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Vitor Farinha Luz

The Private Memory of Aggregate Shocks

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Dissertação apresentada à Escola de Pós-Graduação em Economia da Fundação Getulio Vargas como requisito parcial para obtenção de título de Mestre em Economia.

Orientador: Carlos Eugênio da Costa

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FUNDAÇÃO GETULIO VARGAS
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ATA DA DEFESA DE DISSERTAÇÃO DO CURSO DE MESTRADO EM ECONOMIA

Vitor Farinha Luz

THE PRIVATE MEMORY OF AGGREGATE SHOCKS

Dissertação apresentada como requisito parcial para obtenção do grau de Mestre em Economia no curso de Mestrado em Economia desta Escola de Pós-Graduação em Economia (EPGE) da Fundação Getulio Vargas (FGV), pela banca examinadora composta pelos professores a seguir. O aluno tem um prazo máximo de **3 meses** para envio da versão final da Dissertação de Mestrado, incluindo eventuais modificações sugeridas pela banca examinadora.

Orientador: Prof. Carlos Eugênio Ellery Lustosa da Costa
EPGE-FGV

Humberto Moreira
Prof. Humberto Luiz Ataide Moreira
EPGE-FGV

Prof. Vinicius Carrasco
PUC-Rio

Parecer da Banca Examinadora

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Assinatura do Mestrando: Vitor Farinha Luz

Praia de Botafogo, 190 10 e 11º andares
22250-900 Rio de Janeiro RJ Brasil
Tel.: (55 21) 2559 5860 Fax: (55 21) 25524898
E-mail: epge@fvg.br Site: http://epge.fgv.br/portal

Aos meus pais, por todo amor,
apoio incondicional e pela
vida.
Para minha noiva, Fernanda,
por tomar os meus sonhos
também como seus.
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companheirismo e incentivo em
todos os momentos.

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Resumo

Em economias caracterizadas por choques agregados e privados, mostramos que a alocação ótima restrita pode depender de forma não-trivial dos choques agregados. Usando versões dos modelos de Atkeson e Lucas (1992) e Mirrlees (1971) de dois períodos, é mostrado que a alocação ótima apresenta memória com relação aos choques agregados mesmo eles sendo i.i.d. e independentes dos choques individuais, quando esses últimos choques não são totalmente persistentes. O fato de os choques terem efeitos persistentes na alocação mesmo sendo informação pública, foi primeiramente apresentado em Phelan (1994). Nossas simulações numéricas indicam que esse não é um resultado pontual: existe uma relação contínua entre persistência de tipos privados e memória do choque agregado.

Palavras-chave: Repeated Moral Hazard, Dynamic Mirrlees Economy, Aggregate Shocks.

The Private Memory of Aggregate Shocks

Vitor F. Luz*
Fundação Getulio Vargas

Dissertação submetida à banca examinadora como requerimento para obtenção do título de mestre em economia.

Abstract

In economies characterized by both aggregate and privately observed idiosyncratic risks we show that constrained efficient allocations may display non-trivial dependence on aggregate shocks. Using two period versions of both a Atkeson and Lucas (1992) preference shock model and a dynamic Mirrlees (1971) economy we show that constrained optimal allocations have memory with respect to *aggregate* shocks despite their being i.i.d. and independent from idiosyncratic shocks, whenever the latter are not perfectly persistent. The fact that shocks may have persistent effects on allocations despite their public and i.i.d nature, was first shown by Phelan (1994) in a dynamic moral hazard economy with CARA preference. Our numerical simulations indicate that these are not knife-edge results: there is a monotonic relationship between private persistence and aggregate memory in many different environments.

Introduction

Following Wilson (1968)'s landmark contribution, much have been learnt about optimal risk sharing. In an environment in which all idiosyncratic shocks are observed, the mutualization of private risks and its consequences for the dynamics of private consumption as a function of aggregate consumption are now well understood.

In the presence of private information, the early contribution of Rogerson (1985) followed by the methodological advances found in Green (1987), Spear and Srivastava (1987) and Thomas and Worrall (1990) allowed a better understanding of fundamental issues of dynamic insurance schemes. Contrary to the case with no private information, (constrained) optimal schemes exhibit memory which leads to extreme results in the long run. Most of this latter literature, however, focuses on the case in which only

*This is joint work with Carlos da Costa

private risk exists, making assumptions on the nature of idiosyncratic shocks that ultimately lead to the elimination of aggregate risk.[1] In a static setting, optimal allocations will display non-trivial interactions between aggregate and private risk, if and only if one drops the independence assumption between private risks, as a consequence of the findings of Holmstrom (1982).

In this paper we consider a different source of interaction between aggregate shocks and private allocations that arises in a dynamic environment. We assume that aggregate and private shocks are independent from one another and explore how constrained optimal insurance may still endogenously generate links in dynamic allocations. That is, we eliminate all exogenous interactions between shocks and investigate the interconnections displayed by the corresponding allocations.

We focus on two different environments. First, we examine a variation of Atkeson and Lucas (1992) economy in which aggregate risk is added to the problem. Exploiting the intuition of a two period model, we show a strong form of dependence. Allocations exhibit memory with respect to aggregate shocks. That is, today's allocation depends not only on current shock, as in Wilson (1968), and on yesterday's idiosyncratic shock, as in Rogerson (1985), but also on yesterday's aggregate shock.

One may wonder whether this form of dependence is unique to Atkeson and Lucas (1992) environment. What is special about this environment is that individuals can only be separated using a dynamic incentive scheme, since in each period allocations are one-dimensional.

To show that this is not the whole story, we consider a two-period Mirrlees economy and show that the same type of dependence arises there. In this model, our argument is more subtle than the Atkeson and Lucas (1992) case because incentives can be generated within each period, since there are two dimensions of allocations, consumption and labor, in which individuals have different marginal rates of substitution and we show that intertemporal distortions are key to the argument. Our results are potentially important to understand how the very nature of constrained efficient insurance for private risk may generate non-trivial dynamics for aggregate variables. It may also shed light on the macro implications of endogenously incomplete private insurance markets. The two aforementioned models are presented in a two period setting. We strongly believe that many of the presented results are valid for any number of periods when idiosyncratic shocks are a Markov process of order one.

In our environment we are able to define perfect insurance with respect to aggregate shock—i.e., allocations vary *with aggregate shocks* in the same fashion as in a first best world. In a Mirrlees world, this condition implies perfect tax smoothing. We show this property to characterize situations with private information but no idiosyncratic risk—

[1] Phelan (1994) is a noteworthy exception.

replicating the finding of Werning (2007)—and the last period of finite economies.[2] In contrast with Werning (2007), we also consider environments with evolving private information, both in an Atkeson and Lucas (1992) and in a dynamic Mirrlees (1971) setting.

This work is closely related to Phelan (1994), which considers a dynamic moral hazard OLG economy subject to aggregate shocks. In an environment where individuals have constant absolute risk aversion—CARA—preferences it is implied that the allocation may potentially depend on past aggregate variables. We show that this is true in our finite period CRRA economy and try to provide some insight regarding the driving forces for this result.

In recent years, a relatively large body of research has arisen on the Macroeconomics implications of private information. Kocherlakota and Pistaferri (2007) and Kocherlakota and Pistaferri (2009) have shown how the pricing kernel associated with a dynamic Mirrlees economy is compatible with the behavior of excess returns in foreign and domestic markets. Using the fact that the pricing kernel is a function of the cross-sectional harmonic mean of marginal consumption—as derived by Kocherlakota (2005). They use cross-sectional data on consumption to estimate the preference parameters associated with a CRRA specification for three economies. Kocherlakota and Pistaferri (2009) show that a low coefficient of relative risk aversion is needed to account for the equity premium found in the data, while Kocherlakota and Pistaferri (2007) find similar low coefficients can account for the forward exchange rate premium.

A natural next step in this agenda is to calibrate such models to generate pricing data compatible with the underlying primitives of the economy. It, thus, becomes important to understand how private information regarding idiosyncratic shocks interact with public aggregate ones to generate optimal allocations from which the pricing kernels are derived. This paper considers very simple two period economies that display these elements to explore how the dynamic nature of incentives induces persistence in allocations with respect to aggregate shocks.

The rest of the paper is organized as follows. Section 1 describes a general setting that encompasses both economies that we investigate. The Atkeson and Lucas (1992) framework is studied in section 2 while the Mirrlees (1971) setting is dealt with in section 3. Some numerical exercises in the spirit of Golosov et al. (2006) are conducted in section 4. Section 5 concludes the paper.

[2] Also presented in the Appendix.

1 Basic Setting

The economy is inhabited by a continuum of measure one of ex-ante identical individuals, each living for 2 periods. In every period agents' preferences are subject to 'taste' shocks $\theta \in \Theta = \left\{\theta^H, \theta^L\right\}$, with $\theta^H > \theta^L$. We use $\theta^2 = (\theta_1, \theta_2)$ to denote a history of shocks idiosyncratic shocks up to period 2. Agents have preferences over a consumption set L, and rank deterministic allocation streams $\{x_t\}_{t=1}^2 \in \mathbb{R}_+^{L^2}$, given taste shocks θ^2, according to

$$\sum\nolimits_{t=1,2} \beta^{t-1} U\left(x_t, \theta_t\right).$$

We let $U\left(., \theta\right) : \mathbb{R}_+^L \to \mathbb{R}$ be a smooth, strictly increasing and strictly concave function, for all $\theta \in \Theta$.

Taste shocks are not the only source of uncertainty in this economy. In each period, an aggregate shock represented by a random variable $z \in Z \equiv \{z\left(1\right), ..., z\left(M\right)\} \subset \mathbb{R}_{++}$, with $z\left(i\right) > z\left(j\right)$ for $i > j$, affects the economy's technology. Let also $z^2 = (z_1, z_2)$ denote the history of aggregate shocks. We consider i.i.d. aggregate shocks that are distributed according to μ_Z^1 (μ_Z^2 is the product measure induced on Z^2).

Conditional on z^2, idiosyncractic shocks θ^2 are drawn from the probability measure μ_Θ^2 over Θ^2. Note that we are not restricting the joint distribution of z^2, thus we allow any kind of serial correlation. We assume that a law of large numbers apply so that, at period 2 and state z^2, the cross-sectional distribution of agents coincides with the ex-ante distribution μ_Θ^2. Denote by $\mu_\Theta^1\left(\theta\right)$ the marginal distribution of first period shocks $\sum_{\theta'} \mu_\Theta^2\left(\theta, \theta'\right)$.

Idiosyncratic shocks are private information while aggregate shocks are publicly observed.

An *allocation* is in this case $x = \{x_t\}_{t=1}^2$, with $x_t : Z^t \times \Theta^t \to \mathbb{R}_+^L$ for each $t \in \{1, 2\}$. $x_t\left(z^t, \theta^t\right)$ denotes the bundle allocated to an agent with history θ^t at period t, when aggregate history is z^t. Agents' preferences are represented by a von Neumann-Morgenstern utility function,

$$\mathbb{E}\left[\sum_{t=1,2} \beta^{t-1} U\left(x_t(\theta^t, z^t), \theta_t\right)\right] = \sum_{t=1,2} \beta^{t-1} \sum_{z^t} \sum_{\theta^t} \mu_\Theta^t\left(\theta^t\right) \mu_Z^t(z^t) U\left(x_t(\theta^t, z^t), \theta_t\right). \quad (1)$$

Technology is represented by a transformation function $G : \mathbb{R}_+^L \times Z \to \mathbb{R}$. We say that an allocation x is *resource feasible* if

$$G\left(\sum\nolimits_{\theta^t} \mu_\Theta^t\left(\theta^t\right) x_t(\theta^t, z^t), z_t\right) \leq 0, \text{ for all } t, z^t. \quad (2)$$

The presence of private information makes it necessary to write the planner's program as a mechanism design problem. Throughout the analysis we assume that the

planner is endowed with a commitment technology which will allow us to restrict implementation to direct revelation mechanisms, in which agents are asked to report their types at each period, and are assigned corresponding bundles.

Define, a *reporting strategy* $\sigma = \{\sigma_t\}_{t=1}^2$ as a sequence of mappings $\sigma_t : Z^t \times \Theta^t \to \Theta$, which associate to every history $\left(z^t, \theta^t\right)$ an announcement $\hat{\theta}$. Two things are worth mentioning here. First, since individuals cannot lie about the aggregate state of the economy, reports are restricted to idiosyncractic shocks. Second, we assume that the agent only announces the current shock, θ_2, in the second period, and not the history θ^2. Indeed, the mechanism could be described by requiring the agent to announce θ^2 instead. However, given that we assume perfect recall by the government, we could rule out strategies that do not respect the condition $\sigma^2\left(\theta^2\right) = \left(\sigma^1(\theta_1), \theta'\right)$ for some θ'. As a consequence, asking θ_t each period is without loss of generality. The set of all strategies is Σ. We use $\sigma^{TT} = \left\{\sigma_t^{TT}\right\}_{t=1}^2$ to denote the truth-telling strategy $\sigma_t^{TT}\left(\theta^t\right) = \theta_t$ for all t.

We abuse notation and let $U(x, \sigma)$ be the utility derived from an agent choosing reporting strategy σ given allocation x, i.e.,

$$U(x, \sigma) \equiv \mathbb{E}\left[\sum\nolimits_{t=1,2} \beta^{t-1} U\left(x_t\left(\sigma^t(\theta^t, z^t), z^t\right), \theta_t\right)\right].$$

An allocation x is *incentive compatible* if

$$U\left(x, \sigma^{TT}\right) \geq U(x, \sigma), \text{ for all } \sigma \in \Sigma. \tag{3}$$

The goal of this paper is to characterize some features of the solution to the problem of maximizing (1) subject to (2) and (3). Since the sets of all possible shocks is finite, existence and uniqueness of this solution is trivially verified for the cases of interest. From now one we denote this allocation $x^* = \{x_t^*\}_{t=1}^2$. Two relevant special cases of commodity spaces, taste shocks—Section 2—and technological shocks—Section 3—, are discussed.

2 The Preference Shock Case

We first consider an economy with consumption only. It is, basically, an aggregate risk version of Atkeson and Lucas (1992)' economy. This means that L is a singleton. We represent an allocation x as a consumption stream, denoted by $c = \{c_t\}_{t=1}^2 \in \mathbb{R}_+^2$. We also assume i.i.d. idiosyncratic shocks for simplicity. This model presents the basic elements generating the results, but maintains the analysis at a relatively tractable level.

There is a continuum of individuals that live for 2 periods and are exposed to taste shocks each period that determine marginal utility of consumption across states.

Generalized versions of the presented results here are valid for an arbitrary number of periods, even though the analysis here is restricted to this case for simplicity. We focus on the simplest case that isolates some of the driving forces underlying the results.

Individuals are assumed to have constant relative risk aversion preferences for any level of the idiosyncractic parameter θ, i.e., the utility of a θ-type agent is given by

$$U(c,\theta) = \begin{array}{ll} \theta c^{1-\rho}/(1-\rho) & \text{, if } \rho > 0 \text{ and } \rho \neq 1, \\ \theta \log c & \text{, if } \rho = 1. \end{array}$$

This preference ordering is useful because it presents a simple characterization of optimal risk sharing,[3] that allows us to further analyze the relationship between insurance over different risks (idiosyncractic and aggregate), and because it keeps the problem tractable in the interesting dimensions. We will keep a similar restriction in the following sections.

Since this section will not consider production frictions, resource constraints are simply given by a fixed aggregate endowment that is determined by current period's shock z_t.[4] Then, we have the following technology constraint,

$$G\left(\sum\nolimits_{\theta^t} \mu\left(\theta^t\right) c(\theta^t, z^t), z_t\right) = \sum\nolimits_{\theta^t} \mu\left(\theta^t\right) c(\theta^t, z^t) - z_t \leq 0, \text{ for all } t, z^t.$$

The optimal allocation, as defined in the previous section will be denoted by c^*.

As a benchmark case, consider the first best allocation, that is given by

$$c_t^{FB}(\theta^t, z^t) = z_t \theta_t^{\frac{1}{\rho}} \mathbb{E}\left[\theta^{\frac{1}{\rho}}\right]^{-1}.$$

Four important features of this allocation are noteworthy: the allocation is *i)* independent of θ^{t-1}; *ii)* increasing in θ_t, *iii)* independent of z^{t-1} and; *iv)* linear in z_t.

With private information, things change quite a bit. It is already known that independence of θ-history is ruled out in almost every example of repeated moral hazard or informational friction, for efficiency reasons. Private information is revealed *only* through intertemporal distortions, i.e., agents with higher θ will demand more consumption today in exchange for any reduction in consumption in the future, which imposes history dependence on consumption. This is the only available way to provide any insurance for this non-observable shock in the presented setting. Monotonicity in θ_t follows immediately from incentive compatibility since single-crossing is obviously valid here. By single-crossing, we mean that, considering only first period shocks, individuals can be ordered by the marginal rate of substitution between present and future consumption.

[3] In the sense of Wilson (1968).

[4] We could consider a general function $e(.)$, in which case aggregate endownment in state z would be given by $e(z)$. Considering $e(z) = z$ is without loss of generality.

The last two features are more subtle and less explored in the moral hazard literature. Since aggregate shocks are public information and independent across time, allocation independence on previous aggregate shocks seems a natural result. This is not true, however, in a limited insurance environment. Optimal allocations are driven basically by the trade-off between insurance and incentives, and this is generally affected by these aggregate shocks.[5] As a consequence, the whole distribution of future promised utility (the variable used to screen agents) depends on past aggregate shocks as they affected previous allocations.

Linearity, on the other hand, seems a rather natural characteristic to expect from optimal allocation. Linear dependence on z_t is a direct characteristic of optimal risk sharing related to aggregate shocks in settings with the constant absolute risk aversion assumption, with each agent taking a fixed share of the risk.

Lemma 1 *Consider any resource feasible allocation $c = \{c_1, c_2\}$. If*

$$c_t(\theta^t, z^t) \neq z_t \eta(\theta^t, z^{t-1})$$

for $t = 1, 2$, then there is another allocation which generates the same ex-ante utility and such that

$$\sum \mu\left(\theta^t\right) c'(\theta^t, z^t) \leq z_t, \text{ for all } z_t \in Z,$$

with a strict inequality for some z_t.

Throughout the analysis, we assume a sole provider of insurance. Public observability of aggregate risk induces one to assume that such gains as presented in Lemma 1 will be incorporated by this insurer.

When the allocation is linear in z_t (a generalized definition applies in the Mirrlees' environment) we say it is *separable.* Lemma 1 that last period allocation will be separable. As we shall see, this will not be the case in any period that precedes further information revelation in the future; in our case, $t = 1$.

This raises the question of what exactly is different about period 2. Last period contains no information revelation, because there's no further consumption to allow for screening the agents. Take any fixed $\theta', \theta'' \in \Theta$ and any $\left(\bar{\theta}, \bar{z}^2\right) \in \Theta \times Z^2$, then consider $\sigma = \left(\sigma_1^{TT}, \sigma_2\right)$ where $\sigma_2\left(\bar{\theta}, \theta', \bar{z}^2\right) = \theta''$ and $\sigma = \sigma^{TT}$ otherwise, then

$$c_2\left(\bar{\theta}, \theta', \bar{z}^2\right) \geq c_2\left(\bar{\theta}, \theta'', \bar{z}^2\right).$$

Analyzing the reverse deviation we get that $c\left(\bar{\theta}, \theta', \bar{z}^2\right) = c_2\left(\bar{\theta}, \theta'', \bar{z}^2\right)$. We, then, realize that c_2 can be represented as a function $c_2' : \Theta \times Z^2 \rightarrow \mathbb{R}_+$. From now on we

[5] It may also be driven by the trade-off between inequality and incentives, depending on whether the analysis is carried on before or after individual shock values are sorted across agents.

restrict the analysis to functions of this form (consumption and reporting strategies). This conclusion simplifies the analysis restricting incentive compatibility to one period only.

At period T, the only important things inherited form the previous period are utility promises that must be fulfilled. Now we properly define them,

$$w^{*}\left(\theta_{1}, z_{1}\right) \equiv \mathbb{E}(\theta) \sum_{z_{2} \in Z} \mu_{Z}^{1}\left(z_{2}\right) \frac{c_{2}^{*}\left(\theta_{1}, z_{1}, z_{2}\right)^{1-\rho}}{1-\rho}.$$

Given that there is no screening problem in the last period, there is no idiosyncractic risk insurance. Given this, no reason remains to distort allocations relative to aggregate shock.

Proposition 1 *Conditional on period* 1 *promises, consumption in period* 2 *presents optimal risk sharing relative to the aggregate shock, i.e., for any* $\left(\theta_{1}, z_{1}, \theta_{1}^{\prime}, z_{1}^{\prime}\right)$, $c_{2}^{*}\left(\theta_{1}, z_{1}, .\right)$ *and* $c_{2}^{*}\left(\theta_{1}^{\prime}, z_{1}^{\prime}, .\right)$ *are perfectly correlated. Moreover, utility promises fully define consumption distribution in period* 2,

$$c_{2}^{*}\left(\theta_{1}, z_{1}, z_{2}\right)=z_{2} \frac{w^{*}\left(\theta_{1}, z_{1}\right)^{\frac{1}{1-\rho}}}{\sum_{\theta_{1} \in \Theta} \mu_{\Theta}^{1}\left(\theta_{1}\right) w^{*}\left(\theta_{1}, z_{1}\right)^{\frac{1}{1-\rho}}}.$$

One important feature is that there is perfect aggregate risk insurance, conditional on previous utility promises. Consumption in period 2 is perfectly correlated across agents and the consumption distribution is determined, except for a constant, solely by period 1 distribution of utility promises.

The direct relationship between utility promises made in period 1 and consumption levels in period two imply that we can anticipate resource constraints in the last period by considering a feasible subset of promises, in order to characterize the planner's problem in the first period as a static one, given by the restriction:

$$\sum \mu_{Z}^{1}\left(z_{2}\right) \frac{1}{1-\rho}\left[\frac{z_{2} w\left(\theta_{1}, z_{1}\right)^{\frac{1}{1-\rho}}}{\sum_{\theta_{1} \in \Theta} \mu_{\Theta}^{1}\left(\theta_{1}\right) w^{*}\left(\theta_{1}, z_{1}\right)^{\frac{1}{1-\rho}}}\right]^{1-\rho}=w\left(\theta_{1}, z_{1}\right) \text { for all } \theta_{1}, z_{1}.$$

This is equivalent to

$$\sum_{\theta_{1} \in \Theta} \mu_{\Theta}^{1}\left(\theta_{1}\right) w\left(\theta_{1}, z_{1}\right)^{\frac{1}{1-\rho}}=\left[\sum \mu_{Z}^{1}\left(z_{2}\right)\left(\frac{1}{1-\rho}\right) z_{2}^{1-\rho}\right]^{\frac{1}{1-\rho}} \equiv \kappa.$$

Since any distribution of promises that satisfy this equation can be fulfilled in the following period, the problem can be stated as the one of consumption and promises choices.

Proposition 2 *First period optimal allocations* (c_1^*, w^*) *solve the following problem*

$$\max_{w,c_1} \sum_{z_1 \in Z} \mu_Z^1(z_1) \left[\sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) \left[\theta_1 \frac{c_1(z_1,\theta_1)^{1-\rho}}{1-\rho} + \beta w(z_1,\theta_1) \right] \right] \tag{4}$$

s.t.

$$\sum_{\theta \in \Theta} \mu_\Theta^1(\theta) c_1(\theta_1, z_1) \leq z_1, \forall z_1 \in Z, \tag{5}$$

$$\theta_1 \frac{c_1(\theta_1,z_1)^{1-\rho}}{1-\rho} + \beta w(z_1,\theta_1) \geq \theta_1 \frac{c_1(\theta_1',z_1)^{1-\rho}}{1-\rho} + \beta w(z_1,\theta_1'), \forall (\theta_1,\theta_1',z_1) \in \Theta^2 \times Z, \tag{6}$$

and

$$\sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) w(\theta_1,z_1)^{\frac{1}{1-\rho}} = \kappa. \tag{7}$$

The simple characterization of the problem facilitates solving first period consumption, since this is a conventional static adverse selection problem.

To determine whether the insurance scheme relative to idiosyncractic insurance entails distortions relative to aggregate risk remains to be analyzed. This point is made in the next proposition. The proofs are in the Appendix.

Proposition 3 *First period consumption is* ***not*** *separable on aggregate shock* z_1*, i.e., there exist* ***no*** *functions* $\tilde{c}(\theta_1)$ *and* $\eta(z_1)$ *such that* $c_1^*(\theta_1,z_1) = \tilde{c}(\theta_1)\eta(z_1)$.

The central issue is that of characterizing the trade-off between consumption inequality in the two periods. Endowment shocks in period 1 could be accommodated by increasing proportionately each agent's consumption, which is the possibility rejected in this proposition. The reason for this is that more endowment changes the incentive versus inequality conflict in the first period.

Another interesting feature that arises from limited insurance in the presence of aggregate shocks is that allocations may present persistence regarding aggregate shocks. In other words, even though previous aggregate shocks are independent of present information structure of the economy, current consumption distribution may depend on the history of the economy. Therefore aggregate variables may present time dependence due to the presence of long term contracts, having nothing to do with primitive persistence in the model. This is proved in the next proposition.

Proposition 4 *Period 2 consumption depends on period* 1 *aggregate shocks, i.e.,* $c_2 : Z^2 \times \Theta^2 \to \mathbb{R}_+$ *cannot be independent of* z_1.

3 Mirrlees Economy

In this section, we consider a dynamic Mirrlees economy, in which case, $x = (c, y)$ where c is consumption and y denotes efficiency units of work. Now, θ is a productivity parameter which, combined with z, determines how many efficiency units an agent produces per unit of effort, l. An agent with productivity θ produces $y = l\theta z$ with effort l if the aggregate state is z.

We consider instantaneous utility of the form

$$U(c, l) \equiv u(c) - v(l)$$

with $u(.)$ strictly concave and increasing and $v(.)$ strictly convex and increasing.

The social Planner's problem is

$$\max \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_\Theta^t(\theta^t) \left[u\left(c(\theta^t, z^t)\right) - v\left(\frac{y(\theta^t, z^t)}{\theta_t z_t}\right) \right]$$

subject to

$$\sum_{\theta^t} \mu_\Theta^t(\theta^t) \left[c(\theta^t, z^t) - y(\theta^t, z^t) \right] \leq 0$$

and

$$\sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_\Theta^t(\theta^t) \left[u\left(c(\theta^t, z^t)\right) - v\left(\frac{y(\theta^t, z^t)}{\theta_t z_t}\right) \right] \geq$$

$$\sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_\Theta^t(\theta^t) \left[u\left(c\left(\sigma(\theta^t, z^t), z^t\right)\right) - v\left(\frac{y\left(\sigma(\theta^t, z^t), z^t\right)}{\theta_t z_t}\right) \right]$$

for all $\sigma \in \Sigma$.

We shall now investigate how the optimal allocation (c^*, l^*) depends on the history of aggregate shocks z^t for each t. More specifically, whether aggregate shocks generate persistent effects on the distribution of consumption and labor. In other words we ask: what is the prescription for optimal tax response to aggregate shocks?

It is a general feature of repeated screening and moral hazard problems that allocations keep track of individuals' private shocks histories. This allows the principal to go beyond repeating the static optimal allocation by linking future allocations to present type reports. The extreme poverty result presented in Atkeson and Lucas (1992) and Phelan (1998) is a long run consequence of this feature of optimal allocations. At each period, the cross-sectional distribution of consumption and labor inherits the heterogeneity of previous period and brings a new source of variation, since you have to generate further distortions to separate agents with different types at the current period.[6] This pattern generates ever increasing inequality across agents.

[6] The new dispersion in allocations is independent of previous heterogeneity, assuming iid individual shocks, an assumption present in most of the literature.

This whole argument is false for the aggregate shock for one simple reason: it is observable. Since the aggregate state of the economy is assumed to be known by everyone, and is unrelated with idiosyncratic shocks, there is no obvious reason to believe that the optimal allocation will present history dependence with respect to this variable.

The main finding of this section is that aggregate shocks may present persistence, even in this simple and, in many senses, "independent" setting. We show that, for our simple example, this dependence is indeed optimal.

At any given period, individuals care about the current allocation and how the current report will affect their prospects in further periods. Lemma 2, below, states that, since it is always possible to distort future consumption to separate individuals today, differences in current consumption are preserved in the future in such a way as not to interfere with future incentive compatibilities.

Lemma 2 *The optimal allocation satisfies*

$$\frac{u'\left(\bar{\theta}^t, z^t\right)}{u'\left(\theta^t, z^t\right)} = \frac{\mathbb{E}\left[u'\left(c_{t+1}\right)^{-1} | \bar{\theta}^t, z^{t+1}\right]^{-1}}{\mathbb{E}\left[u'\left(c_{t+1}\right)^{-1} | \theta^t, z^{t+1}\right]^{-1}},$$

for every z^t *and* $\bar{\theta}^t, \theta^t$*.*

Proof. From Proposition 1 in Golosov et al. (2003), we know that there exists $\{\lambda_t\}_{t=1}^T$,with $\lambda_t : Z^t \to \mathbb{R}$, such that

$$\lambda_{t+1}\left(z^{t+1}\right) = \frac{\mathbb{E}\left[u'\left(c_{t+1}\right)^{-1} | \theta^t, z^{t+1}\right]^{-1}}{u'\left(c_t\left(\theta^t, z^t\right)\right)},$$

for all z^{t+1} and θ^t. Dividing this equation for $\bar{\theta}^t$ and θ^t implies the Lemma. ■

One interesting and straightforward example of this result is the case in which $u(\cdot)$ is log.

Corollary 1 *Consider* $u = \ln$*, then, letting* $Y\left(z^t\right)$ *denote total production, we have*

$$\frac{c_t\left(\bar{\theta}^t, z^t\right)}{Y\left(z^t\right)} = \frac{\left[\sum_{\theta'} \mu\left(\theta' | \bar{\theta}^t\right) c_{t+1}\left(\left(\bar{\theta}^t, \theta'\right), \left(z^t, z\right)\right)\right]}{\left[\sum_{\theta^{t+1}} \mu\left(\theta^{t+1}\right) c_{t+1}\left(\theta^{t+1}, \left(z^t, z\right)\right)\right]} = \frac{\mathbb{E}\left[c_{t+1} | \bar{\theta}^t, z^{t+1}\right]}{Y\left(z^{t+1}\right)}.$$

This means that if at any period aggregate shocks affect the consumption share of any agent, then subsequent consumption will also present dependence on *this* aggregate shock, even though it is not related with future aggregate endowments.

In order to further characterize how this intertemporal programming problem is related to shock persistence, we restrict analysis to specific utility functions with constant absolute risk aversion in the consumption side and a power disutility of labor effort. We consider

$$u(c) = \frac{c^{1-\rho}}{1-\rho} \text{ and } v(l) = \frac{l^\gamma}{\gamma},$$

for $\rho > 0$, $\rho \neq 1$ and $\gamma > 1$. For $\rho = 1$ take $u(c) = \log c$.

Given the low tractability of incentive compatibility constraints in even simple environments, it has become relatively common to restrict analysis substantially in terms of preferences and stochastic distribution of shocks in order to obtain clear-cut results. We believe this is an interesting case, because it is a common preference structure in the Macroeconomic literature and, on both the static Mirrlees and autarky problems, which are very common alternate models, aggregate shocks have very clear effects on allocations. In both models, optimal allocations can be written $c_t\left(\theta^t\right)\eta(z_t)$ and $y\left(\theta^t\right)\eta(z_t)$, where $\eta(z_t) \equiv z_t^{\gamma/(\rho+\gamma-1)}$. This means that idiosyncratic and aggregate states have separable effects in individual consumption and labor allocations. It is also an interesting point to compare how the dynamic structure of private information and the optimal social insurance scheme will affect this behavior.

We now show that it is optimal to generate dependence on previous aggregate shocks as an incentive generation mechanism.

Proposition 5 *Let $\mu_\Theta^2\left(\theta^2\right) > 0$ for all θ^2. Then, period 2 allocations depend on z_1.*

Proof. Toward a contradiction, assume that period 2 allocation does not depend on z_1. Define as $w_t^{z^t}\left(\tilde{\theta}^t|\theta^t\right)$ the instantaneous utility at time t, of an agent θ^t that claims to be $\tilde{\theta}^t$ in the aggregate state z^t. From Lemma 2 and the utility functions, it is clear that first period consumption takes the form $c(\theta, z) = \tilde{c}(\theta)\eta(z)$.

Consider the social planner's problem and define $\phi(z)$ as the multiplier of the high-type incentive compatibility in the case of first period shock z. Then, the first order conditions for type θ^H consumption and production are

$$u'\left(\theta^H, z\right)\left[1 + \frac{\phi(z)}{\mu(z)\,\mu\left(\theta^H\right)}\right] = \left[\sum_\theta \mu(\theta)\,u'(\theta, z)^{-1}\right]^{-1},$$

and,

$$v'\left(\frac{l_1\left(\theta^H, z\right)}{z\theta^H}\right)\frac{1}{z\theta^H}\left[1 + \frac{\phi(z)}{\mu(z)\,\mu\left(\theta^H\right)}\right] = \left[\sum_\theta \mu(\theta)\,u'(\theta, z)^{-1}\right]^{-1},$$

respectively.

The first equation, together with 2, implies that $\phi(.)/\mu(.)\,\mu(\theta^H)$, does not depend on z. This means that, considering $v'(y) = y^{\gamma-1}$, $l(\theta, z)$ is of the form $\tilde{l}(\theta)\,\eta(z)^{\frac{\rho}{\gamma-1}} z^{\frac{\gamma}{\gamma-1}}$. From the binding resource constraint for every aggregate shock, we have that $\eta(z) = z^{\frac{\gamma}{\rho+\gamma-1}}$.

The argument above implies that $w_0^z\left(\theta|\hat{\theta}\right) = z^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}} w\left(\theta|\hat{\theta}\right)$, for some function $w(.|.)$. Then, the incentive compatibility constraint becomes

$$z^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}}\left[w_1(\theta|\theta) - w_1(\theta'|\theta)\right] \geq \sum_{i=H,L;z_2\in Z} \mu\left(\theta^i|\theta\right)\mu(z_2)\left[w_2^{z^2}\left(\theta', \theta_i|\theta, \theta_i\right) - w_2^{z^2}\left(\theta, \theta_i|\theta, \theta_i\right)\right].$$

The high-type incentive compatibility always binds, since, otherwise, it would be welfare increasing to transfer consumption from the high-type to the low-type, who is poorer. Since, by assumption, the right-hand side of the equation above does not depend on z, first period incentives are not being generated through second period allocations, i.e.,

$$\sum_{i=H,L;z_2\in Z} \mu\left(\theta^i|\theta\right)\mu(z_2)\left[w_2^{z^2}\left(\theta', \theta_i|\theta, \theta_i\right) - w_2^{z^2}\left(\theta, \theta_i|\theta, \theta_i\right)\right] = 0,$$

for all θ, θ' and z.

Because second period allocations are not contingent on first period reports, the planner's problem can be separated between the two period and the optimal allocation is always the static Mirrlees' problem solution. Define $\left(c^M(.|z), l^M(.|z)\right)$ as the optimal allocation in the static Mirrlees model with shock z. Lemma 2 implies

$$\frac{c^M\left(\theta^H|z\right)^{-\rho}}{c^M\left(\theta^L|z\right)^{-\rho}} = \frac{\left[\sum_{\theta'}\mu\left(\theta|\theta^H\right)c^M(\theta|z)^{\rho}\right]^{-1}}{\left[\sum_{\theta'}\mu\left(\theta|\theta^L\right)c^M(\theta|z)^{\rho}\right]^{-1}},$$

which is true only if $\mu\left(\theta^H|\theta^H\right) = \mu\left(\theta^L|\theta^L\right) = 1$. This is a contradiction. ■

3.1 Perfectly Persistent Shocks

Now consider the case in which $\mu_\Theta^2\left(\theta^H, \theta^H\right) + \mu_\Theta^2\left(\theta^L, \theta^L\right) = 1$, i.e., individual productivity levels are constant.Then, the first order conditions of the Planner's problem are given by

$$\begin{aligned}
\left[1 + \sum_{\theta'}\left[\phi\left(\theta'|\theta\right) - \phi\left(\theta|\theta'\right)\right]\right] c_t\left(\theta, z^t\right)^{-\rho} &= \lambda\left(z^t\right), \\
\left[1 + \sum_{\theta'}\left[\phi\left(\theta'|\theta\right) - \phi\left(\theta|\theta'\right)\left(\frac{\theta}{\theta'}\right)^{\gamma}\right]\right] \frac{y_t\left(\theta, z^t\right)^{\gamma-1}}{(\theta z_t)^{\gamma}} &= \lambda\left(z^t\right),
\end{aligned}$$

where ϕ are the IC constraints multipliers.

Then we have that for arbitrary κ and ω, $c_t\left(\theta, z^t\right) = \lambda\left(z^t\right)^{\frac{-1}{\rho}} \kappa\left(\theta\right)$ and $y_t\left(\theta, z^t\right) = \left(\theta z_t\right)^{\frac{\gamma}{\gamma-1}} \lambda\left(z^t\right)^{\frac{1}{\gamma-1}} \omega\left(\theta\right)$. Substituting this in the resource constraints, that always binds, we have that

$$\sum \mu\left(\theta\right)\left[\lambda\left(z^t\right)^{\frac{-1}{\rho}} \kappa\left(\theta\right) - \left(\theta z_t\right)^{\frac{\gamma}{\gamma-1}} \lambda\left(z^t\right)^{\frac{1}{\gamma-1}} \omega\left(\theta\right)\right] = 0,$$

which means that $\lambda\left(z^t\right) = a z_t^{-\frac{\gamma\rho}{\rho+\gamma-1}}$, and then,

$$c_t\left(\theta, z^t\right) = \tilde{c}\left(\theta\right) z_t^{\frac{\gamma}{\rho+\gamma-1}},\ y_t\left(\theta, z^t\right) = \tilde{y}\left(\theta\right) z_t^{\frac{\gamma}{\rho+\gamma-1}}.$$

Suppose that the choice of variables for consumption and labor are $\tilde{c}$ and $\tilde{y}$, while the *de facto* allocation is given by $x_t\left(\theta^t, z^t\right) = z_t^{\frac{\gamma}{\rho+\gamma-1}} \tilde{x}_t\left(\theta^t\right)$. Since this does not affect the degree of freedom in the planner's problem, it is basically the same choice. Nonetheless, the problem can be rewritten as

$$\max \mathbb{E}\left[z_t^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}}\right] \sum_{t=1,2} \beta \sum_{\theta} \mu_{\Theta}^1\left(\theta\right)\left[\frac{\tilde{c}(\theta)^{1-\rho}}{1-\rho} - \frac{\tilde{y}(\theta)^{\gamma}}{\gamma\theta^{\gamma}}\right]$$

subject to

$$z_t^{\frac{\gamma}{\rho+\gamma-1}} \sum_{\theta^t} \mu_{\Theta}^1\left(\theta\right)\left[\tilde{c}(\theta) - \tilde{y}(\theta)\right] \leq 0$$

and

$$\mathbb{E}\left[z_t^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}}\right] \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \mu_{\Theta}^1\left(\theta\right)\left[\frac{\tilde{c}(\theta)^{1-\rho}}{1-\rho} - \frac{\tilde{y}(\theta)^{\gamma}}{\gamma\theta_t^{\gamma}}\right] \geq$$
$$\mathbb{E}\left[z_t^{\frac{\gamma(1-\rho)}{\rho+\gamma-1}}\right] \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \mu_{\Theta}^1\left(\theta\right)\left[\frac{\tilde{c}\left(\theta'\right)^{1-\rho}}{1-\rho} - \frac{\tilde{y}\left(\theta'\right)^{\gamma}}{\gamma\theta^{\gamma}}\right].$$

It is not hard to see that Pareto optimal allocations are given by

$$x_t^*\left(\theta, z^t\right) = x^M\left(\theta\right) z_t^{\frac{\gamma}{\rho+\gamma-1}},$$

where x^M is the optimal allocation in the static Mirrlees problem in the analogous economy with $z = 1$. This result does not depend on the number of periods, however it is very sensitive to the assumption about preferences. In the case of non isoelastic disutility of labor, for example, it is not valid anymore.

From this analysis, two immediate results follow.

Proposition 6 *If $P\left(\theta^2\right) = 0$ whenever $\theta_1 \neq \theta_2$, then period 2 allocations do not depend on z_1.*

Define the implicit optimal marginal labor tax as

$$\tau_t^*\left(\theta^t, z^t\right) = 1 - \frac{y_t^*\left(\theta^t, z^t\right)^{\gamma-1}}{\left(\theta_t z_t\right)^{\gamma} c_t^*\left(\theta^t, z^t\right)^{-\rho}}.$$

And it is simple to conclude that optimal tax schedule presents perfect tax smoothing in every period.

Proposition 7 *If $P\left(\theta^2\right) = 0$ whenever $\theta_1 \neq \theta_2$, then*

$$\tau_t^*\left(\theta^t, z^t\right) = 1 - \frac{y^M\left(\theta^T, z^{T-1}\right)^{\gamma-1}}{\theta_t^{\gamma} c^M\left(\theta^T, z^{T-1}\right)^{-\rho}}$$

These results suggest a kind of complementarity between individual shock persistence and aggregate allocation persistence, at least for the extreme case of i.i.d and perfectly persistent shocks. Our objective in the next section is to show that these are not knife-edge cases. We provide a numerical example in which aggregate memory declines continuously with persistence in idiosyncratic shocks.

4 Numerical Examples

In this section we conduct numerical exercises that help illustrate some of the properties that we have derived in section 1. This exercise is similar to the one presented in Golosov et al. (2006), in which a two period Mirrlees economy with government spending shocks, that are public information. However, they have a different focus and only characterize how last period shocks affect optimal labor tax wedges. More precisely, Golosov et al. (2006) do not allow public shocks in the first period, which would allow them to address the persistence issues that is the focus of our work.

Kocherlakota (2005) provides a similar numerical example in many ways, but analyses the impact of individual shock persistence and the size of the public shocks on capital taxation, which is the main focus of his paper, and does not mention labor tax nor the possibility of persistent effects of aggregate shocks.

In our exercises we fix $1/\rho = \gamma = 2$, $\theta(1) = 1$, $\theta(2) = 2$, $\mu_\Theta^1(\theta(1)) = \mu_\Theta^1(\theta(2)) = .5$, $\mu_Z^1(z(1)) = \mu_z^1(z(2)) = .5$, $z(1) = 1$ and $z(2) = 2$. Our measure of persistence, $\pi \in [0,1]$, is the value of the non-unit eigenvalue of the transition matrix for idiosyncratic shocks.[7] The probability of maintaining the same productivity in the next period,

[7] We use the term transition matrix in the traditional sense for Markov chains. In our case the Markov nature of idiosyncratic shock is trivial.

i.e., $P((\theta,\theta)|\theta) = \pi$ for all θ is directly associated with our measure in this two type context. In particular, because we vary π in $[0.5, 1]$, our persistence measure ranges between 0 and 1: 0 in the i.i.d. case and 1 in the case of perfectly persistent shocks.

We analyze how relevant individual shock persistence is on the allocation and what is driving the non-existence of persistence of aggregate shocks effects in the model for the perfectly persistence case.

Figure 1[8] shows how consumption $c(\theta, z)$ and income $y(\theta, z)$ varies in the first period as we vary idiosyncratic shock persistence. A very apparent effect is that the higher the persistence of individual types the higher the dispersion in consumption, whilst not much effect is found in individual labor. This means that at lower persistence levels, it is possible to generate work incentives without too much inequality in consumption.

Figure 2 presents first period instantaneous utilities and future utility promises. The previous analysis of first period consumption and labor is reflected in figure 2-A. When individual types are less persistent, lower type individuals work less and are not so harshly punished in terms of current consumption, and they attain higher instantaneous utility. The high-type individuals, on the other hand, receive more instantaneous reward for their greater effort when there is more persistence. The "crossing" behavior present in Figure 2-A may seem counterintuitive at first, given that incentives for information revelation must be provided. Figure 2-B presents the rest of the history. Differences in utility promises represent how future allocation is being used as a tool to screen individuals in the first period. If individual productivity shock is permanent, there is no room for generating intertemporal insurance schemes. When future allocations are actively used in the first period screening problem, first period aggregate shocks are also transmitted to future allocation, since the incentives versus redistribution trade-off is affected.

Finally, Figure 3 provides some evidence on optimal tax dependence on these shocks. It displays the marginal income tax for individuals that are of low type in the second period (high type individuals in the last period are never distorted) for the low second period aggregate shock, varying first period aggregate and idiosyncratic shocks. It is apparent from this figure that second period marginal tax depends on z_1, a fact that also implies that allocations depend on previous aggregate shocks. It is a fact that this dependence disappears as $\pi \to 1$, as we have seen in section 3.

[8] In the plotted charts, if X is a period one variable, $X1H$ stands for the value for the low type (represented by 1) in the high aggregate state (represented by H). For period two variables, $X12HL$ represents the variable for an agent with idiossincratic history (θ^L, θ^H) in the aggregate state $(z(2), z(1))$.

5 Conclusion

We have studied economies with both aggregate and privately observed idiosyncratic risks. We have shown that constrained efficient allocations depend on aggregate shocks in a non-trivial fashion. Allocations display memory with respect to *aggregate* shocks although we have imposed independence across periods and from idiosyncratic shocks.

The two models presented herein are very simple and do not present a full dynamic model with many periods. There is still much to be done to advance our comprehension of the intricate relationship between these two forms of insurance and their macroeconomic implications. Indeed, it is our belief that the properties emphasized here applies to more general settings and are of great interest and applicability to general macroeconomic models with any kind of aggregate stochastic shocks and some source of limited insurance related to unobservable characteristics.

As of this moment, not much research on the interrelation between idiosyncratic and public shocks at this level. As far as we know a similar exploration of any model with only consumption, as presented in section 2, has only been considered by Phelan (1994), in a context with CARA preferences. Phelan (1994) is focused on showing how some simplifying assumptions (CRRA preferences are crucial here) allows for the applicability of recursive methods to a setting with both idiosyncratic and aggregate shocks. His analysis does provide results regarding the interaction between the two shocks in the i.i.d. case, for his environment although the result is not emphasized.

The Mirrlees economy presented in section 3 is a special case of the model studied in Kocherlakota (2005). We have made more restrictive assumptions since the analysis of the effects of aggregate shocks and its relations to limited insurance depend on the nature of the shocks and its stochastic process. We must, however, mention the numerical exercises conducted by Golosov et al. (2006) in a two period Mirrlees economy with aggregate risk. Some of our results may be viewed as providing theoretical grounds for their numerical findings. Golosov et al. (2006) do not exploit models that generate memory of aggregate shocks. Also related are the numerical exercises found in Kocherlakota (2005). Since his concern is tax implementation, he does not focus on the effects of persistence.

We have tried to motivate further study of dynamic settings with mixed shocks because we still do not have clear conclusions about how optimal allocations look like in general. If we are to advance serious tests about, say, the existence of institutional networks that provide optimal allocation of risk with micro level data, as in Townsend (1994), a better understanding of how observable implications are dependent or sensitive to this assumptions is needed.

A The Atkeson and Lucas (1992) Economy

Proof of Proposition 1. Given promises (w^*, c_2^*) is *sequentially efficient* in the following sense: there is no other consumption c_2' such that

$$w^*(\theta_1, z_1) = \mathbb{E}(\theta) \sum_{z_2 \in Z} \mu_Z^1(z_2) \frac{c_2'(\theta_1, z_1, z_2)^{1-\rho}}{1-\rho};$$

$$\sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) c_2'(\theta_1, z_1, z_2) \leq z_2, \text{ for all } z^2 \in Z^2;$$

with the resource constraint slack for at least one $z \in Z$.
Then, there are nonnegative state prices $\lambda(z^2)$ such that

$$c_2^* \in \arg\min_{c_2} \sum_{z^2 \in Z^2} \lambda(z^2) \sum_{\theta_1 \in \Theta} \mu_\Theta^1(\theta_1) c_2(\theta_1, z_1, z_2)$$

subject to

$$\mathbb{E}(\theta) \sum_{z_2 \in Z} \mu_Z^1(z_2) \frac{c_2(\theta_1, z_1, z_2)^{1-\rho}}{1-\rho} = w^*(\theta_1, z_1).$$

Considering multipliers $\eta(\theta_1, z_1)$ for the promise keeping constraints,[9] we have the Lagrangian

$$\sum_{z^2 \in Z^2} \sum_{\theta_1 \in \Theta} \left[\lambda(z^2) \mu_\Theta^1(\theta_1) c_2(\theta_1, z_1, z_2) - \eta(\theta_1, z_1) \mathbb{E}(\theta) \mu_Z^1(z_2) \frac{c_2(\theta_1, z_1, z_2)^{1-\rho}}{1-\rho} \right].$$

Solving the Lagrangian and fixing shadow prices such that the solution respects utility promises and resource constraints generates the result. ■

Proof of Proposition 3. First, note that total endowment will always be exhausted in period 1, otherwise, utility could be increased by the same amount to every agent without changing incentive constraints. Then, considering $c_1^*(\theta_1, z_1) = \tilde{c}(\theta_1)\eta(z_1)$,

$$\sum_{\theta \in \Theta} \mu(\theta) c_1^*(\theta, z) = z, \forall z \in Z,$$

i.e.,

$$\eta(z_1) = \frac{z}{\sum_{\theta \in \Theta} \mu(\theta) \tilde{c}(\theta_1)}, \forall z_1 \in Z.$$

Then we can redefine $\tilde{c}$ so that $c^*(\theta, z) = z\tilde{c}(\theta)$. Thus, we conclude that separability, in this model, is intrinsically linked to linear functions of endowments.
Consider, then, (c^*, w^*) the candidate optimal consumption plan and promises. Fix a

[9] Notice that there are no incentive compatibility constraints in the last period.

given aggregate endowment $z_1 \in Z$ and only two types $\left(\theta^H, \theta^L\right)$, define γ and τ such that

$$\mu_\Theta^1\left(\theta^L\right)\left[w^*\left(z_1, \theta^L\right) - x\right]^{\frac{1}{1-\rho}} + \mu_\Theta^1\left(\theta^H\right)\left[\gamma\left(x\right) + w^*\left(z_1, \theta^H\right)\right]^{\frac{1}{1-\rho}} = \kappa, \tag{8}$$

and

$$\begin{aligned} & \theta^L \frac{\left[c_1^*\left(\theta^H, z_1\right) - \varepsilon\right]^{1-\rho}}{1-\rho} + \beta\left[w^*\left(z_1, \theta^H\right) + \gamma\left(\tau\left(\varepsilon\right)\right)\right] \\ = \ & \theta^L \frac{\left[c_1^*\left(\theta_L, z_1\right) + \frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta_L\right)}\varepsilon\right]^{1-\rho}}{1-\rho} + \beta\left[w^*\left(z_1, \theta^L\right) - \tau\left(\varepsilon\right)\right]. \end{aligned} \tag{9}$$

where κ is defined as in the text.

Then, it can be easily checked that

$$\begin{aligned} \gamma'\left(x\right) &= \frac{\mu_\Theta^1\left(\theta^L\right)}{\mu_\Theta^1\left(\theta^H\right)}\left[\frac{w^*\left(z_1, \theta^L\right) - x}{\gamma\left(x\right) + w^*\left(z_1, \theta^H\right)}\right]^{\frac{\rho}{1-\rho}}; \\ \gamma'\left(0\right) &= \frac{\mu_\Theta^1\left(\theta^L\right)}{\mu_\Theta^1\left(\theta^H\right)}\left[\frac{w^*\left(z_1, \theta^L\right)}{w^*\left(z_1, \theta^H\right)}\right]^{\frac{\rho}{1-\rho}}; \end{aligned}$$

and that

$$\begin{aligned} \tau'\left(\varepsilon\right) = \ & \frac{1}{\beta\left[1 + \gamma'\left(\tau\left(\varepsilon\right)\right)\right]}\left\{\theta^L\left[c_1^*\left(\theta^L, z_1\right) + \frac{\mu\left(\theta^H\right)}{\mu\left(\theta^L\right)}\varepsilon\right]^{-\rho}\frac{\mu\left(\theta^H\right)}{\mu\left(\theta^L\right)}\right. \\ & \left. + \theta^L\left[c_1^*\left(\theta^H, z_1\right) - \varepsilon\right]^{-\rho}\right\}. \end{aligned}$$

Hence,

$$\tau'\left(0\right) = \frac{\theta^L}{\beta\left[\gamma'\left(0\right) + 1\right]}\left\{\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta_L\right)}c_1^*\left(\theta_L, z_1\right)^{-\rho} + c_1^*\left(\theta^H, z_1\right)^{-\rho}\right\}.$$

We also know that, from optimality of c^* and proposition 2, that for given z_1,

$$\begin{aligned} 0 \in \arg\max_\varepsilon & \left\{\mu_\Theta^1\left(\theta^L\right)\left[\frac{\theta^L}{1-\rho}\left[c_1^*\left(z_1, \theta^L\right) + \frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)}\varepsilon\right]^{1-\rho} + \right.\right. \\ & \left. \beta\left[w^*\left(z_1, \theta^L\right) - \tau\left(\varepsilon\right)\right]\right] + \\ & \left.\mu_\Theta^1\left(\theta^H\right)\left[\theta^H\frac{\left[c_1^*\left(z_1, \theta^H\right) - \varepsilon\right]^{1-\rho}}{1-\rho} + \beta\left[w^*\left(z_1, \theta^H\right) + \gamma\left(\tau\left(\varepsilon\right)\right)\right]\right]\right\}. \end{aligned}$$

Therefore, the following first order condition holds for all $z_1 \in Z$,

$$\begin{aligned} & \mu_\Theta^1\left(\theta^L\right)\left[\theta^L\left[c_1^*\left(z_1, \theta^L\right)\right]^{-\rho}\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)} - \beta\tau'\left(0\right)\right] + \\ & \mu_\Theta^1\left(\theta^H\right)\left[-\theta^H\left[c_1^*\left(z_1, \theta^H\right)\right]^{-\rho} + \beta\gamma'\left(\tau\left(0\right)\right)\tau'\left(0\right)\right] = 0, \end{aligned}$$

that can be re-written as

$$\mu_\Theta^1\left(\theta^H\right)\left[\theta^L\left[c_1^*\left(z_1,\theta^L\right)\right]^{-\rho}-\theta^H\left[c_1^*\left(z_1,\theta^H\right)\right]^{-\rho}\right]+$$
$$+\frac{\mu_\Theta^1\left(\theta^H\right)\gamma'(0)-\mu_\Theta\left(\theta^L\right)}{1+\gamma'(0)}\theta^L\left\{\left[c_1^*\left(\theta^L,z_1\right)\right]^{-\rho}\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)}+\left[c_1^*\left(\theta^H,z_1\right)\right]^{-\rho}\right\}=0$$

$$\frac{\mu_\Theta^1\left(\theta^H\right)\gamma'(0)-\mu_\Theta^1\left(\theta^L\right)}{1+\gamma'(0)}=\frac{\mu_\Theta^1\left(\theta^H\right)\left[\theta^H\left[c_1^*\left(z_1,\theta^H\right)\right]^{-\rho}-\theta^L\left[c_1^*\left(z_1,\theta^L\right)\right]^{-\rho}\right]}{\theta^L\left\{\left[c_1^*\left(\theta^L,z_1\right)\right]^{-\rho}\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)}+\left[c_1^*\left(\theta^H,z_1\right)\right]^{-\rho}\right\}}$$

Now, suppose that $c_1^*(\theta,z)=z\tilde{c}(\theta)$. Then, the equation above implies

$$\frac{\mu_\Theta^1\left(\theta^H\right)\gamma'(0)-\mu_\Theta^1\left(\theta^L\right)}{1+\gamma'(0)}=\frac{\mu_\Theta^1\left(\theta^H\right)\left[\theta^H\tilde{c}\left(\theta^H\right)^{-\rho}-\theta^L\tilde{c}\left(\theta^L\right)^{-\rho}\right]}{\theta^L\left\{\tilde{c}\left(\theta^L\right)^{-\rho}\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)}+\tilde{c}\left(\theta^H\right)^{-\rho}\right\}}$$

We can see that

$$\frac{\mu_\Theta^1\left(\theta^H\right)\gamma'(0)-\mu_\Theta^1\left(\theta^L\right)}{1+\gamma'(0)}$$

should be independent of z. As a consequence,

$$\frac{w^*\left(z_1,\theta^L\right)}{w^*\left(z_1,\theta^H\right)}$$

is independent of z, meaning that $w^*(z,\theta)=\xi(\theta)\psi(z)$, but, since

$$\sum_{\theta\in\Theta}\mu_\Theta^1(\theta)\,w^*(z,\theta)^{\frac{1}{1-\rho}}=\kappa,\text{ for all } z\in Z.$$

w^* should be independent of z. Downward incentive compatibility constraints would be slack for every aggregate shock higher then $\min\{z\in Z\}$: a contradiction. ■

Proof of Proposition 4. Consider the reallocation presented in the proof of Proposition 3. The first order equation is

$$\frac{\mu_\Theta^1\left(\theta^H\right)\gamma'(0)-\mu_\Theta^1\left(\theta^L\right)}{1+\gamma'(0)}=\frac{\mu_\Theta^1\left(\theta^H\right)\left[\theta^H\left[c_1^*\left(z_1,\theta^H\right)\right]^{-\rho}-\theta^L\left[c_1^*\left(z_1,\theta^L\right)\right]^{-\rho}\right]}{\theta^L\left\{\left[c_1^*\left(\theta^L,z_1\right)\right]^{-\rho}\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)}+\left[c_1^*\left(\theta^H,z_1\right)\right]^{-\rho}\right\}}.$$

Consumption in period 2 is totally dependent on period 1 utility promises. Then considering w_1 independent on z_1, the left hand side of the equation above is independent on z_1, define ω as this term. This means that

$$\left[c_1^*\left(z_1,\theta^H\right)\right]\left(\theta^H\right)^{-\frac{1}{\rho}}\left[\mu\left(\theta^H\right)-\omega\right]^{-\frac{1}{\rho}}=\left[c_1^*\left(z_1,\theta^L\right)\right]\left[\theta^L\mu\left(\theta^H\right)+\omega\theta^H\frac{\mu_\Theta^1\left(\theta^H\right)}{\mu_\Theta^1\left(\theta^L\right)}\right]^{-\frac{1}{\rho}}.$$

And therefore first period allocations would be proportional to z_1. This, together with second period independence on z_1, is impossible since downward incentive compatibility constraints would become slack when increasing first period shock realization.

This replicates the finding of Phelan (1994) in a setting with CARA utility ■

B Mirrlees' period T allocation

Here we characterize the last period allocation of a T period version of the model presented in Section 3. Consider the problem being analyzed,

$$\max_{c,y} \sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^1(z^t) \mu_\Theta^t\left(\theta^t\right) \left[u\left[c(\theta^t, z^t)\right] - v\left[\frac{y(\theta^t, z^t)}{\theta_t z_t}\right]\right]$$

subject to

$$\sum_{\theta^t} \mu_\Theta^t\left(\theta^t\right)\left[c(\theta^t, z^t) - y(\theta^t, z^t)\right] \leq 0$$

and

$$\sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_\Theta^t\left(\theta^t\right) \left[u\left[c(\theta^t, z^t)\right] - v\left[\frac{y(\theta^t, z^t)}{\theta_t z_t}\right]\right] \geq$$
$$\sum_{t=1,2} \beta^{t-1} \sum_{\theta^t} \sum_{z^t} \mu_z^t(z^t) \mu_\Theta^t\left(\theta^t\right) \left[u\left[c\left(\sigma(\theta^t, z^t), z^t\right)\right] - v\left[\frac{y\left(\sigma(\theta^t, z^t), z^t\right)}{\theta_t z_t}\right]\right]$$

Suppose that the variable of choice is $\tilde{c}$ and $\tilde{y}$, but that the actual consumption and labor implemented in period T are given by $c_T\left(.\right) = \tilde{c}_T\left(.\right) z_T^{\frac{\gamma}{\gamma+\rho-1}}$ and $y_T\left(.\right) = \tilde{y}_T\left(.\right) z_T^{\frac{\gamma}{\gamma+\rho-1}}$, since $\tilde{c}_T$ and $\tilde{y}_T$ also depend on z_T potentially, this change of variable is without loss of generality. Assume that $c_t = \tilde{c}_t$ and $y_t = \tilde{y}_t$ for $t < T$. Given that we are in a finite period model, the incentive compatibility constraint can be written as a restriction that agents do not lie at period t given any history and that they do not plan to lie from t onwards. Then the last period incentive compatibility becomes

$$z_T^{\frac{(1-\rho)\gamma}{\gamma+\rho-1}} \left\{u\left[\tilde{c}(\theta^{T-1}, \theta, z^t)\right] - v\left[\frac{\tilde{y}(\theta^{T-1}, \theta, z^T) z_T^{\frac{\gamma}{\gamma+\rho-1}}}{\theta_T}\right]\right\} \geq$$
$$z_T^{\frac{(1-\rho)\gamma}{\gamma+\rho-1}} \left\{u\left[\tilde{c}\left(\theta^{T-1}, \theta', z^t\right)\right] - v\left[\frac{\tilde{y}\left(\theta^{T-1}, \theta', z^T\right)}{\theta_T}\right]\right\}.$$

And the resource constraints turns into

$$z_T^{\frac{\gamma}{\gamma+\rho-1}} \sum_{\theta^T} \mu_\Theta^T\left(\theta^T\right)\left[\tilde{c}_T(\theta^T, z^T) - \tilde{y}_T(\theta^T, z^T)\right] \leq 0.$$

Then the constraints on $\tilde{c}$ and $\tilde{y}$ do not depend explicitly on the aggregate shock in the last period. From this and the concavity of the objective function, we get that $\tilde{c}$ and $\tilde{y}$ do not depend on z_T. This implies the following Lemma.

Lemma 3 *Consider a finite Mirrlees economy with T periods with i.i.d. θ-shocks. Then, there exist functions $\tilde{c}$, $\tilde{y}$ such that*

$$c_T^*\left(\theta^T, z^T\right) = \tilde{c}_T\left(\theta^T, z^{T-1}\right) z_T^{\frac{\gamma}{\gamma+\rho-1}},$$

and

$$y_T^*\left(\theta^T, z^T\right) = \tilde{y}_T\left(\theta^T, z^{T-1}\right) z_T^{\frac{\gamma}{\gamma+\rho-1}}.$$

Since in the last period there is no future allocations to be also used to screen agents, allocations depend on aggregate shocks as in a static Mirrlees model. We say that the last period allocation is *separable* in this case. An important implication of this separability of individual and public risk is that marginal distortions, which mean marginal labor tax, are independent of shocks.

Lemma 4 *Period T marginal labor tax does not depend on z_T.*

Proof. From the definition

$$\tau_T^*\left(\theta^t, z^t\right) = 1 - \frac{y_T^*\left(\theta^T, z^T\right)^{\gamma-1}}{\left(\theta_T z_T\right) c_T^*\left(\theta^T, z^T\right)^{-\rho}},$$

it follows immediately that

$$\begin{aligned} \tau_t^*\left(\theta^t, z^t\right) &= 1 - \frac{z_T^{\frac{\gamma(\gamma-1)}{\gamma+\rho-1}} \tilde{y}_t\left(\theta^T, z^{T-1}\right)^{\gamma-1}}{\left(\theta_T z_T\right)^{\gamma} z_T^{\frac{-\rho\gamma}{\gamma+\rho-1}} \tilde{c}_T\left(\theta^T, z^{T-1}\right)^{-\rho}} \\ &= 1 - \frac{\tilde{y}_t\left(\theta^T, z^{T-1}\right)^{\gamma-1}}{\theta_T^{\gamma} \tilde{c}_T\left(\theta^T, z^{T-1}\right)^{-\rho}}. \end{aligned}$$

■

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Figure 1

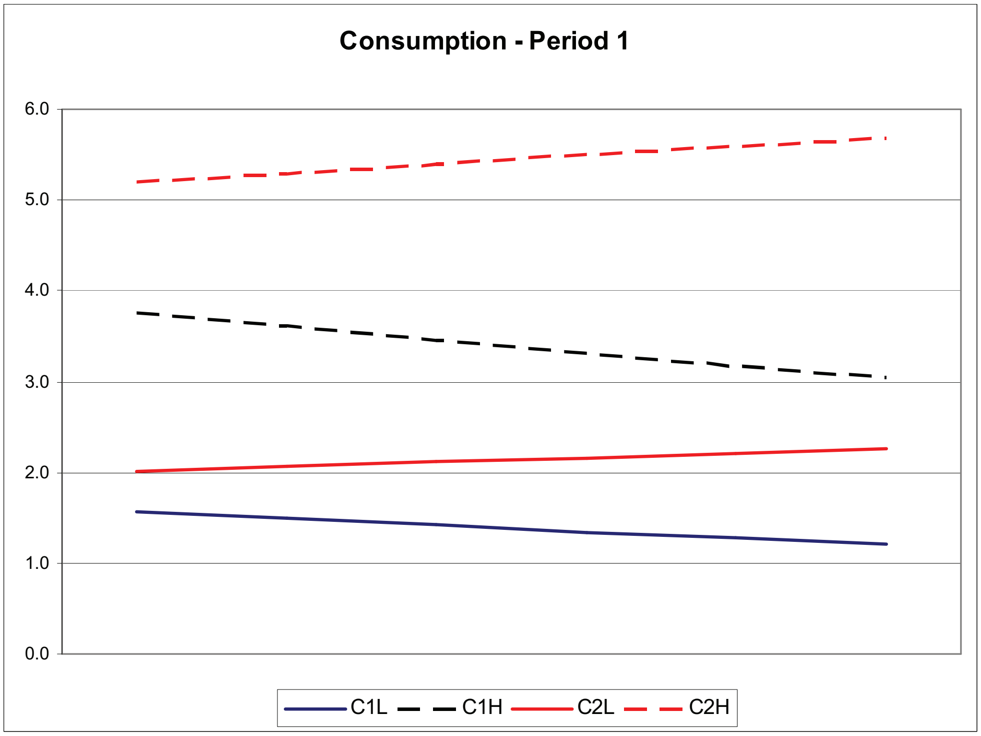


Figure 1-A

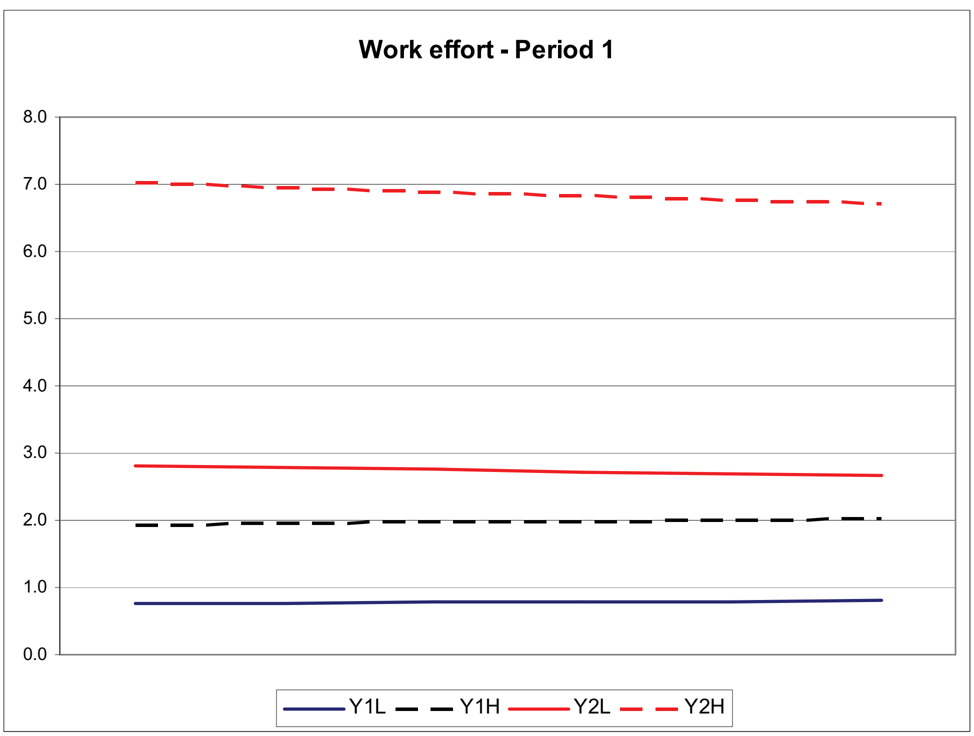


Figure 1-B

Figure 2

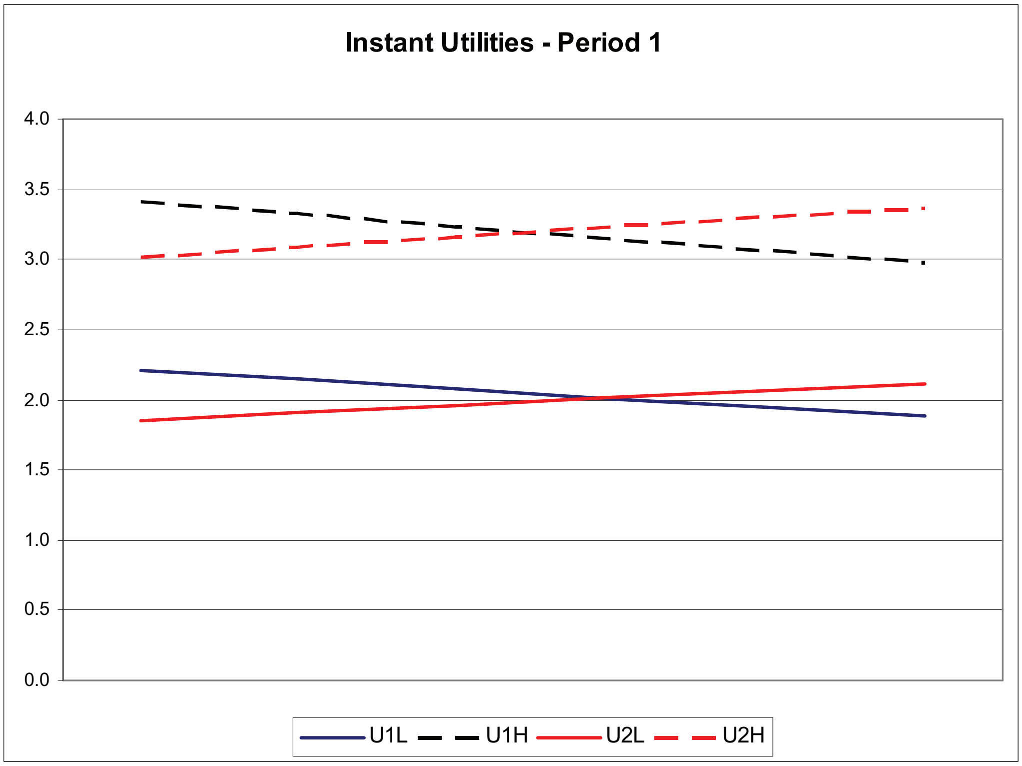


Figure 2-A

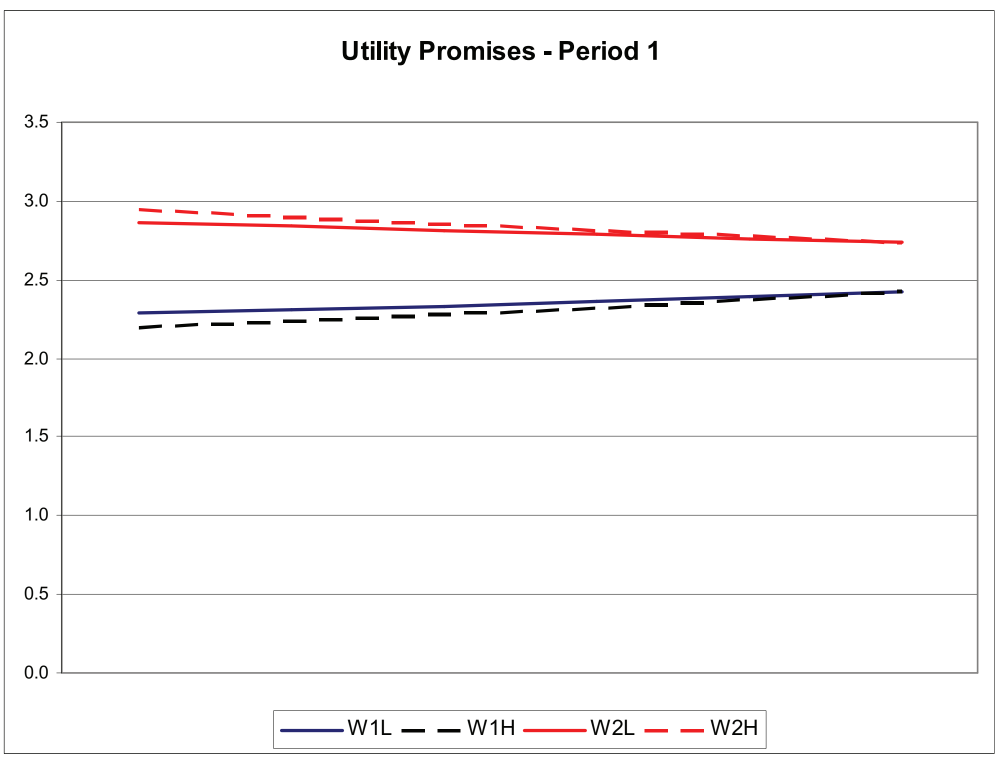
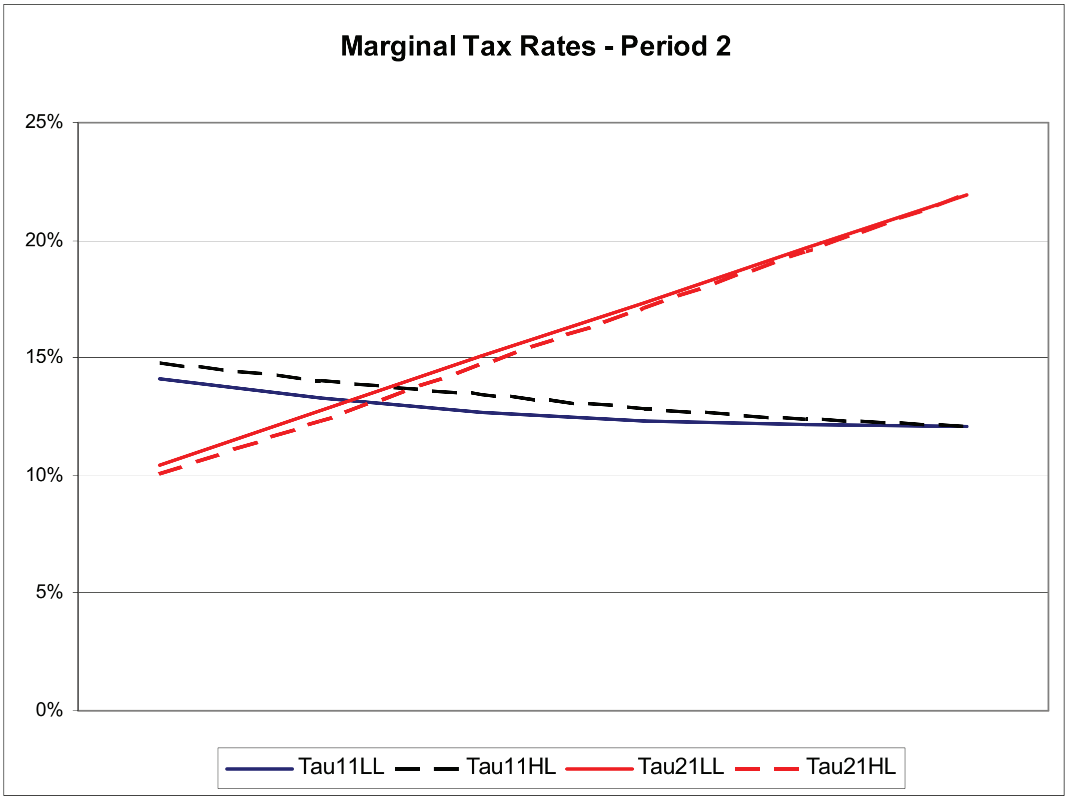


Figure 2-B

Figure 3


Marginal Tax Rates - Period 2
25%
20%
15%
10%
5%
0%
Tau11LL
Tau11HL
Tau21LL
Tau21HL