FUNDAÇÃO GETULIO VARGAS

ESTCOLA DE ADMINISTRAÇÃO DE EMPRESAS DE SÃO PAULO

MIGUEL STEVANATO JACOB

AN ESTIMATION OF SHORT- AND LONG-TERM PRICE ELASTICITY OF BUS DEMAND IN SÃO PAULO AND A STUDY OF ITS IMPLICATIONS ON FARE SUBSIDIES POLICY

SÃO PAULO

2018

MIGUEL STEVANATO JACOB

AN ESTIMATION OF SHORT- AND LONG-TERM PRICE ELASTICITY OF BUS DEMAND IN SÃO PAULO AND A STUDY OF ITS IMPLICATIONS ON FARE SUBSIDIES POLICY

Dissertação de mestrado apresentada à Escola de Administração de Empresas de São Paulo da Fundação Getulio Vargas para a obtenção do título de Mestre em Administração Pública pelo Curso de Mestrado em Administração Pública e Governo

Linha de Pesquisa: Política e Economia do Setor Público

Orientador: Prof. Dr. Ciro Biderman

SÃO PAULO

2018

Jacob, Miguel Stevanato.

An estimation of short-and long-term price elasticity of bus demand in São Paulo and a study of its implications on fare subsidies policy / Miguel Stevanato Jacob. - 2018.

142 f.

Orientador: Ciro Biderman

Dissertação (CMAPG) - Escola de Administração de Empresas de São Paulo.

1. Transporte urbano - Tarifas. 2. Transportes coletivos - São Paulo (SP). 3. Política de transporte urbano. I. Biderman, Ciro. II. Dissertação (CMAPG) - Escola de Administração de Empresas de São Paulo. III. Título.

CDU 656.1(816.11)

MIGUEL STEVANATO JACOB

AN ESTIMATION OF SHORT- AND LONG-TERM PRICE ELASTICITY OF BUS DEMAND IN SÃO PAULO AND A STUDY OF ITS IMPLICATIONS ON FARE SUBSIDIES POLICY

Dissertação de mestrado apresentada à Escola de Administração de Empresas de São Paulo da Fundação Getulio Vargas para a obtenção do título de Mestre em Administração Pública pelo Curso de Mestrado em Administração Pública e Governo

Linha de Pesquisa: Política e Economia do Setor Público

Data da aprovação:
08/02/2018

Banca examinadora:

____
Prof. Dr. Ciro Biderman (Orientador)
FGV-EAESP

____
Dr. Bernardo Guatimosim Alvim
BANCO MUNDIAL

____
Profa. Dra. Monica Pinhanez
INSPER

AGRADECIMENTOS

Ainda que este trabalho seja fruto de uma grande dose de tarefas individuais e introspecção, não me senti solitário durante sua elaboração. Parcialmente, pois “a verdadeira solidão está em um lugar que vive por si e que para você não tem nem voz nem feição, onde o estranho é você” e, por mais que eu tenha me sentido fora de lugar ao meu primeiro contato com a academia, a ciência nunca me pareceu estranha. A outra parte vem do fato de eu não ter chegado até aqui sozinho ou montado exclusivamente em meus esforços. Não me refiro, entretanto, ao contexto social – tomo por óbvia sua culpa –, mas às pessoas que estiveram ao me lado e me apoiaram incontáveis vezes. Conquanto não mencione todas elas abaixo, pensei em cada uma durante a elaboração desse trabalho.

Agradeço à minha família, especialmente aos meus pais, Ana e Ibraim, que me ensinaram, talvez inconscientemente, o ceticismo crítico e a curiosidade que são necessários ao método científico. Sem seu suporte e carinho – e de meus tios Páscoa, Armando e Leo – nada do que sou eu seria.

Ao Ciro Biderman, sou grato além das formalidades pela orientação e oportunidade; à Ana Guerrini, agradeço a mentoria e a amizade. São dois guias no campo profissional, pessoal e acadêmico, duas pessoas brilhantes cuja confiança em mim depositada só posso retribuir em forma de agradecimentos intangíveis. Talvez, se um dia inspirar uma pessoa tal qual os dois me inspiram, terei quitado minha dívida de gratidão.

Aos colegas do mestrado, do Cepesp e do trabalho, Lari, Arthur, Ivan, Lycia, Adriano, Claudinha, Fred, Schneider, Dani Stucchi e Caio Franco, agradeço a parceria do cotidiano paralelo à dissertação. Devo um agradecimento especial ao Léo Bueno, que desenvolveu boa parte do (meu e do seu) trabalho ao meu lado.

Agradeço também também os comentários e críticas feitas por Bernardo Alvim, Monica Pinhanez, Renato Vieira e André Chagas.

À Marcella e ao Paçoca, os almoços que seguraram várias crises nos últimos meses. Ravi, Léo, Phelps, André e Luquita, agradeço a força.

À Hannah, agradeço o apoio e lembro de cada gesto de seu amor. Não fosse por você, eu seria incapaz de lembrar que há estrelas além do céu nublado.

O presente trabalho foi realizado com apoio do CNPq, Conselho Nacional de Desenvolvimento Científico e Tecnológico - Brasil

Versões do trabalho, as bases de dados e os códigos utilizados para as estimações (em linguagem R) estão disponíveis em qualquer um dos links abaixo.

Other versions of the present work, the datasets and the R scripts used for the estimations are available at any of the links below.

https://goo.gl/kpucLe

https://www.dropbox.com/sh/yvhnjogfrjpj4hl/AABUfbXFJIb10WUs7-c2NKOqa?dl=0

Garoa do meu São Paulo

- Timbre triste de martírios-

Um pobre vem vindo, é rico!

Só bem perto fica pobre,

Passa e torna a ficar rico

Garoa, sai dos meus olhos.

RESUMO

São Paulo se expandiu rapidamente durante o Século XX e se tornou uma das maiores cidades do mundo, com aproximadamente 12 milhões de habitantes que realizam cerca de 25 milhões de deslocamentos urbanos diariamente. Seu sistema de transporte público (ônibus e metrô) é responsável por 37% dessas viagens e é notavelmente importante, especialmente para seus usuários intensivos – majoritariamente pessoas pobres cujos deslocamentos dependem dele. Os subsídios ao transporte e o valor da tarifa vêm se colocando no centro de um debate sobre política urbana durante os últimos anos. A Prefeitura de São Paulo gasta quase 7% de seu orçamento em subsídios diretos à tarifa de ônibus que se mantém estagnada em termos reais desde 2005 – empreendendo um valor três vezes maior do que era há dez anos. Ao mesmo tempo, o sistema de ônibus em São Paulo aparenta ser inefetivo em tirar carros das ruas. O ambiente urbano da cidade e a sustentabilidade fiscal desse sistema podem ser colocados em risco se essa situação permanecer, uma vez que um ciclo vicioso de quedas no nível de usuários e aumentos no subsídio podem comprometer o transporte público. O preço e a forma de precificação da tarifa são pontos centrais nessa questão, uma vez que a literatura em finanças púbicas diz que um serviço público pode ser fiscalmente sustentável e ensejar eficiência alocativa à economia se a cobrança por elefor precificada corretamente. O presente trabalho estima a elasticidade preço da demanda por ônibus em São Paulo, uma informação importante para responder se sua tarifa ajuda a: gerar eficiência alocativa na economia; atingir sustentabilidade financeira para o sistema de ônibus e fazer com que as pessoas priorizem o ônibus em detrimento do automóvel privado – e, assim, atingir sustentabilidade urbana. Para tal, modelos de Escolha Discreta são estimados para os anos de 1997 e 2007. Utilizando-se a Pesquisa Origem-Destino do Metrô calculam-se as elasticidades de curto prazo para ambos os anos. Posteriormente, a implementação do Bilhete Único (2004) é considerada um choque exógeno no preço das passagens para aqueles que usam mais de um ônibus para seus deslocamentos, sendo assim uma oportunidade para a estimação da elasticidade de longo-prazo na medida em que é virtualmente um choque exógeno de preço. Os resultados sugerem que a demanda por ônibus é inelástica com respeito ao preço tanto no curto quanto no longo-prazo, o que corrobora literatura prévia. Ainda que mais estudos sejam necessários para avaliar se os subsídios devem ser diminuídos, outras políticas além da forma de precificação devem ser consideradas a fim de se tornar o transporte público mais atrativo.

Palavras-chave: Transporte urbano – Tarifas, Transportes coletivos - São Paulo (SP), Política de transporte urbano

ABSTRACT

São Paulo expanded rapidly during the 20^{th} Century and became one of the biggest cities in the World, with almost 12 million inhabitants that make around 25 million urban trips per day. Its transit system (bus and subway) accounts for 37% of those trips and is remarkably important, especially for its heavy users – mainly poor people whose commuting might depend on it. Not by chance, subsidies and fare price have been at the heart of an urban policy debate during the last years. Nowadays, São Paulo's local government spends almost 7% of its budget in bus subsidies - a threefold increase in real terms in ten years - since costs are soaring and fare remains almost constant in real terms since 2005. Despite high subsidies, the city's bus system seems to be ineffective in taking cars out of the street and ridership is slightly decreasing. São Paulo's bus system's fiscal sustainability might be put at risk if things remain unchanged, in that a vicious cycle of ridership decreasing and fare or subsidies increasing might jeopardize transit and harm urban environment. Fare price and its pricing form are central in this question, since literature on public finance says that one public service's system can be fiscally sustainable and causes allocative efficiency if fare is priced correctly. The present work calculates price elasticity of bus demand in São Paulo, an important piece of information to answer whether fare helps achieving allocative efficiency for the economy, reaching fiscal sustainability on bus system, and making commuters shift from car to transit – and, hence, keeping the city's urban sustainability. Discrete Choice Models are estimated for the years of 1997 and 2007 using a household survey on commuting. They directly provide short-term elasticities for both years. Then, Bilhete Único implementation (2004) is considered an exogenous shock on trips' cost for those who use two buses or more on their commuting, therefore being used as an opportunity for estimating long-term elasticity. The results suggest that bus demand is inelastic with respect to price both in short- and long- term, which corroborates previous literature and provides insight for public policies. This indicates that fare is ineffective in taking cars off the streets, but more studies should be conducted to assess whether subsidies should be reduced, especially for reasons of affordability. Policies other than the pricing form should be conducted to achieve transportation sustainability by modal shifting from cars to transit.

LISTA DE ILUSTRAÇÕES

Figure 25 - Boxplots for mode's speed after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

LISTA DE TABELAS

SUMÁRIO

1 INTRODUCTION

In 1872, São Paulo was 318 years old and its population was 31,385. The village, composed by "poorly hedged hovels, unpaved streets, puddles, few people on the street, animals sleeping by the corners" (Toledo, 2015, p. 16) began to be crossed by trams – actually, donkey-drawn vehicles, replaced by electric trams in 1899. 82 years later, in 1954, São Paulo's 2.9 million inhabitants eclipsed Rio de Janeiro's and it became Brazil's largest city – and, in 2017, it still is.

Nowadays, São Paulo is a global city, a financial centre that accounts for 12% of Brazil's GDP. It is home to 12 million people and the heart of a 20.7 million people conurbation of 39 municipalities - the São Paulo Metropolitan Region (SPMR). As in many other metropolises around the world, fast populational growth – and other factors – caused urban sprawl (Meyer, Grostein, & Biderman, 2004a). Meyer, Grostein and Biderman also affirm that the high extension of the urban area and its radial structure generate an urban environment that does not favor 'road rationality.' Consequently, rail was made secondary to roads designed for cars, which are very dependent on infrastructure availability, a structural limitation that for its time imposes pressure over urban life and limits public transportation modes' possibilities.

As a consequence, urban transportation is, then, an issue of great concern to society and congestion is a common subject of conversations in elevators (usually followed by comments about the weather). Moving around São Paulo's streets might be challenging and stressful since traffic jams are recurrent and chaotic. A radio station dedicated to inform drivers about traffic status, 24h per day, was launched in 2007 and local news daily talks about traffic conditions.

A good picture of how people commute within the SPMR is provided by the Origin-Destination Survey of 2007 – a household survey conducted by the São Paulo Metropolitan Company (Metro)[1] that collects, every ten years, information about a sample of residents' daily urban trips and their socioeconomic characteristics[2]. This survey is quite important for the present work since it is the source of data used here. From a sample of almost 30 thousand families – 91 thousand

[1] The state-owned company that runs city's subway system.

[2] Although this survey is conducted by the Metro Company, it collects information about all commuting modes, and not just subway.

people and 169 thousand trips – it is possible to infer that the daily number of trips in SPMR was, in 2007, around 38 million (METRO, 2008).

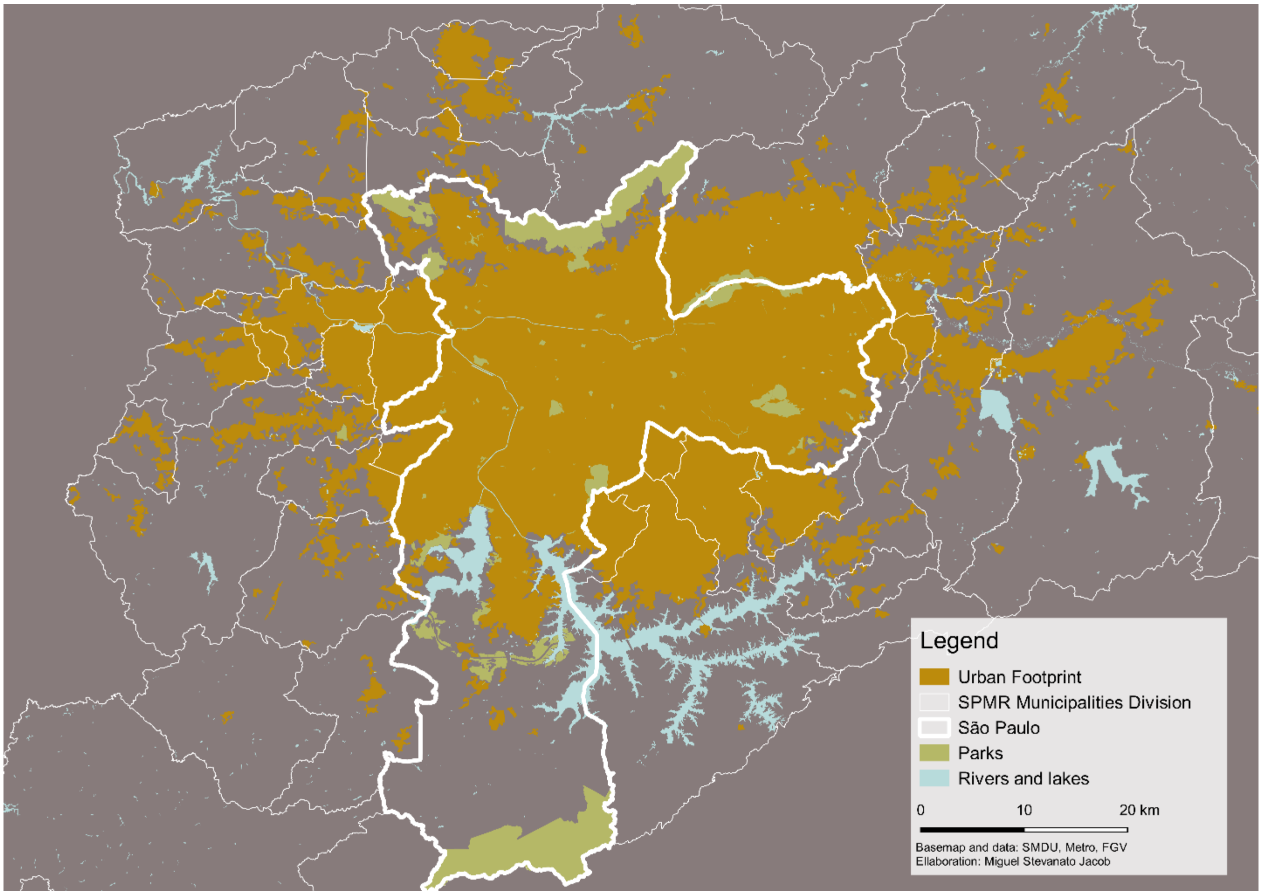

Figure 1 - SPMR urban footprint and political division. Source: São Paulo City Hall and Fundação Getulio Vargas. Prepared by the author.

The figure is 21% higher than it was in 1997 (METRO, 1999) – while the population grew by 16% during those ten years. This surge on total trips, alone, did not necessarily impose pressure over traffic and probably reflects that people are moving around town more frequently – and, therefore, assessing more services and amenities. Actually, usual congestion might be better explained if one notice that 28.5% of those trips are made by people in cars. As of 2015, SPMR's vehicles fleet was around 12 million – São Paulo accounts for 7.5 million of them, of which 70%

are cars.[3] If all of them go out on the street – the city has 17,000 kilometers of paved streets - at the same time, it would very likely cause an instant gridlock.

Abstractions aside, São Paulo accounts for approximately 25 million, or 65%, of SPMR daily trips. 31.6% of the trips in São Paulo Municipality (SPM) are made using individual modes and 31.3% use some kind of active transportation. The other 37.1% rely on public collective modes, as Figure 2 shows. The total number of trips in São Paulo grew by 18.9% during the 1997-2007 period.

Figure 3, containing information on the principal mode[4] taken, also shows that the top chosen modes - walking, driving car, and riding bus - grew up in absolute terms. However, only the latter increased as a proportion of all trips, from 24.05% to 24.22% - a change too small to infer that people are using transit more frequently. If São Paulo wants to mitigate congestion problems, a significative change of these numbers is required: transit must be strengthened and used more frequently, and automobile dependency should decrease (Brinco, 2006).

---

[3] See SEADE (2015) and IBGE - Brazilian Institute of Geography and Statistics

[4] A hierarchy of modes for principal mode classification of multimodal trips is defined by the Metro Company, according to its planning needs. It is as follows: Subway > Train > Bus > Chartered bus > School bus > Taxi > Driving > Car Passenger > Motorcycle > Bicycle > Other > Walk. For example, if one cycled for one hour and then took the subway for ten minutes, the latter will be considered the principal mode, because it has a higher position in the hierarchy. For the purpose of this study, the author elaborated another form of classification, which will be presented when necessary.

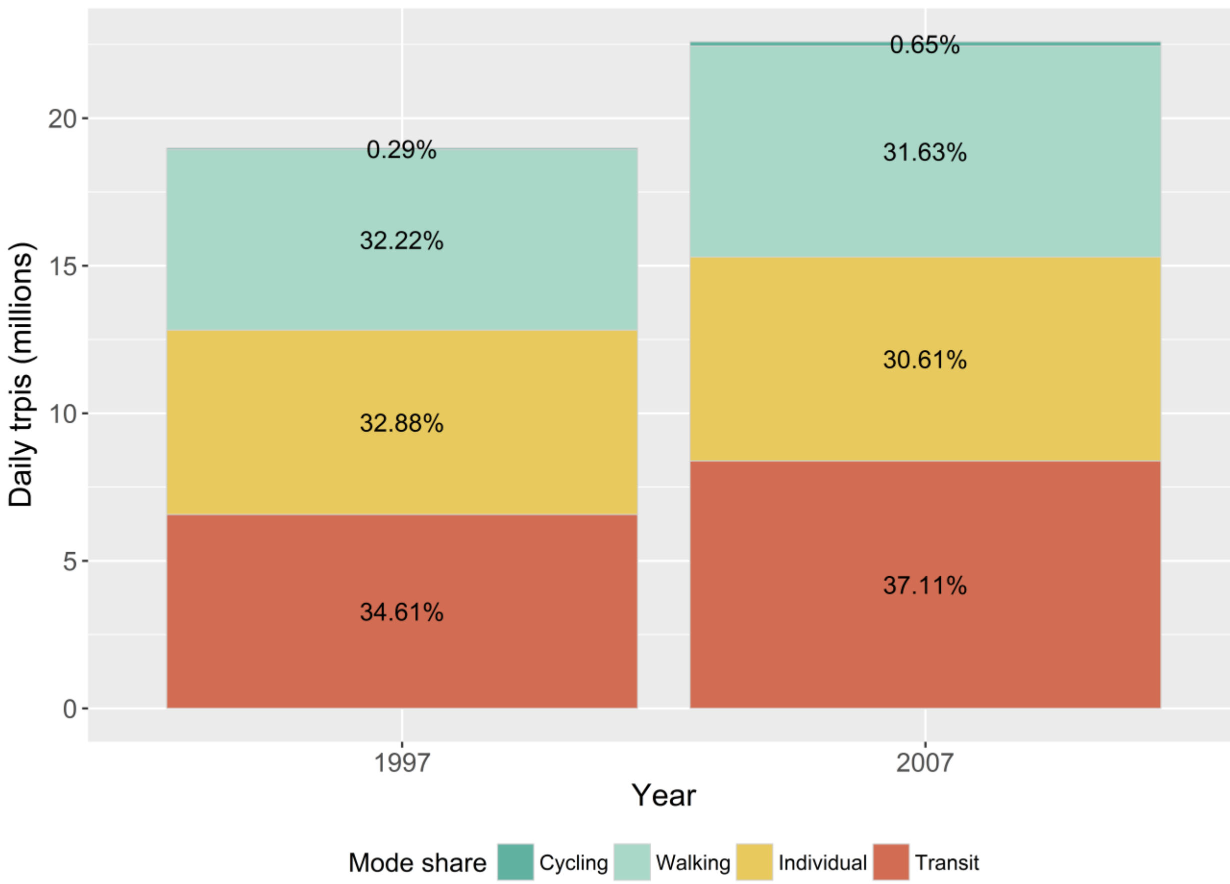

Figure 2 - Daily trips in São Paulo (city) by transport mode - 1997 and 2007. Total is 18.9MM for 1997 and 22.5MM for 2007. The numbers on the bars represent the mode share for each year. Source: OD Surveys 1997 and 2007. Prepared by the author.

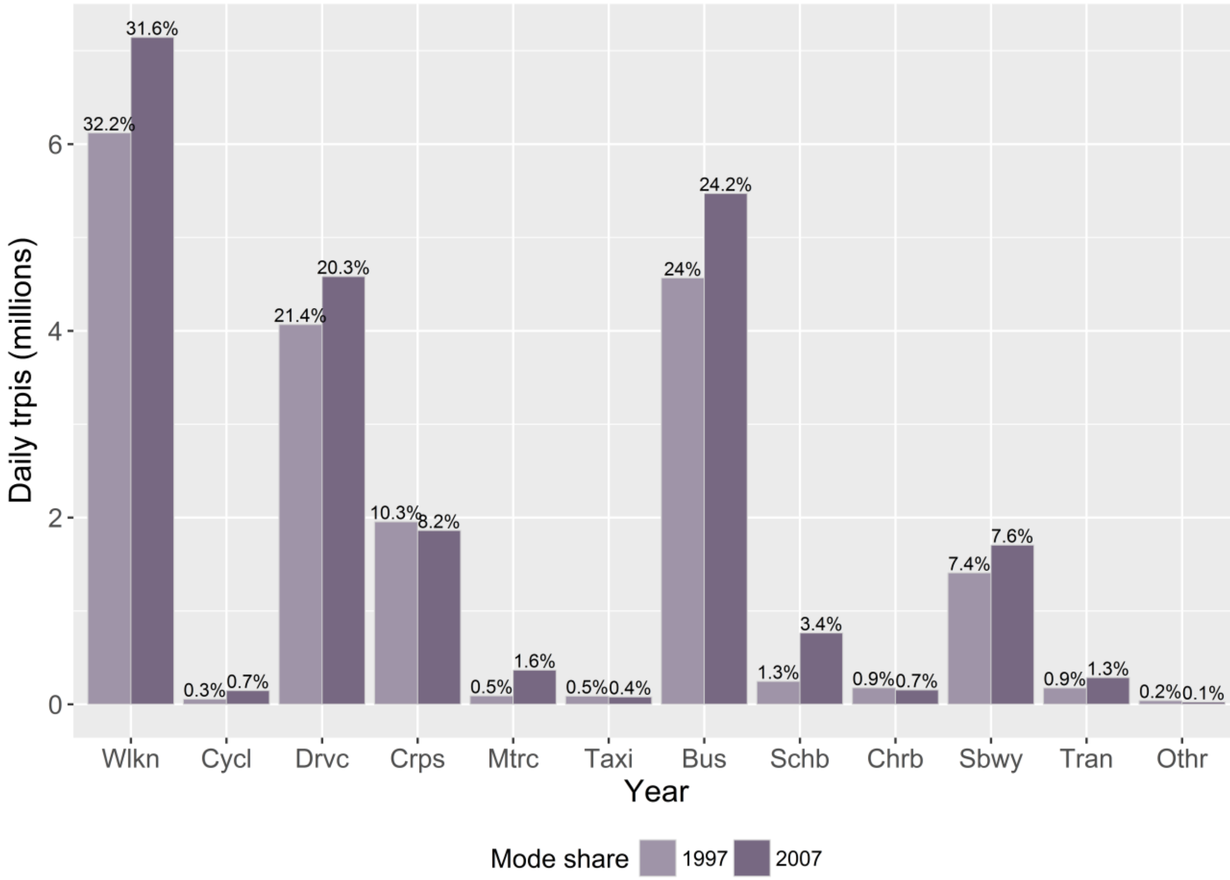

Figure 3- Daily trips in São Paulo (city) divided by principal mode - 1997 and 2007. Total is 18.9MM for 1997 and 22.5MM for 2007. The numbers on the bars represent the mode share for each year. Modes are: Walking, Cycling, Driving car, Car passenger, Motorcycle, Taxi, Bus, School bus, Chartered bus, Subway, Train and Other. Source: OD Surveys 1997 and 2007. Prepared by the author.

There is still one more piece of information to infer from the OD survey which reinforces the role of cars in both congestion and the understanding of São Paulo's commuting pattern: of the trips made in São Paulo by car, 34% have the destination point within a 2.5km radius of the origin point, as shown in Figure 4. They are, probably, short enough to walk or cycle. Those trips are mainly realized by wealthier people since both commuting pattern and mode share are markedly different when considering commuters' income level, as seen in Figure 5.

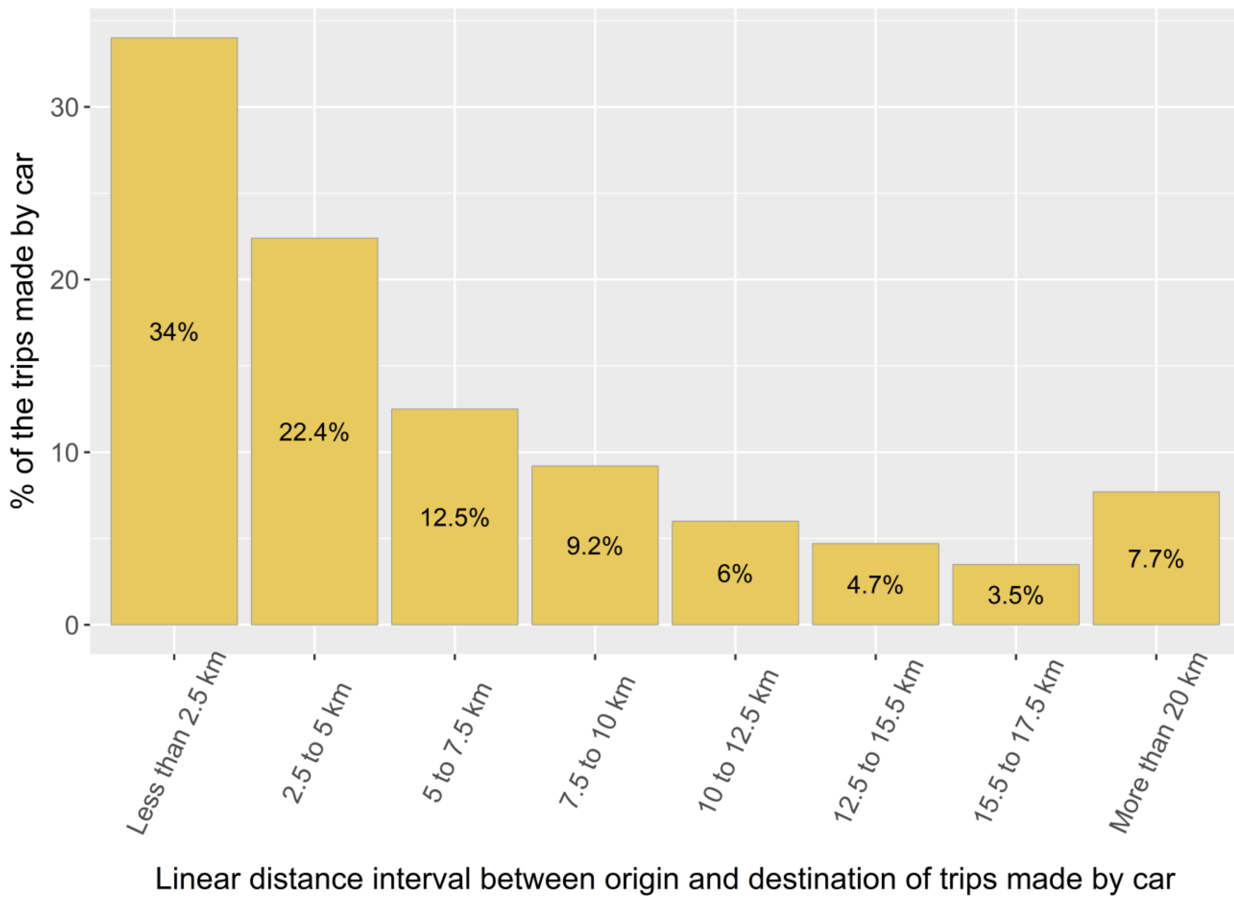

Figure 4 - Linear distance distribution of the trips made entirely by car in 2007. Source: OD Survey of 2007. Prepared by the author.

Following the same logic of Figure 2, in Figure 5 commuters are divided in deciles of familiar income. Two facets are instantly noticeable when looking at this graph: first, the number of daily trips grows as income increases. The 20% richer commute 2.5 times more than the 20% poorer. Second, the mode share is completely dissimilar between rich and poor people.

While the poorest commuters rely on walking and transit, more than half of the top quintile of individuals' trips are made by car. This simple analysis denotes the importance of public collective transportation not only for transportation system itself but also because sometimes it is the only way poor people can move around town. That is to say: transit is remarkably important for São Paulo's urban transportation since it can make urban environment more sustainable. It is even more important for poor people's commuting, whose transportation alternatives are eventually limited.

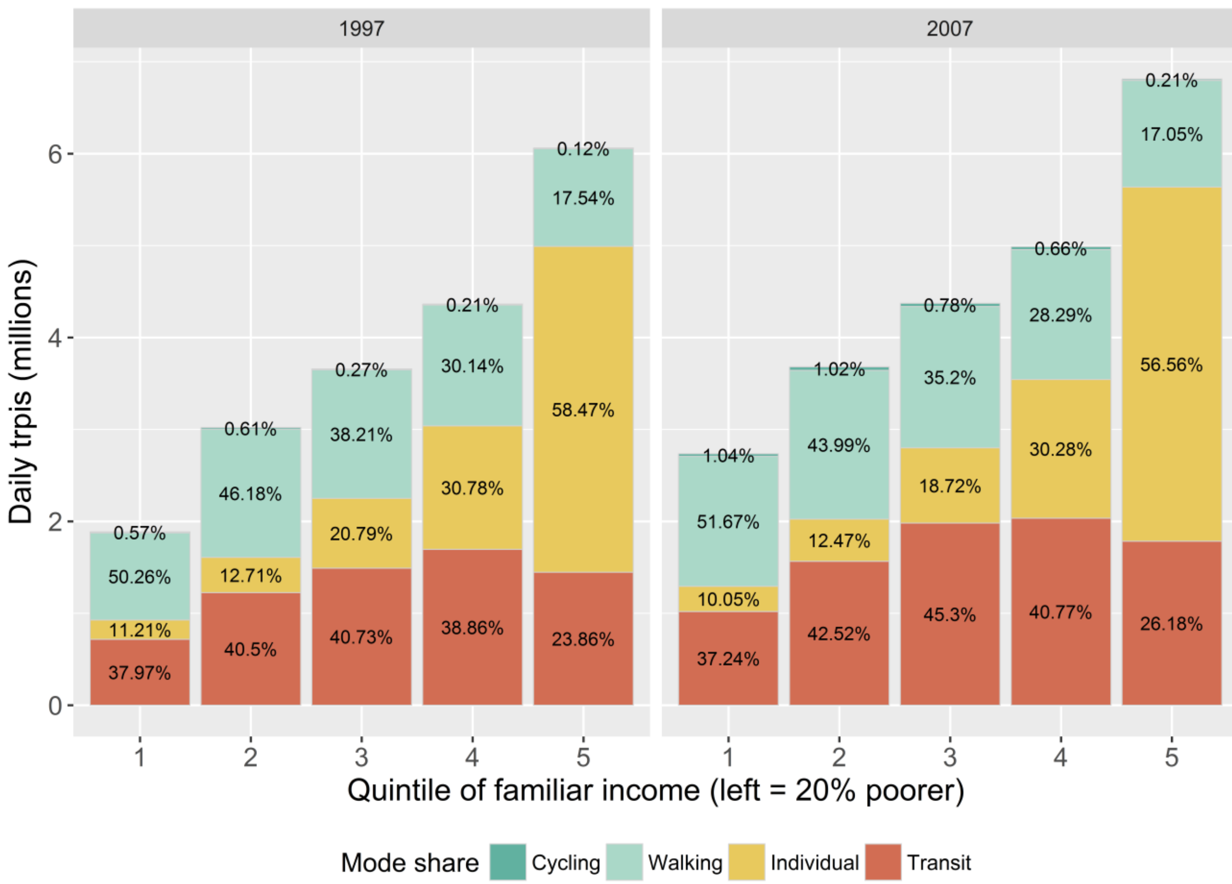

Figure 5 - Daily trips in São Paulo (city) divided by principal mode and familiar income quintile - 1997 and 2007. The numbers on the bars represent the mode share for each quintile, for the correspondent year. Data: OD Surveys 1997 and 2007. Prepared by the author

An important disclaimer must be made (specifically for non-Brazilian readers): bus is the most common form of transit provided by Brazilian local governments. Only a few cities rely on subway and other forms of rapid transit, which are usually run by States' governments. That is the case in São Paulo, where the subway has an insufficient network that also happens to be concentrated in wealthier areas. An indicator ranging from 1 to 0 (perfect and totally poor, respectively) developed by *The Institute for Transportation and Development Policy (ITDP)*, called *People Near Transit (PNT)*, that measures the degree of one city’s inhabitants access to mass transit (ITDP, 2017) evidences this situation. São Paulo’s PNT is 0.25, as opposed to 0.6 for Beijing, 0.77 for New York and 1 for Paris. This means that only 25% of São Paulo’s inhabitants live within 1.5 kilometer from a mass transit station, a maximum distance threshold considered adequate for access. Nevertheless, the index does not include BRT.

Subway and other forms of rapid transit are important forms of urban transportation, for sure. However, the present work is mainly dedicated to bus service, which is considerably more widespread in Brazil and is more operationally versatile than rail, which depends on great infrastructure. Thus, the present work talks about "transit's" benefits as a whole, but cares about bus provision - and studies the case of São Paulo, which importance is evidenced below.

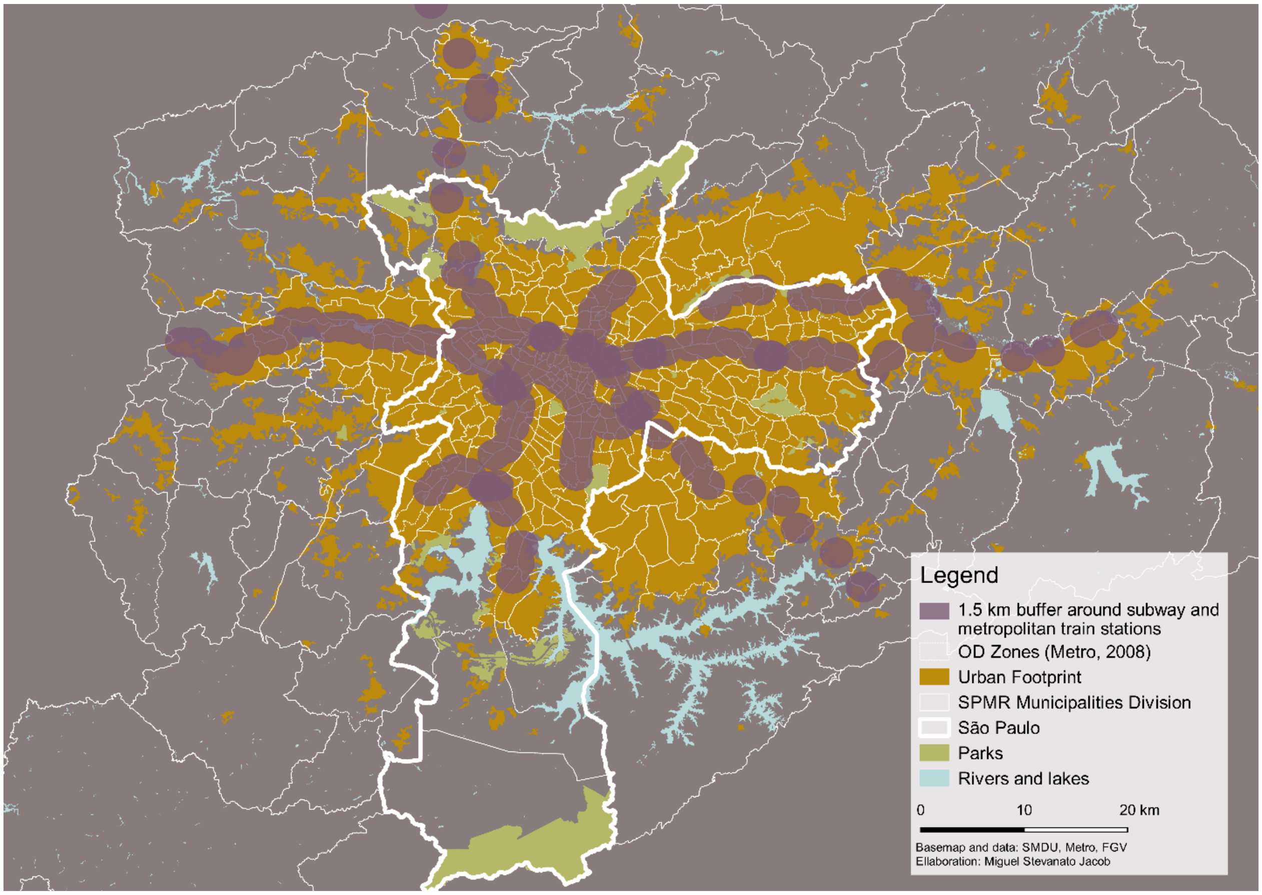

Figure 6 - SPMR urban footprint and political division, plus a 1.5km buffer around subway and metropolitan train stations. Source: São Paulo City Hall and Fundação Getulio Vargas. Prepared by the author.

Rapid urban growth during the last decades has imposed difficulties to public transportation. As pointed by Kenneth Gwilliam (2002) in a World Bank review of transportation paradigms, urban sprawl increases land prices. While poor people are forced to live in the outskirts, “[as] average income grow[s] and car ownership [increases], the patronage, financial viability and eventually [the] quality and quantity of public transport diminishes” (Gwilliam, 2002, p. xii). This is true for Brazil: São Paulo is an archetype of a city that grew very fast (Haddad et al., 2017; Meyer,

Grostein, & Biderman, 2004b) in a kind of urbanization which was extensive and precarious (Meyer et al., 2004a). Between 1940 and 1960, while the population grew 171%, MRSP's urban footprint expanded 364% (Meyer et al., 2004a). As a consequence, to move around São Paulo, specifically by bus or subway, is far from trivial. This might be evidenced by movements in ridership.

Carlos Carvalho and Rafael Pereira (2011) follow what Ricardo Brinco (2006) has stated and point out that bus system's costs and fare prices have increased all over Brazil during the last three decades, while ridership has sequentially plunged, slightly surged, and finally stabilized. In Brazil, between 1995 and 2003, fare price grew 60% above inflation, at average; during this period, ridership decreased by 30%. From 2003 to 2008, ridership roughly increased by 10%. According to the authors, the aforementioned plunge was due to a combination of increasing fares and decreasing familiar income, and the subsequent surge was possible only when real terms increases in minimum wage gave people more purchasing power (Carvalho & Pereira, 2011). They conclude, however, that the gains in purchasing power are not enough to keep a sustainable level of passengers in urban bus systems in Brazil due to its rising costs and lack of competitiveness with private modes. Although this might happen due to subway growth and substitution between modes, it is still a sign that traditional transit is in risk.

Many authors in Brazilian and international literature are concerned with the fact that high transit costs – usually transferred to fare price, since the vast majorities of transit authorities in Brazil do not subsidize fare directly - and low quality might spoil its ridership and cause a vicious cycle. A plunge in ridership makes average transit prices more and more expensive because fewer people pay for the system's cost - since service level takes time to adjust to ridership demand and fare is usually priced at the average cost. The rising costs of Brazilian cities' transit systems, if transferred to fare, might spoil service quality and financial management - and thus jeopardize transit itself and endanger the commuting of low-income sectors of the population, since those people are the ones who need public transport the most. This is, somehow, happening at Rio de Janeiro's BRT by the time this work is developed. Both judicial and administrative decisions did not allow companies to adjust fare and this fact, added to high unemployment rates and bribery scandals, made BRT's system collapse financially. Service-level and quality plunged, and, today, shadow transportation and evasions put 14 thousand bus lines in danger (O GLOBO, 2017).

Also, there is no major policy in Brazil to assist in keeping transit costs at a lower level – for example, cross-subsidies between gas and transit, such subsidies might alleviate the pressures cited by both discouraging car usage and keeping fare at an affordable level. Brinco (2006) point out that that Brazilian transportation policy is highly car-oriented: there is a development paradigm centered on cars, which has deleterious effects on public collective transportation.

Besides a need for a paradigm shift, there is an urgent policy problem, pointed by Carvalho and Pereira (2011), which is closely related to fare policy: there is a trend imbalance of system's costs and potential levies, given that costs are increasing and (a) fare's pricing modes do not comply to attract people to public transportation and (b) local governments do not have the capacity of subsidize public transportation.

Ticketing data of São Paulo's bus system presents a few problems before 2005, therefore, it is hard to carry on with the same analysis as above to assess the city’s ridership trend. However, during the last years, ridership decreased by 5% between 2005 and 2016, while fare decreased 2% in real terms, as can be seen next.

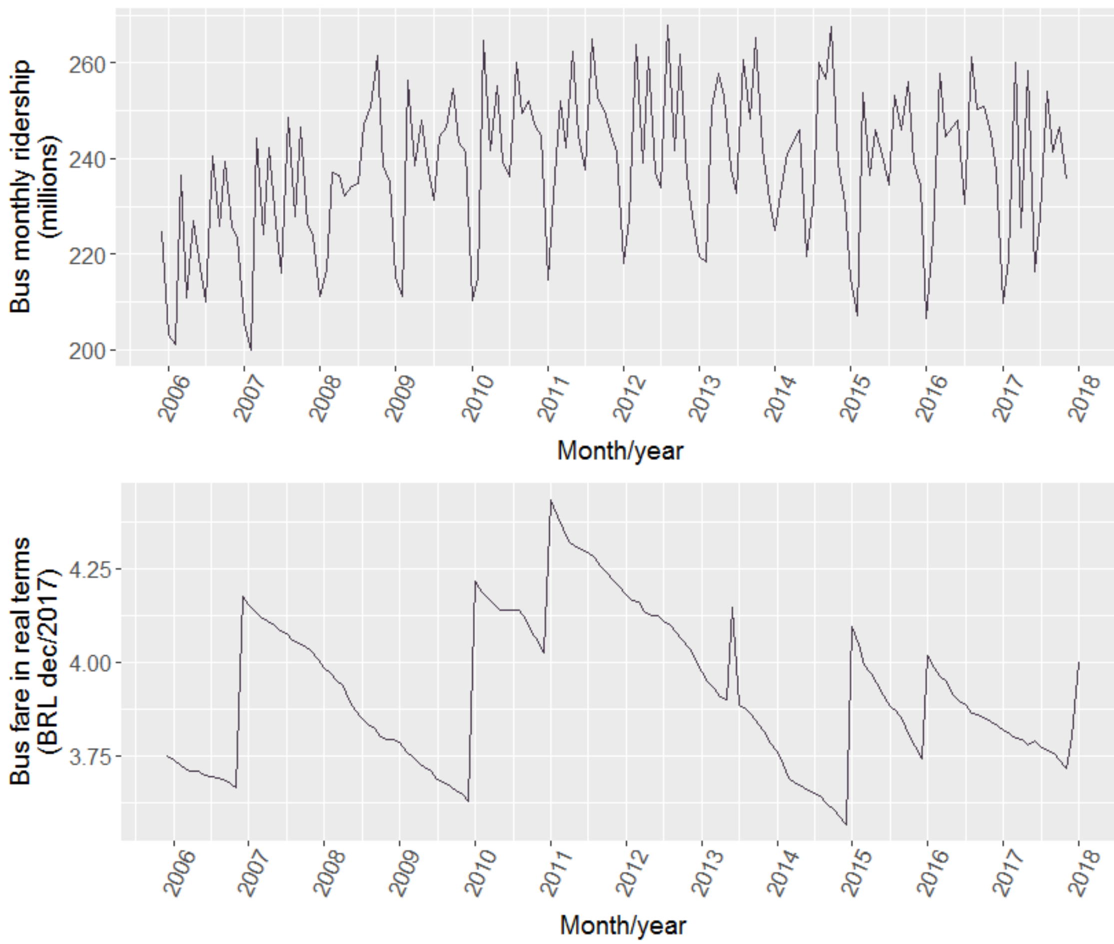

Figure 7 - São Paulo (city) bus ridership and fare prices (in BRL of Dec/2017, IPCA/BCB adjusted; index available at <https://sidra.ibge.gov.br/tabela/1419>) between 2005 and 2016. Source: São Paulo local government and Brazilian Central Bank. Prepared by the author.

In São Paulo, the bus system is operated by private companies under government permissions. Those companies are tendered in public bids and, differently from the vast majority of Brazilian cities, São Paulo's local government does subsidizes bus fares and, lately, it has been doing so in a way that seems out of control.

In 2004, local government used to spend around 900 million Reais in fare subsidies – around 3.4% of its total budget[5]. This value surged over the last years and reached around 3 billion Reais (approximately 1 billion US Dollars) in 2016, a sum that represents almost 7% of the total budget.

---

[5] Whole budget. Including salaries, pensions, transfers to legislative power, other policies and so on.

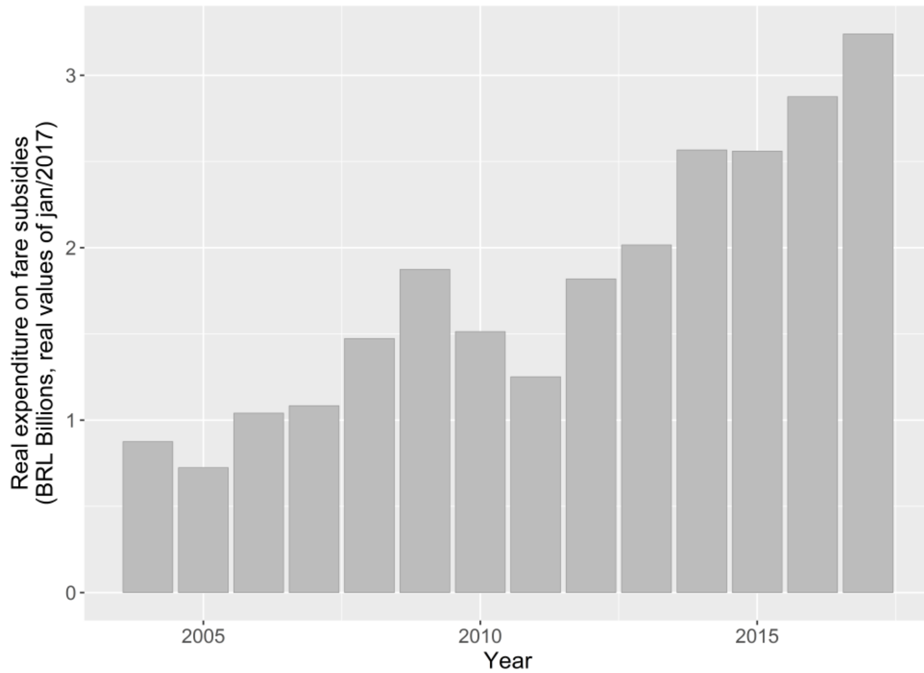

Figure 8 - São Paulo's subsidies to bus fare. Data: São Paulo local government Available at <http://orcamento.sf.prefeitura.sp.gov.br/orcamento/execucao.php> . Prepared by the author.

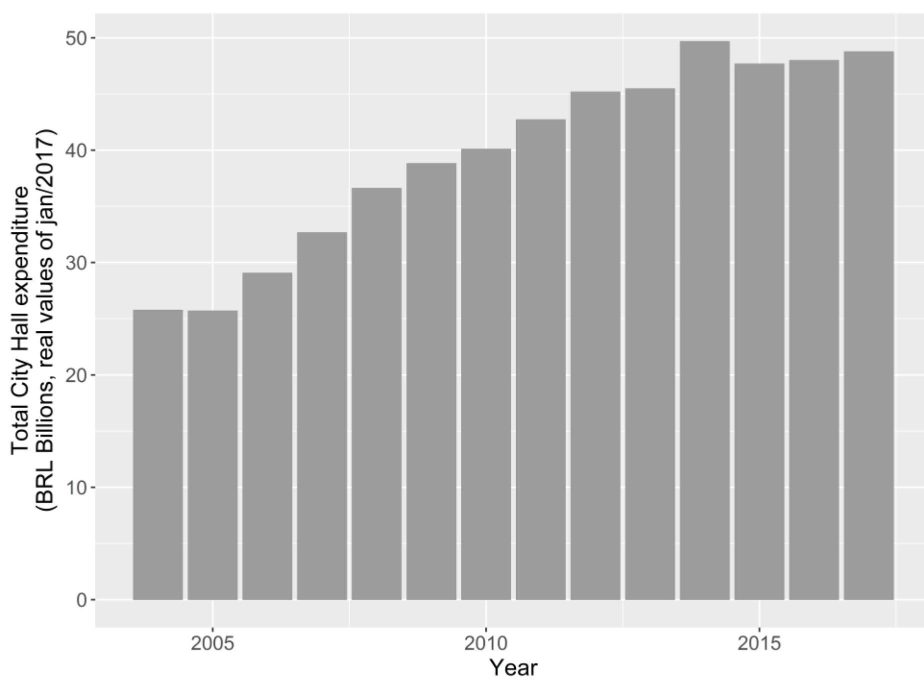

Figure 9 - São Paulo's total expenditure. Source: data from São Paulo local government Available at <http://orcamento.sf.prefeitura.sp.gov.br/orcamento/execucao.php>. Prepared by the author.

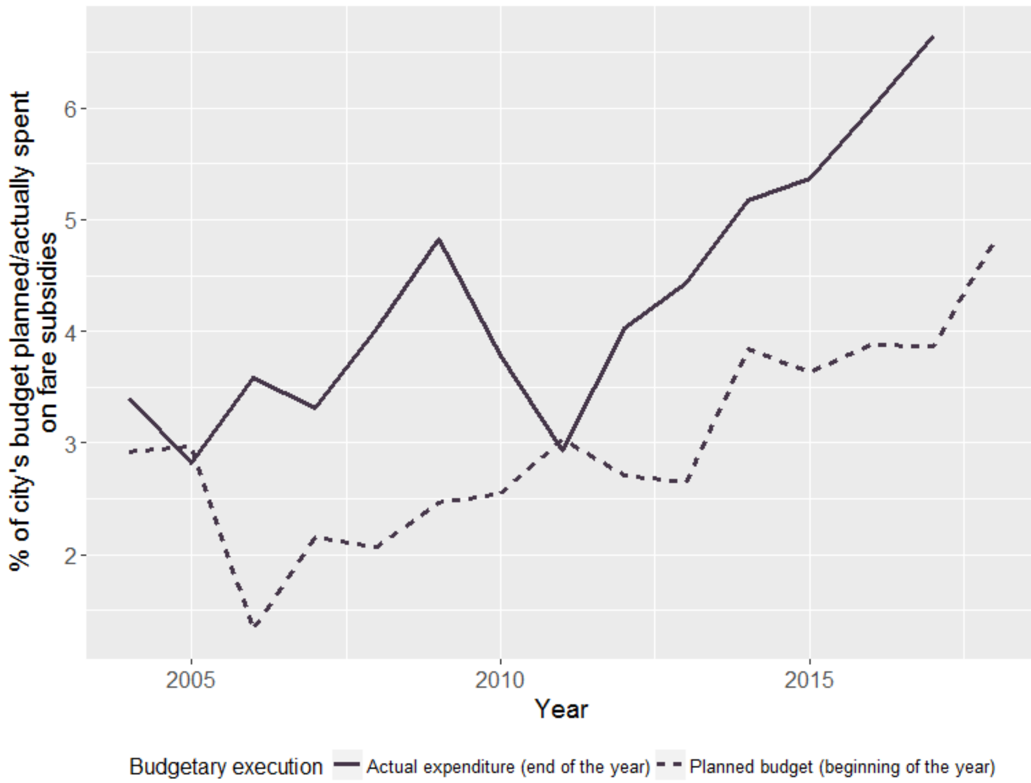

Figure 10 - São Paulo's subsidies to bus fare, compared to total expenditure. Source: São Paulo local government Available at <http://orcamento.sf.prefeitura.sp.gov.br/orcamento/execucao.php>. Prepared by the author.

Since both ridership and fare price in real terms remained relatively stable during those years this rise in subsidies might be caused by increasing costs. Human resources (driver and ticket checker) costs, fuel costs and direct taxes grew above the average inflation in Brazil during the last decades (Carvalho & Pereira, 2011).

This situation is of concern since rising costs of transit system might put it in risk. Soaring costs might either reduce transit quality or, if passed on in fare price, might jeopardize ridership. The first can discourage ridership in an environment where car is the most competitive option for those who can afford it. The latter imposes pressure over people's budget spent on transportation, already at high levels.

São Paulo's local government's subsidies for the bus system represents almost 7% of city's budget but covers only 32% of its cost. Meanwhile, fare remained stagnated in real terms and

ridership was quite stable. This indicates that São Paulo’s government is keeping fare at constant levels by covering soaring costs. Also, subsidies are funded by general levies.

According to Ian Parry and Kenneth Small (2009), the classic rationales for subsidizing transit are: (a) that scale economies imply that the marginal cost of supply falls below the average cost and that (b) low fares would discourage automobile use, being then a second-best for congestion charging and mitigating negative externalities caused by transportation.

Negative externalities caused by transportation and potentialized by traffic congestion affect both individual and collective modes’ commuters – and even those who are not commuting at a given moment, since air and noise pollution and road safety affects everyone in cities. Those are problems well documented and there is even a consensus of how to tackle them: literature usually proposes congestion pricing and/or prioritizing public transportation (Basso & Jara-Díaz, 2012). Theoretically, São Paulo is doing so by subsidizing bus fare. Nevertheless, it is not possible to say that São Paulo complies with those recommendations in practice: lower fares seems to be ineffective in taking cars out of the streets, transit is still not the better option for long and peak-hour commuting (Gomide, 2003), and families spend a good amount of their income on transportation costs (Carvalho & Pereira, 2012), to the point that half of poor families declares that they choose walking since fare is unaffordable (Gomide, 2003).

Additionally, yet literature recommends that fare should be adequately priced in that to keep allocative efficiency to the economy, there is no evidence on the rationales for São Paulo’s bus fare subsidization.

The problem gets more intricated when recalling that urban transportation is a factor that can either catalyzes economic growth (Haddad et al., 2017) or cause productivity losses (Haddad & Vieira, 2015), reduce poverty (Gwilliam, 2002) and promote people’s access to the city (Albalate & Bel, 2009). Whether to invest or not in transit – both directly, through fare or infrastructure subsidies, or indirectly, through regulation, for instance – is a discussion relevant for public policy’s practice and academic works.

Thus, one substantial problem placed in the intersection of transport economics and public policy, specifically if those are inserted in a developing country context such as that of Brazil, is **how to**

provide or regulate public transportation keeping in mind that "urban transport can contribute to poverty reduction both indirectly through its impact on the city economy and hence on economic growth, and directly through its impact on the daily needs of poor people"? (Gwilliam, 2002, p. xii) Of course, this reflection is influenced by a paradigm in urban transport strategy, which essentially comprises promoting system's fiscal sustainability and the highest possible economic welfare while keeping fare affordable. This is part of a broader view, which main interest is on cities' sustainability. The role of transportation in this view is closely attached to transit strengthness, competitiveness, financial sustainability and adequate governance and management (Carruthers, Dick, & Saurkar, 2005), in that it might diminish transportation negative externalities, since its use, compared to automobile's, causes less congestion, less pollution per capita and possibly good impact on the urban environment due to the less parking demand. The present work is influenced by this paradigm and considers that incentivizing people's use of transit is socially desirable.

The present work approaches a factor that is central for the problems exposed until now: price-elasticity of bus demand.

Understanding the effects of price change over ridership is important for both public policy's practice and for further academic works. This information is crucial for transit planning since it might help on service-level definition, and even for lines placement and modifications. For public finance, transportation economics, and public policy, price-elasticity of demand is central for planning or evaluating the impact of subsidies or pricing on policy. It is one of the variables that helps to understand whether transit system is leading to system's and cities' sustainability, through pricing fare at an efficient level and getting car out of the streets, respectively.

The estimations made in the present work - price elasticity of bus demand, cross-price elasticity of bus/car demand (with respect to car/bus) and the Discrete Choice Model itself - do not fully address all the questions presented here in that a final answer on how transit policy should be formulated is given at the end. However, they might provide good insights for policymakers and/or serve as basis for other models (such as the Four-Step model or computable general equilibrium models) that could provide robust answers for questions about transit allocative efficiency and affordability. The present work hopes to provide insightful knowledge on whether

subsidies are efficient on keeping fare affordable taking cars off the streets and keeping people's transportation capacity strong.

The present work embraces economic aspects of this analysis: once demand for bus variation in São Paulo given a fare change is known, a discussion of the meanings and adequacy of fare's subsides is conducted, supported by the literature on the mentioned themes. Results follow what has been found in literature until now: bus demand is price inelastic, most likely indicating that marginal cost pricing rule would not promote welfare maximization and that a subsidies policy is very likely to be ineffective in taking cars off the street.

Thus, the present work aims to contribute to the debate by calculating São Paulo's price elasticity of bus demand, for both short (two cross sections for the years of 1997 and 2007) and long-term (ten years period). To do so, the work estimates two Discrete Choice Models, as proposed by Daniel McFadden (1973, 1974), using OD Surveys' data of 1997 and 2007 (and some simulation over the microdata).

Discrete Choice Models postulates that, given a set of alternatives, "the probability of individuals choosing a given option is a function of their socioeconomic characteristics and the relative attractiveness of the option" (Ortúzar & Willumsen, 2011, p. 227). That is, these models consider that one decides her commuting mode in a choice set in which price and duration of the trip in each one of the modes is known. Thus, one part of her utility function is considered random and unobserved by the scientist and the other part has observed elements of the individual and of the alternatives. It is supposed that the commuter will pick up the mode that provides her the greatest utility.

McFadden's Discrete Choice Model differs from other similar formulations in the error terms' assumptions – supposed to be independent (Hensher & Button, 2000; Koppelman & Vaneet, 2000; McFadden, 1973, 1974; Ortúzar & Willumsen, 2011). The model is then estimated with a multinomial logit regression - the coefficients represent changes in probabilities of one choosing each mode, if some independent variable changes and everything else remain constant - from which aggregate probabilities of each mode being chosen can be derivate, as well marginal effects of independent variables on these probabilities. The latter provide short-term elasticities. The first is the aggregate demand that, if estimated for 1997 and 2007, can be compared to

calculate long-term elasticity. However, an exogenous price shock is required for estimating the elasticity. Here resides one innovation proposed in this paper: to use the implementation of Bilhete Único as the source of an exogenous price shock.

Bilhete Único, implemented in 2004 by São Paulo's local government, is a transit smart ticketing card. It allows users to pay for one bus ride and make up to three bus transfers within three hours - it means that users can start four bus trips within 180 minutes paying only the first one. Commuters can also use it to pay 23% less when taking the subway or train before or after taking up to three buses (compared to paying for both subway and bus fares).

OD Surveys provide us evidence that Bilhete Único influenced a change on riders' behavior: in 2007, around 24% of bus users made one or more transfers, against 15% in 1997. As it is showed further in the present work, the number of trips using more than one bus (and bus only) in São Paulo surged 145% in this period. One can also argue that Bilhete Único is a progressive public policy since it makes the total cost of commuting smaller for those who spend more time and distance riding - and those happen to be the poorest people, who live in São Paulo's outskirts and work at the city center.

One limitation of the proposed approach is that the long-term elasticities can only be calculated for one segment of the population that uses more than one bus for commuting. Nonetheless, it is useful in the terms said above - especially once one considers that the affected users are workers who live in São Paulo's outskirts and commute for two or more hours to São Paulo's downtown every day.

This work is organized in six sections including this introduction. In the following section, a revision of previous literature is conducted going over economics works to qualify the hypotheses and research questions. After that, the methodological section presents the econometric strategy to answer the questions of interest discussed above. The results are presented in the section so named and discussed thereafter while considering the consequences for public policy. The conclusion is the last section and summarizes the work alongside the presentations of caveats and suggestions for further analysis.

Figure 11 - Bilhete Único being used in a bus. Source: ViaTrolebus <http://viatrolebus.com.br/2014/07/recarga-do-bilhete-unico-podera-ser-feita-dentro-dos-onibus-de-sao-paulo/>. Credit: José Patrício/AE.

2 LITERATURE REVIEW

Transportation is immensely important in our daily lives. The act of moving is required for getting somewhere for doing something. It is also important at the aggregate level of one economy. For instance, Emile Quinet and Roger Vickerman (2004) point out that, besides its importance for people's lives, transportation is a production factor, since it is means for manufacturing, commerce, and the consumption of services. If analyzed as an economic sector, it accounts for approximately eight percent of European Union countries' GDP. Transport is recognized as central to economic activity from Adam Smith's works in 1776 to present works (De Palma, Lindsey, Quinet, & Vickerman, 2011b).

Given transportation's inherent importance in both individuals' routines and countries' economy, it is also massively explored academically. It is a subject covered by many fields of knowledge: it is a core subject for urbanism and human geography, vastly queried and modeled by engineers[6] and is popular among sociologists. These studies cover the subject and rely on diverse research lenses: human behavior, traffic simulations, social networks, and so on. In economics, while some authors emphasize the aggregate level and transportation roles in the productive chain, others opt to analyze the individual level. **The present work fits with the latter analytic approach since it relies on formulations of individuals' behavior to estimate bus price elasticity of demand in São Paulo. This information, however, is useful on higher levels, since it provides insights for transit policy design, particularly with respect to fare pricing. To provide sound evidence that might support policy analysis is particularly important for the transportation sector – especially in developing countries -** since the path adopted by policymakers and society is not always the best one.

Sturdiness of research is accompanied by methodological sophistication which have, in tandem, led to very robust results and theoretical progress. Many of them result in policy recommendations since transportation itself or the infrastructure required for its operation are often provided or regulated by the State.

---

[6] As much as it is by economists, whose concerns are usually with respect to production and consumption, that is, the equilibrium of supply and demand

However, looking closer at implemented public policies, it is possible to say that they struggle to follow academic recommendations (Quinet & Vickerman, 2004). Public policies of transport find themselves in the middle of complicated theoretical models and a "world which does not comply with its requirements for optimality" (Quinet & Vickerman, 2004, p. 341). Public collective transportation is a subfield affected by the same but even more critical adversity, since the difficulties related to it are exacerbated by problems intrinsic to public provision or regulation of goods and services: funding for infrastructure is even more scarce and decisions involve an almost unpredictable political process – which might be either good or bad for service itself. This is notably grievous in developing countries where state-built capacities[7] are far from the ideal (Bräutigam, 2008; Fjeldstad & Moore, 2007; Keen, 2012). Thus, there is a huge imbalance between what is recommended by specialists and academics, and what governments really do, either because of a lack of public will to change public transportation for the good, or because of inadequate public policies or deficiency of resources. Implemented policies for public transportation are then likely to be inadequate. These are part of the reasons that have brought the author to the present work, evidenced below.

Work's **motivation** emerged from the combination of (a) transportation's inherent importance for both individuals and society (including academia) and (b) unsatisfactory public policies in the sector, particularly in developing countries. These characteristics are even more important for public, collective transportation. Besides them, transit system is core for cities' sustainability, due to its power to mitigate congestion and transportation externalities per capita. Also, it is the only transportation mode that many people - specifically the poorer in developing countries – can rely on to move around urban areas and carry on with their daily activities.

This work's literature review has the following logic: it departs from a broad, general literature, in which the transportation topic is inserted. It then revises works of transportation economics and public finance, and then approaches the transit system's financing problem, which reaches the level of specificity desired to get as close as possible to the problem's details. The author hopes

[7] *State-building* is defined as the process that generates state's administrative, fiscal and institutional capacity of interaction and societal support achievement to accomplish public objectives in an efficient way through consistent and constructive actions.

that, by the time the reader gets there, the work's importance will have been shown. Hypotheses and research questions are then formulated and presented. A brief methodological literature review is made, arguing that the chosen methodology helps to answer the questions proposed.

It is organized as follows:

- First, the **Substantive review** evidences the problem dealt with in this work and revises the theory on transportation economics and public finance which supports it. Previous works that deal with the same subject are presented, hypotheses, and research questions are formulated. It is divided in the following subsections:

    - **Rationales for state intervention in urban transportation: general theory and the Brazilian case** briefly reviews the logic behind state provision or regulation of transit. The conclusion is that this intervention, if well designed, might lead the market to a higher welfare, in that market failures are mitigated. However, one problem is evidenced: state intervention must make transit both allocatively efficient and affordable without menacing system's fiscal sustainability. These are not taken for granted in literature. Allocative efficiency is a condition for transit's (and, therefore, cities') sustainability. Affordability is means for poor people's transportation and should be considered for social reasons.

    - **Fare pricing, efficiency and system's fiscal sustainability** reviews literature about achieving allocative efficiency through fare pricing. Previous works present the most efficient – that is, the one which provides the greatest possible welfare to society – way of pricing fare is to equalize fare price to its marginal cost. This is, however, hard to implement due to practical obstacles such as increasing returns of scale in transit systems. Consequently, fare's marginal cost frequently falls below the average cost, implying deficits in the system. This, summed up with the social necessity of keeping fare affordable, frequently entails state subsidies to fare. That is the case of São Paulo, where fare subsidies soared lately. But does such subsidization succeed in keeping transit affordable and mitigating transportation's negative externalities in a way to recover its cost to society? Works that deals with this are reviewed to show which part of literature defends subsidies.

  - Since the present work does not intend to answer whether subsidies in São Paulo are cost-effective, the subsection **Other forms of mitigating negative externalities through ridership increase** revises other ways of transit enhancement in that to keep cities sustainable and to make transit possible for people. The questions approached here are used in the Discussion section to present possible policies that can help in this task.

  - **Research hypotheses and questions: transit price elasticities calculation in literature** puts present work's objectives in perspective, based on what has been found up until now in the literature. It also present hypotheses raised during the literature review process. It also discusses how the findings affect public policy, as a basis for the Discussion chapter.

- In the last subsection, **Methodological review**, the offered model's subjacent logic is presented along with its epistemology and ontological features for the sake of providing the reader with the necessary basis for better understanding its formulations. It also provides insights about individual's choice process. Its purpose is to argue that the chosen methodology answers the questions proposed and adequately approaches the hypotheses brought up in this literature review.

2.1 Substantive review

2.1.1 Rationales for state intervention in urban transportation: general theory and the Brazilian case

In the foreword of *A Handbook of Transport Economics,* a transportation economics manual edited by André De Palma, Robin Lindsey, Emile Quinet and Roger Vickerman (2011), Nobel Prize winner Daniel McFadden shares his thoughts about transportation as a field studied with economic lenses and the principal attributes of such field (McFadden, 2011).

His opinion is that there is a specific subfield of "economics of transport" in the sense that the problems addressed by it – as pointed out by Quinet and Vickerman (2004): supply, demand, and regulation - are fundamentally of economics. But why is it so special? De Palma, Lindsey & Quinet (2011b) claim that “the transport sector holds a special place in economics for a number of reasons. For example, several basic concepts that are widely used in economic analysis originated from the study of developments and policy issues in transport” (De Palma, Lindsey, Emile, et al., 2011, p. 1). McFadden (2011), for instance, claims that transportation is special not only because of subjective importance given its economical magnitude in every country or contributions for other fields, but also because its idiosyncrasies might require adjustments to usual models.

The most remarkable specific features that bring on these adjustments are space, time, the multiplicity of agents' decisions, and the “relationship between the public and private sectors in the provision and management of transport” (2011, p. xvii). The following paragraphs explore in detail the importance of each one of these characteristics.

Regarding *space*: activities are located in territories. Gaps between them affect the costs related to these actions and, therefore, influence land cost and generate spatial inequalities. This, consequently, influences cities’ formation and their growth until they become metropolises. There is also a great impact on an individual’s and firms’ location and movement in space. Space is, then, an attribute that might change the production and consumption of transportation.

This is emphatically stated in seminal works such Edward Glaeser's (2011) and Jan Brueckner's (2011), whose cited work schematize an urban economics model known as Alonso-Muth-Mills[8]. This urban economics model provides a good understanding of individual's actions regarding transport and its effects over cities' formation – and, therefore, its importance in a World which gets more urban every day. In this model, there is one core factor: space.

Basically, Brueckner states that cities form themselves due to job location. The main rationale here is that jobs are concentrated within a territory and, then, residences locate near those spots, forming and enlarging cities. Spatial concentration of jobs, in its turn, is explained by two major forces: *scale economies* and *agglomeration economies*.

The first regards the fact that specialization leads to more productivity and economic gains due to labor division. A larger firm produces more output per unit than a smaller one since larger scale procedures are more efficient. Increasing returns of scale "can explain the formation of 'company towns', but it cannot explain how truly large urban agglomerations arise" (Brueckner, 2011, p. 4). This is where the second feature, *agglomeration economies* enters the scene.

They are divided in *technological* and *pecuniary, the first* referring to the gains that happen when some innovation makes production more efficient and *the latter* are economies generated when inputs become cheaper, with no effects over productivity. Both tend to happen in cities since these gains arise when firms are located in close proximity. Easiness of hiring specialized workers, competition and/or cooperation between firms, and knowledge spillovers are a few examples of agglomeration economies.

Transportation cost savings within pecuniary-agglomeration economies are particularly singular in that final product shipping cost is (very likely) reduced when a firm locates in a city - closer to its final market, workers, and, perhaps, to its inputs supplier. Those forces not only explain cities formation and enlargement, but they also explain why many cities are monocentric (a town in which employment is mainly located close to the city center) as firms tend to concentrate where

---

[8] Since it was originated in 1964 William Alonso's work, 1969's Richard Muth and 1967's Edwin Mills. William Wheaton and Brueckner himself in 1974 and 1987, respectively, worked in this model's derivations.

others are already located (Brueckner, 2011; Quinet & Vickerman, 2004) – a kind of agglomeration economy which emerges within a formed city.

In the classical model of cities, all jobs are supposed to be concentrated in a central point (the Central Business District, or CBD), and, given that people will commute to the CBD to get to their jobs, they will choose where to live through maximizing a welfare function in which commuting time is quite important. Individuals' utility is supposed to be a function of money spent on three goods: rent, commuting, and all other goods consumed.

The further homes are located from the CBD, the more expensive it would be to commute to work – given the assumption that transport costs are a function of distance which is linear and increasing with distance. Since people would spend less time and money in transportation if they live closer to their jobs, they agree to pay more for housing. Then, people either demand less housing (in household size) or dwellings become cheaper in the outskirts (Brueckner, 2011; Quinet & Vickerman, 2004).

If applied to the real world, the model explains why houses are smaller in cities' old towns, higher density areas in city centers, part of the formation of suburbs in the United States, and urban sprawl in the recently formed megacities: families decide to place their residences where housing and transportation are jointly affordable and, somehow, maximize their welfare – whatever it is and however it is assessed and calculated rationally or not.

Thus, both transportation costs and concentration of jobs have a central role in urban economics and cities' formation. They are directly influenced by the role of territory in human activities and, then *space* is key for both cities formation and subsequent urban transportation.

Back to McFadden's (2011) foreword, *time*, the next characteristic, is seen as an attribute of all consumption, but is even more important for transport, since it is a feature of the act of moving itself. That is, time is a necessary attribute and a limited resource for the very act of moving around. In the same way that one has budgetary restrictions for buying goods and services, time might limit one's consumption of transportation. Also, this factor could have been employed in other activities. These might alter individual's decision process, the quantities of the service that is offered to consumers, and its price.

The role of the next attribute (*individual's decision process*) in transportation unfolds in two parts: the choices regarding *transportation consumption* itself, as mentioned above, and those that are part of *other consumption decisions* - since transportation is necessary for (or a consequence of) other activities. In other words, the time consumed when moving around and the time that is necessary to be spent to access other goods or services to be consumed. For instance, one must choose where one is going to live. A person’s home location is, therefore, influenced by commuting costs to her workplace, as it is expressed above. This long-run decision unfolds itself in many short-run, daily choices: to buy a car or to use transit, what time to leave, whether to park on the street or to pay for a spot in a private parking lot, etc. Also, in order to purchase goods or services, someone (and/or the thing consumed) must get somewhere. The act of getting somewhere costs an individual’s time and money, and this should be considered alongside with one’s budgetary constraint. That is,

> decisions relating to transport are part of a much wider set of decisions relating to the choices between a range of activities, or to the sequential decisions determined by experience or memory, all filtered by psychological attitudes (McFadden, 2011, p. xvi)

The role of decisions is particularly important for this work’s methodology since Discrete Choice Models rely on a few assumptions that are related to the choice process. This is explored in detail in the methodological subsection of this literature review.

The last special attribute mentioned by McFadden is the *relationship between the public and private sectors in the provision and management of transport*, which is said to be deeply explored in transportation economics literature. In other words: State is an active, usual player in the market – and this relationship is frequently analyzed in this topic’s studies.

As mentioned by Roy Bahl and Johannes Linn (1992), transit market for instance hardly has free entry in practice. According to McFadden, there are two main explanations for this usual intervention:

> The first relates to the importance and nature of externalities, in particular congestion externalities which are endemic to transport. It is thus the role of the regulator or state to take measures to control the undesirable effects. Such measures can include policies on prices or quantities, changes of legislation, and the use of new information and communication technologies such as flexible pricing based on current or forecast levels of aggregate usage. The second explanation arises from the fact that for several reasons, both institutional and technical, public authorities are deeply involved in the supply of transport services. From this has arisen the development of public–private partnerships as well as the need to consider imperfect competition, indirect taxation, contracts and regulation under asymmetric information (McFadden, 2011, p. xvii)

That is: transportation is a vastly explored field since the State is frequently involved in its provision or regulation, both of which call for further studies in the area. Externalities seems to be one of the main rationales for state intervention - as mentioned above - but this is actually just one of many market failures that entail government intervention.

The term “market failure” is usually used to refer to cases in which an unregulated market fails to meet its greatest possible outcome (Sobel, 2005). These failures are usually inefficiencies in market products that are objective, positive, and measurable - and not the subjective, normative criteria, which usually has to do with equity (Bator, 1958). Bator depicts the usual market failure[9] definition as

> the failure of a more or less idealized system of price-market institutions to sustain ‘desirable’ activities or to estop ‘undesirable’ activities … The desirability of an activity, in turn, is evaluated relative to the solution values of some explicit or implied maximum-welfare problem. It is the central theorem of modern welfare economics that under certain strong assumptions about technology, tastes, and producers’ motivations, the equilibrium conditions which characterize a system of competitive markets will exactly correspond to the requirements of Paretian efficiency. (Bator, 1958, p. 351)

In other words, there is a preconceived production level of an economy – implied in a positive, quantifiable, welfare-maximization function that might be constrained to a Paretian optimality

---

[9] Later in his work, Bator states that the static “market efficiency is neither sufficient nor necessary for market institutions to be the preferred mode of social organization. Quite apart from institutional considerations, Pareto efficiency as such may not be necessary for bliss” (Bator, 1958, p. 378).

condition in which is impossible to reallocate resources without worsening at least one individual's situation - that might be not achieved due to undesirable factors. In such cases government intervention might lead to a higher outcome. In the transportation market this higher outcome given state intervention, might arise because of three reasons – one of them being closely related to externalities (Quinet & Vickerman, 2004).

First, transportation tends to be *fragmented* since a free-market is less cohesive because of asymmetry of information and high transaction costs. Governments, on the other hand, can provide some rationality that leads to more *integration*, which is desirable.

Second, as natural occurrences in *economies of scale*, *monopolies* might arise, appearing in the provision of transport services and infrastructure. In this case, the monopolist will produce a smaller level of output than the competitive, welfare maximizing, market combined.

Third, and here we are back to market failures, *externalities* generated by urban transportation might be exacerbated by congestion, which is tightly attached to a high use of private transportation modes (namely, cars, whose users almost never internalize those costs). For instance, since congestion charges are difficult to implement, states' intervention in transit is a second-best for car user's internalization of its costs. A high rate of car usage is more likely to happen in the absence of a good and functional collective transport - usually failed to be efficiently produced by a free-market, bringing the economy to a final output that is inferior to the optimal one.

In Brazil,[10] state intervention in the transportation sector is set in the Constitution. Transportation is one of citizen's rights[11] (BRASIL, 1988, Art. 6º). It is of municipal responsibility to organize and provide, directly or under concession or permission, the public services of local interest - including transit, which is said to be essential (BRASIL, 1988, Art. 30º).

---

[10] Brazil's form of government is that of a federative republic (composed of 26 states - plus a federal district - and 5,570 municipalities).

[11] Actually, it was not listed as one of the fundamental entitlements until 2015, when one constitutional amendment in this way was made by Congress and ratified by the Senate.

Central government, in its turn, should establish guidelines for urban transportation – such as for urban development, housing, and basic sanitation (BRASIL, 1988, Art. 21º). The guidelines made by the central government, as demanded by the Constitution, took 20 years to be regulated, passed as a law in late 2000's. Gomide (2008) shows that the factor which put public transportation on the Federal Government's agenda in the beginning of the 2000's was concessionaire's financial crisis – and not social needs entailed by urban, social and demographic changes during the second half of 20th Century that caused urban sprawl (pushing poor people to the outskirts) and, consequently, more demand for transportation.

"Organizing and providing", however, does not mean that governments provide citizens transit for free or subsidize it, nor that the quality of transit is one of the government's main concerns. According to Carvalho and Pereira (2011), São Paulo is one of the few cities in Brazil which can afford directly subsidizing its bus service, while the other municipalities simply price the fare by dividing system's total cost by ridership[12], a form of *uniform tariff* (Bahl & Linn, 1992), which will be discussed further in the present work.

Brazilian local governments provision of urban collective transportation meets a problem addressed by Gwilliam (2002). According to him, there is a “paradox of urban strategy” in that there is an excess of transit demand over supply, even though there is a huge involvement of private suppliers and governmental interests on the sector. Different the supply of other services’ (such as telecommunications) good quality, and low costs are not found in the provision of collective transportation, especially in developing countries.

The current paradigm in transportation economics addresses these questions by recommending that urban transit provision should be allocatively efficient, and fare must be affordable. Gwilliam (2002) reports this view by what he calls “the World Bank’s suggestions”, which is summarized as follows:

---

[12] Generally, rapid transit in metropolitan areas is subsidized by States, but only a few cities in Brazil have rapid transit infrastructure. Also, there are laws in Brazil that guarantee a free pass for retired people and students, which might be subsidized by the government or included on fare price and paid by all other users.

> Public transport is for all. Concentrating on the transport modes of poor people in middle-income countries essentially means the provision of affordable forms of public transport, both formal and informal. But it should not be viewed as only for the poor, as the importance of public transport to all income groups in many rich European cities demonstrates. Improving efficiency in public transport must be concerned not only with keeping costs down but also with providing a flexible framework within which the less poor as well as the very poor can use public transport with confidence and comfort. (Gwilliam, 2002, p. xiv)

There is also evidence of a lack of governmental policies that properly provide or stimulate the provision of good and inclusive public transportation in Brazil (Gomide, 2003). This is reflected in plunging ridership that deteriorates transit quality even more (Carvalho & Pereira, 2011). This is summed up as a lack of efficiency, quality, and affordability.

Thus, the main issue raised in the literature is how to promote (allocative) efficiency and affordability to a transit system, without jeopardizing that transit system's fiscal sustainability. The first might be addressed through adequate fare pricing, which is approached in the next subsection. The second and third entail a discussion about subsidies and other policies which can enhance transit ridership, approached subsequently.

2.1.2 Fare pricing and system's fiscal sustainability

When transport economists refer to *efficiency* in transit system, they usually mean allocative efficiency. That is, welfare maximization, where every unit that is produced is therefore consumed and no one suffers from deprivation of consumption, as it was presented before in this literature review. This is how this term shall be understood in the present work.

Once a good part of a transit service is under state responsibility, it is fair to ask whether it should or should not be provided free from direct charges (therefore completely funded by State). One of the greatest concerns in the literature is how to provide this service so the most welfare is achieved with least expenditure. The first direct recommendation is that transit should not be provided free of charge. The most part of economics literature says that if a public manager wants to achieve allocative efficiency in urban services provisions by governments, user charges - a direct price for a service or good which is paid by the identified user of it – must be considered (Bahl & Linn, 1992; Bird, 2001).

User charges are said to maximize allocative efficiency because they rationalize the usage of a scarce resource. Bird affirms that "the main economic rationale of user charges is (…) not to produce revenue, but to promote economic efficiency" (Bird, 2001, p. 172), in that people will over-consume a scarce resource if it is funded by general taxes and free of charges.

In detail, people choose to pay for a good or service until its marginal cost equals the marginal benefits of consumption. If citizens are not directly charged when pursuing a good or service, they do not perceive the cost they pay for a good (i.e., when it is financed by general taxes) and the cost of consuming an additional unit is virtually zero. Underpricing or the provision of goods or services “free” is then tightly associated with over-consumption. This represents a problem because of the very simple reason that resources are scarce, particularly in developing countries.

Although literature recommends that user charges must be employed for optimality’s sake, it is not a sufficient condition for its achievement. Bahl and Linn dedicate a chapter of their seminal book *Urban Public Finance in Developing Countries* (1992) to discuss issues that exist in public services pricing, and user charging is included.

They list four main concerns in the pricing of urban services that affect the provision of goods and services, from pricing form to quality, and, therefore, **should be considered together**: *efficiency*, *fiscal constraints*, *equity and growth* and *political and administrative feasibility*. These principles can be applied to any kind of public service and, therefore, to public collective transportation.

On *equity and growth* questions, the authors assert that income distribution should not be the first objective when pricing urban service. Nevertheless, they allege that it can determine low-income people's access to fundamental services and hence can help achieving a more equitable country.

On the *importance of political agents, institutional setting, and historical characteristics of the matter of public service pricing*, the authors share the vision of the public finance school that sees the State as an arena in which agents act exerting influence over one another (Backhaus & Wagner, 2005) – and over many extensions of policies, from budget formulation to execution. Public policy is naturally developed involving multiple actors and interests on user charges would not be different. Bahl and Linn highlight six groups of actors usually identified on this topic: (1)

beneficiaries, those who use the service and, naturally, have interest to pay the less for it, (2) non-beneficiaries, those who do not use the service but are general taxpayers and thus might prefer its self-financing, (3) managers of public service, (4) local politicians, (5) higher (national) level authorities and (6) international institutions, particularly active to provide loans (or grants) and technical expertise to developing countries. The first group generally exerts pressure over the managers to pay as little as they can, while the second often does not has incentives to do so - or is poorly articulated. Managers and politicians, seeking, respectively, influence and patronage provision to satisfy the electors, might have incentives to expand service as rapidly as they can keep prices as low as possible[13].

On *efficiency*, Bahl and Linn share literature's view that has been presented before in this subsection. The provision of public goods or services must be charged from the identified consumer or user. There is, however, a furthest discussion of user charging: how will do goods and services be priced? That is, at what price should the good or service be provided?

Literature usually recommends marginal cost rule. That is, goods or services prices (user charges) should be exactly service's marginal economic cost[14] (Bird, 2001). This would enable the achievement of a socially desirable level of prices – that is, one through which welfare would be maximized as no one is excluded from consuming, and prices would be at the equilibrium level, where allocative efficiency would be reached:

> The justification of this rule is that welfare is maximized when the benefit of an additional unit of the service to the consumer - which is reflected by his willingness to pay the price - is equal to the cost of producing this additional unit. (Bahl & Linn, 1992, p. 241)

---

[13] There is, however, theories that see the State as a revenue maximizing agent, as described by Rodden (2003). The present work does not review this discussion and see the budgetary question as a result of many interactions between actors, that can either raise or diminish levies, depending on interests and political power.

[14] Economic cost as referred by Bird is not the same as accounting cost. While the latter reflect *"identifiable monetary outlays incurred in the process of carrying out a particular activity"* (Bird, 2001, p. 173), the first also takes into account opportunity costs, for both the resource and the input. Nevertheless, this is more a rationale for theory than a practical recommendation, since it is very difficult - if not virtually impossible - to calculate economic prices empirically.

Although marginal cost pricing rule is clearly the scheme of taxation recommended by literature because of efficiency reasons, the authors make a disclaimer about its application. Marginal cost pricing rule requires a few assumptions which are hard to find in practice: perfect information, zero economic surplus, and the absence of externalities, distortions in the economy, and transactions costs. Besides these, there is another characteristic of it that is **a necessary condition for marginal cost pricing rule to be truly efficient: demand cannot be perfectly price-inelastic and should respond to price changes** (Bahl & Linn, 1992; Bird, 2001)**.**

The following excerpt is essential for the present work. Bird says that

> [u]nless there is *some* responsiveness to prices, such charges serve no allocative or efficiency function. If there is no price responsiveness – the demand for the service is virtually inelastic – any price charged for the service will simply “tax” those who use the service. User charges may of course still be justified in terms of fairness in such cases. Why should the cost of providing a service, even a mandatory one, be paid for by someone other than the user? (…) But such charges have no efficiency rationale because, by definition, they do not affect choices or activities. Determining appropriate user charges when demand is inelastic is thus a quite different task than in more normal circumstances. (Bird, 2001, p. 174)

This is important for the present work since marginal cost pricing rule is widely recommended for public services pricing, and there is a claim for efficiency increases in the provision of transit in Brazil. Nevertheless, the rule should only be applied if demand is quite elastic and that is what the present work aims to answer.

Even though those aspects impose several limits to the rule's application, Bahl and Linn argue that to be aware of it still provides relevant implications, such as drawing government attention to economic costs (and not to sunken or "historical" costs), and to the necessity of price readjustment during inflationary periods. There are three other aspects to that the present work can be directly related to.

First, when demand is almost perfectly price inelastic, to change the price of an incorrectly priced service causes losses of welfare in that the response of operational costs can take a long time to adjust to new levels of demand, what causes oversupply and fiscal imbalance.

Second, particularly in developed countries, there might be incentives for government to expand services beyond what is necessary in order to achieve more influence and/or votes. In developing countries, however, there are also political and spurious incentives to do so, but it is also important - especially in a big metropolis - to price services properly to keep public and private competitors on an equal footing.

Third, if there are easily identifiable groups of users, welfare can be maximized through distinct prices among them. One example is the provision of a good which must be transported and, thus, cost is different depending on where consumers live. For instance, water provision, a service in which the provision of pipes is far from being costless and grows with the distance of the place where it must be provided. The other case that allows groups to be identified is when the consumption pattern of a clusters is clearly different even though the price is the same for them. That is, the price is the same for everyone, but there are different behaviors associated with demand, and these individuals share some common characteristic. Implicitly, they have different demand elasticities for that service. This is the case when the government can act as a "discriminatory monopolist" and guarantee a higher welfare through pricing groups differently and this principle works better if one group has a perfectly inelastic demand and the other does not. Bahl and Linn say that, usually, richer people tend to have a more inelastic demand for public services, that means that there might be room for extracting a higher price from this group if this is socially desirable.

Fiscal constraints, the other concern on public services pricing, emerge when pricing rule leads to deficits. It happens essentially when marginal cost is below average cost, either caused by the decline of average costs with service growth (economies of scale) or capital indivisibility (capacity excess). Simply, the cost of providing one extra unit of transit is so low that the consumers would virtually want to pay zero for it.

Economies of scale are demonstrated to be present in transit service by Herbert Mohring (1972). This is now known as *Mohring effect*[15]. These cases are the classical leads to fiscal deficits in

---

[15] The Mohring effect is the fact that in bus systems, if the frequency increases with demand, a rise in demand will, counterintuitively, decreases passengers waiting times in bus stops. That happens because dispatch increases and, then, waiting time shortens.

transit system. Nevertheless, there is another deeper factor that might lead prices to fall below the average marginal cost, and therefore, causes economic deficits: the occasion when users' non-monetary costs (i.e. time) are considered.

Basso and Jara-Díaz (2012) schematize a model replicated here for didactic reasons[16]. In Figure 12, below, AC_T is the average cost of transit (that is, the total cost divided by the ridership), MC_T is the marginal total cost and AC_{TU} is the average cost for transit users (which is composed of waiting, access, and in-vehicle times). $X(Y_T)$ is the inverse demand function (that is, ridership). The general cost of transportation for users is $AC_{TU} + P_T$, where P_T is the value of transit fare.

The efficient generalized price of transit would be in the point where $X(Y^*_T) = MC(Y^*_T)$. However, since part of the costs fall on the user, the optimal fare the equilibrium happens at $X(Y^*_T) = MC(Y^*_T) – AC(Y^*_{TU})$, that is, naturally, under the average total cost AC_T. The basis for this formulation also derives from the work of Mohring (1972).

This leads to two possible fare rules: (a) one in which a subsidy, S, of magnitude AC_T - MC_T is introduced and (b) the other in which price is set at AC_T.

The first, subsidies, are generally financed by general taxes funding. The second might appear in the form of a *multipart tariff* (in which the fare is composed by a lump-sum part and a variable part in that the deficit is covered by part of users) or a *uniform tariff*, which abandons marginal cost rule and price the fare by its average cost (Bahl & Linn, 1992).

Although Bird (2001) calls uniform tariff "obviously inefficient" in that it might lead governments to miss the opportunity to allocate resources optimally, this is the most common practice in Brazil (Carvalho & Pereira, 2011). Nonetheless, both uniform tariffs and subsidies by general funds are said to be sub-optimal conditions (Basso & Jara-Díaz, 2012).

---

[16] Because the formulation is not originally used to analyse fare pricing policies exhaustively, neither work's results suggest that transit should be priced at its marginal cost. Nonetheless, supposing that user costs should enter in these formulations is plausible. Hence, it is useful to present a marginal cost scheme that address the question such as made by these authors.

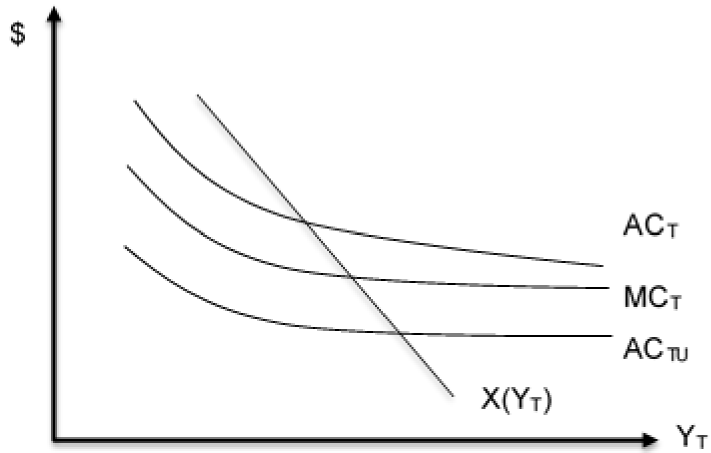

Figure 12- Transit marginal cost pricing rule and optimal subsidy. Source: Basso & Jara-Díaz, 2012, p. 892.

According to Bahl and Linn and Bird, general fund financing might be acceptable if the following conditions are met: a deficit still exists after marginal cost pricing and if the fiscal deficit does not affect investment and management decisions. There is also a social concern: is it fair and/or socially desirable to fund someone's consumption/access to service? One other issue pointed by the authors is that general fund finance is only acceptable if taxes cause no distortion to resource allocation.

Distortion-free levies are virtually nonexistent and thus the question becomes whether general fund financing causes a smaller distortion than the marginal (or full) cost of service pricing[17] or, moreover, whether tax inversions cause a product increase higher than the welfare reduction caused by levies. These questions add to Bird's (2001) opinion that positive externalities (or the mitigation of negative ones) caused by the provision of a service can justify government subsidies.

[17] The latter might be advantageous to bear governments in mind of a service affordability when they are designing the systems - that is, governments should never design services which the population cannot afford.

As a counterpoint to Bahl, Linn and Bird, who defend marginal cost pricing and theoretically discourage general funds financing of public services or goods, there are works in transport economics that have found economic gains due to transit subsidization[18].

For instance, Basso and Jara-Díaz (2012) argue that financing public transportation with congestion tolls is feasible. The logic is the following: they suppose that total demand (private and collective, jointly) for transportation is inelastic, such as it happens in peak hours. That is, people will move anyway, they can only choose which modal. Hence, there is an optimal modal split, a division where no one is excluded, and society achieves the highest welfare level possible. Thus, a calculation of a congestion toll charged from cars whose funds go integrally into transit financing is automatic. Congestion price is then totally dependent on transit price since the difference between the two costs is what matters. "It is possible to establish prices that, while keeping the optimal price difference, make the total subsidy for transit equal to the total congestion tolls collected" (Basso & Jara-Díaz, 2012, p. 898). This proposition relies on what is called the *Lewis-Mogridge position*[19] (that, in turn, relies on the *Downs-Thomson hypothesis*[20]), which has been evidenced by Martin Mogridge (1997) in an analysis of the city of London: the speed of door-to-door travels tends to be very similar or equal among modals during congested hours.

Parry and Small (2009) seek to evaluate whether urban transit subsidies should be reduced. They propose an analysis of cost (subsidies) and benefit (welfare gains due to externalities mitigation)

---

[18] This research line mostly draws on Mohring's (1972) formulations of a transit cost that includes users' costs. "Transport demand can be dealt with as if the price of a trip equals whatever fare is charged plus the money value the traveler attaches to the time his trip requires" (Mohring, 1972, p. 591). This is the basis of what was said above with respect to marginal costs always falling below average costs.

[19] *Lewis-Mogridge position* explain the fact that more road infrastructure does not alleviate congestion, in that building more roads will increase private modes' attractiveness and demand will shift from collective modes to cars. The new equilibrium position will have more cars in the streets, therefore worsening traffic flows.

[20] *Downs-Thomson hypothesis* (also known as *Downs-Thomson paradox*) states that, at equilibrium, the generalized cost of transportation (that accounts for time and out of pocket costs) is the same for car and transit. This is said to be a "golden-rule" of urban transport, that quality of peak hour (that is, inelastic demand) travels is the same for cars and transit, since there are always people who will be indifferent from one mode or another and those will choose the one that is marginally better.

of transit subsidization. The authors make use of "tractable formulas for the welfare effects of fare adjustments in passenger peak and off-peak rail and bus transit, and for optimal pricing of those services" (Parry & Small, 2009, p. 700). These formulas account for transportation externalities, such as congestion, pollution, accidents, scale economies and service level adjustments. After applying the model to 50 American cities, the authors find a result that goes categorically against those who presume that system deficits should not be subsidized. Their results show that subsidies are welfare improving at the margin across modes, and this means that the total output of cities is maximized due to congestion alleviation, especially during peak hours. Nevertheless, Parry and Small (2009), implicitly assume that transit demand happens due to the (subsidized) price, since they estimate the social benefit that transit usage generates, and they do not care about the fact that those users not necessarily would change modes if price change. That is, they ignore price elasticity of demand. If demand is inelastic, this estimation is less precise and might change author's final conclusion. There are other works that seek models to theoretically overcome the Downs-Thomson paradox through subsidies and/or transit authorities' strategies, such as Wang et al. (2017) and Zhang et al. (2014).

Thus, there are clearly two research lines that were identified during literature review. One - here mostly represented by the works of Bahl and Linn (1992) and Bird (2001) due to their didacticism and exhaustiveness – that clearly recommends user charges and marginal cost pricing rule to provide allocative efficiency to public goods or services provision and do not recommend general funds to subsidize state delivery of services. These authors do not draw specifically on transit and urban transportation, but they do mention that the basic principles are the same for any kind of service. They do, however, mention that peculiarities of each kind of service must be considered for such decisions. On the other hand, there is a research line that specifically treats about transit pricing and relies on Mohring's (1972) formulations, which include users costs into system. It is worthy to mention: as McFadden (2011) well notes, transportation peculiarities (such time involved in consumption) might change general theoretical constructions. A few papers – represented here by Basso and Jara-Díaz (2012) and Parry and Small (2009) - suggest that cross-subsidies between private modes and transit might even increase total welfare through the mitigation of externalities. Nonetheless, these authors also do not directly suggest general levies

as a source of transit funding. There is no consensus in this discussion (Sun, Guo, Schonfeld, & Li, 2016).

It is worth to mention that, regarding transit, there are works who advocate for a totally "free" public transportation, such as Cools, Fabbro, & Bellemans (2016). These authors justify that there might be a "zero-price effect" that induce people's usage of transit in a more intensive way than just reducing fares – what they also say to be ineffective in shifting users chosen modes. Nevertheless, these authors do not care about the problems addressed by most of the literature reviewed in this subsection (i.e. rational use of scarce resources and optimization of both public funds' use and economic welfare). User charges and adequate fares seems to be a must in public finance and transportation economics literature.

Transportation literature also says that to price fare adequately means not only to promote efficiency but also to keep transit affordable. This is part of a paradigm that emerged in the beginning of the 2000's, which supplanted a previous one, more focused on financial and economic viability, and optimality, and less on cities' sustainability – which depends on transit strength, competitiveness, financial sustainability, and adequate governance and management (Carruthers et al., 2005).

Carruthers, Dick, & Saurkar (2005) refer to *affordability* of public transit as **whether "the financial cost of journeys put an individual or household in the position of having to make sacrifices to travel or the extent to which they can afford to travel when they want to"** (Carruthers et al., 2005, p. 1). The concept of affordability itself might be seen as a degree of transit accessibility in that it jointly incorporates users' income, quantity of travel, and fares' level. This concept is particularly important for developing countries such as Brazil, where the commuting of the poor depends on transit (Carruthers et al., 2005; Carvalho & Pereira, 2011; Gwilliam, 2002).

Therefore, one of the main justifications for urban transit subsidies is to keep fare affordable. Nevertheless, the positive effects implied by this affirmation are not taken for granted in literature. Nicolás Estupiñán et al. (2007) point out that there are virtually no quantitative assessments of subsidies distributional incidence, making it very hard – or impossible – to determine whether subsidies are pro-poor and increase affordability.

The authors also note that reasons for transit subsidization are not clear in developing countries – hence, policy suggestions with respect to fare are hard to find in works that deal with those places. Also,

> it is not clear that transport subsidies that operate in isolation from more general welfare programs have been effective in alleviating the problems faced by the poor. Whether they can be made more effective is an open question, but current policies are not achieving this goal. (Estupiñán et al., 2007, p. 3)

In this sense, the authors say that subsidies are usually justified with allocative efficiency arguments, such as presented in this subsection: transit subsidies are a second-best option for congestion pricing, a must when scale economies and users' costs are considered and – more specifically for developing countries – a social or distributive policy.

Nevertheless, the broad view about subsidies for public services, in which transit service is included, might be summarized by the following excerpt:

> the appropriate initial position in formulating sound public policy is that any public service with an easily identifiable direct beneficiary should be paid for by that beneficiary, unless sound and convincing arguments in favor of a particular degree of explicit public subsidy can be produced. (Bird, 2001, p. 178)

Bird is not referring to affordability. A "convincing argument" for subsidization highlighted by Bird is positive externalities caused by the consumption of a good or service. He mentions that if a service or good causes positive externalities when consumed, its provision by a public agent would bring a higher welfare to society and can even help reach a socially optimum level of its consumption - thereafter analyzed by transportation literature and presented before in the present work. These arguments are used to legitimate state market intervention and also work for the reduction of negative externalities.

To mitigate negative externalities is also one of the rationales that Parry and Small (2009) list for the typical justification of transit subsidization - besides the model in which subsidies are always necessary if one considers that marginal cost of supplying this service is usually less than the

average cost, due to scale economies and users' costs contemplation, as explained previously in the present work. It is generally believed that a lower fare discourages automobile use, what mitigates congestions and negative transportation externalities (such as car crashes, air pollution, productivity losses, and so on) per person.

Estupiñán et al. (2007) also refer to transit as a reducer of negative externalities as one of the main justifications for the ubiquity of public transportation subsidization[21].

If transferring money to poor families is one of society's goals, São Paulo's bus subsidies seems to be adequate and a good welfare-seeking intervention. However, it is worth to remember Bahl and Linn's (1992) comments on institutional settings. Bird (2001) seems to agree with them in that he states that how a service is priced by government happens less on rational basis and more because of a "historical accident and administrative convenience" (Bird, 2001, p. 171). Recently, a situation occurred in São Paulo might corroborate to this point.

In June 2013, São Paulo's local government raised bus fare by 6.6% (BRL 0.20 or USD 0.061). In a few days, big demonstrations against the readjustment became riots and the government - even after arguing that keeping the fare at BRL 3.00 would require budgetary reallocation, removing money from other areas - backed down. Tartaroti (2015) argues that these events achieved one of its organizers' original - yet implicit - objectives: transit prioritization over other policies, such as health and education. Keeping in mind that the events of June 2013 are a good example of a historical accident, user charges topics are more important to set a rationale on how the service is (or should be) financed than for a deep analysis of it.

Although analyzing bus subsidies in São Paulo with political lenses seems to suit well the reality – and one can find works that proceeds from deep historical analysis (de Vasconcellos, 1999) to attempts to understand how suppliers of transit (concessionaries) and criminal organizations

---

[21] Nevertheless, the authors point out that there is a social choice implied in this argument: transit is more important – or at least as important as – that other policies such as education or healthcare. If so, transit subsidies are a form of achieving "society's goals relative to a money transfer to the head of household" (Estupiñán et al., 2007, p. 8).

operate to influence official or shadow transportation (Campos, 2016; Hirata, 2011) – the present work is limited to the analysis of this problem with economics lenses.

Thus, the main rationale used for the justification of transit subsidies is to reduce negative externalities – or, more adequately, to mitigate the negative ones caused by transportation itself and especially by the high usage of individual modes – a typical second-best policy for congestion charges. Nevertheless, there are a few questions to be addressed by literature in that to check whether subsidies are effective to achieve this goal, such as the one mentioned in the following excerpt:

> if the actual consumption of transport services is the justification for sectoral transport subsidies, then more emphasis needs to be placed on measuring the number of trips undertaken by individuals and how certain policy interventions affect this quantity, including studies to determine the elasticity of demand with respect to fare pricing. (Estupiñán et al., 2007, p. 8).

There are also concerns on equity and affordability and those must be considered in a public policy (political) decision. The present work, however, is limited to answer whether subsidies in São Paulo seems to be functional for a ridership increase in order to reduce negative externalities through transit usage. Nevertheless, there are other policies that might help to increase ridership. They might be used isolated or jointly with fare prices policies. A few are reviewed next.

2.1.3 Other forms of mitigating negative externalities through ridership increase

Besides works that approach a transit ridership boost through fare pricing, there are others that have analyzed other forms of transit strengthening. The following paragraphs review case studies or simulations that try to find such cases and their subsequent or theoretical impacts.

According to Basso and Jára-Diaz (2012), “the two most popular ways to deal with congestion that have been suggested in the literature are congestion pricing and giving priority to public transportation” (Basso & Jara-Díaz, 2012, p. 890). The latter is a typical second-best policy, the first being the optimal one (Bahl & Linn, 1992; Basso & Jara-Díaz, 2012). As mentioned before in the present work, policies that try to enhance transit ridership are usually a second-best for congestion pricing.

In a theoretical framework highly stressed by economists, the proper way to price congestion is to charge each new user the cost that his or her entry adds to other users' prices (Knight, 1924; Pigou, 1920, as cited in Albalate & Bel, 2009). Thus, the price paid by each user would equal the marginal cost faced by all users, not his or her own cost only. Although it would be better to directly tackle the causes of the socially undesired effects of congestion - that is, charging for the negative externalities caused by congestion - this have always been hard to implement due to either technical (Basso, Guevara, Gschwender, & Fuster, 2011) or political reasons (Albalate & Bel, 2009).

Nowadays the technical questions are supplanted, and political feasibility is found in policies such as urban tolls that are found in cities around the world (Albalate & Bel, 2009; Quigley & Hårsman, 2010). Also, there are other effective policies of congestion charges, such as parking fees. Gillen (1977) show that one effective kind of congestion pricing is to charge for parking times (and not quantity), and that this might be more effective in discouraging automobile usage.

Also, welfare might increase because of tax inversions and mitigating externalities (Basso & Jara-Díaz, 2012) after congestion charging being applied. Models followed by empirical tests are frequently found in the literature (Basso et al., 2011; Haddad et al., 2017; Haddad, Hewings, Porsse, Van Leeuwen, & Vieira, 2015; Haddad & Vieira, 2015). Haddad et al. (2017) use a general equilibrium model to estimate what would happen to São Paulo's economy if policies of congestion charging and/or investments in public transit infrastructure were conducted. They show that policies that increase the costs of private transportation have a negative overall impact on economic growth.

These authors also show, however, that investments in transport infrastructure causes increases in the product of the economy and Haddad and Vieira (2015) found out that subway infrastructure in São Paulo has considerable impacts on workers productivity and spillover effects that reaches São Paulo State and even the country. This is also made by Basso et al. (2011) and the authors defend that dedicated bus line implementation is a superior policy to transit subsidization or congestion price in terms of welfare. Also, dedicated bus lanes might have more political support than urban tolls or even high investments on transit infrastructure (Smart, 2014).

The analysis in Alvim, Darido, Lippe, & Mehndiratta (2013) takes the impact of building one subway line in São Paulo in a more tangible way than simulations that care about welfare. They estimate that this new line has a significant positive impact for lower-income households' commuting in terms of travel time and jobs accessed. There is also positive impact over job creation and little effect over land prices, and so such construction is probably worth the investment in infrastructure. Similarly, El-Geneidy et al. (2016) conduct a case study of lower-income households in Montreal to evaluate job accessibility with respect to transit as a form of social policy with equity concerns. However, these authors explore transit costs (time and out of pocket) and conclude that fare is as important as infrastructure for job accessibility of poorer people.

Fergusson (1992), analyzing American fuel shortages and strikes during the 1970's, conclude that elasticity with respect to service level is higher (closer to zero) than the one related to prices. His conclusion is that, if transit authorities face a tradeoff between service level and prices, the first should be favored for system's sustainability sake.

Beside transit fares, from the individual perspective, it is also important for governments to know what factors make commuters prefer transit rather than private modes. A classical characteristic that augments the probability of one individual choosing transit as the main modal to travel to work is proximity to mass transit stations (Lindsey, Schofer, Durango-Cohen, & Gray, 2010) and service level (Mohring, 1972). Shyr, Andersson, Cheng, and Hsiao (2017) verified that not only service level matters, but also the variety of places one can get by transit – that is, people are more prone to use transit once they can access places of any purpose (including for leisure purposes) through it.

Chiou, Jou and Yang (2015) conduct a similar analysis for Taiwan and find that areas with lower-income households and higher service level are those that use transit the most, while bus age, motorcycle ownership, and the percentage of minors in the population affect ridership negatively. Also, they have found that elements varied among regions: more minors in the population is an example, as it varies from negative impact in higher populational density areas to positive impact in lower ones.

As mentioned by Basso and Jara-Díaz (2012),

> demands for car and buses have cross-elasticities not only with respect to price, but also with respect to quality of service…. In making a choice, people would consider not only direct monetary costs but also travel times, comfort and so on. Therefore, congestion tolls, transit subsidies, prices and service variables – such frequency – are closely interrelated (Basso & Jara-Díaz, 2012, p. 891).

Service level seems to be important in determining whether one individual will ride a bus or choose to drive, specifically in higher-density areas (Currie & Wallis, 2008). But there might be other policies that complement it. Watkins, Ferris, Borning, Rutherford and Layton (2011) conducted a survey with bus users after the implementation of a real-time information system for mobile devices in Seattle, US. They wanted to figure out whether users of this app perceived their waiting time in bus stops to be lower than traditional users. Their conclusion is that real-time information usage decreases users perceived waiting time by 13%. Most strikingly, they found that those users actual waiting time is 2 minutes less than the other users. That is, implementing real-time information system buses location might decrease both perceived and actual waiting time for buses – one critical factor in users' decision. There are other works that found similar results (Brakewood, Macfarlane, & Watkins, 2015; Tang & Thakuriah, 2012). This simple policy might be as effective as increasing bus services in order to lower riders' total trip time, as traditionally proposed and exemplified by Mohring (1972).

There is another kind of information, one related to fare price, that can induce mode shift. Drevs, Tscheulin, Lindenmeier and Renner (2014) verify that users' willingness to pay for transit changes if they have the information that the fare is subsidized. That is, once users know that transit has government subsidies, two movements of crowding-in and -out are seen since part of users get concerns of fairness (such as preconized by theory on user charges) and part of them think of the combination of subsidies and user charges as double charging or free-riding. Since the latter is smaller than the first, authors say that transit authorities can rely on information as one strategy of ridership enhancement. It also might bring forth users' consciousness on the importance of service charging.

Summing up, literature says that ridership can be maintained through adequate fare pricing and there is a debate on how this require subsidies or not. Nevertheless, there are other policies which can be applied aiming to increase ridership and a few works that explore those ways were

reviewed in this subsection. Next, research hypotheses raised and/or qualified during the literature review are presented alongside with price elasticities of transit demand estimations found in the literature analyzed.

2.1.4 Research hypotheses and questions: transit price elasticities calculation in literature

To this point, literature suggests that public collective transportation has an important role to play in mitigating negative externalities caused by transportation. Moreover, it is usually a government's responsibility to provide or regulate it adequately.

One way to do so is to make it attractive, in that commuters will tend to choose it rather than private modes. There are a few policy's characteristics that address this question, such as informational incentives, service level, good infrastructure, among others. Literature reviewed in the last subsection suggests that those policies are welfare improvers.

The other way to make sound policy decisions with respect to transit has to do with fare pricing. This topic has been explored most in literature. First, there seems to exist a consensus that transit should not be provided free off charges. However, there is neither an unified point of view, nor robust academic findings on how transit should be priced. This includes a discussion about fare subsidization as a second-best policy for congestion charging (Basso & Jara-Díaz, 2012; De Palma et al., 2011b; Estupiñán et al., 2007), a form of keeping transit affordable for those who cannot pay much for it (Carvalho & Pereira, 2011; Estupiñán et al., 2007; Gomide, 2003; Gwilliam, 2002), or, if priced adequately, because of its capacity for poverty alleviation and social development (Estupiñán et al., 2007; Gomide, 2003; Gwilliam, 2002). On one hand, public finance literature recommends marginal cost pricing rule (Bahl & Linn, 1992; Bird, 2001); these authors themselves, however, advocate for policy decisions that encompass the provided service's idiosyncrasies. On the other hand, there are a few studies that suggest that subsidizing transit is welfare improving (Basso et al., 2011; Basso & Jara-Díaz, 2012; Mohring, 1972; Small, 1987). Nonetheless, Estupiñán et al. (2007) suggest that supply-side subsidies are neutral or regressive, while demand-side subsidies might be more efficient.

The arguments revised to this point suggest that bus provision in São Paulo must be approached on allocative efficiency grounds. There is a huge amount of the City's budget being allocated on

it and fare is still expensive - or, at least, it has power to ignite a lively public debate and riots - which suggest an inefficient user charges policy. Nevertheless, São Paulo's transportation levels seems to be on the edge: if policy goes wrong, the city might become unsustainable and people's commuting capacity might be put in danger. This situation calls for works in the area.

The present work addresses one characteristic that helps in the debate about fare pricing: price elasticity of bus demand. As was mentioned during this literature review, to know about this is important since:

1. Some demand responsiveness with respect to price is a necessary condition for marginal cost pricing rule to be truly efficient (Bahl & Linn, 1992; Bird, 2001). If one thinks of pricing fare under this rule, this information is substantial. Nevertheless, even though in the case of almost perfect inelasticity (elasticity between -1 and 1 and not equal to 0), welfare losses are quite high if fare is incorrectly priced (Bahl & Linn, 1992). This situation demands attention.

2. If policy's intention is to mitigate the negative externalities of transportation through transit usage, how this policy affects the quantity consumed becomes important since the final output to be measured is exactly bus ridership. To know price elasticity of demand is core for this kind of analysis (Estupiñán et al., 2007).

In a developing country where resources are scarce such as in Brazil, it is also important to know how many people will be excluded from the system if fare changes. Such information is core for discussions about how socially desirable it is to have bus subsidies – and also for debates on affordability. Thus, once transit strengthen might consolidate urban development and sustainability and mitigate congestion externalities, a key factor to be known in any transit system is price elasticity of demand, because (a) it helps in the debate of fare pricing and (b) it helps to mitigate effects on demand shifts caused by fare variation that can jeopardize people's commuting capacity.

It is possible that fare subsidies mitigate urban transportation's negative externalities for society, and so that expenditure of general funds here might be justifiable. Also, cutting subsidies might put fare's affordability in danger. On the other hand, there is a debate in the literature about how

efficient subsidies are in urban transportation to best help the poorest within society. The present work does not address these questions, although it recognizes their importance. Nevertheless, to know bus price elasticity of demand is fundamental for the debate on these topics. Next, a few works that does such calculations – and their results – are presented.

For all passenger transport (not only just public collective), price elasticity is usually small or inelastic in the short-term and larger in the long-term (Quinet & Vickerman, 2004). The authors review a series of studies and find that urban public transport's elasticities are, at average, -0.3, varying from -0.1 to -0.6, in the short-term. In the long-term, they are usually about double these values.

Wang and Skinner (1984) calculate these values for seven American cities and achieved values ranging from -0.042 to -0.62. They use a time-series model, considering monthly ridership as the demand and controlling for many variables, real price being one of them. The literature reviewed by them find values from -0.04 to -0.87 through various methods. Ferguson (1992), also analyzing American cities, similarly, finds price elasticities of -0.09.

Shyr et al. (2017) analyze 97 cities around the world to estimate factors that explain transit use. They found an average price elasticity of -0.141.

Hensher and Bullock (1979) calculate price elasticity of rail service Sidney, Australia. The value found was -0.57.

Storchmann (2003, as cited in Brinco, 2006) mentions a price elasticity of demand of -0.35 in Germany. He also mentions that most new users brought to transit in an eventual fare reduction are actually old users who have opted out sometime before, former cyclists or pedestrians – rather than former drivers, given the cross-price elasticity of car and transit demand of 0.03.

In Brazilian literature, Carvalho and Pereira (2011) get price elasticities for 10 cities together. At average, the number they get is around -0.6. Nevertheless, this estimation is considerably generalist.

Thus, work's **research question is "what are short- and long-term price elasticities of bus demand in São Paulo?"**. The present work aims to contribute to the debate on fare pricing

policy by answering this question. This was the **gap** found in literature, which the present work aims to close.

The **hypotheses** raised during the literature review are the following:

1. Price elasticity of bus trip demand in São Paulo is negative and smaller than one, that is, inelastic, in the short-term.
2. Price elasticity of bus trips (with one or more transfers) demand in São Paulo is negative and closer to -1 than the short-term one, that is - less inelastic - in the long-term.

It is worth mention that the approach for verifying the second hypothesis is, somehow, innovative since it uses Bilhete Único's implementation as an exogenous price shock in trips that use more than one bus. In 2004, São Paulo's city government changed the system of payment for buses and implemented a public policy called Bilhete Único, a smart card that allows users to pay for the first ride and make up to three bus transfers - that is, taking up to four buses - within three hours. Imagine an individual that takes two buses to work. In 2004, this individual spent half of she formerly spent on transit overnight. This also presents a limitation for the work: long-term elasticity can only be estimated for the trips that took more than one bus. Nevertheless, it is a valid approximation and not less important, since riding more than one bus within a trip is usually demanded by people who live in city's outskirts.

The proposed way to check the hypotheses and to answer the research question is presented at the next subsection.

2.2 Methodological review

One way to answer the stated question is through a model of individual decision that captures individual's responsiveness to changes in one variable, all other set constant. Such models usually consider that one can choose one or another transport mode, and this choice is function of other variables, one of them being price.

The present work uses Daniel McFadden's *Discrete Choice Model* (1973, 1974), which accounts for these questions. McFadden's work is considered to have ″fundamentally changed the way empirical economists study individual behavior″ (Manski, 2001, p. 224). His 1973 article's ″most striking characteristic (...) is its smooth progression from a conceptual contribution of great generality to a practical contribution of immediate usefulness″ (Manski, 2001, p. 224) - which also advocates for this model’s adequacy for the present work.

The purpose of the mentioned theory is to model one's choice. McFadden starts his 1973 work stating that “a fundamental concern of economics is understanding human choice behaviour. Models of hypothesis are formed on the nature of decision processes and are evaluated in the light of observed behaviour” (McFadden, 1973, p. 105). Economists have been doing this for decades at that time, indeed, using rational choice theory, but McFadden proposes a slightly different approach, called Random Utility Model.

This theoretical lens is a bit different from models derived from the traditional Rational Choice Theory[22]: the decision maker is said to be far from fully rational. He or she responds to stimulus but might respond differently to the same incentive because of any irrational, volatile condition, which is context dependent. The broad, old view of rational choice acknowledge this volatile process, indeed. However, it lacks the explanation for it and fails into adapting its models to receive it, what is addressed by McFadden in assuming that one has many utility functions and picks one up at random whenever he or she has to make a decision (McFadden, 1999). The utility is considered to be the well-being achieved when consuming a good or service (Nicholson & Snyder, 2005) and it can be a function of product’s or individual’s variables.

---

[22] Which assumes that social behavior comes from aggregate individual agents, acting to maximize their own utility. It also assumes that individuals have preferences, which are complete and transitive.

Mogridge (1997) mentions that "the achieved utility on a given mode is not the same as the offered utility, but generally appears higher. This is because only those travelers for whom the mode is better will actually choose it" (Mogridge, 1997, p. 12). Also, alternatives, per se, do not produce utility: this is derived from the characteristics of alternatives (Lancaster & Patterson, 1990) as well as those of the individual; for example, the observable utility is usually defined as a linear combination of variables (Ortúzar & Willumsen, 1994).

The early basis for Random Utility Models are found in psychology (McFadden, 1973). McFadden's formulation on the process of choice might be well understood in Figure 13 and in the excerpt below, which I fully reproduce due to its clearness and plainness:

> choice behavior can be characterized by a decision process, which is informed by perceptions and beliefs based on available information, and influenced by affect, attitudes, motives, and preferences. [Figure 13] depicts these elements in the decision process and their linkages. A few brief definitions are needed. *Perceptions* are the cognition of sensation. I will use 'perceptions' broadly to include beliefs, which are mental models of the world, particularly probability judgments. *Affect* refers to the emotional state of the decision-maker, and its impact on cognition of the decision task. *Attitudes* are defined as stable psychological tendencies to evaluate particular entities (outcomes or activities) with favor or disfavor. Technically, attitudes are often defined as latent factors that explain the variation of a battery of indicators (most commonly semantic differentials). The domain of attitudes may be very broad, including for example comparative judgments, but an attitude itself is a unitary valuation. *Preferences* are comparative judgments between entities. Under certain technical conditions, including completeness and transitivity, preferences can be represented by a numerical scale, or *utility*. *Motives* are drives directed toward perceived goals. The *cognitive process* for decision making is the mental mechanism that defines the cognitive task and the role of perceptions, beliefs, attitudes, preferences, and motives in performing this task to produce a *choice*. (McFadden, 1999, p. 74)

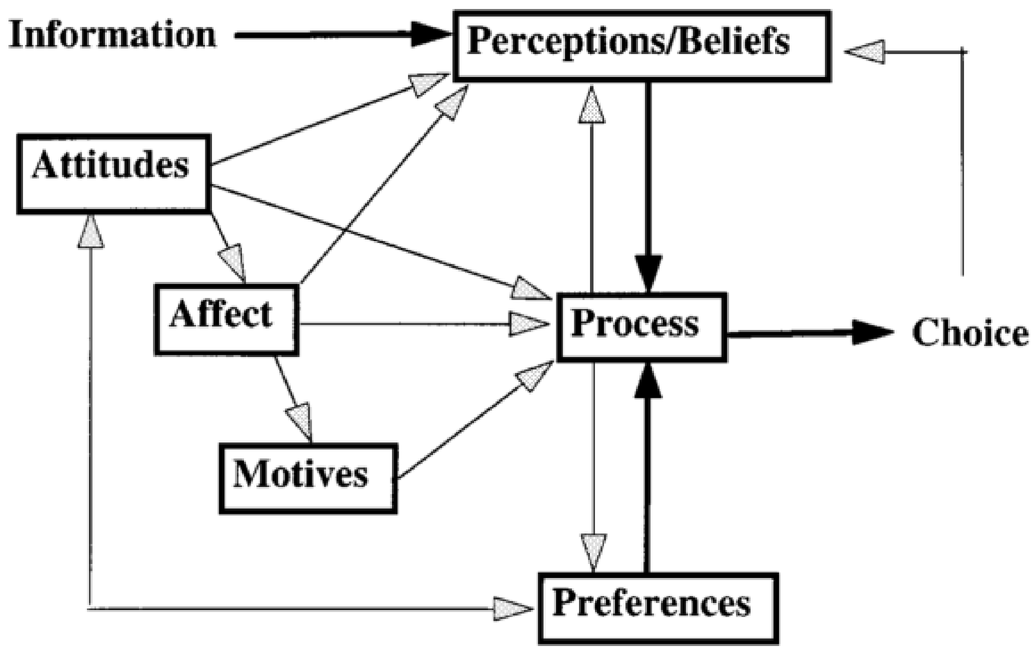

Figure 13 - Choice process, as it is in McFadden's formulations. Source: McFadden (1999, p. 74).

As mentioned in the first part of this literature review, choice process is also influenced by locational constrained determinants. Nevertheless, modal choice feedback influences all the decision process, therefore also influencing where one decides to live. Those decisions have different horizons of time and are shown in Domencich and McFadden (1975), as reproduced in Figure 14.

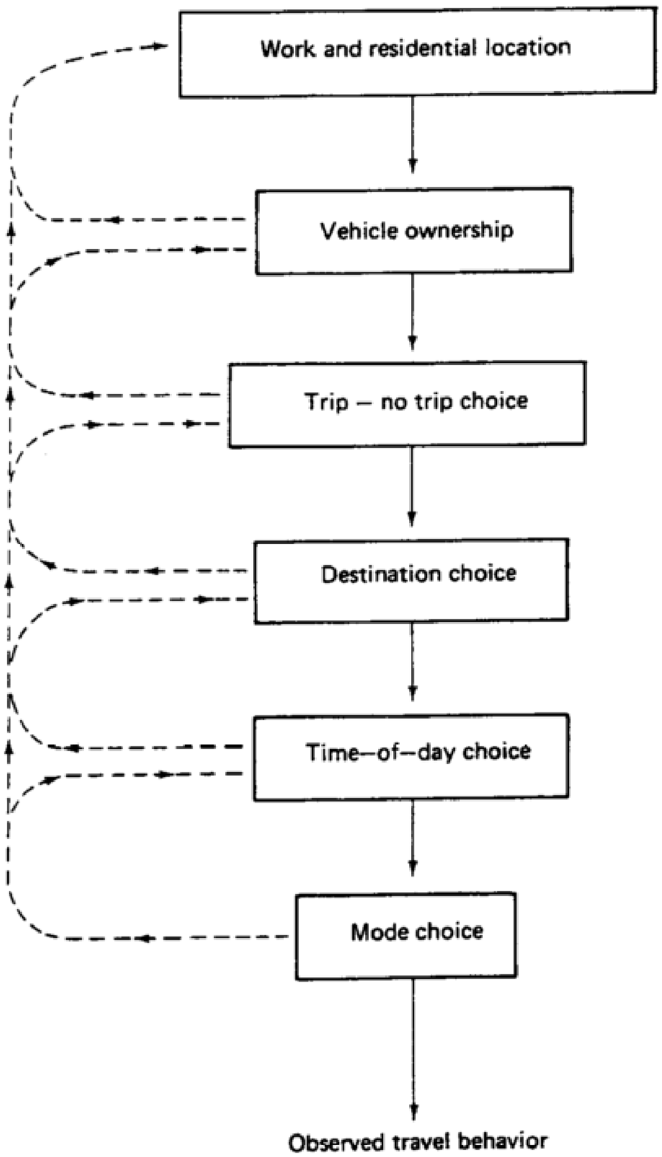

Figure 14- Choice process from another, time-oriented, perspective. Solid arrows represent the decision sequence, and dashed arrows represent flows of information succeeding choices and re-entering the decision process. Source: Domencich & McFadden (1975, p. 43).

In practical terms, McFadden (1974) says that sampled data from a population will fail to fit a traditional utility maximization function "because of measurement errors in (...) [demographic variables], consumer optimization errors and unobserved variations in the population" (McFadden, 1974, p. 308). However,

> McFadden's simple insight was to re-interpret the randomness as arising from cross-sectional variation in utility functions across the population rather than from time-series variation within a given individual (Manski, 2001, p. 221).

Thus, the model assumes that one's utility has a random, unobserved, term. Hence, McFadden's proposition of a viable approach is to model on the known aspects of individuals and choices. The derivation of how to estimate discrete choice models through multinomial logit is mostly attributed to McFadden's 1974 work (Ben-Akiva & Lerman, 1985).

"A viable approach does emerge if the researcher has partial knowledge of the form of $U(.,.)$ (...). In particular, he supposed that $U(.,.)$ has the linear-in-parameters separable form" (Manski, 2001, p. 222), that is, $U = V + \varepsilon$, where the separable form $V(.,.)$ is observed and support estimation of associated coefficients once some assumptions on ε are made - these assumptions are listed on the Methods' section.

Considering that one individual, i, must choose a mode, m, for commuting. Also considering that she will choose the mode that maximizes her utility, U_{im}, constrained to her budget, R. It is plausible to assume that the mode chosen will be the one that provides her more utility - or less disutility, her choice being made in the terms presented above, in a semi-rational way.

$$Max U_{im}, p_m x_m \leq R \quad (1)$$

The model proposes that U_{im} has the following form:

$$U_{im} = V_{im} + \varepsilon_{im} \quad (2)$$

and that the first term, V_{im}, is a function of explanatory variables (demographic, economic, sociological, or any kind of variable that influences an individual's choice, in the terms presented

before), x_{im}, which can also embody mode's characteristics (Domencich & McFadden, 1975), t_{im}, coming from the budget constraint, in the form of an indirect utility:

$$\begin{aligned} U_{im} &= V_{im} + \varepsilon_{im} \\ &= \alpha_i + \beta x_{im} + \varepsilon_{im} \\ &= \alpha_i + \beta x_{im} + \delta_m t_{im} + \varepsilon_{im} \end{aligned} \tag{3}$$

Since ε_{im} are not observed, the model proposes estimating this in terms of probabilities. Then, it gives us the probability that one will chose mode $m1$ rather than $m2$. The representative utility, described above, allows us to check how this probability varies, given a change in any of the independent variables.

Using the OD Survey mentioned before, one that has information on both peoples' commuting choice and socioeconomic characteristics, it is possible to build a dataset for São Paulo whereupon the model might be applied. Similar work have been conducted with the same dataset and model, but proposes to answer different questions (Pacheco & Chagas, 2016). This kind of analysis allows us to answer the proposed questions and is backed up by recent literature which has been analyzed here.

The method is derived in the next section. The dataset construction and the results are presented subsequently.

3 METHODS

3.1 Discrete Choice Model and elasticities estimation

Elasticity is basically assessed through the formula below:

$$e_P = \frac{\frac{\Delta Q}{Q}}{\frac{\Delta P}{P}}, \qquad (4)$$

which explicitly shows demand responsiveness to changes in price. However, in practical situations, it has major limitations. It is not possible, for example, to know if an eventual variation of Q was really due to ΔP or caused by any other factor (Wang & Skinner, 1984). It also requires an exogenous price shock, which is hardly met empirically.

To overcome these limitations, the present work considers:

- For short-term elasticity, the use of a Discrete Choice Model, as developed by Daniel McFadden (1973, 1974), the intrinsic features of which have been discussed in the literature review section and whose application is explained in this section. This allows for calculating price elasticity of any mode demand in a given period of time, with cross-sectional data.

- For long-term elasticity, the application of this model for 1997 and 2007 (OD Survey years) and comparing the aggregate demand that resulted from each one. This corresponds to the ΔQ on equation 4. To obtain ΔP, I consider that Bilhete Único implementation works as an exogenous shock on bus prices of those trips which are made with at least one transfer, that is, with two or more buses, once it allows users to pay for the first ride and make up to three bus transfers within three hours - in fact, this is one of the innovations this work proposes. However, this approach allows us to calculate long-term elasticity of the mode which had the exogenous price shock only - that is, trips that used two or more buses.

In a visual scheme, the research design is as follows:

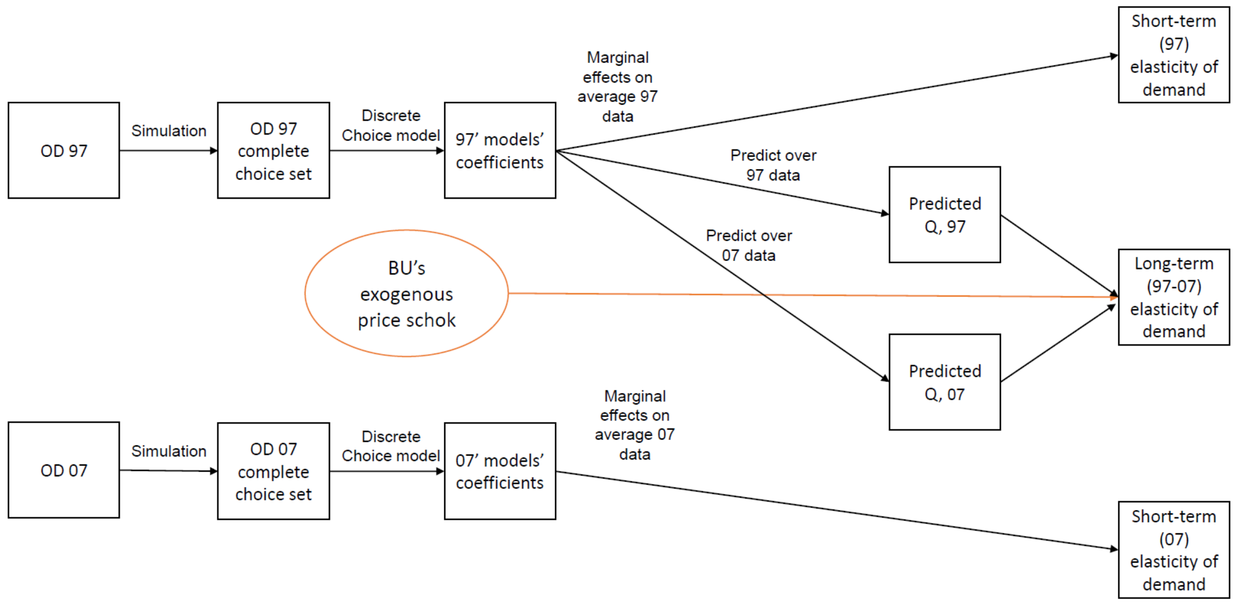

Figure 15 - Research Design

Since the method proposed uses the same model for calculating both short and long-term elasticities, this section describes it.

3.1.1 Models' basic characteristics

The model is as follows: consider that an individual i chooses her commuting mode, called here m (which belongs to her choice set[23], M). Consider also that she does so maximizing her utility U (the well-being achieved when consuming the good (Nicholson & Snyder, 2005; Quinet & Vickerman, 2004), which is subject to a budget constraint:

$$MaxU_{im}, p_m x_m \leq R, \tag{5}$$

[23] The choice set is the group of alternatives that might be chosen by one individual in a discrete choice process.

where p_m and x_m are the price and quantity consumed of the alternative m (which stands for mode) and R is the budget to be spent on commuting.

On the model proposed here, the utility function is not fully observable or quantifiable by the researcher; nevertheless, it can be modeled in the following way, suggested by McFadden (1973). First, dividing U into two terms: one that is observed and one that is not. The first, V_{im}, is usually called *representative utility* or *systematic component*. The latter, ε_{im}, is the unobserved part and it is considered *random* or *stochastic* (Train, 2009). According to McFadden (1973, p. 108), the first term *"reflects the 'representative' tastes of the population"* and the second, *"the idiosyncrasies of this individual in tastes for the alternative"*. Then, U is defined as:

$$U_{im} = V_{im} + \varepsilon_{im}, \tag{6}$$

where V_{im} can also be interpreted as the indirect utility, that incorporate budgetary constraints and price. Let's say that b and c are possible choices and $P_b = Pc$, one might choose the latter when $U_b > Uc$. This is equivalent to maximizing her utility, and it is postulated that the individual knows why U_b is greater than Uc.

However, the researcher does not has full information and since ε_{im} is not observable, "choices can only be modeled in terms of probabilities" (Croissant, 2012, p. 11). Then, considering ε_{im} stochastic, the joint density of vector $\varepsilon_i = \langle \varepsilon_{i1}, \dots, \varepsilon_{im} \rangle$ is $f(\varepsilon_i)$, this allows for making probabilistic statements about an individual's choice (Hensher & Button, 2000; Ortúzar & Willumsen, 2011; Train, 2009). In other words, from decision maker's point of view, the choice is purely deterministic; however, from the researcher's point of view, there is an unobserved part that composes utility and it can only be formulated in probabilistic terms (Croissant, 2012). In a similar way as proposed by McFadden (1973), probability P_{ib} that i chooses b over other modes m in the choice set M is then:

$$
\begin{aligned}
P_{ib} &\equiv P(b|i,M) \\
&= P(U_b > U_m), \forall m \neq b \\
&= P(V_{ib} + \varepsilon_{ib} > V_{im} + \varepsilon_{im}), \forall\ m \neq b \\
&= P(\varepsilon_{im} - \varepsilon_{ib} > V_{ib} - V_{im}), \forall\ m \neq b \\
&= \int_{\varepsilon} I\ (\varepsilon_{im} - \varepsilon_{ib} > V_{ib} - V_{im}) f(\varepsilon_i) d\varepsilon_i, \forall\ m \neq b,
\end{aligned} \tag{7}
$$

where $I(\cdot)$ is an indicator function which is equal to 1 if the expression is true and 0 if is not (Train, 2009).

For better understanding of equation 7, consider that b and c stands for bus and car and suppose that $V_c > V_b$. In this case, by observing the deterministic part only, one would say that bus would not be chosen at all. However, if the individual chooses bus instead, this is because "the unobserved factors for bus are sufficiently better than those for car to overcome the advantage that car has on observed factors" (Train, 2009, p. 16). In other words, if the car's deterministic part is greater than that of the bus, the chance that the person chooses bus instead of car is the probability that the unobserved factors are greater than the observed ones for each mode, or vice-versa.

Considering all m modes available in the choice set, the probability of b being chosen (i.e., a general form that relates all the choice set of equation 7) is, therefore:

$$
\begin{aligned}
P_{ib} &= Pr(U_b > U_1, \ldots, U_b > U_c, \ldots, U_b > U_m), \forall\ m \neq c) \\
&= Pr(\varepsilon_{i1} - \varepsilon_{ib} > V_{ib} - V_{i1}, \ldots, \varepsilon_{ic} - \varepsilon_{ib} > V_{ib} - V_{ic}, \ldots, \varepsilon_{im} - \varepsilon_{ib} > V_{ib} - V_{im}), \forall\ m
\end{aligned} \quad (8)
$$

Being F_{-b} the multivariate distribution of all but b's error terms, the unconditional probability can also be written in the following terms:

$$
P_b = \int F_{-b}[(V_b - V_1) + \varepsilon_b, \ldots, (V_b - V_m) + \varepsilon_b] f(\varepsilon_b) d\varepsilon_b \tag{9}
$$

3.1.2 Assumptions for probabilities' calculation

McFadden (1974) then presents 3 assumptions that are necessary to calculate probabilities' closed forms, that can be estimated through a multinomial logit. They are organized by Croissant (2012) in a simpler way, which is presented below.

H1: Independence of errors, that states a univariate distribution of the errors terms. The conditional and unconditional probabilities are shown in equations 10 and 11, respectively. They allow the calculation of only a one-dimensional integral to compute probabilities, later.

$$(P_b|\varepsilon_b) = \prod_{m \neq b} F_m \left(V_b - V_m + \varepsilon_b\right) \tag{10}$$

$$(P_b) = \int \prod_{m \neq b} F_m \left(V_b - V_m + \varepsilon_b\right) f(\varepsilon_b) d\varepsilon_b \tag{11}$$

Independence of errors creates an adjacent hypothesis: Independence of Irrelevant Alternatives (IIA). It postulates that the probability ratio between two alternatives depends only on the characteristics of these two alternatives, all other alternative’s characteristics being irrelevant (Croissant, 2012; Hensher & Button, 2000; Train, 2009). In practice, if IIA holds – what is necessary for proper estimation of Multinomial Logit Models – omitting an irrelevant subset from estimation “altogether will not change parameters systematically” (Greene, 2012, p. 767). This can be tested as proposed in Hausman and McFadden (1984).

H2: Gumbel distribution, which says that all ε follow a standard Weibull distribution – also called Gumbel or Extreme Value Type I (Ortúzar & Willumsen, 2011) –, where F is the cumulative and f is the density functions:

$$F(x) = e^{-e^{-x}} \tag{12}$$

$$f(x) = e^{-(x+e^{-x})} \tag{13}$$

H3: Homoscedasticity, which says that errors are identically distributed.

The three put together, the conditional and unconditional probabilities shown in equations 10 and 11 become 14 and 15 below, where F is the cumulative and f is the density functions of the standard Gumbel Distribution.

$$(P_b|\varepsilon_b) = \prod_{m \neq b} F\,(V_b - V_m + \varepsilon_b) \tag{14}$$

$$(P_b) = \int \prod_{m \neq b} F\,(V_b - V_m + \varepsilon_b) f(\varepsilon_b) d\varepsilon_b \tag{15}$$

3.1.3 Probabilities' calculation

A logit transformation of the deterministic utility, V_{im}, is thus equal to the closed form of the probabilities. Departing from equation 7 and summing up hypothesis 1 and 2,

$$P(\varepsilon_b < V_b - V_m + \varepsilon_b) = e^{-e^{-(V_{ib} - V_{im} + \varepsilon_b)}} \tag{16}$$

From H1,

$$(P_b|\varepsilon_b) = \prod_{m \neq b} e^{-e^{-(V_{ib} - V_{im} + \varepsilon_b)}} \tag{17}$$

A didactic demonstration of the unconditional probability of b being chosen is found in Croissant (2012), and the result is:

$$P_b = \frac{1}{\sum_m e^{-(V_m - V_b)}}, \tag{18}$$

which is exactly the logit probability below:

$$P_b = \frac{e^{V_b}}{\sum_m e^{V_m}}, \tag{19}$$

which can be estimated with maximum likelihood by a multinomial logit regression, as mentioned by Train & McFadden (1978).

3.1.4 Approaching the representative utility

The subsections above basically showed that, under some assumptions about the random term of the utility of an individual, the representative utility is used to calculate the probabilities of one mode being chosen by a multinomial logit. Then, the present work shall concentrates on specifying this part of utility as a function of alternative's and individual's characteristics.

V_{im} (the indirect utility function of individual *i* choosing mode *m*) can be described in terms of:

- Individual specific variables z_i with an alternative specific coefficient γ_m,
- Alternative specific variables t_{im} with an alternative specific coefficient, δ_m and
- Alternative specific variables x_{im} with a generic coefficient β,

The second and third set of variables differ on whether their coefficients are different for each alternative. This allows for evaluating one attribute differently for each mode. To illustrate the second and third set, respectively, take time and cost: for instance, one minute spent on car may cause less "disutility" than one minute spent on transit; in an opposite way, one Real spent on transportation causes the same disutility whatever mode is taken. The representative utility is generically specified as:

$$V_{im} = \alpha_i + \beta x_{im} + \gamma_m z_i + \delta_m t_{im}, \forall m \qquad (20)$$

Thus, the following specifications are suggested. First, one with cost and time associated with generic coefficients:

$$V_{im} = \alpha_i + \beta_1 cost_{im} + \beta_2 time_{im} + \boldsymbol{\gamma}_{\mathbf{m}} \mathbf{Z}_{\mathbf{i}}, \forall m \qquad (21)$$

The second one below treats time's coefficient as mode specific:

$$V_{im} = \alpha_i + \beta_1 cost_{im} + \boldsymbol{\gamma}_{\mathbf{m}} \mathbf{Z}_{\mathbf{i}} + \delta_m time_{im}, \forall m \qquad (22)$$

The difference between them is: in (21), one minute spent on whatever mode causes the same disutility for the individual; (22) allows time spent on one mode to cause different disutility than time spent on other mode. $\mathbf{Z}_{\mathbf{i}}$ is a set of socio-demographic variables concerning individuals, and $\boldsymbol{\gamma}_{\mathbf{m}}$ represents the coefficients associated with each variable, for each mode.

The present work chooses not to estimate models in which cost’s coefficient is mode specific for two reasons. First, and most importantly, walking is considered to be free. That is, its cost is zero (for all trips), and it is not possible to estimate a model in which the observed variable does not vary. Second, and this is frequently found in literature, it makes more sense that one individual values time differently from mode to mode than money.

The variables are found in (or are possible to simulate based on) the Metro's OD surveys. They also inform which mode was used on the trip – forming, then, the independent variable in the form of 0 or 1 for each alternative. This is described on the Data section.

3.1.5 Summing up: marginal effects and demand estimation

Once the model is estimated, the marginal effects with respect to individual-specific variables is:

$$\frac{\partial P_{im}}{\partial z_i} = P_{im}(\beta_m - \sum_b P_{ib}\,\beta_b), \tag{23}$$

and the marginal effects with respect to mode-specific variables is:

$$\frac{\partial P_{im}}{\partial x_{im}} = \gamma P_{im}(1 - P_{ib}), \tag{24}$$

Applying 24 with respect to price, in this context where there is one model estimated for each OD Survey year, gives exactly the **short-term price elasticity of demand**.

We can also easily justify that a weighted sum of each individual's P_{im} equals the demand Q_{im} of trips by the modal m (in terms of all trips), as suggested by Domencich and McFadden (1975) and applied by Hensher and Bullock (1979) and Train (2009):

$$\hat{Q_m} = \sum_i w_i P_{im}\,, \tag{25}$$

which provides the demand (Q) necessary for 4, the **long-term price elasticity of demand estimation**.

These methods are applied over OD Surveys (1997 and 2007) and the results are shown in the next section.

3.2 Nested Logit model

Independence of Irrelevant Alternatives (IIA) brings advantages for the Multinomial Logit, such as easiness of estimation and the possibility of inquiry on choices among a subset of alternatives and not among all of them (Train, 2009). Also, if choice probabilities, in fact, exhibit IIA, they provide an accurate representation of reality (Train, 2009). Nevertheless, this is a strong assumption and not rarely it is inappropriate, such as in the famous case of the blue-bus red-bus problem (Ben-Akiva & Lerman, 1985; Koppelman & Vaneet, 2000; Quinet & Vickerman, 2004; Train, 2009; Walker & Ben-Akiva, 2011; Wen & Koppelman, 2001).

In this case, the addition of one modal that has the same characteristics of other pre-existent ones will cause the joint probability of choosing them to be higher than it was before. For simplicity, supposing that one individual chooses between commuting by a blue-bus or a car. If she chooses at random, the probability of choosing bus or car is 0.5. If a red-bus is introduced and the IIA assumption is maintained, the probability of choosing a blue-bus, a red-bus or a car is of 1/3 each. In this case, the probability of this individual choosing transit is now 2/3, what is unrealistic since the difference between one type of bus and another is merely their color. This usually happens when there is one variable which is omitted or non-observed and, at the same time, important enough to make U_{i1} and U_{i2} correlated.

In the multinomial logit model presented until now, IIA is considered as a hypothesis. There is a way of relaxing this hypothesis for estimations' sake: the Nested Logit model, which allows for correlation of error terms among predefined groups that are believed to have similar characteristics (Ben-Akiva & Lerman, 1985; McFadden, 1978; Train, 2009; Walker & Ben-Akiva, 2011; Wen & Koppelman, 2001).

As schematized by Croissant (2012) now it is supposed that each alternative belongs to a group, called nest, N, which distribution for the error term is:

$$exp\left(-\sum_{n=1}^{N}\left(\sum_{j\in n} e^{-\epsilon_j/\lambda_n}\right)^{\lambda_n}\right) \tag{26}$$

And the probability of the individual *i* choosing alternative *m* that belongs to nest *l* is, therefore:

$$P_{im} = \frac{e^{V_m/\lambda_l}\left(\sum_{k\in B_l} e^{V_k/\lambda_l}\right)^{\lambda_l - 1}}{\sum_{n=1}^{N}\left(\sum_{k\in B_n} e^{V_k/\lambda_l}\right)^{\lambda_l}} \tag{27}$$

Once the specification presented before might incur in the blue-bus red-bus paradox, the present work also estimates one set of Nested Logit models – even though the discrete model passes the IIA test. This is as another form of a possible more realistic representation of reality.

3.3 Data preparation

As mentioned before, Metro's OD surveys ask each resident of a sample of households about their trips on the previous day of the interview and his or her characteristics. Therefore, it collects information about the respondent (as age, gender, income, educational level, occupation, and other socioeconomic and demographic information), and about each one of the trips that the person made on the day before the interview (duration, starting and ending point, motivation, hours, among others).

By collecting data of the realized trips only, the survey's datasets do not present all the mode options (and each option's features, as price and time it would take) that are available to this person before she makes her decision about the mode to be used on a trip. For example, if someone has made a trip using a bicycle, the researcher knows how long did this trip take, where and when did it started, etc., but there is no information on the other modes that this person could have taken. Also, the researcher cannot know, for instance, if this same trip is feasible by bus. Therefore, it is not known how long the counterfactual trip could have taken if this individual has chosen to go by other modes.

Thereby, a few assumptions and simulations are required to fill this gap on data, allowing the present work to estimate the model presented on equations 21 and 22. They are presented in the paragraph below. Simulations, naturally, have caveats since they are a rough representation of the reality or counterfactual and rely on many approximations. Nonetheless, even if all options were available for respondent's answer (for instance, in a declared preferences survey), caveats would

exist: people tend to report unrealistically negatively about the modes they didn't choose, mainly to justify their decisions, and they do so unconsciously (Mogridge, 1997). Furthermore, assumptions and simulations are widely found in the literature, including the works cited here.

First, it is assumed that all modes are available to be chosen by every individual for every trip reported on the OD survey. This is, perhaps, the strongest assumption in the present work. Since the surveys do not contain such information, one possible workaround is to infer one's options given her socioeconomic characteristics (such as home location, income, etc.). For instance, it is plausible to say that one's home distance to a subway station is a good proxy for rail presence on her choice set. To have or not a car is another clue, and information on bus lines is available for inference. Nonetheless, there are other factors that can influence the choice set formation that are unknown. For example, it is impossible to know whether this person is disabled and cannot walk to the bus stop. Also, to own a car does not mean that the respondent is allowed to use it – Svab (2016) shows that if one individual is married, the impact of having one vehicle at home on commuting by car is positive if the individual is male and negative if she is female; the probability of commuting by car once the family owns one is only positive for women if the household possesses two or more cars. Therefore, the simple assumption that anyone could have chosen any of the modes has advantages for this work's easiness of understanding and models' estimation.

The second assumption is on trip's price and duration for all mode options available, taken or non-taken – information required by the model. Of these, the only observed is the time taken on the chosen mode. Hence, the cost and duration of hypothetical trips on non-taken modes must be estimated, as well as the cost of the trip by the chosen mode. The present work used the average speed of each mode to predict the latter, and the first was a combination of fixed (transit fares are fixed and known) and variable (car costs are function of trip distance) prices. Simulations' description is find in the Appendix section.

Thus, these steps provide a sample of trips in São Paulo collected by the ODs surveys. For each trip then there are individual's demographic characteristics (observed), a set of alternative modes and its respective price and time (either observed or simulated), and the mode chosen by the individual (observed). This last feature classifies the specification proposed as a ***revealed***

preferences[24] model. This is superior in comparison to *stated preferences' models* – a model in which a respondent is provided the choice set and alternatives' characteristics and has to choose one, hypothetically - since people do not necessarily do what they respond, and mislead the researcher (Ben-Akiva & Lerman, 1985).

It is worth also describing some other features of data. Walking is only considered as a mode if the person walked any distance to work or school or, for other trip purposes, if distance was greater than 500m. Also, the ODs surveys register up to four modes taken in a (multimodal) trip. For example, if one took the subway and one bus, both modes will be registered[25]. This entails another problem: which mode to consider on multimodal trips?

Since the "main mode" hierarchy defined by the Metro Company[26] is not well suited for such models, the present work opts to group trips in conceived modes. Many grouping combinations were conducted, and a few were appropriated to the model. Hence, it was opted for the one which makes further analysis easier to interpret. Modes are reported in Table 1, below.

Two other features of data preparation are worth mentioning. First, OD Survey of 1997 does not contains the distance of trips, in that would be impossible to adequately simulate the counterfactual trips or calculate the price on the modal taken. There is not information on the coordinates of origin and destination, only it is provided the zone and subzone (which are sociodemographic territorial aggregations) of residence, origin, destination, etc. To overcome this caveat, the present considered that each trip started and finished in the centroid of its respective subzone and, using the adequate files, estimated the distance of the trip. For intrazone trips, a

---

[24] It works as if the researcher becomes an observer of one person - the decision maker whose age, income, gender, etc. are known – who must choose a mode to commute among a set of alternatives. This person also knows how long trip would take and how much it would cost in every mode, and chooses one of them – the one that maximizes her utility. Her choice preference is revealed through the survey.

[25] Walking, however, does not figure as one mode on multimodal trips. It is also, an only, a unique mode taken. For example, if one walked from home to the bus stop, took a bust, and then walked again from the bus stop to work. It is registered on the data that this person walked for x minutes on the origin and y minutes on the destination, but on the (up to four) modes fields, only bus is registered.

[26] Described on the Introduction section. This hierarchy makes sense for transport planning and does not fit future work's methodology and objectives.

linear model (that has distance as the dependent variable and, as independent ones, modal and a set of zones dummies) was estimated and its coefficients fed 1997 intrasubzones trips. The codes for those estimations is available in a weblink provided in the beginning of the present work.

Second, car costs were estimated based on previous works (ANTP, 2010), and refined using panel data of oil prices. The sheets used for this are also available alongside with this work's files. Simply, however, there were estimated a price per kilometer for car trips for 1997 and 2007. Since transit has a fixed price, its cost is intrinsic to the trip. Multimodal trips' costs consider both (as an example, if a trip was made by car and bus, the share of each one's distance was considered to calculate the share of car price, and transit price was added wholly).

Table 1 - Choice set, respective modes combination (if multimodal), and nest grouping

Grouped mode (mode set)	**(Multi)modal combination**	**Nest (if apply)**
Car	Car (driver) only	Auto
Multi_Transit	Combinations that include bus and subway and/or train and/or another mode combined with them	Transit
One_Bus	One bus. Bus only.	Transit
One_Plus_Bus	More than one bus. Bus only.	Transit
Rapid_Transit	Subway and/or train only	Transit
Walking	Walking only	-

The distribution of trips according to such modes is presented in Figure 16, which follows the same logic of Figure 3. It is important to notice that the number of trips that uses two bus or more went from approximately 600 thousand in 1997 to 1.48 million in 2007, that is, a surge of 145%.

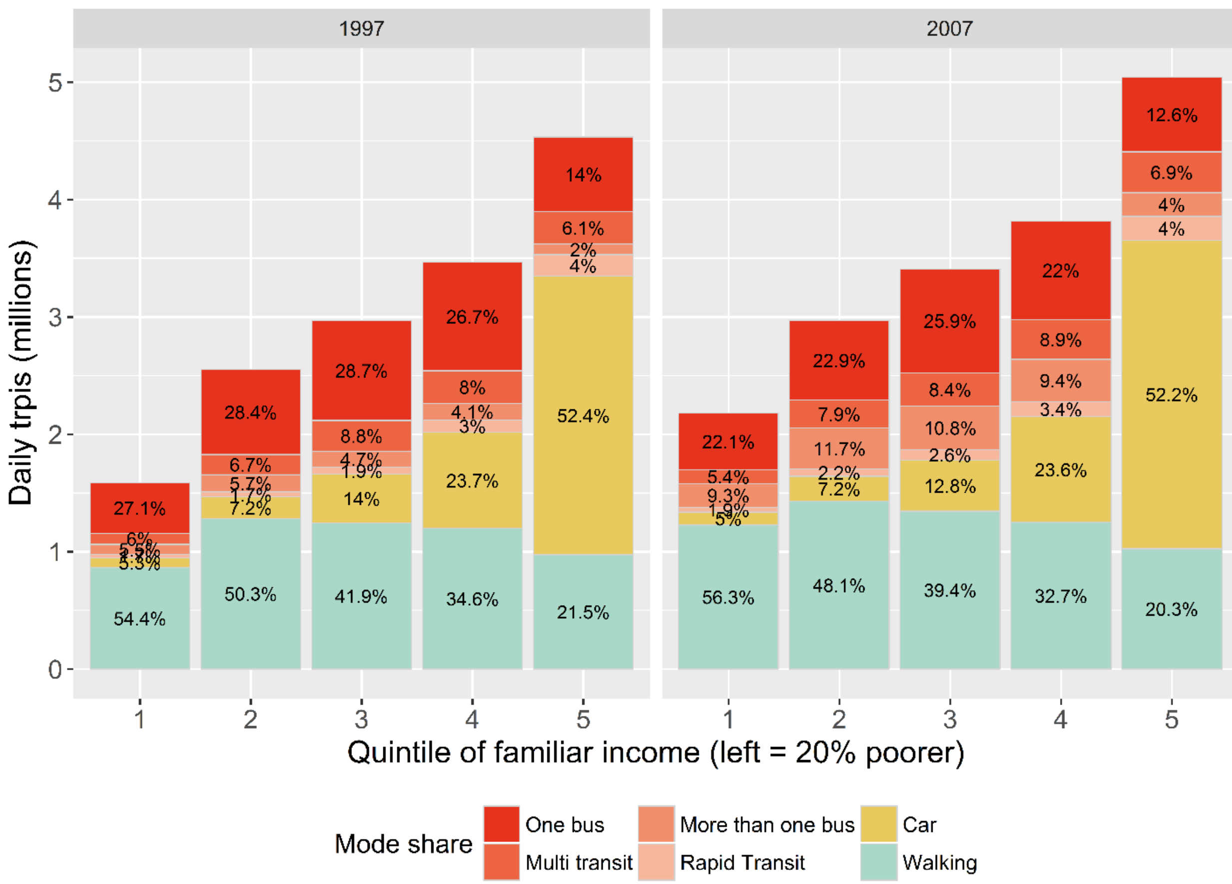

Figure 16 - Daily trips in São Paulo (city) divided by mode combination and familiar income quintile - 1997 and 2007. The numbers on the bars represent the mode share for each quintile, for the correspondent year. Source: data from OD Surveys 1997 and 2007, adapted. Prepared by the author.

3.3.1 Descriptive statistics

Average cost, cost per kilometer, distance and speed, for each mode observed in OD Surveys data are shown in Table 2. Boxplots of variables are presented in the Appendix section.

Table 2 - Descriptive statistics. Average variables related to the mode.

	Cost 2007	Cost 1997	Cost per km 2007	Cost per km 1997	Distance 2007	Distance 1997	Speed 2007	Speed 1997
Car	3.5106	4.2079	0.6079	0.7363	5,774.4753	5,714.1791	10.4209	11.7594
Multi_Transit	3.4653	3.7456	0.3030	0.3499	16,325.7648	15,797.7298	10.6429	11.3847
One_Bus	2.3000	1.5089	0.6519	0.4488	6,602.0635	6,414.3594	6.9077	7.4983
One_Plus_Bus	2.3000	3.1285	0.3101	0.4248	11,732.0243	11,437.4683	7.3131	7.6332
Rapid_Transit	2.3000	2.0415	0.4740	0.5711	9,997.7028	7,964.0617	10.6101	10.5676
Walking	0.0000	0.0000	0.0000	0.0000	816.4988	681.8458	3.1132	2.8522

Cost is in BRL (dec/2016). Cost per km is in BRL (dec/16) per km. Distance is in meters. Speed is in km/h.

4 RESULTS

4.1 Multinomial Logit

4.1.1 Multinomial Logit: Regressions results

The results of the regressions described on Equations 21 and 22 are presented next, for 1997 and 2007. The estimations of coefficients and standard errors are presented in separate tables for better formatting.

Table 3, Table 4, Table 5 and Table 6, with **regressions numbered from (1) to (8), present estimations made with only a few restrictions on data.** First, were eliminated the observations whose survey respondent did not travel or those whose respondent do not live in within the limits of the city of São Paulo. Second, were also excluded trips for which modals were hard to fit into any of the categories defined or were made by an less representative modal (such as bike and scholar bus). Still, there is a sample of approximately 90 and 100 thousand trips for 1997 and 2007, respectively – there were eliminated 69 and 67 thousand trips for the mentioned years. The trips have all kind of purposes, origins, and destinations.

Table 7, Table 8, Table 9 and Table 10, for their time, with **regressions numbered from (9) to (16), present estimations made with one major change on data: the elimination of the trips whose respondent declared that his or her employer paid by the trip.** It is a right of Brazilian workers to have their commuting by public transit partially paid by the employer through a voucher. In practice, the employer must finance workers' trips from home to work and back home and, in exchange, might discount up to 6% of workers' salary. This is colloquially called *Vale Transporte*. The problem that *Vale Transporte* imposes to the present estimations is that workers are not responsive to fare changes: in practice, they pay a fixed amount - 6% of their salary – and get all trips necessary to go back and forth to work. Hence, they should not be affected by an eventual change in fares and the models might be incorrectly estimated if commuters are considered homogenous[27].

---

[27] This was the main appointed problem to the present work during candidacy exams and during the 45th National Economics Conference.

However, it is not clear-cut how to deal with this problem, since including the pursuing of *Vale Transport* as a control variable would forbid models' estimation since this benefit is almost totally concentrated on transit takers, so that there is not enough variation for the dependent variables when it is considered. Also, it is very strong to say that those workers get free transit or are totally non-responsive to fare changes since they must compensate part of it and there is no further information that allow to infer how they internalize that in their choice process. Last, this is a characteristic that has confusing effects with respect to commuters' income level, since it is a benefit that is only available for formal workers and since the worker contributes with 6% of his or her salary, it is only advantageous for the worker if 6% of his or her earnings is less than what he or she spend on commuting.

Vale transporte use is depicted in Figures 17,18 and 19, below. After that, Table 3 and Table 4 present regressions coefficients for 1997 and 2007 for the first option, with no segmentation on data. Table 5 and Table 6 respectively present their standard errors. The logic is the same for Table 7 and Table 8 (coefficients) and Table 9 and Table 10 (standard errors), but they present the results of regressions using trips that have not used *Vale Transporte*.

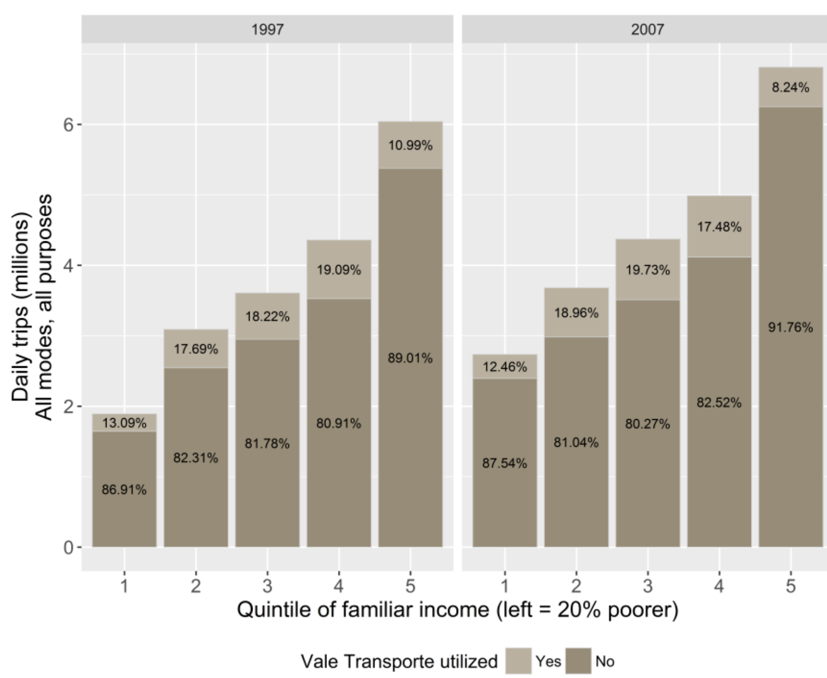

Figure 17 - Vale Transporte utilization, commuters divided by familiar income. All modes, all purposes. Source: OD Surveys 1997 and 2007. Prepared by the author.

Figure 18 - Vale Transporte utilization, commuters divided by familiar income. Transit modes, all purposes. Source: OD Surveys 1997 and 2007. Prepared by the author.

Figure 19 - Vale Transporte utilization, commuters divided by familiar income. Transit modes, work purpose. Source: OD Surveys 1997 and 2007. Prepared by the author.

Table 3 – Coefficients for multinomial logit model - 1997 - no segmentation on data.

Multinomial logit model - results for 1997								
Logistic function odds coefficients. No segmentation on data. Car is the base mode.								
	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept								
Multi_Transit	-1.633***	3.438***	3.472***	5.518***	-3.453***	2.269***	2.284***	4.061***
One_Bus	-0.001	6.077***	6.053***	8.486***	-0.222***	5.967***	5.936***	8.473***
One_Plus_Bus	-1.507***	3.145***	3.258***	6.122***	-2.939***	2.220***	2.252***	4.992***
Rapid_Transit	-2.725***	2.075***	2.060***	3.344***	-2.666***	1.921***	1.920***	3.317***
Walking	1.522***	8.340***	8.139***	10.538***	3.585***	10.837***	10.665***	13.263***
Cost: generic								
Generic	-0.190***	-0.212***	-0.210***	-0.203***	-0.034***	-0.034***	-0.034***	-0.023***
Duration: generic								
Generic	-0.037***	-0.035***	-0.035***	-0.035***				
Duration: mode specific								
Car					0.002***	-0.002**	-0.002***	-0.003***
Multi_Transit					0.041***	0.040***	0.040***	0.039***
One_Bus					0.002***	0.001***	0.001***	-0.001
One_Plus_Bus					0.017***	0.015***	0.015***	0.014***
Rapid_Transit					0.011***	0.010***	0.010***	0.008***
Walking					-0.098***	-0.098***	-0.098***	-0.103***
Familiar income								
Multi_Transit		-0.108***	-0.110***	-0.054***		-0.092***	-0.092***	-0.044***
One_Bus		-0.185***	-0.185***	-0.103***		-0.191***	-0.190***	-0.109***
One_Plus_Bus		-0.208***	-0.216***	-0.115***		-0.202***	-0.208***	-0.114***
Rapid_Transit		-0.031***	-0.031***	0.001		-0.036***	-0.036***	-0.003
Walking		-0.174***	-0.169***	-0.086***		-0.182***	-0.178***	-0.088***
Age								
Multi_Transit		-0.208***	-0.220***	-0.219***		-0.248***	-0.253***	-0.245***
One_Bus		-0.253***	-0.250***	-0.255***		-0.256***	-0.250***	-0.256***
One_Plus_Bus		-0.184***	-0.219***	-0.240***		-0.210***	-0.231***	-0.248***
Rapid_Transit		-0.224***	-0.219***	-0.194***		-0.213***	-0.212***	-0.188***
Walking		-0.305***	-0.282***	-0.284***		-0.319***	-0.299***	-0.287***
Age^2								
Multi_Transit		0.002***	0.002***	0.002***		0.003***	0.003***	0.003***
One_Bus		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
One_Plus_Bus		0.002***	0.002***	0.003***		0.002***	0.003***	0.003***
Rapid_Transit		0.003***	0.003***	0.002***		0.002***	0.002***	0.002***
Walking		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
Dummy gender: 1 = male								
Multi_Transit		-1.000***	-1.024***	-1.026***		-1.167***	-1.173***	-1.187***
One_Bus		-1.134***	-1.114***	-1.128***		-1.082***	-1.057***	-1.094***
One_Plus_Bus		-0.787***	-0.892***	-0.901***		-0.938***	-0.994***	-1.022***
Rapid_Transit		-0.851***	-0.828***	-0.825***		-0.763***	-0.754***	-0.767***
Walking		-0.994***	-0.901***	-0.918***		-1.046***	-0.953***	-0.932***
Dummy employment: 1 = employed								
Multi_Transit			0.233***	0.198***			0.101**	0.089*
One_Bus			-0.044	-0.139***			-0.104***	-0.201***
One_Plus_Bus			0.713***	0.575***			0.491***	0.390***
Rapid_Transit			-0.087	-0.063			-0.024	-0.015
Walking			-0.394***	-0.505***			-0.364***	-0.496***
Dummy study: 1 = student								
Multi_Transit				0.302***				0.502***
One_Bus				0.144***				0.176***
One_Plus_Bus				-0.166***				-0.035
Rapid_Transit				0.798***				0.751***
Walking				0.203***				0.302***
Dummy car ownership: 1 = own a car								
Multi_Transit				-2.915***				-2.795***
One_Bus				-3.333***				-3.349***
One_Plus_Bus				-3.489***				-3.380***
Rapid_Transit				-2.715***				-2.772***
Walking				-3.439***				-3.926***
Observations	93,748	93,748	93,748	93,748	93,748	93,748	93,748	93,748
R2	0.161	0.255	0.258	0.320	0.313	0.394	0.395	0.458
Log Likelihood	-114,394.000	-101,569.300	-101,236.800	-92,712.370	-93,685.550	-82,671.200	-82,538.530	-73,947.310
LR Test	43,955.070*** (df = 7)	69,604.490*** (df = 27)	70,269.380*** (df = 32)	87,318.310*** (df = 42)	85,371.940*** (df = 12)	107,400.600*** (df = 32)	107,666.000*** (df = 37)	124,848.400*** (df = 47)

Table 4 - Coefficients for multinomial logit model - 2007 - no segmentation on data.

Multinomial logit model - results for 2007

Logistic function odds coefficients. No segmentation on data. Car is the base mode.

	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept								
Multi_Transit	-1.581***	4.614***	4.681***	5.782***	-3.485***	3.203***	3.232***	4.210***
One_Bus	0.057***	7.273***	7.211***	8.214***	-0.265***	7.271***	7.209***	8.378***
One_Plus_Bus	-0.931***	5.095***	5.260***	6.525***	-2.213***	4.362***	4.425***	5.657***
Rapid_Transit	-2.730***	3.349***	3.358***	3.878***	-2.875***	3.128***	3.144***	3.869***
Walking	1.337***	9.542***	9.294***	10.437***	3.399***	12.285***	12.092***	13.232***
Cost: generic								
Generic	-0.223***	-0.233***	-0.231***	-0.231***	-0.060***	-0.026***	-0.025***	-0.028***
Duration: generic								
Generic	-0.033***	-0.032***	-0.031***	-0.031***				
Duration: mode specific								
Car					0.001	-0.005***	-0.005***	-0.005***
Multi_Transit					0.037***	0.036***	0.036***	0.036***
One_Bus					0.0002	-0.001***	-0.001***	-0.002***
One_Plus_Bus					0.015***	0.013***	0.013***	0.012***
Rapid_Transit					0.016***	0.014***	0.014***	0.013***
Walking					-0.088***	-0.091***	-0.091***	-0.094***
Familiar income								
Multi_Transit		-0.276***	-0.280***	-0.169***		-0.243***	-0.245***	-0.139***
One_Bus		-0.424***	-0.422***	-0.280***		-0.445***	-0.442***	-0.298***
One_Plus_Bus		-0.518***	-0.535***	-0.363***		-0.511***	-0.518***	-0.350***
Rapid_Transit		-0.142***	-0.142***	-0.059***		-0.152***	-0.152***	-0.062***
Walking		-0.420***	-0.410***	-0.269***		-0.413***	-0.406***	-0.251***
Age								
Multi_Transit		-0.210***	-0.224***	-0.204***		-0.249***	-0.254***	-0.231***
One_Bus		-0.253***	-0.243***	-0.219***		-0.265***	-0.254***	-0.235***
One_Plus_Bus		-0.176***	-0.205***	-0.191***		-0.206***	-0.217***	-0.203***
Rapid_Transit		-0.236***	-0.233***	-0.194***		-0.236***	-0.236***	-0.203***
Walking		-0.306***	-0.277***	-0.256***		-0.324***	-0.300***	-0.266***
Age^2								
Multi_Transit		0.002***	0.002***	0.002***		0.003***	0.003***	0.002***
One_Bus		0.003***	0.003***	0.002***		0.003***	0.003***	0.002***
One_Plus_Bus		0.002***	0.002***	0.002***		0.002***	0.002***	0.002***
Rapid_Transit		0.002***	0.002***	0.002***		0.002***	0.002***	0.002***
Walking		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
Dummy gender: 1 = male								
Multi_Transit		-1.128***	-1.140***	-1.126***		-1.281***	-1.281***	-1.277***
One_Bus		-1.211***	-1.182***	-1.166***		-1.185***	-1.157***	-1.156***
One_Plus_Bus		-1.120***	-1.165***	-1.150***		-1.226***	-1.236***	-1.241***
Rapid_Transit		-0.959***	-0.946***	-0.936***		-0.933***	-0.927***	-0.925***
Walking		-1.233***	-1.154***	-1.135***		-1.268***	-1.195***	-1.131***
Dummy employment: 1 = employed								
Multi_Transit			0.230***	0.246***			0.080*	0.121**
One_Bus			-0.160***	-0.146***			-0.203***	-0.207***
One_Plus_Bus			0.490***	0.466***			0.202***	0.188***
Rapid_Transit			-0.046	0.064			-0.008	0.066
Walking			-0.483***	-0.493***			-0.423***	-0.416***
Dummy study: 1 = student								
Multi_Transit				0.533***				0.738***
One_Bus				0.628***				0.617***
One_Plus_Bus				0.424***				0.560***
Rapid_Transit				1.042***				0.971***
Walking				0.515***				0.724***
Dummy car ownership: 1 = own a car								
Multi_Transit				-2.552***				-2.566***
One_Bus				-2.718***				-2.752***
One_Plus_Bus				-2.790***				-2.776***
Rapid_Transit				-2.575***				-2.642***
Walking				-2.758***				-3.182***
Observations	102,045	102,045	102,045	102,045	102,045	102,045	102,045	102,045
R2	0.126	0.247	0.250	0.287	0.284	0.390	0.391	0.430
Log Likelihood	-134,045.200	-115,451.000	-114,996.500	-109,376.800	-109,841.500	-93,474.800	-93,336.870	-87,442.540
LR Test	38,624.020*** (df = 7)	75,812.470*** (df = 27)	76,721.420*** (df = 32)	87,960.950*** (df = 42)	87,031.580*** (df = 12)	119,764.900*** (df = 32)	120,040.800*** (df = 37)	131,829.400*** (df = 47)

Table 5 - Standard errors for multinomial logit model - 1997 - no segmentation on data.

Multinomial logit model - results for 1997

Logistic function standard errors. No segmentation on data. Car is the base mode.

	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept								
Multi_Transit	(0.016)	(0.095)	(0.096)	(0.127)	(0.040)	(0.108)	(0.108)	(0.140)
One_Bus	(0.011)	(0.071)	(0.072)	(0.101)	(0.018)	(0.074)	(0.075)	(0.105)
One_Plus_Bus	(0.018)	(0.109)	(0.111)	(0.144)	(0.037)	(0.118)	(0.120)	(0.151)
Rapid_Transit	(0.023)	(0.122)	(0.122)	(0.164)	(0.036)	(0.124)	(0.125)	(0.167)
Walking	(0.012)	(0.071)	(0.071)	(0.102)	(0.023)	(0.085)	(0.086)	(0.121)
Cost: generic								
Generic	(0.002)	(0.003)	(0.003)	(0.003)	(0.007)	(0.007)	(0.007)	(0.007)
Duration: generic								
Generic	(0.0003)	(0.0003)	(0.0003)	(0.0003)				
Duration: mode specific								
Car					(0.001)	(0.001)	(0.001)	(0.001)
Multi_Transit					(0.001)	(0.001)	(0.001)	(0.001)
One_Bus					(0.0004)	(0.0004)	(0.0004)	(0.0004)
One_Plus_Bus					(0.0005)	(0.0005)	(0.0005)	(0.0005)
Rapid_Transit					(0.001)	(0.001)	(0.001)	(0.001)
Walking					(0.001)	(0.001)	(0.001)	(0.001)
Familiar income								
Multi_Transit		(0.004)	(0.004)	(0.004)		(0.004)	(0.004)	(0.004)
One_Bus		(0.003)	(0.003)	(0.003)		(0.003)	(0.003)	(0.003)
One_Plus_Bus		(0.007)	(0.007)	(0.007)		(0.007)	(0.007)	(0.007)
Rapid_Transit		(0.004)	(0.004)	(0.004)		(0.004)	(0.004)	(0.004)
Walking		(0.003)	(0.003)	(0.003)		(0.003)	(0.003)	(0.003)
Age								
Multi_Transit		(0.005)	(0.005)	(0.006)		(0.005)	(0.006)	(0.007)
One_Bus		(0.004)	(0.004)	(0.005)		(0.004)	(0.004)	(0.005)
One_Plus_Bus		(0.006)	(0.006)	(0.007)		(0.006)	(0.006)	(0.007)
Rapid_Transit		(0.006)	(0.007)	(0.008)		(0.006)	(0.007)	(0.008)
Walking		(0.004)	(0.004)	(0.005)		(0.004)	(0.004)	(0.005)
Age^2								
Multi_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
One_Bus		(0.00004)	(0.00005)	(0.0001)		(0.00004)	(0.00005)	(0.0001)
One_Plus_Bus		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Rapid_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Walking		(0.00004)	(0.00005)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Dummy gender: 1 = male								
Multi_Transit		(0.032)	(0.033)	(0.034)		(0.035)	(0.036)	(0.037)
One_Bus		(0.022)	(0.022)	(0.025)		(0.022)	(0.022)	(0.025)
One_Plus_Bus		(0.037)	(0.038)	(0.040)		(0.039)	(0.040)	(0.042)
Rapid_Transit		(0.044)	(0.045)	(0.046)		(0.043)	(0.044)	(0.045)
Walking		(0.022)	(0.023)	(0.025)		(0.027)	(0.028)	(0.031)
Dummy employment: 1 = employed								
Multi_Transit			(0.043)	(0.044)			(0.047)	(0.048)
One_Bus			(0.028)	(0.030)			(0.028)	(0.030)
One_Plus_Bus			(0.054)	(0.056)			(0.056)	(0.058)
Rapid_Transit			(0.056)	(0.057)			(0.055)	(0.056)
Walking			(0.027)	(0.029)			(0.034)	(0.036)
Dummy study: 1 = student								
Multi_Transit				(0.050)				(0.053)
One_Bus				(0.037)				(0.037)
One_Plus_Bus				(0.064)				(0.064)
Rapid_Transit				(0.065)				(0.064)
Walking				(0.038)				(0.047)
Dummy car ownership: 1 = own a car								
Multi_Transit				(0.045)				(0.048)
One_Bus				(0.037)				(0.038)
One_Plus_Bus				(0.050)				(0.051)
Rapid_Transit				(0.056)				(0.055)
Walking				(0.038)				(0.044)
Observations	93,748	93,748	93,748	93,748	93,748	93,748	93,748	93,748
R2	0.161	0.255	0.258	0.320	0.313	0.394	0.395	0.458
Log Likelihood	-114,394.000	-101,569.300	-101,236.800	-92,712.370	-93,685.550	-82,671.200	-82,538.530	-73,947.310
LR Test	43,955.070*** (df = 7)	69,604.490*** (df = 27)	70,269.380*** (df = 32)	87,318.310*** (df = 42)	85,371.940*** (df = 12)	107,400.600*** (df = 32)	107,666.000*** (df = 37)	124,848.400*** (df = 47)

Table 6 - Standard errors for multinomial logit model - 1997 - no segmentation on data.

Multinomial logit model - results for 2007

Logistic function standard errors. No segmentation on data. Car is the base mode.

	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Intercept								
Multi_Transit	(0.015)	(0.103)	(0.103)	(0.124)	(0.040)	(0.119)	(0.119)	(0.141)
One_Bus	(0.010)	(0.079)	(0.079)	(0.098)	(0.023)	(0.084)	(0.084)	(0.104)
One_Plus_Bus	(0.013)	(0.100)	(0.100)	(0.121)	(0.028)	(0.109)	(0.109)	(0.130)
Rapid_Transit	(0.022)	(0.123)	(0.123)	(0.152)	(0.036)	(0.126)	(0.127)	(0.155)
Walking	(0.012)	(0.077)	(0.077)	(0.097)	(0.022)	(0.093)	(0.093)	(0.115)
Cost: generic								
Generic	(0.003)	(0.003)	(0.003)	(0.003)	(0.008)	(0.008)	(0.008)	(0.008)
Duration: generic								
Generic	(0.0002)	(0.0002)	(0.0002)	(0.0002)				
Duration: mode specific								
Car					(0.001)	(0.001)	(0.001)	(0.001)
Multi_Transit					(0.001)	(0.001)	(0.001)	(0.001)
One_Bus					(0.0004)	(0.0004)	(0.0004)	(0.0004)
One_Plus_Bus					(0.0004)	(0.0004)	(0.0004)	(0.0004)
Rapid_Transit					(0.001)	(0.001)	(0.001)	(0.001)
Walking					(0.001)	(0.001)	(0.001)	(0.001)
Familiar income								
Multi_Transit		(0.006)	(0.006)	(0.006)		(0.006)	(0.006)	(0.006)
One_Bus		(0.005)	(0.005)	(0.004)		(0.005)	(0.005)	(0.005)
One_Plus_Bus		(0.008)	(0.008)	(0.008)		(0.008)	(0.008)	(0.008)
Rapid_Transit		(0.006)	(0.006)	(0.005)		(0.006)	(0.006)	(0.006)
Walking		(0.004)	(0.004)	(0.004)		(0.005)	(0.005)	(0.004)
Age								
Multi_Transit		(0.005)	(0.005)	(0.006)		(0.005)	(0.006)	(0.006)
One_Bus		(0.003)	(0.004)	(0.004)		(0.004)	(0.004)	(0.004)
One_Plus_Bus		(0.005)	(0.005)	(0.005)		(0.005)	(0.005)	(0.006)
Rapid_Transit		(0.006)	(0.006)	(0.007)		(0.006)	(0.006)	(0.007)
Walking		(0.003)	(0.004)	(0.004)		(0.004)	(0.004)	(0.005)
Age^2								
Multi_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
One_Bus		(0.00004)	(0.00004)	(0.00005)		(0.00004)	(0.00004)	(0.00005)
One_Plus_Bus		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Rapid_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Walking		(0.00004)	(0.00004)	(0.00004)		(0.00004)	(0.00005)	(0.0001)
Dummy gender: 1 = male								
Multi_Transit		(0.031)	(0.031)	(0.032)		(0.035)	(0.035)	(0.036)
One_Bus		(0.022)	(0.023)	(0.023)		(0.022)	(0.022)	(0.024)
One_Plus_Bus		(0.029)	(0.030)	(0.031)		(0.031)	(0.031)	(0.032)
Rapid_Transit		(0.040)	(0.041)	(0.041)		(0.040)	(0.040)	(0.040)
Walking		(0.021)	(0.021)	(0.022)		(0.026)	(0.026)	(0.027)
Dummy employment: 1 = employed								
Multi_Transit			(0.044)	(0.045)			(0.049)	(0.050)
One_Bus			(0.029)	(0.030)			(0.028)	(0.030)
One_Plus_Bus			(0.042)	(0.043)			(0.043)	(0.044)
Rapid_Transit			(0.053)	(0.054)			(0.052)	(0.053)
Walking			(0.026)	(0.027)			(0.032)	(0.033)
Dummy study: 1 = student								
Multi_Transit				(0.047)				(0.051)
One_Bus				(0.036)				(0.036)
One_Plus_Bus				(0.047)				(0.048)
Rapid_Transit				(0.058)				(0.056)
Walking				(0.035)				(0.044)
Dummy car ownership: 1 = own a car								
Multi_Transit				(0.044)				(0.048)
One_Bus				(0.036)				(0.036)
One_Plus_Bus				(0.042)				(0.043)
Rapid_Transit				(0.054)				(0.053)
Walking				(0.036)				(0.041)
Observations	102,045	102,045	102,045	102,045	102,045	102,045	102,045	102,045
R2	0.126	0.247	0.250	0.287	0.284	0.390	0.391	0.430
Log Likelihood	-134,045.200	-115,451.000	-114,996.500	-109,376.800	-109,841.500	-93,474.800	-93,336.870	-87,442.540
LR Test	38,624.020*** (df = 7)	75,812.470*** (df = 27)	76,721.420*** (df = 32)	87,960.950*** (df = 42)	87,031.580*** (df = 12)	119,764.900*** (df = 32)	120,040.800*** (df = 37)	131,829.400*** (df = 47)

Table 7 - Coefficients for multinomial logit model - 1997 - Vale Transporte users excluded.

Multinomial logit model - results for 1997								
Logistic function odds coefficients. Only travels made with employer subsidies (Vale Transporte). Car is the base mode.								
	Cost and duration coefficients are generic				**Cost coefficient in generic, duration is mode-specific**			
	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Intercept								
Multi_Transit	-2.051***	3.330***	3.216***	5.068***	-3.730***	2.318***	2.172***	3.704***
One_Bus	-0.204***	6.133***	5.958***	8.264***	-0.502***	5.926***	5.734***	8.103***
One_Plus_Bus	-1.936***	3.278***	3.355***	6.164***	-3.502***	2.226***	2.210***	4.829***
Rapid_Transit	-2.834***	2.002***	1.875***	3.140***	-2.882***	1.784***	1.680***	3.081***
Walking	1.550***	8.376***	8.183***	10.598***	3.502***	10.678***	10.500***	13.093***
Cost: generic								
Generic	-0.181***	-0.207***	-0.208***	-0.197***	0.017**	0.005	0.006	0.015*
Duration: generic								
Generic	-0.040***	-0.037***	-0.037***	-0.037***				
Duration: mode specific								
Car					0.004***	-0.0001	-0.0004	-0.0005
Multi_Transit					0.041***	0.040***	0.040***	0.040***
One_Bus					0.005***	0.004***	0.005***	0.003***
One_Plus_Bus					0.021***	0.019***	0.019***	0.018***
Rapid_Transit					0.015***	0.013***	0.013***	0.012***
Walking					-0.094***	-0.094***	-0.094***	-0.099***
Familiar income								
Multi_Transit		-0.103***	-0.100***	-0.054***		-0.092***	-0.089***	-0.048***
One_Bus		-0.175***	-0.171***	-0.096***		-0.177***	-0.172***	-0.098***
One_Plus_Bus		-0.201***	-0.205***	-0.107***		-0.191***	-0.193***	-0.105***
Rapid_Transit		-0.030***	-0.028***	0.002		-0.033***	-0.031***	-0.001
Walking		-0.167***	-0.163***	-0.081***		-0.175***	-0.171***	-0.083***
Age								
Multi_Transit		-0.236***	-0.219***	-0.214***		-0.274***	-0.251***	-0.239***
One_Bus		-0.275***	-0.252***	-0.252***		-0.277***	-0.251***	-0.250***
One_Plus_Bus		-0.223***	-0.235***	-0.253***		-0.249***	-0.249***	-0.260***
Rapid_Transit		-0.232***	-0.213***	-0.189***		-0.222***	-0.206***	-0.183***
Walking		-0.309***	-0.285***	-0.285***		-0.321***	-0.297***	-0.283***
Age^2								
Multi_Transit		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
One_Bus		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
One_Plus_Bus		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
Rapid_Transit		0.003***	0.003***	0.002***		0.003***	0.002***	0.002***
Walking		0.003***	0.003***	0.003***		0.003***	0.003***	0.003***
Dummy gender: 1 = male								
Multi_Transit		-1.006***	-0.921***	-0.909***		-1.169***	-1.064***	-1.054***
One_Bus		-1.145***	-1.032***	-1.027***		-1.108***	-0.985***	-0.986***
One_Plus_Bus		-0.803***	-0.836***	-0.819***		-0.973***	-0.956***	-0.938***
Rapid_Transit		-0.875***	-0.784***	-0.771***		-0.783***	-0.702***	-0.696***
Walking		-1.005***	-0.885***	-0.876***		-1.047***	-0.917***	-0.868***
Dummy employment: 1 = employed								
Multi_Transit			-0.336***	-0.335***			-0.459***	-0.427***
One_Bus			-0.445***	-0.505***			-0.511***	-0.571***
One_Plus_Bus			0.227***	0.110*			0.039	-0.023
Rapid_Transit			-0.368***	-0.326***			-0.321***	-0.293***
Walking			-0.450***	-0.551***			-0.470***	-0.583***
Dummy study: 1 = student								
Multi_Transit				0.372***				0.606***
One_Bus				0.192***				0.238***
One_Plus_Bus				-0.192**				-0.021
Rapid_Transit				0.760***				0.696***
Walking				0.160***				0.253***
Dummy car ownership: 1 = own a car								
Multi_Transit				-2.741***				-2.614***
One_Bus				-3.261***				-3.277***
One_Plus_Bus				-3.470***				-3.322***
Rapid_Transit				-2.597***				-2.665***
Walking				-3.445***				-3.912***
Observations	79,346	79,346	79,346	79,346	79,346	79,346	79,346	79,346
R2	0.161	0.273	0.275	0.340	0.312	0.407	0.409	0.474
Log Likelihood	-90,180.150	-78,204.220	-77,995.630	-70,947.100	-73,958.710	-63,757.220	-63,548.630	-56,562.250
LR Test	34,734.650*** (df = 7)	58,686.510*** (df = 27)	59,103.680*** (df = 32)	73,200.760*** (df = 42)	67,177.540*** (df = 12)	87,580.520*** (df = 32)	87,997.700*** (df = 37)	101,970.400*** (df = 47)

Table 8 - Coefficients for multinomial logit model - 2007 - Vale Transporte users excluded.

Multinomial logit model - results for 2007								
Logistic function odds coefficients. Only travels made with employer subsidies (Vale Transporte). Car is the base mode.								
	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Intercept								
Multi_Transit	-2.164***	4.261***	4.087***	4.651***	-4.061***	2.901***	2.702***	3.051***
One_Bus	-0.256***	7.159***	6.849***	7.184***	-0.795***	6.974***	6.656***	7.150***
One_Plus_Bus	-1.573***	4.784***	4.643***	5.294***	-2.967***	3.967***	3.757***	4.399***
Rapid_Transit	-3.012***	2.938***	2.726***	2.562***	-3.217***	2.634***	2.449***	2.459***
Walking	1.420***	9.372***	9.187***	10.149***	3.290***	11.837***	11.684***	12.609***
Cost: generic								
Generic	-0.190***	-0.211***	-0.216***	-0.209***	0.052***	0.050***	0.048***	0.038***
Duration: generic								
Generic	-0.037***	-0.035***	-0.035***	-0.035***				
Duration: mode specific								
Car					0.004***	-0.001	-0.001	0.0001
Multi_Transit					0.042***	0.041***	0.041***	0.041***
One_Bus					0.006***	0.005***	0.005***	0.004***
One_Plus_Bus					0.021***	0.019***	0.019***	0.018***
Rapid_Transit					0.021***	0.018***	0.019***	0.018***
Walking					-0.082***	-0.084***	-0.084***	-0.088***
Familiar income								
Multi_Transit		-0.251***	-0.244***	-0.131***		-0.222***	-0.213***	-0.108***
One_Bus		-0.390***	-0.375***	-0.243***		-0.403***	-0.386***	-0.249***
One_Plus_Bus		-0.447***	-0.441***	-0.288***		-0.438***	-0.426***	-0.274***
Rapid_Transit		-0.108***	-0.102***	-0.029***		-0.112***	-0.106***	-0.030***
Walking		-0.401***	-0.395***	-0.254***		-0.384***	-0.379***	-0.229***
Age								
Multi_Transit		-0.243***	-0.215***	-0.176***		-0.279***	-0.246***	-0.198***
One_Bus		-0.278***	-0.236***	-0.189***		-0.293***	-0.249***	-0.203***
One_Plus_Bus		-0.214***	-0.190***	-0.154***		-0.244***	-0.210***	-0.173***
Rapid_Transit		-0.250***	-0.218***	-0.155***		-0.246***	-0.215***	-0.153***
Walking		-0.299***	-0.272***	-0.242***		-0.319***	-0.292***	-0.248***
Age^2								
Multi_Transit		0.003***	0.002***	0.002***		0.003***	0.003***	0.002***
One_Bus		0.003***	0.002***	0.002***		0.003***	0.003***	0.002***
One_Plus_Bus		0.002***	0.002***	0.002***		0.003***	0.002***	0.002***
Rapid_Transit		0.003***	0.002***	0.002***		0.003***	0.002***	0.002***
Walking		0.003***	0.003***	0.002***		0.003***	0.003***	0.002***
Dummy gender: 1 = male								
Multi_Transit		-1.119***	-1.036***	-1.020***		-1.275***	-1.182***	-1.166***
One_Bus		-1.254***	-1.128***	-1.109***		-1.248***	-1.121***	-1.113***
One_Plus_Bus		-1.220***	-1.148***	-1.126***		-1.372***	-1.275***	-1.271***
Rapid_Transit		-0.936***	-0.841***	-0.832***		-0.860***	-0.767***	-0.763***
Walking		-1.223***	-1.135***	-1.104***		-1.244***	-1.156***	-1.083***
Dummy employment: 1 = employed								
Multi_Transit			-0.510***	-0.453***			-0.612***	-0.563***
One_Bus			-0.760***	-0.719***			-0.816***	-0.812***
One_Plus_Bus			-0.429***	-0.421***			-0.622***	-0.625***
Rapid_Transit			-0.591***	-0.473***			-0.585***	-0.518***
Walking			-0.484***	-0.501***			-0.492***	-0.481***
Dummy study: 1 = student								
Multi_Transit				0.939***				1.181***
One_Bus				1.099***				1.092***
One_Plus_Bus				0.862***				0.970***
Rapid_Transit				1.475***				1.431***
Walking				0.625***				0.847***
Dummy car ownership: 1 = own a car								
Multi_Transit				-2.672***				-2.653***
One_Bus				-2.733***				-2.796***
One_Plus_Bus				-2.773***				-2.758***
Rapid_Transit				-2.547***				-2.639***
Walking				-2.781***				-3.187***
Observations	89,595	89,595	89,595	89,595	89,595	89,595	89,595	89,595
R2	0.140	0.283	0.286	0.331	0.296	0.420	0.423	0.471
Log Likelihood	-104,206.400	-86,932.150	-86,598.790	-81,076.360	-85,387.180	-70,303.550	-69,911.220	-64,181.800
LR Test	34,062.490*** (df = 7)	68,610.880*** (df = 27)	69,277.610*** (df = 32)	80,322.460*** (df = 42)	71,700.820*** (df = 12)	101,868.100*** (df = 32)	102,652.800*** (df = 37)	114,111.600*** (df = 47)

Table 9 - Standard errors for multinomial logit model - 1997 - Vale Transporte users excluded.

Multinomial logit model - results for 1997								
	Logistic function standard errors. Only travels made with employer subsidies (Vale Transporte). Car is the base mode.							
	Cost and duration coefficients are generic				**Cost coefficient in generic, duration is mode-specific**			
	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Intercept								
Multi_Transit	(0.020)	(0.108)	(0.109)	(0.148)	(0.046)	(0.118)	(0.120)	(0.159)
One_Bus	(0.012)	(0.075)	(0.077)	(0.111)	(0.020)	(0.079)	(0.080)	(0.115)
One_Plus_Bus	(0.023)	(0.122)	(0.124)	(0.166)	(0.045)	(0.132)	(0.134)	(0.172)
Rapid_Transit	(0.026)	(0.133)	(0.135)	(0.185)	(0.040)	(0.135)	(0.137)	(0.186)
Walking	(0.013)	(0.074)	(0.075)	(0.110)	(0.024)	(0.088)	(0.090)	(0.128)
Cost: generic								
Generic	(0.003)	(0.003)	(0.003)	(0.003)	(0.007)	(0.008)	(0.008)	(0.008)
Duration: generic								
Generic	(0.0003)	(0.0003)	(0.0003)	(0.0003)				
Duration: mode specific								
Car					(0.001)	(0.001)	(0.001)	(0.001)
Multi_Transit					(0.001)	(0.001)	(0.001)	(0.001)
One_Bus					(0.001)	(0.001)	(0.001)	(0.001)
One_Plus_Bus					(0.001)	(0.001)	(0.001)	(0.001)
Rapid_Transit					(0.001)	(0.001)	(0.001)	(0.001)
Walking					(0.001)	(0.001)	(0.001)	(0.001)
Familiar income								
Multi_Transit		(0.005)	(0.005)	(0.005)		(0.005)	(0.005)	(0.005)
One_Bus		(0.003)	(0.003)	(0.003)		(0.003)	(0.003)	(0.003)
One_Plus_Bus		(0.009)	(0.009)	(0.009)		(0.009)	(0.009)	(0.009)
Rapid_Transit		(0.004)	(0.004)	(0.004)		(0.004)	(0.004)	(0.004)
Walking		(0.003)	(0.003)	(0.003)		(0.003)	(0.003)	(0.003)
Age								
Multi_Transit		(0.006)	(0.006)	(0.007)		(0.006)	(0.006)	(0.007)
One_Bus		(0.004)	(0.004)	(0.005)		(0.004)	(0.004)	(0.005)
One_Plus_Bus		(0.006)	(0.007)	(0.008)		(0.007)	(0.007)	(0.008)
Rapid_Transit		(0.007)	(0.007)	(0.009)		(0.007)	(0.007)	(0.008)
Walking		(0.004)	(0.004)	(0.005)		(0.004)	(0.005)	(0.006)
Age^2								
Multi_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
One_Bus		(0.00005)	(0.00005)	(0.0001)		(0.00005)	(0.0001)	(0.0001)
One_Plus_Bus		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Rapid_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Walking		(0.00005)	(0.00005)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Dummy gender: 1 = male								
Multi_Transit		(0.040)	(0.041)	(0.042)		(0.043)	(0.045)	(0.046)
One_Bus		(0.024)	(0.025)	(0.028)		(0.024)	(0.025)	(0.028)
One_Plus_Bus		(0.047)	(0.049)	(0.051)		(0.049)	(0.051)	(0.053)
Rapid_Transit		(0.050)	(0.052)	(0.052)		(0.049)	(0.051)	(0.051)
Walking		(0.023)	(0.024)	(0.027)		(0.028)	(0.030)	(0.032)
Dummy employment: 1 = employed								
Multi_Transit			(0.048)	(0.049)			(0.052)	(0.053)
One_Bus			(0.030)	(0.032)			(0.029)	(0.032)
One_Plus_Bus			(0.060)	(0.063)			(0.061)	(0.064)
Rapid_Transit			(0.060)	(0.061)			(0.058)	(0.059)
Walking			(0.028)	(0.031)			(0.035)	(0.038)
Dummy study: 1 = student								
Multi_Transit				(0.063)				(0.065)
One_Bus				(0.042)				(0.041)
One_Plus_Bus				(0.082)				(0.081)
Rapid_Transit				(0.077)				(0.075)
Walking				(0.041)				(0.051)
Dummy car ownership: 1 = own a car								
Multi_Transit				(0.053)				(0.056)
One_Bus				(0.041)				(0.041)
One_Plus_Bus				(0.059)				(0.061)
Rapid_Transit				(0.064)				(0.063)
Walking				(0.040)				(0.047)
Observations	79,346	79,346	79,346	79,346	79,346	79,346	79,346	79,346
R2	0.161	0.273	0.275	0.340	0.312	0.407	0.409	0.474
Log Likelihood	-90,180.150	-78,204.220	-77,995.630	-70,947.100	-73,958.710	-63,757.220	-63,548.630	-56,562.250
LR Test	34,734.650*** (df = 7)	58,686.510*** (df = 27)	59,103.680*** (df = 32)	73,200.760*** (df = 42)	67,177.540*** (df = 12)	87,580.520*** (df = 32)	87,997.700*** (df = 37)	101,970.400*** (df = 47)

Table 10 - Standard errors for multinomial logit model - 2007 - Vale Transporte users excluded.

Multinomial logit model - results for 2007								
Logistic function standard errors. Only travels made with employer subsidies (Vale Transporte). Car is the base mode.								
	Cost and duration coefficients are generic				**Cost coefficient in generic, duration is mode-specific**			
	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Intercept								
Multi_Transit	(0.019)	(0.117)	(0.118)	(0.150)	(0.048)	(0.134)	(0.135)	(0.168)
One_Bus	(0.011)	(0.082)	(0.082)	(0.106)	(0.026)	(0.088)	(0.089)	(0.113)
One_Plus_Bus	(0.017)	(0.114)	(0.115)	(0.145)	(0.035)	(0.125)	(0.125)	(0.156)
Rapid_Transit	(0.026)	(0.136)	(0.138)	(0.179)	(0.042)	(0.140)	(0.142)	(0.182)
Walking	(0.012)	(0.078)	(0.079)	(0.100)	(0.022)	(0.093)	(0.094)	(0.119)
Cost: generic								
Generic	(0.003)	(0.004)	(0.004)	(0.004)	(0.009)	(0.009)	(0.009)	(0.010)
Duration: generic								
Generic	(0.0003)	(0.0003)	(0.0003)	(0.0003)				
Duration: mode specific								
Car					(0.001)	(0.001)	(0.001)	(0.001)
Multi_Transit					(0.001)	(0.001)	(0.001)	(0.001)
One_Bus					(0.001)	(0.001)	(0.001)	(0.001)
One_Plus_Bus					(0.001)	(0.001)	(0.001)	(0.001)
Rapid_Transit					(0.001)	(0.001)	(0.001)	(0.001)
Walking					(0.001)	(0.001)	(0.001)	(0.001)
Familiar income								
Multi_Transit		(0.007)	(0.007)	(0.007)		(0.007)	(0.007)	(0.007)
One_Bus		(0.005)	(0.005)	(0.005)		(0.005)	(0.005)	(0.005)
One_Plus_Bus		(0.009)	(0.009)	(0.009)		(0.010)	(0.010)	(0.010)
Rapid_Transit		(0.006)	(0.006)	(0.006)		(0.006)	(0.006)	(0.006)
Walking		(0.004)	(0.004)	(0.004)		(0.005)	(0.005)	(0.004)
Age								
Multi_Transit		(0.005)	(0.006)	(0.007)		(0.006)	(0.007)	(0.007)
One_Bus		(0.004)	(0.004)	(0.004)		(0.004)	(0.004)	(0.005)
One_Plus_Bus		(0.005)	(0.006)	(0.006)		(0.006)	(0.006)	(0.007)
Rapid_Transit		(0.006)	(0.007)	(0.008)		(0.006)	(0.007)	(0.008)
Walking		(0.003)	(0.004)	(0.004)		(0.004)	(0.004)	(0.005)
Age^2								
Multi_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
One_Bus		(0.00004)	(0.00004)	(0.00005)		(0.00004)	(0.00004)	(0.00005)
One_Plus_Bus		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Rapid_Transit		(0.0001)	(0.0001)	(0.0001)		(0.0001)	(0.0001)	(0.0001)
Walking		(0.00004)	(0.00004)	(0.00004)		(0.00004)	(0.00005)	(0.0001)
Dummy gender: 1 = male								
Multi_Transit		(0.040)	(0.040)	(0.041)		(0.044)	(0.045)	(0.045)
One_Bus		(0.025)	(0.025)	(0.026)		(0.025)	(0.025)	(0.027)
One_Plus_Bus		(0.039)	(0.039)	(0.040)		(0.041)	(0.041)	(0.042)
Rapid_Transit		(0.048)	(0.049)	(0.049)		(0.046)	(0.047)	(0.047)
Walking		(0.022)	(0.022)	(0.023)		(0.026)	(0.027)	(0.028)
Dummy employment: 1 = employed								
Multi_Transit			(0.048)	(0.049)			(0.053)	(0.054)
One_Bus			(0.030)	(0.031)			(0.029)	(0.031)
One_Plus_Bus			(0.046)	(0.047)			(0.048)	(0.049)
Rapid_Transit			(0.056)	(0.057)			(0.054)	(0.055)
Walking			(0.027)	(0.028)			(0.032)	(0.033)
Dummy study: 1 = student								
Multi_Transit				(0.061)				(0.064)
One_Bus				(0.041)				(0.040)
One_Plus_Bus				(0.060)				(0.061)
Rapid_Transit				(0.070)				(0.068)
Walking				(0.037)				(0.046)
Dummy car ownership: 1 = own a car								
Multi_Transit				(0.052)				(0.057)
One_Bus				(0.038)				(0.039)
One_Plus_Bus				(0.050)				(0.052)
Rapid_Transit				(0.064)				(0.061)
Walking				(0.036)				(0.042)
Observations	89,595	89,595	89,595	89,595	89,595	89,595	89,595	89,595
R2	0.140	0.283	0.286	0.331	0.296	0.420	0.423	0.471
Log Likelihood	-104,206.400	-86,932.150	-86,598.790	-81,076.360	-85,387.180	-70,303.550	-69,911.220	-64,181.800
LR Test	34,062.490*** (df = 7)	68,610.880*** (df = 27)	69,277.610*** (df = 32)	80,322.460*** (df = 42)	71,700.820*** (df = 12)	101,868.100*** (df = 32)	102,652.800*** (df = 37)	114,111.600*** (df = 47)

The regressions presented on the tables above are multinomial logits, so the coefficients are not directly interpretable as marginal effects. Prior to any odds-ratio transformation, the analysis of coefficients' signals and magnitudes helps interpreting the results of regressions.

The present work is most interested in alternatives' characteristics effects over the probability of the modal being chosen. Hence, special attention is given for those variables. Nevertheless, the other variables - related to the individual - are briefly interpreted below since they can provide policy insights. Once *Car* is the base mode in all estimations, coefficients are interpreted in relation to it. For instance, if familiar income varies, holding everything else constant, the probability of the indicated mode being chosen varies with respect to car, in the direction of the coefficient signal.

The models estimated with no segmentation on data have the signals of the coefficients as expected. Cost (generic) and duration (generic) coefficients are negative, and duration (specific) is mostly negative for *Car* and *Walking*, and mostly positive for the other modes, except for *One Bus* in 2007. For the models in which the trips consider only out-of-the-pocket (non-*Vale Transporte* takers*)* paying commuters, the same is observed for duration (generic) and for cost when duration is generic. When duration is mode-specific, cost becomes positive for 2007 and positive, but not statistically significant for 1997. This does not make sense since it is unexpected that one would become more prone to travel when the trip gets more expensive and the reasons for that are hard to understand. This might be caused by some noise in the data, or since the models do not allow the individual not to travel – once she has to choose a modal, probabilities might be contaminated if there are much more observations that choose the most expensive mode. The present work gives itself the benefit of these doubts and only the regressions that make sense are used for later predictions. Thus, the interpretation is as follows:

- Alternative specific variables (t_{im} and x_{im}):
    - If commuting gets more expensive (cheaper), the probability of choosing any mode decreases (increases) – except in the case explained above.
    - If commuting gets longer (shorter), the probability of choosing any mode decreases (increases). If we compare commuting times between modes (that is,

alternative specific coefficients), if the trip gets longer, the probability of choosing *Car* and *Walking* in 1997 and *Car*, *Walking* and *One Bus* in 2007 decreases while the other modes' odds increases. Probably this has to do with the average speed of other modes: since the regressions only control for travel distance in an implicit way (since cost and duration are function of it), faster modes such as subway or some multimodal combination become a better choice. If a person lives far away from his or her job there might be no alternative other than taking more than one bus.

- Individual specific variables (z_i):
  - When familiar income increases, the probability of any mode but *Car* being chosen decreases. This confirms what is presented on the figures that depict mode distribution in São Paulo: the wealthier the person, the higher is the chance that he or she will commute by car.
  - The effect is the same for age: when it increases, the more likely one is to choose *car*. *Age*^{2}, however, has the opposite effect, indicating that this phenomenon has an upper limit. After a certain age is reached, individuals are more likely to walk or to choose transit.
  - Being male or owning a car also increases the probability of choosing *car*.
  - Being a student, on the other hand, decreases the probability of choosing *car* for all the modes estimated for 2007. Interestingly, however, this coefficient has the opposite sign for *more than one bus* (One_Plus_Bus) in the 1997's models. That is, for 1997, it was expected that being a student - everything else held constant – would decreases the probability of choosing *more than one bus* compared to car; in 2007, the opposite is expected, although coefficients do not present statistical significance when duration is mode-specific. This might indicate that students are more prone to ride transit in 2007 than they were in comparison to 1997 and this might be Bilhete Único's effect. Nevertheless, Brazil has changed a lot during those years and it is hard to identify what phenomena caused this change.

- Last, being employed is the characteristic that varies most among models. For 1997 and 2007 and no segmentation on data, it decreases the probability of *one bus* or *walking* being chosen and increases the odds of multimodality and *more than one bus*. It has no significant effect over *rapid transit*'s chances. For the models segmented for *Vale Transporte* only, it has negative and statistically significant effects over all modes – except for *more than one bus* in the models in which duration is mode-specific.

4.1.2 Multinomial Logit: test statistics

The first test to be applied bears on IIA. It departs from the formulation that a "model can be reestimated on a subset of the alternatives (and) under IIA, the ratio of probabilities for any two alternatives is the same whether or not other alternatives are available." (Train, 2009, p. 49). That is, a subset of alternatives is defined, and the null hypothesis is that the parameters of the subset are the same as the parameters on the full set. This is proposed in Hausman and McFadden (1984). The null hypothesis is that parameters are the same – that is, IIA holds and choice ratio between two alternatives is independent from other alternatives. The alternative hypothesis is that coefficients' ratio is different and hence IIA does not hold. If the null hypothesis is rejected, the dependent variables are not uncorrelated in the models used for the test.

During the author's master's candidacy exam, it was proposed to do so excluding alternatives *Car* and *Walking* to form a subset. The results are presented in

Table 11 for the models numbered with (8) for 1997 and 2007. Since the chi-squared is negative for these tests, it is not possible to reject the null hypothesis. This is an evidence that IIA has not been violated (Hausman & McFadden, 1984, p. 1226; Long & Freese, 2014, p. 409).

Nevertheless, Long & Freese themselves said that IIA tests are frequently inconclusive and "that these tests are not useful for assessing violations of the IIA property" (2014, p. 208). The author chooses to go on with the analysis and present a Nested Logit Model, for exhaustiveness sake.

Table 11 - Hausman-McFadden test for Multinomial Logit Models (8), conducted with *mlogit package* in R.

```
> hmftest(model_8_97,z = c("Rapid_Transit","One_Plus_Bus","Multi_Transit","One_Bus"))

        Hausman-McFadden test

data:  long1997
chisq = -183130000, df = 29, p-value = 1
alternative hypothesis: IIA is rejected

> hmftest(model_8_07,z = c("Rapid_Transit","One_Plus_Bus","Multi_Transit","One_Bus"))

        Hausman-McFadden test

data:  long2007
chisq = -40592, df = 29, p-value = 1
alternative hypothesis: IIA is rejected
```

The other three tests - Wald, Lagrange Multiplier and Likelihood ratio, as suggested by Croissant (2012) - presented on tables 12, 13 and 14 evaluate whether the covariates (presented on the model numbered with (8)) enter the model significantly. Model (8) is compared with a covariates-constrained model, (5).

Table 12 - Wald, Lagrange multiplier and Likelihood ratio tests for Multinomial Logit Models (8), conducted with *mlogit package* in R.

```
> waldtest(model_8_97, model_8_97_constrained)
Wald test

Model 1: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY +
    D_STUDENT + D_CAR | DURATION
Model 2: MODE ~ COST | 1 | DURATION
  Res.Df  Df Chisq Pr(>Chisq)
1  93701
2  93736 -35 20111  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> waldtest(model_8_07, model_8_07_constrained)
Wald test

Model 1: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY +
    D_STUDENT + D_CAR | DURATION
Model 2: MODE ~ COST | 1 | DURATION
  Res.Df  Df Chisq Pr(>Chisq)
1 101998
2 102033 -35 24312  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
> scoretest(model_8_97_constrained, model_8_97)

        score test

data:  MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY +     D_STUDENT + D_CAR | DURATION
chisq = 28699, df = 35, p-value < 2.2e-16
alternative hypothesis: unconstrained model

> scoretest(model_8_07_constrained, model_8_07)

        score test

data:  MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY +     D_STUDENT + D_CAR | DURATION
chisq = 31608, df = 35, p-value < 2.2e-16
alternative hypothesis: unconstrained model

>
> lrtest(model_8_97, model_8_97_constrained)
Likelihood ratio test

Model 1: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY +
    D_STUDENT + D_CAR | DURATION
Model 2: MODE ~ COST | 1 | DURATION
  #Df LogLik  Df Chisq Pr(>Chisq)
1  47 -73945
2  12 -93684 -35 39477  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> lrtest(model_8_07, model_8_07_constrained)
Likelihood ratio test

Model 1: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY +
    D_STUDENT + D_CAR | DURATION
Model 2: MODE ~ COST | 1 | DURATION
  #Df  LogLik  Df Chisq Pr(>Chisq)
1  47  -87443
2  12 -109841 -35 44798  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> |
```

Similarly, the tests are presented in for the regressions numbered with (12) in the specification that excludes the trips paid by the employer through the *Vale Transporte*.

Table 13 - Hausman-McFadden test for Multinomial Logit Models (12), conducted with mlogit package in R.

```
> hmftest(model_12_97,z = c("Rapid_Transit","One_Plus_Bus","Multi_Transit","One_Bus"))

        Hausman-McFadden test

data:  long1997
chisq = 6810.2, df = 26, p-value < 2.2e-16
alternative hypothesis: IIA is rejected

> hmftest(model_12_07,z = c("Rapid_Transit","One_Plus_Bus","Multi_Transit","One_Bus"))

        Hausman-McFadden test

data:  long2007
chisq = 10235, df = 26, p-value < 2.2e-16
alternative hypothesis: IIA is rejected
```

Table 14 - Wald, Lagrange multiplier and Likelihood ratio tests for Multinomial Logit Models (12), conducted with *mlogit package* in R.

```
> waldtest(model_12_97, model_9_97_constrained)
Wald test

Model 1: MODE ~ COST + DURATION | INC_FAMILIAR + AGE + AGE_2 + D_MALE + 
    D_EMPLOY + D_STUDENT + D_CAR
Model 2: MODE ~ COST + DURATION | 1 | 1
  Res.Df  Df Chisq Pr(>Chisq)    
1  34571                         
2  34606 -35  8074  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> waldtest(model_12_07, model_9_07_constrained)
Wald test

Model 1: MODE ~ COST + DURATION | INC_FAMILIAR + AGE + AGE_2 + D_MALE + 
    D_EMPLOY + D_STUDENT + D_CAR
Model 2: MODE ~ COST + DURATION | 1 | 1
  Res.Df  Df  Chisq Pr(>Chisq)    
1  42165                          
2  42200 -35 9411.7  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> 
> scoretest(model_9_97_constrained, model_12_97)

        score test

data:  MODE ~ COST + DURATION | INC_FAMILIAR + AGE + AGE_2 + D_MALE +     D_EMPLOY + D_STUDENT + D_CAR | 1
chisq = 12485, df = 35, p-value < 2.2e-16
alternative hypothesis: unconstrained model

> scoretest(model_9_07_constrained, model_12_07)

        score test

data:  MODE ~ COST + DURATION | INC_FAMILIAR + AGE + AGE_2 + D_MALE +     D_EMPLOY + D_STUDENT + D_CAR | 1
chisq = 14654, df = 35, p-value < 2.2e-16
alternative hypothesis: unconstrained model

> 
> lrtest(model_12_97, model_9_97_constrained)
Likelihood ratio test

Model 1: MODE ~ COST + DURATION | INC_FAMILIAR + AGE + AGE_2 + D_MALE + 
    D_EMPLOY + D_STUDENT + D_CAR
Model 2: MODE ~ COST + DURATION | 1 | 1
  #Df LogLik  Df Chisq Pr(>Chisq)    
1  42 -41362                         
2   7 -48974 -35 15225  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> lrtest(model_12_07, model_9_07_constrained)
Likelihood ratio test

Model 1: MODE ~ COST + DURATION | INC_FAMILIAR + AGE + AGE_2 + D_MALE + 
    D_EMPLOY + D_STUDENT + D_CAR
Model 2: MODE ~ COST + DURATION | 1 | 1
  #Df LogLik  Df Chisq Pr(>Chisq)    
1  42 -47151                         
2   7 -55653 -35 17004  < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
> 
```

4.2 Nested Logit

4.2.1 Nested Logit: regressions results

Once it is hard to assess IIA through traditional tests (Long & Freese, 2014), the author also chooses to present the results of a Nested Logit Model - much more as a robustness check. They are presented below. lthough their results seem good, the post-estimations are not in line with literature and other models and therefore are hard to trust.

For the Nested Logit model, since “there is no well-defined testing procedure for discriminating among tree structures” (Greene, 2012, p. 770), many nest structures and combinations of modes within nests were tested. Table 15 and Table 16 present the regressions’ results for the nests such as they are presented in Table 1.

One will notice that Walking was excluded from the choice set. This was the only identification strategy that resulted in a well-specified model. The active modal exclusion represents a limitation for external validity. Nevertheless, once the literature reviewed (and therefore the present work) cares more about the substitution between car and transit, the estimation is still valid for exhaustiveness’ sake.

Moreover, the regressions were estimated with a degenerate nest, that is, one set of nests that contains one branch with one modal only in it. It is the case of the Auto nest, that considers only Car as a choice option. The other branch, Transit, includes the options of public collective transportation – including the multimodal option since its characteristics were more similar to the alternatives constrained to Bus and Rail only, as is noticed in Figure 20 to Figure 24. As presented by Croissant (2012), this is not a limitation for estimation, but makes the nest elasticity hard to interpret “as it is related to the degree of correlation of the alternatives within the nests and that there is only one alternative in this nest” (Croissant, 2012, p. 35).

Table 15 - Coefficients for Nested Logit Model - 1997

Nested logit model - results for 1997

Logistic function odds coefficients. Car is the base mode.

	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Intercept								
Multi_Transit	1.857***	80.551***	80.144***	114.168***	-12.866***	241.292***	240.218***	345.747***
One_Bus	-1.102***	79.422***	78.901***	113.180***	-13.593***	241.234***	240.098***	346.637***
One_Plus_Bus	0.304**	79.029***	78.648***	113.319***	-13.245***	240.602***	239.546***	346.135***
Rapid_Transit	-2.506***	76.341***	75.875***	109.105***	-15.305***	238.043***	236.946***	342.280***
Nest elasticity								
Generic	0.057***	0.075***	0.076***	0.072***	0.015***	0.024***	0.024***	0.023***
Cost: generic								
Generic	-2.052***	-1.851***	-1.850***	-1.801***	-2.309***	-2.171***	-2.171***	-2.137***
Duration: generic								
Generic	-0.012***	-0.012***	-0.012***	-0.012***				
Duration: mode specific								
Car					-0.821***	-0.579***	-0.580***	-0.584***
Multi_Transit					0.034***	0.035***	0.035***	0.036***
One_Bus					-0.015***	-0.015***	-0.015***	-0.016***
One_Plus_Bus					0.007***	0.006***	0.006***	0.005***
Rapid_Transit					-0.005***	-0.005***	-0.005***	-0.005***
Familiar income								
Multi_Transit		-2.087***	-2.075***	-1.173***		-6.487***	-6.455***	-3.628***
One_Bus		-2.144***	-2.129***	-1.210***		-6.614***	-6.580***	-3.715***
One_Plus_Bus		-2.176***	-2.168***	-1.231***		-6.619***	-6.591***	-3.718***
Rapid_Transit		-2.013***	-2.000***	-1.108***		-6.473***	-6.440***	-3.607***
Age								
Multi_Transit		-3.273***	-3.234***	-3.317***		-10.436***	-10.282***	-10.498***
One_Bus		-3.337***	-3.271***	-3.355***		-10.430***	-10.260***	-10.494***
One_Plus_Bus		-3.265***	-3.241***	-3.340***		-10.400***	-10.260***	-10.497***
Rapid_Transit		-3.292***	-3.235***	-3.293***		-10.398***	-10.233***	-10.437***
Age^2								
Multi_Transit		0.036***	0.035***	0.036***		0.114***	0.112***	0.115***
One_Bus		0.036***	0.036***	0.037***		0.114***	0.111***	0.115***
One_Plus_Bus		0.036***	0.035***	0.037***		0.113***	0.112***	0.115***
Rapid_Transit		0.036***	0.035***	0.036***		0.113***	0.111***	0.115***
Dummy gender: 1 = male								
Multi_Transit		-14.081***	-13.874***	-14.661***		-44.402***	-43.716***	-46.089***
One_Bus		-14.302***	-14.038***	-14.827***		-44.296***	-43.574***	-45.950***
One_Plus_Bus		-13.955***	-13.792***	-14.583***		-44.127***	-43.475***	-45.850***
Rapid_Transit		-13.944***	-13.699***	-14.479***		-43.978***	-43.267***	-45.636***
Dummy employment: 1 = employed								
Multi_Transit			-0.613**	-2.361***			-2.796***	-8.429***
One_Bus			-1.100***	-2.874***			-3.109***	-8.825***
One_Plus_Bus			-0.290	-2.121***			-2.497***	-8.218***
Rapid_Transit			-0.937***	-2.615***			-3.017***	-8.621***
Dummy study: 1 = student								
Multi_Transit				3.961***				13.291***
One_Bus				3.972***				12.852***
One_Plus_Bus				3.623***				12.746***
Rapid_Transit				4.500***				13.480***
Dummy car ownership: 1 = own a car								
Multi_Transit				-44.472***				-139.809***
One_Bus				-44.802***				-140.501***
One_Plus_Bus				-44.972***				-140.567***
Rapid_Transit				-44.268***				-139.947***
Observations	58,777	58,777	58,777	58,777	58,777	58,777	58,777	58,777
R2	0.021	0.127	0.129	0.221	0.103	0.209	0.210	0.303
Log Likelihood	-72,918.530	-65,024.540	-64,837.880	-58,018.470	-66,741.930	-58,922.150	-58,841.690	-51,891.800
LR Test	3,055.571*** (df = 7)	18,843.540*** (df = 23)	19,216.880*** (df = 27)	32,855.680*** (df = 35)	15,408.770*** (df = 11)	31,048.330*** (df = 27)	31,209.250*** (df = 31)	45,109.030*** (df = 39)

Table 16 - Coefficients for Nested Logit Model - 2007

Nested logit model - results for 2007							
Logistic function odds coefficients. Car is the base mode.							
	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific		
	(17)	(18)	(19)	(20)	(22)	(23)	(24)
Intercept							
Multi_Transit	-0.628***	0.960***	0.538***	8.364***	137.192***	132.687***	124.044***
One_Bus	-0.223***	3.683***	3.127***	10.607***	141.293***	136.669***	128.275***
One_Plus_Bus	-1.144***	1.368***	0.967***	8.684***	137.991***	133.491***	125.093***
Rapid_Transit	-2.419***	-0.272**	-0.649***	6.408***	136.828***	132.306***	123.326***
Nest elasticity							
Generic	0.244***	2.313***	2.657***	0.843***	0.052***	0.054***	0.066***
Cost: generic							
Generic	-0.747***	-0.095***	-0.082***	-0.256***	-0.614***	-0.626***	-0.681***
Duration: generic							
Generic	-0.010***	-0.005***	-0.004***	-0.009***			
Duration: mode specific							
Car					-0.333***	-0.323***	-0.255***
Multi_Transit					0.027***	0.027***	0.028***
One_Bus					-0.016***	-0.016***	-0.016***
One_Plus_Bus					0.005***	0.005***	0.004***
Rapid_Transit					0.002***	0.002***	0.002***
Familiar income							
Multi_Transit		-0.107***	-0.084***	-0.261***	-7.918***	-7.650***	-3.952***
One_Bus		-0.263***	-0.227***	-0.373***	-8.165***	-7.895***	-4.175***
One_Plus_Bus		-0.378***	-0.368***	-0.455***	-8.186***	-7.922***	-4.185***
Rapid_Transit		0.054***	0.063***	-0.136***	-7.892***	-7.624***	-3.923***
Age							
Multi_Transit		-0.079***	-0.080***	-0.268***	-5.090***	-4.798***	-3.583***
One_Bus		-0.152***	-0.119***	-0.297***	-5.097***	-4.787***	-3.580***
One_Plus_Bus		-0.063***	-0.067***	-0.256***	-5.049***	-4.761***	-3.554***
Rapid_Transit		-0.103***	-0.086***	-0.259***	-5.073***	-4.778***	-3.554***
Age^2							
Multi_Transit		0.001***	0.001***	0.003***	0.052***	0.049***	0.037***
One_Bus		0.002***	0.001***	0.003***	0.052***	0.049***	0.037***
One_Plus_Bus		0.001***	0.001***	0.003***	0.052***	0.048***	0.036***
Rapid_Transit		0.001***	0.001***	0.003***	0.052***	0.049***	0.036***
Dummy gender: 1 = male							
Multi_Transit		-0.440***	-0.401***	-1.401***	-23.666***	-22.567***	-18.426***
One_Bus		-0.653***	-0.535***	-1.513***	-23.566***	-22.444***	-18.303***
One_Plus_Bus		-0.571***	-0.553***	-1.492***	-23.626***	-22.533***	-18.393***
Rapid_Transit		-0.225***	-0.142***	-1.217***	-23.365***	-22.263***	-18.121***
Dummy employment: 1 = employed							
Multi_Transit			0.324***	0.116***	-2.976***		-2.826***
One_Bus			-0.333***	-0.495***	-3.285***		-3.171***
One_Plus_Bus			0.378***	0.134***	-2.901***		-2.787***
Rapid_Transit			-0.063	-0.076	-3.029***		-2.845***
Dummy study: 1 = student							
Multi_Transit				0.639***			10.347***
One_Bus				0.849***			10.129***
One_Plus_Bus				0.642***			10.145***
Rapid_Transit				1.127***			10.556***
Dummy car ownership: 1 = own a car							
Multi_Transit				-3.155***			-41.770***
One_Bus				-3.300***			-41.915***
One_Plus_Bus				-3.350***			-42.020***
Rapid_Transit				-3.198***			-41.823***
Observations	69,200	69,200	69,200	69,200	69,200	69,200	69,200
R2	-0.041	0.105	0.109	0.164	0.184	0.185	0.242
Log Likelihood	-92,900.150	-79,869.720	-79,556.990	-74,584.610	-72,832.900	-72,753.610	-67,625.390
LR Test	-7,309.283 (df = 7)	18,751.580*** (df = 23)	19,377.040*** (df = 27)	29,321.810*** (df = 35)	32,825.210*** (df = 27)	32,983.800*** (df = 31)	43,240.240*** (df = 39)

Table 17 - Standard errors for Nested Logit Model - 1997

Nested logit model - results for 1997								
Logistic function standard errors. Car is the base mode.								
	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific			
	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
Intercept								
Multi_Transit	(0.138)	(1.271)	(1.265)	(2.060)	(1.112)	(11.076)	(10.998)	(19.990)
One_Bus	(0.134)	(1.258)	(1.253)	(2.049)	(1.114)	(11.071)	(10.994)	(19.986)
One_Plus_Bus	(0.137)	(1.269)	(1.264)	(2.058)	(1.113)	(11.075)	(10.997)	(19.989)
Rapid_Transit	(0.135)	(1.261)	(1.256)	(2.051)	(1.115)	(11.072)	(10.995)	(19.987)
Nest elasticity								
Generic	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Cost: generic								
Generic	(0.013)	(0.013)	(0.013)	(0.013)	(0.014)	(0.014)	(0.014)	(0.014)
Duration: generic								
Generic	(0.0002)	(0.0002)	(0.0002)	(0.0002)				
Duration: mode specific								
Car					(0.067)	(0.039)	(0.039)	(0.050)
Multi_Transit					(0.0003)	(0.0003)	(0.0003)	(0.0003)
One_Bus					(0.0002)	(0.0003)	(0.0003)	(0.0003)
One_Plus_Bus					(0.0003)	(0.0003)	(0.0003)	(0.0003)
Rapid_Transit					(0.001)	(0.001)	(0.001)	(0.001)
Familiar income								
Multi_Transit		(0.038)	(0.038)	(0.035)		(0.311)	(0.309)	(0.233)
One_Bus		(0.037)	(0.037)	(0.035)		(0.311)	(0.309)	(0.232)
One_Plus_Bus		(0.038)	(0.038)	(0.035)		(0.311)	(0.309)	(0.233)
Rapid_Transit		(0.038)	(0.038)	(0.035)		(0.311)	(0.309)	(0.233)
Age								
Multi_Transit		(0.054)	(0.054)	(0.068)		(0.485)	(0.478)	(0.617)
One_Bus		(0.054)	(0.054)	(0.068)		(0.485)	(0.478)	(0.617)
One_Plus_Bus		(0.054)	(0.054)	(0.068)		(0.485)	(0.478)	(0.617)
Rapid_Transit		(0.054)	(0.055)	(0.068)		(0.485)	(0.478)	(0.617)
Age^2								
Multi_Transit		(0.001)	(0.001)	(0.001)		(0.005)	(0.005)	(0.007)
One_Bus		(0.001)	(0.001)	(0.001)		(0.005)	(0.005)	(0.007)
One_Plus_Bus		(0.001)	(0.001)	(0.001)		(0.005)	(0.005)	(0.007)
Rapid_Transit		(0.001)	(0.001)	(0.001)		(0.005)	(0.005)	(0.007)
Dummy gender: 1 = male								
Multi_Transit		(0.245)	(0.251)	(0.310)		(2.056)	(2.032)	(2.699)
One_Bus		(0.243)	(0.250)	(0.309)		(2.056)	(2.031)	(2.699)
One_Plus_Bus		(0.244)	(0.250)	(0.309)		(2.056)	(2.031)	(2.699)
Rapid_Transit		(0.245)	(0.252)	(0.311)		(2.056)	(2.032)	(2.699)
Dummy employment: 1 = employed								
Multi_Transit			(0.258)	(0.303)			(0.812)	(1.054)
One_Bus			(0.257)	(0.301)			(0.812)	(1.054)
One_Plus_Bus			(0.260)	(0.304)			(0.813)	(1.055)
Rapid_Transit			(0.262)	(0.306)			(0.814)	(1.056)
Dummy study: 1 = student								
Multi_Transit				(0.328)				(1.280)
One_Bus				(0.328)				(1.280)
One_Plus_Bus				(0.330)				(1.281)
Rapid_Transit				(0.329)				(1.280)
Dummy car ownership: 1 = own a car								
Multi_Transit				(0.783)				(8.189)
One_Bus				(0.786)				(8.189)
One_Plus_Bus				(0.785)				(8.189)
Rapid_Transit				(0.786)				(8.190)
Observations	58,777	58,777	58,777	58,777	58,777	58,777	58,777	58,777
R2	0.021	0.127	0.129	0.221	0.103	0.209	0.210	0.303
Log Likelihood	-72,918.530	-65,024.540	-64,837.880	-58,018.470	-66,741.930	-58,922.150	-58,841.690	-51,891.800
LR Test	3,055.571*** (df = 7)	18,843.540*** (df = 23)	19,216.880*** (df = 27)	32,855.680*** (df = 35)	15,408.770*** (df = 11)	31,048.330*** (df = 27)	31,209.250*** (df = 31)	45,109.030*** (df = 39)

Table 18 - Standard errors for Nested Logit Model - 2007

Nested logit model - results for 2007							
Logistic function standard errors. Car is the base mode.							
	Cost and duration coefficients are generic				Cost coefficient in generic, duration is mode-specific		
	(17)	(18)	(19)	(20)	(22)	(23)	(24)
Intercept							
Multi_Transit	(0.061)	(0.105)	(0.099)	(0.485)	(14.605)	(13.601)	(10.900)
One_Bus	(0.032)	(0.099)	(0.093)	(0.475)	(14.575)	(13.570)	(10.868)
One_Plus_Bus	(0.034)	(0.104)	(0.098)	(0.476)	(14.574)	(13.570)	(10.868)
Rapid_Transit	(0.036)	(0.115)	(0.108)	(0.482)	(14.577)	(13.572)	(10.870)
Nest elasticity							
Generic	(0.012)	(0.063)	(0.077)	(0.038)	(0.005)	(0.005)	(0.006)
Cost: generic							
Generic	(0.032)	(0.002)	(0.002)	(0.010)	(0.042)	(0.042)	(0.042)
Duration: generic							
Generic	(0.0001)	(0.0001)	(0.0001)	(0.0001)			
Duration: mode specific							
Car					(0.038)	(0.036)	(0.025)
Multi_Transit					(0.0003)	(0.0003)	(0.0003)
One_Bus					(0.0002)	(0.0002)	(0.0002)
One_Plus_Bus					(0.0002)	(0.0002)	(0.0002)
Rapid_Transit					(0.0004)	(0.0004)	(0.0004)
Familiar income							
Multi_Transit		(0.006)	(0.006)	(0.015)	(0.841)	(0.782)	(0.352)
One_Bus		(0.006)	(0.006)	(0.015)	(0.841)	(0.782)	(0.352)
One_Plus_Bus		(0.007)	(0.007)	(0.015)	(0.841)	(0.782)	(0.352)
Rapid_Transit		(0.005)	(0.004)	(0.016)	(0.841)	(0.783)	(0.352)
Age							
Multi_Transit		(0.004)	(0.004)	(0.013)	(0.531)	(0.481)	(0.309)
One_Bus		(0.003)	(0.003)	(0.013)	(0.531)	(0.481)	(0.309)
One_Plus_Bus		(0.004)	(0.004)	(0.013)	(0.531)	(0.481)	(0.309)
Rapid_Transit		(0.005)	(0.005)	(0.014)	(0.531)	(0.481)	(0.309)
Age^2							
Multi_Transit		(0.00005)	(0.00004)	(0.0001)	(0.005)	(0.005)	(0.003)
One_Bus		(0.00004)	(0.00003)	(0.0001)	(0.005)	(0.005)	(0.003)
One_Plus_Bus		(0.00004)	(0.00004)	(0.0001)	(0.005)	(0.005)	(0.003)
Rapid_Transit		(0.0001)	(0.0001)	(0.0001)	(0.005)	(0.005)	(0.003)
Dummy gender: 1 = male							
Multi_Transit		(0.021)	(0.020)	(0.069)	(2.444)	(2.243)	(1.566)
One_Bus		(0.017)	(0.016)	(0.067)	(2.444)	(2.243)	(1.565)
One_Plus_Bus		(0.019)	(0.018)	(0.068)	(2.444)	(2.243)	(1.565)
Rapid_Transit		(0.030)	(0.029)	(0.073)	(2.444)	(2.243)	(1.565)
Dummy employment: 1 = employed							
Multi_Transit			(0.023)	(0.038)		(0.495)	(0.431)
One_Bus			(0.010)	(0.029)		(0.493)	(0.429)
One_Plus_Bus			(0.017)	(0.033)		(0.493)	(0.429)
Rapid_Transit			(0.039)	(0.052)		(0.497)	(0.433)
Dummy study: 1 = student							
Multi_Transit				(0.049)			(0.935)
One_Bus				(0.045)			(0.934)
One_Plus_Bus				(0.048)			(0.935)
Rapid_Transit				(0.058)			(0.936)
Dummy car ownership: 1 = own a car							
Multi_Transit				(0.151)			(3.586)
One_Bus				(0.150)			(3.586)
One_Plus_Bus				(0.150)			(3.586)
Rapid_Transit				(0.153)			(3.586)
Observations	69,200	69,200	69,200	69,200	69,200	69,200	69,200
R2	-0.041	0.105	0.109	0.164	0.184	0.185	0.242
Log Likelihood	-92,900.150	-79,869.720	-79,556.990	-74,584.610	-72,832.900	-72,753.610	-67,625.390
LR Test	-7,309.283 (df = 7)	18,751.580*** (df = 23)	19,377.040*** (df = 27)	29,321.810*** (df = 35)	32,825.210*** (df = 27)	32,983.800*** (df = 31)	43,240.240*** (df = 39)

- Alternative specific variables (t_{im} and x_{im}):
    - As in multinomial logit models, if commuting gets more expensive (cheaper), the probability of choosing any mode decreases (increases).
    - If commuting gets longer (shorter), the probability of choosing any mode decreases (increases). If we compare commuting times between modes, if the trip gets longer, the probability of choosing Car or One Bus in 1997 and Car, One Bus or Rapid Transit in 2007, decreases, the other modes' odds increasing.
- Individual specific variables (z_i):
    - When familiar income increases or age increases (up to a certain level, due to the opposite effect in the Age^2 variable) the probability of any mode but car being chosen decreases. The same is observed for being male, owning a car or not being a student in 1997 and 2007 and for being employed in 1997 and most of regressions' trials for 2007. This result follows what was found before.

4.2.2 Nested Logit: test statistics

As suggested by Croissant (2012), the Likelihood Test for the nesting structure is presented below. This works as a test of "no nests", which meaning is that the nest elasticity is equal to 1 and, therefore, nests are needless. For the structure proposed here in the regressions numbered with (24), the Likelihood Test shows that the nests make sense.

Table 19 - Likelihood ratio tests for Nested Logit Models (24), conducted with *mlogit package* in R.

```
> lrtest(model_24_97)
Likelihood ratio test

Model 1: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY + 
    D_STUDENT + D_CAR | DURATION
Model 2: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY + 
    D_STUDENT + D_CAR | DURATION
  #Df LogLik Df  Chisq Pr(>Chisq)    
1  39 -51892                         
2  38 -53730 -1 3676.9  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> lrtest(model_24_07)
Likelihood ratio test

Model 1: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY + 
    D_STUDENT + D_CAR | DURATION
Model 2: MODE ~ COST | INC_FAMILIAR + AGE + AGE_2 + D_MALE + D_EMPLOY + 
    D_STUDENT + D_CAR | DURATION
  #Df LogLik Df  Chisq Pr(>Chisq)    
1  39 -67625                         
2  38 -68346 -1 1440.8  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> 
```

4.3 Elasticities estimation

To this point, the present work estimated 24 regressions divided in 3 forms of specifications. From now on, for elasticities estimations, one of each is chosen: regressions (8), (12) and (24). They have been chosen since they presented the best combination of fit and sense. Research design is presented in Figure 15, which might provide a broad, easier view of it.

4.3.1 Short-term elasticities

Applying Equation 24 to every mode - over the price average of each alternative - provides Table 19, which present elasticities with respect to price for models (8), 1997 and 2007, and (12), 1997 and 2007. Their interpretation is: "how much does the probability of one individual, picked at random, choosing the *column* mode changes given a 1% change at *line* mode price's average".

The figures of interest are on the diagonals. It is possible to notice that price elasticity of bus demand in São Paulo is estimated to be:

- -0.013 for one bus in 1997 and -0.036 in 2007 if estimated with model (8). For two (or more, up to four) bus, it is -0.067 for 1997 and -0.059 for 2007.

- If these figures are estimated with regressions (12), which are restricted to non-users of *Vale Transporte* only, they are considerably higher: -0.194 and -0.377 for one bus (1997 and 2007) and -0.591 and -0.452 for two or more buses.

Another useful piece of information is given by the cross-price elasticities of demand. They tell us that if bus fare decreases, there will not be so many people shifting from other modes to bus. They are:

- Models (8): 0.021 (one bus, 1997), 0.028 (one bus, 2007), 0.004 (more than one bus, 1997) and 0.06 (more than one bus, 2007)

- Models (12): 0.106 (one bus, 1997), 0.103 (one bus, 2007), 0.026 (more than one bus, 1997) and 0.028 (more than one bus, 2007)

It is also useful to note that car price elasticity of demand with respect to car price itself is always higher than the cross-price elasticity of car demand and bus price.

It is also worth to note time elasticity of demand, as presented in Table 21. If we analyze models (12), they are -0.81 (1997) and -0.98 (2007) for one bus and -1.137 (1997) and -1.108 (2007) for more than one bus. If this model holds, then, demand is elastic (or almost elastic) with respect to trip duration.

Two points are worth commenting here. First, it is clear that people changed from other modes combinations to two or more bus from 1997 to 2007 and they probably did not do this at random: this alternative might have provided more utility to commuters. Nevertheless, since bus demand seems to be inelastic, those people might have changed their mode choice due to factors other than price. Thus, subsidies seem to have a quite limited capacity of attracting commuters to public transportation. Second, elasticity with respect to time is higher than the price one. Put together, reducing time spent in transit commuting seems to be more effective in increasing ridership than reducing fare *via* subsidies.

Table 20 - Marginal effects with respect to price for models (8), (12) and 24, 1997 and 2007

Marginal effects with respect to price: Model (8), 1997

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.062**	0.016	0.016	0.016	0.016	0.016
Multi_Transit	0.005	**-0.08**	0.005	0.005	0.005	0.005
One_Bus	0.021	0.021	**-0.013**	0.021	0.021	0.021
One_Plus_Bus	0.004	0.004	0.004	**-0.067**	0.004	0.004
Rapid_Transit	0.003	0.003	0.003	0.003	**-0.043**	0.003
Walking	0	0	0	0	0	**0**

Marginal effects with respect to price: Model (8), 2007

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.053**	0.027	0.027	0.027	0.027	0.027
Multi_Transit	0.005	**-0.092**	0.005	0.005	0.005	0.005
One_Bus	0.028	0.028	**-0.036**	0.028	0.028	0.028
One_Plus_Bus	0.006	0.006	0.006	**-0.059**	0.006	0.006
Rapid_Transit	0.004	0.004	0.004	0.004	**-0.06**	0.004
Walking	0	0	0	0	0	**0**

Marginal effects with respect to price: Model (12), 1997

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.485**	0.103	0.103	0.103	0.103	0.103
Multi_Transit	0.046	**-0.691**	0.046	0.046	0.046	0.046
One_Bus	0.106	0.106	**-0.194**	0.106	0.106	0.106
One_Plus_Bus	0.026	0.026	0.026	**-0.591**	0.026	0.026
Rapid_Transit	0.014	0.014	0.014	0.014	**-0.389**	0.014
Walking	0	0	0	0	0	**0**

Marginal effects with respect to price: Model (12), 2007

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.355**	0.148	0.148	0.148	0.148	0.148
Multi_Transit	0.038	**-0.687**	0.038	0.038	0.038	0.038
One_Bus	0.103	0.103	**-0.377**	0.103	0.103	0.103
One_Plus_Bus	0.028	0.028	0.028	**-0.452**	0.028	0.028
Rapid_Transit	0.015	0.015	0.015	0.015	**-0.465**	0.015
Walking	0	0	0	0	0	**0**

Table 21 - Marginal effects with respect to duration for models (8) and (12), 1997 and 2007

Marginal effects with respect to duration: Model (8), 1997

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.055**	0.014	0.014	0.014	0.014	0.014
Multi_Transit	-0.058	**0.927**	-0.058	-0.058	-0.058	-0.058
One_Bus	0.012	0.012	**-0.008**	0.012	0.012	0.012
One_Plus_Bus	-0.029	-0.029	-0.029	**0.476**	-0.029	-0.029
Rapid_Transit	-0.014	-0.014	-0.014	-0.014	**0.212**	-0.014
Walking	0.012	0.012	0.012	0.012	0.012	**-10.146**

Marginal effects with respect to duration: Model (8), 2007

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.102**	0.053	0.053	0.053	0.053	0.053
Multi_Transit	-0.053	**0.911**	-0.053	-0.053	-0.053	-0.053
One_Bus	0.04	0.04	**-0.051**	0.04	0.04	0.04
One_Plus_Bus	-0.045	-0.045	-0.045	**0.426**	-0.045	-0.045
Rapid_Transit	-0.025	-0.025	-0.025	-0.025	**0.338**	-0.025
Walking	0.035	0.035	0.035	0.035	0.035	**-8.562**

Marginal effects with respect to duration: Model (12), 1997

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.642**	0.136	0.136	0.136	0.136	0.136
Multi_Transit	0.051	**-0.772**	0.051	0.051	0.051	0.051
One_Bus	0.443	0.443	**-0.81**	0.443	0.443	0.443
One_Plus_Bus	0.051	0.051	0.051	**-1.137**	0.051	0.051
Rapid_Transit	0.031	0.031	0.031	0.031	**-0.85**	0.031
Walking	1.05	1.05	1.05	1.05	1.05	**-2.115**

Marginal effects with respect to duration: Model (12), 2007

	Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
Car	**-0.597**	0.249	0.249	0.249	0.249	0.249
Multi_Transit	0.043	**-0.778**	0.043	0.043	0.043	0.043
One_Bus	0.267	0.267	**-0.983**	0.267	0.267	0.267
One_Plus_Bus	0.068	0.068	0.068	**-1.108**	0.068	0.068
Rapid_Transit	0.025	0.025	0.025	0.025	**-0.796**	0.025
Walking	0.946	0.946	0.946	0.946	0.946	**-1.746**

Since neither Stata software nor R libraries provide an automatic calculation of marginal effects for the Nested Logit Model, the short-term price elasticities (*via* marginal effects) were only estimated for One Bus, because this is the alternative of main concern in the present work.

The elasticities presented in Table 22 were estimated following the suggestion of Cameron & Trivedi (2009) in the book *Microeconometrics using stata*. Basically, they propose applying the coefficients to the dataset and predicting the probabilities of an alternative being chosen. Then, the price is altered at the margin, the probabilities are predicted again with the new values and the difference provide the elasticities. The results, as they can be seen below, are a price elasticity of

bus demand of -0.048 for 1997 and -0.011 for 2007. They are closer to the models estimated with almost no segmentation on data (8), and are quite smaller (more inelastic) than the ones estimated with data excluding *Vale Transporte* (12).

Table 22 - Marginal effects for models (24) with respect to bus prices, 1997 and 2007

Marginal effects with respect to bus price for 1997 (Model 24 - Nested logit)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit
0.001	0.017	-0.048	0.016	0.013

Marginal effects with respect to bus price for 2007 (Model 24 - Nested logit)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit
0.001	0.003	-0.011	0.005	0.002

4.3.2 Long-term elasticities

Applying Equation 25 to one model provides one alternative's aggregate probability of choice, for one year. Results are reported in Table 21.

To control for spurious effects on demand caused by other variables (either observed or not), before inserting these values on Equation 4, the present work proposes to apply 1997’s regression’s coefficients on the 2007 dataset, and, then, calculating the aggregate probabilities of each mode again. This is equivalent to predicting the demand for 2007, using individual's behavior on 1997.

Thus, for model (8), Equation 4 becomes Equation (28), which is estimated supposing that *Bilhete Único* caused an exogenous price shock on bus trips with transfers[28], and the demands for 1997 and 2007 (estimated with 1997's coefficients) are the ones found in Table 21.

[28] Price value for 2007 is one fare price, BRL 2.30, because any commuter taking two or more bus in 2007 payed for just one. For 1997, it is the average cost of those trips, BRL 3.50, in 2007's real terms.

Table 23 - Predict aggregate probabilities for specifications (8), (12) and (24), sequentially for 1997, 2007 and 2007 with 1997 coefficients

Predict aggregate probabilities of each mode for 1997 : Model (8)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
0.2675	0.06462	0.23163	0.03688	0.02738	0.37199

Predict aggregate probabilities of each mode for 2007 : Model (8)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
0.33795	0.05686	0.18403	0.06427	0.0305	0.32639

Predict aggregate probabilities of each mode for 2007 : Model (8), using coefficients of 1997

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
0.3282	0.07132	0.2135	0.03641	0.03142	0.31916

Predict aggregate probabilities of each mode for 1997 : Model (12)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
0.30053	0.04409	0.18846	0.0267	0.02371	0.41651

Predict aggregate probabilities of each mode for 2007 : Model (12)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
0.38502	0.03719	0.13821	0.03928	0.0227	0.37759

Predict aggregate probabilities of each mode for 2007 : Model (12), using coefficients of 1997

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit	Walking
0.3707	0.04592	0.1465	0.02893	0.02882	0.37913

Predict aggregate probabilities of each mode for 1997 : Model (24)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit
0.426	0.1	0.376	0.057	0.041

Predict aggregate probabilities of each mode for 2007 : Model (24)

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit
0.501	0.081	0.283	0.093	0.042

Predict aggregate probabilities of each mode for 2007 : Model (24), using coefficients of 1997

Car	Multi_Transit	One_Bus	One_Plus_Bus	Rapid_Transit
0.481	0.13	0.089	0.27	0.031

Thus, the estimated price elasticity of two or more buses demand in the long-term is 0.04, if it is estimated through the strategy of specification that result on regression **(8):**

$$e_P = \frac{\frac{\Delta\hat{Q}}{\hat{Q}}}{\frac{\Delta P}{P}} = \frac{\frac{(0.03641 - 0.03688)}{0.03688}}{\frac{(2.30 - 3.128)}{3.128}} = 0.04874 \qquad (28)$$

Instead, if regression (12) - estimated through a restricted Multinomial Logit Model, for those trips which the respondent declared to pay for his or her own trip (that is, excluding users of *Vale Transporte)* - is used, the result is -0.3153:

$$e_P = \frac{\frac{\Delta\hat{Q}}{\hat{Q}}}{\frac{\Delta P}{P}} = \frac{\frac{(0.02893 - 0.0267)}{0.0267}}{\frac{(2.30 - 3.128)}{3.128}} = -0.3153 \qquad (29)$$

Last, if regression (24) - estimated through a Nested Logit Model - is used, the result is:

$$e_P = \frac{\frac{\Delta\hat{Q}}{\hat{Q}}}{\frac{\Delta P}{P}} = \frac{\frac{(0.27 - 0.057)}{0.057}}{\frac{(2.30 - 3.128)}{3.128}} = -14.11 \qquad (30)$$

This last result seems quite absurd when compared to others or literature. However, recalling that long-term elasticities rely on a prediction (coefficients from 1997, data from 2007) and that the highest McFadden's (pseudo) R^2 found on the nested logit estimations were around 0.3, it is hard to trust the post estimations of this model. The present work would need more time to achieve a better specification for the nested logit, but the author chooses to present the results much more as a robustness check.

In summary, with exception of the Nested Logit Model: short-term elasticities derived from first model estimated (8) are as inelastic as the long-term one. For models (12), that exclude *Vale*

Transporte, the short-term elasticities are higher in *modulus* than the long-term one (that is, more elastic for more than one bus). This result follows what was expected in the literature.

5 DISCUSSION

São Paulo grew rapidly during the 20th Century, in that its urbanization process resulted in a sprawling metropolis of 12 million people with huge economic importance where social and urban inequalities are frequently observed (Meyer et al., 2004a). Likewise, its transportation system is not more organized or uniform: almost 30% of the 25 million daily trips are made by those who belong to the 20% richest families, while the 20% bottom account for around 12%. While 56% of the first group's trips are made by car, only 10% of the latter use it, those relying most on active modes and transit. Transit is slower than autos; pedestrians and cyclists are not treated well in the present urban environment.

In spite of its questionable quality, transit has had a central role to play in both the city's sustainability and people's (especially the poorer) commuting capacity. Bus, for its time, is very significant not only for those who opt for transit since 24.2% of all of the city's trips have it as their main mode. Lately, bus' fare price has been in the center of a public debate about urban accessibility.

São Paulo's Municipal Government is one of the only in Brazil which directly subsidizes bus fare. It spent, in 2017, more than 3 billion Reais (or 1 billion US Dollars) in direct subsidies, an amount that represents almost 7% of its budget. Five years before, in 2012, this figure was as low as 3%.

Increasing subsidies result from growing operational costs – mainly motivated by drivers and ticket checker salaries' readjustments (Carvalho & Pereira, 2011) – and by fare's price which has remained constant in real prices for about ten years. However, resources are limited, and government probably will not be able to keep this policy longer. This might be a risky situation: literature on transportation warns that decreasing (or stable) ridership and/or increasing costs might jeopardize transit (Carvalho & Pereira, 2011; Gwilliam, 2002). The rationale behind this is that an expensive transit system would repel passengers and – if fare is not priced adequately – transit will get either more expensive for the remaining riders or its quality will decrease (Bahl & Linn, 1992; Bird, 2001; Carvalho & Pereira, 2011; Gwilliam, 2002; Mohring, 1972).

In this vicious cycle, both situations might transform bus into a less attractive option, therefore, the city's sustainability would be endangered (Carvalho & Pereira, 2011; Gwilliam, 2002) because of negative externalities (McFadden, 2011) and congestion caused and/or augmented by private modes. Besides (a) affordability (Estupiñán et al., 2007; Parry & Small, 2009) and (b) transportation as a right of citizens (BRASIL, 1988), (c) mitigating transportations' negative externalities is one of the main reasons pointed by literature why transit should be prioritized in cities (Parry & Small, 2009). And one form of preventing transit from deteriorating would be keeping ridership at high levels and, at the same time, at fiscally sustainable levels. The latter is closely related to the pricing form and the first might be achieved through a system's design or state subsidies that keep fares at low levels (Carruthers et al., 2005; Gwilliam, 2002).

Nevertheless, literature on economics and public finance is reticent on subsidizing public services. Taxing one activity for other's funding is seen as allocatively inefficient and when the subject is public services this might also be financially dangerous for the reasons presented above (Bahl & Linn, 1992). Literature clearly recommends user charges and services' pricing by marginal cost pricing rule (Bahl & Linn, 1992; Bird, 2001), which in the case of public transportation can also mean fiscal sustainability for transit operators (Carvalho & Pereira, 2011; Gwilliam, 2002). However, there are two situations in which marginal cost pricing rule does not hold.

The first is when positive externalities are generated by the service in question (Basso & Jara-Díaz, 2012; Bird, 2001; Estupiñán et al., 2007; Mohring, 1972; Parry & Small, 2009). At first, transit would not be eligible since any kind of transportation generates negative externalities. Nevertheless, collective transportation causes less externalities per capita than private modes. This is, indeed, a second-best for congestion charging (Albalate & Bel, 2009; Basso & Jara-Díaz, 2012; Estupiñán et al., 2007; Gardner, Unnikrishnan, & Waller, 2010). However, there are works which have found that subsidizing transit might increases society's welfare through externalities mitigation (Basso & Jara-Díaz, 2012; Mohring, 1972; Parry & Small, 2009). However, the only way transit may indeed improve welfare is substituting individual modes for collective.

The second situation in which marginal cost pricing rule is not effective in creating an environment which is fiscally sustainable and allocatively efficient is when demand is perfectly

inelastic (Bahl & Linn, 1992; Bird, 2001). In such cases, demand is not responsive to price, and there will exist no gain in efficiency by marginal cost pricing rule.

There is no evidence on how fare price is decided in São Paulo. Thereby, it is not possible to state whether the city prices its bus system by (a) first deciding how much it is going to be spent on subsidies and then dividing the remaining costs by ridership (a form of uniform tariff) or (b) marginal cost pricing considering users' costs, in that marginal cost falls below the average cost and subsidies are necessary. One can also imagine that, in a totally opposite way, authorities decide a (almost random, politically feasible or inertial) fare for the system and complement the residual cost with subsidies. Nonetheless, subsidies are a reality that might or might not comply with optimality idealized in literature.

Additionally, the bus system in São Paulo seems to fail in taking cars off the street. Nowadays the city meets itself in an unsustainable edge where congestion causes many losses (Brinco, 2006; Haddad et al., 2017, 2015). Also, once car is mainly used by richer commuters and transit usage is expected to decrease as familiar income increases, it is hard to believe that people will use more transit proportionally if they get wealthier, everything else held constant.

Thus, there is a need of governmental action in order to make transit more attractive and literature says that this might be done by raising funds through adequate priced fares (Gwilliam, 2002). Nevertheless, transit cannot become fiscally sustainable and allocatively efficient and, at the same time, an unaffordable alternative for commuters, particularly because vulnerable people would be the most affected (Estupiñán et al., 2007).

One piece of information that might help towards figuring out how to solve this puzzle is price elasticity of bus demand. As it was presented before,

1. Some demand responsiveness with respect to price is a necessary condition for marginal cost pricing rule to be truly efficient (Bahl & Linn, 1992; Bird, 2001). If one thinks of pricing fare under this rule, this information is substantial. Nevertheless, even though in the case of almost perfect inelasticity (elasticity between -1 and 1 and not equal to 0), welfare losses are quite high if fare is incorrectly priced (Bahl & Linn, 1992). This is a situation that demands attention.

2. If policy's intention is to mitigate transportation negative externalities through transit usage, how this policy affects the quantity consumed becomes important since the final output to be measured is exactly bus ridership. To know price elasticity of demand is core for this kind of analysis (Estupiñán et al., 2007).

3. In a resource-constrained environment such as Brazil, it is important to know how many people will be excluded from the system if fare changes. Such information is core for discussions about how socially desirable is to have bus subsidies – and also for debates on affordability.

The present work does such estimation. Using a discrete choice econometric framework (Ben-Akiva & Boccara, 1995; Domencich & McFadden, 1975; Greene, 2009; Koppelman & Vaneet, 2000; Manski, 2001; McFadden, 1973, 1974, 1978; Train, 2009) and data from the OD Surveys of 1997 and 2007, price elasticities of bus demand for short- and long-term in São Paulo are calculated.

Many trials were made during the work's process, but only three slightly different specification strategies estimated with two different models are presented: (a) one multinomial logit model with little exclusions on data, eliminating only less important modals' combinations, (b) one multinomial logit model excluding the trips that were paid with *Vale Transporte* voucher, that is, subsidized by commuter's employer and (c) one nested logit model, using the same data form the first model, except that *Walking* alternative was excluded, since the version containing it did not present good results.

The values found are presented in the table below.

Table 24 - Summary of price elasticity of bus demand estimations

		Short-term 1997	Short-term 2007	Long-term 1997-2007
One bus	Multinomial logit - less constrained data (Regressions (8))	-0.013	-0.036	-
	Multinomial logit - *Vale Transporte* restriction (Regressions (12))	-0.194	-0.377	-
	Nested Logit (Regressions (24))	-0.048	-0.011	-
More than one bus	Multinomial logit - less constrained data (Regressions (8))	-0.067	-0.059	0.048
	Multinomial logit - *Vale Transporte* restriction (Regressions (12))	-0.591	-0.452	-0.315
	Nested Logit (Regressions (24))	-	-	-14.11

First, it is worth commenting that these figures follow those that have been found in literature. The pattern for São Paulo is the same of developed cities and other major Brazilian metropolitan regions: **demand for bus is price inelastic in both short- and long-term**. That is, demand is not very responsive to price fluctuation. Elaborated hypotheses are then corroborated.

Elasticities estimated with model (12) are quite higher (that is, demand is less price inelastic) than the ones estimated with model (8). The latter results are more in line with literature. This might happen due to a model advance (as was intended), but might also be caused by some noise on simulations and approximations. If one believes that it is a more adequate mode, it is possible to affirm that, if commuters have out-of-the-pocket costs, elasticity of bus demand is between -0.19 and -0.59 for short- and long-term, respectively. This means that a 10% rise on fare will cause a 1.9% to 5.9% ridership reduction. However, if one prefers the models (8), a 10% fare increase would decrease ridership by 0.5% in the short-term and by 1% in the long term.

The same is seen on the nested logit models, estimated with models (24). They follow models (8), except for long-term price elasticity of more-than-one-bus demand, which resulting value is quite absurd. However, recalling that long-term elasticities rely on a prediction (coefficients from

1997, data from 2007) and that the highest McFadden's R^2 found on the nested logit estimations were around 0.3, it is hard to trust the Nested Logit models here conducted post estimations. Also, and simply, since long-term elasticity is notably absurd, the Nested models should be better specificated and are ignored in the following discussion. The author chooses to present the results much more as a robustness check.

Demand price inelasticity has two major consequences for public policy. First, as was mentioned here before, marginal cost pricing rule does not induce allocative efficiency. Second – and since demand as a whole is assumed fixed for those models – higher subsidies (and therefore a smaller fare) will not guarantee that the bus becomes more attractive in comparison to other modes.

The most superficial meaning of these results is that government might cut bus subsidies in São Paulo. Such action would not cause a plunge in ridership and would probably bring fiscal sustainability for bus system and some relief for governmental budgetary allocation. Also, evidence found in literature suggest that supply-side subsidies are not necessarily a pro-poor policy (Estupiñán et al., 2007; Gwilliam, 2002) and also make the system inefficient (Gwilliam, 2002).

Nonetheless, it is clear in Brazil that urban transportation is a delicate question. It is possible to argue that 2013's riots and the later justifications for urban transportation inclusion as citizen's right in Constitution make clear that Brazilians need affordable transit and can influence governmental budget so that subsidies will hold (BRASIL, 1988; Tartaroti, 2015). On the other hand, literature also points that concessionaires and criminal organizations operate to influence official or shadow transportation provision or regulation, at both subnational and national level (Campos, 2016; Gomide, 2008; Hirata, 2011). Moreover, there is no evidence that there is technical refinement on how fare is priced and subsidies are set in São Paulo. Its bus system is potentially inefficient, for both operational and financial reasons.

Also, affordability is said to be important for transit sustainability (Carvalho & Pereira, 2011; Gwilliam, 2002) and people's commuting (Estupiñán et al., 2007; Gwilliam, 2002), particularly in a city such as São Paulo, where poor people's commuting relies on the bus.

Thus, it is unclear if transit subsidies should be reduced in São Paulo. Deeper analysis must be conducted in order to assess (a) whether directly subsidizing fare is welfare enhancing and (b) exactly who will be impaired in the advent of a subsidies plunge. These questions are mainly concerned with affordability, system's sustainability, and economic efficiency, and bus price elasticity of demand will be a valuable piece of information when addressing these questions. Nevertheless, literature suggests actions other than pricing fare efficiently in order to make transit more attractive and, then, sustainable through ridership levels.

For example, buses' geolocation information in real time, which was already implemented in São Paulo, might enhance user's confidence in transit (Brakewood et al., 2015; Tang & Thakuriah, 2012; Watkins et al., 2011), a core factor for ridership maintenance (Chair et al., 2016). Also, there are other creative policies that can help. For example, there is no need of a ticket checker in each bus, since Bilhete Único accounts for almost all of fare payments. This money can turn into investment on transit infrastructure, for example, which is solidly said to be a welfare improver policy (Alvim et al., 2013; Basso et al., 2011; Haddad et al., 2017, 2015; Haddad & Vieira, 2015; Smart, 2014). One might also suggest other innovative policies, such as congestion pricing or parking fees. Moreover, there is evidence that congestion pricing can fund public transportation in a welfare-enhancing way (Basso & Jara-Díaz, 2012; Grin, 2011; Parry & Small, 2009; Quigley & Hårsman, 2010).

It is clear that affordability is an important question and should be addressed with sound public policies. According to the literature reviewed and to the results of the present work, affordability perhaps is better tackled if treated with direct money transfers to those citizens who need an income complement to pay for transit. This would also helps decreasing the inefficiencies to the system and distortions to the economy generated by supply-side subsidies.

This work also presents rich information about time elasticities of demand. It has been seen in the Results' Section that commuters are more responsive to variations in time than in price, to the point of existing some elasticity (slightly higher than 1). This possibly means that **making transit faster might be more effective in reducing car usage than making it cheaper.** Investing in infrastructure - such as bus dedicated lanes (Basso et al., 2011; Smart, 2014) - is, again, a sound public policy decision.

The present work contributions can be summarized as follows.

First, it has provided a way of modelling the Metro OD Survey in that a dataset adequate for Discrete Choice estimations is structured. This survey is widely used as a source of information and data for research and public policy in Brazil. Nevertheless, it has many caveats, such as not asking questions about the trips the individuals did not make and commuting time. The preparation of the dataset for the models conducted here is an example of how counterfactual trips or the price of the realized one might be estimated.

Second, it proposes an innovation by using Bilhete Único as a source of exogenous shock in prices, what might be replicated in other studies.

Third, the estimations themselves. As it was discussed in the present section, demand seems to be very price inelastic in the first estimation conducted. The second one presents figures considerably higher, but yet – and especially long-term's – demand is price inelastic. Also, there were estimated elasticities with respect to trip duration, and those are considerably higher to the point of being elastic.

Fourth, the implications for public policy. It is possible to state that fare price – perhaps the main characteristic alongside with service level and quality – in São Paulo is not effective in influencing people to shift modes from individual to collective, and thus taking cars off the street, and thus creating a more sustainable environment for urban transportation. However, directly fare subsidies are high and have surged in the last ten years, what seems to be cost-ineffective given what was discussed above. Nevertheless, subsidies might help keeping fare affordable. If this is the goal of this policy, government should consider transferring money directly to commuters, since their welfare would be improved, and a systemic inefficiency caused by supply-side subsidies would be mitigated. Last, but not least, the present work suggests that people are more responsive to time improving than fare reductions. Thus, more attention should be given to policies that make transit commuting faster.

6 CONCLUSION

The present work aimed to estimate short- and long-term price elasticities of bus demand in São Paulo. In order to do so, it has used a discrete choice econometric framework and data from the OD Surveys of 1997 and 2007. There were estimated short-term elasticities (1997 and 2007) for one bus (and other modes) and long-term elasticities for more than one bus (2007 – 1997 period). The latter estimations limit the work's external validity, but its importance is not smaller since many people, especially the poor, live far from jobs and rely on this commuting alternative. Nonetheless, its results might be generalized for São Paulo's bus system as a whole, since estimations do not vary much between those combinations of transit modes.

The main finding is that the bus is moderately price inelastic in São Paulo. This follows what literature on transportation economics says. By estimating bus price elasticities for São Paulo, the present work helps to close a gap in Brazilian transportation literature and also provide insights for public policy. Also, an innovative identification strategy was proposed by treating Bilhete Único's implementation as an exogenous price shock on fare.

It is clear from the literature that bus system should be competitive, financially sustainable, well governed and well managed. However, São Paulo's local government spends a great part of its budget toward directly subsidizing bus fare, and the figures surged in the last years. While this helps keeping fare affordable, it seems to fail on influencing commuters' modal choice, in that cross-price elasticity of bus and car demand is also close to zero and urban transport is chaotic. Bus ridership, by its time, is relatively constant during the last years.

Soaring costs and stable ridership might jeopardize transit because of fiscal reasons: agencies' or operators' revenue might plunge, causing its quality to decrease. Less attractive or expensive public transportation might cause even more plunges on ridership, in that a vicious cycle is created and cities' sustainability might be endangered. the economics response to these problems is pricing bus efficiently, possibly through equalizing bus fare to its marginal cost.

Nevertheless, there are a few problems in this rule. First, it is not certain that marginal cost pricing rule succeeds in maximizing economic welfare, especially when demand is inelastic. Second, subsidies might either mitigate transportation's negative externalities and keep fare

affordable, which is desirable for social reasons. Since there are also a few studies that find gains on economic welfare by subsidizing transit, this logic does not provide clear recommendations on what should be done with subsidies for São Paulo's case. However, the present work's findings help create a pathway in that it is very likely that cutting subsidies would not cause a plunge in ridership. It might, however affect mostly poor people. Increasing subsidies, for its time, will not succeed in creating a more sustainable urban environment. At least it is possible to forecast impacts of fare increase on ridership using the parameters presented in this work.

For mitigating car's negatives externalities, dynamic congestion and/or parking charging must be considered. There are also sound public policies that might enhance bus ridership, such as informational improvements and investment in infrastructure, since commuters seem to be more responsive to transit quality and – as estimations in the present work suggests – travel duration (time). For instance, if the cost of reducing commuting time by 1% is up to 3 times larger than the cost of a 1% subsidy, investments in speed reduction will be more efficient in keeping ridership stable than price subsidies. Thus, it is probably better to invest in policies that reduce commuting time than in those that reduce fare price. If subsidies are maintained for social reasons, they should be given directly to commuters, since supply-side subsidies causes system to be inefficient, especially when bus system is operated by private firms.

Further work should be conducted in order to assess (a) whether directly subsidizing fare is welfare enhancing in São Paulo and (b) exactly who will be excluded from consumption in the event of a subsidies plunge. The present work provide basis for the first, since dividing the value of each coefficient in the regressions by the coefficient associated with cost (generic), the coefficients are directly interpretable and helps on calculating consumer surplus (Train, 2009). Also, the dataset organized for the estimations presented here can be used to calculate price elasticities of demand of different groups, allowing government to charge each one differently, acting as a "discriminatory monopolist". This can guarantee a higher welfare and works better if one group has a perfectly inelastic demand and the other do not. Probably richer people has a more inelastic demand for public services, meaning that there might be room for extracting a higher price of this group to finance the poorest's comuting – supposing that this is socially desirable.

These questions are mainly concerned with affordability, transit system's sustainability, and economic efficiency. Bus price elasticity of demand will be a valuable information when addressing these questions. Additional efforts should also be made to understand exactly how fare is priced and the political framework and actors behind bus provision in São Paulo. Also, more focus shall be given to complementary policies (such as good information and infrastructure implementation) that might increase transit attractiveness in developing countries.

7 APPENDIX

7.1 Appendix A: OD Data format and simulations for models

One small sample of OD survey's data is shown in Table 10.

Table 25 - OD Data sample for exemplificaiton

Trip ID	Individual ID	Demographics individual	Mode chosen	Trip length	Trip duration	Trip cost (simulated)	Other trip variables
199700121002010161825	001210020101	Zi	Car	14660	50	6.0819691929	Tt
199700121002010161826	001210020101		Walking	100	3	0	Tt
199700121005010178154	001210050101	Zi	One Bus	8434	30	0.8500000000	Tt

As mentioned before, there is no information on one's choice set of modes, nor information on trip's cost. Then, this data is simulated. Table 11 shows the logic for trips' choice set simulation, considering the first line of table 10. Table 12 shows how trip's cost was simulated.

Table 26 - Choice set simulation for one trip

Trip ID	Mode	Chosen	Trip length	Trip duration	Trip cost (simulated)
199700121002010161825.1	Car	1	14660	50 (observed)	See table below
199700121002010161825.2	Multi_Transit	0	14660	lenght*avg. speed of the respective mode	
199700121002010161825.3	One_Bus	0	14660	lenght*avg. speed of the respective mode	
199700121002010161825.4	One_Plus_Bus	0	14660	lenght*avg. speed of the respective mode	
199700121002010161825.5	Rapid_Transit	0	14660	lenght*avg. speed of the respective mode	
199700121002010161825.6	Walking	0	14660	lenght*avg. speed of the respective mode	

Table 27 - Cost simulation rules for trips

Mode	Year	Cost	
		If mode chosen (modes are observed)	**If mode simulated in choice set (modes are not observed)**
Car	1997	Price per kilometer * lenght	Price per kilometer * lenght
	2007	Price per kilometer * lenght	Price per kilometer * lenght
Multi_Transit	1997	Bus fare * number of buses taken + Rapid transit fare	Avg. cost of this mode when observed
	2007	Bus fare * number of buses taken + Rapid transit fare	Avg. cost of this mode when observed.
One_Bus	1997	Bus fare	Bus fare
	2007	Bus fare	Bus fare
One_Plus_Bus	1997	Bus fare * number of buses taken	Avg. cost of this mode when observed.
	2007	Bus fare	Bus fare
Rapid_Transit	1997	Rapid transit fare	Rapid transit fare
	2007	Rapid transit fare	Rapid transit fare
Walking	1997	0	0
	2007	0	0

7.2 Appendix B: Boxplots for variables after mode grouping, divided by year

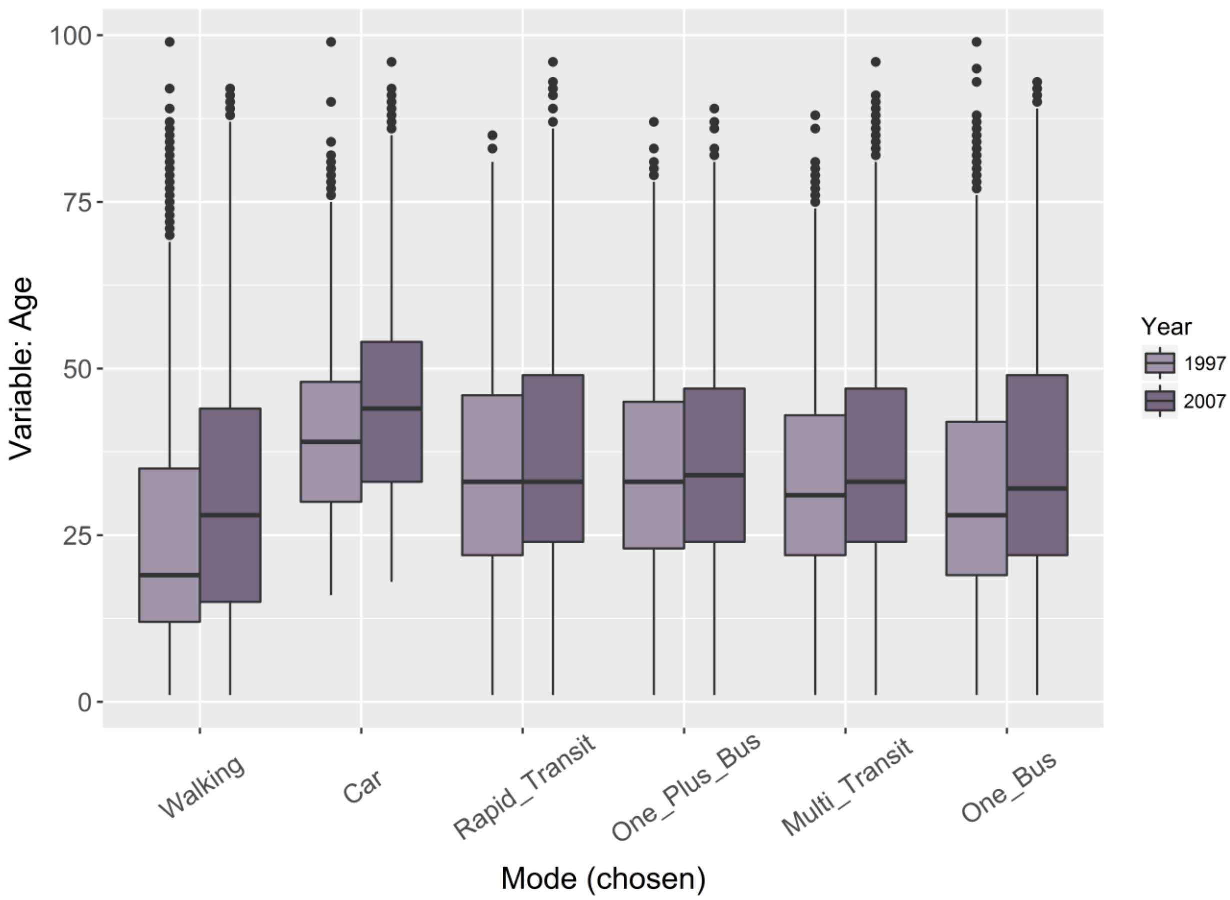

Figure 20 - Boxplots for individual's age after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

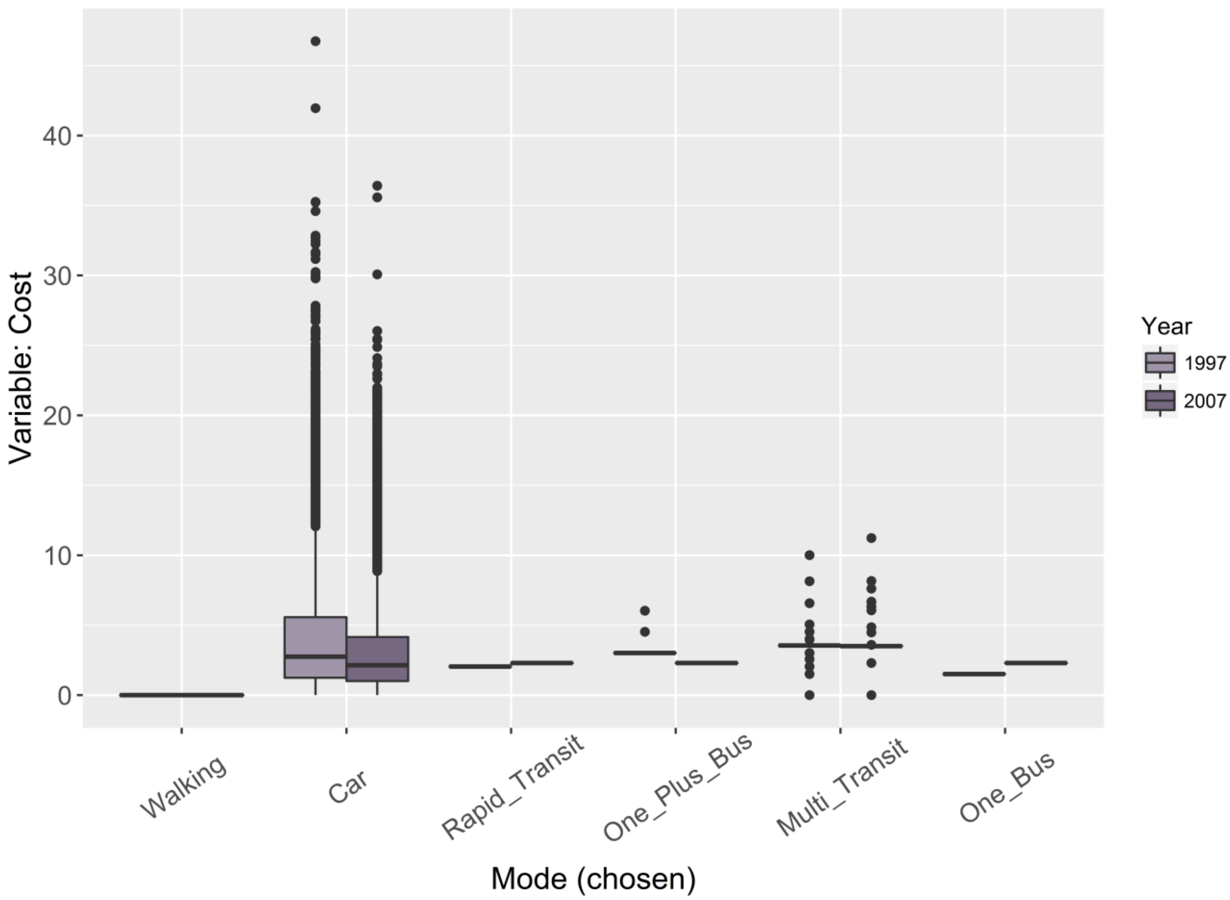

Figure 21 - Boxplots for modes' (chosen) cost after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

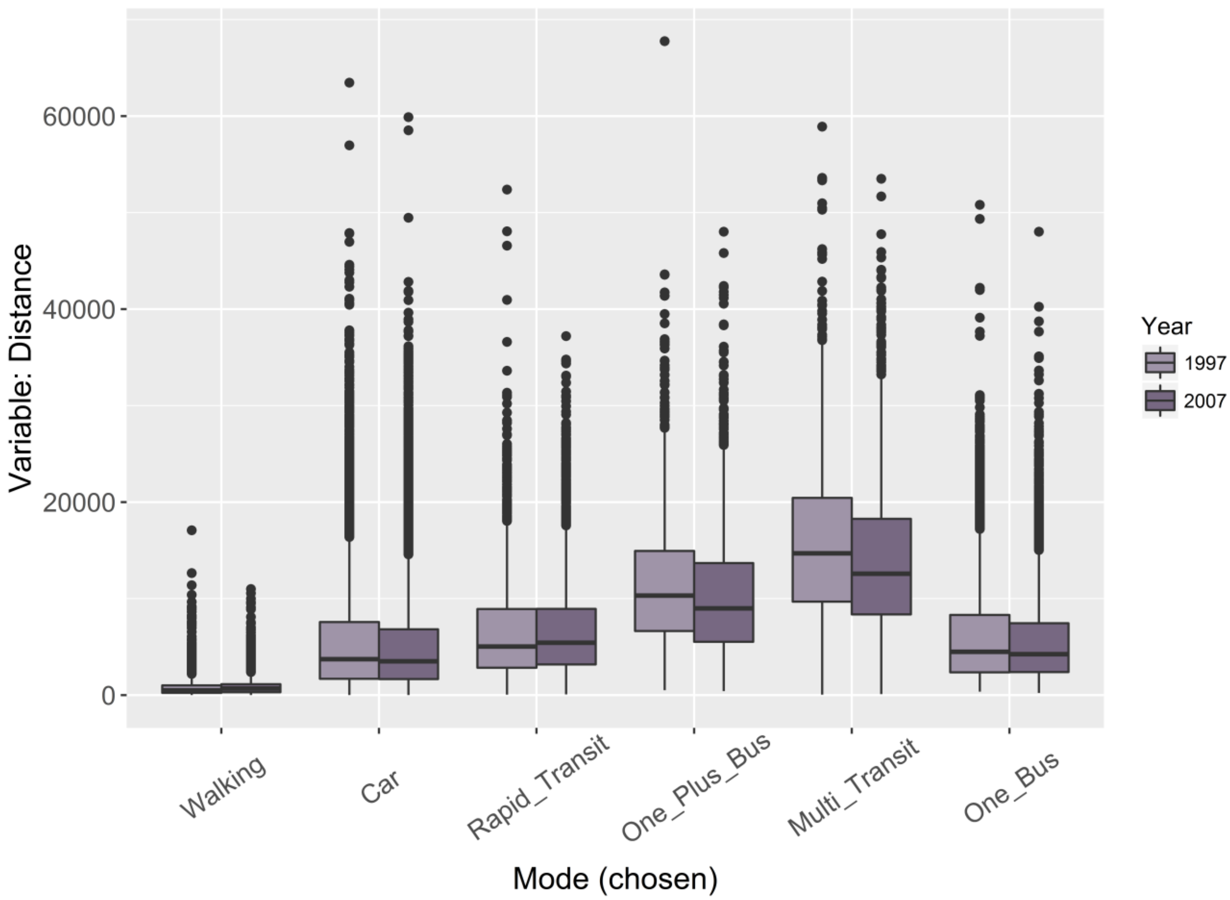

Figure 22 - Boxplots for trips' distance after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

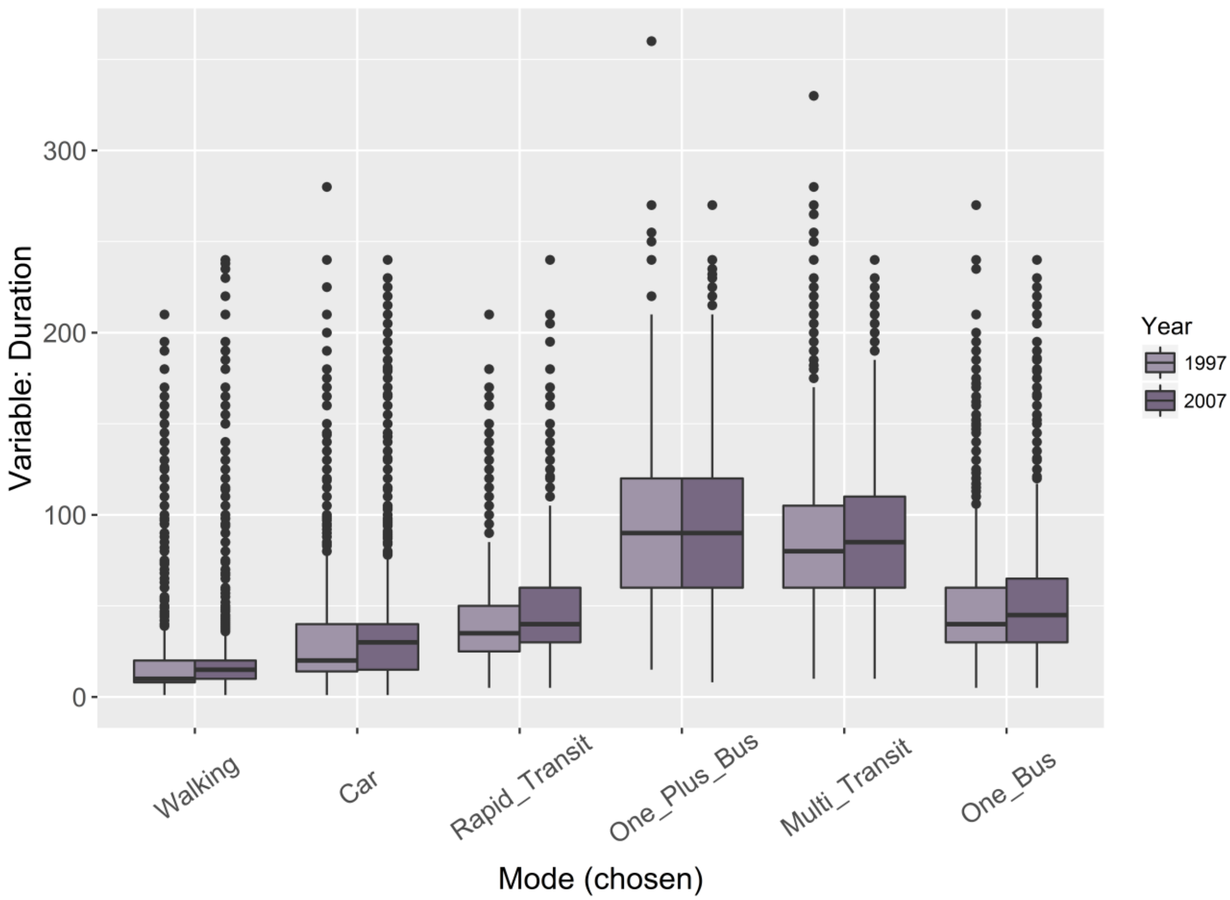

Figure 23 - Boxplots for modes' (chosen) duration after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

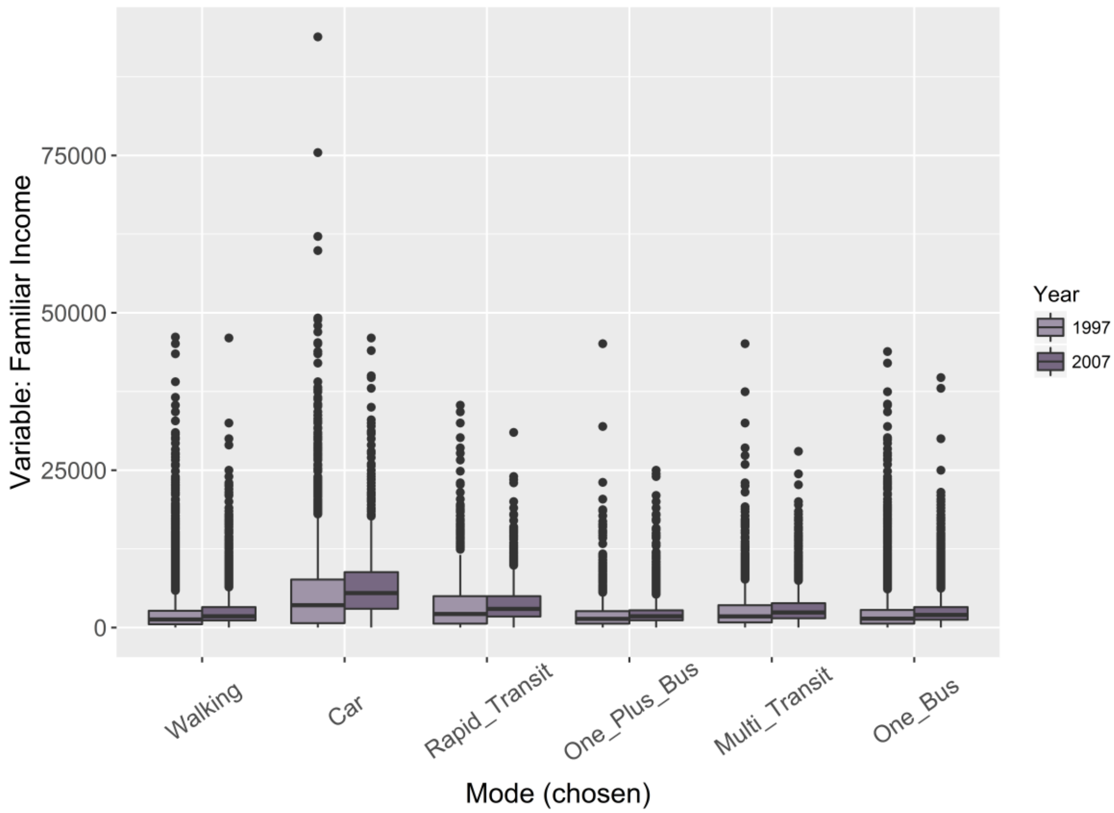

Figure 24 - Boxplots for individual's familiar income after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

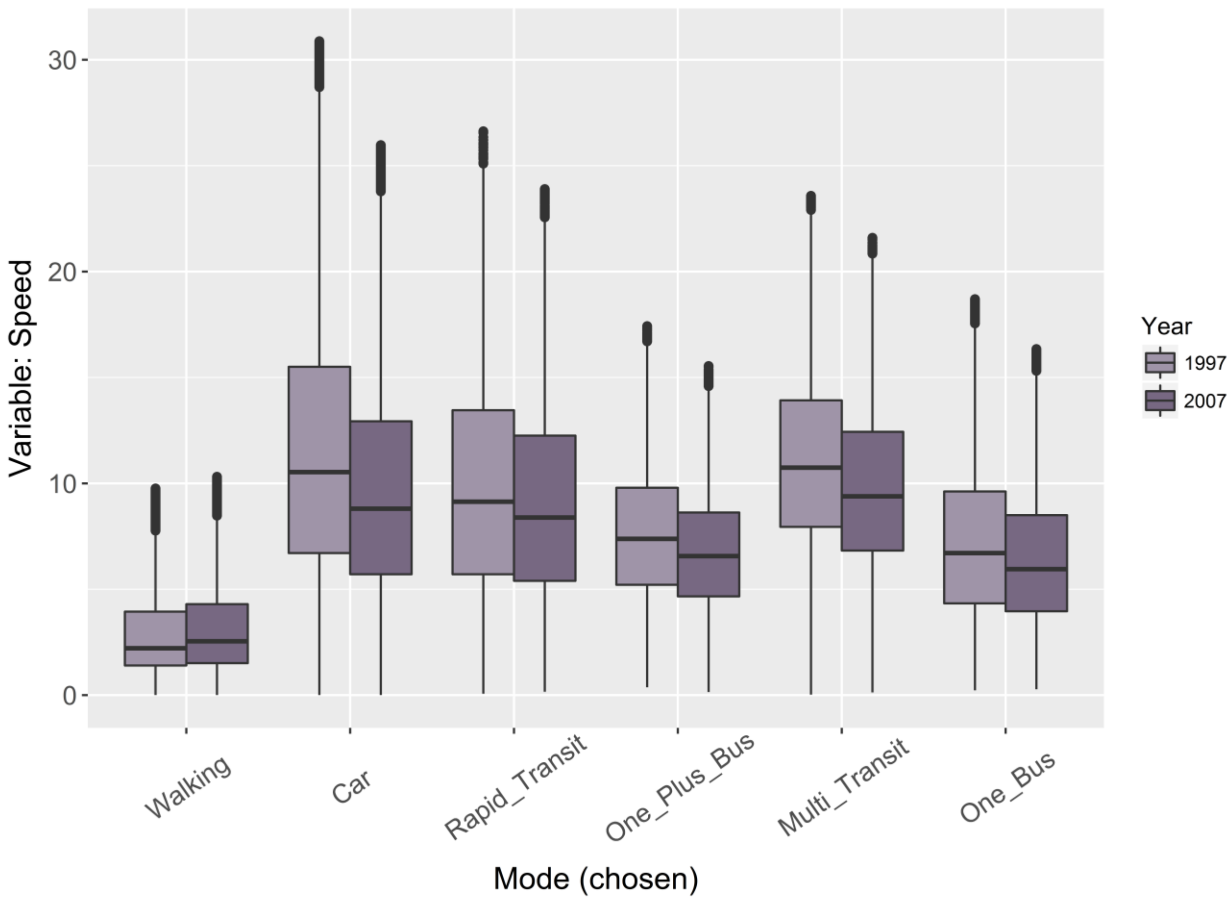

Figure 25 - Boxplots for mode's speed after mode grouping, divided by year. Source: data from OD Surves 1997 and 2007, adapted. Prepared by the author.

8 REFERENCES

Albalate, D., & Bel, G. (2009). What Local Policy Makers Should Know about Urban Road Charging: Lessons from Worldwide Experience. *Public Administration Review*, *69*(5), 962–975. Retrieved from http://www.jstor.org/stable/40468975

Alvim, B. G., Darido, G., Lippe, M. I. G., & Mehndiratta, S. R. (2013). Integrating land use considerations to maximize the benefits of a metro rail project for the bottom 40%: A case of Sao Paulo. In *41º Encontro Nacional de Economia* (p. 25).

ANTP. (2010). *Sistema de Informações da Mobilidade Urbana: Custos dos Deslocamentos*.

Backhaus, J. G., & Wagner, R. E. (2005). Society, state, and public finance: Setting the analytical stage. In *Handbook of public finance* (pp. 1–18). Springer.

Bahl, R. W., & Linn, J. F. (1992). *Urban public finance in developing countries*. New York: The World Bank, Oxford University Press.

Basso, L. J., Guevara, C. A., Gschwender, A., & Fuster, M. (2011). Congestion pricing, transit subsidies and dedicated bus lanes: Efficient and practical solutions to congestion. *Transport Policy*, *18*(5), 676–684. https://doi.org/https://doi.org/10.1016/j.tranpol.2011.01.002

Basso, L. J., & Jara-Díaz, S. R. (2012). Integrating congestion pricing, transit subsidies and mode choice. *Transportation Research Part A: Policy and Practice*, *46*(6), 890–900. https://doi.org/10.1016/j.tra.2012.02.013

Bator, F. M. (1958). The Anatomy of Market Failure. *The Quarterly Journal of Economics*, *72*(3), 351–379. https://doi.org/10.2307/1882231

Ben-Akiva, M., & Boccara, B. (1995). Discrete choice models with latent choice sets. *International Journal of Research in Marketing*, *12*(1), 9–24. https://doi.org/10.1016/0167-8116(95)00002-J

Ben-Akiva, M., & Lerman, S. R. (1985). *Discrete Choice Analysis. Theory and Application to Travel Demand*. https://doi.org/10.1016/0191-2607(88)90064-7

Bird, R. M. (2001). User Charges in Local Government Finance. In M. Freire & R. Stren (Eds.), *The Challenge of Urban Government* (pp. 171–182). Washington, DC: World Bank Institute. Retrieved from http://www1.worldbank.org/publicsector/decentralization/June2003Seminar/Bird2.pdf

Brakewood, C., Macfarlane, G. S., & Watkins, K. (2015). The impact of real-time information on bus ridership in New York City. *Transportation Research Part C: Emerging Technologies*, *53*, 59–75. https://doi.org/http://dx.doi.org/10.1016/j.trc.2015.01.021

BRASIL. Constituição (1988). Brasília, DF. Retrieved from http://www.planalto.gov.br/ccivil_03/constituicao/constituicao.htm

Bräutigam, D. (2008). Introduction: taxation and state-building in developing countries. In *Taxation and state-building in developing countries: Capacity and consent* (pp. 1–33).

Cambridge, UK: Cambridge University Press.

Brinco, R. (2006). *Transporte urbano e dependência do automóvel*. Porto Alegre. Retrieved from http://cdn.fee.tche.br/documentos/documentos_fee_65.pdf

Brueckner, J. (2011). *Lectures on urban economics*. MIT Press Books.

Cameron, A. C., & Trivedi, P. K. (2009). *Microeconometrics using stata*. Stata Press.

Campos, M. V. L. (2016, December 20). *Ferramentas de governo: instrumentação e governança urbana nos serviços de ônibus em São Paulo*. Universidade de São Paulo, São Paulo. https://doi.org/10.11606/D.8.2016.tde-20122016-141805

Carruthers, R., Dick, M., & Saurkar, A. (2005). *Affordability of public transport in developing countries*.

Carvalho, C. H. R., & Pereira, R. H. M. (2011). *Efeitos da variação da tarifa e da renda da população sobre a demanda de transporte público coletivo urbano no Brasil*. Texto para Discussão, Instituto de Pesquisa Econômica Aplicada (IPEA).

Carvalho, C. H. R., & Pereira, R. H. M. (2012). *Gastos das famílias brasileiras com transporte urbano público e privado no Brasil: uma análise da POF 2003 e 2009*. Texto para Discussão, Instituto de Pesquisa Econômica Aplicada (IPEA).

Chair, R. S., Gopal, N. V., Lee, E. S., Young, D., Youssouf, E., Dill, J., & Newman, C. (2016). *Who's On Board: What Today's Riders Teach Us About Transit That Works*. New York.

Chiou, Y. C., Jou, R. C., & Yang, C. H. (2015). Factors affecting public transportation usage rate: Geographically weighted regression. *Transportation Research Part A: Policy and Practice*, *78*, 161–177. https://doi.org/10.1016/j.tra.2015.05.016

Cools, M., Fabbro, Y., & Bellemans, T. (2016). Free public transport: A socio-cognitive analysis. *Transportation Research Part A: Policy and Practice*, *86*, 96–107. https://doi.org/10.1016/j.tra.2016.02.010

Croissant, Y. (2012). *Estimation of multinomial logit models in R: The mlogit Packages*. *R package version 0.2-2*. Retrieved from https://cran.r-project.org/web/packages/mlogit/vignettes/mlogit.pdf

Currie, G., & Wallis, I. (2008). Effective ways to grow urban bus markets – a synthesis of evidence. *Journal of Transport Geography*, *16*(6), 419–429. https://doi.org/https://doi.org/10.1016/j.jtrangeo.2008.04.007

De Palma, A., Lindsey, R., Quinet, E., & Vickerman, R. (2011a). *A Handbook of Transport Economics*. (A. De Palma, R. Lindsey, E. Quinet, & R. Vickerman, Eds.). Cheltenham, UK. Northampton, MA, USA: Edward Elgar.

De Palma, A., Lindsey, R., Quinet, E., & Vickerman, R. (2011b). Introduction. In A. De Palma, R. Lindsey, Q. Emile, & R. Vickerman (Eds.), *A Handbook of Transport Economics* (1st ed., pp. 1–20). Cheltenham, UK. Northampton, MA, USA: Edward Elgar.

de Vasconcellos, E. A. (1999). *Circular é preciso, viver não é preciso: a história do trânsito na cidade de São Paulo*. Annablume.

Domencich, T. A., & McFadden, D. (1975). *Urban travel demand: a behavioral analysis*. (D. W. Jorgenson & J. Waelbroeck, Eds.) (1st ed.). Amsterdam; Oxford: North-Holland Publishing Company.

Drevs, F., Tscheulin, D. K., Lindenmeier, J., & Renner, S. (2014). Crowding-in or crowding out: An empirical analysis on the effect of subsidies on individual willingness-to-pay for public transportation. *Transportation Research Part A: Policy and Practice*, *59*, 250–261. https://doi.org/10.1016/j.tra.2013.10.023

El-Geneidy, A., Levinson, D., Diab, E., Boisjoly, G., Verbich, D., & Loong, C. (2016). The cost of equity: Assessing transit accessibility and social disparity using total travel cost. *Transportation Research Part A: Policy and Practice*, *91*, 302–316. https://doi.org/10.1016/j.tra.2016.07.003

Estupiñán, N., Gómez-Lobo, A., Muñoz-Raskin, R., & Serebrisky, T. (2007). *Affordability and Subsidies in Public Urban Transport: What Do We Mean, What Can Be Done?* (4440). *Transport Reviews*. Washington, DC. https://doi.org/10.1080/01441640902786415

Ferguson, E. (1992). Transit ridership, incident effects and public policy. *Transportation Research Part A*, *26*(5), 393–407. https://doi.org/10.1016/0965-8564(92)90003-P

Fjeldstad, O., & Moore, M. (2007). *Taxation and State-Building: Poor countries in a Globalised World* (2007 No. 11). Bergen: Chr. Michelsen Institute.

Gardner, L. M., Unnikrishnan, A., & Waller, S. T. (2010). Solution Methods for Robust Pricing of Transportation Networks under Uncertain Demand. *Transportation Research Part C: Emerging Technologies*, *18*(5), 656–667. https://doi.org/http://dx.doi.org/10.1016/j.trc.2009.09.004

Gillen, D. W. (1977). Alternative Policy Variables to Influence Urban Transport Demand. *The Canadian Journal of Economics / Revue Canadienne d'Economique*, *10*(4), 686–695. https://doi.org/10.2307/134299

Glaeser, E. (2011). *Triumph of the city: How our greatest invention makes us richer, smarter, greener, healthier, and happier*. Penguin Books.

Gomide, A. de Á. (2003). Transporte urbano e inclusão social: elementos para políticas públicas. Brasília: Instituto de Pesquisa Econômica Aplicada (Ipea): Texto para discussão, n. 960.

Gomide, A. de Á. (2008). Agenda Governamental e o Processo de Políticas Públicas: O Projeto de Lei de Diretrizes da Política Nacional de Mobilidade Urbana. *Ipea*. Brasília: Instituto de Pesquisa Econômica Aplicada (Ipea): Texto para discussão, n. 1334. https://doi.org/ISSN 1415-4765

Greene, W. H. (2009). Discrete Choice Modeling. In *Palgrave Handbook of Econometrics* (pp. 473–556). London: Palgrave Macmillan UK. https://doi.org/10.1057/9780230244405_11

Greene, W. H. (2012). *Econometric analysis* (7th ed.). Prentice Hall.

Grin, E. J. (2011). Caminhos e descaminhos das subprefeituras na cidade de São Paulo no governo Marta Suplicy (2001-2004).

Gwilliam, K. M. (2002). *Cities on the Move*. Washington, DC: The World Bank. Retrieved from http://documents.worldbank.org/curated/pt/928301468762905413/pdf/multi0page.pdf

Haddad, E. A., Gracia, N. L., Germani, E., Vieira, R. S., Aguilera, A. I., Alves, B. B., … Skoufias, E. (2017). Mobility in Cities: Distributional Impact Analysis of Transportation Improvement in São Paulo Metropolitan Region. In *45º Encontro Nacional da Associação Nacional dos Centros de Pós-Graduação em Economia* (pp. 1–19).

Haddad, E. A., Hewings, G. J. D., Porsse, A. A., Van Leeuwen, E. S., & Vieira, R. S. (2015). The underground economy: Tracking the higher-order economic impacts of the São Paulo Subway System. *Transportation Research Part A: Policy and Practice*, *73*(Supplement C), 18–30. https://doi.org/https://doi.org/10.1016/j.tra.2014.12.011

Haddad, E. A., & Vieira, R. S. (2015). Mobilidade, Acessibilidade e Produtividade: Nota sobre a Valoração Econômica do Tempo de Viagem na Região Metropolitana de São Paulo. *Revista de Economia Contemporânea*, *19*, 343–365. Retrieved from http://www.usp.br/nereus/wp-content/uploads/TD_Nereus_08_2015.pdf

Hausman, J., & McFadden, D. (1984). Specification Tests for the Multinomial Logit Model. *Econometrica*, *52*(5), 1219–1240. Retrieved from https://econpapers.repec.org/RePEc:ecm:emetrp:v:52:y:1984:i:5:p:1219-40

Hensher, D. A., & Bullock, R. G. (1979). Price elasticity of commuter mode choice: Effect of a 20 per cent rail fare reduction. *Transportation Research Part A: General*, *13*(3), 193–202. https://doi.org/10.1016/0191-2607(79)90071-2

Hensher, D. A., & Button, K. J. (2000). *Handbook of Transport Modelling*. (D. A. Hensher & K. J. Button, Eds.) (1st ed.). Oxford: Elsevier.

Hirata, D. V. (2011). Produção da desordem e gestão da ordem: Notas para uma história recente do transporte clandestino em São Paulo. *Dilemas-Revista de Estudos de Conflito E Controle Social*, *4*(3), 441–465.

ITDP. (2017). *Mobility as Equity: Low-Income People Near Transit*. Rio de Janeiro. Retrieved from https://www.itdp.org/wp-content/uploads/2017/01/4.-Mobility-as-Equity-Low-Income-People-Near-Transit.pdf

Keen, M. (2012). *Taxation and Development - Again* (12/220). *IMF Working Paper*. https://doi.org/10.5089/9781475510294.001

Koppelman, F. S., & Vaneet, S. (2000). Closed-form Discrete-choice Models. In D. A. Hensher & K. J. Button (Eds.), *Handbook of Transport Modelling* (1st ed., pp. 211–228). Oxford: Elsevier.

Lancaster, T. D., & Patterson, D. W. (1990). Comparative Pork Barrel Politics: Perceptions from

the West German Bundestag. *Comparative Political Studies*, *22*(4), 458–477.

Lindsey, M., Schofer, J. L., Durango-Cohen, P., & Gray, K. A. (2010). Relationship between proximity to transit and ridership for journey-to-work trips in Chicago. *Transportation Research Part A: Policy and Practice*, *44*(9), 697–709. https://doi.org/10.1016/j.tra.2010.07.003

Long, J. S., & Freese, J. (2014). *Regression Models for Categorical Dependent Variables Using STATA. Sociology The Journal Of The British Sociological Association* (3rd ed.). Press, Stata.

Manski, C. F. (2001). Daniel McFadden and the Econometric Analysis of Discrete Choice. *The Scandinavian Journal of Economics*, *103*(2), 217–229. Retrieved from http://www.jstor.org/stable/3440992

McFadden, D. (1973). Conditional logit analysis of qualitative choice behavior. In P. Zarembka (Ed.), *Frontiers in Econometrics* (pp. 105–142). New York: Academic Press. https://doi.org/10.1108/eb028592

McFadden, D. (1974). The measurment of urban travel demand. *Journal of Public Economics*, *3*, 303–328. Retrieved from https://eml.berkeley.edu/reprints/mcfadden/measurement.pdf

McFadden, D. (1978). Modeling the choice of residential location. *Transportation Research Record*, (673).

McFadden, D. (1999). Rationality for Economists? *Journal of Risk and Uncertainty*, *19*(1–3), 73–105. https://doi.org/10.1023/A:1007863007855

McFadden, D. (2011). Foreword. In A. De Palma, R. Lindsey, Q. Emile, & R. Vickerman (Eds.), *A Handbook of Transport Economics* (pp. xv–xvii). Cheltenham, UK. Northampton, MA, USA: Edward Elgar.

METRO. (1999). *Pesquisa Origem e Destino 1997 - Região Metropolitana de São Paulo: Síntese das Informações - Pesquisa Domiciliar*. São Paulo. Retrieved from http://www.metro.sp.gov.br/metro/arquivos/OD1997/sintese_OD_1997.pdf

METRO. (2008). *Pesquisa Origem e Destino 2007 - Região Metropolitana de São Paulo: Síntese das Informações - Pesquisa Domiciliar*. São Paulo. Retrieved from http://www.metro.sp.gov.br/metro/arquivos/OD2007/sintese-od2007.pdf

Meyer, R. M. P., Grostein, M. D., & Biderman, C. (2004a). Capítulo 2 - Parte 1. In *São Paulo Metrópole*.

Meyer, R. M. P., Grostein, M. D., & Biderman, C. (2004b). Capítulo 2 - Parte 2. In *São Paulo Metrópole*.

Mogridge, M. J. (1997). The self-defeating nature of urban road capacity policy: A review of theories, disputes and available evidence. *Transport Policy*, *4*(1), 5–23. https://doi.org/10.1016/S0967-070X(96)00030-3

Mohring, H. (1972). Optimization and Scale Economies in Urban Bus Transportation. *The American Economic Review*, *62*(4), 591–604. Retrieved from http://www.jstor.org/stable/1806101

Nicholson, W., & Snyder, C. (2005). *Microeconomic Theory: Basic Principles and Extensions* (10th ed.). Mason: South Western/Thomson.

O GLOBO. (2017, December 29). Com BRT depredado e vans liberadas, transporte municipal do Rio ruma para o colapso. Retrieved from https://oglobo.globo.com/rio/com-brt-depredado-vans-liberadas-transporte-municipal-do-rio-ruma-para-colapso-22237347

Ortúzar, J. de D., & Willumsen, L. G. (1994). Introduction. In *Modelling transport* (4a ed., pp. 1–28). Clichester: Wiley.

Ortúzar, J. de D., & Willumsen, L. G. (2011). *Modelling Transport*. *Modelling Transport* (4th ed.). Chichester, UK: John Wiley & Sons, Ltd. https://doi.org/10.1002/9781119993308

Pacheco, T. S., & Chagas, A. L. S. (2016). Demanda por transporte na Região Metropolitana de São Paulo e política de pedágio urbano para redução de congestionamento. In *Anais do 44º Encontro Nacional de Economia*.

Parry, I. W. H., & Small, K. A. (2009). Should Urban Transit Subsidies Be Reduced? *The American Economic Review*, *99*(3), 700–724. Retrieved from http://www.jstor.org/stable/25592479

Quigley, J. M., & Hårsman, B. (2010). Political and Public Acceptability of Congestion Pricing: Ideology and Self-Interest. *Journal of Policy Analysis and Management*, *29*(4), 854–874. Retrieved from http://www.jstor.org/stable/40802194

Quinet, E., & Vickerman, R. (2004). *Principles of Transport Economics*. Edward Elgar Publishing.

Rodden, J. (2003). Reviving Leviathan: fiscal federalism and the growth of government. *International Organization*, *57*(4), 695–729.

Shyr, O. F., Andersson, D. E., Cheng, Y. H., & Hsiao, Y. H. (2017). What explains rapid transit use? Evidence from 97 urbanized areas. *Transportation Research Part A: Policy and Practice*, *100*, 162–169. https://doi.org/10.1016/j.tra.2017.04.019

Small, K. A. (1987). A Discrete Choice Model for Ordered Alternatives. *Econometrica*, *55*(2), 409–424. https://doi.org/10.2307/1913243

Smart, M. J. (2014). A volatile relationship: The effect of changing gasoline prices on public support for mass transit. *Transportation Research Part A: Policy and Practice*, *61*, 178–185. https://doi.org/10.1016/j.tra.2014.01.011

Sobel, R. S. (2005). Welfare economics and public finance. In *Handbook of public finance* (pp. 19–51). Springer.

Sun, Y., Guo, Q., Schonfeld, P., & Li, Z. (2016). Implications of the cost of public funds in

public transit subsidization and regulation. *Transportation Research Part A: Policy and Practice*, *91*, 236–250. https://doi.org/10.1016/j.tra.2016.06.029

Svab, H. (2016, October 3). *Evolução dos padrões de deslocamento na região metropolitana de São Paulo: a necessidade de uma análise de gênero.* Universidade de São Paulo, São Paulo. https://doi.org/10.11606/D.3.2016.tde-30092016-142308

Tang, L., & Thakuriah, P. (Vonu). (2012). Ridership effects of real-time bus information system: A case study in the City of Chicago. *Transportation Research Part C: Emerging Technologies*, *22*, 146–161. https://doi.org/http://dx.doi.org/10.1016/j.trc.2012.01.001

Tartaroti, R. (2015). Dissertação de Mestrado: Modelos de concessão de transporte coletivo por ônibus: o caso da cidade de São Paulo e suas alternativas. São Paulo.

Toledo, R. P. de. (2015). *A Capital da Vertigem* (1st ed.). Rio de Janeiro: Objetiva.

Train, K. (2009). *Discrete Choice Methods with Simulation* (2nd ed.). Cambridge: Cambridge University Press.

Train, K., & McFadden, D. (1978). The goods/leisure tradeoff and disaggregate work trip mode choice models. *Transportation Research*, *12*(5), 349–353. https://doi.org/10.1016/0041-1647(78)90011-4

Walker, J. L., & Ben-Akiva, M. (2011). Advances in discrete choice: mixture models. In A. De Palma, R. Lindsey, E. Quinet, & R. Vickerman (Eds.), *A Handbook of Transport Economics* (pp. 160–187). Cheltenham, UK. Northampton, MA, USA: Edward Elgar.

Wang, G. H. K., & Skinner, D. (1984). The impact of fare and gasoline price changes on monthly transit ridership: Empirical evidence from seven U.S. transit authorities. *Transportation Research Part B*, *18*(1), 29–41. https://doi.org/10.1016/0191-2615(84)90004-3

Wang, W. (Walker), Wang, D. Z. W., Zhang, F., Sun, H., Zhang, W., & Wu, J. (2017). Overcoming the Downs-Thomson Paradox by transit subsidy policies. *Transportation Research Part A: Policy and Practice*, *95*, 126–147. https://doi.org/10.1016/j.tra.2016.11.003

Watkins, K. E., Ferris, B., Borning, A., Rutherford, G. S., & Layton, D. (2011). Where Is My Bus? Impact of mobile real-time information on the perceived and actual wait time of transit riders. *Transportation Research Part A: Policy and Practice*, *45*(8), 839–848. https://doi.org/10.1016/j.tra.2011.06.010

Wen, C.-H., & Koppelman, F. S. (2001). The generalized nested logit model. *Transportation Research Part B: Methodological*, *35*(7), 627–641. https://doi.org/http://dx.doi.org/10.1016/S0191-2615(00)00045-X

Zhang, F., Yang, H., & Liu, W. (2014). The Downs-Thomson Paradox with responsive transit service. *Transportation Research Part A: Policy and Practice*, *70*, 244–263. https://doi.org/10.1016/j.tra.2014.10.022