Empirical Selection of Optimal Portfolios and its Influence in the Estimation of Kreps-Porteus Utility Function Parameters*

Adriano Faria**
Rafael Ornelas***
Caio Almeida****

Abstract

This paper investigates the effects on the estimation of parameters related to the elasticity of intertemporal substitution and risk aversion, of the selection of different portfolios to represent the optimal aggregate wealth endogenously derived in equilibrium models with Kreps-Porteus recursive utility. We argue that the usual stock market wide index is not a good portfolio to represent optimal wealth of the representative agent, and we propose as an alternative the portfolio from the Investment Fund Industry. Especially for Brazil, where that industry invests most of its resources in fixed income, the aforementioned substitution of the optimal proxy portfolio caused a significant increase in the risk aversion coefficient and the elasticity of the intertemporal substitution in consumption.

Keywords: CCAPM, Risk aversion, Equity premium puzzle, GMM.

JEL Codes: C32, E21, G12.

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**EPGE/FGV and ANBIMA, Rio de Janeiro, Brazil. E-mail: afaria@fgvmail.br
***EPGE/FGV, Rio de Janeiro, Brazil. E-mail: rafael.ornelas@fgvmail.br
****EPGE/FGV, Rio de Janeiro, Brazil. E-mail: calmeida@fgv.br

1. Introduction

This paper analyzes the behavior of the Brazilian consumer in face of the risk in intertemporal substitution in consumption through the estimation of CCAPM (consumption capital asset pricing model) models. For the American economy, Mehra and Prescott (1985) use a CCAPM model with a CCRA (constant relative risk aversion) utility function and find that the risk premium of the stock market can not be replicated using the conventional parameters in the economy and finance models. This result is known as Equity Premium Puzzle (EPP). More precisely, to obtain such replication the risk aversion coefficient would have to be between 30 and 40.

One of the answers to this puzzle was the generalization of the utility function of the representative consumer.[1] Epstein and Zin (1989) analyze a generalization of the conventional time-additive, Kreps and Porteus (1978) expected utility. This generalized specification allows the intertemporal substitution and the risk aversion to be separable, correcting a deficiency noted in other papers such as Hall (1987). Epstein and Zin (1991) use this specification to carry out different empirical tests which show that the utility function proposed can in fact help solve the EPP, providing especially more reasonable values for the risk aversion parameter. However, these authors use a stock market index (S&P500) as a proxy for the aggregated wealth of the representative agent, which could be questioned in some dimensions.

It is known that a stock market index does not represent a good approximation of aggregated wealth in CAPM representative agent models. For example, Jagannathan and Wang (1996) argue that the return on human capital must necessarily be included in the return measure of aggregated wealth of a representative agent, given that a large part of this wealth, in practice, does not stem from shares. These authors obtain results that indicate that a CAPM conditional model is capable of capturing the cross-section of expected returns of American stocks, when the aggregated wealth is represented as a combination between returns of a stock market index and returns on human capital. Additionally, Dittmar (2002) corroborate the need to include alternative elements in the measure of aggregate wealth of the agent, when it shows that polynomial stochastic discount factors in the aggregate wealth (again measured as return of a market index plus return on human capital) are admissible in cross-sections of assets portfolios of the American industry.

In fact, the papers above show that the usual choice of optimal aggregate wealth, approximated using a stock market index, is highly questionable when we understand that the access to the stock market is limited even in developed

[1] An alternative solution for the EPP is adopting a new, more volatile and more correlated with stock returns consumption measure. Savov (2011) proposes trash production as an alternative consumption measure, since consumption produces trash. Da and Yun (2010) use electricity as a measure, because that is a real time measure of consumption and electricity consumption is measured precisely and with different frequencies.

countries, such as the USA.[2] The point we want to make in this paper is complementary to the one observed in the aforementioned papers. The idea is modify the aggregate wealth in CCAPM-type models with recursive utility. In particular, as we turn our attention to Brazil, were the culture of investing in fixed income is strong; the choice of a stock market index to represent aggregate wealth is even more biased. Therefore, we propose as an alternative the investment fund industry portfolio to represent aggregate wealth (in other words, the agent's optimal portfolio). This choice of investment fund portfolio seems adequate since such portfolio includes several types of investment, with greater weight to the most usual ones in the country, which in the case of Brazil are fixed income.

More objectively, we attempt to bring some contributions to the literature. The main one is a critique of the way CCAPM with Kreps-Porteus utility is implemented. Usually a portfolio of the broad stock market index is used to represent the endogenously derived optimal portfolio in the problem of agent optimization and, consequently, the return of that index is used a return of wealth invested by this agent. Despite uncontested until now, in the specific case of the CCAPM with recursive utilities, the choice of this market portfolio greatly affects the results of the model, especially the ability to return more reasonable risk aversion coefficients, such as those found by Epstein and Zin (1991).

Our paper approaches a vast literature investigating the implications of models with heterogeneous agents with limited participation in the stock market for asset pricing. As an example we can mention Basak and Cuoco (1998), Guvenen (2003) and Gomes and Michaelides (2008). One of the seminal papers in this literature is Mankiw and Zeldes (1991), which describes that a large part of the American population does not hold stocks and that the consumption standard between individuals who hold stocks and does who do not is significantly different. This literature uses, in general, data from a panel exploring the differences between those groups as a possible explanation for part of the EPP. In this manner, methodically, the present paper complements the literature, since we do not have such panel of data and therefore seek, in an aggregated manner, to limit the access of the stock market by removing a large part of the weight of stocks in the composition of the optimal portfolio of the representative agent.

As a marginal contribution at a national level, we use new consumption measures with monthly frequencies and separation between durable and nondurable goods. As far as we know, such frequency and separation of consumption have not been used in scientific papers applied to the Brazilian economy yet.

This paper is composed by 7 sections including this introduction. In Section 2 we present a brief description of the group of papers which analyze the consumption models with Brazilian data. In Section 3, we present the CCAPM model with two specifications of the utility function, CRRA and Kreps-Porteus. The use of the CRRA function yields estimates of parameters to be used in comparison with those

[2] See also Mankiw and Zeldes (1991) and Guiso et al. (2000).

obtained using the Kreps-Porteus function. In Section 4, we discuss the Investment Fund Industry and the selection of optimal aggregated wealth. Section 5 presents the data description and estimation methods used. Section 6 presents the empirical results. Finally, the conclusion is presented in Section 7.

2. Literature Regarding Brazil

Some papers have already dealt with the consumption model for the Brazilian market. Among those we can mention: Reis et al. (1998), Issler and Piqueira (2001), Bonomo and Domingues (2002) and Cysne (2005). Reis et al. (1998) use the basic model with linearized CRRA and assets return without risk, and obtain a risk aversion coefficient between 3 and 5. Then, Issler and Piqueira (2001) carried out a broader study, using three different parametrizations of the utility function (CRRA, External Habit and Kreps-Porteus). The main conclusion is that there is no EPP for Brazil. Estimates for the risk aversion coefficient present values of 4.89 for CRRA and between 0.5 and 1 for the other models. The estimates for the elasticity of intertemporal substitution of consumption present values close to 0.25.

Bonomo and Domingues (2002) use a recursive and additive utility function and impose the endowment process to follow a Markovian chain. They do not find evidence of EPP in Brazil. Despite that, when attempting to reproduce the no-risk interest rates, the authors found a low intertemporal discount rate, and therefore, concluded that the Brazilian market presents Inverted Risk-Free Puzzle (IRFP).

Cysne (2005) uses two different methods (approximation under the hypothesis of log-linearity and calibration). The author concludes that there is EPP in Brazil, but that the model in Mehra and Prescott (1985), with recursive or additive utility, is not able to generate Equity Premium for the Brazilian market. Additionally, the author did not find the IRFP found by Bonomo and Domingues (2002) or the Risk-Free Rate Puzzle (RFRP), found by Weil (1989) in the North American market when separating the relation between risk aversion and intertemporal substitution.

3. Utility Function

Consider a problem with a representative agent living an infinite life; this agent derives utility when consuming a single good. In each period t, the consumption of this good is deterministic and given by C_t, however the future consumption is uncertain and unknown. The problem of this representative agent is given by:

$$Max_{[\{C_{t+j}\}_{j=0}^{\infty},\{\theta_{t+j+1}\}_{j=0}^{\infty}]} U_t(\cdot) \quad s.t.$$

$$C_{t+j} + q_{t+j}\theta_{t+j+1} \leq [q_{t+j} + q^*_{t+j}]\theta_{t+j} + y_{t+j}, \forall j = 0, 1, 2...;$$

where N assets are available in this economy, $\theta_{t+j(Nx1)}$ is the vector regarding the quantity in each of these assets in $t+j$, $q_{t+j(Nx1)}$ is the price vector of these

assets and $q^*_{t+j(Nx1)}$ is the value of the assets dividends, also in the period $t+j$. Additionally, in each period the agent receives an exogenous income y_{t+j}.

Once the general problem of the representative agent is presented, let us specify two models with different utility functions that we use to solve this problem. Firstly the CRRA utility function and then the model with Kreps-Porteus utility.

3.1 CRRA utility function

The intertemporal function in the basic model is of the Neumann-Morgenstern type, that is, additively separable; namely:

$$U_t = E_t \left[\sum_{j=0}^{\infty} \beta^j u(C_{t+j}) \right], \tag{1}$$

where C_t is the per capita aggregated consumption, $E_t[\cdot]$ is the conditional expectation operator of the information available at the period t and $0 < \beta < 1$ is the intertemporal discount factor. Maximizing (1) subject to budget constraints, we find the following N Euler equations:

$$u^{'}(C_t) = E_t[\beta u^{'}(C_{t+1}) R_{i,t+1}], \quad \forall i = 1, ...N, \tag{2}$$

where $R_{i,t+1} := (q_{t+1} + q^*_{t+1})/q_t$ is the gross real return of asset i in period t.

We can rewrite the last equation using the stochastic discount factor, $M_{t+1} = (\beta u^{'}(C_{t+1}))/u^{'}(C_t)$;

$$1 = E_t[M_{t+1} R_{i,t+1}], \quad \forall i = 1, ...N. \tag{3}$$

Now, the instantaneous utility function is parametrized for the type CRRA:

$$u(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}, \tag{4}$$

where γ is the risk aversion coefficient, which in this model is connected to the elasticity of the intertemporal substitution, Ψ. Observe that

$$\begin{aligned} \Psi &\equiv \frac{\partial \ln(C_{t+1}/C_t)}{\partial \ln R_t} \\ &= \frac{1}{\gamma}. \end{aligned}$$

This way, in this specification, the risk aversion coefficient and the elasticity of the intertemporal substitution are reciprocal, when the first one is high (low), the second, is necessarily low (high). This is the main critique to this model.

Finally, observe that using the parametrization of the utility function in (2), we can rewrite (3) so that

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma}, \tag{5}$$

that is, the stochastic discount factor is a consumption growth function.

3.2 Model with Kreps-Porteus type utility

In this model, we have two main hypotheses. First, we assume that the representative agent uses her preference on risk to form the certainty equivalent in future utility. Second, this certainty equivalent is combined with the deterministic consumption using an aggregating function, to find the intertemporal utility. The intertemporal utility is given by:

$$U_t = W(C_t, \mu[\widetilde{U}_{t+1}|I_t]). \tag{6}$$

where, $\widetilde{U}_{t+1}$ is the unknown utility of the agent in the period $t+1$, the certainty equivalent, given the information of the agent in the period t, I_t, is $\mu[\widetilde{U}_{t+1}|I_t]$. W is the aggregating function that combines the current consumption, C_t, with the certainty equivalent.

This form of utility with recursive structure generalizes the deterministic models studied by Kreps and Porteus (1978). Epstein and Zin (1989) show that this class of preferences have the ability to separate the risk aversion from the intertemporal substitution, with the degree of risk aversion being given by the certainty equivalent $\mu[\cdot]$, while the intertemporal substitution is contained in $W(\cdot)$. Now we describe the functional forms of $W(\cdot)$ and $\mu[\cdot]$. First, we have that the aggregating function W is:

$$W(C,z) = [(1-\beta)C^\rho + \beta z^\rho]^{1/\rho}, \quad 0 \neq \rho < 1, \tag{7}$$

$$W(C,z) = (1-\beta)\log(C) + \beta\log(z), \quad \rho = 0, \tag{8}$$

where, $C, z \geq 0$ and $\beta = 1/(1+\delta), \delta > 0$. When the future consumption is deterministic, the aggregating function results in the CES utility function with elasticity of substitution constant at $\psi = 1/(1-\rho)$ and the time preference rate is δ. For the functional form of the certainty equivalent we use the specification known as "α-mean":

$$\mu[\widetilde{U}_{t+1}|I_t] = [E_t\widetilde{U}^\alpha_{t+1}]^{1/\alpha}, \quad 0 \neq \alpha < 1, \tag{9}$$

$$\log(\mu[\widetilde{U}_{t+1}|I_t]) = E\log(\widetilde{U}_{t+1}), \quad \alpha = 0, \tag{10}$$

where E_t is the conditional expectation operator, given I_t. Joining the two functional forms, we have that the intertemporal utility (if $\alpha \neq 0$ and $\rho \neq 0$) is given by:

$$U_t = [(1-\beta)C^\rho + \beta(E_t\widetilde{U}^\alpha_{t+1})^{\rho/\alpha}]^{1/\rho}. \tag{11}$$

As shown on Epstein and Zin (1989), the relative risk aversion coefficient is given by $\gamma = 1 - \alpha$, where the degree of risk aversion increases when α decreases. Additionally, we have that the quick resolution of uncertainty is preferable when $\alpha < \rho$, the opposite occurs when $\alpha > \rho$. Finally, notice that, when $\alpha = \rho$ we enter the familiar case of specifying the expected utility.

Using (11) to solve the problem of the representative consumer, we find the following N Euler equations:

$$E_t\left[\beta^{\eta}\left(\frac{C_{t+1}}{C_t}\right)^{\eta(\rho-1)}\widetilde{M}_t^{\eta-1}R_{i,t+1}\right] = 1, \quad \forall i = 1, ...N; \tag{12}$$

where, $\eta = \alpha/\rho$ and $\widetilde{M}_t$ is the gross return of the optimal portfolio. In the logarithmic specification of preferences, $\alpha = 0$ and $\rho \neq 0$, where we have that the Euler equations would be given by:

$$E_t[\widetilde{M}_t^{-1}R_{i,t+1}] = 1, \quad \forall i = 1, ...N. \tag{13}$$

Observe that, to distinguish the logarithmic model of expected utility ($\alpha = \rho = 0$) of the logarithmic model of non-expected utility ($\alpha = 0$ and $\rho \neq 0$), the parameter of interest is ρ. To test the hypothesis of separability, we can rewrite the equation (12) as follows:

$$E_t\left[\frac{[\beta(C_{t+1}/C_t)^{\rho-1}\widetilde{M}_t]^{\eta} - 1}{\eta}\right] = 0. \tag{14}$$

This equation allows us to make a distinction between the logarithmic model of expected utility ($\alpha = \rho = 0$) and the logarithmic model of non-expected utility ($\alpha = 0$ and $\rho \neq 0$).

Equation (13) implies that the stochastic discount factor in this model is equal to

$$\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{\rho-1}\right]^{\eta}\left(\frac{1}{\widetilde{M}_t}\right)^{1-\eta}; \tag{15}$$

which is the weighted geometric mean of the stochastic discount factors of the linear and logarithmic models of expected utility. Observe that the parameter which determines the weight of each stochastic discount factor in the equation is η. When $\eta = 0$ we have that the market returns are enough to discount the payoffs of the assets, in this case we have a simple or static CAPM. When $\eta = 1$, we have that the growth of consumption is what discounts the payoffs of the assets, in this case the intertemporal CAPM, the same as the one presented in the previous subsection. Finally for other values of η, both market return and growth of consumption are necessary to obtain the stochastic discount factor.

To assist the comparison of empirical forecast of the expected utility model and the model of equation (12), we consider a linear approximation of the stochastic discount factor, in such way that we can rewrite equation (12) as:

$$\eta E_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{\rho-1} R_{j,t} \right] + (1-\eta) E_t[\widetilde{M}_t^{-1} R_{j,t}] \approx 1. \tag{16}$$

It is clear that in this model with non-expected utility, the risks associated with the asset depend both on the covariance of returns of the market portfolio and on the covariance of returns of consumption growth rates. To make this relation even clearer, Epstein and Zin (1991) describe that, assuming that the vector of assets returns and consumption growth are jointly distributed following a lognormal distribution, the following equation holds:

$$E_t[R^*_{j,t} - R^*_{i,t}] \approx \frac{\Sigma_{i,t} - \Sigma_{j,t}}{2} + (1-\eta) cov_t(\widetilde{M}^*_t, R^*_{j,t} - R^*_{i,t}) - \eta(\rho-1) cov_t(\widetilde{X}^*_{t+1}, R^*_{j,t} - R^*_{i,t}), \tag{17}$$

where $\Sigma_{i,t}$ is the conditional variance of $R^*_{i,t}$, given I_t. The asterisk denotes logarithms.

In the equation above, being R_j the return of the stock market and R_i the return of the risk free rate, we have that the greater the covariance between the return of the optimal portfolio with excess return, $R_j - R_i$, the greater the capacity of the model to explain the Equity Premium. Additionally, the selection of the optimal portfolio will influence the estimative of the risk aversion coefficient and the elasticity of consumption intertemporal substitution.

As we can see, the selection of the optimal portfolio is crucial to the model. The usual implementation uses the broad stock market index as a representation of the optimal portfolio, in Brazil the IBRX100 could be this index. Most likely, the covariance of return of this index with the excess return of a stock portfolio is greater than that observed between the last and the return of a portfolio composed of 50% government bonds and 50% stocks. The question is which would be a reasonable measure of an optimal portfolio. We strongly believe that such index is not, especially for the Brazilian market. The next section deals with precisely that.

4. The Investment Fund Industry

The Investment Fund Industry (IndFI)[3] is one of the great magnets of private savings. To understand the importance of this industry in the allocation of wealth, firstly we compare, in the 2002-2013 window, the net worth (NW) of the investment fund industry with the NW of the savings account. On average in the period the

[3]Raw data on the investment fund industry come from the Brazilian Financial and Capital Markets Association (ANBIMA).

NW of savings account represent only 23% of the NW of the aforementioned industry.

Comparing now the net funding (variation of NW discounting profitability) normalized by the national private savings, we have that the investment fund industry had a net funding of approximately 40% of the national private savings. Savings account had a net funding of approximately 25%. That is, the net funding of the investment fund industry was 60% greater than that of savings account. Additionally, the amount of funding in the industry represents a large part of private savings.

The other two important destinations of investment in the financial market are the pension plans and the private credit in the form of CD (certificate of deposit). These last allocations have lower risk than the investment fund industry since they concentrate in the investment of fixed income assets with investment ratings. Therefore, we believe that by taking the investment fund industry portfolio as the representative agent portfolio we are allocating more than enough risk taking. However, the assumed risk for this portfolio is extremely lower than the inherent risk of Ibovespa Index or IBRX100.

One of the clearer ways to visualize this is opening the composition of the investment fund industry NW by type of asset, found in Table 1. There we can see the average percentage of allocations in the period from 2003 to 2013. In the table it is possible to see that only 14.5% of the resources are invested in variable income, and that the largest part of the 85.5% of resources invested in fixed income are in low risk investment (repurchase agreements, federal government bonds (FGB) and CD).

Therefore, at least for the Brazilian economy, we believe that using as the optimal portfolio of the representative agent a portfolio composed only by stocks means imposing an unrealistic risk taking behavior.

Table 1
Percentage allocation of investment fund industry NW – values in %

Fixed income						Variable income		
Repurchase agreement backed by FGB	FGB	CD	Corporate bounds	Others	Sub total fixed income	Stocks	Others	Sub total variable income
15.60	46.71	8.05	3.84	11.29	85.49	14.28	0.23	14.51

Now, a pertinent point is if the evidence mentioned above is also present in other countries. The ICI (Investment Company Institute), despite not divulging the investment industry portfolio, does divulge the allocation of NW for type of investment fund. The information is available from 2005 onward, therefore we calculated the average percentage allocation for the period of 2005 to 2013, and condensed the information on Table 2. Not surprisingly, Brazil has a percentage of allocation in stock funds that is much lower when compared to most continents,

while the situation is the complete opposite when it comes to fixed income investment funds. However, we can infer that even for more developed regions and countries the percentage invested in stocks is not greater than 60% (stock funds + mixed). Therefore it is unlikely that index consisting of only stocks represent the portfolio of the representative agent.

Table 2
Participation of type of fund in the net worth of the investment fund industry

	Stock	Fixed income	Money market	Mixed	Others
World	44%	22%	20%	11%	4%
America	46%	22%	22%	9%	1%
Europe	36%	26%	18%	16%	4%
Asia and Pacific	51%	13%	14%	5%	17%
Brazil	10%	58%	4%	23%	6%
USA	50%	20%	24%	7%	0%

5. Database and Estimation Method

5.1 Database

The interval used for the study is from October 2003 to March 2014. The frequencies used were quarterly and monthly. As a measure of optimal portfolio return we use two measures: the return of IBRX100 and the return of the investment fund industry (IndFI), which will enable us to evaluate alterations in the estimates of the parameters caused by portfolio selection.

We use four series of returns that we believe are the best ones available in the sample to carry out a span of investment opportunities of the representative agent. Specifically, the return of the three following classes of funds: stocks, multimarket and fixed income. We also use the return of DI (Certificado de Depósito Interbancário) as risk free return. There are two reasons for using the return of classes of funds. First, for the period considered, Ibovespa had only two sector sub index, IEE (Index for Electrical Energy) and INDX (Index for Industrial Sector). Those sub-indexes differed very little from Ibovespa, and therefore generate a very restrictive span, and also that the fixed income investments are disproportionately larger. Second, the calculation of returns of investment funds is done in quotas, with all costs inherent to the business, except for taxes, already deducted (e.g.: transactional cost, administration and performance fees). While the calculation of the index returns does not take into consideration the transactional and operational costs for the replication of the index of an actual portfolio.

The selection of these three classes of funds among all the possible ones was due to their participation in the NW of the industry as a whole, as related in

Table 3, which shows the average percentage for the analyzed period. The class referenced as DI is able to replicate the DI with a very small margin of error, and, because of this, we replaced this class with the actual DI series. All the return measures were deflated using the IGDP-DI index. Monthly returns were composed to obtain quarterly returns.

Table 3
Participation of type of fund in the net worth of the investment fund industry

Stock	Multimarket	Pension	Ref. DI	Fixed income	Others
10%	22%	10%	16%	33%	10%

For the consumption series we use the indicator of industrialized consumer goods (CG) and the indicator of industrial semi-durable and non-durable consumer goods (SNDCG), available from IPEA. Both measures refer to real consumption, since the data used in the calculations are quantity data. Similarly to all consumption measures already used in papers about Brazil, these are related to apparent consumption (total of goods produced, plus imports minus exports) and not effective consumption.

To obtain per capita consumption growth rates, first we build a monthly index of population growth. To that end, using growth rates for each year, obtained from IBGE, we decompose those into constant monthly growth rates for each year. After that, to avoid staggering at the end of the years, we run a cubic spline in the monthly series. Finally, we build a monthly index of the number of inhabitants, which is used to calculate the real consumption per capita index. From this resulting monthly index, we calculate the monthly growth rates, which are accumulated to obtain the quarterly growth rates. Table 4 contains the descriptive statistics for our data. It is possible to observe that, for both frequencies and both consumption measures, the variance of consumption growth is:

(i) lower than the variance of the stock funds returns and the IBRX100 index (variable income index);

(ii) greater than the variance of the DI returns and the multimarket funds and fixed income returns (fixed income investments).

Since we saw that the investment fund industry portfolio is composed mostly by fixed income investment, it is not surprising that the variance of consumption growth is also greater than the variance of return in that industry.

Table 5 confirms the supposition that the excess of return in a stock portfolio is more closely related to the return of a general market index than with the return of a mixed portfolio, composed of fixed and variable income investments, such as the investment fund industry portfolio. We observe that both for the quarterly and the monthly frequencies, the correlation between excess returns of the stock funds with

Table 4
Descriptive Statistics – Gross Returns

		Mean	
		Monthly	Quarterly
Funds	Stocks	1.0092	1.0292
		(0.0453)	(0.0938)
	Multimarket	1.0052	1.0158
		(0.0089)	(0.0203)
	Fixed Income	1.005	1.0152
		(0.0064)	(0.0169)
	Ref. DI	1.0046	1.0142
		(0.0066)	(0.0177)
	IndFI	1.0051	1.0155
		(0.0094)	(0.0218)
Index	IBRX100	1.0093	1.03
		(0.0642)	(0.1298)
	DI	1.0046	1.0139
		(0.0065)	(0.0175)
Consumption	CG	1.0025	1.0074
		(0.0265)	(0.0419)
	SNDCG	1.0013	1.0036
		(0.0223)	(0.029)

Note: Standard deviation in parenthesis.

the IBRX100 is greater than that of the investment fund industry return. Table 5 also shows that for the monthly frequency, the growth of our consumption measure is practically not correlated with the excess of return. A possible reason for this is the nature of the data. Because this is apparent consumption of industrialized goods (and therefore with the ability of being stored), there is the possibility that the movements in an interval of a month is stock adjustment and not consumption.

Table 5
Relation between excess return of stock funds with pricing kernel forming variables

	Quarterly				Monthly			
	IBRX100	IndFI	CG	SNDCG	IBRX100	IndFI	CG	SNDCG
Covarriance	0.0103	0.0013	0.0011	0.0004	0.0025	0.0003	0.0001	0
Correlation	0.9108	0.6834	0.3048	0.1463	0.8855	0.7271	0.0827	0.0399

5.2 GMM Estimation

Take R_1, R_2 and R_3 as the gross returns of stock funds, multimarket fund and fixed income funds, respectively. And take R_4 as the returns of DI. The estimation of the model with CRRA utility function comprises the estimation of the system of the four equations described in (3), for $N = 4$, and using (5). The estimation of the model with Kreps-Porteus utility comprises the estimation of the system constituted by 5 equations: The four equations in the form (12), for $N = 4$, and equation (14).

The technique used for the estimation is GMM (generalized method of moments). To implement this technique it is necessary to identify the group of instruments. The estimator is based on the fact that the forecast errors associated with the Euler equations are not correlated with any information available to the agent during decision making. Under this hypothesis, any variable belonging to the information group of the individual is a potential instrument. Based on the form used in Epstein and Zin (1991), we built 4 groups of instruments for each model, introducing lags of order 1 and 2. Precisely for the CRRA specification, the groups of instruments are I1=$\{1,\ C_t/C_{t-1},\ R_{1,t},\ R_{2,t},\ R_{3,t},\ R_{4,t}\}$, I2=$\{1,\ C_t/C_{t-1}, C_{t-1}/C_{t-2},\ R_{1,t},\ R_{2,t},\ R_{3,t},\ R_{4,t}\}$, I3=$\{1,\ C_t/C_{t-1}, C_{t-1}/C_{t-2},\ R_{1,t},\ R_{1,t-1},\ R_{4,t},\ R_{4,t-1}\}$ e I4=$\{1,\ C_{t-1}/C_{t-2},\ R_{1,t-1},\ R_{2,t-1},\ R_{3,t-1},\ R_{4,t-1}\}$.

The groups of instruments for the Kreps-Porteus specification are: I5=$\{1,\ M_t,\ C_t/C_{t-1},\ R_{1,t},\ R_{2,t},\ R_{3,t},\ R_{4,t}\}$, I6=$\{1,\ M_t,\ M_{t-1},\ C_t/C_{t-1},\ R_{1,t},\ R_{2,t},\ R_{3,t},\ R_{4,t}\}$, I7=$\{1,\ M_t,\ M_{t-1},\ C_t/C_{t-1},\ C_{t-1}/C_{t-2},\ R_{1,t},\ R_{1,t-1},\ R_{4,t},\ R_{4,t-1}\}$, I8=$\{1,\ M_{t-1},\ C_{t-1}/C_{t-2},\ R_{1,t-1},\ R_{2,t-1},\ R_{3,t-1},\ R_{4,t-1}\}$.

The groups of instruments I4 and I8 are of great importance in our specific problem, since they are instruments built with second order lag of the variables. Hall (1988) showed that several empirical studies found high values for the consumption intertemporal coefficient due to problem of temporal aggregation of the series. If such problem occurs, the forecasting errors of the Euler equations can be correlated with the more recent variables annexed to the group of the agent information. Therefore, Hall (1988) argues that variables with lag below 2 should not be used as instruments. In our case, due to the stocking ability of our consumption measure, the intertemporal aggregation can occur.

Because we have more orthogonality conditions than parameters to be estimated, our models are overidentified. We use the method of Newey and West (1994) with variable window as a ponderation matrix of such restrictions.

6. Empirical Results

Firstly we present the results of the model for the quarterly frequency compared to the estimates between the two utility function specifications, measures of consumption and optimal investment portfolio. Table 6 reports the estimates for the CRRA function for the two consumption measures. In general terms it is possible to say that the model behaved within expectations, with estimate values close to those found by Issler and Piqueira (2001): $\beta \approx 0.985$, risk aversion coefficients lower than 1 and consequently elasticity of intertemporal consumption substitution (EIS) greater than 1. Additionally, the Hansen test of overidentifying restrictions did not reject the validity of the Euler equations in any of the cases.

However, it is possible to identify some differences between the consumption measure and the instruments. Regarding the different consumption measures, the risk aversion coefficient remained higher for non-durable goods than for consumption goods in the aggregate. At first sight, this result is not intuitive, however, the

EIS is the inverse of the risk aversion coefficient, and therefore the elasticity is lower for non-durable consumer goods. This result presents greater intuition, since non-durable goods have lower stocking capacity, and tend to be more essential (such as food industry) than durable goods, therefore their intertemporal substitution is hindered. In regards to the instruments, we have that I4 (instruments lag 2) present lower substitution elasticity, as defended by Hall (1988).

Table 6
Estimated parameters for CRRA utility – Quarterly

	β	γ	J	β	γ	J
		CG			$SNDCG$	
I1	0.9854***	0.2167***	12.6358	0.9851***	0.4379***	7.2671
	(0.0016)	(0.0359)	[0.9428]	(0.0017)	(0.0784)	[0.9986]
I2	0.9840 ***	0.2635***	6.4568	0.9836***	0.5745***	6.4623
	(0.0007)	(0.0151)	[1.0000]	(0.0006)	(0.0301)	[1.0000]
I3	0.9839***	0.2367***	6.4177	0.9834***	0.4625***	6.4185
	(0.0006)	(0.0162)	[1.0000]	(0.0005)	(0.0271)	[1.0000]
I4	0.9858***	0.6484***	6.20541	0.9844***	0.9192***	7.1407
	(0.0007)	(0.0774)	[0.9996]	(0.0007)	(0.1479)	[0.9988]

Standard deviation values are between parenthesis and p-values between brackets.
J is the Hansen test for overidentification.

We now present the estimate for the Kreps-Porteus function. Table 7 presents the estimates for the two measures of consumption and optimal portfolio. The first panel of this table reports the estimates using CG as a measure of consumption and of the two distinct measures of the optimal portfolio: The usual IBRX100 and the one using the return of the Investment Fund Industry. Again we have $\beta \approx 0.985$, $\psi > 1$, and the Euler equation system validity. In general term η is also more distant from the unitary value, highlighting that the optimal portfolio returns are useful for pricing different types of returns. However, the differences implied by the optimal portfolio selection are noticeable. Maybe the main one is the one we attempt to show since the presentation of the models and the descriptive statistics: the use of the industry portfolio causes the estimates of the risk aversion coefficient to be greater than when we use stock market indexes. For the present result the risk aversion for the IndFI is almost twice as high as the one estimated using IBRX100.

Another crucial difference found in the EIS, it is greater when we use the returns of the investment fund industry as the optimal portfolio returns. This result is in agreement with the estimates found by Vissing-Jorgensen (2002). This paper find that the differences in the estimate of EIS among stock holders and non-stock holder are meaningful and statistically significant, with greater elasticity for the last ones.

All results presented so far do not depend on the group of instruments used. However, it must be observed that once again the use of the group of instruments with lag 2, in this case I8, caused an alteration in the substitution elasticity, making it smaller and with values closer to the ones observed in the literature.

Table 7
Estimated parameters for the Kreps-Porteus utility – Quarterly

	β	η	ρ	ψ	γ	J
Consumer goods (CG)						
$IBRX100$						
I5	0.9831***	0.7731***	0.7427***	3.8877***	0.3928***	10.4761
	(0.0006)	(0.0134)	(0.0220)	(0.3328)	(0.0111)	[0.9999]
I6	0.9824***	0.7541***	0.7071***	3.4152***	0.4666***	5.2534
	(0.0006)	(0.0058)	(0.0135)	(0.1576)	(0.0082)	[1.0000]
I7	0.9821***	0.7271***	0.7509***	4.0150***	0.4539***	5.2438
	(0.0002)	(0.0049)	(0.0088)	(0.1433)	(0.0066)	[1.000]
I8	0.9872***	0.7872***	0.1762	1.2140***	0.86121***	24.8561
	(0.0019)	(0.0301)	(0.1321)	(0.1948)	(0.1054)	[0.8118]
$IndFI$						
I5	0.9881***	-0.1788***	0.8795***	8.3028***	1.1572***	5.3115
	(0.0004)	(0.0061)	(0.0119)	(0.8241)	(0.0051)	[1.0000]
I6	0.9880 ***	-0.2076***	0.8743***	7.9555***	1.1815***	4.8070
	(0.0004)	(0.0033)	(0.0109)	(0.6923)	(0.0034)	[1.0000]
I7	0.9880***	-0.2203***	0.8804***	8.3659***	1.1939***	4.8281
	(0.0003)	(0.0041)	(0.0052)	(0.3705)	(0.0033)	[1.0000]
I8	0.9910***	-0.6784***	0.4647***	1.8683***	1.3153***	6.1862
	(0.0018)	(0.0234)	(0.1296)	(0.4527)	(0.0884)	[1.000]
Semi-durable and non-durable consumer goods (SNDCG)						
$IBRX100$						
I5	0.9831***	0.7743***	0.5575***	2.2599 ***	0.5683***	10.7677
	(0.0018)	(0.0219)	(0.1136)	(0.5806)	(0.0792)	[0.9998]
I6	0.9816***	0.7413***	0.5841***	2.4046***	0.5669***	5.8321
	(0.0006)	(0.0069)	(0.0323)	(0.1873)	(0.0205)	[1.0000]
I7	0.9816***	0.7241***	0.5403***	2.1756***	0.6086***	5.2338
	(0.0003)	(0.0035)	(0.0217)	(0.1027)	(0.0148)	[1.0000]
I8	0.9833***	0.7091***	0.2680	1.3662***	0.8099***	24.8698
	(0.0015)	(0.0248)	(0.2014)	(0.3759)	(0.1426)	[0.8113]
$IndFI$						
I5	0.9882***	-0.1751***	0.7536***	4.0590***	1.1319***	5.3362
	(0.0004)	(0.0053)	(0.0105)	(0.1737)	(0.0031)	[1.0000]
I6	0.9878***	-0.2066***	0.7898***	4.7588***	1.1632***	4.7887
	(0.0004)	(0.0031)	(0.0256)	(0.5819)	(0.0058)	[1.0000]
I7	0.9879***	-0.2284***	0.7637***	4.2327***	1.1744***	4.8197
	(0.0003)	(0.0042)	(0.0166)	(0.2975)	(0.0043)	[1.0000]
I8	0.9887***	-0.3406***	0.71048***	3.4540***	1.2419***	4.6912
	(0.0006)	(0.0061)	(0.0683)	(0.8156)	(0.0251)	[1.00]

Standard deviation values are between parenthesis and p-values between brackets.
J is the Hansen test for overidentification.

This suggests that the consumption measure used could present intertemporal aggregation problems.

Using the Kreps-Porteus utility function, the second panel of Table 7 presents the results using semi and non-durable consumer goods as a consumption measure. It is possible to see that the same conclusions drawn for aggregate consumer goods are kept for non-durable goods. Despite that, due to the separability between intertemporal consumption substitution elasticity and risk aversion, it is possible to obtain an even more important conclusion. Precisely, the use of non-durable consumer goods affects greatly the aforementioned elasticity and not the degree of risk aversion. This makes the benefits of the separation of these parameters clear.

After the analysis of models with quarterly frequency, we move to the monthly frequency. In general terms, the implications of the choice optimal portfolio selection between IBRX100 and the investment fund industry portfolio are maintained. The difference presented in terms of intertemporal substitution elasticity when selecting the aggregated measure of consumer goods or semi and non-durable consumer goods is also kept. Therefore, these facts point to the robustness of the previous conclusions. However, the problem of temporal aggregation is more present when we increase the frequency to monthly. In a way, if this aggregation problem was already present in the quarterly frequency, it is natural that the same problem would increase when we change the frequency to monthly. In this way, only the group of instruments with lag 2, the I4 and I8, are able to return acceptable estimates for the elasticity of intertemporal substitution. Tables 8 and 9 are equivalent to tables 6 and 7 for the monthly frequency.

Table 8
Estimated parameters for CRRA utility – Monthly

	β	γ	J	β	γ	J
		CG			$SNDCG$	
I1	0.9938***	-0.1302***	9.9253	0.9939***	−0.2122***	10.5110
	(0.0004)	(0.0261)	[0.9870]	(0.0005)	(0.0349)	[0.9810]
I2	0.9941 ***	−0.0873***	11.3984	0.9941***	−0.0945***	11.3771
	(0.0003)	(0.0208)	[0.9941]	(0.0003)	(0.0199)	[0.9942]
I3	0.9943***	−0.0111	11.4769	0.9942***	−0.0533 ***	11.3121
	(0.0004)	(0.0119)	[0.9938]	(0.0004)	(0.0163)	[0.9944]
I4	0.995***	0.2799***	12.2958	0.9951***	0.5469***	14.596
	(0.0005)	(0.0517)	[0.951]	(0.0006)	(0.132)	[0.879]

Standard deviation values are between parenthesis and p-values between brackets.
J is the Hansen test for overidentification.

Table 9
Estimated parameters for the Kreps-Porteus utility – Monthly

	β	η	ρ	ψ	γ	J
			Consumer goods (CG)			
			$IBRX100$			
I5	0.9931***	0.7213***	1.1503***	-6.6504***	0.1701***	9.4955
	(0.0003)	(0.0097)	(0.021)	(0.9321)	(0.022)	[1.000]
I6	0.9935***	0.7106***	1.0662***	-15.0960***	0.2423***	9.9479
	(0.0003)	(0.0099)	(0.0147)	(3.3621)	(0.0181)	[0.9999]
I7	0.9936***	0.7042***	1.0539***	-18.5253***	0.2577***	10.0241
	(0.0003)	(0.0085)	(0.012)	(4.1201)	(0.0152)	[0.999]
I8	0.9947***	0.7576***	0.6301***	2.7041 ***	0.5225***	12.4259
	(0.0004)	(0.0161)	(0.0501)	(0.3665)	(0.0362)	[0.9993]
			$IndFI$			
I5	0.9957***	-0.2566***	1.0098***	-101.4150	1.2591***	8.4335
	(0.0002)	(0.0088)	(0.0097)	(100.3266)	(0.009)	[0.9998]
I6	0.9959***	-0.2708***	0.9897***	97.6485	1.2681***	9.3818
	(0.0002)	(0.0081)	(0.0092)	(87.8539)	(0.0083)	[1.000]
I7	0.9959***	-0.2610***	0.9677***	30.9965***	1.2526***	9.3414
	(0.0003)	(0.0099)	(0.0093)	(9.0187)	(0.0093)	[1.000]
I8	0.9961***	-0.2211***	0.8772***	8.1461***	1.1939***	8.29308
	(0.0003)	(0.0096)	(0.0271)	(1.799)	(0.0114)	[0.9998]
			Semi-durable and non-durable consumer goods (SNDCG)			
			$IBRX100$			
I5	0.9931***	0.7456***	1.2845***	-3.5145***	0.0421	9.9804
	(0.0003)	(0.0114)	(0.0321)	(0.3972)	(0.0322)	[0.9999]
I6	0.9934***	0.7320***	1.2147***	-4.6561 ***	0.1107***	10.7311
	(0.0004)	(0.012)	(0.0269)	(0.5836)	(0.0289)	[0.9998]
I7	0.9937***	0.6964***	1.0865***	-11.5580 ***	0.2433***	9.7666
	(0.0004)	(0.0138)	(0.0179)	(2.3921)	(0.0247)	[0.9999]
I8	0.9947***	0.7531***	0.3101*	1.4495***	0.7664***	18.6554
	(0.0007)	(0.0255)	(0.1691)	(0.3553)	(0.1238)	[0.9709]
			$IndFI$			
I5	0.9956***	-0.2465***	1.0599***	-16.6869***	1.2613***	8.4647
	(0.0002)	(0.0073)	(0.0126)	(3.5087)	(0.0084)	[1.000]
I6	0.9958***	-0.2709***	1.0296***	-33.7280***	1.2790 ***	9.3805
	(0.0002)	(0.0084)	(0.0106)	(12.1475)	(0.0095)	[1.000]
I7	0.9958***	-0.2610 ***	1.0000***	-45894.3	1.2610 ***	9.3821
	(0.0003)	(0.0091)	(0.0097)	(206000)	(0.0096)	[1.000]
I8	0.9960***	-0.2608***	0.8192***	5.5322***	1.2136***	9.46975
	(0.0003)	(0.0096)	(0.0436)	(1.3365)	(0.012)	[1.000]

Standard deviation values are between parenthesis and p-values between brackets.
J is the Hansen test for overidentification.

7. Conclusion

Based on the investment profile of the Investment Fund Industry, we seek to contest the usual way the CCAPM model with Kreps-Porteus utility function is implemented. More precisely, we criticize the selection of stock market index as a proxy for the investment portfolio of a representative agent. Since the investment fund industry portfolio is composed of both stocks and fixed income securities, it presents a lower risk exposure than the portfolio composed of stock market indexes. Therefore, assuming that the returns of accumulated wealth of the representative agent is guided by stock market indexes is the equivalent of assuming that such agent takes more risks than the investment industry in his or her country. We propose the use of the investment fund industry portfolio as an alternative for the optimal portfolio.

We showed, through the approximations derived by Epstein and Zin (1991), how the selection of this portfolio can affect the results of the model, which could even revert the widely published result that this modeling is able to provide more realistic (lower) estimations for the coefficient of risk aversion. As an empirical exercise, we implemented the CCAPM model with Kreps-Porteus utility function for the Brazilian economy in the period from 2003 to 2014, considering the usual measure (IBRX100) and the proposed measure (investment fund industry portfolio) as optimal portfolio. The results indicate that regardless of the data frequency and the different types of consumption measurement used, the usual measure of the of the optimal portfolio generates risk aversion coefficients that are markedly lower than those obtained when the optimal portfolio mirrors the investment fund industry portfolio. That is, the usual estimates for such coefficient might present a downward bias. It is also possible to observe that the elasticity of the intertemporal substitution is higher when the optimal portfolio used is the one suggested in this paper.

In regards to the less expressive, but useful results for the study of the Brazilian economy, we have that the elasticity of the intertemporal substitution for consumption of non-durable consumer goods is lower than that observed for the whole group of consumer goods. Finally, the present paper did not identify the presence of an equity premium puzzle for the Brazilian economy, which is in agreement with most previous papers on the topic.

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