.17) | (2.17) |
| Equity Market Neutral | 0.38 | 0.36 | 0.34 | 0.33 | 0.33 | 0.33 | 0.33 | 0.33 |
| T (Asym) | (4.38) | (4.43) | (4.55) | (4.58) | (4.53) | (4.46) | (4.42) | (5.75) |
| T (Boot) | (4.45) | (4.79) | (5.53) | (5.54) | (5.75) | (5.8) | (5.92) | (5.52) |
| Event Driven | 0.45 | 0.42 | 0.39 | 0.38 | 0.38 | 0.38 | 0.39 | 0.39 |
| T (Asym) | (2.7) | (2.17) | (1.79) | (1.73) | (1.72) | (1.72) | (1.72) | (5.08) |
| T (Boot) | (3.74) | (3.81) | (4.26) | (4.49) | (4.71) | (4.96) | (5.15) | (4.95) |
| Fixed Income Arbitrage | 0.25 | 0.30 | 0.32 | 0.32 | 0.33 | 0.33 | 0.33 | 0.34 |
| T (Asym) | (1.09) | (1.58) | (1.9) | (1.94) | (1.99) | (2.03) | (2.05) | (3.17) |
| T (Boot) | (1.8) | (2.41) | (3) | (3.12) | (3.32) | (3.47) | (3.76) | (3.35) |
| Fund of Funds | 0.10 | 0.14 | 0.15 | 0.15 | 0.16 | 0.16 | 0.17 | 0.17 |
| T (Asym) | (0.58) | (0.85) | (0.86) | (0.88) | (0.9) | (0.94) | (0.97) | (2.29) |
| T (Boot) | (1.01) | (1.54) | (1.91) | (1.98) | (2.07) | (2.3) | (2.36) | (2.36) |
| Global Macro | 0.26 | 0.35 | 0.37 | 0.38 | 0.38 | 0.39 | 0.40 | 0.41 |
| T (Asym) | (1.25) | (2.11) | (2.5) | (2.61) | (2.74) | (2.89) | (3.02) | (3.66) |
| T (Boot) | (1.71) | (2.54) | (3.17) | (3.45) | (3.63) | (3.82) | (3.78) | (3.81) |
| Long/Short Equity Hedge | 0.29 | 0.31 | 0.33 | 0.34 | 0.34 | 0.35 | 0.36 | 0.36 |
| T (Asym) | (0.96) | (1.04) | (1.13) | (1.17) | (1.2) | (1.22) | (1.24) | (4.09) |
| T (Boot) | (3.68) | (3.95) | (4.41) | (4.59) | (4.53) | (4.82) | (4.69) | (4.1) |
| Managed Futures | 0.43 | 0.60 | 0.67 | 0.70 | 0.71 | 0.72 | 0.73 | 0.72 |
| T (Asym) | (2.07) | (2.96) | (3.17) | (3.28) | (3.4) | (3.48) | (3.53) | (3.77) |
| T (Boot) | (1.37) | (2.18) | (2.92) | (3.3) | (3.44) | (3.47) | (3.42) | (3.65) |
| Multi-Strategy | 0.32 | 0.35 | 0.35 | 0.35 | 0.36 | 0.36 | 0.36 | 0.37 |
| T (Asym) | (2) | (2.4) | (2.36) | (2.33) | (2.32) | (2.31) | (2.31) | (5.76) |
| T (Boot) | (4.18) | (4.95) | (5.84) | (5.96) | (5.86) | (6.31) | (6.26) | (5.82) |

This table presents the estimated non-parametric $\alpha$ measure for Hedge Fund indexes associated with the Cressie Read stochastic discount factor when we use the Agarwal and Naik (2004) factors as the benchmark assets. In the last column we also add the OLS Jensen's alpha for comparison. Additional columns indicate the Cressie Read $\gamma$ used in the estimation. We report the estimated $\alpha$, the asymptotic $t-statistics$ as well as the bootstrapped t-statistics (calculated using bootstrapped standard errors from 1000 re-samples). Bootstrapped t-statistics for the linear model follow the residual bootstrap suggested by Kosowski et al. (2006). Asymptotic T-statistics are calculated using Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors with the optimal number of lags given by $\lfloor 4(T/100)^{2/9} \rfloor$. Hedge Fund indexes are equally weighted based on self reported primary strategy using all available data, after applying our filters, for each month. The estimation use the full sample from January 1994 to June 2015.

Table 8: **CAPM Non Linear Alpha for Hedge Fund Indexes**

| | CAPM | (2) | (3) | (4) | $\gamma = -3.5$ | $\gamma = -1$ | $\gamma = 0.5$ |
|---|---|---|---|---|---|---|---|
| Convertible Arbitrage | 0.21 | 0.44 | 0.31 | 0.08 | 0.05 | 0.19 | 0.21 |
| T (Asym) | (1.16) | (2.61) | (2.22) | (0.54) | (0.13) | (0.72) | (0.84) |
| T (Boot) | (1.17) | (2.5) | (2.19) | (0.55) | (0.25) | (1.32) | (1.57) |
| CTA | 0.43 | 0.16 | 0.22 | 0.03 | 0.49 | 0.45 | 0.43 |
| T (Asym) | (3.58) | (1.08) | (1.5) | (0.15) | (3.26) | (3.38) | (3.34) |
| T (Boot) | (3.51) | (1.08) | (1.46) | (0.16) | (3.57) | (3.35) | (3.38) |
| Emerging Markets | 0.2 | 0.66 | 0.55 | 0.42 | 0.09 | 0.17 | 0.19 |
| T (Asym) | (0.68) | (2.4) | (1.99) | (1.43) | (0.17) | (0.4) | (0.48) |
| T (Boot) | (0.7) | (2.38) | (2.04) | (1.41) | (0.33) | (0.79) | (0.94) |
| Equity Market Neutral | 0.34 | 0.38 | 0.38 | 0.36 | 0.34 | 0.34 | 0.34 |
| T (Asym) | (5.92) | (6.16) | (6.22) | (6.29) | (4.71) | (4.94) | (5.02) |
| T (Boot) | (6.09) | (6.26) | (6.73) | (5.96) | (6.9) | (6.69) | (6.79) |
| Event Driven | 0.37 | 0.57 | 0.53 | 0.46 | 0.31 | 0.36 | 0.37 |
| T (Asym) | (3.37) | (6.03) | (5.56) | (4.37) | (1.2) | (1.85) | (2.02) |
| T (Boot) | (3.5) | (6.02) | (5.82) | (4.42) | (3.01) | (4.36) | (4.66) |
| Fixed Income Arbitrage | 0.33 | 0.52 | 0.47 | 0.34 | 0.23 | 0.32 | 0.33 |
| T (Asym) | (2.78) | (4.18) | (4.53) | (2.92) | (1.08) | (2.19) | (2.39) |
| T (Boot) | (2.96) | (4.3) | (4.5) | (3.08) | (1.74) | (3.46) | (3.75) |
| Fund of Funds | 0.11 | 0.23 | 0.22 | 0.08 | 0.07 | 0.1 | 0.11 |
| T (Asym) | (1.13) | (2.68) | (2.56) | (0.84) | (0.36) | (0.66) | (0.73) |
| T (Boot) | (1.19) | (2.66) | (2.47) | (0.8) | (0.75) | (1.22) | (1.36) |
| Global Macro | 0.35 | 0.22 | 0.26 | 0.15 | 0.39 | 0.36 | 0.35 |
| T (Asym) | (3.36) | (1.89) | (2.29) | (1.18) | (3.53) | (3.25) | (3.2) |
| T (Boot) | (3.32) | (1.95) | (2.24) | (1.2) | (3.79) | (3.46) | (3.59) |
| Long/Short Equity Hedge | 0.38 | 0.4 | 0.41 | 0.3 | 0.37 | 0.38 | 0.38 |
| T (Asym) | (3.66) | (3.76) | (3.81) | (2.68) | (1.28) | (1.6) | (1.67) |
| T (Boot) | (3.76) | (3.95) | (3.69) | (2.57) | (4.09) | (4.31) | (4.51) |
| Managed Futures | 0.45 | 0.03 | 0.16 | -0.18 | 0.55 | 0.48 | 0.46 |
| T (Asym) | (2.73) | (0.14) | (0.82) | (-0.79) | (2.67) | (2.68) | (2.61) |
| T (Boot) | (2.67) | (0.14) | (0.76) | (-0.74) | (2.78) | (2.53) | (2.53) |
| Multi-Strategy | 0.36 | 0.41 | 0.42 | 0.33 | 0.35 | 0.36 | 0.36 |
| T (Asym) | (4.32) | (4.93) | (5.06) | (3.89) | (2.15) | (2.66) | (2.78) |
| T (Boot) | (4.78) | (5.05) | (5.06) | (3.83) | (5.34) | (5.83) | (6.29) |

This table presents the estimated OLS Jensen's $\alpha$ measure for Hedge Fund indexes when we artificially add higher order moments of the CRSP value weighted market portfolio in the linear regression framework. For completeness we also include the same estimates for selected values of $\gamma$. The first column present the result for the CAPM alpha. Subsequent columns add, one at the time, polynomial terms of increasing order. We report the estimated $\alpha$, the asymptotic $t - statistics$ as well as the bootstrapped t-statistics follow the residual bootstrap suggested by Kosowski et al. (2006). Hedge Fund indexes are equally weighted based on self reported primary strategy using all available data, after applying our filters, for each month. The estimation use the full sample from January 1994 to June 2015.

Table 9: **Summary Statistics Individual Fund Alpha**

| | -3.5 | -2 | -1 | -0.5 | 0 | 0.5 | 1 | Lin |
|---|---|---|---|---|---|---|---|---|
| | | | | CAPM: | | | | |
| Mean | 0.21 | 0.21 | 0.21 | 0.21 | 0.21 | 0.22 | 0.22 | 0.22 |
| Var | 0.63 | 0.59 | 0.57 | 0.56 | 0.55 | 0.54 | 0.53 | 0.53 |
| Skew | -0.32 | -0.33 | -0.31 | -0.31 | -0.26 | -0.20 | -0.22 | -0.22 |
| Kurt | 7.77 | 7.73 | 7.57 | 7.57 | 7.42 | 7.25 | 7.12 | 7.12 |
| Perc. | 0.15 | 0.16 | 0.16 | 0.16 | 0.16 | 0.16 | 0.17 | 0.21 |
| | | | | Fama, French and Carhart: | | | | |
| Mean | 0.18 | 0.18 | 0.17 | 0.17 | 0.16 | 0.16 | 0.16 | 0.16 |
| Var | 0.75 | 0.68 | 0.63 | 0.60 | 0.58 | 0.58 | 0.57 | 0.57 |
| Skew | -0.15 | -0.06 | -0.02 | 0.03 | 0.06 | 0.04 | 0.01 | 0.06 |
| Kurt | 8.40 | 8.10 | 7.95 | 7.97 | 7.80 | 7.90 | 8.05 | 7.97 |
| Perc. | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.13 | 0.17 |
| | | | | Fung and Hsieh: | | | | |
| Mean | 0.23 | 0.24 | 0.24 | 0.25 | 0.25 | 0.25 | 0.26 | 0.26 |
| Var | 0.87 | 0.88 | 0.85 | 0.83 | 0.75 | 0.71 | 0.69 | 0.70 |
| Skew | -0.14 | -0.13 | -0.09 | 0.04 | 0.08 | 0.15 | 0.16 | 0.21 |
| Kurt | 7.44 | 7.39 | 7.32 | 7.22 | 6.98 | 6.92 | 6.84 | 6.89 |
| Perc. | 0.11 | 0.11 | 0.12 | 0.12 | 0.12 | 0.13 | 0.13 | 0.22 |
| | | | | Agarwal and Naik: | | | | |
| Mean | 0.28 | 0.26 | 0.25 | 0.23 | 0.23 | 0.23 | 0.23 | 0.23 |
| Var | 1.04 | 1.06 | 1.02 | 0.98 | 0.90 | 0.88 | 0.83 | 1.00 |
| Skew | 0.64 | 0.53 | 0.46 | 0.35 | 0.31 | 0.11 | 0.15 | 0.44 |
| Kurt | 8.85 | 8.92 | 8.51 | 8.43 | 8.07 | 7.71 | 7.38 | 9.49 |
| Perc. | 0.13 | 0.12 | 0.12 | 0.13 | 0.13 | 0.13 | 0.13 | 0.20 |

This table presents the summary statistics for both Cressie-Read and Jensen's $\alpha$ (estimated via OLS) for the individual Hedge Funds in our sample. Each panel indicate the properties of the cross sectional alpha for a given set of basis assets. Perrc. represents the percentage of alphas that were positive and statistically significant at 10%. To calculate all statistics we winsorize the sample at 0.05% level.

Table 10: **Fung and Hsieh Cressie-Read Alpha: Selected Percentiles**

| | Percentiles | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Gamma | 1% | 5% | 10% | 25% | 50% | 75% | 90% | 95% | 99% |
| -3.5 | -2.02 | -1.27 | -0.99 | -0.39 | 0.04 | 0.58 | 1.58 | 0.76 | 4.01 |
| | (-0.63) | (-2.35) | (-0.75) | (-0.47) | (0.06) | (3.98) | (1.43) | (1.26) | (1.38) |
| -2 | -2.34 | -1.26 | -0.98 | -0.31 | 0.23 | 0.58 | 1.55 | 0.87 | 4.00 |
| | (-0.76) | (-2.37) | (-0.76) | (-0.43) | (0.38) | (4.2) | (1.42) | (1.46) | (1.49) |
| -1 | -2.62 | -1.23 | -0.94 | -0.23 | 0.26 | 0.58 | 1.50 | 1.21 | 3.77 |
| | (-0.88) | (-2.44) | (-0.75) | (-0.38) | (0.45) | (4.35) | (1.42) | (1.94) | (1.55) |
| -0.5 | -2.69 | -1.15 | -0.82 | -0.20 | 0.23 | 0.58 | 1.45 | 1.37 | 3.58 |
| | (-0.91) | (-2.58) | (-0.7) | (-0.36) | (0.38) | (4.43) | (1.42) | (2.12) | (1.58) |
| 0 | -2.64 | -1.07 | -0.61 | -0.17 | 0.20 | 0.59 | 1.38 | 1.48 | 3.34 |
| | (-0.91) | (-2.67) | (-0.58) | (-0.31) | (0.33) | (4.51) | (1.4) | (2.29) | (1.61) |
| 0.5 | -2.49 | -1.02 | -0.56 | -0.12 | 0.21 | 0.59 | 1.31 | 1.57 | 3.08 |
| | (-0.89) | (-2.73) | (-0.55) | (-0.21) | (0.33) | (4.56) | (1.36) | (2.45) | (1.64) |
| 1 | -2.33 | -1.00 | -0.57 | -0.10 | 0.23 | 0.60 | 1.14 | 1.61 | 2.84 |
| | (-0.86) | (-2.77) | (-0.58) | (-0.19) | (0.37) | (4.56) | (1.21) | (2.5) | (1.65) |

This table presents the estimated Cressie-Read $\alpha$ measure for selected percentiles. Estimation of the alphas use the Fung and Hsieh ten factor model as the basis assets for the SDF estimation. Percentiles breakpoints are defined by the cross sectional alphas estimated using the Hansen and Jagannathan SDF ($\gamma = 1$). Additionally, for the same breakpoints, we report the alpha performance estimated for a variety of $\gamma s$. T-statistics, in parenthesis, are calculated using Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors with the optimal number of lags given by $\lfloor 4(T/100)^{2/9} \rfloor$.

Table 11: **Cressie-Read Alpha per Category**

| | -3.5 | -2 | -1 | -0.5 | 0 | 0.5 | 1 |
|---|---|---|---|---|---|---|---|
| Convertible Arbitrage | 0.16 | 0.17 | 0.20 | 0.20 | 0.22 | 0.23 | 0.24 |
| 10% | -0.54 | -0.42 | -0.35 | -0.31 | -0.35 | -0.31 | -0.28 |
| 90% | 0.76 | 0.76 | 0.70 | 0.70 | 0.71 | 0.75 | 0.85 |
| CTA | 0.39 | 0.38 | 0.38 | 0.40 | 0.41 | 0.42 | 0.41 |
| 10% | -0.74 | -0.78 | -0.79 | -0.78 | -0.74 | -0.79 | -0.78 |
| 90% | 1.89 | 1.84 | 1.86 | 1.78 | 1.69 | 1.58 | 1.57 |
| Emerging Markets | 0.11 | 0.14 | 0.13 | 0.18 | 0.18 | 0.19 | 0.21 |
| 10% | -1.14 | -1.22 | -1.21 | -1.12 | -1.10 | -1.07 | -0.94 |
| 90% | 1.27 | 1.34 | 1.28 | 1.36 | 1.25 | 1.22 | 1.29 |
| Equity Market Neutral | 0.19 | 0.20 | 0.21 | 0.20 | 0.20 | 0.20 | 0.19 |
| 10% | -0.64 | -0.56 | -0.54 | -0.55 | -0.54 | -0.55 | -0.58 |
| 90% | 1.07 | 1.07 | 1.09 | 1.10 | 1.02 | 0.93 | 0.88 |
| Event Driven | 0.38 | 0.39 | 0.38 | 0.39 | 0.39 | 0.39 | 0.39 |
| 10% | -0.30 | -0.30 | -0.29 | -0.31 | -0.27 | -0.25 | -0.23 |
| 90% | 1.13 | 1.11 | 1.10 | 1.07 | 1.10 | 1.09 | 1.04 |
| Fixed Income Arbitrage | 0.31 | 0.30 | 0.30 | 0.31 | 0.29 | 0.30 | 0.30 |
| 10% | -0.48 | -0.38 | -0.31 | -0.30 | -0.28 | -0.25 | -0.27 |
| 90% | 1.02 | 1.11 | 0.96 | 0.97 | 0.88 | 0.88 | 0.89 |
| Fund of Funds | 0.07 | 0.08 | 0.09 | 0.09 | 0.09 | 0.09 | 0.10 |
| 10% | -0.49 | -0.46 | -0.44 | -0.44 | -0.43 | -0.41 | -0.42 |
| 90% | 0.60 | 0.60 | 0.60 | 0.61 | 0.59 | 0.59 | 0.60 |
| Global Macro | 0.26 | 0.23 | 0.22 | 0.21 | 0.24 | 0.27 | 0.29 |
| 10% | -0.86 | -0.85 | -0.91 | -0.77 | -0.71 | -0.56 | -0.51 |
| 90% | 1.19 | 1.17 | 1.19 | 1.22 | 1.19 | 1.18 | 1.13 |
| Long/Short Equity Hedge | 0.26 | 0.32 | 0.32 | 0.31 | 0.33 | 0.33 | 0.33 |
| 10% | -0.78 | -0.79 | -0.76 | -0.72 | -0.67 | -0.67 | -0.67 |
| 90% | 1.31 | 1.27 | 1.21 | 1.21 | 1.22 | 1.21 | 1.21 |
| Managed Futures | 0.41 | 0.41 | 0.41 | 0.42 | 0.42 | 0.43 | 0.43 |
| 10% | -0.98 | -0.94 | -0.94 | -0.86 | -0.86 | -0.75 | -0.72 |
| 90% | 1.79 | 1.71 | 1.59 | 1.60 | 1.61 | 1.60 | 1.62 |
| Multi-Strategy | 0.22 | 0.20 | 0.20 | 0.20 | 0.19 | 0.21 | 0.21 |
| 10% | -0.68 | -0.65 | -0.70 | -0.64 | -0.61 | -0.56 | -0.54 |
| 90% | 1.07 | 1.04 | 1.02 | 1.01 | 0.95 | 0.97 | 0.94 |

This table presents the average alpha, 10th percentile, and 90th percentile across hedge fund categories and Cressie-Read gamma. Each column indicate the gamma used in the SDF estimation. Estimation of the alphas use the Fung and Hsieh ten factor model as the basis assets for the SDF estimation.

Table 12: **The Cross Section of Alphas**

| | Cressie-Read $\gamma$: | | | | | | |
|---|---|---|---|---|---|---|---|
| | -3.5 | -2 | -1 | -0.5 | 0 | 0.5 | 1 |
| HF Mean | 0.58 | 0.57 | 0.58 | 0.58 | 0.61 | 0.63 | 0.65 |
| | (26.37) | (22.19) | (24.93) | (24.10) | (25.58) | (24.87) | (26.57) |
| Co-Skewness | 0.01 | -0.03 | -0.04 | -0.03 | -0.04 | -0.04 | -0.05 |
| | (0.16) | (-0.69) | (-0.98) | (-0.72) | (-0.86) | (-1.03) | (-1.41) |
| Co-Kurtosis | -0.16 | -0.20 | -0.20 | -0.20 | -0.21 | -0.22 | -0.23 |
| | (-6.90) | (-5.41) | (-4.82) | (-5.03) | (-5.60) | (-6.07) | (-6.64) |
| I. Vol. | -0.11 | -0.11 | -0.11 | -0.10 | -0.10 | -0.08 | -0.08 |
| | (-2.76) | (-2.80) | (-2.62) | (-2.23) | (-2.04) | (-1.49) | (-1.21) |
| HF Skew | 0.05 | 0.06 | 0.07 | 0.07 | 0.07 | 0.07 | 0.07 |
| | (3.49) | (3.79) | (3.80) | (3.56) | (3.16) | (2.94) | (2.63) |
| HF Kurt | 0.01 | 0.02 | 0.03 | 0.03 | 0.03 | 0.03 | 0.03 |
| | (0.66) | (1.23) | (1.58) | (1.74) | (1.62) | (1.38) | (1.17) |
| AR(1) | 0.08 | 0.08 | 0.09 | 0.09 | 0.08 | 0.10 | 0.09 |
| | (7.91) | (8.92) | (7.29) | (8.11) | (6.45) | (8.31) | (8.50) |
| Size | 0.01 | 0.00 | 0.01 | 0.01 | 0.02 | 0.02 | 0.02 |
| | (1.50) | (0.77) | (2.46) | (1.66) | (2.63) | (2.52) | (2.55) |
| Age | 0.06 | 0.06 | 0.05 | 0.05 | 0.06 | 0.06 | 0.06 |
| | (2.13) | (2.37) | (2.16) | (2.23) | (2.81) | (2.98) | (3.07) |
| $R^2$ | 0.38 | 0.38 | 0.38 | 0.39 | 0.43 | 0.46 | 0.48 |

This table presents the linear regression estimates for the following regression: $\alpha_i^\gamma = \beta_0 + \sum_{i=1}^{N} \beta_i X_i + \epsilon_i$. Here $X_i$ denotes the control variables in the regression (lines in the table above). Columns indicate the gammas used in the SDF estimation. All alphas are computed using the SDF implied from the Fung and Hsieh ten factor model. Although not reported all regressions include dummies for the hedge fund primary category. T-statistics are computed using robust standard errors clustered by primary category.

Table 13: **The Cross Section of Alphas: Divergence**

| | Basis Assets: | | | |
|---|---|---|---|---|
| | CAPM | FFC | FH | AN |
| HF Mean | 0.01 | -0.05 | -0.08 | -0.07 |
| | (0.43) | (-3.46) | (-2.88) | (-2.44) |
| Co-Skewness | 0.14 | 0.13 | 0.07 | 0.04 |
| | (3.02) | (3.71) | (2.81) | (1.80) |
| Co-Kurtosis | 0.27 | 0.15 | 0.02 | 0.02 |
| | (5.86) | (9.13) | (0.64) | (0.60) |
| I. Vol. | 0.19 | 0.33 | 0.34 | 0.38 |
| | (3.73) | (5.41) | (4.89) | (5.31) |
| HF Skew | -0.10 | -0.07 | -0.02 | -0.00 |
| | (-3.76) | (-2.37) | (-0.78) | (-0.01) |
| HF Kurt | -0.08 | -0.06 | -0.03 | -0.02 |
| | (-4.76) | (-2.42) | (-1.21) | (-0.66) |
| AR(1) | -0.10 | -0.08 | -0.06 | -0.02 |
| | (-4.18) | (-3.50) | (-2.42) | (-1.01) |
| Size | 0.02 | -0.00 | -0.00 | -0.01 |
| | (1.53) | (-0.15) | (-0.08) | (-0.50) |
| Age | -0.06 | -0.22 | -0.33 | -0.17 |
| | (-2.95) | (-5.98) | (-22.49) | (-7.67) |
| $R^2$ | 0.11 | 0.24 | 0.34 | 0.28 |

This table presents the linear regression estimates for the following regression: $Div_i = \beta_0 + \sum_{i=1}^{N} \beta_i X_i + \epsilon_i$. Here $X_i$ denotes the control variables in the regression (lines in the table above) and $Div_i$ is the logarithm of the cross sectional variance of possible Cressie-Read alphas. Columns indicate the factors used in the alpha estimation (CAPM stands for the CRSP value weighted market portfolio, FFC for the Fama, French and Carhart four factor model, FH for the Fung and Hsieh ten factor model, and AN for the Agarwal and Naik nine factor model). Although not reported all regressions include dummies for the hedge fund primary category. T-statistics are computed using robust standard errors clustered by primary category.

## Weights on Mixed Moments - Alpha Taylor Expansion

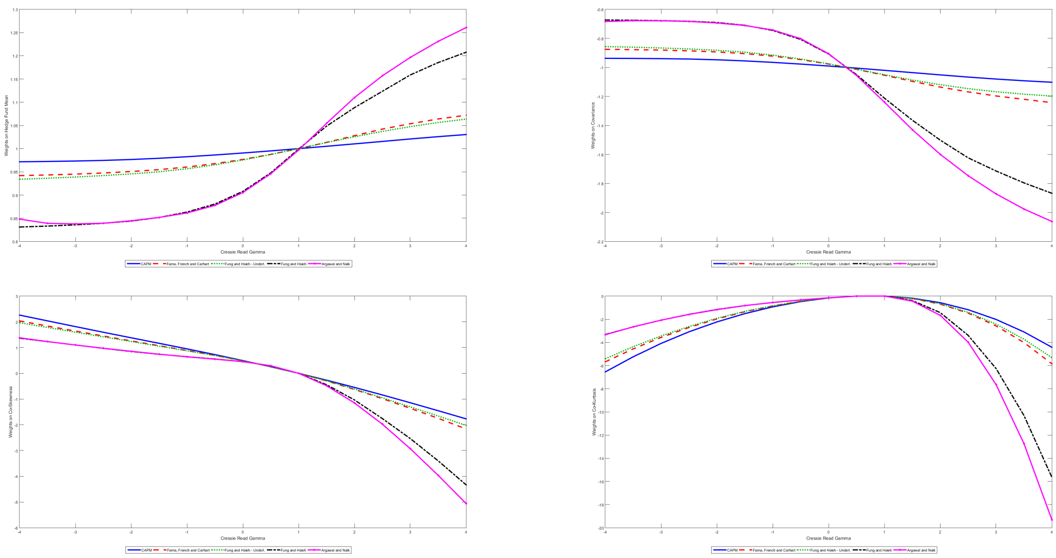

Figure 1: This figure plots the implied weights on the hedge fund average return, the co-variance with the endogenous portfolio, the co-skewness and co-kurtosis based on a Taylor expansion of the SDF alpha. Weights are calculated for five possible combinations of basis assets: the CRSP value weighted market return, the Fama and French (1993) and Carhart (1997) factors, the Fung and Hsieh (2001) underlying factors plus the full set of ten factors, and the Agarwal and Naik (2004) factors. Weights are calculated using the optimal $\lambda$ for a grid of $\gamma's$ between -4 and 4 with 0.5 increments.

## Implied Risk Neutral Distribution

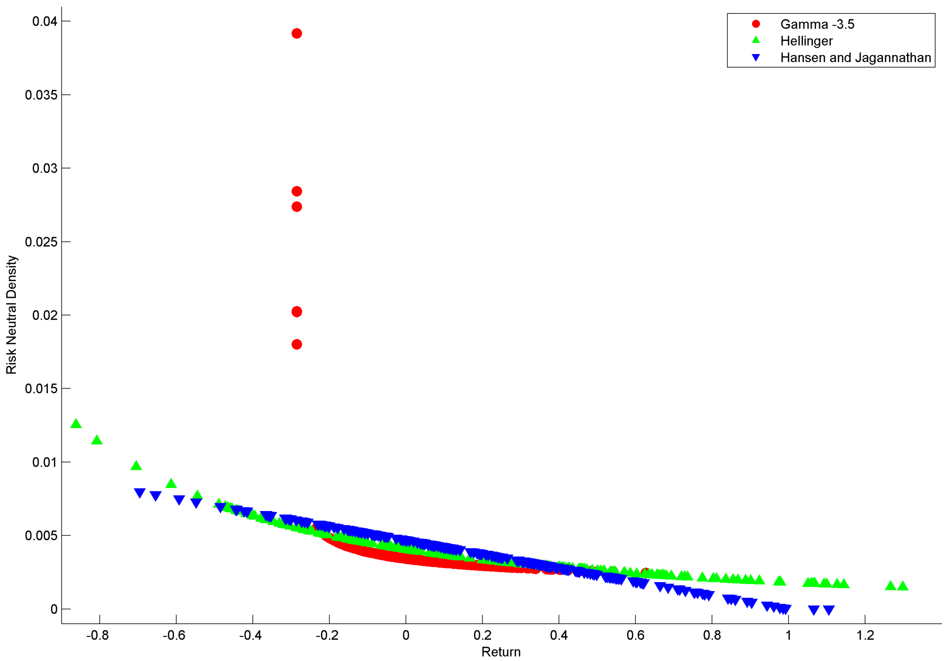


Figure 2: This figure plots the estimated risk neutral distribution for $\gamma = \{-3.5, -0.5, 1\}$ against the endogenous portfolio $(\lambda(\gamma, R)'R)$. To estimate the stochastic discount factor we consider the full sample for the Fung and Hsieh (2001) factors and $E[m] = 1$.

## Estimated Alpha - Toy Example

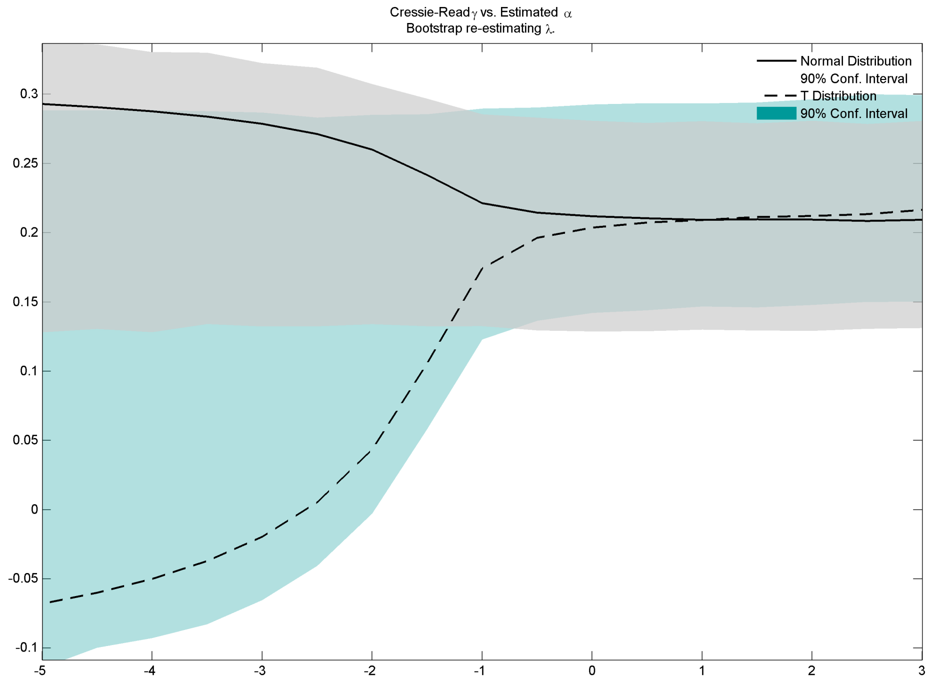


Figure 3: This figure presents the estimated alpha for various values of $\gamma$ when we use a unique asset as basis for the stochastic discount factor estimation. To generate the underlying data we simulate the market returns from a non-central T distribution with noncetrality parameter equal 0.2 and $\nu = 6$. The hedge fund returns are simulated from a non-central T distribution with noncetrality parameter equal 0.2 and $\nu = 4$ and from a Gaussian distribution that matches the mean and variance of the non-central T distribution. When generating our sample we force the sample mean and variance for both hedge funds to be equal. Confidence intervals are calculated via bootstrap with 10,000 draws.

## Stochastic Discount Factor Time Series

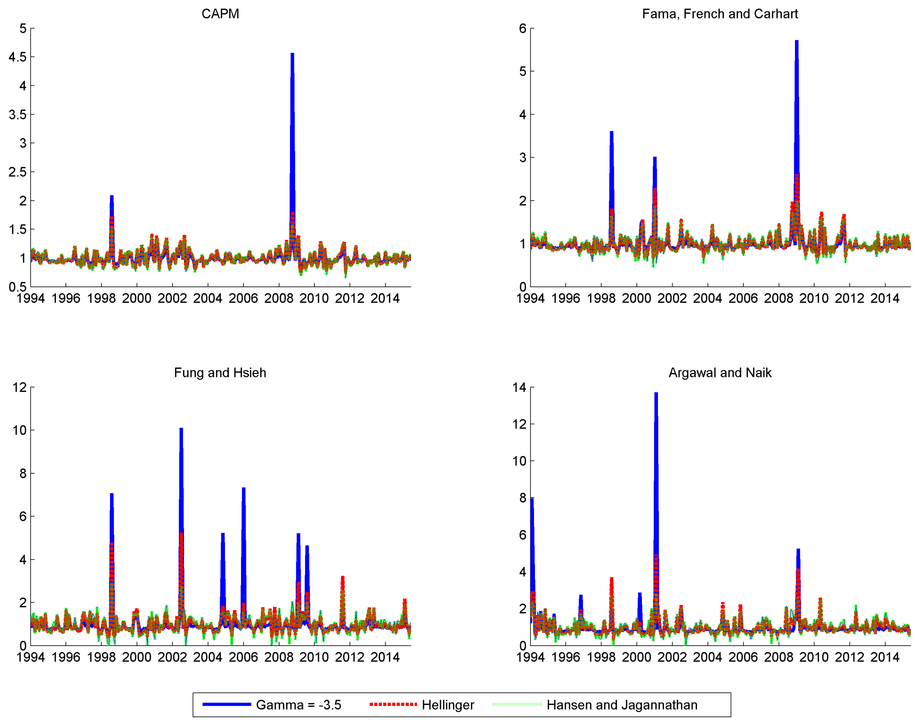


Figure 4: This figure plots the time series of the estimated stochastic discount factor for $\gamma = \{-3.5, -0.5, 1\}$ when we consider four sets of basis assets: the CRSP value weighted market return, the Fama and French (1993) and Carhart (1997) factors, the Fung and Hsieh (2001) factors, and the Agarwal and Naik (2004) factors.

## Alphas Across Basis Assets and Gammas

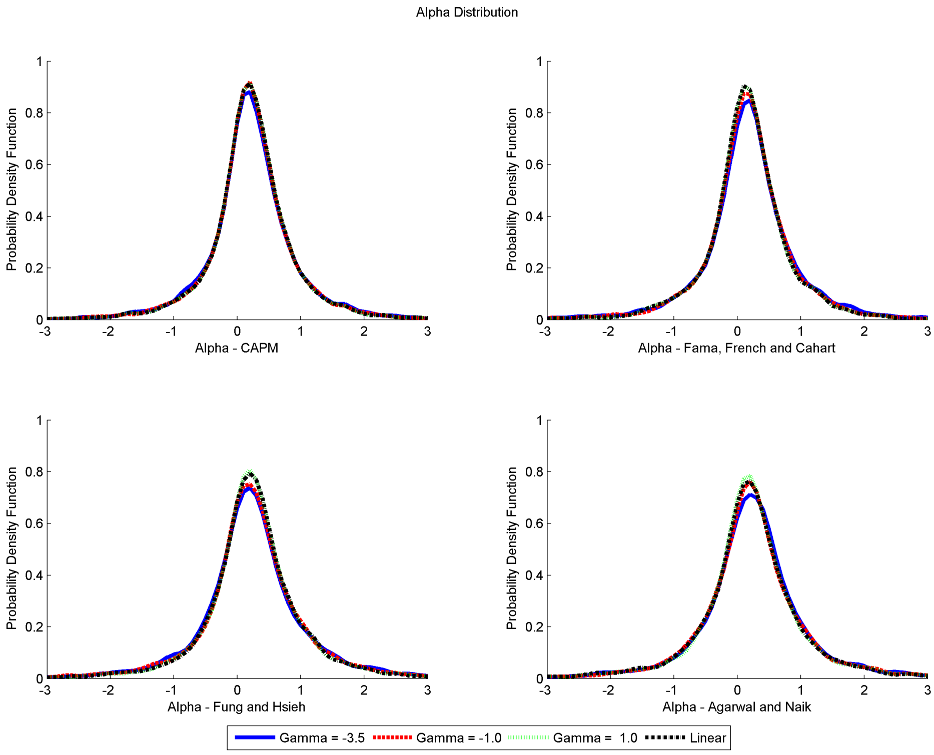


Figure 5: This figure plots the kernel density for estimated Cressie-Read alphas for four sets of basis assets and three possible gammas as well as the Jensen's Alpha. v The values of gammas used are indicated in the legends. Alphas are calculated for each hedge fund using an estimated SDF matching the fund time series.

## T-Statistics Comparison: Asymptotic Distribution

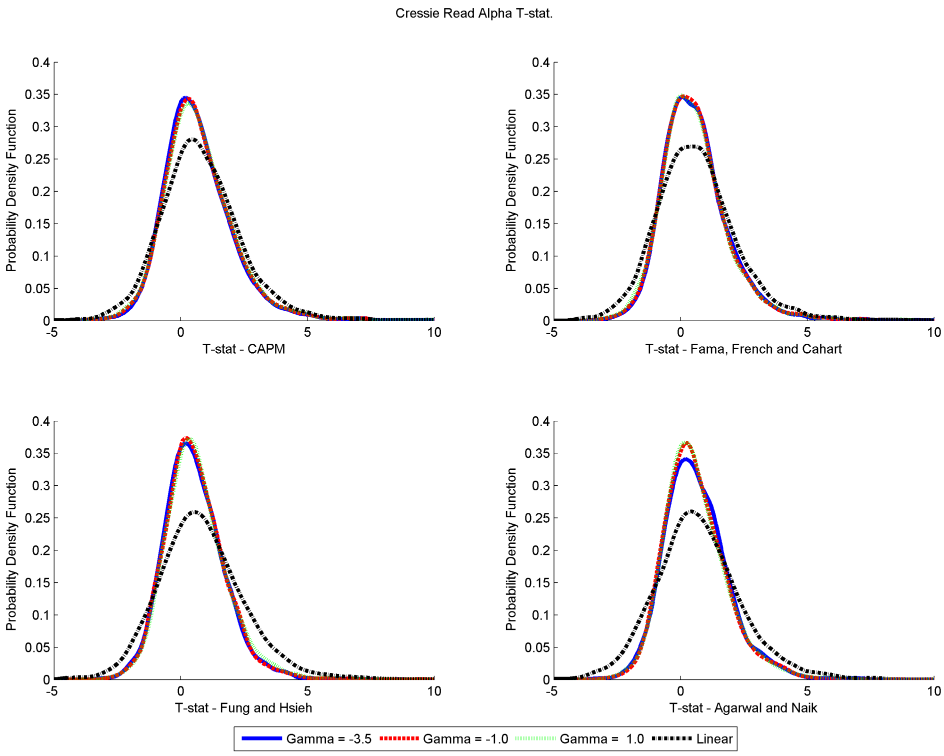


Figure 6: This figure plots the kernel density for the Alpha T-statistics for four sets of basis assets: the CRSP value weighted market return, the Fama and French (1993) and Carhart (1997) factors, the Fung and Hsieh (2001) underlying factors plus the full set of ten factors, and the Agarwal and Naik (2004) factors. Each plot contains the estimated T-statistics density for four values of gamma ($\gamma = \{-3.5, -1, 0, 1\}$) and the Jensen's Alpha (estimated via OLS). Alphas, and t-statistics, are calculated for each hedge fund using an estimated SDF matching the fund time series.

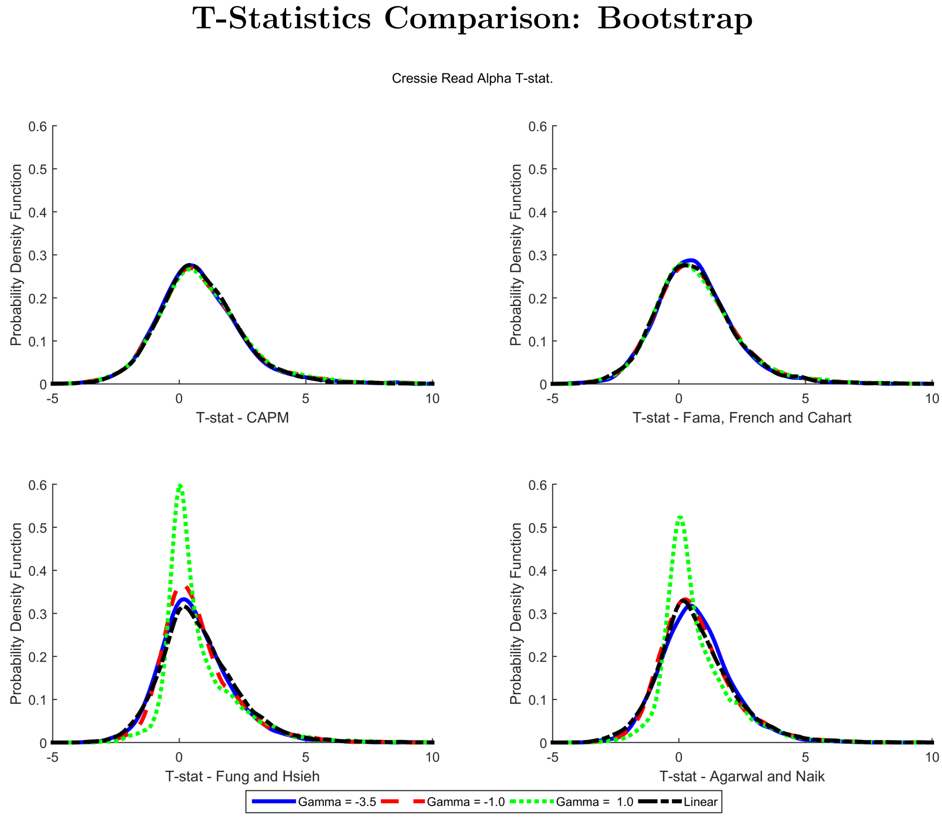


Figure 7: This figure plots the kernel density for the Alpha Bootstrapped T-statistics for four sets of basis assets: the CRSP value weighted market return, the Fama and French (1993) and Carhart (1997) factors, the Fung and Hsieh (2001) underlying factors plus the full set of ten factors, and the Agarwal and Naik (2004) factors. Each plot contains the estimated T-statistics density for three values of gamma ($\gamma = \{-3.5, -1, 1\}$) and the Jensen's Alpha (estimated via OLS). Cressie-Read bootstrapped T-statistics are based on a non-parametric bootstrap with 1000 re-samples for both hedge fund returns and basis factors. In each bootstrap we re-estimate the SDF $\lambda$. Jensen's Alpha bootstraps are based on the residual bootstrap of Kosowski et al. (2006).

## Cressie-Read Alpha vs. Jensen's Alpha

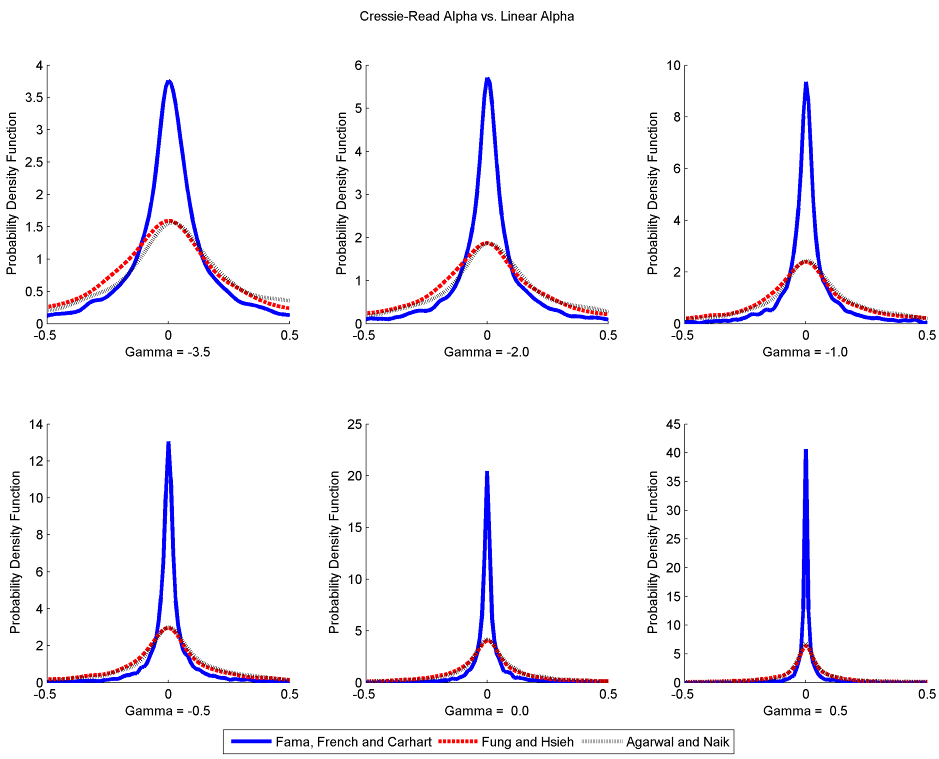


Figure 8: This figure plots the kernel density for the difference between Cressie-Read alphas and Jensen's Alpha (estimate via OLS) for a combination of three sets of basis assets and six gammas. The basis assets are, respectively, the Fama and French (1993) and Carhart (1997) factors, the Fung and Hsieh (2001) underlying factors plus the full set of ten factors, and the Agarwal and Naik (2004) factors. The CAPM plot is omitted since alpha estimates for most models are similar. The values of gammas used are indicated below each plot. Alphas are calculated for each hedge fund using an estimated SDF matching the fund time series.

**Crssie-Read Alpha vs. Jensen's Alpha - T-statistics**

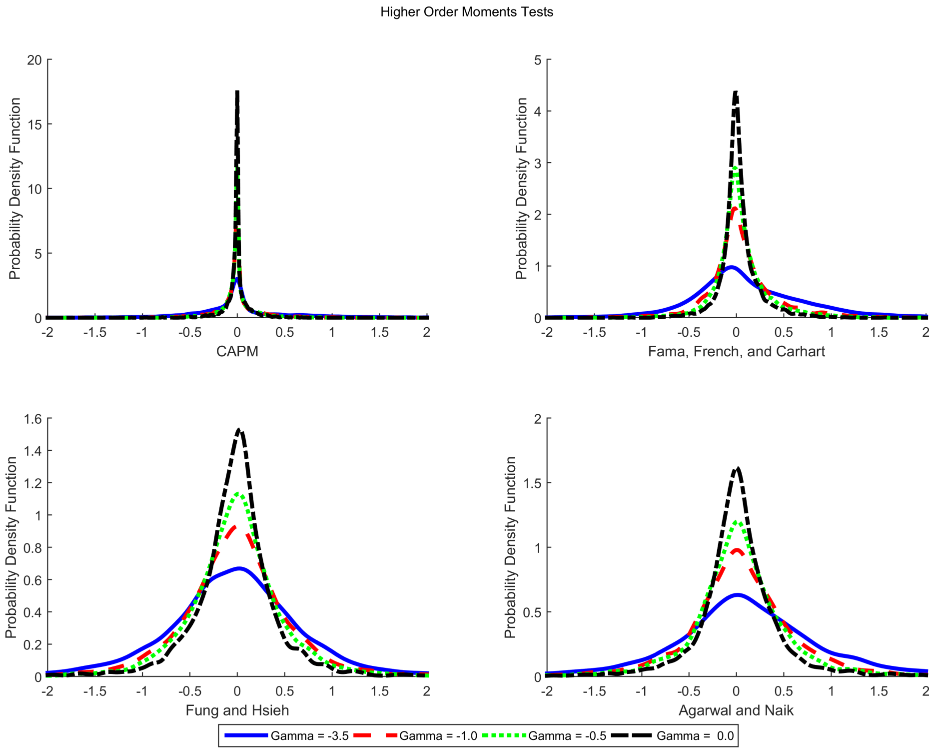


Figure 9: This figure plots the kernel density for the cross sectional t-statistics for a test of equality between Cressie-Read and Jensen's alphas estimates. Each panel plots the distribution for a given set of basis assets and four values for the Cressie-Read $\gamma$. The basis assets are, respectively, the CRSP value weighted market portfolio (CAPM), the Fama and French (1993) and Carhart (1997) factors, the Fung and Hsieh (2001) underlying factors plus the full set of ten factors, and the Agarwal and Naik (2004) factors. The values of gammas used are indicated below each plot. Alphas are calculated for each hedge fund using an estimated SDF matching the fund time series. T-statistics are calculated based on a paired test using Newey and West (1987) standard errors.

**Jensen's vs. Cressie-Read Alpha: Bootstrapped T-Statistics**

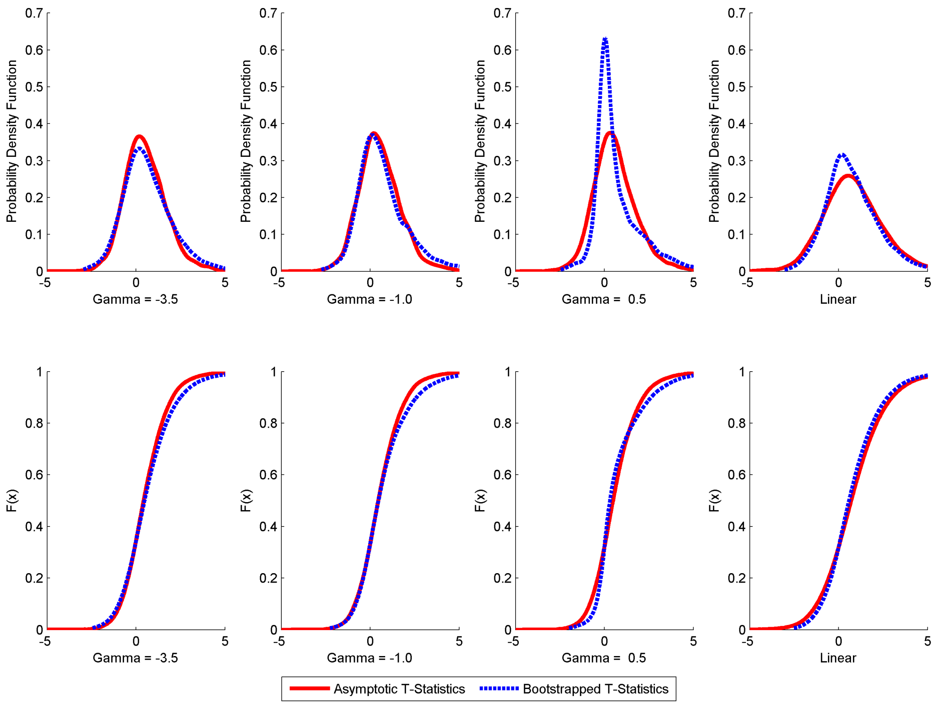


Figure 10: This figure plots the kernel density and the cumulative distribution function for the asymptotic and bootstrapped t-statics for selected Cressie-Read gammas as well as the Jensen's alpha estimates. Alphas are calculated accroding to the Fung and Hsieh (2001) model. Cressie-Read bootstrapped T-statistics are based on a non-parametric bootstrap with 1000 re-samples for both hedge fund returns and basis factors. In each bootstrap we re-estimate the SDF $\lambda$. Jensen's Alpha bootstraps are based on the residual bootstrap of Kosowski et al. (2006). Asymptotic T-statistics are calculated using Newey and West (1987) heteroskedasticity and autocorrelation consistent standard errors with the optimal number of lags given by $\lfloor 4(T/100)^{2/9} \rfloor$.