

The Revealed Social Welfare Function: USA \times Brazil*

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Abstract

This paper proposes the use of the optimal nonlinear tax formula to derive the social welfare function of policymakers. In particular, it uses PSID and PNAD for 1990 to estimate the implicit social welfare functions associated with income tax imposition for the USA and Brazil. Under assumptions on the household preferences, the estimations suggest that both U.S. and Brazilian social welfare functions are concave and Paretian, but that only the Brazilian function is utilitarian, i.e., the Brazilian social planner is less inequity-averse than her U.S. counterpart.

Keywords: Social Welfare Function, Optimal Tax, Pareto Efficiency and Nonparametric Estimation.

JEL Codes: H20, H21, C14, C15.

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1. Introduction

Mirrlees (1971) proposed a solution for the problem of nonlinear income tax structure using general utility and social welfare functions. He also numerically computed the optimal marginal tax rates assuming a utilitarian social welfare function and an additive logarithmic utility function for the households. Many different authors have extended these theoretical results to an optimal nonlinear income tax model suggesting that the results derived in Mirrlees depended upon assumptions on the social welfare function, the preferences of the households and the skill (or pre-tax income) distribution.¹

For instance, Atkinson and Stiglitz (1980) pointed out that different results could be obtained for optimal marginal tax rates depending on the choice of the social welfare function. The authors observed on the one hand that if a utilitarian social welfare function were used, a marginal tax rate, which is slightly decreasing in income, could be an optimal tax structure. On the other hand, using the same utility function for the agents, if a Rawlsian social welfare objective function were considered, a very high effective marginal rate for high abilities was the optimal marginal tax structure. These results reflect the sensitivity of the optimal tax structure in relation to the selected social welfare function.

With respect to the household's utility function, Tuomala (1990) stressed that different levels and shapes for the tax structure could be obtained if the elasticity of the substitution between labor and the consumption good was lower than the one assumed in Mirrlees (1971). Tuomala's numerical calculation using consumers' preferences that satisfy the condition that the elasticity of substitution between consumption and leisure is 0.5 revealed marginal taxes for low skilled households greater than the one computed in Mirrlees (1971). In addition, Diamond (1998) using Mirrlees' approach and imposing quasi-linearity on individuals' preferences, derived a set of conditions which could be achieved by a U-shaped pattern of optimal marginal tax rates. One of those conditions was related to the skill distribution which was assumed to be either Pareto or exponential distributed.

Even though these papers suggest that the optimal tax structure depends on the assumed form of the household's utility function, the assumed form of the skill distribution and the used social welfare function, none of them has considered the estimation of or inference about the social welfare function. The exception is Bourguignon and Spadaro (2002, 2005a,b). Under several assumptions, the authors numerically calculated the shape of the social welfare function for European countries. Bourguignon and Spadaro imposed quasi-linearity on the agents' utility function and also used an isoelastic utility function. That was one of the conditions in Diamond's paper that, if satisfied, could generate the U-shaped optimal marginal tax rate.

¹See Tuomala (1990) for a survey on this subject.

Following Bourguignon and Spadaro (2002), this paper estimates the social welfare functions from observed American and Brazilian data under the assumption that the household's utility function is quasi-linear in consumption and that the labor supply elasticity is constant. These assumptions make the results comparable to Bourguignon and Spadaro (2002).²

The paper differs from the previous one in many aspects. First, the paper estimates the behavior of the social planner for two contrasting but still comparable countries. For instance, while Brazil has a very high income inequality, Gini coefficient equal to 61.3, and the USA has a much lower value, 40³ the ratio of tax revenues to the GDP is very similar (around 12%, see Table 1). Second, the actual marginal income tax rate is used here, instead of tax-benefit data for European countries, as the variable that represents the marginal income tax in Mirrlees' model.⁴ Third, since the observed marginal tax function can be adjusted to a step function, no estimation of the derivative of the tax function is necessary. It is assumed to be zero everywhere. Fourth, I provide the confidence interval of the estimates, which allows us to make hypothesis tests on the estimated social welfare function. Last, the estimated social welfare functions are assumed to be continuous and the kernel regression method is used to compute them.

The objective of the paper is threefold. First, to use the optimal nonlinear income tax formula as a tool to help reveal the government's preferences with respect to its welfare function. Second, to test if the marginal social welfare weights for different households are equal (i.e., if the social welfare function is utilitarian). Third, to investigate if the marginal social welfare weights are positive for all households (i.e. if the social welfare function is Paretian).

²A different approach proposed by Saez (2001) incorporates labor market responses into Mirrlees' model.

³See www.worldbank.org.

⁴This is due to data limitation for the Brazilian case.

Table 1
Descriptive statistics of the data

		Single		Whole Sample	
		USA	Brazil	USA	Brazil
obs		1, 117	2, 962	5, 111	16, 653
Income	min	175	0.210	175	0.210
	median	18, 000	7.569	31, 074	11.440
	mean	21, 925	15.377	37, 591	20.457
	max	300, 000	235.50	563, 500	619.95
T(wL)	min	0	0	0	0
	median	1, 845	0	2, 934	0
	mean	2, 824	0.974	4, 568	1.533
	max	46, 276	4.435	134, 934	127.69
T'(wL)	min	0	0	0	0
	median	0.15	0	0.15	0
	mean	0.17	0.04	0.18	0.06
	max	0.33	0.275	0.33	0.275
skills (w)	min	31.29	348.3	31.29	348.3
Elast.0.5	median	725.1	3, 797	1, 064	4, 999
	mean	802.1	5, 541	1, 156	6, 828
	max	5, 000	41, 340	7, 612	78, 820
skills (w)	min	53.14	857.1	53.14	857.1
Elast.0.3	median	1, 948	13, 490	2, 998	18, 530
	mean	2, 210	21, 550	3, 359	27, 810
	max	17, 720	203, 000	28, 620	427, 300
skills (w)	min	109.4	2, 926	109.4	2, 926
Elast.0.1	median	7, 496	76, 050	12, 400	110, 700
	mean	8, 857	139, 600	14, 480	182, 100
	max	98, 210	177, 700	174, 200	4284, 000

Note: Income for USA and Brazil is in US 1990 dollars.

With respect to Pareto efficiency of income tax, Werning (2007) derives the conditions under which income tax schedule can be Pareto efficient. In particular, the paper identifies the skill distribution as the key variable to characterize efficient tax schedules. The paper argues that for any increasing income tax structure, “...there exists a set of skill distributions for which the allocation can be Pareto efficient and another set which is not...” (Werning, 2007, p. 1), i.e., the empirical determination of skill distribution is fundamental to know whether specific tax schedules are Pareto efficient or not. As a consequence, one has to impose constraints on the tax function and the allocation it generates in order to have a Pareto efficient tax system.⁵

⁵Werning (2007) maximizes aggregate net resources subject to incentive compatibility constraints and that none worker is made worse off. This problem is different from the standard optimal income tax problem (Mirrlees, 1971) in the sense that it characterizes the conditions under which an income tax schedule can be pursued as a Pareto efficient mechanism.

The results demonstrate that both U.S. and Brazilian social welfare functions are concave and Paretian, but that only the Brazilian one is utilitarian, i.e., the Brazilian social planner is less inequity-averse than her U.S. counterpart. This suggests that the Brazilian economy can achieve a lower income inequality level by imposing an income tax structure similar to the one adopted in the United States.

As a preliminary step, five other important characteristics and limitations of the estimation procedure should be mentioned. First, because I assume a particular preference for the households, the study captures only comparative differences between countries. Since the same assumption on household's preferences is made across countries, the estimations are comparable. Second, this study only considers income tax, that is, it excludes payroll taxes, social security and other indirect taxes. While the first two are interpreted as a future payment back to the households, the latter can be easily introduced in the model.⁶ The introduction of indirect taxes increases the marginal tax faced by every household, however the marginal effect is proportionately higher in low-skilled households.⁷ That is a distorted effect once poor households tend to consume goods that are either tax-exempt or that face a low tax due to the income effect. In addition, as will be discussed later, the survey used for Brazilian data does not include the disposable income variable.⁸ This variable (disposable income) could be used to compute the amount of total taxes paid and then calculate the effective marginal tax. Third, even though the political economy literature emphasizes the role of political considerations in determining actual tax policies, it seems reasonable to recover the social weights assigned to the population in accordance with the imposed income tax. The estimated social welfare functions can be interpreted as the resulting social welfare function driven by political forces. However, one cannot tell what causes our results. Whether individualism or independence is preferred over social cohesion or what the policymaking process is cannot be answered here, only the empirical results. Fourth, there are not public available actual tax data for households in both countries. Therefore, the data are estimations of the true tax payment and the results are conditioned on the assumption that this computation is correct. Last, there are no data available on the cost of tax collection, therefore

⁶The interpretation that social security is paid back to the individual is loosely made to simplify the analysis. Actually, some but not all contributions return to the agents and the marginal tax rate faced by them might be quite high depending on the level of their income, and in a dynamic perspective, on GDP growth versus interest rate and the life cycle stage of each individual.

⁷Since there exists only one consumption good in this model observe that $\partial C/\partial Y = (1 - T'(wL))/(1 + \theta)$, where θ is the indirect tax. However, for large θ as the average commodity tax, as it is the case for Brazil, that relation could be approximated by $(1 - T'(wL))/(1 + \theta) - 1$. We thank the referee for pointing that out.

⁸However, income tax payments are consistently estimated considering gross income tax for U.S. and Brazilian households. It is worth mentioning here that we use only formal workers in Brazil to compute income tax payments.

the analysis ignores this effect.

The paper is structured as follows. Section 2 lays out the model that is estimated considering the case that the isoelastic quasi-linear utility function is used; Section 3 discusses the empirical specification. The data, which consist of an annual observation of households for the year 1990 in the USA and Brazil, are described in Section 4. The results are discussed in Section 5. The last section concludes the analysis.

2. Theoretical Model

The model follows the optimal nonlinear tax structure. Agents choose consumption (c^*) and labor (L^*) to maximize the utility ($u(c, L)$) given the constraint determined by the net of tax income ($c = wL - T(wL)$), where w is the productivity of the household and $T(wL)$ is the amount of tax paid by this household.

The government's concern is to maximize a social welfare function (G) on the support of the productivity range $[\underline{w}, \bar{w}]$, given the household's optimal choice of consumption and labor supply,⁹

$$\begin{aligned} & \max_{T(wL^*)} \int_{\underline{w}}^{\bar{w}} G(v(w)) f(w) d(w) \\ & \text{s.t.} \\ & v(w) = u(c^*(w, T), L^*(w, T)) \\ & \dot{v}(w) = \frac{-wL}{w^2} \frac{\partial u(c^*(w, T), L^*(w, T))}{\partial L} \\ & \int_{\underline{w}}^{\bar{w}} T(wL) f(w) d(w) \geq E \end{aligned} \quad (1)$$

where $f(w)$ is the density of household population for productivity level w , E is the government's expenditure level and $v(w) = u(c^*(w, T), L^*(w, T))$ is the indirect utility level achieved by the household that has productivity level w . Assuming that the utility of the consumption good and leisure is separable and equal for every agent, i.e., $U(c, L) = u(c) + h(1 - L)$, the maximization condition for the optimal tax problem can be obtained.¹⁰

This paper considers the estimation of two related functions to recover the social planner's preferences: the average social welfare and the marginal social welfare functions. The first one concerns the estimation of the implicit average social weight imposed on agents with productivity equal to or greater than w and the second function captures the implicit marginal weight imposed on the household with productivity w .¹¹ Therefore, for empirical purpose it is convenient

⁹This is a partial equilibrium model, therefore the real incidence of any tax is the same as its nominal incidence.

¹⁰See Bourguignon and Spadaro (2005a) for the computation of first-order conditions.

¹¹This is not a redundant exercise since these are different data. Also, see Figures 5 and 9 as an example of the case that the interpretation of the results of the marginal and the average

to rewrite the solution of the problem by considering an isoelastic quasi-linear households' preference,

$$-\frac{\int_{-\infty}^{\bar{w}} G'(u(c^*, L^*)) dF}{p(1 - F(w))} = \frac{f(w)T'(wL)[\frac{e(w)}{w}]^{-1}}{(1 - T'(wL))(1 - F(w))} + 1 \quad (2)$$

where $T'(wL)$ is the marginal tax rate for the agent with productivity w , $F(w)$ is the cumulative skill distribution, $e(w) \equiv (1 - h''(1 - L)L/h'(1 - L))$, $G'(v(w))$ is the marginal social weight of an individual with productivity-type w , and p is the Lagrange multiplier on the government's budget constraint. The term $u'(w)$ stands for the marginal utility of the consumption good, which depends on the wage (productivity) of the individual and the tax.^{12,13}

The LHS represents average social value of the utility of individuals above the marginal tax rate being analyzed. On the right-hand side is the "efficiency" counterpart: increasing the tax rate implies 1 dollar of lump-sum taxation on individuals above this marginal tax rate, corrected by the average efficiency loss, which is equal to the loss of income (obtained by the multiplication of the tax rate and the output at this wage level) multiplied by the compensated labor elasticity. This loss is averaged according to the number of individuals above this wage level and is also normalized to the net income. Thus, this formula shows that at the optimum one can infer the social weights of the social planner by looking at the parameters on the right-hand side: the wages or skills, w , density function of the skills ($f(w)$, $F(w)$, Saez (2001) argues that this distribution comes from the data), actual marginal tax rates, ($T'(wL)$), the compensated labor elasticity, $e(w)$ and the preferences of the household, $u(w)$.

Note that in principle the RHS can be negative. For example, during the 1960s, the marginal tax rates for high income levels were very high in the USA. Saez (2004) reports a rate of more than 80% in the upper part of distribution of taxpayers, which implies a ratio of $T'/(1 - T')$ around 4. This may cause the RHS to be negative. If that is the case, then one may infer that the social planner is not Paretian, i.e., he might impose a negative weight to the upper 0.01%.

The marginal social welfare function $G'(u(w))$ is obtained by taking the derivatives of equation 2 with respect to w

$$G'(u(w)) = 1 + \frac{(1}{e(w))} \left(\frac{T'(wL)}{1 - T'(wL)} \right) \left(1 + \frac{wT''(wL)y'(w)}{T'(wL)(1 - T'(wL))} + \frac{wf'(w)}{f(w)} \right) \quad (3)$$

social welfare function might differ for the same labor supply elasticity assumed.

¹²To avoid supnotation, the optimal utility level of the household for the consumption good is denoted by either $u(c(w, T))$ or $u(w)$.

¹³It can be easily shown that $p = \frac{\int_{\bar{w}} G'(w)dF(w)}{\int_{\bar{w}} 1/u'(c(w, T))dF(w)}$.

where $T''(wL)$ denotes the derivative of the marginal tax function and $f'(w)$ denotes the derivative of the density function with respect to w .¹⁴

This paper focuses on three related questions. Primarily, to guarantee the existence of a solution to the social planner's problem such as equation 2, $G''(w)$ must be negative or zero. This corresponds to the examination of whether $G'(w)$ (marginal welfare function) and $\int_w^{\bar{w}} G'(u(c^*, L^*)) dF$ (average social welfare function) are non-increasing functions everywhere. In a more intuitive approach, the question is whether the marginal social weights are non-increasing with the productivity levels, so that the social weight of high-productivity individuals is lower than or equal to the lower-productive ones.

Secondly, in the case that the social welfare function is concave, are the most productive workers still being penalized because their social weight is lower than that of unskilled households? That is equivalent to asking whether the social welfare function is utilitarian or not. This can be tested by looking at equations 2 and 3 and comparing the estimated social weights. In the case that the social weight of the low skilled households falls within the confidence region of high-skilled households, then the utilitarian social welfare function hypothesis cannot be rejected. On the other hand, if it falls out, then it can be said that the social planner is not utilitarian.

Thirdly, does the increasing income marginal tax structure adopted in the USA and Brazil imply that the social welfare function is necessarily Paretian? This will not be the case if the most productive workers are being "penalized" so severely that their marginal social weights are negative. In terms of the estimated functions, that consists in checking if the functions (marginal and average social welfare functions) assume negative values.

3. The Data

The PSID data provide the marginal tax rate ($T'(wL)$) and the amount of income tax paid ($T(wL)$) by each household for the U.S. data. That is not the case for the Brazilian data – PNAD. The marginal tax and amount of income tax paid imposed on each household are recovered using the information in the PNAD survey matching with Law 7799 published on 07/10/1989. However, it is not possible to recover the total amount of taxes paid by each household.

There are 1,117 observations for the U.S. case and 2,962 observations for the Brazilian case. For the Brazilian data, informal household data (almost 40% of the sample) are excluded. Table 1 presents the data.

¹⁴See Bourguignon and Spadaro (2002) for the derivation of equations 2 and 3.

In addition, I use four additional assumptions. First, I use the data on earned income as a proxy for income. Second, the time period of one year is considered the long-run equilibrium. Third, there is no human capital accumulation, and fourth, I ignore the administrative cost of tax collection.¹⁵

Using quasi-linear isoelastic individuals' preferences makes the terms w , $e(w)$ and $e'(w)$ easily computed as explained below. In addition, normalizing the term $\int_{\underline{w}}^{\bar{w}} G'(w) dF$ to one automatically guarantees that $\lambda = 1$, where λ is the shadow price of the maximization problem.

In the real world, governments determine the tax rate according to characteristics other than productivity, such as marital status and number of children in the family – in the case of the USA and Brazil, which change the exemption and deduction levels. There are two extreme views on how to deal with this problem. As the first option I separate each class of agents according to their marital status and number of children. The second option is to ignore these other characteristics and compute the social welfare function as if families with different characteristics were treated identically. The main assumption underlying this option is that the government maximizes the social welfare function separately from each class of household. I consider only the results for the single household in this paper.¹⁶

3.1 Marginal tax for the USA

The marginal tax table schedule provided by the PSID takes into consideration the exemptions and deductions necessary to evaluate into which bracket the tax unit income falls. This definition matches the IRS's definition of marginal tax rate¹⁷ and the concept behind Mirrlees' formulation. Therefore, our analysis uses the imputed marginal federal taxes from PSID, which corresponded to the following marginal taxes: 0%, 15%, 28% and 33% and depending upon the annual earned income of the household an additional 0% or 10% is computed due to the inclusion of that household in the EITC program.¹⁸ In particular, up to US\$ 19,450.00 as an annual income, there was a marginal tax rate of 15.0%. From US\$ 19,450 to US\$ 47,050.00 annual income the rate was 28.0% and from US\$ 47,050.00 to US\$ 97,620.00 the rate was 33.0%. The standard exemption is US\$ 2050 and deductions can go to US\$ 3250.00 in 1990 for single individuals.¹⁹

¹⁵Those seem to be very restrictive, but the main concern of the paper is to verify the Mirrlees' model strictly using compatible data.

¹⁶The estimations for the other samples are available upon request. Additionally, another segmentation is tried such as restricting the analysis only to households that pay rent. The results are similar and are available upon request.

¹⁷For instance, see *Individual Income Tax Rates and Shares*, 1999 (Appendix D), page 22.

¹⁸For more references and computation on this issue, see *Panel Studies of Income and Dynamics: Procedures and Tape Code 1985 Wave (XVIII), A Supplement*.

¹⁹Above that annual income, the rate would go to 31.0%, but we did not find a single individual which had that marginal tax rate estimated by the PSID.

3.2 Marginal tax for Brazil

The PNAD data provide neither the amount nor the marginal tax imposed on the household; however, Law 7799 published on 07/10/1989 discriminates the marginal income taxes imposed on the households depending upon their gross earned income. For 1990, they were: 0%, 10% and 25% in Brazil.²⁰ Law 7799 also provided the deduction and exemption amounts allowed by marital status and by the number of children, which enables us to compute the amount of income taxes paid by each household. Therefore, the PNAD information about the households, combined with the information provided by Law 7799, recovers both the amount of taxes and the marginal tax paid by each household. This method is a simplified version of the one used in the PSID for the U.S. data.²¹ In particular, our estimates suggest that for an income up to US\$ 178.17, the consumer has to pay no income tax. From US\$ 178.17 to US\$ 593.90, there is a marginal income tax of 10%. Above that value, the consumer has to pay a marginal income tax of 25%.

3.3 Productivity and the $e(w)$ term

For a particular utility function for the agents, $u(c, L) = c - L^{(1+1/\epsilon)}/(1 + 1/\epsilon)$, and given the gross income (y), marginal tax ($T'(wL)$) and amount of taxes paid ($T(wL)$), it is possible to find the maximized level of labor supply for the agents. Substituting the optimal labor supply ($L^* = w^\epsilon [1 - T'(wL^*)]^\epsilon$) back in the budget constraint ($y = wL^*(w, T'(wL))$) and consequently solving for the wages (productivity) ($w = y^{\frac{1}{1+\epsilon}} [1 - T'(wL)]^{\frac{-\epsilon}{1+\epsilon}}$),²² productivity will depend on the gross labor income (y), the marginal tax ($T'(wL)$) and the constant labor supply elasticity, ϵ . Given these preferences, $e(w) = 1 + 1/\epsilon$.²³ Observe that in this case $e'(w) = 0$. In my computations $\epsilon = 0.5, 0.3$ and 0.1 are considered.²⁴

²⁰For the Brazilian sample, there is no information on disposable income of the households and therefore using the 'legal marginal taxes' is the only viable alternative to compute marginal income taxes.

²¹The author is aware of the possibility of underestimating the households' tax payment; however, this can make the conclusions of the paper even stronger. The POF-96 is used to verify the percentage of total income spent on every deductible item. If it is greater than the maximum possible for deduction, then the second is used; otherwise, the first is chosen.

²²This equation implies that the higher the marginal tax, the higher the productivity level should be.

²³See equation 2 and how the term $e(w)$ is defined below it.

²⁴The choice of this range was based on a survey of the estimations provided in (Blundell and Macurdy, 1999, pp. 1646–1651). For Brazil, Avelino and Menezes-Filho (2003) find a labor supply elasticity from -0.21 to 0.05 .

The social welfare functions (average and marginal) are estimated for $F(w)$, ranking in the cumulative distribution of the productivity, and not for w (productivity) *per se*. The term $F(w)$ denotes the case that the rank of the productivity in its cumulative distribution is used instead of the productivity itself. This means that after the productivities are computed they are ordered in ascending fashion within the $[0, 1]$ interval. Their order is used as an X variable. This is a reasonable approach, not only because it is comparable to the literature, but also because the data points are too distant from each other at the very right end of the skill distribution.²⁵

4. Estimation Results

The questions posed in Section 1 are answered by investigating the estimated functions corresponding to equations 2 and 3. Equation 3 explicits the relation between the marginal social weight imposed on the households for each ranked productivity ($F(w)$) and the marginal tax structure. Therefore, testing for concavity, the utilitarian and Paretian hypotheses consist only in examining the slope and the sign at each productivity level for the estimated function. If it is non-increasing in the ranked productivity, the social welfare function is concave and a solution to the social problem exists. If it is equal for every productivity level, then it suggests that the social welfare function is utilitarian. In addition, if it is never negative, a Paretian specification of the social welfare function can be considered.

On the other hand, the advantage of estimating equation 2 is the fact that it does not rely on any estimate of derivatives either for the regression or for density functions, and its estimate is used as a verification of the results obtained for equation 3.

The results for the USA are presented first, followed by the Brazilian estimates. A comparison between the U.S., Brazilian and the previous literature is presented in the last subsection. The conclusion follows next.

4.1 USA

Figure 1 presents the density estimation for single individuals with a constant labor supply elasticity evaluated for the possibilities: 0.1, 0.3 and 0.5 and the productivities are rescaled to have a mean equal to one. The density estimation considers the data points described by the equation $w = Ay^{\frac{1}{1+\epsilon}}[1 - T'(wL)]^{\frac{\epsilon}{1+\epsilon}}$ which can be seen as a weighted function of gross income and the inverse of the $[1 - T'(wL)]$. The higher the labor supply elasticity, the more concentrated the estimated distribution at mean (1). That is the case because more weight is being given to the inverse of the $[1 - T'(wL)]$ which has less variation than the gross income when high labor supply elasticity is being considered.

²⁵In a previous version of the paper, both (ranked and non-ranked productivities) approaches are used and the main results do not change, but the results of the former paper are visually more appealing.

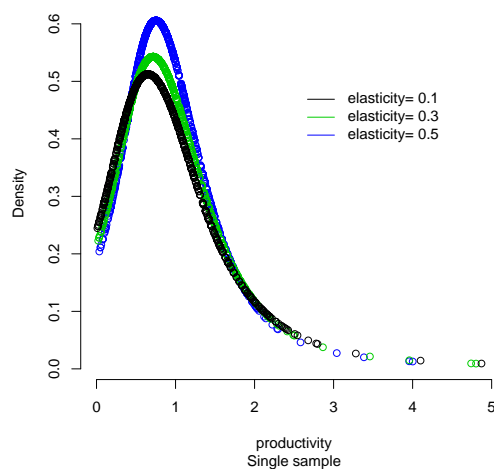


Figure 1
Density estimation US data

Figure 2 displays the true marginal tax rates for each agent's type. The figure presents the structure that the higher the productivity, the higher the marginal tax; however, there are households with similar productivity and a different marginal tax on the extreme range of each tax bracket. That may happen here because these households may face different deduction levels such as educational expenses, previous losses or debts, and consequently they may face different marginal tax rates.²⁶ As explained before, the tax structure presents the shape of a step function and this characteristic is explored for the estimation of the term $T''(wL)$, which is zero.

²⁶The estimated tax function did not change significantly, either with changes in labor supply elasticities or with changes in the sample under analysis. Note also that there is no tie in the households' $F(w)$ position.

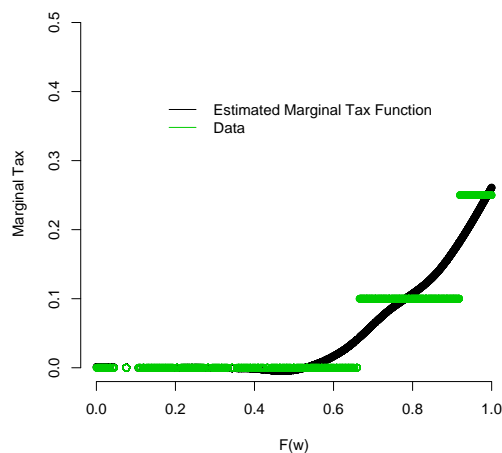


Figure 2
Tax function estimation US-single sample

Figure 3 leads to a comparative inference analysis where the marginal and average estimates are put together for the single sample, considering different labor supply elasticities. Again, the estimates for the other samples are omitted due to similarities in the results. This picture shows that the higher the labor supply elasticities, the more negatively sloped the estimated marginal and average social welfare functions. In deriving the optimal income tax schedule (equation 2), we observe that, given a social welfare function, the more elastic the labor supply, the lower the optimal tax rate should be (because of efficiency consideration). Consequently, it follows that for a given (optimal) tax rate, a higher labor supply elasticity must be associated with a smaller social welfare weight given to high-skilled workers. In other words, for the same redistribution package offered by the government, the revealed social preferences seem more averse to inequality when the labor supply elasticity is high.

Figures 4, 5 and 6 present the estimates of the social weights for each $F(w)$ (ranked productivity-type), described by the specialized representation of equation 3, in the social planner's problem, i.e., it is the estimate of "marginal social welfare function", the respective confidence interval and what I define as "test lines" for labor supply elasticities 0.1, 0.3 and 0.5, considering single households.

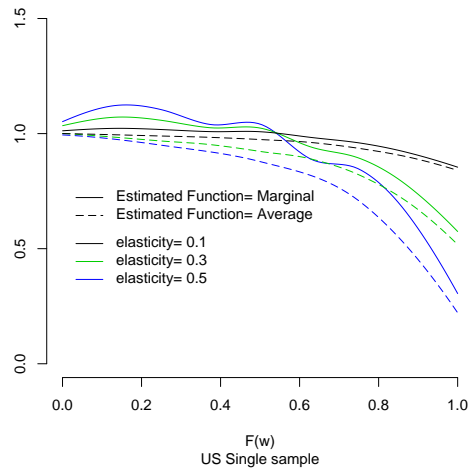


Figure 3
Average and marginal social welfare function

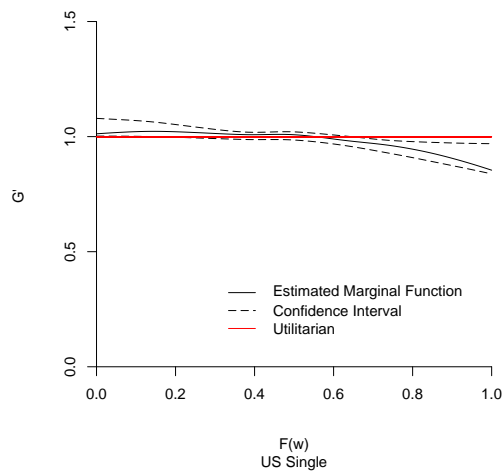


Figure 4
Marginal social welfare function-elasticity=0.1

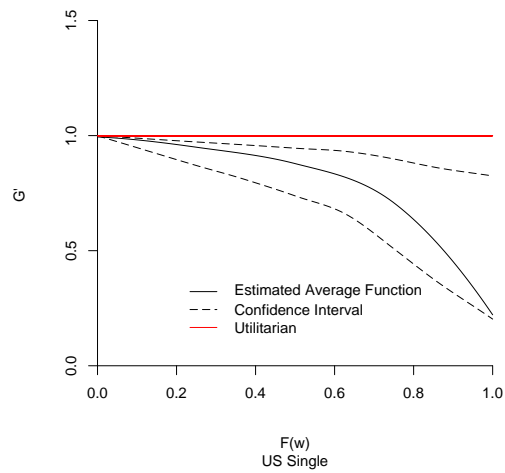


Figure 5
Marginal social welfare function-elasticity=0.3

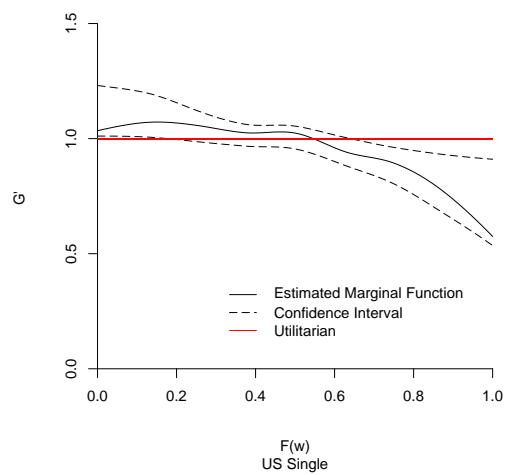


Figure 6
Marginal social welfare function-elasticity=0.5

Figures 7, 8 and 9 represent the estimation of the “average social welfare function”, its respective confidence interval and the “test lines”, for the individuals with productivity above or equal to that specific productivity type represented by the specialized version of equation 2 for the same cases.

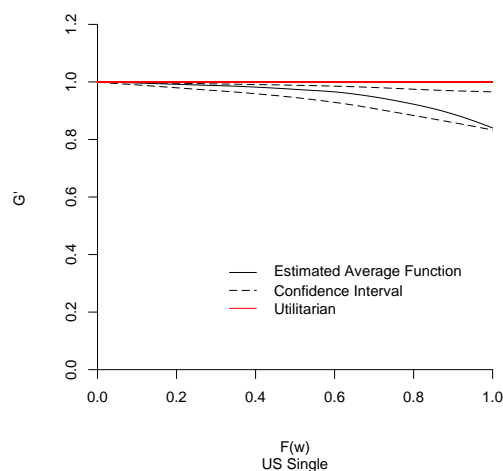


Figure 7
Average social welfare function-elasticity=0.1

The “test lines” are used to test if the estimated functions are concave, utilitarian or Paretian. The Paretian line assumes value equal to zero and is represented by the horizontal axis (x-axis). Whenever the estimated function crosses the Paretian line, the function is said to be no longer Paretian. In addition, if only the confidence interval estimates, but not the function, cross the horizontal axis, then it can be said that the non-Paretian hypothesis cannot be rejected.

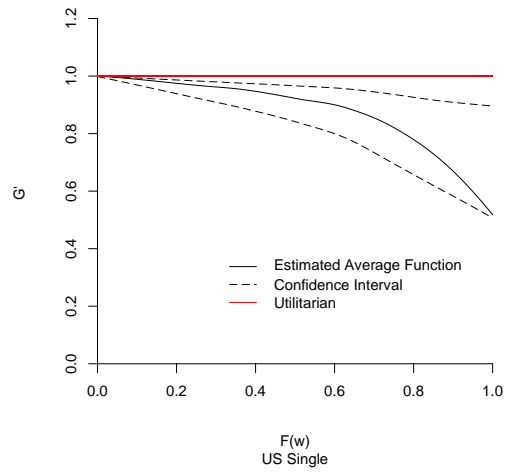


Figure 8
Average social welfare function-elasticity=0.3

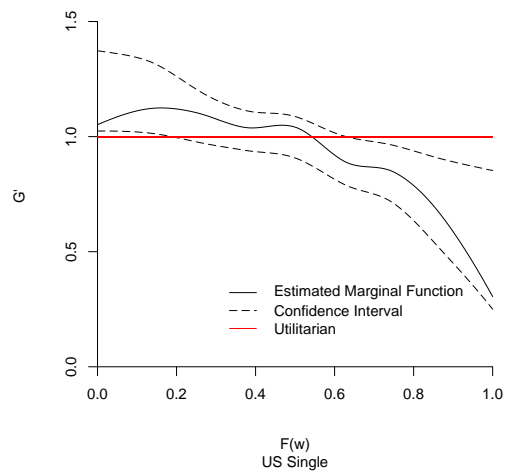


Figure 9
Average social welfare function-elasticity=0.5

The utilitarian test line assumes value equal to one for every household.²⁷ As the utilitarian line starts within the confidence interval of the lowest skilled household, the estimated social welfare function rejects the utilitarian hypothesis if that line falls out of the confidence interval provided for any household. However, it can be concave if the estimated function has a negative slope compared to the utilitarian line. Moreover, if the utilitarian line falls within the confidence interval everywhere, the estimated function is not only utilitarian, but also concave.²⁸

The first of the U.S. data results for the isoelastic case is that the “revealed” social welfare function is Paretian and concave no matter which sample is used, nor the labor supply elasticity used.

Figures 4, 5 and 6 show that if the labor supply elasticity is low, then the social weights of the households are roughly flat; however, the hypothesis of no discrimination against high productive households cannot be accepted within the 1990’s tax schedule. The estimations are clearer when Figures 7, 8 and 9 are observed. In other words, this result suggests that there is equity consideration in terms of social weights in the sense that low-skilled individuals have a higher social weight than high-skilled ones for the U.S. data. The same conclusion is reached for higher labor supply elasticity, but the graphical result is much more obvious. Therefore, the estimations suggest that the “revealed” U.S. social welfare function is concave, Paretian and not utilitarian.

4.2 Brazil

The constant labor supply is evaluated for the possibilities: 0.1, 0.3 and 0.5 and the productivities are rescaled to a mean equal to one. Figure 10 shows that the highest peak of the estimated density is to the left of the mean of the productivities (equal to 1) and it is achieved when the assumed labor supply elasticity is low (0.1). Even though this seems to run contrary to the results estimated for the U.S. data, that may not be the case. Given the functional form for the estimation of w , the higher the elasticity of labor supply, the more concentrated the dispersion of w becomes around its mean value. For Brazil, it is clear that as the labor supply elasticity is higher, the estimated density moves towards the mean, reducing inequality measured by the difference between the median and the mean of the distribution.

²⁷A social welfare function is utilitarian if all the indirect utilities receive the same social weight, however as I am normalizing the average social weight to be one, all the estimations of either the marginal or the average social welfare function have a low end equal or very close to one. Therefore, the choice of a utilitarian test line equal to one seems to be reasonable.

²⁸Alternatively, I estimated the social and marginal social welfare functions for each subsample of the bootstrap and counted that at least 95% were concave for each labor supply elasticity used.

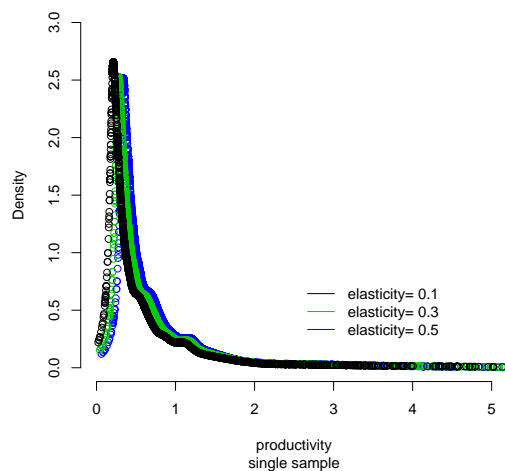


Figure 10
Density estimation Brazilian data

When the low labor supply elasticity is used, the estimated density is the far left of the mean compared to the other cases and also presents the longest right tail. This suggests the same conclusion presented for the U.S. case: the higher the labor supply elasticity, the less unequal the skill distribution.

Conclusions about Figures 11 and 12 are similar to those stated for the U.S. sample and are omitted here except for Figure 11:

- (i) It shows that more than 50% of the households face a zero marginal income tax rate;²⁹ and
- (ii) Agents with close position in the cumulative distribution of the skills ($F(w)$) present the same marginal tax, that is, there is no “point overlapping” in the marginal tax structure.

²⁹Remember that this is an estimated computation. That characteristic might push the government to rely on indirect taxes to increase revenues for the growing debt.

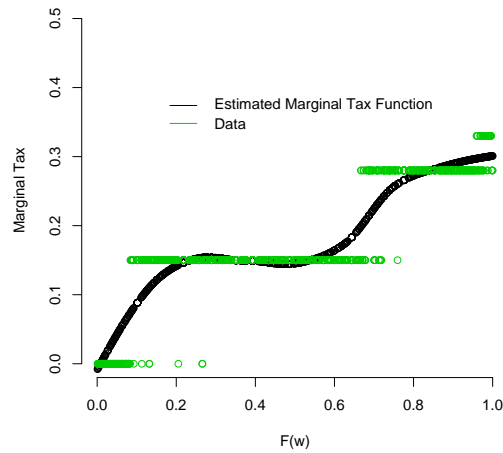


Figure 11
Tax function estimation BraUS-single sample

Figures 13, 14, 15 present the estimate of the social weights of each productivity type in the social planner's problem, i.e., it is the estimate of "marginal social welfare function" described by the specialized representation of equation 3. The respective confidence intervals and the "test lines" are also presented for labor supply elasticities 0.1, 0.3 and 0.5, considering single households.

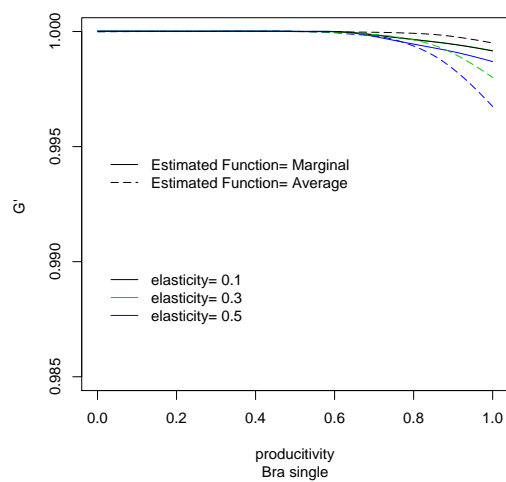


Figure 12
Average and marginal social welfare function

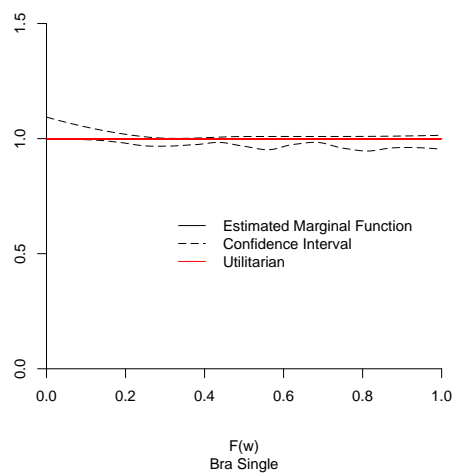


Figure 13
Marginal social welfare function-elasticity=0.1

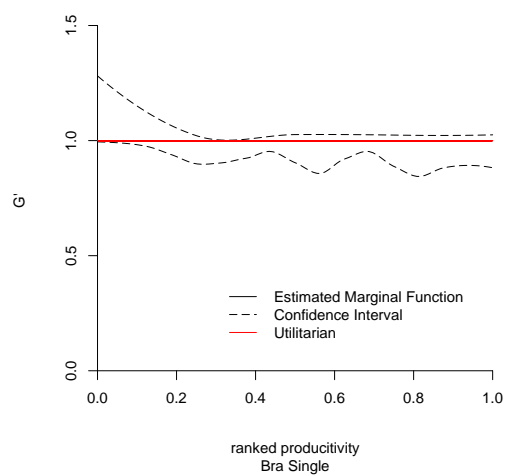


Figure 14
Marginal social welfare function-elasticity=0.3

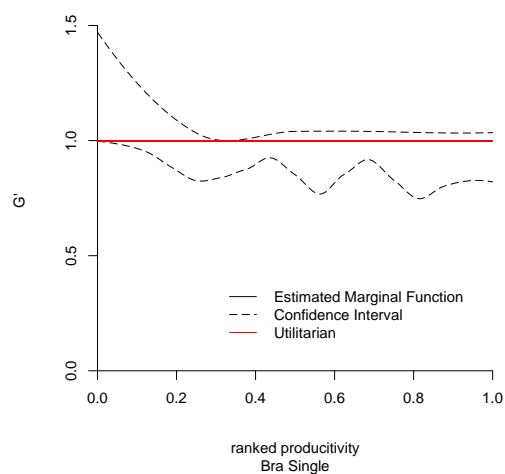


Figure 15
Marginal social welfare function-elasticity=0.5

Figures 16, 17, 18 represent the estimation of the “average social welfare function” for individuals with productivity above or equal to that specific productivity type represented by the specialized version of equation 2, considering only single households.

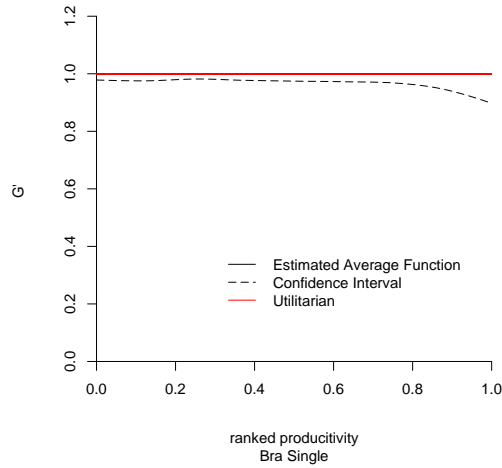


Figure 16
Average social welfare function-elasticity=0.1

The first results for the “revealed” Brazilian social welfare function are that the utilitarian hypothesis cannot be rejected. This suggests not only the utilitarian aspect of the estimated function, but also its concavity. This result does not change with the sample chosen. The result about Brazil is related to the fact that more than 65% of the population have a zero marginal tax rate. Thus, although the planner is utilitarian, it is under the fact that the poor are getting a high welfare weight and the rich are not getting a sufficiently low weight compared to the USA.

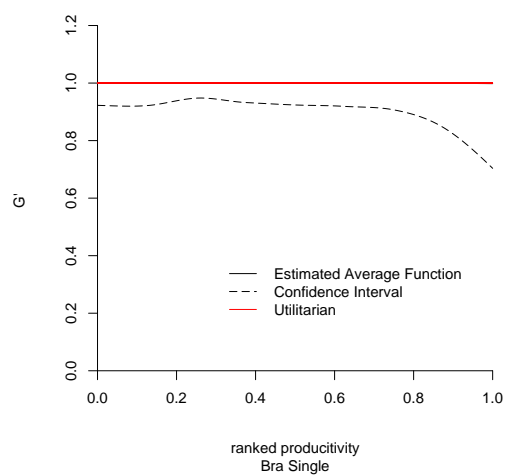


Figure 17
Average social welfare function-elasticity=0.3

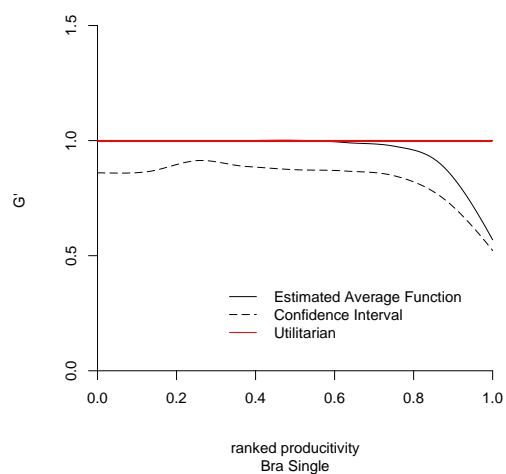


Figure 18
Average social welfare function-elasticity=0.5

Last but not least, the estimations suggest that every household has a positive weight for the Brazilian social planner for every sample chosen. Therefore, the “Brazilian social planner” can be considered Paretian.

5. Conclusion

Diamond (1998) has established conditions under which a U-shaped marginal tax function in productivity is optimal. In the case of a household with a quasi-linear utility function in consumption good, Diamond derived that a U-shaped tax rate is optimal for constant labor elasticity and for a Pareto distribution of skills.

Under the constraints mentioned when describing the model, our results suggest that tax structure adopted in the USA and Brazil for 1990 does not have a U-shaped schedule (Figures 2 and 11). The lowest skilled agents have usually zero marginal income tax and the marginal tax becomes progressive in skills. The question addressed in most of the previous papers was whether this tax structure is optimal or not from a social planner’s point of view. An alternative view of the issue is to find (if there is any) the shape of the social welfare function under this tax structure. This can be seen as a general approach, in the sense that the concavity (optimization), Paretian and utilitarian equity characteristic of the social planner are being analyzed. However, this paper attempts to present one particular case (isoelastic and quasi-linear preferences for the households) under which the rationalization of optimal income tax literature can take place. Other assumptions on the utility functions for the households remain to be tested in order to qualify the results of this paper. Moreover, given data availability in both countries for individual consumption surveys (POF-Brazil and CES-USA), future research could be easily conducted in order to incorporate indirect taxes in the model allowing for a more general estimation of the social welfare function for these two countries.

The estimations for both countries, USA and Brazil, suggest a concave and Paretian social planner. However, the estimated U.S. social planner is not utilitarian, which in this case means that high-skilled households have a lower social weight relative to low-skilled ones. This conclusion is not obtained for the Brazilian data. The Brazilian results could not reject the hypothesis of utilitarianism.

This result has to be interpreted with caution. First, the Brazilian data only correspond to formal workers, and second, other indirect taxes imposed on the households might change the previous comparison between the USA and Brazil. Both effects are not captured in the model. However, the estimation suggests that, if both countries are in fact choosing the optimal marginal tax and if the skill distribution is more unequal in Brazil than in the USA, it must be the case that the “Brazilian social planner” is less averse to inequality than the “U.S. social planner”. That means that Brazilian skilled households have relatively more

weight than the U.S. ones.³⁰ Third, the results come from a first-order condition and no assumption is made on the ratio of income taxes to GDP collected with redistributive purpose in these countries (term E in equation 1). Even if there are differences in these proportions between the USA and Brazil, that does not change the tax structure, but it may imply a higher optimal marginal income tax for all taxpayers whose country happens to have larger tax revenues to be raised.³¹ The USA has a similar tax revenue ratio to GDP to the one observed in Brazil (12.15% and 12.7% respectively), as shown in Table 1.

In order to compare the U.S. to the French and British results reported in Bourguignon and Spadaro (2002), only the scenarios where the labor supply elasticity is set equal to 0.1 and 0.5 are considered. In addition, I have to restrict the comparison to their “mean estimated function”, since Bourguignon and Spadaro did not provide the confidence intervals for the estimations.³²

First, our results confirm the concavity of the social welfare function under the isoelastic quasi-linear preferences for the households. Second, in Bourguignon and Spadaro’s paper, when the elasticity of labor supply is 0.5, the social welfare function is not Paretian. In our case, however, the social welfare function is Paretian, but social weights decrease with productivity levels. Third, if the elasticity of labor supply is 0.1, Bourguignon and Spadaro’s results suggest a Paretian social welfare function with social weights decreasing in skills, however, they provide no empirical test. In our results, even though the estimated function is roughly flat, the results suggest a non-utilitarian U.S. social planner. This suggests that the Brazilian economy can achieve a lower income inequality level by imposing an income tax structure similar the one adopted in the United States.

The fact that the “U.S. social planner” is Paretian as opposed to the French and British planners can be thought of as less ‘equitable’ than the other two. The same conclusion applies to the Brazilian data, with one difference, the estimated Brazilian social planner is also utilitarian, a result that is not clear in Bourguignon and Spadaro but definitely comparable to the U.S. estimation in this paper.

³⁰This is not a striking result given that the result of an optimal tax problem was to generate more unequal income redistribution to one country than to the other – see Figures 1 and 10, it must be the case that the country with higher income inequality should be applying less redistributive income tax policy. Unfortunately, one cannot respond to the political economy literature that causes these results. Whether it reflects preference for individualism and independence than social cohesion in Brazil or due to the policymaking process that favors politically powerful and rich stakeholders in that country.

³¹See Tuomala (1990, p. 101).

³²The figures for the ratio of tax revenues to GDP are different in the USA, France and Great Britain (12%, 23% and 27%; see world development indicators at www.worldbank.org). Therefore, the comparison of the results has to be made more carefully, but it still can be said, given the higher tax revenue, that European countries are collecting mainly from skilled households.

In conclusion, the U.S. social planner can be seen as more ‘equitable’ than the Brazilian planner with respect to income tax decision. However, since the British and French social planners are estimated to be non-Paretian, the result follows the received wisdom and yields that the USA appears to be less concerned about redistribution than its European counterpart.

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