

Microfoundation of Inflation Persistence in the New Keynesian Phillips Curve

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Outline

- Background and Motivation
- This paper: DSGE model with two sources of price stickiness
- Literature Review
- The Model
- Results
 - Estimation
 - IRFs, correlation, distribution of price changes
- Conclusion

Background

- Standard New Keynesian Phillips curve (NKPC) based on optimizing behavior of price setters in the presence of nominal rigidities. Mostly based on:
 - staggered contracts of Taylor (1979, 1980), Calvo (1983), and quadratic adjustment cost model of Rotemberg (1982)
- Framework used in analysis of monetary policy: price rigidity main transmission mechanism through which it impacts the economy:
 - when firms face difficulties in changing some prices, they may respond to monetary shocks by changing instead their production and employment levels

Background

- Popular frameworks to derive NKPC
 - Calvo (1983)'s staggered price setting: only a fraction of firms completely adjusts their prices to optimal level at discrete time intervals in response to changes in various costs
 - Rotemberg (1982): firms set prices to minimize deviations from optimal price subject to quadratic frictions of price adjustment
- Both designed to model sticky prices:
 - Calvo pricing related to the frequency of price changes
 - Rotemberg pricing associated with size of price changes

Motivation

- Although NKPC has some theoretical appeal, growing literature on its empirical shortcomings
- Criticism on ability to match some stylized facts on inflation dynamics and effects of monetary policy:
 - Failure to generate inflation persistence. Although price level responds sluggishly to shocks, inflation rate does not
 - NKPC does not yield the result that monetary policy shocks first impact output, and subsequently cause a delayed and gradual effect on inflation

Motivation

- Fuhrer and Moore (1995) and Nelson (1998), etc.:
 - In order for a model to fully explain the time series properties of aggregate inflation and output it requires that *not only the price level but also the inflation rate be sticky*

This Paper

- DSGE model that endogenously generates inflation persistence
- We consider that firms face two sources of price rigidities, related to both the inability to change prices frequently and to the cost of sizeable adjustments
 - although firms change prices periodically, they face convex costs that preclude optimal adjustment
- In essence, model assumes that price stickiness arises from both the frequency and size of price adjustments

This Paper

- Monetary policy shocks first impact economic activity, and subsequently inflation but with a long delay, reflecting inflation inertia
- The model captures cross-dynamic correlation between inflation and output gap

Literature

- Alternative NKPC models that can account for some of the empirical facts on inflation and output. Most popular ones are extensions of Calvo's staggered prices or contracts:
 - Sticky information
 - Backward rule-of-thumbs

Phillips curve

$$\pi_t = \beta \pi_{t-1} + \lambda y_t \quad \text{the econometric Phillips curve}$$



$$\pi_t = \beta E_t \pi_{t+1} + \lambda mc_t$$

Based On Taylor (1980), Rotemberg (1982) and Calvo (1983)



1. Inflation persistence
2. Hump-shaped responses
3. Disinflation Boom (Ball, 1994)

$$\pi_t = \alpha^f E_t \pi_{t+1} + \alpha^b \pi_{t-1} + \lambda mc_t$$

Gali and Gertler (1999) and CEE (2005) : **Indexation assumption**

Literature

- **Sticky information** (Mankiw and Reis 2002) - information is costly and, therefore, disseminated slowly:

- Prices adjust continuously but information does not
- Model is consistent with inflation persistence
- Empirical implication: prices change frequently, which contradicts widespread micro-data studies

- Evidence found across countries and different data sources is that firms keep prices unchanged for several months:

e.g. Bils and Klenow 2004, Angeloni et al. 2006, Alvarez 2008, Nakamura and Steinsson 2008, Klenow and Malin 2010, etc.

Literature

Backward Rule of Thumb

- **Hybrid NKPC** - Gali and Gertler (1999): Fraction of firms changes prices based on past prices of other firms with a correction for recent inflation
- **Indexation Models** - Christiano, Eichenbaum, Evans (2005), Steinsson (2003), and Smets and Wouters (2003, 2007): part of the firms adjusts their prices by automatic indexation to past inflation:
 - ❖ Models explain inflation inertia as they incorporate a lagged inflation term into the resulting hybrid NKPC
 - ❖ Arbitrary role given to past inflation as at least some agents are backward-looking in the process of setting prices
 - firms do not reoptimize prices each given period

Literature

•Hybrid NKPC Models

- Generally criticized for being ad-hoc as they lack a theory of firms to motivate their backward looking behavior
(e.g. . Angeloni et al (2006), Rudd and Whelan 2007, Woodford 2007, Cogley and Sbordone 2008, Benati 2008, etc.)

•Indexation Models

- More structure as a fixed share of random Calvo-type firms could set their prices optimally each period while the rest changes according to past aggregate inflation
 - Lagged inflation term is derived from firms' decisions.

Literature

- General Indexation models and Sticky Information models:
 - imply that prices are adjusted continuously
- Evidence not supported by microdata evidence of price stickiness - both infrequent and small price adjustments
- Continuously price updating is an implication of many NKPC models including Reis (2006), Christiano et al (2005), Smets and Wouters (2003), Rotemberg (1982), Kozicki and Tinsley (2002), among many others

This Paper

- Proposes a microfounded theoretical model that endogenously generates inflation persistence as a result of optimizing behavior of the firms
- Combines staggered price setting (Calvo) and quadratic costs of price adjustment (Rotemberg) in a unified framework

This Paper

- Phillips curve derived from DSGE model, and relates current inflation to inflation expectations, lagged inflation, and real marginal cost or output gap
- Lagged inflation term is endogenously generated in a forward-looking framework:
 - Agents remain forward looking and follow an optimizing behavior

This Paper

- In contrast to the general indexation models and sticky information models, in the proposed model:
 - prices are not continuously adjusted and firms that are able to change prices do not fully adjust them due to convex costs of adjustment
 - New Phillips curve based on dual stickiness nests the standard NKPC as a special case (Calvo pricing or quadratic cost)
 - Model as an alternative to ad-hoc hybrid NKPC and sticky information Phillips curve

This Paper

- Price stickiness
 - direct microeconomic evidence
 - firms' decisions (frequency and size of price changes)

Firms face two problems

1. When to change prices? → the frequency of price changes

Fixed costs of price adjustment:

- Physical menu costs
- Implicit and Explicit Contracts (ranked the 1st in the EU area and 2nd U.S.)

2. How much to change prices? → the size of price changes

Convex, Variable costs price adjustment :

- a) Managerial costs (information gathering costs, decision making, and internal communication costs)
- b) Customer costs (communication and negotiation costs)
- c) Other costs – antagonizing customers

Zbaracki, Ritson, Levy, Dutta, Bergen (2004)

- Firm investigated changes prices “once a year”
- However, “the firm reacted to major changes in supply and demand conditions **slowly and/or partially** because of the convexity of costs [of price adjustment]...”

The Model

Firms' Problems and the Phillips Curve

- Two types of firms:

Representative final goods-producing firm

Continuum of intermediate goods-producing firms

Firms' Problems and the Phillips Curve

Final Goods-Producing Firm

The final goods-producing firm purchases a continuum of intermediate goods, Y_{it} , at input prices, P_{it} , indexed by $i \in [0, 1]$. The final good, Y_t , is produced by bundling the intermediate goods:

$$Y_t = \left[\int_0^1 Y_{it}^{1/\lambda_f} di \right]^{\lambda_f} \quad 1 \leq \lambda_f < \infty.$$

final-good-producing firm chooses Y_{it} to maximize profit in a perfectly competitive market taking both input (P_{it}) and output prices (P_t) as given,

$$\text{Max}_{P_t} \left[\int_0^1 Y_{it}^{1/\lambda_f} di \right]^{\lambda_f} - \int_0^1 P_{it} Y_{it} di$$

Firms' Problems and the Phillips Curve

Final Goods-Producing Firm

FOC
$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\lambda_f/(\lambda_f-1)} Y_t.$$

$\lambda_f/(\lambda_f - 1)$ measures the constant price elasticity of demand for each intermediate good.

relationship between the prices of the final and intermediate goods can be obtained by integrating FOC

$$P_t = \left[\int_0^1 P_{it}^{1/(1-\lambda_f)} di \right]^{1-\lambda_f}$$

derived from the fact that the final goods-producing-firm earns zero profits.

The final good price can be interpreted as the aggregate price index.

Firms' Problems and the Phillips Curve

Intermediate Goods-Producing Firm – Calvo pricing

$$P_t = \left[(1 - \theta) \tilde{P}_{it}^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f}$$

where \tilde{P}_t denotes the optimal price set by the intermediate good-producing firms.

fraction of firms that reoptimizes their price at time t choose the same price in equilibrium

$$\text{thus } \tilde{P}_{it} = \tilde{P}_t \text{ for all } i$$

Since individual prices are optimized infrequently, the aggregate price level also adjusts sluggishly, making current price level depend on its own lag

Firms' Problems and the Phillips Curve

Intermediate Goods-Producing Firm – Adjustment cost

each intermediate goods-producing firm faces a quadratic adjustment cost of adjusting its nominal price given by

$$QAC = \frac{c}{2} \left(\frac{\tilde{P}_t}{P_t} - \frac{\tilde{P}_{t-1}}{P_{t-1}} \right)^2 Y_t.$$

that it is costly for current individual price to deviate from past price level, which makes prices sticky.

The intermediate goods-producing firm maximizes real profit from selling its output in a monopolistically competitive market assuming that price is fixed with the Calvo probability θ in any given period. In addition, firms face a quadratic adjustment cost in adjusting their prices

Firms' Problems and the Phillips Curve

Intermediate Goods-Producing Firm – Adjustment cost

The intermediate goods-producing firm maximizes real profit from selling its output in a monopolistically competitive market assuming that price is fixed with the Calvo probability θ in any given period. In addition, firms face a quadratic adjustment cost in adjusting their prices.

The firm chooses \tilde{P}_t to maximize:

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[\frac{(\tilde{P}_t - mc_{t+k}P_{t+k})Y_{it+k}}{P_{t+k}} \right] - \frac{c}{2} \left(\frac{\tilde{P}_t}{P_t} - \frac{\tilde{P}_{t-1}}{P_{t-1}} \right)^2 Y_t$$

Firm i 's profit depends on \tilde{P}_t as long as it cannot re-optimize

The Model

$$\tilde{P}_t \equiv P_{it} = \operatorname{argmax} E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[\frac{(P_{it} - mc_{t+k})Y_{it+k}}{P_{t+k}} \right] - \frac{c}{2} \left(\frac{P_{it}}{P_t} - \frac{P_{it-1}}{P_{t-1}} \right)^2 Y_t$$

subject to $Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\lambda_f / (\lambda_f - 1)} Y_t$

→ *The first order condition and*

$$P_t = \left[(1 - \theta) \tilde{P}_t^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f} \text{ yield}$$

$$\rightarrow \pi_t = \Lambda_1 E_t \pi_{t+1} + \Lambda_2 \pi_{t-1} + \lambda mc_t$$

Christiano, Eichenbaum and Evans (JPE, 2005)

$$\tilde{P}_t \equiv P_{it} = \arg \max E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[\frac{(P_{it} - mc_{t+k})Y_{it+k}}{P_{t+k}} \right]$$

subject to the demand function,
$$Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\lambda_f / (\lambda_f - 1)} Y_t$$

The F.O.C. and
$$P_t = \left[(1 - \theta) \tilde{P}_t^{1/(1-\lambda_f)} + \theta \pi_{t-1} P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f} \text{ yield}$$

$$\rightarrow \pi_t = \alpha^f E_t \pi_{t+1} + \alpha^b \pi_{t-1} + \lambda mc_t$$

Log-linearizing FOC shows how inflation persistence is generated in the model

$$E_t \sum_{k=0}^{\infty} (\theta\beta)^k \left[\hat{p}_t - \hat{X}_{t+k} - \hat{m}c_{t+k} \right] = \frac{c}{1-a} \Delta \hat{p}_t.$$

$$P_t = \left[(1-\theta) \tilde{P}_t^{(1-\lambda_f)} + \theta P_{t-1}^{(1-\lambda_f)} \right]^{1/(1-\lambda_f)}$$

$$\hat{p}_t = \frac{\theta}{1-\theta} \hat{\pi}_t \longrightarrow \Delta \hat{p}_t = \frac{\theta}{1-\theta} (\hat{\pi}_t - \hat{\pi}_{t-1})$$

$\tilde{X}_{t+k} \equiv 1/\pi_{t+1}\pi_{t+2}\dots\pi_{t+k}$ and \hat{X}_{t+k} is the percentage deviations from \tilde{X}_{t+k} .

$a \equiv \lambda_f / (\lambda_f - 1)$, $p_t = \tilde{P}_t / P_t$.

Intuition behind lagged inflation term

$$\begin{aligned}
 \frac{c}{2} \left(\frac{P_{it}}{P_t} - \frac{P_{it-1}}{P_{t-1}} \right)^2 Y_t &\longrightarrow P_t = f^q(P_{t-1}, \dots) \\
 &\quad + \\
 P_t = \left[(1 - \theta) \tilde{P}_t^{1/(1-\lambda_f)} + \theta P_{t-1}^{1/(1-\lambda_f)} \right]^{1-\lambda_f} &\longrightarrow P_t = f^c(P_{t-1}, \dots) \\
 &\quad \parallel \\
 &\quad P_t = f(P_{t-2}, \dots)
 \end{aligned}$$

$$\pi_t = \Lambda_1 E_t \pi_{t+1} + \Lambda_2 \pi_{t-1} + \lambda m c_t$$

if $c = 0$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda^* m c_t$$

where $\Lambda_1 \equiv \eta/\tau$, $\Lambda_2 \equiv \kappa/\tau$, $\lambda \equiv (1-\theta\beta)/\tau$, $\tau \equiv (\theta/(1-\theta) + (1+\theta\beta)\kappa)$, $\eta \equiv \theta\beta(1/(1-\theta) + \kappa)$,
 $\kappa \equiv c(1-\theta\beta)\theta/(a-1)(1-\theta)$.

Model

$$U(C_t, 1 - N_t) = \frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

$$C_t + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right)(N_t) + (1 + i_{t-1})\left(\frac{B_{t-1}}{P_t}\right) + \Pi_t$$

where C_t is the composite consumption good, N_t is hours worked, Π_t is real profits received from firms, and B_t is the nominal holdings of one-period bonds that pay a nominal interest rate i_t . The IS curve is:

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_t^y$$

where y_t and i_t denote the output gap and the nominal interest rate, respectively.

$$\varepsilon_t^y = \delta_\pi \varepsilon_{t-1}^y + \nu_t^y \text{ with } \nu_t^y \sim N(0, \sigma_y^2)$$

DSGE Model

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + \varepsilon_t^y$$

$$\pi_t = \Lambda_1 E_t \pi_{t+1} + \Lambda_2 \pi_{t-1} + \lambda mc_t + \varepsilon_t^\pi$$

$$i_t = \rho i_{t-1} + (1 - \rho)[\alpha_\pi E_t \pi_{t+1} + \alpha_y y_t] + \varepsilon_t^i$$

$$mc_t = \left(\frac{1}{\sigma} + \varphi\right) y_t$$

Estimation

- Small-scale DSGE model estimated using Bayesian techniques

Table 1: Estimation Results - Double Sticky Price DSGE model

parameters	prior dist.	prior mean	prior st.dev.	posterior mean	95% of confidence interval
θ	beta	0.5	0.10	0.75	[0.69 , 0.82]
c	normal	30	30.0	171.0	[143.2 , 197.4]
σ	invgauss	1	∞	0.16	[0.13 , 0.18]
ρ	beta	0.7	0.05	0.72	[0.69 , 0.76]
α_π	normal	1.5	0.25	1.72	[1.58 , 1.86]
α_y	normal	0.5	0.1	0.48	[0.33 , 0.62]
δ_y	beta	0.5	0.2	0.94	[0.91 , 0.97]
σ_π	invgauss	0.1	2	0.73	[0.66 , 0.80]
σ_y	invgauss	0.1	2	0.19	[0.15 , 0.22]
σ_i	invgauss	0.1	2	1.36	[1.23 , 1.48]

Results

Common ways in the literature to evaluate the model success in capturing inflation dynamics

- Examining the estimated parameter associated with lags of inflation in the Phillips curve
- Autocorrelation functions for inflation
- Impulse response functions

Results

Model Parameters

- Evidence of intrinsic inflation persistence
 - Parameter estimates associated with the two types of price stickiness are highly significant, supporting the proposed model
 - Coefficients associated with lags of inflation in the Phillips curve are positive and highly significant

Results

Frequency of price changes

- Calvo parameter θ , *degree of nominal rigidity* = 0.75. 1/4 firms reset prices to optimize profit. Average length of time between price changes is 4 quarters
- Estimates closely match micro-data evidence - prices change on avg once a year:
 - Klenow and Malin (2010) Alvarez (2008) (18 countries 11 months), Alvarez et al (2006) Euro area (4 to 5 quarters). Eichenbaum, Jaimovich and Rebelo (2008) sticky reference prices among shorter-lived new prices (11.1 months)

Results

Size of price adjustments

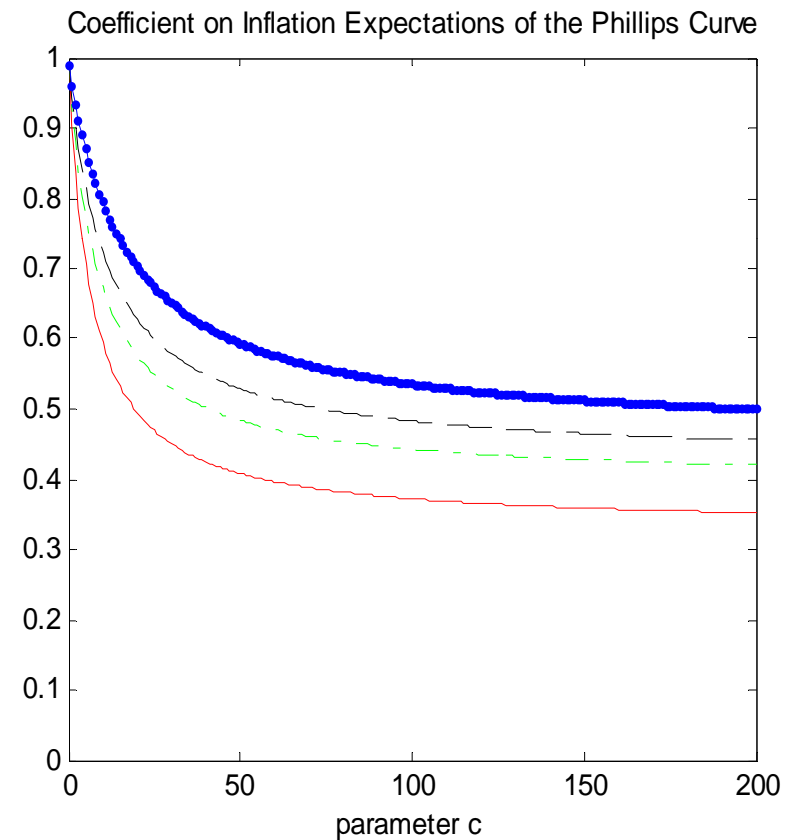
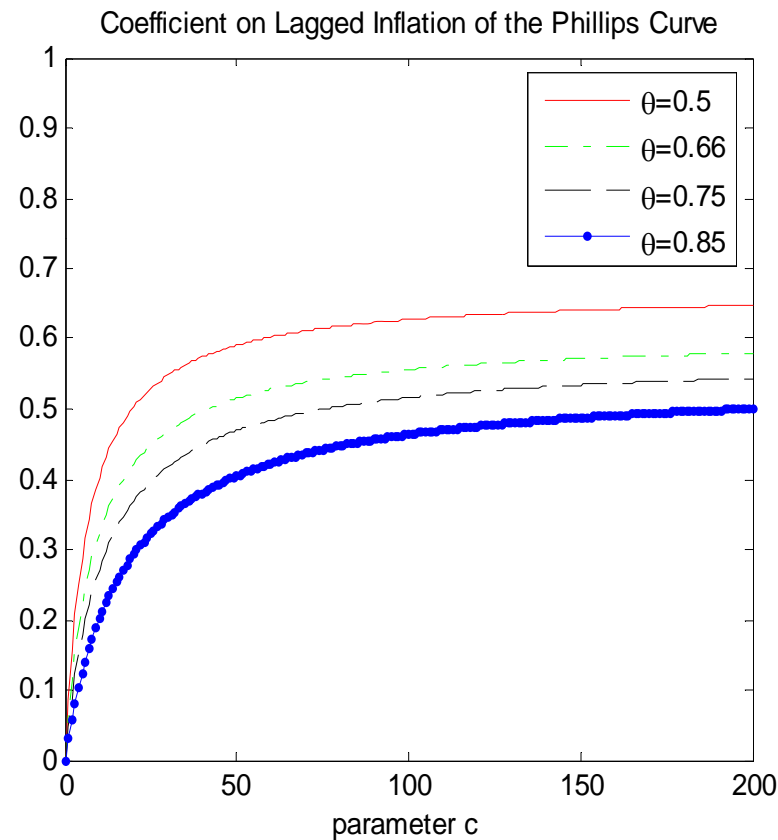
- Magnitude of adjustment costs large and statistically significant: *quadratic adjustment cost*, $c = 171.0$, 95% confidence (143.2, 197.4)

Empirical findings: price changes are mostly smaller than the size of aggregate inflation (e.g. Dhyne et al. 2005, Alvarez et al (2006), Klenow and Kryvtsov 2008, etc.)

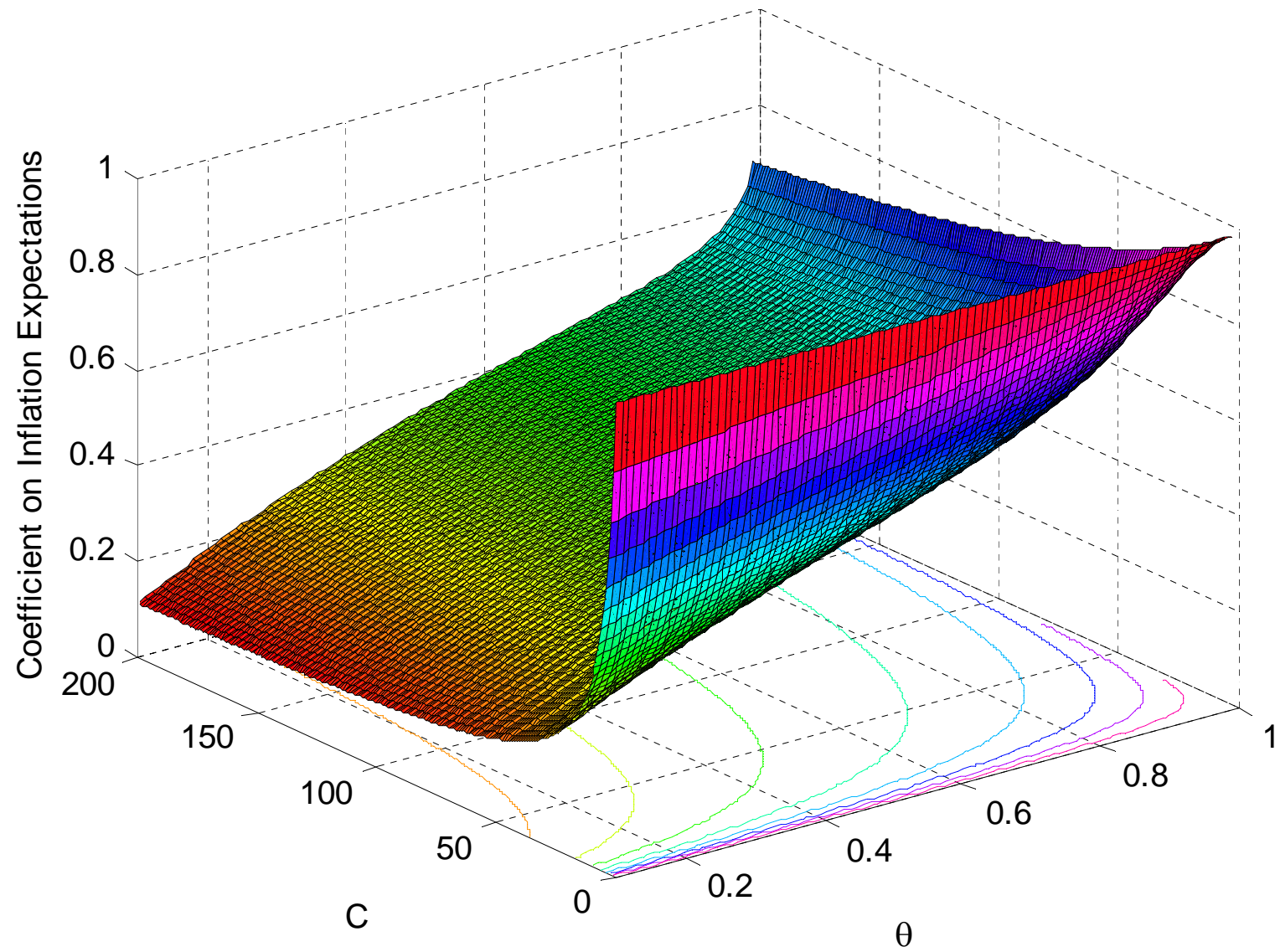
- Klenow and Kryvtsov (2008) - U.S. consumer price changes (absolute value): 44% < 5% 25% < 2.5% 12% < 1%
- Vermeulen et al. (2007) - Euro area producer price:
25% < 1% Mean price change only 4%

Properties of the Model - Coefficients

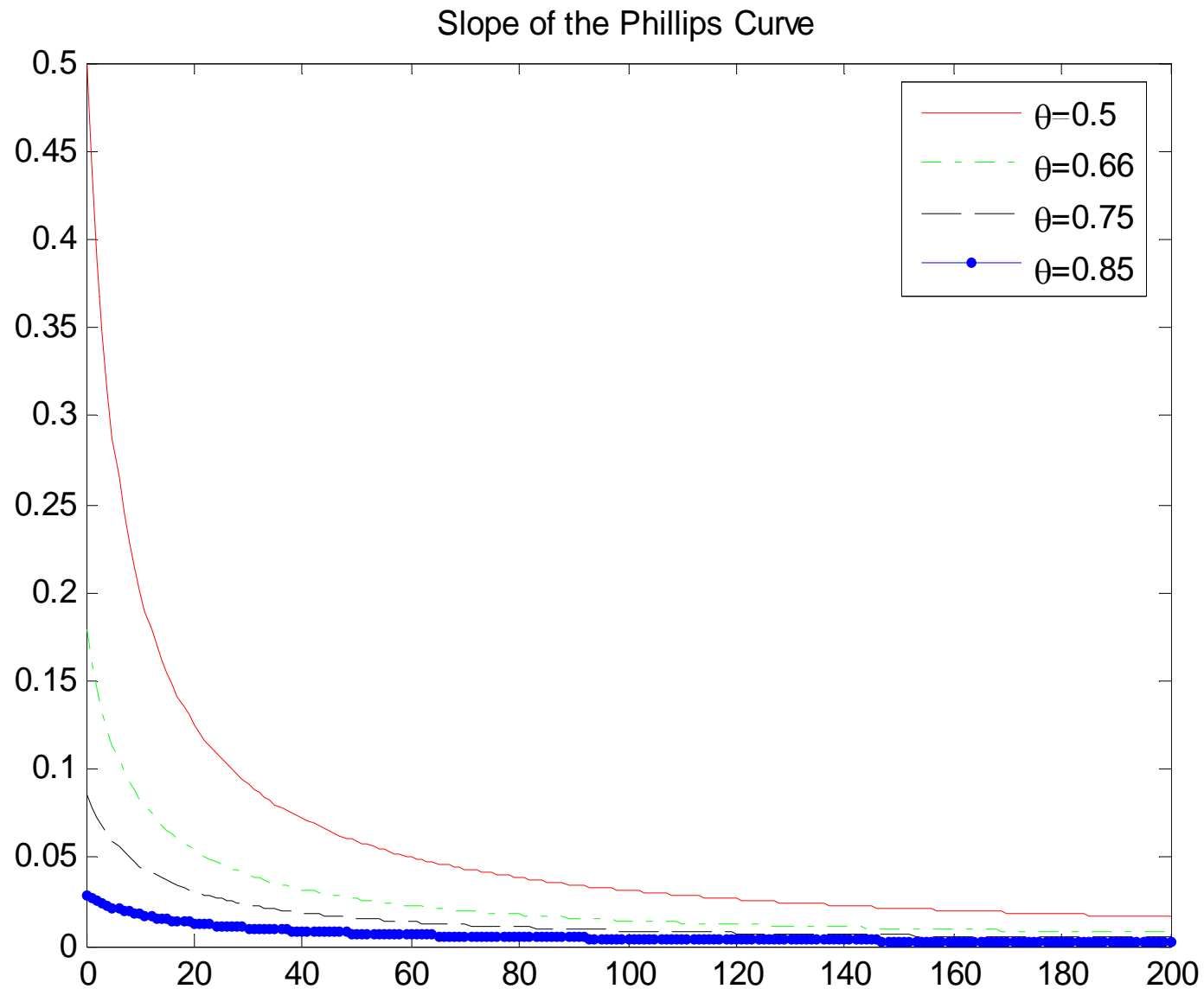
Coefficients on Inflation Expectations and Lagged Inflation



The coefficient on inflation expectations increases with the Calvo parameter θ and decreases with the adjustment cost parameter c



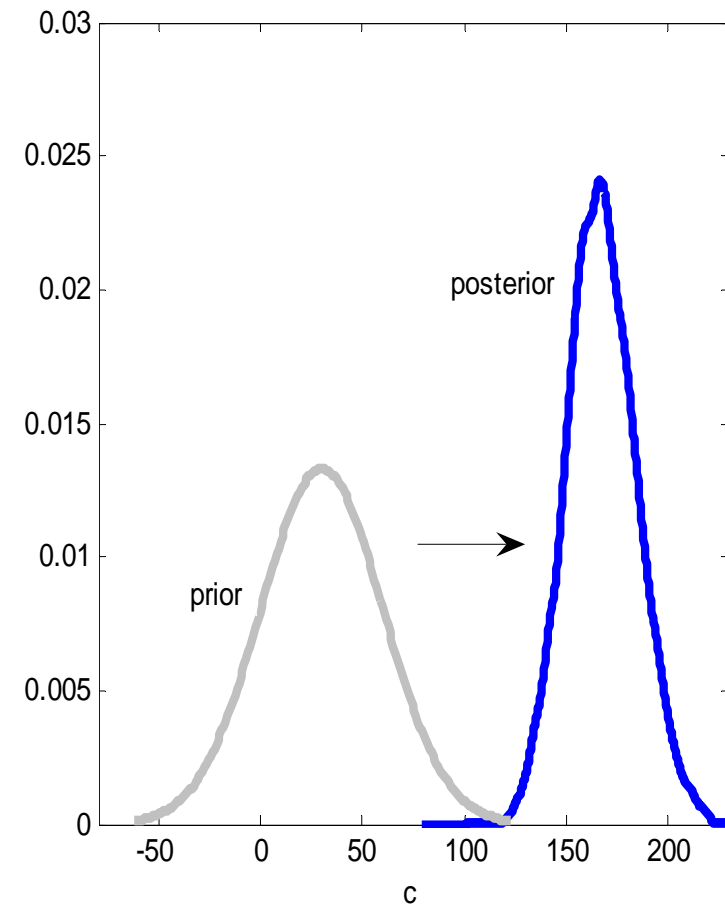
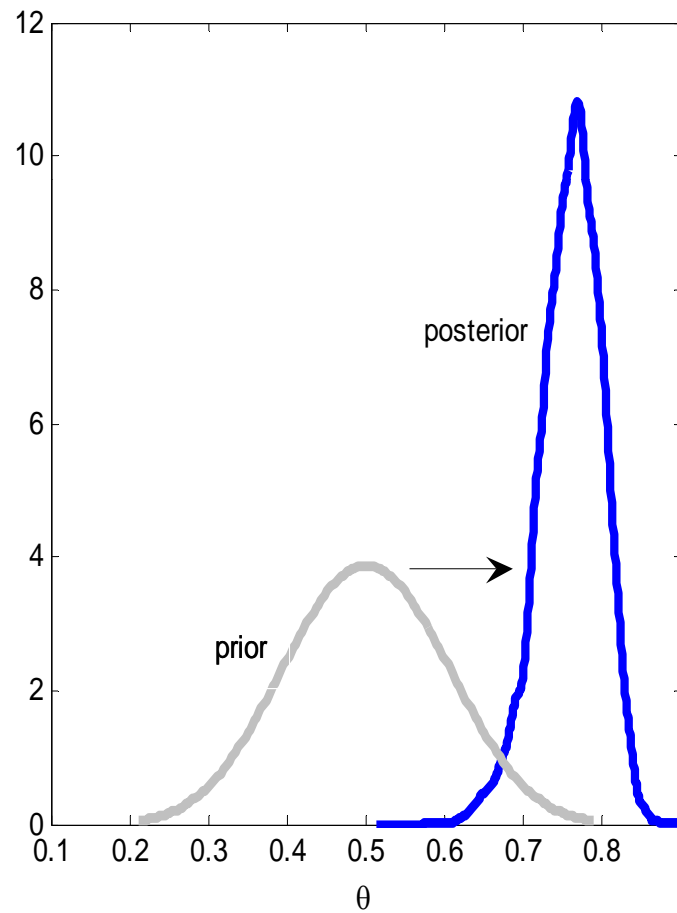
Slope of the Phillips Curve



Estimation Results

Output Gap	1960-2008		
	likelihood	c	theta
CBO output gap	-922.3	167.3 (140.0, 195.6)	0.76 (0.70, 0.82)
HP filtered gap (two-sided)	-879.1	121.7 (97.4, 146.3)	0.72 (0.63, 0.79)
CF filtered gap (one-sided)	-836.3	127.0 (102.5, 152.3)	0.73 (0.66, 0.81)

Priors and Posteriors of Key Parameters



Restrictions on c : likelihood estimates (1960-2008)

Output gap measure	$c \neq 0$			$c=0$		
	likelihood	c	theta	likelihood	c	theta
CBO output gap	-922.3	167.3 (140.0, 195.6)	0.76 (0.70, 0.82)	-1119.8	-	0.89 (0.87, 0.91)
HP filtered gap (two-sided)	-879.1	121.7 (97.4, 146.3)	0.72 (0.63, 0.79)	-1025.8	-	0.85 (0.83, 0.88)
CF filtered gap (one-sided)	-836.3	127.0 (102.5, 152.3)	0.73 (0.66, 0.81)	-1008.3	-	0.88 (0.86, 0.90)

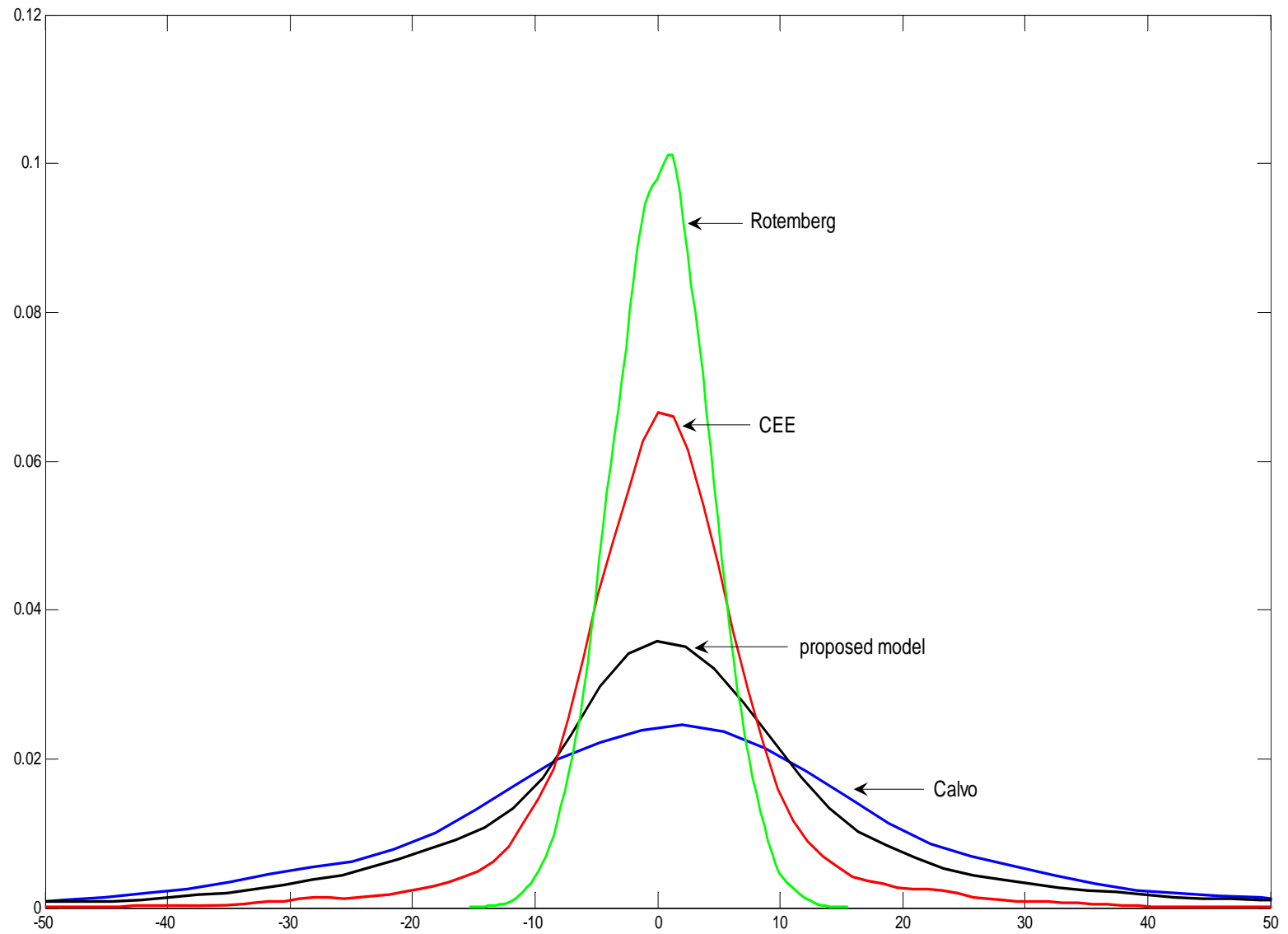
Sub-sample Estimates

1960-1979			1983-2008		
likelihood	c	theta	likelihood	c	theta
-399.2	117.5 (89.4, 149.4)	0.73 (0.65, 0.82)	-407.5	143.5 (112.3, 173.9)	0.72 (0.64, 0.81)
-384.3	83.8 (55.6, 107.0)	0.68 (0.58, 0.79)	-379.2	110.7 (78.6, 138.5)	0.69 (0.58, 0.79)
-370.4	98.5 (70.8, 129.8)	0.69 (0.59, 0.79)	-368.8	111.4 (80.4, 140.0)	0.69 (0.59, 0.80)

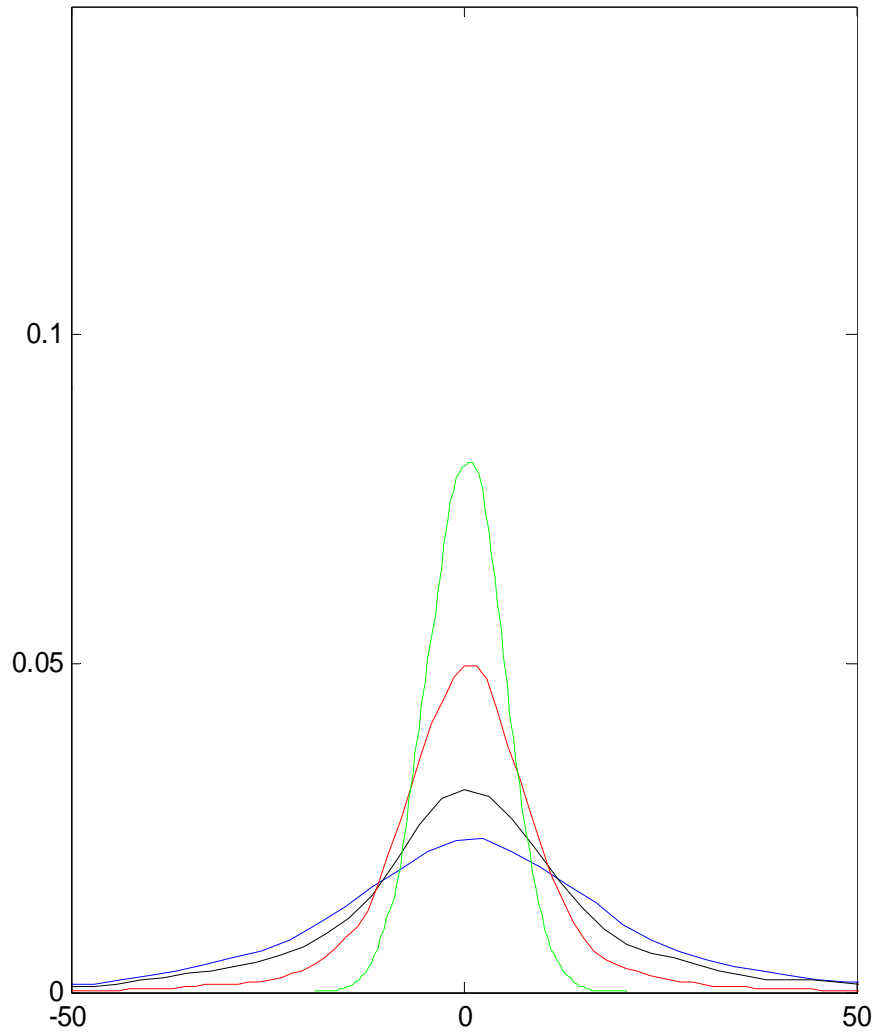
Cost shocks ~ ARMA(4,1): 1960-2008

Output gap measure	GDP deflator			NFB deflator		
	Likel.	c	theta	likelihood	c	theta
CBO output gap	-906.1	117.7 (75.9, 161.1)	0.68 (0.56, 0.80)	-954.7	55.7 (28.7, 82.8)	0.66 (0.54, 0.78)
HP filtered gap (two-sided)	-867.3	140.1 (105.8, 170.6)	0.72 (0.64, 0.79)	-867.4	139.9 (107.2, 171.7)	0.72 (0.65, 0.80)
CF filtered gap (one-sided)	-824.8	130.4 (95.0, 165.9)	0.73 (0.64, 0.81)	-879.9	90.6 (61.4, 121.0)	0.73 (0.66, 0.82)

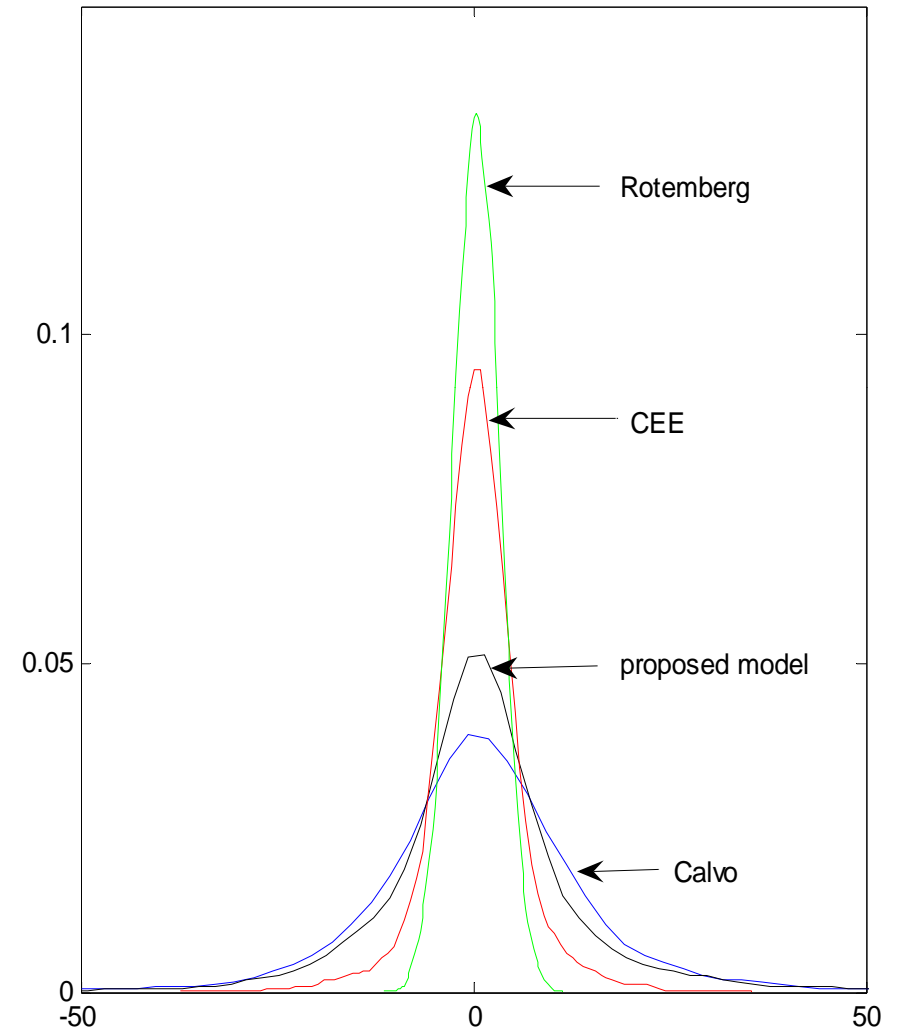
Distributions of Price Changes



pre-1980



post-1980



Average - absolute values of price changes (%)

	Calvo	Our model	CEE	Rotemberg
1960q1-2008q4	15.51	12.42	5.98	3.18
1960q1-1979q4	16.65	14.61	7.58	3.93
1983q1-2008q4	9.90	8.58	4.07	2.40

Data: Absolute size of price changes

- Klenow and Kryvtsov (2008):

US CPI: mean (median) of absolute price changes

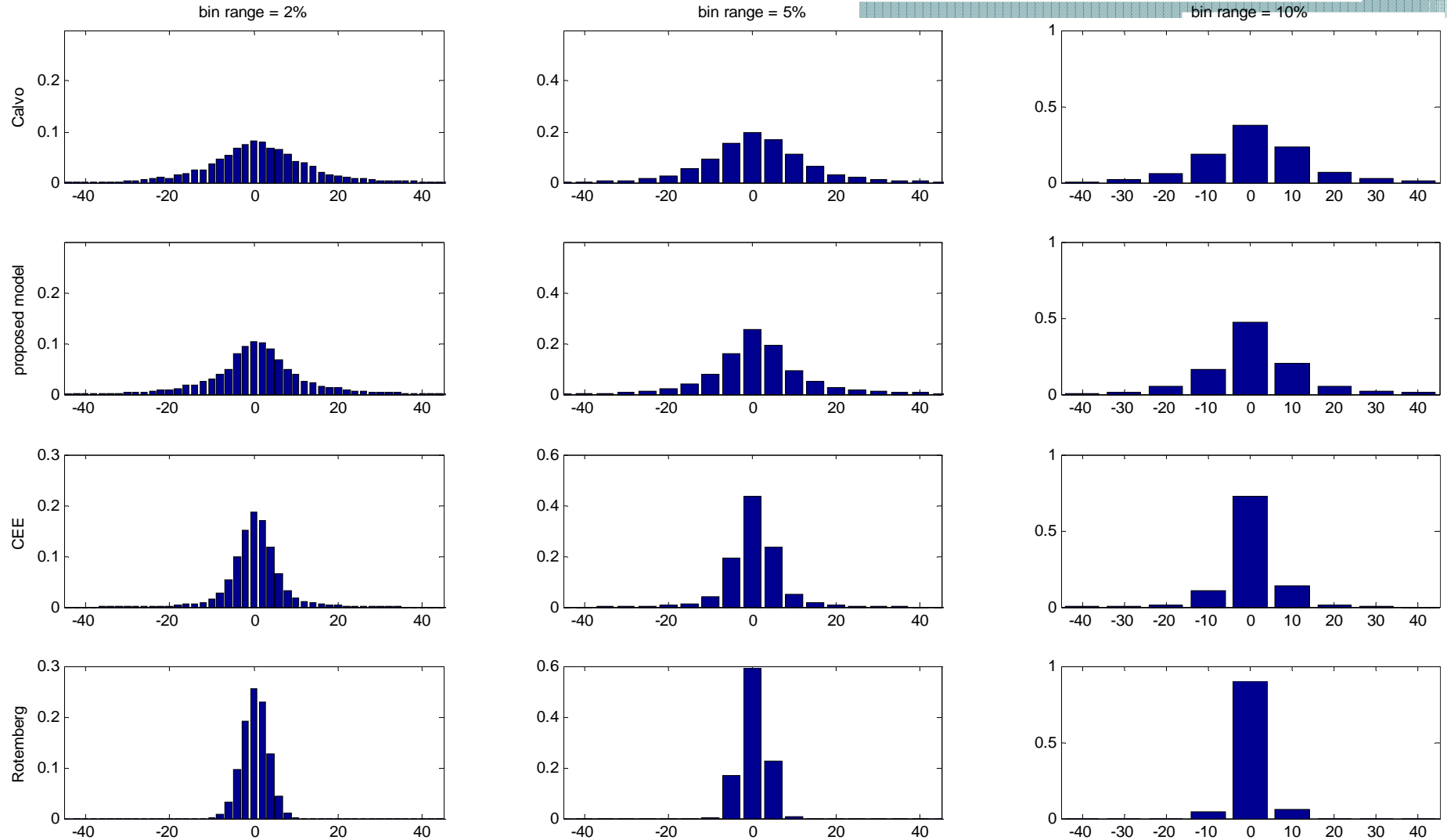
- posted price changes: 14% (11.5%)
- regular price changes: 11.3% (9.7%)

- Nakamura and Steinsson (2008):

U.S. finished goods producer prices

- median of price changes: 7.7%

Klenow: median data are more meaningful for macro.



U.S. CPI: 44% of regular price changes $< 5\%$, 25% $< 2.5\%$, and 12% $< 1\%$ in absolute value (Klenow and Kryvtsov 08)

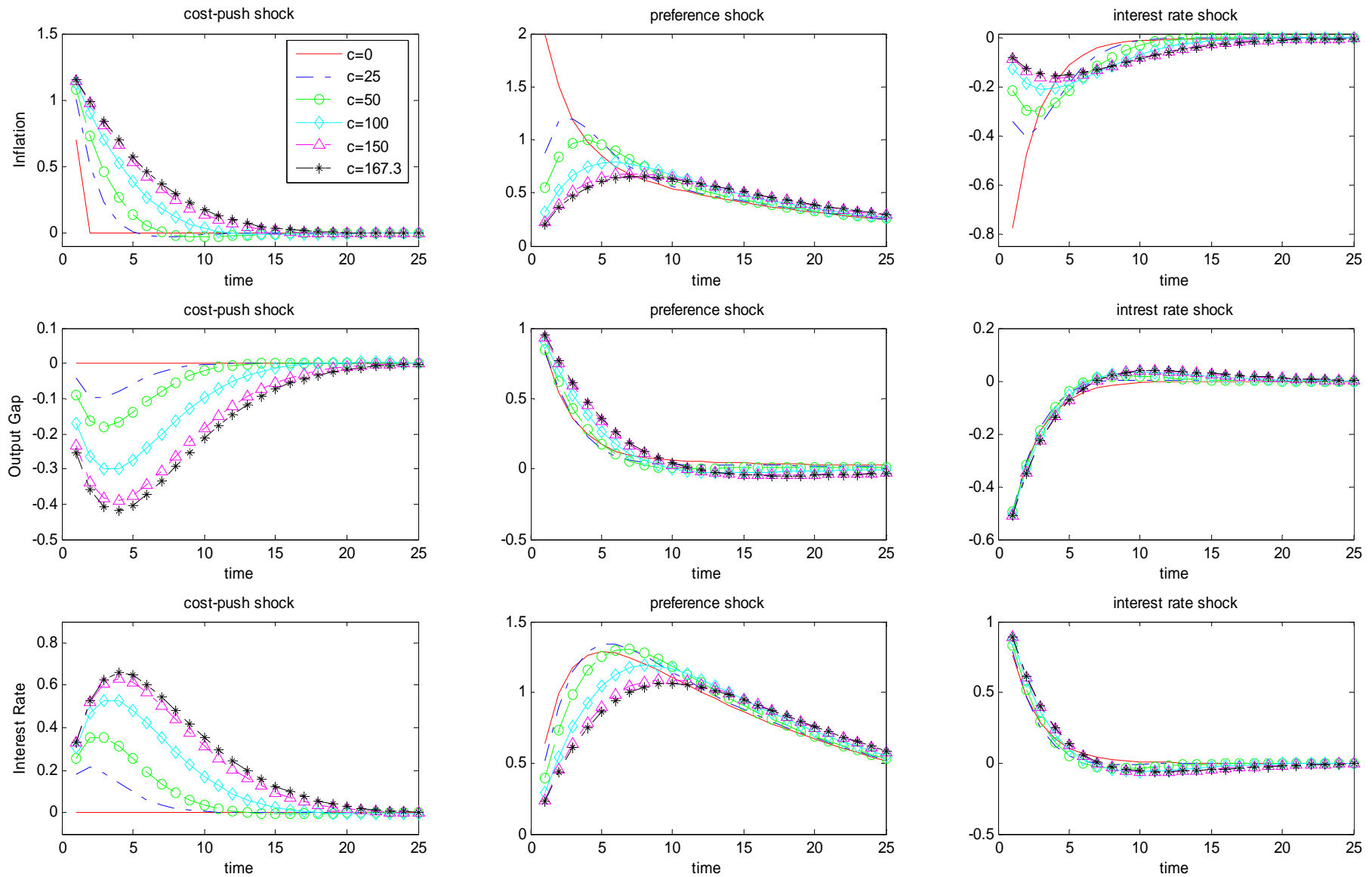
Calvo : 35.4% $< 5\%$ (Histogram for bin range with 10%.

Our model : 46.9% $< 5\%$.

IRF Results

- Response of inflation to a contractionary monetary policy shock:
 - Standard NKPC model, impact of a monetary policy shock on inflation is immediate.
 - Proposed model the response of inflation is gradual, displaying significant inertia

Impulse Response Functions and the Parameter c

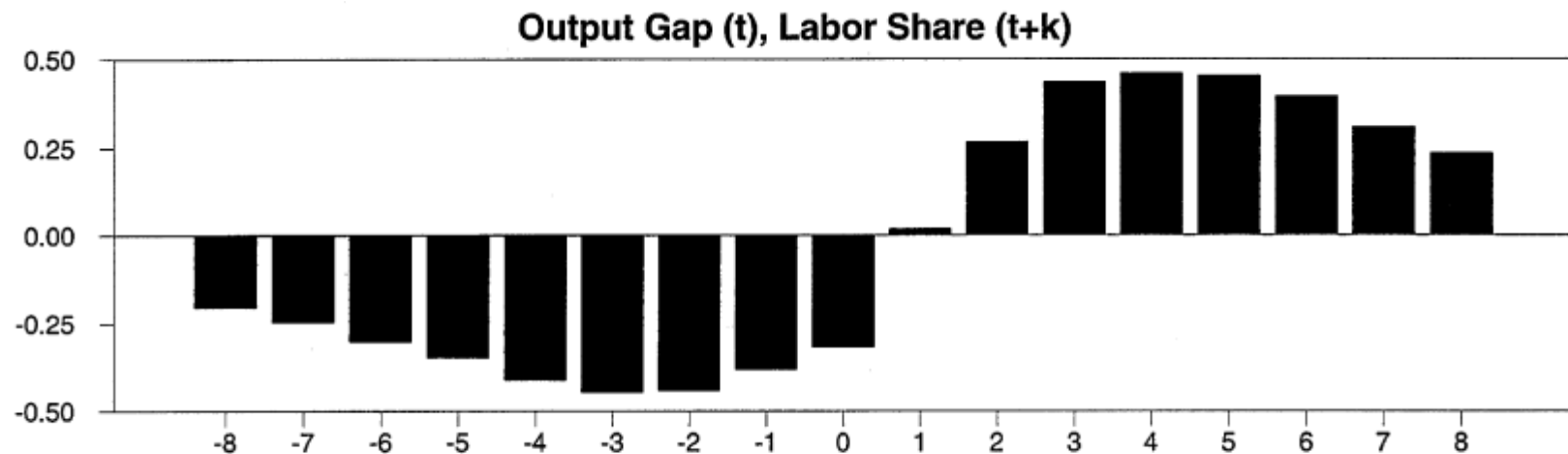


Results

- Taylor (1999) considers as a yardstick of a success of monetary models their ability to generate the “reverse dynamic” cross-correlation between output gap and inflation.

Can the output gap version Phillips curve explain the dynamic correlation between output gap and inflation?

- It is well known the labor income share version hybrid Phillips curve is able to explain the observed dynamic correlation
- Since labor's share of income is countercyclical, we need to investigate whether the output gap version Phillips curve is able to account for the dynamic correlation

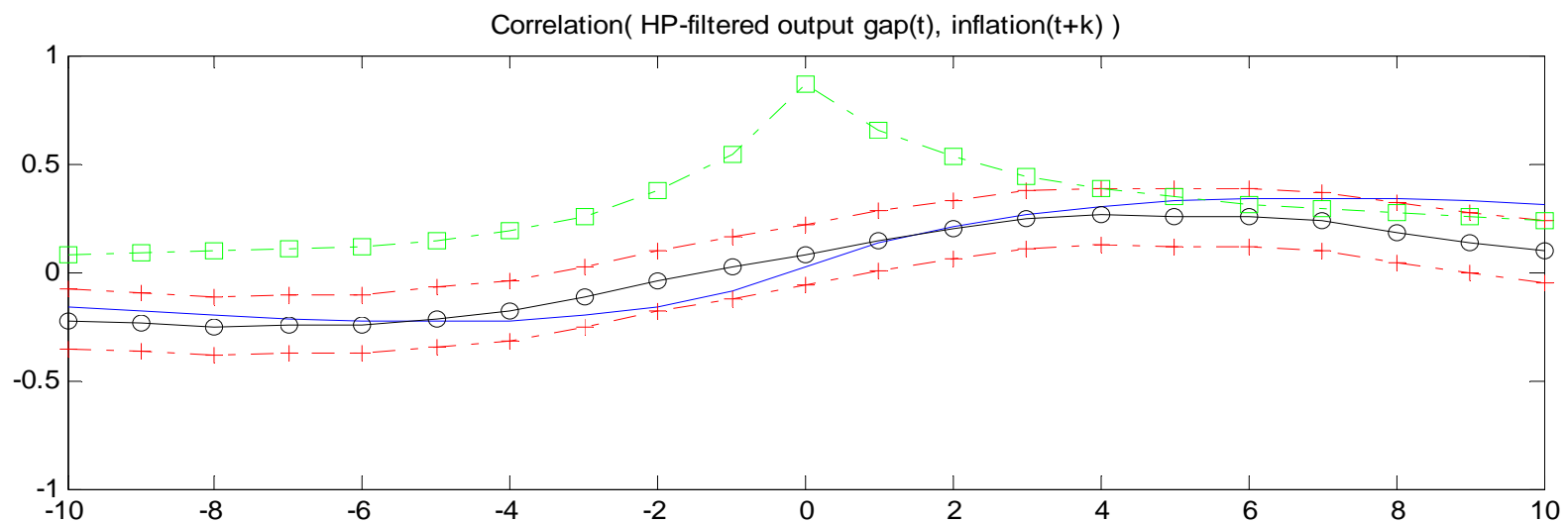
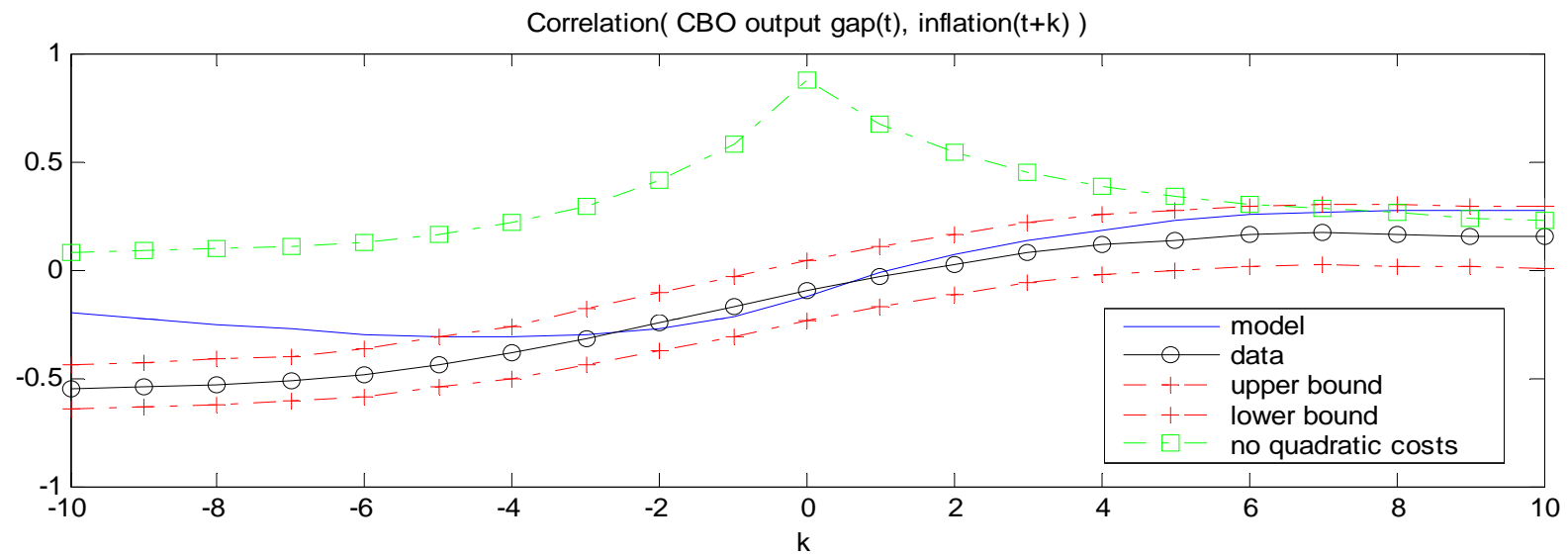


Source: Gali and Gertler (1999)

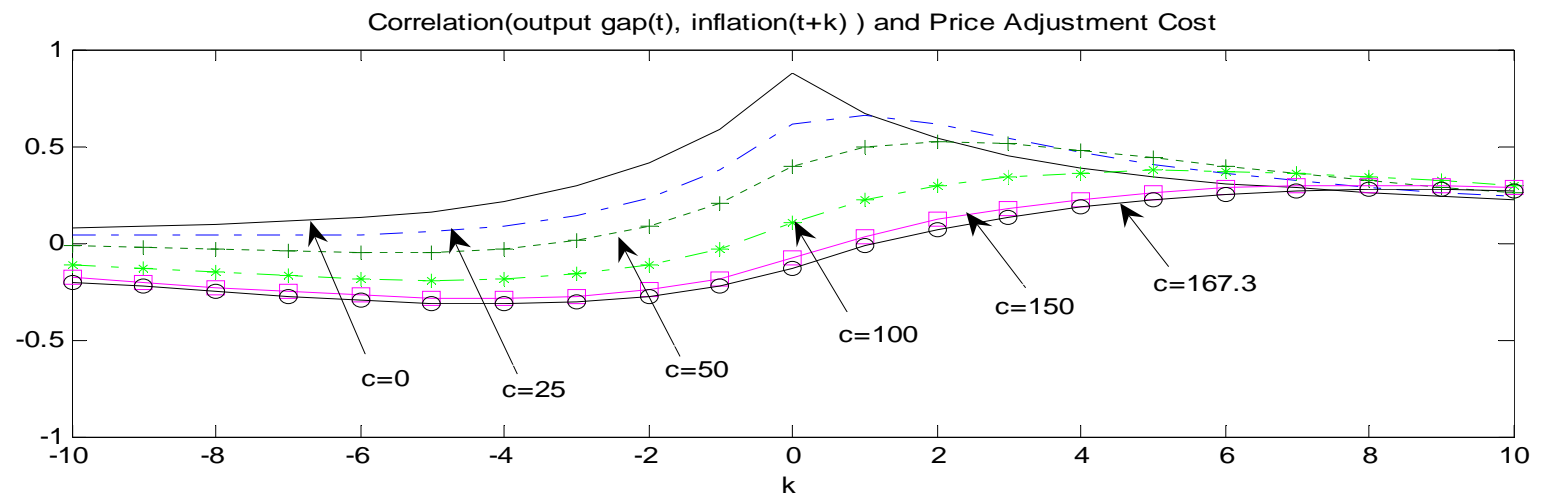
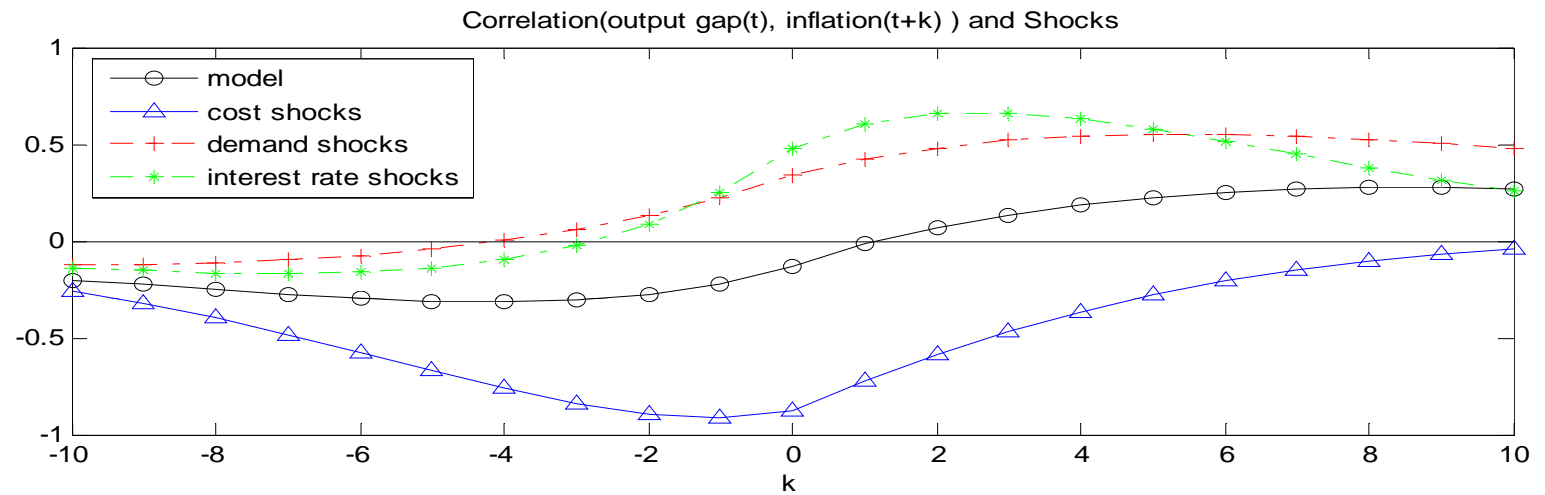
Results

- Model yields “reverse dynamic” result:
 - current output gap tends to be positively related with future inflation, whereas past inflation tends to be negatively associated with current output gap
- Autocorrelation function for the estimated inflation is high and decay gradually, also indicating that inflation is highly persistent
- The estimated inflation autocorrelation closely tracks the observed data.

Dynamic correlation between output gap measures and inflation



continued



Conclusions

- Inflation persistence endogenously generated in the model without relying on indexation assumption
- Proposed sticky price model is supported by the data.
- Results are robust to sub-samples, output gap measures, and inflation measures.
- Distribution of price changes, IRFs, correlation are plausible.

Thank You

Model

$$C_t + \frac{B_t}{P_t} = \frac{W_t}{P_t} N_t + \exp\{-\varepsilon_t^\nu\} (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t$$

.....(2) When equation (1) is replaced with equation (2) and production function (without technology),

$$Y_t = N_t$$