

# Market Depth at the BM&FBovespa\*

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## Abstract

The objective of this paper is to estimate a dynamic market depth measure, called VNET, for Brazilian stocks using transactions data. VNET gauges the difference between the numbers of buyer- and seller-initiated trades within the time it takes for the stock price to change by at least a certain amount. It is a realized measure of liquidity for a given price deterioration, which one may track throughout the trading day to capture liquidity's short-term dynamics. More specifically, we model the price duration using an autoregressive conditional duration (ACD) model. The predetermined nature of the ACD process is convenient because it makes it possible to forecast future changes in the liquidity of a stock. By identifying the best moment to buy or sell, the VNET is an excellent starting point for any optimal execution strategy. Our empirical findings indicate that the VNET measure of market depth depends on the bid-ask spread, volume traded, number of trades, and both expected and unexpected price durations. Finally, we also estimate the price impact of a trade by varying the increment in the definition of price duration.

*Keywords:* ACD models, Execution, Liquidity, Price duration, Price impact, Trade.

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## 1. Introduction

In the last few years, there has been a substantial increase not only in trading, but also in price volatility in the Brazilian capital markets. This rise in the number of trades and volume is a consequence of the recent dynamic of the Brazilian economy and of the modernization of the technological infrastructure of BM&FBovespa. This creates investment opportunities in the country, more specifically in the stock market, attracting more sophisticated investors, who use quantitative and high-frequency models to operate.

At first glance, one could well think that the ability of the market to process an ever increasing number of trades would imply an increase in liquidity. However, by observing the great fluctuations in prices that occur on high volume days, we notice that the trade absorption capacity has also brought a relative increase in the price impact. Despite this link, it is difficult to appreciate the resulting impact in the trading cost. Clearly, neither volume nor volatility are direct measures of liquidity, even though they are both intimately connect to it, inasmuch as the bid-ask spread.

There are several definitions of liquidity. The most popular states that liquidity measures how fast and easily one can purchase or sell an asset at a certain price. Alternatively, one can view liquidity as the expected deterioration in price if we wish to buy or sell an asset within a given time horizon. Unfortunately, it is impossible to measure liquidity directly for there is no proxy that reflects every aspect of liquidity. For example, the bid-ask spread measures only price deterioration for small trades given that it assumes that the trade has no price impact.

In this paper, we estimate a specific market depth proxy, VNET, which measures the difference between the numbers of buys and sells within a price duration (i.e., time needed for the price to move beyond a given increment). Engle and Lange (2001) argue that VNET works as a quantitative measure of liquidity, representing the depth of the order book corresponding to stock-specific price deterioration. In other words, it is a measure of the price impact of a trade, just as Amihud's (2002) liquidity measure. The advantage of the VNET is that it is based on a high frequency predictive model, and therefore, capable of forecasting real time price curve movements.

Trading costs have some fixed components such as brokerage fees, stock exchange fees and infrastructure costs as a whole. But it also has random components driven by the price dynamic. According to the optimal execution model proposed by Almgren and Chriss (2000), trading costs depend on both exogenous (e.g., volatility) and endogenous (e.g., price impact) factors. If all other factors are constant, the greater the urgency of trade, the greater the market impact, whereas the greater the execution time, the greater the susceptibility to volatility. Only after execution, one may compute the realized implementation deficit due to the effects of volatility and the market impact in the initial price. Before the execution, this deficit is a random variable, with both mean and variance depending

directly on the execution time interval. This means one can build an efficient frontier to minimize the deficit for a given level of variance and minimize the variance for a given level of deficit. Because the price impact depends on market depth, any execution strategy that seeks to minimize it, must take into consideration the depth dynamic.

VNET gauges the difference in the numbers of buys and sells within a price duration. It is therefore a measure of realized liquidity for a specific price deterioration that one may compute throughout a given day to capture the liquidity short-term dynamic. We use the autoregressive conditional duration (ACD) model proposed by Engle and Russell (1998) to forecast the time necessary for a stock price to move beyond a predetermined barrier (i.e. price duration). This allows us to measure the level of urgency in the execution of certain volumes. When imbalanced volume is traded in a period of time that is much greater than expected by the ACD model, the depth is greatly reduced, generating an estimated value of impatience, which can be understood as the presence of informed agents.

This paper, motivated by the market microstructure literature, seeks to provide a market depth forecasting model for 14 Brazilian stocks. The results show that market depth varies with market conditions. For example, when volume increases, the imbalance necessary to move prices also increases, however less than proportionally. Similarly, market depth reduces as the bid-ask spread increases. The expected price duration, which is inversely proportional to volatility, has a positive impact on VNET. This confirms that, the greater the flow of information, the lower the depth. On the other hand, unpredicted shocks in the price duration increase market depth, because it indicates to some extent a lower probability of informed trading.

This paper contributes to the recent literature on Brazilian financial market microstructures and, in particular, on liquidity aspects of the BM&FBovespa. Machado and Medeiros (2011) examine the liquidity premium in the Brazilian equity market. See also Mussa (2012) and references therein. Victor et al. (2013) investigate the existence of a common factor that governs the liquidity of every stock in the BM&FBovespa. Perlin (2013) conducts a very interesting analysis of the entry of liquidity agents in the BM&FBovespa. Our market depth analysis is therefore complementary to the existing literature on the Brazilian stock market liquidity.

The remainder of this paper is as follows. Section 2 describes the structure of the Brazilian stock market. Section 3 discusses some market microstructure aspects of equity markets. Section 4 describes the BM&FBovespa data. Section 5 introduces the models and their estimation results. Section 6 builds a market reaction curve for one of the stocks. Section 7 summarizes our contribution and offers some final considerations.

## 2. The Brazilian Stock Market

In Brazil, there is only one stock exchange, namely the BM&FBovespa. Once listed, stocks are available in the secondary market, with an electronic trading environment kept by the BM&FBovespa. The structure of the electronic trading is based on a limited order book that registers all buys and sells, creating a transparent mechanism for price formation.

The trading day begins with a pre-opening trading (i.e., opening call) 15 minutes before the official opening. In this period, buy and sell offers are recorded so as to benchmark the opening price. After the opening call, trading is continuous from 10 am to 5pm (or 11am to 6pm during daylight-saving time). For stock exchange index constituents (see [www.bmfbovespa.com.br](http://www.bmfbovespa.com.br) for details on index portfolio composition), there is also a closing call between 4.55pm and 5pm (or 5.55pm and 6pm during daylight-saving time).

The after-market period is from 5.45pm to 7.00pm (or 6.30pm to 7.30pm during daylight-saving time), with a pre-opening trading of 15 minutes. During this time, one may trade only Ibovespa and IBrX100 constituents that have been traded on that day. Additionally, there is a maximum limit of 2% (positive or negative) variation in the price of the regular trading closing prices. All stocks are traded in standard lots, of 1, 100, 1000 or 100,000 stocks, with price per share or per lot of one thousand stocks. The most common combination is the standard lot of 100 stocks with price per share. All 14 stocks in our sample follow this combination.

For amounts smaller than the standard lots, there is a fractional market at the BM&FBovespa. However, liquidity is very low and hence we do not contemplate it in this study. Nevertheless, it is worth pointing out that the interaction between full lots and fractional amounts in the same market can have a positive impact on liquidity, since it opens space for trading between different sizes of investors. Additionally, it allows the use of algorithms based in predetermined execution plans to place more precise orders.

The electronic trading environment is governed by a prioritization algorithm based on price and time. As a general principle, if there are orders with better prices, or at the same prices, but registered before, both for buying and selling, the execution of orders at worse prices, or orders that were registered later before the order with the highest priority is executed, will not be allowed. If an order has its quantity increased or its price altered, it loses chronological priority in the order book.

The only exception is direct trading, that is, when the same brokerage firm proposes to buy and sell the same asset to two different clients. The trading registered as direct must always have their prices specified, which must necessarily be among the best buy order and the best sell order. This type of order has priority in the price/time algorithm, even if price is the same as the best order registered in the exchange. Once the direct trading is accepted, the BM&FBovespa announces the transaction, but can only conclude the trading after a predetermined period

of time. If, during this time, there is an interference of another brokerage firm that offers to buy for a higher price or sell for a lower price, the brokerage that initiated the direct trading can propose a new price. This process takes as many interactions as necessary to conclude the deal at the best price.

The BM&FBovespa electronic trading allows limit, market, at-the-opening-price, stop and direct orders. A limit order is sent with a price limit for execution, that is, the order can only be executed at that price or better. A market order is fulfilled entirely or partially, depending on the available quantity, for the best price on the other end of the order book. If it cannot be entirely fulfilled the remaining part is sent to the same side of the book at the closing price for the last trading. At-the-opening-price order are executed at the opening price of either opening or closing calls. Stop orders entertain a trigger that, if hit, activates the order at a predetermined price. Direct orders are when two identical orders in opposite ends, from different clients, are registered by the same brokerage firm. In addition, it is also possible to send orders with or without minimum amounts. An order with minimum amount is automatically canceled if the minimum amount cannot be reached. For orders without a minimum amount (iceberg orders), only a part of the total amount is disclosed to the market, and the minimum amount disclosed must be at least ten times larger than the standard lot of that asset.

In regards to duration, orders can be divided in: day order, good-till-canceled (GTC), immediate-or-cancel (IOC), fill-or-kill (FOK). Day orders are valid until a date specified upon registration, whereas GTC orders are valid until they are canceled. IOC must be immediately filled, either partially or entirely, and any unfulfilled parts of the order will be canceled. FOK orders must be filled in its entirety, otherwise it is completely canceled.

There are a few trading parameters in the Brazilian stock exchange, which, if reached, cause the trading of that asset to be suspended. After suspension, there is an intraday auction in which every market participant may intervene and contribute to price formation. Additionally, the BM&FBovespa has the power to cancel or to implement a call session for any asset it deems necessary. During intraday auctions, orders are registered, but no trades are executed. Intraday auction orders cannot be canceled or made worse.

The BM&FBovespa also features a circuit break mechanism. The objective is to dampen sudden movements and to rebalance buy and sell orders in the presence of extreme volatility. When the main market index, Ibovespa, falls 10% in relation to the closing price in the previous day, all trading in the market is interrupted for 30 minutes. After reopening, the limit for negative price variation becomes 15%, with a trading halt of one hour. If, after this second suspension, the stock exchange reopens and the negative price variation reaches 20%, trading may be interrupted indeterminately.

### 3. Market Microstructures

The VNET model empirically explores the connection between trading activity, price volatility, and cost of trading. In most cases, price changes are due to an imbalance between buy and sell orders. Indeed, signed volume typically indicates the direction of private information, even in efficient markets (Easley and O'Hara, 1992).

In practice, news does not cause price to instantly move to a new level. It implies an update of current beliefs, generating a sequence of trades and, consequently, a volatility clustering until prices stabilize at a new level. Additionally, the order book is composed of several limited orders, which are not necessarily canceled after some news. What happens is that several trades are registered, sometimes at different prices, until the price consolidates at a new level.

The evidence on information heterogeneity and adverse selection abounds in the literature. Several studies aim to distinguish between informed and uninformed agents. If these two types of agents place their orders differently, then the distribution of information in the market can be partially revealed by the trade activity at any given moment. Easley and O'Hara (1987) and Hasbrouck (1988) find a positive correlation between trade size and price impact, whereas McNish and Wood (1992) show that the bid-ask spread tends to increase after high volume trading.

Information asymmetry also helps explain the connection between trading frequency and market depth. Agents with privileged information usually face time restrictions due to the sensitivity of information. This generates a positive correlation of trade intensity and volatility with the presence of informed agents (Foster and Viswanathan, 1995).

By relying on price durations, VNET differs from the classic assumption of fixed time intervals. It is thus closer to Easley and O'Hara's (1992) framework, which attributes informational content to time. In particular, higher trade intensity moves in tandem with a higher probability of informed trading, and hence with a wider bid-ask spread.

### 4. Data Description

Quote and transactions data are from the BM&FBovespa market data sector from October 2009 to December 2010. For each stock, we split the data into two data sets. The first comprises all trades, whereas the second focuses on offers as well as buy and sell orders. Both databases contain the ticker, date of the event (trade or offer), time of the event (down to the second), number of contracts, and price, apart from a sequence number. Transactions data also include information on the buy and sell offers that initiated the trade, as well as their sequence numbers. We use the latter to connect both databases.

We start with the 14 most traded stocks in the year 2010, using the Ibovespa

criteria. We limit the sample period to the third quarter of 2010, namely July, August and September. During this period, the BM&FBovespa does not change trading hours and hence it is only one hour ahead of the New York Stock Exchange.

Extracting relevant information from the databases require some amount of data management. The first step is to exclude trades during opening call, closing call and intraday auctions, as well as those in the after-market sessions. The main goal is to isolate the impact of the trade activity in market depth, and hence we must exclude these periods for they have different trade patterns than the continuous trading period.

To exclude opening call trades, we simply eliminate the first trades of the day with the same time stamp and price. To exclude closing call and after-market trades, we delete any observation with a time stamp after 4.55pm (when closing call begins). As intraday auctions do not have a direct indicator, we exclude any observation that follows a period of at least 5 minutes without trades, with at least five trades at the same time stamp and price.

In determining price durations, we exclude overnight price movement relative to the closing call from the previous day and their corresponding price durations. The relevant initial price for each day is the price of the first trade after the opening call. For each observation, we record the average price between the buy and sell offers (MIDQUOTE) as well as the bid-ask spread (SPREAD). We then compute price durations based on the variation of MIDQUOTE because it is the best proxy for the efficient price. In particular, MIDQUOTE avoids the bid-ask bounce that typically haunts transaction prices at the high frequency.

In the transactions database, we create a variable that indicates whether the trade is buyer- or seller-initiated. For each trade, we match the time stamps of the offers that originate that trade with the time stamps in the offer database to define the initiator. In other words, a buy offer will come after the sell offer if it is a buyer-initiated trade, whereas a sell offer will hit a previous buy offer if seller-initiated.

In what follows, we describe how exactly we compute the price durations and describe the main variables we investigate in our empirical analysis of market depth at the BM&FBovespa.

#### 4.1 Durations

The usual definition says that market depth is the number of shares that one may buy or sell at a certain price interval. Thus, we will not use a fixed time interval, but a stochastic one, defined by a given price deterioration. In contrast to the time between trades, we define price durations as the time necessary for prices to move beyond a predetermined point. For example, suppose that the share price goes up 15 cents and then falls 15 cents within a certain fixed time interval. The price variation is zero within this time interval, despite the occurrence of two price movements of 15 cents each.

This is why it is more convenient to determine durations by fixing the price interval (INTERVAL). For each stock, we set the value of INTERVAL to the average size of the bid-ask spread. We then build two barriers, adding and subtracting the INTERVAL from the first MIDQUOTE of the day to compute the first price duration of that day by tracking the first MIDQUOTE that either hits or cross one of the barriers. We then restart the process to compute the next price duration using the MIDQUOTE price at the end of the first price duration as the basis for the new barriers (namely,  $\text{MIDQUOTE} \pm \text{INTERVAL}$ ). To ensure that we are indeed singling out real price events, we only register a price duration if we observe at least two subsequent MIDQUOTE observations outside the original price interval.

Table 1 reports some descriptive statistics about the 14 stocks in the sample. The average MIDQUOTE is between 12 and 50, whereas the average bid-ask spread, which we use to set the value for INTERVAL, varies from 0.03 to 0.14. We observe price durations in nearly every day of the 64 days in the sample period for all the 14 stocks. There are on average 6 to 22 durations per day. Finally, the average duration ranges from approximately 700 to 3000 seconds, establishing a healthy heterogeneity in liquidity across stocks.

## 4.2 Variables of Interest

From the offer data set, we register the following information:

- (1) DUR is the price duration in seconds;
- (2) TIME is the time stamp at the end of the price duration, which we use to define time-of-day dummy variables;
- (3) SPREAD is the difference between the buy and sell prices immediately before the end of a price duration;
- (4) SELL\_OFFER and BUY\_OFFER are the average sizes of the buy and sell offers in the duration, which gauge the realized depth in each side of the market.

From the transactions database, we record the following variables:

- (1) TRADES is the total number of trades within a price duration;
- (2) VOLUME is the number of shares traded within a price duration;
- (3) PJUMP is the absolute price variation within a price duration;
- (4) Q\_BUY and Q\_SELL are respectively the amount of stock shares bought and sold within a price duration.

We calculate the VNET measure as the logarithm of the absolute value of the difference between Q\_BUY and Q\_SELL. To avoid seasonality, we standardize



DUR by the average duration in that time of the day. To do so, we divide the trading day in 30-minute intervals and calculate the unconditional mean of the price duration for each interval.

For operational reasons, we estimate the conditional expectation of the price duration in the next section using PTIME, the squared root of the adjusted duration. Table 2 reports some descriptive statistics for the above variables. There is not a lot of cross-sectional dispersion in the average and standard deviation of PTIME across stocks. The same applies to SPREAD to some extent. Interestingly, Q\_BUY and Q\_SELL have very similar average and standard deviations for any given stock, though with a lot of variation in the cross-section. We observe a similar pattern linking TRADES and VOLUME. Finally, PJUMP and VNET also display great variation across stocks.

## 5. Market Depth Determinants

The intraday liquidity model seeks to explain the connection between trade dynamics and price movements. The difference in volume between buy and sell orders that drives price changes depends on the market perception about informed trading. VNET captures exactly the net directional volume during an interval:

$$VNET = \ln \left| \sum_i d_i vol_i \right|$$

where  $d$  is a direction indicator (buy=1 and sell=-1) and  $vol$  is the total number of stocks traded during a specific price duration.

Before trying to identify the determinants of market depth, as measured by VNET, we first decompose PTIME into its expected and unexpected parts given that their impacts on market depth are not necessarily the same. In particular, the expected price duration is inversely proportional to the volatility (Engle and Russell, 1998) and hence it is interesting to observe how it affects market depth on its own.

### 5.1 Decomposing price durations

Figure 1 exhibits the autocorrelograms of the price durations (PTIME) for the 14 stocks in our sample. It shows that the PTIME series are very persistent, justifying the autoregressive nature of ACD-type model. Our ACD specification entertains that the expected duration depends not only on past durations (expected and realized), but also on the bid-ask spread. Because we expect the bid-ask spread to entail a negative impact, we add the inverse of the bid-ask spread (INV\_SPREAD) as a control. We then model the price durations using the following ACD-type specification:

$$\begin{aligned}
PTIME_t &= EPTIME_t \times UPTIME_t, \text{ with } E[UPTIME_t | I_{t-1}] = 1 \\
EPTIME_t &= \omega + \alpha PTIME_{t-1} + \beta EPTIME_{t-1} + \phi INV\_SPREAD_{t-1}
\end{aligned}$$

where  $EPTIME$  is the expected squared root of the price duration (i.e., the conditional expectation of  $PTIME$ ) and  $UPTIME$  is the unexpected squared root of the price duration given the information set  $I_{t-1}$  available at time  $t-1$ . Note that we employ a multiplicative decomposition so as to preserve the nonnegativeness of the price duration. To estimate the parameters  $(\omega, \alpha, \beta, \phi)$ , we evoke quasi-maximum likelihood as in Drost and Werker (2004) by treating the baseline distribution of the unexpected duration as exponential. Note that the resulting quasi-likelihood function for  $PTIME$  is proportional to the likelihood function of a GARCH model with a Gaussian distribution and hence we may carry out the estimation using any standard econometric package that enables GARCH estimation.

Table 3 shows that the coefficient estimates for the past price duration ( $PTIME$ ) are significant at the 5% level for 10 of the 14 stocks. Similarly, we find significant coefficient estimates for the past expected duration ( $EPTIME$ ) for 13 of the 14 stocks. Interestingly, the bid-ask spread does not seem to affect much price durations (or volatility) given that its coefficient estimates are insignificant at the 5% level, except to CSN, Itaúsa and, possibly, Usiminas.

In the next section, we examine the determinants of  $VNET$ . Apart from the liquidity controls ( $SPREAD$ ,  $VOLUME$  and  $TRADES$ ), we include both expected and unexpected durations ( $EPTIME$  and  $UPTIME$ ) in the analysis. The expected duration reflects the price volatility, whereas the innovations in the price duration mirror unexpected changes in the trade flow. As both volatility and trade flow could well have a direct impact on the market depth, we control  $VNET$  for both  $EPTIME$  and  $UPTIME$ .

## 5.2 VNET

$VNET$  is a measure of realized market depth. The goal of this section is to predict the  $VNET$  over a price duration using a number of liquidity and volatility measures. More specifically, we control for the past values of  $SPREAD$ ,  $VOLUME$  and  $TRADES$  as well as for the contemporaneous  $EPTIME$  and  $UPTIME$ :

$$\begin{aligned}
VNET_t &= \beta_0 + \beta_1 SPREAD_{t-1} + \beta_2 VOLUME_{t-1} + \beta_3 TRADES_{t-1} \\
&+ \beta_4 EPTIME_t + \beta_5 UPTIME_t
\end{aligned}$$

Table 4 reports that the coefficient estimates for  $SPREAD$  are negative in 11 of the 14 stocks. This evinces that stocks with higher liquidity have lower bid-ask spreads and hence more market depth. However, these effects are mostly insignificant. We find only 4 stocks with a significantly negative  $SPREAD$  coefficient at the 5% level of confidence using one-sided  $t$ -tests.

VOLUME represents a measure of trade intensity as well as of relative imbalance between buys and sells given a certain VNET level. Because the VNET is an absolute measure of the difference between purchases and sales, the higher the VOLUME, the lower the relative imbalance, measure in percentages. The estimates of  $\beta_2$  are significantly positive at the 5% level for 12 of the 14 stocks. It can also be observed that the coefficients are always lower than one, signaling that the VNET responds in a less-to-proportional fashion to changes in VOLUME. This response might reflect the increase in the risk of informed trading due to the higher volume.

The number of trades over a price duration is another measure of trade intensity. In the presence of information asymmetry, an increase in trade frequency represents an increase in the probability of agents having privileged information. The negative coefficient of TRADE in 10 of the 14 stocks corroborates the idea that market depth is decreasing in trade frequency. These negative coefficient estimates are mostly significant at the 5% level as opposed to the positive coefficient estimates that are all insignificant at the usual levels of confidence.

Because volatility is associated to the arrival of news and to the presence of informed agents, we expect a positive effect of the expected duration on VNET. In fact, the coefficient estimates are positive for 10 of the 14 stocks. These estimates are mostly at the 5% level in contrast to the insignificant positive coefficient estimates. Finally, the coefficient estimates for the unexpected duration are positive and highly significant for every stock. This indicates that impatience increases in the presence of informed trading.

Note that the effect of the unexpected component of the price duration is contemporaneous. Implicitly, we are assuming that UPTIME is weakly exogenous. Because trade activity instigates price movements, we must condition VNET on PTIME to gauge how many stocks can be traded over a given time interval without moving prices more than the increment value given by INTERVAL. To verify model congruency, we carry out a residual analysis. Visual inspection of the residuals and of their autocorrelagrams indicate no volatility clustering, trend or autocorrelation. LM tests cannot reject the absence of autocorrelation and of conditional heteroskedasticity. In short, the residuals are seemingly white noise for every stock.

## 6. Market Reaction Curve

VNET measures how market depth deteriorates as we move along the book of buy and sell offers. We have so far kept the value of the price increment arbitrarily fixed at the average bid-ask spread (INTERVAL). However, there is no reason why agents should contemplate the same value for the price increment given that their tolerance to trading costs may differ.

In this section, we reestimate the conditional duration model for different price increments to produce a market reaction curve. We carry out this analysis for

the ITUB4 share because it has on average the largest number of price durations over a day. Given that the value of `INTERVAL` is 0.07 cents for Itau-Unibanco, we entertain the following alternative values for `INTERVAL`: 0.03, 0.05, 0.09, and 0.11. For each value in the grid, we estimate the ACD model for the resulting price durations, as well as the VNET regression. Table 5 reports the coefficient estimates for the latter. It is very reassuring to observe that the results are qualitatively very similar across the different values of the price increment.

For each regression, we project VNET conditioning on the average values of `SPREAD`, `VOLUME`, `TRADES`, `EPTIME`, and `UPTIME`. We include in this group of variables the average market depth at the buy side, which is another proxy for the observed market depth. We assign the value of `INTERVAL` to each predicted VNET and the value 0.01 for the observed market depth given that a buyer must bear at least the cost of the bid-ask spread. The minimum price variation for the ITUB4 shares is 0.01 cent and hence this is also the least it will take to put forth the operation.

Figure 2 shows that, as expected, the average VNET increases with the price increment. In particular, the market reaction curve is reasonably linear, exhibiting only a small amount of convexity. The fact that the observed depth is very close to the linear regression line further confirms that VNET gauges market depth pretty well.

## 7. Conclusion

In this empirical study, we investigate one of the least explored components of liquidity in the Brazilian stock market, namely, market depth. To do so, we gauge the price impact of a trade using Engle and Lange's (2001) VNET technology. More specifically, we first decompose price durations into their expected and unexpected components using ACD-type models and then run regressions to understand how the realized market depth varies with them and with other liquidity indicators for 14 stocks traded at BM&FBovespa. As in Engle and Lange (2001), we employ VNET, the net signed volume over a price duration, to proxy for market depth.

We find that market depth is increasing in a number of liquidity measures as well as in the unexpected price duration. We also evince that market depth deteriorates in the presence of information asymmetry and hence it is now always the case that the larger the volume, the larger the market depth. These results are interesting because they pave the way for a VNET-based trade execution strategy that aims to reduce transaction costs.

## References

Almgren, R. & Chriss, N. (2000). Optimal execution of portfolio trade. *Journal of Risk*, 3:5–39.

- Amihud, Y. (2002). Illiquidity and stock returns: Cross-section and time series effects. *Journal of Financial Markets*, 5:31–56.
- Drost, F. & Werker, B. J. M. (2004). Semiparametric duration models. *Journal of Business and Economic Statistics*, 22:40–50.
- Easley, D. & O'Hara, M. (1987). Price, trade size, and information in securities markets. *Journal of Financial Economics*, 19:69–90.
- Easley, D. & O'Hara, M. (1992). Time and the process of security price adjustment. *Journal of Finance*, 47:577–605.
- Engle, R. F. & Lange, J. (2001). Predicting VNET: A model of the dynamics of market depth. *Journal of Financial Markets*, 4:113–142.
- Engle, R. F. & Russell, J. (1998). Autoregressive conditional duration: A new model for irregularly spaced data. *Econometrica*, 66:1127–1162.
- Foster, F. D. & Viswanathan, S. (1995). Can speculative trading explain the volume-volatility relation? *Journal of Business and Economic Statistics*, 13:379–408.
- Hasbrouck, J. (1988). Trades, quotes, inventories, and information. *Journal of Financial Economics*, 22:229–252.
- Machado, M. A. V. & Medeiros, O. R. (2011). Modelos de precificação de ativos e o efeito liquidez: Evidências empíricas no mercado acionário brasileiro. *Revista Brasileira de Finanças*, 9:383–412.
- McInish, T. H. & Wood, R. A. (1992). An analysis of intraday patterns in bid/ask stockss for NYSE stocks. *Journal of Finance*, 47:753–764.
- Mussa, A. (2012). *A liquidez e os modelos de precificação de ativos: Um estudo empírico no mercado acionário brasileiro de 1995 a 2011*. PhD thesis, Faculdade de Economia e Administração, Universidade de São Paulo.
- Perlin, M. S. (2013). Os efeitos da introdução de agentes de liquidez no mercado acionário brasileiro. *Revista Brasileira de Finanças*, 11:281–304.
- Victor, F. G., Perlin, M. S., & Mastella, M. (2013). Comonalidades na liquidez: Evidencias e comportamento intradiário para o mercado brasileiro. *Revista Brasileira de Finanças*, 11:375–398.

Table 1  
Descriptive statistics for share prices, bid-ask spread and price durations

stock	class	ticker	INTERVAL	MIDQUOTE	number of days with duration	number of durations per day	duration length (in seconds)
Banco do Brasil	ON	BBAS3	0.08	29.08	64	19	1,147.92
Bradesco	PN	BBDC4	0.07	31.01	62	18	1,194.6
Brazil Foods	ON	BRFS3	0.09	24.03	63	12	1,674.9
BMFBovespa	ON	BVMF3	0.03	13.10	60	11	1,535.1
CSN	ON	CSNA3	0.14	28.29	63	6	2,710.05
Gerdau	PN	GGBR4	0.05	24.49	63	21	925.11
Itausa	ON	ITSA3	0.03	12.26	62	8	2,249.37
Itau-Unibanco	PN	ITUB4	0.07	37.43	64	22	992.26
MMX	ON	MMXM3	0.07	12.29	60	6	2,939.34
Petrobras	ON	PETR3	0.09	30.90	61	16	1,269.46
	PN	PETR4	0.05	27.22	61	18	713.53
Usiminas	PNA	USIM5	0.13	48.56	62	17	1,137.86
Vale	ON	VALE3	0.12	47.34	63	19	1,150.66
	PNA	VALE5	0.11	41.33	58	9	1,485.22

We report the average values of the bid-ask spread (INTERVAL) and of the midquote price (MIDQUOTE). In addition, we document the number of days with price durations for each stock, as well as the average number of price durations over a day. Finally, we also display the average length of the price durations in seconds.

Table 2  
Some descriptive statistics for the variables of interest

variable	BBAS3	BBDC4	BRFS3	BVMF3	CSNA3	GGBR4	ITSA4	IT	UB4	MMXM3	PETR3	PETR4	USIM5	VALE3	VALE5
PTIME	0.8680	0.8870	0.8790	0.8830	0.8940	0.8940	0.9070	0.8770	0.9020	0.8890	0.8800	0.8800	0.8690	0.9020	0.9070
	0.7640	0.8120	0.7940	0.8070	0.9140	0.8270	0.8330	0.7800	0.8740	0.8010	0.7740	0.7740	0.7660	0.8084	0.8440
SPREAD	0.4970	0.4620	0.4770	0.4700	0.3540	0.4490	0.4210	0.4800	0.4320	0.4570	0.4750	0.4750	0.4960	0.4319	0.4220
	0.0620	0.0540	0.0690	0.0260	0.0730	0.0420	0.0250	0.0550	0.0520	0.0540	0.0380	0.1070	0.0690	0.0530	0.0530
	0.0600	0.0500	0.0600	0.0200	0.0600	0.0400	0.0200	0.0500	0.0500	0.0500	0.0300	0.0300	0.1000	0.0600	0.0400
Q_BUY	0.0370	0.0310	0.0410	0.0140	0.0510	0.0230	0.0130	0.0300	0.0310	0.0330	0.0210	0.0620	0.0410	0.0370	0.0370
	2.8930	1.7820	3.1360	4.9350	3.0400	2.2590	3.5190	2.1250	2.2130	2.4890	3.1020	1.4040	2.0160	2.7010	2.7010
	1.8550	1.6050	2.1740	3.6200	2.7670	1.8750	2.9370	1.8130	1.9750	2.0910	2.6770	1.0140	1.7360	2.5210	2.5210
Q_SELL	3.7610	1.1510	2.9710	4.6360	1.2550	1.4410	2.6240	1.5370	1.5140	1.4530	1.8980	1.3170	1.7160	1.0920	1.0920
	2.3290	1.5760	2.9340	5.2450	3.0110	2.4050	3.7980	2.2770	1.9590	2.6830	3.4910	1.3830	1.8860	2.5560	2.5560
	1.7330	1.3510	1.9240	3.9490	2.8190	2.0280	2.8880	1.9750	1.7450	2.2410	2.9910	1.0460	1.7050	2.4310	2.4310
	2.2880	969.00	3.4440	4.8310	1.3630	1.6650	3.7780	1.3950	1.1560	2.1180	4.8360	1.2270	803.00	977.00	977.00
TRADES	414.43	378.10	265.10	689.55	564.47	262.68	554.15	350.59	442.62	326.11	564.96	281.43	241.86	1041.20	1041.20
	214.00	197.00	147.00	407.00	347.50	149.00	307.00	177.00	266.00	167.00	304.00	155.00	152.00	684.50	684.50
	566.71	578.07	330.42	787.32	600.06	356.14	675.36	623.44	530.47	458.14	1049.90	389.11	275.56	1266.30	1266.30
VOLUME	235.97	188.57	117.16	659.63	312.36	160.74	424.36	190.33	361.02	239.39	583.65	131.75	112.28	937.87	937.87
	110.90	97.50	62.50	416.85	199.90	89.90	254.00	94.20	229.50	108.60	343.20	72.40	71.70	550.35	550.35
	355.03	293.44	146.35	779.51	319.44	217.11	520.57	345.24	415.31	370.60	1084.60	179.67	131.21	1699.70	1699.70
PJUMP	2.0930	1.8700	1.6030	1.7040	3.0490	1.1100	1.6240	1.8420	1.8170	1.6740	2.0440	2.3740	1.8510	4.9480	4.9480
	1.2300	1.1000	1.0300	1.0750	2.0550	0.7100	0.9700	1.0300	1.2400	0.9900	1.2750	1.5300	1.2200	3.3850	3.3850
	2.5850	2.3780	1.7140	1.9510	2.8880	1.2950	1.8270	2.7920	2.0670	1.8940	3.1360	3.3160	1.9070	5.4760	5.4760
VNET	10.104	9.8870	9.5370	11.262	10.367	9.8960	10.761	9.8620	10.579	10.065	11.062	9.6850	9.5910	11.267	11.267
	10.272	9.9570	9.6390	11.432	10.495	10.069	10.900	9.9830	10.754	10.202	11.204	9.8040	9.7470	11.384	11.384
	1.6390	1.4410	1.5800	1.4270	1.5410	1.3710	1.5110	1.5370	1.3600	1.5500	1.3640	1.3690	1.4280	1.4430	1.4430

For all variables, the first row reports their average value across stocks, whereas the second and third rows display their median and standard deviation.

Table 3  
ACD model coefficient estimates for each stock

ticker	intercept	P TIME	E P TIME	INV_SPREAD
BBAS3	0.0578 (0.2573)	0.1046 (0.0316)	0.8520 (0.0000)	-0.0005 (0.3728)
BBDC4	0.0054 (0.9420)	0.0829 (0.3224)	0.8894 (0.0000)	0.0008 (0.4344)
BRFS3	0.3885 (0.0174)	0.1749 (0.0010)	0.4688 (0.0213)	-0.0012 (0.3652)
BVMF3	0.2953 (0.2263)	0.2467 (0.0025)	0.4417 (0.0915)	0.0004 (0.7199)
CSNA3	0.1635 (0.0202)	0.0499 (0.3176)	0.8278 (0.0000)	-0.0016 (0.0357)
GGBR4	0.0691 (0.0438)	0.1221 (0.0000)	0.8048 (0.0000)	0.0002 (0.7289)
ITSA4	0.1943 (0.1116)	0.1474 (0.0027)	0.5112 (0.0001)	0.0028 (0.0423)
ITUB4	0.1154 (0.1492)	0.1045 (0.0198)	0.7949 (0.0000)	-0.0005 (0.5310)
MMXM3	0.1821 (0.4580)	0.1202 (0.1777)	0.7465 (0.0110)	-0.0016 (0.4418)
PETR3	0.0850 (0.1187)	0.1520 (0.0040)	0.7871 (0.0000)	-0.0007 (0.4033)
PETR4	0.1611 (0.1685)	0.1595 (0.0329)	0.7295 (0.0000)	-0.0012 (0.2165)
USIM5	0.0076 (0.7746)	0.1176 (0.0012)	0.8390 (0.0000)	0.0025 (0.0515)
VALE3	0.0392 (0.0463)	0.0799 (0.0000)	0.8767 (0.0000)	0.0002 (0.6879)
VALE5	0.0225 (0.2541)	0.0391 (0.0646)	0.9429 (0.0000)	-0.0001 (0.7529)

We report quasi-maximum likelihood estimates and, within parentheses, the  $p$ -values of the robust  $t$ -test for whether they are equal to zero. P TIME is the price duration, E P TIME is the unobservable conditional expectation of the price duration, and INV\_SPREAD is the inverse of the bid-ask spread.



Table 4  
VNET regressions for each stock

ticker	intercept	SPREAD	VOLUME	TRADES	EPTIME	UPTIME
BBAS3	43.243 (0.0000)	-0.1341 (0.0164)	0.5960 (0.0000)	-0.3852 (0.0000)	0.1119 (0.4210)	0.5366 (0.0000)
BBDC4	61.776 (0.0000)	0.0075 (0.8978)	0.3301 (0.0006)	-0.0966 (0.3410)	0.2993 (0.0319)	0.4734 (0.0000)
BRFS3	56.665 (0.0000)	0.0443 (0.5425)	0.2970 (0.0092)	0.0191 (0.8811)	-0.2631 (0.3724)	0.6437 (0.0000)
BVMF3	58.609 (0.0000)	-0.1814 (0.0402)	0.3209 (0.0043)	0.0194 (0.8645)	0.0699 (0.7601)	0.4722 (0.0000)
CSNA3	10.299 (0.0000)	-0.1876 (0.0820)	-0.2348 (0.3288)	0.3202 (0.1860)	16.963 (0.0162)	0.5183 (0.0000)
GGBR4	68.111 (0.0000)	-0.0693 (0.1942)	0.1324 (0.1869)	0.1766 (0.0860)	0.2976 (0.0371)	0.4634 (0.0000)
ITSA4	67.623 (0.0000)	0.1386 (0.2652)	0.4743 (0.0009)	-0.3316 (0.0285)	11.874 (0.0060)	0.5787 (0.0000)
ITUB4	50.046 (0.0000)	-0.0220 (0.6937)	0.4427 (0.0000)	-0.1481 (0.1336)	-0.3806 (0.0257)	0.4978 (0.0000)
MMXM3	64.383 (0.0000)	-0.0213 (0.8446)	0.4086 (0.0306)	-0.2356 (0.2032)	0.5103 (0.2063)	0.3537 (0.0000)
PETR3	28.169 (0.0000)	-0.0667 (0.3097)	0.7266 (0.0000)	-0.3596 (0.0005)	-0.1390 (0.3027)	0.4212 (0.0000)
PETR4	45.973 (0.0000)	-0.0571 (0.3745)	0.5727 (0.0000)	-0.2308 (0.0269)	-0.0549 (0.6792)	0.3233 (0.0000)
USIM5	67.657 (0.0000)	-0.0533 (0.3184)	0.2493 (0.0096)	-0.0801 (0.4428)	0.1950 (0.0934)	0.4210 (0.0000)
VALE3	48.728 (0.0000)	-0.0858 (0.1111)	0.4747 (0.0000)	-0.2694 (0.0062)	0.3483 (0.0376)	0.5717 (0.0000)
VALE5	58.516 (0.0000)	-0.2403 (0.0033)	0.5264 (0.0043)	-0.4308 (0.0252)	0.7519 (0.0300)	0.5242 (0.0000)

We report OLS coefficient estimates and, within parentheses, the  $p$ -values of the robust  $t$ -test for whether they are equal to zero. SPREAD is the bid-ask spread, VOLUME is the number of shares traded within the price duration, TRADES is the number of trades within the price duration, EPTIME is the expected price duration according to the ACD models in Table 3, and UPTIME is the unexpected price duration.

Table 5  
VNET regressions for different INTERVAL values

INTERVAL	NOBS	Intercept	SPREAD	VOLUME	TRADES	EPTIME	UPTIME
0.03	3395	62.525 (0.0000)	-0.0745 (0.0765)	0.2563 (0.0000)	-0.0768 (0.1889)	0.0189 (0.8547)	0.3623 (0.0000)
0.05	2258	58.519 (0.0000)	-0.0844 (0.0666)	0.3439 (0.0000)	-0.1665 (0.0213)	-0.0260 (0.8054)	0.4499 (0.0000)
0.07	1413	50.046 (0.0000)	-0.0220 (0.6937)	0.4427 (0.0000)	-0.1481 (0.1336)	-0.3806 (0.0257)	0.4978 (0.0000)
0.09	945	62.262 (0.0000)	-0.1366 (0.0339)	0.2286 (0.0522)	0.0639 (0.6169)	-0.2911 (0.1623)	0.5094 (0.0000)
0.11	663	73.185 (0.0000)	-0.0709 (0.3714)	0.1445 (0.3590)	0.0947 (0.5766)	-0.7272 (0.0211)	0.5571 (0.0000)

We report OLS coefficient estimates and, within parentheses, the  $p$ -values of the robust  $t$ -test for whether they are equal to zero. INTERVAL is the value of the price increment we employ to compute the price durations. NOBS is the number of observations of the resulting time series of price durations. SPREAD is the bid-ask spread, VOLUME is the number of shares traded within the price duration, TRADES is the number of trades within the price duration, EPTIME is the expected price duration according to the ACD model, and UPTIME is the unexpected price duration.

Figure 1  
PTIME autocorrelagram for each stock

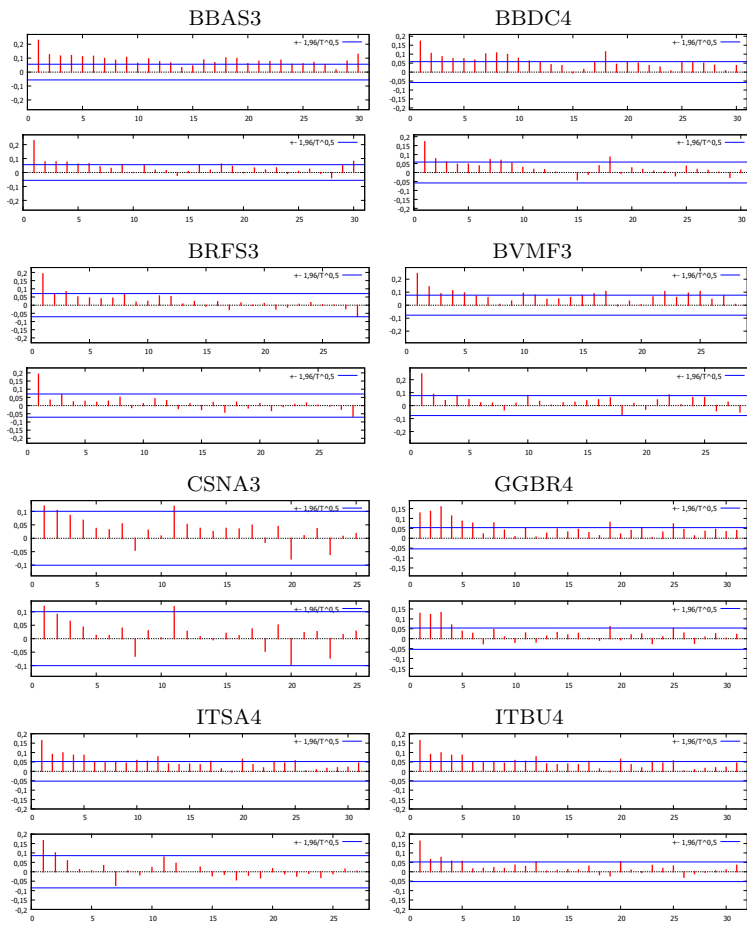


Figure 1 – PTIME autocorrelagram for each stock (cont.)

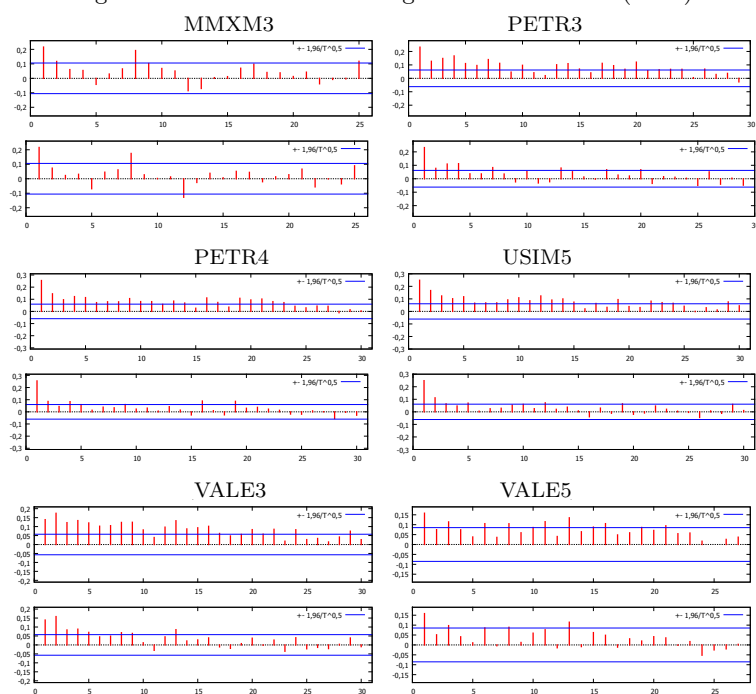


Figure 2  
Market reaction curve

