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DOCTORAL THESIS

Essays on Monetary Theory

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Abstract

Escola de Pós-Graduação em Economia

Doctor of Philosophy

Essays on Monetary Theory

by Caio Augusto Colnago Teles

In this thesis, we use mechanism design approach in order to study economies in which the optimal mechanism bears some resemblance to actual monetary system. More precisely, we study optimal monetary policy in models in which either: money is *essential*, or, money and bonds are *coessential*. In the first chapter, we study an optimal intervention in a model of outside money. Next, we extend the model to include bonds and interpret its role. Finaly, the last chapter we discuss the problems with the usual modeling aproach to monetary policy transition and its implications.

Keywords: Money; Monetary Policy; Inflation; Mechanism Design; Inside Money;

Introduction

In this thesis, we use mechanism design approach in order to study economies in which optimal mechanism bears some resemblance to actual monetary system. More precisely, we study optimal monetary policy in models in which either: money is *essential*, or, money and bonds are *coessential*. For this, we build on search-based models in tradition of Shi-Trejos-Wright and numerical simulation.

In chapter 1, we review and reinterpret the results obtained in Deviatov (2006). In order to test for robustness and extend its generality, we include more general simulations of the Shi-Trejos-Wright model with higher upper bound of fiat money, as well as develop a model of intermediation in tripartite meetings. We find the following unnoticed implications of lack of commitment in matching models of money: savings are inefficiently low; inflation has a negative effect on self-insurance; and although lump-sum transfers should be avoided in many specifications, positive inflation can be optimal with inside money.

In chapter 2, we present a model in which money and bonds are *coessential*. We set up a random matching environment in which money distribution is the centerpiece of welfare considerations. Using mechanism design approach, we present mechanisms with money and illiquid bonds that enlarge the set of implementable allocations. The benefits from the use of bonds arise from its illiquidity feature. Following Kocherlakota (2003), the improvement are achieved by the bonds' commitment capacity. More speciffically, illiquid bonds induce high savings, allowing control spending in meetings that disperse money. Moreover, this control spending device seems much more effective than redistribution.

In chapter 3, we investigate the effects of different modeling approaches to monetary policy transiton. In the mechanism design approach to monetary theory, due to the high complexity of the problem, search-based models in tradition of Shi-Trejos-Wright usually model monetary policy by the composition of two independent matrices. We argue that this modeling approach brings with it serious inconveniences. An active monetary policy would increase the uncertainty faced by agents. Furthermore, in many relevant instances, it

also leads to an undesired increase in money dispersion. Thus, we propose an alternative to this modeling, getting rid of such inconveniences but also respecting its essence. We show that with the proposed approach, an active monetary policy becomes more attractive and it is optimal for a much larger set of parameters. We conclude that the monetary policy in Deviatov (2006) is suboptimal due to this modeling peculiarity.

Chapter 1

A paradox of expansionary policies

1.1 Introduction

A recent consensus in monetary theory is that money is an imperfect mechanism.[1] Absent interventions, it is a poor store of value. And the dynamics of monetary trades produces an uneven distribution of money, exposing traders to liquidity risks. The main lesson seems to be that a monetary program for creating liquidity can lessen these imperfections, hurting one dimension in order to improve the other.

A benchmark experiment involves models of pure currency, with people facing the risk of running out of money due to a particular sequence of trade opportunities. It is then asked whether lump-sum transfers of money improve welfare, relative to a zero-inflation equilibrium. Using variations of the Bewley (1980) model, Levine (1991) and Kehoe, Levine, and Woodford (1992) have tackled this question, thereby constructing an important counterexample to the Friedman (1953) and Friedman (1969) rule. Using particular assumptions, they focus, however, on Markov equilibria in which nominal payments are invariant to the inflation rate.

Keeping the distribution of money tractable and still capturing a negative effect of inflation on output is certainly a step forward. In this paper, however, we argue that there is a key drawback in this forced simplification, because another negative effect of inflation is ignored. It is robust and can change the role of expansionary policies. Using matching models, we ask what happens when inflation is allowed to reduce savings. We then find a paradox: lump-sum transfers can actually make the risk-sharing problem worse when the distribution of money is sufficiently rich. Our point about monetary policy is that it should

[1]This is a join work with Ricardo Cavalcanti and Fernando Barros.

be assessed by similar considerations related to crowding-out of self-insurance [see, for instance, the non-monetary model with private insurance markets and limited enforcement by Krueger and Perri (2011)].

Illustrating our point requires removing simplifications adopted by existing work, forcing us to appeal to numerical methods. Instead of a Bewley model, we use some variations of the basic random-matching model and provide a careful explanation why previous work has missed this paradox. Part is due to the misconception that main issue with inflation is the so called 'hot potato effect'. One can indeed easily see in a cash-in-advance model that inflation reduces the return of money, producing a distortion as the real quantity of goods traded falls. But the ignored effect we are talking about is only present when the quantity of nominal savings is important, and this is not the case with a representative agent, or when the model allows for a quick rebalancing of money balances. In order to study the paradox, it is important to be able to exacerbate the role of nominal savings.[2]

The fact is that in matching models with fiat money, better incentives to save can prevent a negative externality from taking place: when sellers receive a high nominal payment they become less inclined to trade goods for money in the future, aside from the fact that each unit of money may have an inadequate return. In this paper, we illustrate this fact from different but related angles. First, we explain why extensions of the models in Shi (1995) and Trejos and Wright (1995), pursued by Molico (2006) and Deviatov (2006), find a role for inflation: it is because the former does not allow for allocations that provide better insurance to poor money-holders, but which could introduce non-convexities, while the latter restricted the upper-bound on holdings in such a way that inflation was not reducing savings when expansionary policies mattered. Second, we find a way of making savings more important in a commodity-money version of Lagos and Wright (2005). In particular, we allow for tripartite meetings, highlighting the importance of money holdings by intermediaries. Third, we revisit fiat-money exercises with this tripartite structure, discussing restrictions to 'core' allocations and the assumption that intermediaries can now be regulated. We then document the fact that, for many parameter specifications, savings are particularly low with core

[2]Bewley models (see recent application by Lippi, Ragni, and Trachter (2014)) allow poor and rich traders to find a relatively fast regression to mean holdings because they participate in the same market every period. The speed of such redistribution, by contrast, is much slower in general matching models. This difference has not been emphasized in the literature (see Wallace (2014a) for other novelties), but is key for understanding the paradox and may as well be present when inflationary transfers are used to pay interests on money (see Wallace (2014b)). Such policies, although also interesting, lie outside the scope of this paper.

allocations, but regulation of intermediaries can lessen the problem.

If Levine (1991) and Bewley (1980) are offering counterexamples to the Friedman rule, the reader may ask, to what previous research our results give more support? The related literature is a voluminous one. In the conclusion we make few remarks on the Wallace (2014b) conjecture that positive inflation in pure-currency economies should be optimal with the possible exception of knife-edge specifications. In section 1.3 we mention some additional literature, while in section 1.6 we highlight passages of Bagehot (1873) as early references to the importance of keeping a proper distribution of money. Finally, section 1.7 concludes and the appendix contains proofs as well as auxiliary information about simulations.

1.2 A preview

Our counterexamples are selected from steady states, and focus on how traders can privately (self-) insure against shocks using currency, given that meetings are heterogeneous and random, while gains from trade can in principle be divided in a variety of ways, according to wealth profiles in each meeting.[3] They also complement findings by Deviatov (2006), and Deviatov and Wallace (2014), about optimal policy with outside and inside money.[4] Before describing novel ways to reinforce concerns with monetary savings (starting first with a quasi-linear model with commodity money, in the next session, and then with fiat money and more general preferences), we give now an overview of basic simulations to motivate that idea that even the simple pairwise-meetings model can give rise to robust counterexamples.

[3]The idea that policy evaluation depends crucially on self-insurance and thus on available forms to store wealth motivates the empirical study of Krueger and Perri (2008). There is also a large literature featuring numerical work in Bewley models (such as Imrohoroglu (1992)). While in matching models (see an extensive survey by Lagos, Rocheteau, and Wright (2014)) negative effects of inflation on self-insurance have not been singled out, there is a remote relationship between our findings and the notion of constrained inefficiency (see Geanakoplos and Polemarchakis (1986)): an inefficiently high level of capital holdings in the Aiyagari (1994) model, for instance, generates an externality in the form of more labor-income risk (see Davila et al. (2012) and references therein). Finally, an empirical assessment of how redistributive effects of inflation depend on spending possibilities, measured by the maturity structure of nominal assets, is the subject of Doepke and Schneider (2006).

[4]Instead of using a Bewley model, Cavalcanti and Nosal (2009) propose another way of making flat expansions appealing, using a simple matching model with just 0-1 holdings of money, and adding a seasonal pattern. In particular, they specify a utility jump for people specializing in consumption at a particular season and finding welfare gains when expansions target that season (see also Wallace (2014a) and references therein). But negative effects of inflation in their setting are also narrowly defined (in their model policies are introduced to correct the problem that savings are too high in low seasons).

There are two key observations in these simulations. First, because consumption and production of a perishable good takes part in pairwise meetings, the social planner is not restricted to the 'law of one price' as in Bewley models. This means that allocations involving poor traders can be target so as to provide insurance with attractive 'prices' for this group, an alternative to inflationary policies affecting the whole population. Second, self-insurance in the form of money holdings depends on how wealth can be stored. In particular, in numerical simulations, if the upper bound on holdings is too small, expansionary policies gain an artificial edge over self-insurance.

Our simulations (presented in detail later) use lotteries, as in Berentsen, Molico, and Wright (2002), having the interpretation that people are spending (and receiving from the government, when expansionary policies are in effect) non-integer amounts of money. This approach is complemented by a proxy of inflation suggested by Li (1994) and Li (1995): the inflation tax is captured by some random confiscation of money holdings. As in many money models, these economies have people meeting randomly in pairs and trading perishable goods for fiat money as in Shi (1995) and Trejos and Wright (1995). Expected utilities associated with money holdings, v, must be consistent with payments and output produced in single-coincidence meetings. A social planner chooses from stationary allocations according to average utility. For an affordable numerical cost, we assume that money is indivisible and that goods are traded for lotteries on money holdings, with support $\{0, 1, 2, 3, 4\}$. There is a large set of possible stationary distributions of money and terms of trade: output and lotteries for each type of meeting, as indexed by traders' wealth. Exchanges must be better than autarky in meetings for both traders and belong to the core (defined formally later). We have assembled, in Table 1, selected features of optimal allocations.

TABLE 1.1: Insurance when optimal inflation is zero: taxing consumers in meetings with the poorest producer

β	0.9	0.8	0.7	0.6	0.5
i	v / tax	v / tax	v / tax	v / tax	v / tax
0	0.4514 / -	0.0263 / -	0.0089 / -	0.0018 / -	0.0013 / -
1	1.2638 / 0.0000	0.5940 / 0.0000	0.3948 / 0.0000	0.2282 / 0.0000	0.2253 / 0.0000
2	1.5692 / 0.4067	0.7824 / 0.0000	0.5166 / 0.0000	0.3663 / 0.0502	0.2738 / 0.0698
3	1.7538 / 0.2271	0.8794 / 0.0000	0.5675 / 0.0000	0.3892 / 0.0000	0.2861 / 0.0000
4	1.9027 / 0.0552	0.9291 / 0.3964	0.5851 / 0.2921	0.4107 / 0.0000	0.3010 / 0.0000

Based on Table 9. v represents the expected discounted utility for each level of holdings i before meetings take place; In meetings where the consumer holds i and the producer holds 0, 'tax' is computed as $1 - y/x$, where y is actual output and x is the cuttoff level determining zero surplus for the producer, keeping fixed the optimal payment.

By letting a key parameter, the discount factor β, take different values, we allow the discrepancy between private and social objectives to vary: as we shall see later on, as β is increased, concerns about savings loses importance, so that there is more flexibility with respect to how taxation is levied. For now, some interesting insights are revealed by measures of maximum output that poorest producers could be asked to hand out, given the expected utilities they are actually receiving as payments (not displayed). The relative differences between these ceilings and quantities actually delivered, labeled *tax*, represents a loss to consumers that varies with their holdings of money. According to Table 1, for all β, the poorest producer is always receiving a surplus in some meetings (identified by positive taxes). The precise wealth i of whom is taxed depends on β.

Now there are two remarks on previous work in connection with features of Table 1. First, in attempts to reproduce Levine (1991) findings by Molico (2006) and Deviatov (2006), corresponding tax statistics are typically zero. In the former case, the tax is always zero due to a bargaining assumption. In the latter, as shown below, for examples with positive inflation the tax is zero because the support used was too small (a 2-unit upper bound), leading to a flatter v (less affected by Inada conditions). Second, we did allow for a lump-sum transfer of money followed by inflation in the same fashion as Deviatov (2006) did, but no improvements were found. The reason, as mentioned above, is that inflation interacts adversely with self-insurance.

1.3 Adding more risk

We now describe how to make savings even more important in matching models. Adopting the mechanism-design perspective of Hu, Kennan, and Wallace (2009) and Cavalcanti and Puzzello (2010), for the model of Lagos and Wright (2005), the idea is to require a form of intermediation in meetings, highlighting the importance of the distribution of money and, later, new effects when policies include regulation of inside money.

In this section, for simplicity, we avoid the discussion of interventions in the quantity of money. The goal for now is to show externalities in savings decisions. We model commodity money in a version of quasi-linear economies due to Cavalcanti and Puzzello (2010). Random meetings are preceded by a subperiod in which people must decide how much to consume or to save in the form of a durable commodity. Ideally, society would have traders

choosing high savings at that moment. But since traders cannot commit there is an instance of the Jacklin (1987) problem: after people learn their types, and corresponding opportunity costs of carrying money holdings, they tend to choose lower levels of savings.[5] It is then shown how consumption taxes can improve welfare relative to the alternative of giving all the surplus to consumers at the meetings stage.

In order to avoid complicated dynamic effects, we assume that commodity money is valued according to separable, linear utility. Relevant histories of savings are entirely captured by recent preference shocks. When people trade in pairs taxation is not needed and consumers keep all gains from trade. But when intermediation is considered, and meetings include a third trader that can lend to consumers, we find that savings by intermediaries are important and that some interventions can provide a better allocation of risk.

1.3.1 A commodity-money environment

Time is discrete and each period is divided into two subperiods. The economy has a large population living forever and experiencing random meetings in the first subperiod, and preference shocks in the second. Preference shocks are realizations of an iid process. There is a durable good called money that can be consumed and produced in the second subperiod, according to an idiosyncratic marginal utility θ drawn every date from an uniform distribution. For simplicity, we assume a discrete support $\{\theta_1, ..., \theta_n\}$ and let F, such that $F(\theta_i) = \frac{i}{n}$ for all i, denote the cumulative distribution of θ. In addition, we normalize its mean, setting $\sum_i \frac{1}{n}\theta_i = 1$.

Money is hence a commodity, produced and consumed when people are by themselves, according to linear utility that is the realization of a preference shock. Money balances are planned in order to reach ideal savings, for each θ, for use as a medium of exchange in the next period, first subperiod, when random meetings take place. Money holdings are observable in meetings but trade histories are private information and people cannot commit to future actions.

There is no discounting between first and second subperiods, but there is discounting at the common factor β across dates. We assume θ_i is increasing in i with $\theta_1 > \beta$, so that savings

[5] In Jacklin (1987), allowing traders to exchange claims on bank deposits eliminates risk sharing from deposit contracts. Hence trading after types are assigned allows people to avoid taxation. In monetary models, when a bank is not providing the final allocation of goods, it is necessary to discuss if trading is amplified by policy, in a version of the Lucas (1976) critique.

are always costly. There is also the standard specialization of production and consumption in meetings. We assume that every meeting is formed by three people: a producer, an intermediary and a consumer. We assume that a person has equal probability of taking part in a meeting in any of these three occupations. And that the meeting is a single-coincidence meeting, when the first person can produce a perishable good for the third one, with probability 3α, where $\alpha \leq \frac{1}{3}$. With probability $1 - 3\alpha$ there are no potential gains from trade. The utility of consuming $c \in \mathbb{R}_+$ units of the perishable good is $u(c)$, and the utility of producing c units of the perishable good is $-c$. We assume that $u(0) = 0$ and that u is continuous, concave, differentiable and such that $u'(0) = +\infty$ and $u(c) < c$ for c sufficiently large.

We assume that the only feasible trade in a meeting has the intermediary transferring money to the producer, as loan to the consumer, in exchange for goods produced. Then, after production takes place and the producer leaves the meeting, the consumer is able to receive goods and to pay out the loan with the intermediary.

In this economy, the planner's problem is to maximize the present value of average utility by choice of incentive-compatible allocations that provide a suitable level of insurance against shock θ and exchange risk. Following Cavalcanti and Puzzello (2010), we restrict attention to stationary allocations. We also anticipate that, due to the quasi-linear structure, optimal allocations are not functions of past histories. A meeting is a vector $m = (m_1, m_2, m_3)$ describing holdings of money of the producer, m_1, the intermediary, m_2 , and the consumer, m_3. An allocation is a list (s, x, y, z) describing saving plans s in the second subperiod, as a function θ, and trade plans (x, y, z) in the first subperiod, as a function of m. Saving plans say how much money people will take with them when leaving the second subperiod, according to the realization of idiosyncratic shocks. Trade plans describe loan size x, output level y, and payment amount z. That is, z is the reduction in holdings of money suffered by the consumer, x is how much the producer receives, and $z - x$ is the intermediation profit. We require money transfers to be feasible in the sense of $x(m) \leq m_2$ and $z(m) \leq m_3$.

A plan $s : \{\theta_1, ..., \theta_n\} \to \mathbb{R}_+$ generates a distribution of money μ on $\mathbb{R}_+$. It is convenient to denote by μ^3 the distribution of meetings on $\mathbb{R}^3_+$ generated by μ, and by μ^2 it marginal distribution on $\mathbb{R}^2_+$ when one coordinate of m is fixed. The welfare criteria corresponds to

the utility of an ex-ante representative agent and can be written as

$$w(s,y) = -\int(\theta-\beta)s(\theta)dF(\theta) + \alpha\beta\int(u(y(m)) - y(m))d\mu^3(m). \tag{1.1}$$

Notice that the welfare function w does not depend on monetary payments. This is so because expected utility, when leaving meetings, as a function of after-trade holdings, is the same for all traders. Hence, no matter how money is divided by trade, the average discounted value attached to after-trade holdings is $\beta\sum_i \frac{1}{n}s(\theta_i)$.

1.3.2 Implementable allocations

We also follow the notion of implementability adopted by Cavalcanti and Puzzello (2010), that is, that traders agree with (s,x,y,z), given μ associated to s, if autarky in meetings would not make them better off, and if there are no other saving choices that could improve individual utility given (x,y,z) and μ. We shall leave the discussion of group deviations for the fiat money environment of following sections. But while in Cavalcanti and Puzzello (2010) it is optimal to give all surplus to consumers, here this is not so due to an externality associated to savings decisions of people who end up in position to make loans.

In order to be implementable, an allocation must satisfy incentive constraints. Trade incentive constraints are given by

$$y(m) \leq x(m),\ x(m) \leq z(m) \text{ and } z(m) \leq u(y(m)). \tag{1.2}$$

These inequalities ensure that trade surpluses are nonnegative in all meetings. The saving incentive constraint is that $s(\theta)$ must solve the problem of maximizing $-(\theta-\beta)k + \alpha\beta v(k)$ by choice of money holdings k , where the expected gain from trade $v(k)$ is defined by

$$\begin{aligned} v(k) \quad = \quad & \int (u(y(a,a',k)) - z(a,a',k) + z(a,k,a') \\ & -x(a,k,a') + x(k,a,a') - y(k,a,a'))d\mu^2(a,a'). \end{aligned} \tag{1.3}$$

Because the distribution of money μ in turn must be generated by s, an incentive-compatible savings plan is a fixed point for each (x,y,z). Allocations that are feasible and incentive compatible are called implementable.

1.3.3 Welfare bounds

We are now ready to make two basic points about this intermediation economy. In this subsection we show that there is essentially no friction leading to low savings if intermediation is either removed or if the distribution of shocks is degenerate. If constraints associated with intermediation are relaxed then we find no wedge between private incentives to save and the planner's problem. In addition, the solution can be computed recursively: for a given distribution of savings, welfare is maximized by giving all trade surpluses to consumers; and given the implied rate of exchange between money and output, savings are chosen optimally in a separable way according to the costs of supplying money (see proof of next proposition). In the next subsection, we show how a tax system can be used to perturb this kind of allocation in order to increase the incentives to save according to a transfer plan that rewards intermediation.

With intermediation, part of money held by relatively rich consumers cannot be used to weaken incentive constraints of producers. For a given stock of money, average utility in meetings depends only on consumption and production, not on how money is divided among traders. But if intermediation activity receives no compensation, incentives to save can be suboptimal.

Let (s^*, x^*, y^*, z^*) denote the solution of the planner's problem of maximizing $w(s, y)$ in the set of implementable allocations. Let us first turn off the intermediation constraint (the cash-in-advance requirement $x(m) \leq m_2$), denoting by $(\hat{s}, \hat{x}, \hat{y}, \hat{z})$ a solution of the corresponding relaxed problem. Notice that, in this case, the incentive constraints $y(m) \leq x(m)$ and $x(m) \leq z(m)$, together with the feasibility constraint $z(m) \leq m_3$, imply the inequality $y(m) \leq m_3$. It turns out that $w(\hat{s}, \hat{y})$ is the optimal welfare of a random-matching model without intermediaries.

In the following proposition, a comparison is made with another relaxed problem, obtained by imposing $x(m) \leq m_2$ but ignoring saving incentive constraints, as if s can be imposed on individuals. If $(\tilde{s}, \tilde{x}, \tilde{y}, \tilde{z})$ denotes the solution of this second problem, the following holds.

Proposition 1. *For m in the support of distributions of meetings, output is $\hat{y}(m) = m_3$ when intermediation is relaxed, and $\tilde{y}(m) = \min\{m_2, m_3\}$ when savings need not be incentive compatible. In*

these relaxed problems, moreover, welfare satisfies $w(\hat{s}, \hat{y}) \geq w(\tilde{s}, \tilde{y}) \geq w(s^*, y^*)$, *with inequalities replaced by equalities when there is a single type of trader.*

Proof. See appendix. □

1.3.4 Taxes and financial profits

In the absence of intermediation, if consumers extract all surpluses in meetings then $g(k) = \beta[u'(k) - 1]$ is the marginal private gain from bringing an additional unit of money to meetings when savings is k (see Cavalcanti and Puzzello (2010)). The next proposition explores the fact that, with such terms of trade and intermediation, incentive-feasible savings satisfy $\theta - \beta = \alpha F(\theta) g(s(\theta))$.[6] From a social perspective, however, each additional unit saved also affects, with probability α, the volume of resources lent to rich consumers, so that if higher savings could be imposed a welfare gain would follow.

When all surpluses are given to consumers, output in meetings is given by $y(m) = \min\{m_2, m_3\}$ and idle holdings $m_3 - m_2$, when positive, remain with consumers. Taxes can improve savings without reducing intensive margins of consumption for a fixed m. To see this, consider the following perturbation with transfers that increase the incentives to save for all types, except the richest one.

We let $y(m) = x(m) = \min\{m_2, m_3\}$ for all m but allow part of $m_3 - \min\{m_2, m_3\}$ to be transferred to intermediaries. Let $(\bar{s}_1, ..., \bar{s}_n)$ denote incentive-compatible savings for the no-taxation allocation. It is straightforward to show that the saving problem is convex and that $\bar{s}_i > \bar{s}_j$ whenever $\theta_i < \theta_{j.}$ The new allocation is constructed as follows. First a quantity limit $\varepsilon > 0$ and interest rate $r > 0$ are fixed. Then, when consumer with m_3 holdings meets an intermediary with m_2, for $m_3 > m_2$ and $| m_3 - m_2 |> \varepsilon$, then some interest rx is paid to this intermediary if he or she is providing $x \in (\bar{s}_j, \bar{s}_j + \varepsilon)$ in loans. Hence, in the profit allocation, $z(m) = y(m) + rm_2$ in meeting m such that m_2 is discretely lower than m_3. The values of ε and r are chosen sufficiently small so that idle money $m_3 - m_2$ in such meetings is greater than the extra payment rm_2, and also to insure that each type does not envy savings designed for another type.

[6] In the proposition, the first-order condition for the savings problem is written in terms of left derivatives. For numerical examples, incentive-compatible saving s_1, for instance, is found assuming that all other type-1 people are saving a bit more than s_1 and then finding the interior solution $\theta_1 - \beta = \frac{\alpha}{n} g(s_1)$. More generally, savings are still found independently for all grid points.

Proposition 2. *If there is more than one type of trader then welfare is increasing in the profit rate r in a neighborhood of zero, so that it is not optimal to give all surpluses to consumers.*

Proof. See appendix. □

A numerical illustration of welfare gains promoted by taxation is as follows.[7] Table 1.2 displays basic statistics of allocations as r varies. We find that, for a broad range of values of r, taxes are strictly less than $m_3 - m_2$.

TABLE 1.2: Savings according to profit rates

r	s_1	s_2	s_3	s_4	s_5	Welfare
0%	8.5235	4.5027	1.9184	1.0917	0.7832	3.4733
5%	8.5235	4.5621	1.9361	1.0991	0.7876	3.4758
10%	8.5235	4.6228	1.9541	1.1067	0.7921	3.4783
15%	8.5235	4.6850	1.9724	1.1143	0.7966	3.4807

(1) Values multiplied by 100.
(2) If r is increased to 16% then the richest saver is willing to change behavior (to avoid paying next-type profits).

The proof of proposition 2 can be strengthened. Uniform distributions are not needed but we have omitted a more general treatment for ease of exposition. The fact that, with quasi-linear preferences, a third trader is needed to show that taxation has a role may explain why this point has not appeared formally in the literature. The proof, of course, is simplified by the absence of wealth effects: due to the quasi-linear structure, money transferred to intermediaries (or producers) does not change adversely their incentives to produce in the future. In order to allow for such effects we have unfortunately to resort to numerical methods when we discuss economies with fiat money.

1.4 Adding inflation

Fiat and commodity money have in common an externality: after types are learned, consumers may spend too much fiat money, or traders may save too little commodity money, ignoring that their actions can facilitate trades of others. In the case of the commodity-money model they can help by lending money. In the case of the fiat-money model this too can be important but, in addition, there is the problem that a large payment to someone

[7] We set $u(y) = \sqrt[4]{y}$, $\beta = .6$ and $\alpha = .2$. We let the support of θ be $\{.614, .675, .877, 1.215, 1.619\}$ and set $\varepsilon = 1.854 \times 10^{-3}$.

makes that person less willing to produce in the future (capturing this requires dropping the quasi-linearity assumption).

In this section we depart from the quasi-linear structure, taking the tripartite construct to a fiat-money specification with discrete holdings and lotteries. Risk can now be affected by government transfers and a random confiscation of money that resembles the inflation tax. Except in extreme cases, that we interpret as perfect monitoring, money is in exogenous supply (it is *outside* money) and is fiat (it does not provide direct utility). For tractability, we restrict attention to steady states. We consider persistent occupation in sectors, group deviations and a limit case with inside money. To understand why expansionary policies are not needed in some cases, we explore simulations with a smaller support (an upper bound of 2), that introduces corner solutions and cases of optimal intervention on the supply of money.

1.4.1 The fiat-money environment

A steady-state allocation is now $(\mu, y, \lambda, \tau, \pi)$, where μ is a distribution of money, y defines output for each meeting $m \in M$, λ defines payments in terms of lotteries, also for each meeting m, $\tau = (\tau^n, \tau^b)$ describes occupation-dependent transfers, and π is a measure of inflation. People start each period carrying 0, 1 or 2 units of money, so that $M = \{0, 1, 2\}^3$, either in the *bank* (intermediation) sector or in the complement, the *nonbank* sector. The cases of pairwise meetings are straightforward modifications of the specification with intermediation.

After trades occur, holdings of money evolve according to a stochastic process reflecting inflationary transfers. Bank and nonbank occupations are idiosyncratic shocks evolving according to a first-order Markov process. In particular, the probability that bank people keep their occupation in the next period is ρ, and that for nonbank people is $\frac{1+\rho}{2}$. As a result, in a steady state, the bank sector is always composed by one-third of the population.

We let $m = (m_1, m_2, m_3)$ to denote that money holdings are m_1 for the producer, m_2 for the intermediary, and m_3 for the consumer. The ex ante probability that a nonbank person becomes a consumer or a producer in a meeting is $\frac{\alpha}{2}$. Intermediaries, like nonbank people, take part in a no-coincidence meeting with probability $1 - \alpha$. We denote by μ_i^b the fraction of people starting a period in the bank sector and holding i, and by μ_i^n that in the nonbank

sector and also holding i, where $i \in \{0, 1, 2\}$. In what follows, we often omit the qualification 'coincidence meeting' about m whenever it is clear from the context.

Consumer and producer utilities are again $u(c)$ and $-c$, respectively, and the discount factor is also β. In meeting m, output is deterministic and often denoted by $y(m)$, while there is a probability distribution $\lambda(m)$ defining transfers of money among the three traders. More specifically, for $i = 1, 2, 3$, we let $\lambda_i^j(m)$ denote the (marginal) probability that 'person i' (the person starting with m_i) leaves the meeting holding $j \in \{0, 1, 2\}$ units of money. Hence $\lambda_1^j(m)$ denotes the probability that the producer leaves the meeting holding j units of money. In what follows, Bellman equations are more easily expressed by having $\lambda(m)$ written as a vector, so that $\lambda_i^j(m)$ as a particular coordinate of $\lambda(m)$ (see appendix for more details).

We assume initially that no money can be created or destroyed in meetings, and there are physical restrictions on money flows in meetings dictated by intermediation (this assumption is eventually modified when inside money is discussed later). For now, we say that an (outside-money) allocation is *feasible*, reflecting intermediation frictions of the previous section, if for all $m \in M$ two flow conditions are satisfied. As a first condition, we require that $\lambda_1^{m_1+p}(m) = \lambda_2^{m_2-p}(m)$ for all $p \in \{0, 1, 2\}$ and, moreover, if $p > \min\{m_2, 2 - m_1\}$ then $\lambda_1^{m_1+p}(m) = \lambda_2^{m_2-p}(m) = 0$. That is, if payment to producer has mass on p then the intermediary transits to state $m_2 - p$ with the same probability that the producer transits to $m_1 + p$. Likewise, as a second condition, for every realization p for this payment, we require that $\lambda_2^{m_2-p+q}(m) = \lambda_3^{m_2-q}(m)$ for all $q \in \{0, 1, 2\}$ and, moreover, if $q > \min\{m_3, 2 - m_2 + p\}$ then $\lambda_2^{m_2-p+q}(m) = \lambda_3^{m_2-q}(m) = 0$. That is, if a payment to an intermediary has mass on q then the consumer transits to state $m_3 - q$ with the same probability that the intermediary transits to $m_2 - p + q$.

After meetings, but before the period ends, money holdings are affected by policy and new occupation draws take place. We describe policy as transition matrices detailed in the appendix. First there is an inflation shock: a matrix with parameter π is constructed to capture the probability that money disappears, regardless of occupation. A person with one unit has holdings transiting to 0 with probability π, and not transiting with probability $1-\pi$. A person with two units has holdings transiting to 1 with probability $2(1-\pi)\pi$, and to 0 with probability π^2. After the π-shock holdings are updated by a transfer matrix with parameter $\tau = (\tau^b, \tau^n)$. After-inflation holdings j transit to state $j + 1$ with probability τ^b (τ^n), and

remain in state j with probability $1-\tau^b$ $(1-\tau^n)$ if $j<2$, for people in the bank (nonbank) sector. If $j=2$, the probability of transition is zero.

We say that an allocation is *stationary* if, given λ and (τ,π), $\mu=(\mu^b,\mu^n)$ is a time-invariant distribution of money (see details in the appendix).

Notice, for a given λ, the effect on μ of increasing (τ^b,τ^n,π) above $(0,0,0)$ is to reduce the mass of people with holdings in $\{0,2\}$, in exchange for an increase in the mass of people holding one unit. In principle this policy improves extensive margins, although it now has a potentially negative effect on the return of money, that can reduce y. As we shall see, however, one must account for changes in λ that are incentive-compatible with saving/spending decisions and which can worsen extensive margins as well. For this we need to describe incentive constraints, according to continuation values, defined as follows.

1.4.2 Welfare criteria and rationality constraints

We now present the welfare criteria and incentive constraints, whose details are also included in the appendix. At the beginning of a period, the expected discounted utility of a person with i units of money in bank and nonbank sectors take, respectively, the following form

$$v_i^b=(1-\alpha)w_0^b(i)+\alpha\sum_{\{m:m_2=i\}}\mu^n_{m_1}\mu^n_{m_3}w_2^b(m),$$

and

$$v_i^n=(1-\alpha)w_0^n(i)+\frac{\alpha}{2}\left(\sum_{\{m:m_1=i\}}\mu^b_{m_2}\mu^n_{m_3}w_1^n(m)+\sum_{\{m:m_3=i\}}\mu^b_{m_2}\mu^n_{m_1}w_3^n(m)\right),$$

where w_0^b and w_0^n results from transitions after a no-coincidence meeting, while w_1^n results from transitions after a meeting as a producer, w_3^n results from transitions after a meeting as a consumer, and w_2^b results from transitions after a meeting as an intermediary. In the appendix it is presented the system defining value functions in detail. In particular it is shown that for $m\in M$, $w(m)$ takes the form

$$\begin{aligned}
w_1^n(m) &= -y(m)+\beta\lambda_1(m)A^n v\\
w_2^b(m) &= \beta\lambda_2(m)A^b v\\
w_3^n(m) &= u(y(m))+\beta\lambda_3(m)A^n v
\end{aligned}$$

where A^n and A^b are transition matrices reflecting current occupation. Likewise,

$$\begin{aligned} w_0^b(i) &= \beta A_{0i}^b v \\ w_0^n(i) &= \beta A_{0i}^n v \end{aligned}$$

where A_{0i}^b and A_{0i}^n are particular matrices for those holding i units of money in no-coincidence meetings. For a given (μ, λ, y) and policy (τ, π) this system has a contraction property and features an unique solution v.

The welfare criteria is given by average utility, corresponding to an inner product of $\mu = (\mu^b, \mu^n)$ and $v = (v^b, v^n)$, which amounts to

$$w(y, \mu) = \mu \cdot v = \frac{\alpha}{1-\beta} \sum_{m \in M} \mu_{m_1}^n \mu_{m_2}^b \mu_{m_3}^n [u(y(m)) - y(m)]. \tag{1.4}$$

Remark 1. *Lotteries λ and policy parameters (τ, π) have only indirect effects on w. The same can be said about β, since it does not change preference orders over stationary outcomes from the social perspective.*

The previous remark implies that the savings friction is relevant since, ex post, consumers take discounting into account when ranking alternative actions. This kind of *excessive spending* can be singled out in simulations, according to the following objects.

We assume that individuals can deviate during trades from what is proposed for a particular meeting, taking as given value functions and the law of movement for aggregate variables. They can deviate individually, by choosing autarky in the meeting, or in groups, by seeking a trade bundle that dominates the proposed allocation for the meeting, without making trade partners worse off. Given such notion of rationality, implementable allocations must satisfy inequalities corresponding to individual-rationality and (static) core requirements. Trade weakly dominates autarky in meeting m for an intermediary if

$$w_2^b(m) \geq w_0^b(m_2), \tag{1.5}$$

and for producer and consumer if

$$w_1^n(m) \geq w_0^n(m_1) \text{ and } w_3^n(m) \geq w_0^n(m_1). \tag{1.6}$$

Individuals can also consider group deviations in a meeting. One way to define the requirement that trade belongs to the core in meeting m is to allow the consumer to search for an alternative output/lottery pair $(\bar{\lambda}, \bar{y})$, subject to intermediation constraints with preservation of money holdings defined above, so as to find

A feasible and stationary allocation $(\mu, y, \lambda, \tau, \pi)$ is *implementable* if associated values (v, w) satisfy individual-rationality (1.5-1.6) and core constraints

$$w_3^n(m) \geq \bar{w}_3^n(m) \tag{1.7}$$

for all $m \in M$.[8]

Remark 2. *An intuitive description of constraint (1.7) can be given with pairwise meetings (no intermediation), differentiable value functions (divisible money) and degenerate lotteries. First-order necessary conditions for an interior solution to problem (1)[9] can be shown to imply, in this case,*

$$v'(m_3 - p) = u'(y)v'(m_1 + p)$$

where v' is the derivative of the value function, $m_3 - p$ is after-trade consumer holdings of money, $m_1 + p$ is after-trade producer holdings of money, and y is output. Notice that, according to this condition, money payment p is inversely related to output level y when v is concave. In particular, if β is low and, in turn, individual rationality requires low output, then due to the core requirement p must be high. Average trades therefore feature high spending when β is low.

In summary, the core requirement imposes to the planner a level of spending in meetings most favorable to consumers for a given producer surplus. Turning off the core requirement leads therefore to a savings rate more advantageous in terms of ex-ante average utility and to a natural definition of excessive spending.

1.5 Measuring the savings friction

In this section we report the solution of the planner's problem for many specifications, including also the removal of the intermediation friction (which is a particular case of the

[8] Our algorithm (see appendix) is written with a more general formulation for (1.7).

[9] See appendix C.

environment presented in the previous section). We give special attention on how the planner obtains better results if people could commit to not deviate in groups (that is, when the core requirement is removed). This should give an useful interpretation of numerical results: expansionary policies are either compensating for low self-insurance in the form of outside money, or creating credit with inside money.

We compute three sets of simulations with intermediation. The first two concern outside-money economies exactly as described in section 1.4. The third set describes results for extreme values of occupation persistence, allowing for an inside-money interpretation of the model. In terms of parameters introduced in the previews section, we set $\alpha = 1$ and $u(y) = y^{2/10}$. Since results for intermediation with the upper bound of 2 units are easier to interpret (there are fewer output levels), it is convenient to leave the discussion of pairwise meetings with the upper bound of 4 units to subsection 1.5.3.

1.5.1 Outside-money inflation

In the first set of simulations we put $\beta = .9$, while in the second set we put $\beta = .5$. In both cases, we vary the parameter ρ that determines how persistent the intermediation occupation is. In these outside-money examples we find that at most one unit is transferred and take advantage of this fact, reporting in Tables 1.5 and 1.6, the probability λ that the consumer pays a unit of money. Also, in almost every meeting, the payment from intermediaries to producers is equal to the payment from the consumers to the intermediaries; an exception may occur in meeting $(1, 1, 2)$. When that happens, consumers pay exactly one unit and we report with entry 'profit $(1, 1, 2)$' the probability that the intermediary is leaving the meeting with two units. Finally, we report y relative to $\arg\max_x\{u(x) - x\}$, which for our specification is $y^* = .1337$.

We notice first that without persistence in intermediation occupation (*iid* case), as in the pairwise economy of Deviatov (2006) (see appendix), inflationary interventions are only optimal when the discount factor β has a low value. Hence this corner condition, with consumers saving zero, is robust to the introduction of intermediation. Corners are easily hit because, with the small support for holdings, value functions are relatively flat and Inada conditions do not help generating positive savings. In these corners, velocity effects are turned off and stop imposing welfare losses when people have a low propensity to save.

TABLE 1.3: Outside money, $\beta = .9$ and core on

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	1.0000 / 0.19	1.0000 / 0.19	1.0000 / 0.24
(0,1,2)	4.4824 / 1.00	4.4211 / 1.00	1.9903 / 1.00
(0,2,1)	1.0000 / 0.19	1.0000 / 0.19	1.0000 / 0.24
(0,2,2)	4.4824 / 1.00	4.4211 / 1.00	1.9903 / 1.00
(1,1,1)	0.2229 / 0.14	0.2266 / 0.14	0.2842 / 0.19
(1,1,2)	1.0000 / 0.71	1.0000 / 0.72	0.3987 / 1.00
(1,2,1)	0.2229 / 0.14	0.2266 / 0.14	0.2842 / 0.19
(1,2,2)	1.0000 / 0.77	1.0000 / 0.67	1.0000 / 0.65
profit (112)	0	0	0.74
μ_0^n / μ_0^b	0.1452 / 0.1452	0.1455 / 0.1455	0.2277 / 0.0311
μ_1^n / μ_1^b	0.5550 / 0.5550	0.5581 / 0.5581	0.4938 / 0.1084
μ_2^n / μ_2^b	0.2998 / 0.2998	0.2964 / 0.2964	0.2785 / 0.8605
v_0^n / v_0^b	0.0910 / 0.0780	0.1167 / 0.0875	0.4589 / 0.5998
v_1^n / v_1^b	0.9087 / 0.7789	0.9489 / 0.7117	1.1372 / 0.6395
v_2^n / v_2^b	1.1150 / 0.9900	1.2024 / 0.9018	1.3683 / 0.6456
π	0	0	0.0385
τ^n	0	0	0
τ^b	0	0	0.7794

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and μ_i^k-v_i^k is the measure-value pair of people in sector k holding i units of money.

By contrast, when $\beta = .9$, as in Table 1.3, the distribution of money can be considered a good one, as about 56% of people have one unit of money, without any redistributive intervention. Hence it becomes a good thing to have zero inflation, and an average monetary spending of just $.14$ in meetings $(1,1,1)$ and $(1,2,1)$ allows this distribution of money to remain stationary.

Now, if $\beta = .5$ then core constraints, together with producer incentive constraints, push allocations towards low savings and negative effects of inflation are reduced, yielding a measure of optimal inflation of about $.22$, as we can see in Table 1.4. Meetings $(1,1,1)$ and $(1,2,1)$, that are key for keeping a good distribution of money without inflation, feature no savings at all. The $.22$-inflation expansion prevents a very bad distribution of money from taking place, so that about 38% of people hold one unit of money.

Effects of velocity and discounting on saving rates become evident when the core constraint is turned off. If this is done for $\beta = .9$ then about 77% of the population are always holding one unit of money in a better distribution relative to the case with core on. This is due to a smaller monetary payment of $.02$ on average becomes implementable in meetings $(1,1,1)$ and $(1,2,1)$. If $\beta = .5$, turning off the core requirement allows spending in these meetings to fall from maximum levels to $.02$, delivering a good distribution with about 74%

TABLE 1.4: Outside money, $\beta = .5$ and core on

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(0,1,2)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(0,2,1)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(0,2,2)	0.2603 / 1.00	0.5146 / 1.00	0.5692 / 1.00
(1,1,1)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
(1,1,2)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
(1,2,1)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
(1,2,2)	0.0845 / 1.00	0.1571 / 1.00	0.1967 / 1.00
profit (112)	0	0	0
μ_0^n / μ_0^b	0.2357 / 0.2357	0.3053 / 0.1221	0.3479 / 0.0366
μ_1^n / μ_1^b	0.3826 / 0.3826	0.3426 / 0.2427	0.3637 / 0.1252
μ_2^n / μ_2^b	0.3816 / 0.3816	0.3521 / 0.6351	0.2885 / 0.8382
v_0^n / v_0^b	0.0149 / 0.0597	0.0096 / 0.0542	0.0385 / 0.0264
v_1^n / v_1^b	0.1471 / 0.0642	0.2067 / 0.0593	0.2732 / 0.0282
v_2^n / v_2^b	0.1639 / 0.0652	0.2413 / 0.0601	0.3166 / 0.0286
π	0.2241	0.1576	0.2042
τ^n	0	0.0128	0.1751
τ^b	1	1	1

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and μ_i^k-v_i^k is the measure-value pair of people in sector k holding i units of money.

of the population holding one unit, without inflation.

Notice that this pattern is robust to specifications with low persistence in intermediation occupation. When persistence parameters is set as $1/3$ (*iid* case) or $2/3$ (*Markov-low* case), inflation appears only when $\beta = .5$ and the core requirement is on. When $\beta = .9$ or the core requirement is off, low spending in meetings $(1, 1, 1)$ and $(1, 2, 1)$ suffices to generate a good extensive margin. We still find, nevertheless, that a small but robust inflation appears when persistence in intermediation occupation is high. Even when $\beta = .9$ and the core is off, a case of good spending limits, we see the necessity of an inflation measure of $.028$. Here, however, the intermediation friction is adding a role for expansionary policies that is different from the usual insurance explanation.

To see this, notice that such inflation rate arises but the distribution of money among the nonbank public experiences relatively small changes. If $\beta = .9$ and the core is on then spending in meetings $(1, 1, 1)$ and $(1, 2, 1)$ hits .19 and the nonbank sector fraction holding one unit becomes about 49%. It is the distribution of money in the intermediation sector that experiences a significant change: the fraction of intermediaries without money falls from 14% in the low persistence case to about 3% in the high one. Inflation thus appears with high persistence because money transferred to intermediaries stays in the bank sector

TABLE 1.5: Outside money, $\beta = .9$ and core off

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	0.9387 / 1.00	0.9402 / 1.00	0.9556 / 1.00
(0,1,2)	2.6305 / 1.00	2.6746 / 1.00	1.9684 / 1.00
(0,2,1)	0.9387 / 1.00	0.9402 / 1.00	0.9556 / 1.00
(0,2,2)	2.6305 / 1.00	2.6746 / 1.00	1.9684 / 1.00
(1,1,1)	0.0509 / 0.02	0.0524 / 0.02	0.1297 / 0.08
(1,1,2)	1.0000 / 0.34	1.0000 / 0.33	0.4720 / 1.00
(1,2,1)	0.0509 / 0.02	0.0524 / 0.02	0.1297 / 0.08
(1,2,2)	1.0000 / 0.34	1.0000 / 0.33	1.0004 / 0.58
profit (112)	0	0	0.73
μ_0^n / μ_0^b	0.0637 / 0.0637	0.0634 / 0.0634	0.1709 / 0.0390
μ_1^n / μ_1^b	0.7735 / 0.7735	0.7748 / 0.7748	0.5724 / 0.1703
μ_2^n / μ_2^b	0.1628 / 0.1628	0.1618 / 0.1618	0.2567 / 0.7907
v_0^n / v_0^b	0.5735 / 0.4916	0.6120 / 0.4590	0.7285 / 0.3451
v_1^n / v_1^b	0.9839 / 0.8433	1.0266 / 0.7700	1.1546 / 0.5469
v_2^n / v_2^b	1.4433 / 1.2371	1.4994 / 1.1246	1.6662 / 0.7893
π	0	0	0.0280
τ^n	0	0	0
τ^b	0	0	0.4653

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and μ_i^k-v_i^k is the measure-value pair of people in sector k holding i units of money.

for a while, solving in a similar way the externality problem addressed with taxation in section 3.[10]

We also notice that a financial profit exists in some cases in meeting $(1, 1, 2)$. It occurs when persistence is high. It is followed by improvements in the distribution of money in the nonbank sector. Absent profit outcomes in meeting $(1, 1, 2)$, lottery realizations would leave either the producer or the consumer with two units of money, excluding this person from some trades next period. Although profits make consumption goods more expensive, when persistence is sufficiently high the positive effect on the distribution of money across traders is dominating.[11]

1.5.2 Inside-money inflation

In our last set of simulations, we consider specifications displaying no transitions for intermediation occupations ($\rho = 1$). In this case, the planner is not constrained by intermediation

[10]Transfers directed to intermediaries when persistence is low find a quick inflow into the nonbank sector. Giving money first to intermediaries reduce negative effects on producer constraints.

[11]Giving profits to intermediaries holding one unit in other meetings would hurt the distribution of nonbank money. Having intermediaries with 2 units is not important: meetings $(0, 2, 2)$ feature just one unit spent and the same output as $(0, 1, 2)$. Hence profits perform the money destruction feature of inside-money economies discussed later.

TABLE 1.6: Outside money, $\beta = .5$ and core off

Persistence	iid	Markov low	Markov high
m	y / λ	y / λ	y / λ
(0,1,1)	0.5318 / 1.00	0.6395 / 1.00	0.8788 / 1.00
(0,1,2)	0.5318 / 1.00	0.6395 / 1.00	0.8826 / 1.00
(0,2,1)	0.5318 / 1.00	0.6395 / 1.00	0.8781 / 1.00
(0,2,2)	0.5318 / 1.00	0.6395 / 1.00	0.8826 / 1.00
(1,1,1)	0.0097 / 0.02	0.0112 / 0.02	0.0344 / 0.10
(1,1,2)	0.4233 / 1.00	0.4936 / 1.00	0.1997 / 1.00
(1,2,1)	0.0097 / 0.02	0.0112 / 0.02	0.0344 / 0.10
(1,2,2)	0.4233 / 1.00	0.4936 / 1.00	0.3508 / 1.00
profit (112)	0	0	0.43
μ_0^n / μ_0^b	0.0644 / 0.0644	0.0650 / 0.0650	0.2221 / 0.0233
μ_1^n / μ_1^b	0.7412 / 0.7412	0.7404 / 0.7404	0.5246 / 0.0810
μ_2^n / μ_2^b	0.1944 / 0.1944	0.1946 / 0.1946	0.2533 / 0.8957
v_0^n / v_0^b	0.0000 / 0.0000	0.0000 / 0.0000	0.0016 / 0.0276
v_1^n / v_1^b	0.1718 / 0.0711	0.1953 / 0.0488	0.2617 / 0.0323
v_2^n / v_2^b	0.3195 / 0.1278	0.3451 / 0.0865	0.3577 / 0.0325
π	0	0	0.0458
τ^n	0	0	0
τ^b	0	0	1

Values for ρ are 1/3, 2/3 and 9/10 for, respectively, iid, Markov-low and -high. π is the inflation rate, τ^k is the transfer for sector k and μ_i^k-v_i^k is the measure-value pair of people in sector k holding i units of money.

incentives. In this case, even without an explicit description of how intermediaries can be monitored, it is reasonable to assume that the planner can ask intermediaries to finance any spending levels. The economy then gains an inside-money interpretation in the spirit of Cavalcanti and Wallace (1999) and Williamson (1999).

In Tables 1.7 and 1.8, we display results for inside-money economies and according to two scenarios. The first scenario has the core requirement turned off. One view here is that intermediaries are essential for all conceivable transactions. As a result, the ability to perfectly control them implies that producer and consumers cannot deviate as a group (in a sense, therefore, monitoring is removing the core requirement).

The second scenario leaves the producer-consumer pair with the option of not using the intermediary, and preventing this option from being exercised is in fact a constraint imposed to the planner. That is, although the intermediary is perfectly controlled, it is constrained to do financing only in ways that improve what producer and consumer get by themselves. Although this scenario is in a way a change in the environment (since money can flow freely from consumer to producer off the equilibrium path), it is instructive for checking out how

TABLE 1.7: Inside money and core off

β	.5	.6	.7	.8	.9
m	y / λ	y / λ	y / λ	y / λ	y / λ
(0,0)	0.06 / 0.25	0.08 / 0.22	0.10 / 0.22	0.14 / 0.21	0.18 / 0.20
(0,1)	0.25 / 1.00	0.35 / 1.00	0.48 / 1.00	0.66 / 1.00	0.92 / 1.00
(0,2)	0.40 / 1.87	0.53 / 2.00	0.72 / 2.00	0.88 / 1.68	0.98 / 1.16
(1,0)	0.01 / 0.04	0.01 / 0.09	0.02 / 0.10	0.40 / 0.12	0.06 / 0.15
(1,1)	0.10 / 0.59	0.18 / 1.00	0.24 / 1.00	0.32 / 1.00	0.40 / 0.97
(1,2)	0.18 / 1.00	0.18 / 1.00	0.24 / 1.00	0.32 / 1.00	0.41 / 1.00
b_{00}	0.25	0.22	0.22	0.21	0.20
b_{01}	0.00	1.00	1.00	1.00	1.00
b_{02}	0.87	1.00	1.00	0.68	0.16
b_{10}	0.04	0.09	0.10	0.12	0.15
b_{11}	-0.41	1.00	1.00	1.00	0.97
b_{12}	0.00	0.00	0.00	0.00	0.00
μ_0	0.7262	0.6424	0.6430	0.6428	0.6412
μ_1	0.2373	0.3156	0.3142	0.3150	0.3182
μ_2	0.0364	0.0421	0.0427	0.0422	0.0406
v_0	0.2295	0.2868	0.3982	0.6435	1.3914
v_1	0.2692	0.3518	0.4989	0.7678	1.5571
v_2	0.2996	0.3932	0.5234	0.8014	1.5945
π	0.1854	0.3537	0.3543	0.3505	0.3445

π is the inflation rate, μ_i is the fraction of people in nonbank sector holding i units of money, b_m is the probability that one unit of money be created in meeting m and v_i is the value associated to people holding i units of money.

robust our conclusions are.[12]

In these two scenarios, each meeting is fully described by a pair $m = (m_1, m_2)$, where m_1 (m_2) denotes money holdings of the producer (consumer). In Tables 1.7 and 1.8 we report, for each meeting, output y, relative to y^*, as well as a measure of money transferred to the producer λ. In some meetings, the producer is paid 2 units with positive probability, and hence a reported value $\lambda > 1$ indicates that a two-unit payment has probability $\lambda - 1$, while a single-unit payment has probability $2 - \lambda$. We also indicate, using positive values for b_m, the probability that one unit is created by the intermediary in meeting m. When b_m is negative then $|b_m|$ is the probability that a unit of money of the consumer is destroyed (extracted from the consumer but not transferred to the producer).

We find that in simulations leading to Tables 1.7 and 1.8 there is no use of transfers to the nonbank sector, and hence there is no need to report τ in these tables.

In Table 1.7 we find that inflationary policies are implemented in all configurations. Since there is the option to create credit with perfect control according to its social impact, the

[12]While we thank Neil Wallace for suggesting examination of this second scenario, we did not find previous work discussing how monitoring of a subset of traders can change the set of core allocations.

TABLE 1.8: Inside money and core on

β	.5	.6	.7	.8	.9
m	y / λ	y / λ	y / λ	y / λ	y / λ
(0,0)	0.05 / 0.28	0.06 / 0.19	0.09 / 0.22	0.12 / 0.22	0.16 / 0.16
(0,1)	0.16 / 1.00	0.32 / 1.00	0.39 / 1.00	0.54 / 1.00	1.00 / 1.00
(0,2)	0.23 / 2.00	0.35 / 1.22	0.51 / 1.49	0.59 / 1.15	1.00 / 1.00
(1,0)	0.00 / 0.05	0.01 / 0.05	0.01 / 0.06	0.02 / 0.07	0.05 / 0.13
(1,1)	0.07 / 1.00	0.14 / 1.00	0.23 / 1.00	0.32 / 1.00	0.34 / 1.00
(1,2)	0.07 / 1.00	0.14 / 1.00	0.23 / 1.00	0.32 / 1.00	0.38 / 1.00
$b_{0,0}$	0.28	0.19	0.22	0.22	0.16
b_{01}	0.00	0.00	0.00	0.00	1.00
b_{02}	0.00	0.22	0.49	0.15	0.00
b_{10}	0.05	0.05	0.06	0.07	0.13
b_{11}	0.00	0.00	0.00	0.00	0.00
b_{12}	0.00	0.00	0.00	0.00	0.00
μ_0	0.7752	0.7467	0.7395	0.7286	0.6406
μ_1	0.1954	0.2159	0.2198	0.2290	0.3184
μ_2	0.0294	0.0374	0.0407	0.0424	0.0410
v_0	0.2146	0.2821	0.4059	0.6508	1.375
v_1	0.2673	0.3633	0.4931	0.7545	1.5714
v_2	0.2846	0.3921	0.5376	0.8091	1.6073
π	0.1892	0.1210	0.1387	0.1235	0.2433

π is the inflation rate, μ_i is the fraction of people in nonbank sector holding i units of money, b_m is the probability that one unit of money be created in meeting m and v_i is the value associated to people holding i units of money.

concern with distributions of holdings is less important in comparison with the outside-money case. The mass of nonbank people with two units is reduced by inflation. The high magnitude of inflation, of 35%, is necessary to remove money created in credit operations.

In Table 1.8, monetary policy becomes less expansionary. When the producer-consumer pair can deviate as a group, spending increases and generates larger distortions on extensive margins, forcing the planner to create less credit. As a result a lower inflation rate emerges. We find that inside-money inflation is associated to more efficient insurance overall, compensating for negative velocity effects. Computed increases in consumption are in line with simulations reported by Deviatov and Wallace (2014) for inside-money economies with pairwise meetings and 0-1 holdings of money.

1.5.3 Zero inflation with pairwise meetings

We have anticipated some basic results of economies without intermediation (pairwise meetings) in section 2. We have included in the appendix a test case when the upper bound is 2 units. There is no optimal inflation with the core off, but expansionary policies are welfare

improving when the core is on and the discount factor is low, so that in meeting type $(1,1)$ consumers are spending all their holdings (output is 12% of first-best level or less).

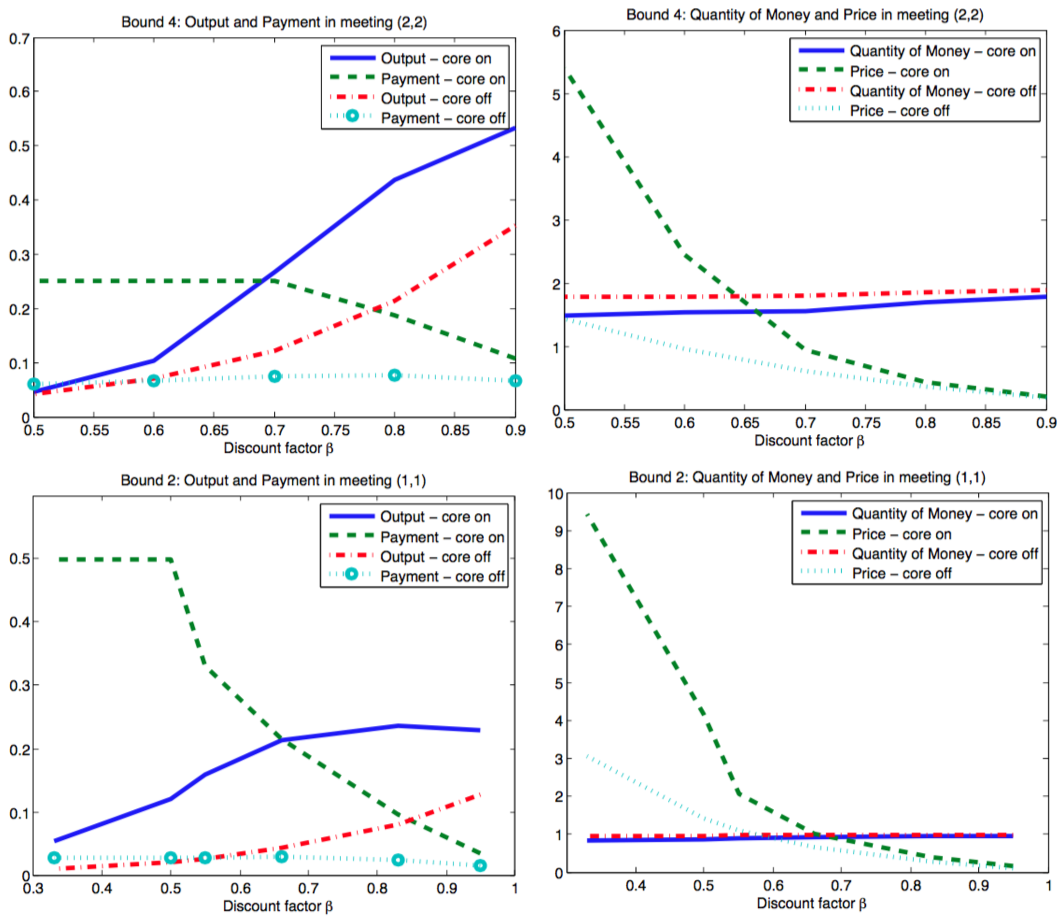

FIGURE 1.1: Salient features of pairwise trades

Tables 1.9 and 1.10 show the effects of increasing the upper bound of outside money, from 2 to 4. Lump-sum transfers are not optimal, regardless of β or the core requirement. In addition to taxes discussed in section 2, when consumers meet the poorest producer, we also find taxation in meeting $(1,3)$, when the producer has one unit and the consumer has three, but only for very high β (of .9). So the main lesson is that with more divisibility of money traders are more conservative in terms of spending money and consumer taxes prove to be more efficient in terms of providing insurance.

Some basic effects of changes in the upper bound are displayed in Figure 1. The top two panels refer to the 4 bound, while the bottom two refer to the 2 bound. On the left we can see the effects of β on output and average payment in meeting $(2,2)$ with the 4 bound (top panel), and in meeting $(1,1)$ with the 2 bound (bottom panel). On the right, curves represent now the mean of the distribution of holdings and the ratio between average payment and

TABLE 1.9: Outside money in pairwise meetings: core on

β	0.9	0.8	0.7	0.6	0.5
m	y / $\lambda(x)$	y / $\lambda(x)$	y / $\lambda(x)$	y / $\lambda(x)$	y / $\lambda(x)$
(0,1)	1.0007 / 0.18 (1)	1.0007 / 0.29 (1)	1.0067 / 0.50 (1)	1.0000 / 0.78 (1)	0.8377 / 1.00 (1)
(0,2)	3.1900 / 1.00 (1)	3.3972 / 1.00 (1)	2.0209 / 1.00 (1)	1.2184 / 1.00 (1)	0.7794 / 1.00 (1)
(0,3)	4.2274 / 1.00 (1)	3.3972 / 1.00 (1)	2.0209 / 1.00 (1)	1.2850 / 0.90 (2)	1.0000 / 0.90 (2)
(0,4)	5.1675 / 1.00 (1)	2.2917 / 0.49 (2)	1.8818 / 1.00 (2)	1.6358 / 1.00 (2)	1.0194 / 1.00 (2)
(1,1)	0.2947 / 0.14 (1)	0.2521 / 0.22 (1)	0.2364 / 0.37 (1)	0.1967 / 0.56 (1)	0.1473 / 0.81 (1)
(1,2)	1.0000 / 0.49 (1)	1.0000 / 0.89 (1)	0.6372 / 1.00 (1)	0.3500 / 1.00 (1)	0.1810 / 1.00 (1)
(1,3)	1.8766 / 1.00 (1)	1.1272 / 1.00 (1)	0.6372 / 1.00 (1)	0.3500 / 1.00 (1)	0.1810 / 1.00 (1)
(1,4)	2.0553 / 1.00 (1)	1.1272 / 1.00 (1)	0.9035 / 1.00 (2)	0.4533 / 1.00 (2)	0.2274 / 1.00 (2)
(2,1)	0.1571 / 0.13 (1)	0.1099 / 0.19 (1)	0.0793 / 0.30 (1)	0.0426 / 0.41 (1)	0.0269 / 0.58 (1)
(2,2)	0.5333 / 0.43 (1)	0.4368 / 0.75 (1)	0.2663 / 1.00 (1)	0.1025 / 1.00 (1)	0.0464 / 1.00 (1)
(2,3)	0.5333 / 0.43 (1)	0.4368 / 0.75 (1)	0.2663 / 1.00 (1)	0.1025 / 1.00 (1)	0.0464 / 1.00 (1)
(2,4)	1.2423 / 1.00 (1)	0.5804 / 1.00 (1)	0.2663 / 1.00 (1)	0.1990 / 1.00 (2)	0.1017 / 1.00 (2)
(3,1)	0.1197 / 0.12 (1)	0.0479 / 0.16 (1)	0.0209 / 0.23 (1)	0.0396 / 0.41 (1)	0.0337 / 0.61 (1)
(3,2)	0.4076 / 0.41 (1)	0.1892 / 0.64 (1)	0.0898 / 0.97 (1)	0.0965 / 1.00 (1)	0.0553 / 1.00 (1)
(3,3)	0.4076 / 0.41 (1)	0.1892 / 0.64 (1)	0.0927 / 1.00 (1)	0.0576 / 0.59 (1)	0.0509 / 0.92 (1)
(3,4)	1.0007 / 1.00 (1)	0.2969 / 1.00 (1)	0.0927 / 1.00 (1)	0.0965 / 1.00 (1)	0.0553 / 1.00 (1)
μ_0	0.0720	0.1171	0.1862	0.2263	0.2695
μ_1	0.3572	0.3708	0.3584	0.3164	0.2801
μ_2	0.3437	0.2806	0.2450	0.2459	0.2415
μ_3	0.1712	0.1605	0.1337	0.1163	0.1083
μ_4	0.0559	0.0710	0.0767	0.0951	0.1006
v_0	0.4514	0.0263	0.0089	0.0018	0.0013
v_1	1.2638	0.5940	0.3948	0.2282	0.2253
v_2	1.5692	0.7824	0.5166	0.3663	0.2738
v_3	1.7538	0.8794	0.5675	0.3892	0.2861
v_4	1.9027	0.9291	0.5851	0.4107	0.3010

$\lambda(x)$ is the optimal probability of transfering x units of money (and paying $x-1$ units with probability $1-\lambda(x)$).

output (called 'price') for the 4 bound (top panel) and the 2 bound (bottom panel).[13]

Meetings $(1,1)$ and $(2,2)$ are important meetings in terms of their impacts on the distribution of money for, respectively, bounds 2 and 4. Curves for output, representing consumption relative to first-best levels, indicate that the economy with 4 units has more trade going on. Curves for payments show that people save more for a large set of preferences with the 4 bound, an indication that the Inada condition has produced steeper value functions around 0 holdings. In this sense, money becomes more valuable (output doubles at high β).

With the 2 bound payments hit the ceiling of consumers' holdings in meeting $(1,1)$ at low β such that positive inflation is optimal. Core effects are important for low β, confirming what has been remarked above. When the core is off, output is smaller but both the meeting's payment and the quantity of money vary little with β.

[13]Payment statistics used in Figure 1 are defined as the average transfer of money in meeting $(2,2)$ (for bound 4) or $(1,1)$ (for bound 2), paid by consumers to producers, divided by the corresponding bound.

TABLE 1.10: Outside money in pairwise meetings: core off

β	0.9	0.8	0.7	0.6	0.5
m	y / $\lambda(x)$	y / $\lambda(x)$	y / $\lambda(x)$	y / $\lambda(x)$	y / $\lambda(x)$
(0,1)	0.8856 / 1.00 (1)	0.7345 / 1.00 (1)	0.5969 / 1.00 (1)	0.4981 / 1.00 (1)	0.4084 / 1.00 (1)
(0,2)	1.1316 / 1.00 (1)	1.1668 / 1.00 (1)	1.0299 / 1.00 (1)	0.8773 / 1.00 (1)	0.6305 / 1.00 (1)
(0,3)	2.5093 / 1.00 (1)	2.2064 / 1.00 (1)	1.4570 / 1.00 (1)	0.9895 / 0.04 (2)	0.9058 / 1.00 (2)
(0,4)	3.2446 / 1.00 (1)	2.2745 / 0.07 (2)	2.1144 / 1.00 (2)	1.4009 / 1.00 (2)	0.9058 / 1.00 (2)
(1,1)	0.2266 / 0.12 (1)	0.1249 / 0.12 (1)	0.0718 / 0.11 (1)	0.0419 / 0.10 (1)	0.0254 / 0.09 (1)
(1,2)	1.0007 / 0.55 (1)	1.0000 / 0.97 (1)	0.6574 / 1.00 (1)	0.4293 / 1.00 (1)	0.2745 / 1.00 (1)
(1,3)	1.8325 / 1.00 (1)	1.0337 / 1.00 (1)	0.6574 / 1.00 (1)	0.4293 / 1.00 (1)	0.2745 / 1.00 (1)
(1,4)	1.8325 / 1.00 (1)	1.0337 / 1.00 (1)	1.0000 / 0.84 (2)	0.6904 / 1.00 (2)	0.4473 / 1.00 (2)
(2,1)	0.1137 / 0.09 (1)	0.0516 / 0.07 (1)	0.0254 / 0.06 (1)	0.0135 / 0.05 (1)	0.0082 / 0.05 (1)
(2,2)	0.3530 / 0.27 (1)	0.2132 / 0.31 (1)	0.1227 / 0.30 (1)	0.0711 / 0.27 (1)	0.0419 / 0.24 (1)
(2,3)	1.0000 / 0.76 (1)	0.6941 / 1.00 (1)	0.4099 / 1.00 (1)	0.2610 / 1.00 (1)	0.1728 / 1.00 (1)
(2,4)	1.3126 / 1.00 (1)	0.6941 / 1.00 (1)	0.4099 / 1.00 (1)	0.2610 / 1.00 (1)	0.1728 / 1.00 (1)
(3,1)	0.0464 / 0.05 (1)	0.0157 / 0.04 (1)	0.0060 / 0.03 (1)	0.0030 / 0.02 (1)	0.0015 / 0.02 (1)
(3,2)	0.0965 / 0.09 (1)	0.0396 / 0.09 (1)	0.0172 / 0.08 (1)	0.0082 / 0.07 (1)	0.0045 / 0.06 (1)
(3,3)	0.2378 / 0.23 (1)	0.1369 / 0.31 (1)	0.0868 / 0.41 (1)	0.0606 / 0.50 (1)	0.0411 / 0.57 (1)
(3,4)	1.0000 / 0.98 (1)	0.4361 / 1.00 (1)	0.2132 / 1.00 (1)	0.1212 / 1.00 (1)	0.0711 / 1.00 (1)
μ_0	0.0361	0.0377	0.0365	0.0335	0.0307
μ_1	0.2906	0.3165	0.3366	0.3443	0.3433
μ_2	0.4432	0.4301	0.4341	0.4455	0.4580
μ_3	0.2030	0.1893	0.1692	0.1558	0.1471
μ_4	0.0271	0.0264	0.0236	0.0209	0.0209
v_0	0.7894	0.2035	0.0706	0.0229	0.0068
v_1	1.2715	0.5723	0.3489	0.2394	0.1754
v_2	1.5437	0.0745	0.4745	0.3351	0.2489
v_3	1.7387	0.8610	0.5527	0.3932	0.2951
v_4	1.8896	0.9339	0.5934	0.4201	0.3141

$\lambda(x)$ is the optimal probability of transfering x units of money (and paying $x-1$ units with probability $1-\lambda(x)$).

It is fair to say that the curve changing the most with the bound is the one reflecting optimal payments in these 'critical' meetings. Based on Figure 1, a reasonable conjecture is that output and payments vary less with the discount factor as the bound is increased further, indicating that more insurance is provided. This perhaps can explain that the quantity of money less than doubles as the bound is increased from 2 to 4: as the insurance problem is alleviated, the planner reduces the quantity of money, relatively, in order to improve the return of money (and the intensive margin of consumption).[14]

Confirming what we have seen with intermediation, the core requirement has a strong effect on exchange risk. This is quite evident in the relationship between β and the optimal distribution of money across tables. When the core is off (Table 10) the distributions do not change much with β and display a bell shape. When the core is on (Table 9), by contrast, reductions in β (and thus in saving rates) have a remarkable effect on the mass of people

[14]Explaining why turning the core off leads to a higher quantity of money and taxation of poor consumers is more challenging. Focusing on poor consumers instead of richer ones helps with the return of money, which perhaps needs more attention when the quantity of money is increased.

with low holdings of money, up to the point that the distribution approaches the uniform case. This is consistent with a strong velocity effect that makes expansionary policies sub-optimal. In addition, consumers in meeting $(2, 4)$ spend more than one unit of money when β is $.5$ or $.6$ and the core is on, but less than one unit when the core is off (which reduces the dispersion of money).

1.6 A classic case for high returns

We have shown that inflation reduces self-insurance, as people trade more consumption in the present for less insurance in the future, so that lump-sum transfers produce a crowding-out effect. Although for a large set of parameters it is best to just avoid expansionary policies altogether, society can provide better insurance with less distortions by supporting inside money with some level of monitoring.

The prescription of preserving the value of money can in fact be linked to old views about the importance of having money flowing into the financial sector, as it can be concluded from introductory passages in Bagehot (1873). In his classic description of money markets and the operation of central banking 150 years ago, the distribution of money seems to play a central role.

> "Everyone is aware that England is the greatest moneyed country in the world; everyone admits that it has much more immediately disposable and ready cash than any other country. But very few persons are aware how much greater the ready balance—the floating loan-fund which can be lent to anyone or for any purpose—is in England than it is anywhere else in the world. [...] Of course the deposits of bankers are not a strictly accurate measure of the resources of a Money Market. On the contrary, much more cash exists out of banks in France and Germany, and in all non-banking countries, than could be found in England or Scotland, where banking is developed. But that cash is not, so to speak, 'moneymarket money': it is not attainable. [...] But the English money is 'borrowable' money. [...] Concentration of money in banks, though not the sole cause, is the principal cause which has made the Money Market of England so exceedingly rich, so much beyond that of other countries."

This is a simple argument that money needs to be in the right hands to exercise its full potential. From the perspective of models in this paper, there is also a flip side that needs to be emphasized: when money is spent then this potential is lost, unless some special mechanism is in place to make liquidity flow back to its origin. Bagehot finds that the financial sector has a delicate function, considered fragile to some extent. We have modeled this tension in a rudimentary way by noting that in some cases intermediation activity needs to be properly compensated. One avenue for future research is to disentangle the advantage of having money ready for lending when it is concentrated in the hands of the banking sector, from the possibility that a well developed financial system can be more responsive to contractive policies when spending needs to be controlled. Bagehot in fact witnessed a gold standard facing episodes of adverse liquidity shocks. His well-known policy recommendation of high interest rates in moments of bank panic, coupled with broad measures of liquidity provision by discounting of bills, are geared at discouraging people from withdrawing funds from the financial sector.

1.7 Conclusion

Mechanism design offers the advantage of replacing definitions of incompleteness by primitives such as imperfect monitoring and lack of commitment to future actions, which are actually necessary for restricting credit transactions that would leave money without a role. With these assumptions, steady states feature a dispersion in money holdings. In this paper, we have explored a particular implication of lack of commitment that has not been singled-out by the literature to date: steady-state savings in matching models are inefficiently low. Since the planner is maximizing ex-ante utility, there is a trade-off between immediate average-utility gains from output traded in meetings, and the distribution of money that results from savings decisions. Good distributions result from high savings, but after people receive idiosyncratic shocks their incentives no longer coincide with social ones.

We have discussed a way to measure this new friction. One can ask how much consumers agree to spend if the alternative is autarky in the meeting (a high punishment), in contrast to the more natural assumption of allowing consumers to select trades as long as they keep producers indifferent (with such a threat of deviation by the pair, the implied lower punishment forces the planner to accept lower savings). As it turns out, with low

discount factors, the implied allocations are very different because the commitment problem becomes more severe. Expansionary policies are sometimes needed in our examples precisely because self-insurance (savings in the form of outside money) are too low.

But what low savings have to do with expansionary policies? Since the distribution of money is endogenous, resulting from savings decisions, we have seen that inflation further reduces self-insurance. In our simulations, considering a large range of discount factors, this finding seem robust in versions with standard pairwise meetings (without intermediation) as well. Due to the high dimensionality of the numerical problem, we cannot offer a definitive answer of whether expansionary policies can be ruled out as a genuine feature of richer outside-money specifications.

We have seen that tripartite meetings are convenient for showing what happens when some monitoring is introduced, so that intermediaries can lessen the problem. We find that monitoring considered by Cavalcanti and Wallace (1999) can potentially promote savings, and inflation with inside money is more likely to be optimal than with outside money.

Wallace (2014b) notes that if the environment has only pure currency (no monitoring whatsoever) then the usual taxation of money holdings is not feasible, but the return of money of richer traders can be improved by a regressive policy that pays interests on holdings above a certain cutoff. His conjecture is that model details should tell whether such policy dominates the lump-sum transfers studied here, that target poor traders. He in fact builds on Kehoe, Levine, and Woodford (1992) to construct an example in which both regressive and lump-sum policies have positive marginal effects on welfare, starting from a no-intervention, zero-inflation equilibrium (without saying which alternative attains the best outcome).

While the conjecture that some intervention (positive inflation) is better than no intervention (zero inflation) seems reasonable for this kind of Bewley model, our results for matching models indicate that assumptions leading to Markov allocations (the result that buyers spend all holdings of money) are not neutral. Of course, this is not to say that matching models in which regressive policies are optimal cannot be constructed. We conjecture instead that for a set of parameters the social desire for consumption smoothing should remain strong, and that in this case monetary policy can reach a corner of no inflation due to negative effects on self-insurance.

In conclusion, with low savings, negative effects of inflation on self-insurance can dominate. Rich effects of expansionary policies already appear in market economies with overlapping generations (see Wallace (1992)), but with matching models the analysis is more complex because risk is in part endogenous: trades are heavily constrained by the distribution of money and therefore by savings behavior more generally.

Chapter 2

Illiquid bonds as a spending control device

2.1 Introduction

"This, as I see it, is really the central issue in the pure theory of money. Either we have to give an explanation of the fact that people do hold money when rates of interest are positive, or we have to evade the difficulty somehow." - Hicks (1935)

In monetary theory, one of the main challenges is explain the coexistence of money and bonds. Why would someone hold money when higher-return risk-free assets exist? To answer this question, the literature usually rests in the better mean of payment feature of money. However, it frequently comes as an assumption inconsistent with the environment described. Moreover, it does not explain the role of coexistence of both assets for the economy. In other words, it does not explain why money and bonds are *coessential*.

Following a mechanism design approach, we provide a possible answer for these questions. For this, we study a monetary economy that builds on Deviatov (2006). It is a pairwise random-matching model realted to Shi (1995) and Trejos and Wright (1995) with an augmented set of money holdings. However, we depart slightly from pure currency environment in order to support a kind of illiquid bond. In this environment, we not only show that coexistence can be supported but also that it is desirable from society point of view.

As bond is introduced with its illiquidity feature, we may imagine that the support to coexistence is as usual: money is more liquid while bond has higher return and, thus, agents carry a mixed portfolio in order to regard both advantages. However, for sake of tractability,

we restrict our attention to no interest-bearing bonds. Therefore, the actual challenge would be sustain the desire for holding bonds instead of money.

For this, we rely on the work of Zhu and Wallace (2007). It is known, following mechanism design approach, that, in pairwise meeting environment, an implementable allocation must have trade outcomes lying in pairwise core. Yet, Zhu and Wallace (2007) show that, as this core is not degenerated, trade surplus division can be dependent on portfolio. Therefore, better trade terms can be provided for certain portfolio choices and, then, it can induce agents carry the dominated asset. This in turn, would support the coexistence of both assets.

For coessentiality issue, we rely on the illiquidity feature of bonds. First, as will be clear, in this pairwise random-matching economy, the frequency of trades and, thus, welfare, is tightly related to money distribution. Indeed, a low dispersion of money holdings[1] seems crucial to improve this margin. However, in order to sustain such a distribution, it is necessary to induce low spending in certain meetings.

Then, we show that in our environment, although low spending in these meetings being very beneficial from society point of view, pairs in these meetings don't perceive it. The disperion of liquid funds is an externality. We show then, that the lack of commitment in environment necessary to support money leads to too low savings. Therefore, illiquid bonds, by inducing commitiment, would work as a spending control device that would enlarge the set of implementable allocations. Illiquid bond will assure the commitment to low spending, then, improving welfare.

Kocherlakota (2003) uses a similar rationale to support the coessentiality of both assets. In particular, they show that the existence of illiquid bonds can improve welfare. Moreover, they show that it cannot be achieved by the use of liquid bonds. This is because, as in our work, the commitment raised by the illiquid bonds is what make possible sustain a larger set of allocation.

The rest of this paper is organized as follows. In section 2.2 we introduce the main environment we work. Section 2.3 we comment about the kind of environments in which illiquid bonds can be supported. Next, in section 2.4, we present the set of implementable allocations and the planner's problem. Section 2.5 presents the numerical approach. In section 2.6, we exhibit and comment the numerical results. Finally, in section 2.7 we conclude.

[1] A homogeneous distribution of money.

2.2 Environment

The environment is analogous to Deviatov (2006). It is a random matching monetary model that builds on Shi (1995) and Trejos and Wright (1995). Time is discrete and infinite. There is a continuum $[0, 1]$ of infinitely-lived agents. Besides, each period is divided in three stages: a portfolio choice stage, a trade stage and a transfer stage.

As in Deviatov (2006), at trade stage, a standard production and consumption specialization pattern is made. An agent becomes a producer (who meets a random consumer), a consumer (who meets a random producer) or stay inactive (no meeting) with, respectively, probabilities $\frac{1}{N}$, $\frac{1}{N}$ and $1-\frac{2}{N}$. In a meeting, the producer can produce y units of a perishable consumption good for the consumer at the cost of disutility $c(y)$, where c is strictly increasing, convex, differentiable and satisfies $c(0) = 0$. The consumer receives period utility $u(y)$ for the consumption of y units of this good, where u is strictly increasing, concave, differentiable and satisfies $u(0) = 0$. Agents maximize the expected sum of discounted period utilities with discount rate $\beta \in (0, 1)$.

There is a durable asset that has no intrinsic value called (fiat) money. Money is indivisible and individual money holdings are restricted to be in $\{0, 1, 2\}$. As in Deviatov (2006), after the trade stage, there is a transfer stage when the central planner can make money transferences to agents - i.e., monetary policy. In order to alleviate the indivisibility issue, we allow the use of lotteries at both, trade and transfer stages. Transfer stage is composed by three substages. First, bond redemption is made (discussed below). Second, there is a probabilistic lump-sum transfer of money. Each agent not constrained by the holdings upper bound receive a unit of money with probability, τ. Then, for normalization purpose, inflation takes place and each unit of money is destroied with probability, π.

Now, unlike Deviatov (2006), we introduce another asset called (illiquid) bonds. At the beginning of each period, at the portfolio choice stage, agents can change any fraction[2] of money for bond in a 1:1 rate. Bonds is then defined by two features: (i) it cannot be traded; (ii) it is redeemed for money at the end of the period. The first point is related to its illiquid feature. For now, we take this as an assumption. The second point is related to the bonds' gross return.

[2]The fraction quality will be discussed furhter in section 3.4. For now, keep in mind that the beginning money holdings will be treated as a continuos variable in the portfolio choice stage.

The economy information structure is as follows. As usual, people cannot commit to future actions and agents are anonymous - i.e., agent's history is private. Also, money holdings are private at the transfer stage. Finally, specialization and portfolios in meetings are known.

2.3 Comments on illiquidity feature

As it is known, in mechanism design approach to monetary theory, we cannot rely on assumptions such as that bonds simply cannot be traded. It must be the result of a well micro founded economic environment. In order to support this view, we present two environments in which these illiquid bonds could arise.

One possible modification in the standard environment that would support the illiquidity is as follows. We introduce a one period record keeping ability to the central planner. At the portfolio choice stage it could be gathered information from agents, that could be used at the end of the period (when bonds are redeemed). This way, the planner could attach an identity to bonds when issued and redeem them only if the bond's owner is the same that bought it. In this sense, it works as a registered bond.

Note, that in this case, the record keeping ability must not take place in the trade stage. Otherwise, monitored trades could be used. In this case, a better spending device would be available and illiquid bonds would have no role.

Another possibility, is that at the portfolio choice stage, agents deposit money with the planner, and, at the end of the period, they withdraw it. However, in order to prevent the trade of claims on deposits, the planner must have an ability to identify agents. Thus, again, some kind of record keeping technology is needed.

Summing up, in order to prevent that bonds will be traded, we must have an environment in which a kind of registered bond is supported. For this, it is necessary to include some kind of record keeping technology and, then, it must depart from a pure currency environment.[3] Nevertheless, attention must be paid to not describe an alternative environment which supports much better mechanisms that would take away the role of money and bonds.

[3] See Wallace (2014b).

2.4 The planner's problem

For sake of tractability, we will restrict our attention only to symmetric and stationary allocations. Besides, and specially important for the model tractability, we will also restrict our attention only to mechanisms in which bonds do not pay interest. In other words, the redemption of money for bonds is in a 1:1 rate.

At first sight, this restriction seems very silly. Because, why would someone buy bonds if it is apparently dominated by money? Moreover, isn't it the main feature of bonds?

For the first question, we can say that with this restriction we are simply inverting the monetary theory puzzle of why money and bonds coexist even though the latter dominates the former. Now, bond is the one dominated in its intrinsic characteristics by money. That said, in order to support the coexistence of these two assets, we rely on the same rationale used by Zhu and Wallace (2007) to support the coexistence of money and a higher return asset.

Zhu and Wallace (2007) introduce two intrinsic useless assets called: money and bonds. Money and bonds have no difference in their intrinsic features other than their return.[4] Then, in order to support the coexistence, they adopt mechanism design approach in a pairwise trade environment. This way, even with a core requirement, the trade surplus division can be dependent on portfolio choice and, then, a expectation of better trade terms can induce agents to carry the supposedly dominated asset - in their case, money.

This endogenous device to make the dominated asset attractive is used in this work. Although bonds are dominated by money in its intrinsic characteristics, better trade terms will be provided to people that carry some bond. This will guarantee that both assets will be used in equilibrium.

For the second question, we argue that is exactly the illiquidity feature of bonds that makes both, money and bonds, coessential. Following Kocherlakota (2003), we assert that it is the commitment feature brought with illiquidity that enlarges the implementable allocation set. Therefore, imposing the restriction of no interest for bonds, we can focus on the main issue that is the spending control device.

Yet, the adoption of this restriction is important for tractability because it circumvents two issues. First, we need not to worry how to model the payment of interest. Since we

[4] Bonds are sold at discount.

work with discrete holdings and probabilistic payments, this would be a challenging task. Moreover, as Wallace (2014b) says, an environment in which money holdings belong to such a small set gives little scope to study regressive policies, such this one would resemble. Second, as will be noted, this restriction will enable us to make the portfolio choice problem trivial.

Finally, showing that a significant improvement can be achieved by the introduction of illiquid bonds with no interest, we are also showing that pure money mechanism is suboptimal. Of course better mechanisms could be achieved by bonds paying interest. But, the optimal will necessarily make use of such kind of bonds.

In this setting, a stationary allocation is defined by the vector $(\mu, y, \lambda, \tau, \pi, \theta)$, where μ is the distribution of money at the beginning of period, (y, λ) is the trading protocol, defined, respectively, by an output and payments in terms of lotteries for each meeting, τ is the money transfer, π is a measure of inflation and θ is the portfolio choice.

The optimal portfolio choice will be a function $\theta : \{0,1,2\} \rightarrow \{0,1,2\} \times \mathbb{R}_+$ where $\theta(m) = (m, \xi)$ and ξ stands for the amount of money taken to trade.[5] Also, λ is a vector $(\lambda^0, \lambda^1, \lambda^2)$ that sets for the probability of consumer transfer, respectively, zero, one and two units of money.

Each meeting is identified by $(m, \xi; m', \xi') \in \{0,1,2\} \times \mathbb{R}_+ \times \{0,1,2\} \times \mathbb{R}_+$, where (m, ξ) and (m', ξ') are, respectively, producer and consumer portfolios. Therefore, output and payments in meetings are function of $(m, \xi; m', \xi')$. As bond is illiquid, we will have the following liqudity constraint:

$$\sum_{i \neq 0} \lambda^i(m, \xi; m', \xi') \leq \xi' \tag{2.1}$$

As Deviatov (2006), the central planner's problem is choosing an allocation to maximize an agent *ex-ante* utility, subject to individual rationality, no pairwise defection in meetings,[6] stationarity requirement and, now, optimal portfolio choice.

Next, we define some auxiliary objects and present the set of implementable allocations.

Let T be the transition matrix from beginning period money holdings to after trade money holdings. Thus, we can write generically:[7]

[5] Thus, $1 - \xi$ is the fraction of bonds bought by the agent.

[6] Liquidity constraint will be included in core requirement.

[7] The matrix element $d_{i,j}$ represents the probability of agent with $i-1$ units of money end up with $j-1$ units of money after the policy. This interpretation is valid for all transition matrices.

$$T = \begin{bmatrix} t_{00} & \frac{1}{N}\int \mu(m')\lambda^1[\theta(0);\theta(m')]dm' & \frac{1}{N}\int \mu(m')\lambda^2[\theta(0);\theta(m')]dm' \\ \frac{1}{N}\int \mu(m')\lambda^1[\theta(m');\theta(1)]dm' & t_{11} & \frac{1}{N}\int \mu(m')\lambda^1[\theta(1);\theta(m')]dm' \\ \frac{1}{N}\int \mu(m')\lambda^2[\theta(m');\theta(2)]dm' & \frac{1}{N}\int \mu(m')\lambda^1[\theta(m');\theta(2)]dm' & t_{22} \end{bmatrix}$$

where t_{mm} denotes a diagonal element of T. Their values are the ones that make each row sums to unity.

Next, let D be the transition matrix from monetary policy. As in Deviatov (2006), it is composed by two matrices. One for transfers, that is given by:

$$D_t = \begin{bmatrix} 1-\tau & \tau & 0 \\ 0 & 1-\tau & \tau \\ 0 & 0 & 1 \end{bmatrix}$$

And other for inflation, given by:

$$D_d = \begin{bmatrix} 1 & 0 & 0 \\ \pi & 1-\pi & 0 \\ \pi^2 & 2\pi(1-\pi) & (1-\pi)^2 \end{bmatrix}$$

Therefore, the matrix associated with the monetary policy transition is given by $D = D_t D_d$. Taking these two transition matrices we have the following stationarity requirement, $\mu T D = \mu$.

Denoting the beginning of period value function by $V \equiv (V_0, V_1, V_2)$, we can write the following matricial Bellman equation:

$$V' = q' + \beta T D V' \tag{2.2}$$

where q is the vector of expected one period utility from trade stage, and, is given by:

$$q' = \begin{bmatrix} -\frac{1}{N}\int \mu(m')c(y[\theta(0);\theta(m')])dm' \\ \frac{1}{N}\int \mu(m')\left\{u(y[\theta(m');\theta(1)]) - c(y[\theta(1);\theta(m')])\right\}dm' \\ \frac{1}{N}\int \mu(m')u(y[\theta(m');\theta(2)]) \end{bmatrix}.$$

Following Deviatov (2006), we can show that equation (2.2) has a unique solution which is given by:

$$V' = (I - \beta TD)^{-1} q'$$

where I is the 3×3 identity matrix.

Now we can write the welfare function as:

$$W = \mu V' = \frac{1}{1-\beta} \int \int \mu(m)\mu(m') \left\{ u(y[\theta(m);\theta(m')]) - c(y[\theta(m');\theta(m)]) \right\} dmdm' \quad (2.3)$$

Note the welfare is a weighted average of meetings surplus. As will be clear, the main trade-off faced by planner is between the called intensive margin - improving each meeting surplus - and the extensive margin - attributing more weight to more productive meetings. In order to improve extensive margin, a less dispersed money distribution - i.e. more homogeneous - is pursued.[8]

As can be noted by the stationarity requirement equation, money distribution is determined by spending pattern and monetary policy. In this work we argue that control spending pattern is much more effective than using monetary policy to attain a good money distribution. As will be clear, the crucial spending necessary to maintain a good distribution is the one in meeting $(1, 1)$. As a good distribution means agents having 1 unit of money,[9] the most frequent meeting will be this one. However, the spending pattern in this meeting must preserve this distribution. For this, we must have low spending to prevent money dispersion.

The individual rationality constraint implies that producers and consumers must expect a higher utility following the trading protocol suggested by the planner than not to trade. Thus, we have for producer and consumer in each (m, ξ, m', ξ') meeting that:

$$\Pi^p(m,\xi;m',\xi') \equiv -c[y(m,\xi;m',\xi')] + \beta \sum_k \lambda^k(m,\xi;m',\xi')(e_{m+k} - e_m)DV' \geq 0 \quad (2.4)$$

[8] Indeed, since rich agents have low incentive to produce and poor ones cannot consume, a high dispersed money distribution - i.e. high heterogeneity - leads to more unproductive meetings. This is due to random matching nature of meetings. Think in the extreme instance in which every person has only one unit of money. In this case, all meetings are productive. On the other hand, in the case we have half people with 0 units of money and the rest with 2, only a quarter of meetings would be productive (no production occurs if consumer has 0 u.m or consumer has 2 u.m). Clearly this result is exacerbated by the money upper bound imposed.

[9] Then, they can always be, effectively, a producer or a consumer.

$$\Pi^c(m', \xi'; m, \xi) \equiv u[y(m', \xi'; m, \xi)] - \beta \sum_k \lambda^k(m', \xi'; m, \xi)(e_m - e_{m-k})DV' \geq 0 \tag{2.5}$$

where, Π^p and Π^c are, respectively, producer and consumer expected utility gain in meeting $(m, \xi; m', \xi')$, and, e_i is a 3×1 indicator vector.[10]

The core constraint implies that, for each meeting, producer and consumer have no incentive to defect together and choose another trading terms. We include here the liquidity constraint. Therefore, we have the core requirement implies that, for each (m, ξ, m', ξ') meeting, trading protocol must satisfies:[11]

$$(y, \lambda) \quad \in \quad \underset{(y,\lambda)\in R_+\times\Delta}{argmax} \quad \Pi^c \quad \text{ subejct to } \sum_{i\neq 0} \lambda^i \leq \xi' \quad \text{and} \quad \Pi^p \geq \gamma, \quad \text{for some} \quad \gamma \tag{2.6}$$

where γ is a meeting specific value that guarantees no trade gain is left on the table.

Finally, the optimal portfolio choice requirement is given by:[12]

$$\theta(m) \quad \in \quad \underset{\xi\in R}{argmax} \quad \int \left[\Pi^p(m, \xi; \theta(m')) + \Pi^c(\theta(m'); m, \xi)\right] dm' \tag{2.7}$$

Therefore, the central planners problem is choose $(\mu, y, \lambda, \tau, \pi, \theta)$ to maximize (2.3), subject to (2.4), (2.5), (2.6), (2.7) and $\mu TD = \mu$.

In order to reduce the dimensionality of the problem we present proposition 3. The idea is to focus only on trade protocols that will prevail in equilibrium.

Proposition 3. *For every implementable allocation $(\mu, y, \lambda, \tau, \pi, \theta)$, there is another implementable allocation $(\mu, \tilde{y}, \tilde{\lambda}, \tau, \pi, \theta)$, such that: (i) it achieves the same welfare;*[13] *(ii) trading protocol $(\tilde{y}, \tilde{\lambda}) = (y, \lambda)$ except for their values in meetings in which there is one-sided deviaton from optimal portfolio choice. In this case, we set a trading protocol such that all surplus goes to the one that has not deviated.*

[10] It values one in the $(i + 1)$-th coordinate and zero in the others.
[11] Omitting subscripts.
[12] Since bonds pay no interest, agents with same beginning of period money holdings have the same reservation utility at trade stage. Thus, the optimal portfolio choice can be achieved maximizing the expected trade surplus.
[13] Actually, it achieves the same resource allocation in equilibrium.

Proof. (i) This is obvious, since we just changed the trading protocol in off-equilibrium contingencies.

(ii) First, we argue that this trading protocol exist. This is true since trading surpluses are strictly increasing in (y, λ).

Second, we show that the alternative allocation is implementable. That said, the alternative allocation trivially satisfies stationarity, individual rationality and core requirements. For optimality of portfolio choice, note that if an agent deviates in portfolio choice stage, he will get its reservation utility thereafter. Since bonds pay no interest, the reservation utility for any choice of portfolio is the same. Therefore, as in trades he must receive at least its reservation value, portfolio choice θ is optimal. □

Proposition 3 guarantees that we can restrict our attention to allocations in which, in meetings where only one agent has deviated from optimal portfolio choice all surplus goes to the other one. Note also that we don't need to specify the trading protocol for meetings where both agents deviate from optimal portfolio.[14] These two ideas lead to the next result.

Corollary 1. *For allocations we restrict our attention, any optimal portfolio choice function, θ, can be supported if the individual rationality constraints are satisfied.*

Proof. Since deviating from suggested portfolio an agent will get its reservation value, the optimal portfolio choice requirement will be embedded in the individual rationality constraints. □

Given these results, we can ignore restriction (2.7) and trading protocol pattern for off-equilibrium portfolios. This will reduce the dimensionality of the problem and make it manageable.

2.5 Computational procedure

For numerical approach, we take section 2.4 results and posit the following problem: choose $(\mu, y, \lambda, \tau, \pi, \xi)$ to maximize (2.3), subject to (2.4), (2.5), (2.6) and $\mu TD = \mu$. Where, trading protocol, (y, λ), will be defined only in equilibrium path and $\xi = (\xi_1, \xi_2)$ stands for liquidity choice from agents with, respectively, one and two units of money. Therefore, the numerical

[14]Since deviation in portfolio choice stage is individual, trading protocol that prevails in meetings where both agents has deviated has no role. Therefore, we can always consider that, in these meetings, all surplus goes to consumer.

problem is analogous to the one in Deviatov(2006), except for the addition of variable ξ and a liquidity constraint requirement.

Then, in order to solve the optimization problem, it is used the KNITRO solver. KNITRO is a local solver for large scale optimization problems. As a local solver, it does not guarantee global optimality. However, due to its relative fast convergence to a local solution,[15] it can be set to try many initial values, taking the best result achieved as a good approximation of global optimum.

In order to proceed with the program, it is necessary to write the problem in a appropriate manner. For this, in addition with the requirement of working with matrices, we must write the constraints as inequalities. Then, we must use a necessary and sufficient Kuhn-Tucker condition associated with the core requirement problem.[16]

First, we replicate the results from Deviatov (2006) without bonds. KNITRO was able to reproduce Deviatov solutions using only 100 different initial points.[17] Then, for the economy with bonds, we use 1000 different initial points. The algorithm seems to find the same solution for several different initial points.

We also use the algorithm to find the solution of a relaxed problem. We solve it for the problem without the core constraint. In this case, bonds are not attractive anymore. For this exercise, we use about 500 different initial points.

2.6 Results

For the rest of paper we work with $u(y) = y^{\kappa}$ and $c(y) = y$. The parameter κ stands for a risk aversion. Higher risk aversion value (i.e., lower κ) is associated to higher utility gains from trade.

Following Deviatov (2006), we work with $\kappa = .2$, $.4$ and $.6$. For $\kappa = .2$, we calculate the optimal allocation for both economies, with and without illiquid bonds. In addition, we solve for $\kappa = .2$ a relaxed (core-off) problem to enlighten the spending control matter. The relaxed problem is the same as Deviatov (2006), but without core constraint. For $\kappa = .4$ and $.6$, we calculate the optimal allocation for an economy with illiquid bonds.

[15] Or stop before reach one.
[16] See appendix E.
[17] For each combination of parameters.

Each table presents the results for economies with different values of β. A column shows the optimal allocation main features of a given economy. As optimal portfolio for agent with 2 units of money is always not be be restricted by liquidity, we ommit his portfolio composition.[18] It contains the output, relative to first-best output,[19] payments[20] for each four meetings, money distribution summarized by the proportion of people with 0 and 1 unit of money, inflation, transfers ,and, for economies with illiquid bonds, portfolio liquidity of agent with 1 unit of money and welfare gain in relation to only money economy.

First, for $\kappa = .2$, we compare tables 2.1 and 2.2. The former shows the optimal allocation for the relaxed problem, while the latter shows the optimal allocation for original, only money, economy. Note that for all β value, observing the higher value in μ_1, money holdings distribution in relaxed problem is much better than in the original one. More, inflation never takes place in relaxed problem. Note, looking to row with λ_{11}, that this better distribution is instead achieved by low spending pattern in meeting $(1, 1)$.

As said before, this meeting is crucial to sustain a good distribution. It is social optimal having high savings in this meeting. However, people in this meeting does not internalize the effects of their spending in this ocassion and, then, this cannot be sustained in a non-monitored trade environment. Finally, observe that this lower spending pattern in relaxed problem is at the cost of lower output in this same meeting.

Now, take a look to table 2.3. Note that with illiquid bonds, we can improve the distribution of money holdings in relation to only money economy, again, for all β values. However, as expected, it cannot improve the same as in relaxed problem. Observe that spending pattern in meeting $(1, 1)$ is much more lower in table 2.3, than in table 2.2. This is how the economies with illiquid bonds achieve a better distribution.

Note also in table 2.3, that spending in meeting $(1, 1)$ is equal to the liquid portion of portfolio. In other words, spending in this meeting is binding. This is how we can guarantee a low spending pattern that assures a good distribution. Notwithstanding, observe that, in contrast to the relaxed problem, lower output in $(1, 1)$ meeting is not the only cost of controling spending pattern. Note that, as portfolio liquidity of agents with 1 u.m. is low,

[18] Any portfolio that not bind his liquidity constraint is optimal. Thus, we can always consider that his optimal portfolio is totally liquid.

[19] The first-best output is the one that would be achieved if we have no frictions. The number reported in tables are as proportion to the first-best output.

[20] Since in all examples agents never pay two units of money, the value of λ_{ij} stands for the probability of payment of one unit of money in meeting (i, j).

their spending in meeting $(0, 1)$ is also binding. Therefore, the output in this meeting is also very low. Thus, the cost of control spending with illiquid bonds is much higher than the one in relaxed problem.

Nevertheless, note that the welfare gain is still significant for all β values. Note also, that the use of illiquid bond becomes more valuable as we descrease β. This is because the money holdings distribution in the only money economy is very poor. Then, illiquid bonds can achieve a much better one. Finally, note that inflation takes no place with illiquid bonds. Thus, it seems to evidence that spending control issues are much more important than redistribution.

TABLE 2.1: Only money - core off: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000	1.0000	1.0000	0.9558*	0.7443*	0.3329*
y_{02}	3.6649*	2.3707*	1.3539*	0.9236*	0.7443*	0.3329*
y_{11}	0.1279*	0.0793*	0.0423*	0.0255*	0.0196*	0.0089*
y_{12}	1.0000*	1.0000*	0.7499*	0.4580*	0.3572*	0.1624*
λ_{01}	1	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0282	0.0458	0.0564	0.0556	0.0550	0.0546
λ_{12}	0.2212	0.5784	1	1	1	1
μ_0	0.1299	0.1619	0.1736	0.1774	0.1797	0.1880
μ_1	0.7484	0.6996	0.6774	0.6783	0.6792	0.6784
inflation	0	0	0	0	0	0
transfer	0	0	0	0	0	0

* Producer's incentive constraint is binding.

TABLE 2.2: Only money: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	1.0000*	1.0000*	0.4353*	0.2109*
y_{02}	3.7614	2.4951	1.7038*	1.0486*	0.4353*	0.2109*
y_{11}	0.2296*	0.2356*	0.2124*	0.1593*	0.1204*	0.0531*
y_{12}	1.0000*	1.0000*	0.4928*	0.2415*	0.1204*	0.0531*
λ_{01}	0.0915	0.2565	0.5872	0.9538	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0682	0.1921	0.4306	0.6607	1	1
λ_{12}	0.2972	0.8155	1	1	1	1
μ_0	0.1932	0.2600	0.3232	0.3679	0.3686	0.3810
μ_1	0.6553	0.5314	0.4302	0.3769	0.3970	0.4016
inflation	0	0	0	0	0.1763	0.2018
transfers	0	0	0	0	0.2498	0.2795

* Producer's incentive constraint is binding.

Finally, in tables 2.4 and 2.5 we present optimal allocation for illiquid bonds economy with, respctively, $\kappa = .4$ and $.6$. Note that, for all β values, in both cases, the liquidity

TABLE 2.3: Illiquid bond: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	0.3583*	0.2128*	0.1070*	0.0603*	0.0454*	0.0547*
y_{02}	3.5436*	2.8017*	1.4997*	0.8609*	0.6583*	0.3225*
y_{11}	0.1302*	0.0874*	0.0528*	0.0334*	0.0264*	0.0221*
y_{12}	1.0000*	1.0000*	0.7394*	0.4770*	0.3828*	0.1304*
λ_{01}	0.0327	0.0552	0.0713	0.0701	0.0689	0.1696
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0327	0.0552	0.0713	0.0701	0.0689	0.1696
λ_{12}	0.2513	1	1	1	1	1
μ_0	0.1287	0.1561	0.1652	0.1613	0.1592	0.2514
μ_1	0.7342	0.6802	0.6515	0.6533	0.6550	0.5469
inflation	0	0	0	0	0	0
transfer	0	0	0	0	0	0
liquidity	0.0327	0.0552	0.0713	0.0701	0.0689	0.1696
welfare gain	7.30%	14.56%	22.53%	28.06%	29.01%	29.97%

* Producer's incentive constraint is binding.

constraint in no meeting is binding.[21] Therefore, there is no role for illiquid bonds in these cases.

This is because higher values of κ reduce the benefits of trade. Insurance is no longer so attractive. Therefore, the cost in output level in meetings (1,1) and (0,1) would be greater than the benefits in money distribution. This result is related to the inaction of monetary policy for higher κ values in Deviatov (2006).

TABLE 2.4: Illiquid bond: $\kappa = .4$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	0.5783*	0.3249*	0.2439*	0.0921*
y_{02}	2.5289	1.4809*	0.5783*	0.3249*	0.2439*	0.0921*
y_{11}	0.3318*	0.3173*	0.1649*	0.0801*	0.0512*	0.0111*
y_{12}	1.0000*	0.7438*	0.1961*	0.0801*	0.0512*	0.0111*
λ_{01}	0.2495	0.6753	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.1605	0.4267	0.8408	1	1	1
λ_{12}	0.4837	1	1	1	1	1
μ_0	0.2437	0.3196	0.4327	0.4678	0.4834	0.5229
μ_1	0.5541	0.4317	0.3412	0.3172	0.3134	0.3023
inflation	0	0	0	0	0	0
transfer	0	0	0	0	0	0
liquidity	1	1	1	1	1	1
welfare gain	0%	0%	0%	0%	0%	0%

* Producer's incentive constraint is binding.

[21] When spending is not binding in the optimal allocation, we take the all liquid portfolio as the optimal one.

TABLE 2.5: Illiquid bond: $\kappa = .6$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	0.6207*	0.1786*	0.0800*	0.0526*	0.0126*
y_{02}	2.0690	0.6207*	0.1786*	0.0800*	0.0526*	0.0126*
y_{11}	0.4073*	0.2370*	0.0641*	0.0212*	0.0119*	0.0016*
y_{12}	1.0000*	0.3489*	0.0641*	0.0212*	0.0119*	0.0016*
λ_{01}	0.4492	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.2621	0.6792	1	1	1	1
λ_{12}	0.6434	1	1	1	1	1
μ_0	0.2689	0.4284	0.5454	0.5875	0.6043	0.6451
μ_1	0.4936	0.3629	0.2950	0.2795	0.2727	0.2545
inflation	0	0	0	0	0	0
transfers	0	0	0	0	0	0
liquidity	1	1	1	1	1	1
welfare gain	0%	0%	0%	0%	0%	0%

* Producer's incentive constraint is binding.

2.7 Conclusion

In this paper we show that the same frictions that make money essential, also induce too low savings. The distribution of liquid funds is an externality. Lack of commitment prevent agents to alleviate the costs of such low savings. Thus, illiquid bonds, by introducing a commitment mechanism, could circumvent this problem and improve welfare. This justifies the coexistence of money and bonds not just from an individual perspective but also from a society point of view.

In order to achieve the results, we have to rely on many simplifying assumptions such as restricting attention only to no interest-bearing bonds. Besides, the description of the environment in which illiquid bonds are supported must be more explored.

For the simplifying assumptions, we argue that allowing for more interest-bearing bonds we would only improve welfare further, but, the use of illiquid bonds will still be optimal. However, for the environment consideration, more attention must be paid to it in order to guarantee that the environment is not so rich that becomes able to support more sofisticated credit arrangements that would take out the role of money.

Finally, it is not clear if results will persist as money holdings increase.

Chapter 3

Monetary policy transition approach in random matching models: investigating its implications

3.1 Introduction

In this paper, it is discussed the optimal management of monetary policy in economies in which money is essential.[1] Money essentiality requirement is what make this problem more interesting. This is because it is generally accompanied by a significant degree of heterogeneity raised by the frictions needed to give money a role in the economy.[2] Then, the redistributive nature of monetary policy plays a role and optimal monetary policy usually departs from Friedman Rule.[3]

In search-based models of money, distributive considerations are even more important. As random pairwise meetings take place, money circulates only slowly and trade opportunities are highly dependent on how money is distributed. Accordingly, the optimal monetary policy has to face a trade-off between: intensive margin, effects on trades for a given distribution of holdings, and extensive margin, effects on distribution of holdings. An active

[1]Money is said essential whenever the implementation of a kind of monetary system induces an improvement in resource allocation. This notion of essentiality goes back to Hahn (1987). For a more recent discussion, see Kocherlakota (1998) and Wallace (2001).

[2]Money essentiality requires incomplete markets. Thus, idiosyncratic shocks would naturally lead to heterogeneity. Although, some tricks, as quasi-linear utilities, may be used to circumvent it and turn the interesting features of heterogeneity off. See Lagos and Wright (2005).

[3]Friedman Rule asserts that optimal monetary policy should maximize money value. The idea is that reducing the opportunity cost of carrying money is crucial to induce trade and, then, achieve the efficient resource allocation.

monetary policy, through its redistributive effects, would improve the extensive margin at the cost of, through inflation, a deterioration in the intensive margin.

With different approaches, Scheinkman and Weiss (1986), Levine (1991), Kehoe, Levine, and Woodford (1992) and Molico (2006) set out models in which some of these features are present and demonstrate that inflation, accomplished through lump-sum transference, is optimal. However, all these works rely on a prespecified trading protocol, such as linear price taking or a bargain rule. We claim that the optimal monetary policy is dependent on trading protocol and, then, the latter must be included as a variable choice in the planner's problem. In other words, we must rely on mechanism design approach.

Green and Zhou (2005) uses this approach in a Bewley economy. He shows that for economies with very patient agents, a deflationary monetary mechanism, sufficient close to Friedman Rule, is nearly optimal. On the other hand, in an environment with impatient agents, he also shows that an inflationary monetary mechanism is preferred to a laissez-faire or deflationary monetary mechanism.

Deviatov and Wallace (2001) and Deviatov (2006) use a random-matching environment to discuss the optimal monetary policy. For sake of tractability, they rely on the assumptions of indivisible money and holdings set in $\{0, 1, 2\}$. This set is the smallest set consistent with monetary policy affecting money distribution. The indivisibility issue is alleviated using lotteries at both, trading and policy stages.

Deviatov and Wallace (2001), using an ex-post individual rationality concept, prove that lump-sum money creation is beneficial for high patient economies. In turn, Deviatov (2006), using an ex-ante individual rationality concept and numerical simulations, shows that lump-sum money creation seems optimal for low patient economies but not for high patient ones.

Wallace (2014b) conjectures that for these kind of economies[4] an active monetary policy would almost always be optimal. Besides, he argues that the examples with no beneficial money creation in Deviatov (2006) may be due to the low scope given to regressive schemes. We argue that he seems right about the conjecture of optimal active policy, but that, in these examples, the issue is not the little scope to regressive schemes.

[4] An economy that, among others features, has: *"significant two-way interaction between the state and trades"*. See Wallace (2014b).

In this paper, we argue that is a monetary policy modeling peculiarity that is driving these non-beneficial money creation results from Deviatov (2006). Indeed, the use of two independent lotteries to model money creation and inflation/normalization makes monetary policy much more unattractive. First, an active monetary policy driven by the two independent lotteries scheme would raise too much uncertainty making carrying money, and thus trading, much less attractive. Second, the supposed benefits in the extensive margin may be not present in significant instances.

In turn, we suggest an alternative modeling approach to monetary policy that circumvent these issues. Specifically, we summarize the two lotteries in only one. This way, we get rid of the excess of uncertainty while maintaining the spirit of the policy.

By means of numerical simulation, using Deviatov (2006) with the alternative policy modeling approach, we find evidence that monetary policy is widely used for a larger set of parameters. Precisely, we show that the many instances in which money creation was not beneficial in Deviatov (2006) was due to its modeling peculiarity. Moreover, the optimal monetary is, indeed, progressive.

The paper is organized as follows. In section 3.2 we introduce the main environment we work. Section 3.3 details the difference in modeling approach to monetary policy between this paper and Deviatov (2006), investigating the issues raised by the latter. Next, in section 3.4, we present the implementable allocations and the planner's problem. Section 3.5 presents the numerical approach. In section 3.6, we exhibit and comment the numerical results. Finally, in section 3.7 we conclude.

3.2 Environment

The environment is the same of Deviatov (2006). It is a random matching monetary model that builds on Shi (1995) and Trejos and Wright (1995). Time is discrete and infinite. There is a continuum $[0, 1]$ of infinitely-lived agents. The production and consumption specialization pattern is standard. In each period, at the trade stage, an agent becomes a producer (who meets a random consumer), a consumer (who meets a random producer) or stay inactive (no meeting) with, respectively, probabilities $\frac{1}{N}$, $\frac{1}{N}$ and $1 - \frac{2}{N}$. In a meeting, the producer can produce y units of a perishable consumption good for the consumer at the cost of disutility $c(y)$, where c is strictly increasing, convex, differentiable and satisfies $c(0) = 0$. The

consumer receives period utility $u(y)$ for the consumption of y units of this good, where u is strictly increasing, concave, differentiable and satisfies $u(0) = 0$. Agents maximize the expected sum of discounted period utilities with discount rate $\beta \in (0, 1)$.

There is a durable asset that has no intrinsic value called (fiat) money. Money is indivisible and individual money holdings are restricted to be in $\{0, 1, 2\}$. In each period, after the trade stage, there is a transfer stage when the central planner can make money transferences to agents - i.e., monetary policy. In order to alleviate the indivisibility issue, we allow the use of lotteries at both stages. People cannot commit to future actions and agents are anonymous - i.e., agent's history is private. Money holdings are private at the transfer stage. Nevertheless, specialization and money holdings in meetings are known.

The distinguish feature from Deviatov (2006) is the modeling approach to the transference lotteries. While Deviatov models this transition by the use of two independent lotteries - one for the lump-sum transfer and other for normalization (inflation), we sum up both in an unique net lottery. In the next section, we describe how these policies are modeled and make a series of exercises to show the main caveats with the usual approach.[5] In addition, we describe the alternative suggested and compare it with the usual modeling. This will sharpen the intuition for the following results.

3.3 Investigating monetary policy trasition

As said before, Deviatov (2006) models the monetary policy transition by two independent lotteries: one for lump-sum[6] transference and other for inflation. The lump-sum transfer matrix is meant to represent an injection of money through lump-sum transfers to agents. To circumvent the indivisibility issue, this must be made probabilistically. Therefore, except for the agents with 2 money holdings[7] each agent will receive a unit of money with probability τ. This leads to the following transition matrix for transference:[8]

[5] We use the term *usual approach* to refer to Deviatov (2006) approach, and, *alternative approach* to refer to the one used in this paper.

[6] For a large subset of the examples, we also allowed to more general money transfers. The only requirement we impose is a truth telling constraint of money holdings. Thus, we allow agent with 1 unit of money receive more transfer than agent with 0 units. But, since this seems not be optimal, we omit it and work only with lump-sum transfers.

[7] Due to upper bound on money holdings. Although this policy is not strictly a lump-sum transfer, it seems to be the best approximation.

[8] The matrix element $d_{i,j}$ represents the probability of agent with $i-1$ units of money end up with $j-1$ units of money after the policy. This interpretation is valid for all transition matrices.

$$D_t = \begin{bmatrix} 1-\tau & \tau & 0 \\ 0 & 1-\tau & \tau \\ 0 & 0 & 1 \end{bmatrix}$$

In turn, the inflation matrix has the role of normalize the economy money holdings. Since we work with the concept of stationarity, after the money injection, normalization (money destruction) is required. For this, we rely on the idea that what matters to decisions are the relative money holdings[9] and, thus, inflation would work just as a money proportional tax. Thus, each unit of money is destroyed with probability ξ. This leads to the following matrix for inflation:

$$D_d = \begin{bmatrix} 1 & 0 & 0 \\ \pi & 1-\pi & 0 \\ \pi^2 & 2\pi(1-\pi) & (1-\pi)^2 \end{bmatrix}$$

Therefore, the monetary policy transition is represented by the composition of these two matrices and is given by:

$$D = D_t D_d = \begin{bmatrix} (1-\tau)+\tau\pi & \tau(1-\pi) & 0 \\ \pi(1-\tau)+\tau\pi^2 & (1-\tau)(1-\pi)+2\tau(1-\pi)\pi & \tau(1-\pi)^2 \\ \pi^2 & 2(1-\pi)\pi & (1-\pi)^2 \end{bmatrix}.$$

Yet, an alternative monetary policy transition is suggested. The idea is as follows. During the policy stage we treat money holdings as a continuous variable, thus, first they receive the money transfers[10] and, then, their money holdings are normalized by inflation. At the end of this stage, the possible continuous money holdings is discretized in the money holdings support. The discretization is made in the neighboring grid points by a lottery whose probabilities are inversely proportional to the distance between the money holdings and the grid point.

Thus, for a lump-sum transference, τ, and a inflation, π, the money holdings balance before discretization for an agent with 0, 1 or 2 units of money is given by:[11]

[9]See Lucas and Woodford (1993). This is possible because our economy has no price rigidities.

[10]It may be even negative. The idea is that this stage comprises both, lump-sum transfer and proportional tax. This way, we are respecting the spirit of a policy that does not distinguish agents.

[11]Where $a^+ = max\{0, a\}$.

$$
\begin{aligned}
0 &\to \tau(1-\pi) &&= 0 + \tau(1-\pi) \\
1 &\to (1+\tau)(1-\pi) &&= 1 - [\pi - \tau(1-\pi)]^+ + [\tau(1-\pi) - \pi]^+ \\
2 &\to (2+\tau)(1-\pi) &&= 2 - [2\pi - \tau(1-\pi)]
\end{aligned}
$$

And the transition matrix is given by:[12]

$$
D = \begin{bmatrix} 1-\tau(1-\pi) & \tau(1-\pi) & 0 \\ [\pi-\tau(1-\pi)]^+ & 1-[\pi-\tau(1-\pi)]^+ - [\tau(1-\pi)-\pi]^+ & [\tau(1-\pi)-\pi]^+ \\ 0 & [2\pi-\tau(1-\pi)] & 1-[2\pi-\tau(1-\pi)] \end{bmatrix}
$$

Remark 3. *The usual monetary policy modeling approach induces an excessive risk for the agents, in addition to promote an undesired higher dispersion of money holdings.*

First, note the second row of the matrix associated with the usual transition would induce a randomization among the three grid points. Thus, the agent with 1 unit of money at the beginning of the stage would face a great uncertainty if the policy is active. Also, the third row of this same matrix shows that the same happens to the agent with 2 units of money. Since this turns money less attractive, this is clearly an undesired result of the usual transition that is not present in the alternative suggested.[13]

Moreover, this also will have negative impact in the money distribution. All rows from the usual transition matrix throw agents to all grid points inducing dispersion in money holdings.

In order to further investigate this issue, we analyze the money distribution behavior for a monetary policy under each transition approach. The exercise is as follows: (i) take an initial distribution; (ii) define a lump-sum transfer level;[14] (iii) calculate the inflation necessary keep the aggregate money holding constant; (iv) calculate the dispersion of the resulting distribution.[15]

[12]Assuming stationarity, we can assure the term $[2\pi - \tau(1-\pi)]$ is positive. Indeed, in steady state, there is no aggregate money creation or destruction. That said, it must be the case that $\tau(1-\pi) = \tilde{\mu}_1\pi + 2\tilde{\mu}_2\pi$, where $\tilde{\mu}_i$ is the proportion of agents with i u.m. after the trades. Then, it is trivial to show that $[2\pi-\tau(1-\pi)] = 2\tilde{\mu}_0\pi + \tilde{\mu}_1\pi > 0$. Also, we assume τ and π sufficient small in order to agents don't receive or lose more than 1 unit of money.

[13]Note that $[\pi - \tau(1-\pi)]^+ > 0$ if, only if, $[\tau(1-\pi) - \pi]^+ = 0$.

[14]Taking τ from zero to one, with grid size of 0.1.

[15]Note we are not imposing stationarity.

The figures below present the results. They show, for a given initial distribution, the resulting money distribution dispersion[16] as function of the money transfer level. The blue and red graphs represents, respectively, the use of the usual and the alternative transitions. Note that the zero in the x-axis is associated with the initial money dispersion, since with no transference, there is no change in money distribution.

In figure 3.1, we take the degenerated distribution as striking case. In this case, initially every agent has one unit of money. Thus, it's expected that a lump-sum transfer doesn't affect it. This is trivially achieved by the alternative approach. Note the money dispersion is zero for any transfer level. However, with the usual transition we can see that, increasing the transference level leads to an increase in money dispersion. This occur exactly because row two of the usual transition matrix induces a dispersion of the agents with one unit of money. Some will end up with 1 unit, others with 0 and others with 2.

This is clearly an undesired property, since it breaks with the lump-sum transference and normalization idea. Besides, this may be an evidence that the usual transition matrix cannot preserve desired money distributions.

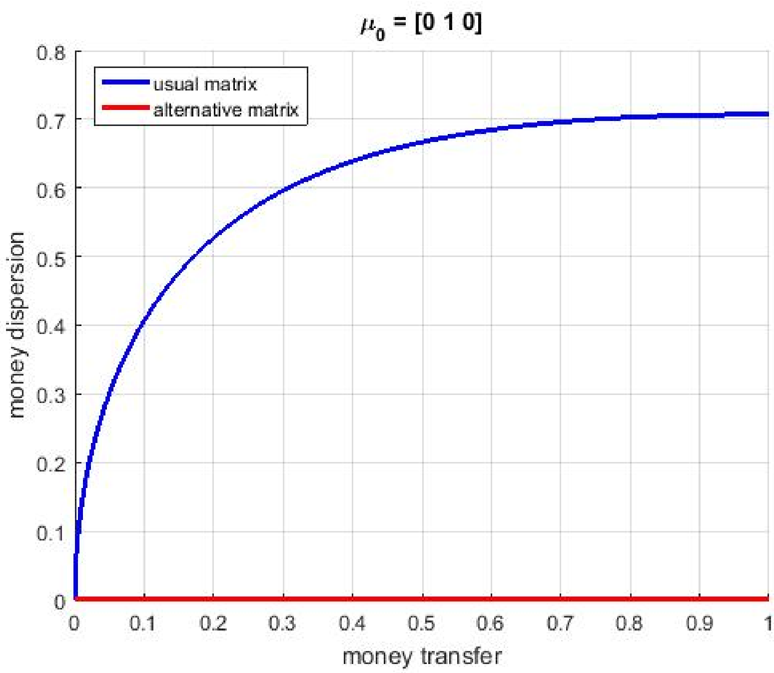

FIGURE 3.1: Money dispersion

Next, since this exercise is sensitive to initial distribution, we repeat it using some notable distributions. They are stationary equilibrium distributions from Deviatov (2006).

[16]Measured by standard deviation.

Figure 3.2 uses stationary distributions for a Deviatov (2006) economy with $\beta = .95$, $.83$, $.66$ and $.50$.[17] Note that for $\beta = .95$, money injection has very distinguish results under each approaches. Note that, as expected, with alternative approach, the money dispersion decreases monotonically as we increase the transfers. However, under the usual approach, this dispersion hikes as the transfers increase. The same occurs for $\beta = .83$.

For $\beta = .66$, the money dispersion under the usual approach decreases slightly for some levels of positive money transfer. However, the decrease under the alternative approach is much more prominent. For $\beta = .50$, money dispersion decreases monotonically as transfers rise under both approaches. However, under the alternative transition, it seems we can achieve a less dispersed distribution.[18]

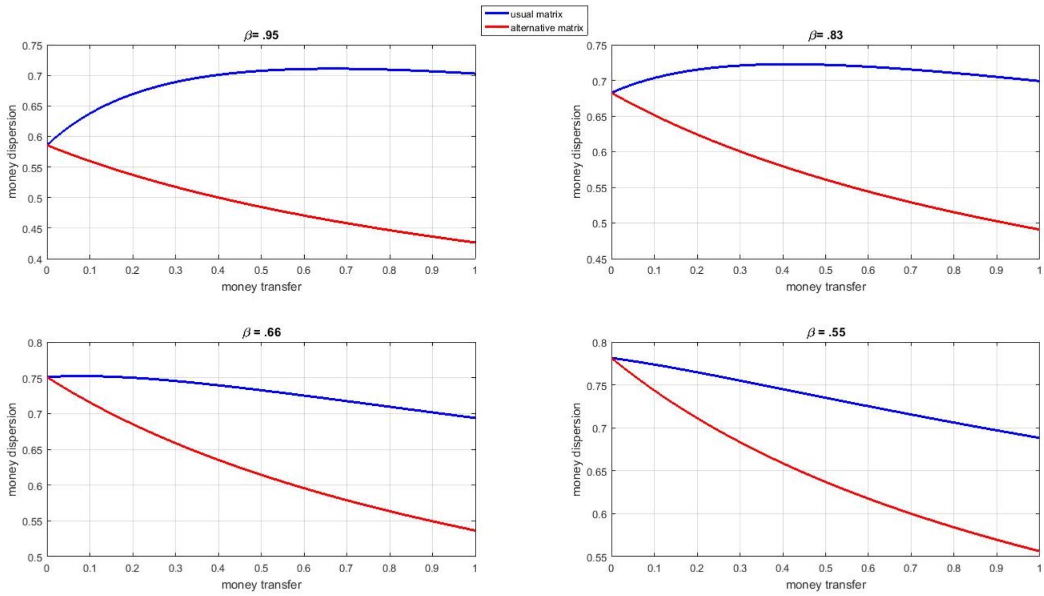

FIGURE 3.2: Money dispersion

These results suggest that not only alternative transition is more effective in reducing money dispersion, but also, active monetary policy induces higher money dispersion for economies with a good money distribution. That said, it is expected that monetary policy under the usual approach is not active for economies with high β while under the new approach the monetary policy becomes much more attractive and optimal for a larger set of parameters.

[17] And curvature $\kappa = .2$.

[18] The resulting dispersion for the same transfer level between approaches may not be comparable. But, the maximum decrease achieved by a policy seems to be higher under the alternative approach.

3.4 The planner's problem

Due to problem complexity, we restrict our attention to stationary and symmetric allocations.[19] A steady-state allocation is defined by the vector $(\mu, y, \lambda, \tau, \pi)$, where μ is a distribution of money, (y,λ) is the trading protocol, defined, respectively, by an output and payments in terms of lotteries for each meeting, τ is the money transfer and π is a measure of inflation.

As Deviatov (2006), the central planner's problem is choosing an allocation to maximize an agent *ex-ante* utility, subject to individual rationality, no pairwise defection in meetings and stationarity requirement.In order to formalize this problem, we will define some auxiliary objects and present the set of implementable allocations.

Each meeting is indexed by $(i, j) \in \{0, 1, 2\} \times \{0, 1, 2\}$, where i and j are, respectively, producer and consumer money holdings. Therefore, y_{ij} is output produced in meeting (i, j) and λ^k_{ij} is the probability of consumer with j money holdings pays k units of money for the producer with i money holdings in this specific meeting. Then, given y_{ij} and λ^k_{ij}, the transition matrix T for money holdings implied by trades can be given by:

$$T = \begin{bmatrix} t_{00} & \frac{1}{N}(\mu_1\lambda^1_{01} + \mu_2\lambda^2_{02}) & \frac{1}{N}\mu_2\lambda^2_{02} \\ \frac{1}{N}(\mu_0\lambda^1_{01} + \mu_1\lambda^1_{11}) & t_{11} & \frac{1}{N}(\mu_1\lambda^1_{11} + \mu_2\lambda^1_{12}) \\ \frac{1}{N}\mu_0\lambda^2_{02} & \frac{1}{N}(\mu_1\lambda^1_{12} + \mu_0\lambda^1_{02}) & t_{22} \end{bmatrix}.$$

where t_{mm} denotes a diagonal element of T. Their values are the ones that make each row sums to unity.

Analogous to trades, monetary policy yields a transition matrix for money holdings. Using D as either, the usual or alternative transition matrix, we have the following stationarity requirement, $\mu T D = \mu$.

Denoting the beginning of period value function by $V \equiv (V_0, V_1, V_2)$, we can write the following matricial Bellman equation:

$$V' = q' + \beta T D V' \tag{3.1}$$

where q is the vector of expected one period utility from trade stage, and, is given by:

[19] Symmetric allocation definition.

$$q' = \begin{bmatrix} -\frac{\mu_1}{N}c(y_{01}) - \frac{\mu_2}{N}c(y_{02}) \\ -\frac{\mu_2}{N}c(y_{12}) + \frac{\mu_1}{N}[u(y_{11}) - c(y_{11})] + \frac{\mu_0}{N}u(y_{01}) \\ \frac{\mu_0}{N}u(y_{02}) + \frac{\mu_1}{N}c(y_{12}) \end{bmatrix}.$$

Following Deviatov (2006), we can show that equation (3.1) has a unique solution which is given by:

$$V' = (I - \beta TD)^{-1} q'$$

where I is the 3×3 identity matrix.

Now we can write the welfare function as:

$$W = \mu V' = \frac{1}{1-\beta} \sum_{i,j} \mu_i \mu_j [u(y_{ji}) - c(y_{ij})] \tag{3.2}$$

Note the welfare is a weighted average of meetings surplus. As will be clear, the main trade-off faced by planner is between the called intensive margin - improving each meeting surplus - and the extensive margin - attributing more weight to more productive meetings. In order to improve extensive margin, a less dispersed money distribution - i.e. more homogeneous - is pursued.[20]

It is expected that monetary policy represented by lump-sum transfers will improve the extensive margin, shifting the distribution of real money balances towards the mean, at the cost of harming the intensive margin, money becomes less valuable. Yet, as argued before, the usual modeling of monetary policy transition doesn't allow it.

The individual rationality constraint implies that producers and consumers must expect a higher utility following the trading protocol suggested by the planner than not to trade. Thus, we have for producer and consumer in each (i, j) meeting that:

$$\Pi^p_{ij} \equiv -c(y_{ij}) + \beta \sum_k \lambda^k_{ij} (e_{i+k} - e_i) DV' \geq 0 \tag{3.3}$$

[20] Indeed, since rich agents have low incentive to produce and poor ones cannot consume, a high dispersed money distribution - i.e. high heterogeneity - leads to more unproductive meetings. This is due to random matching nature of meetings. Think in the extreme instance in which every person has only one unit of money. In this case, all meetings are productive. On the other hand, in the case we have half people with 0 units of money and the rest with 2, only a quarter of meetings would be productive (no production occurs if consumer has 0 u.m or consumer has 2 u.m). Clearly this result is exacerbated by the money upper bound imposed.

$$\Pi_{ij}^c \equiv u(y_{ij}) - \beta \sum_k \lambda_{ij}^k (e_{j-k} - e_j) DV' \geq 0 \tag{3.4}$$

where, Π_p and Π_c are, respectively, producer and consumer expected utility gain in meeting (i, j), and, e_i is a 3×1 indicator vector.[21]

The core constraint implies that, for each meeting, producer and consumer have no incentive to defect together and choose another trading terms. Therefore, we have the core requirement implies that, for each (i, j) meeting, trading protocol must satisfies:

$$(y_{ij}, \lambda_{ij}) \quad \in \quad \underset{(y,\lambda)}{argmax} \quad \Pi_{ij}^c \qquad \text{subejct to} \quad \Pi_{ij}^p \geq \gamma_{ij}, \quad \text{for some} \quad \gamma_{ij} \tag{3.5}$$

where γ_{ij} is a meeting specific value that guarantees no trade gain is left on the table.

Therefore, the central planners problem is choose $(\mu, y, \lambda, \tau, \pi)$ to maximize (3.2), subject to (3.3), (3.4), (3.5) and $\mu TD = \mu$. Note again that the unique distinguish feature from usual to alternative approach is the monetary policy transition matrix D.

3.5 Computational procedure

As Deviatov (2006), due to the high complexity of the problem, we must use a numerical approach. For this, it is used the KNITRO solver. KNITRO is a local solver for large scale optimization problems. As a local solver, it does not guarantee global optimality. However, due to its relative fast convergence to a local solution,[22] it can be set to try many initial values, taking the best result achieved as a good approximation of global optimum.

In order to proceed with the program, it is necessary to write the problem in a appropriate manner. For this, in addition with the requirement of working with matrices, we must write the constraints as inequalities. Then, we must use a necessary and sufficient Kuhn-Tucker condition associated with the core requirement problem.[23] It is also better avoid non differentiability in inequalities. Therefore, for the alternative approach, we work as we know the sign of $[\pi - \tau(1 - \pi)]$ and add it as a restriction. Comparing the results for the two possibilities, we define the optimal allocation.

[21] It values one in the $(i + 1)$-th coordinate and zero in the others.
[22] Or stop before reach one.
[23] See appendix D.

First, we replicate the results from Deviatov (2006) using the usual matrix D. KNITRO was able to reproduce Deviatov solutions using only 100 different initial points. For the alternative approach, we change the matrix D and defines the signal of $[\pi - \tau(1-\pi)]$. Then, we use 1500 different initial points for each case. The algorithm seems to find the same solution for several different initial points. When the optimal seems to have $[\pi - \tau(1-\pi)] = 0$, the two cases achieved the same allocation. This indicates some consistency of the results.

We also use the algorithm to find the solution of a relaxed problem. We solve it for the problem without the core constraint. For the usual approach, we use about 500 different initial points. While for the alternative one, we use 800 initial guesses for each possible signal of $[\pi - \tau(1-\pi)]$.

3.6 Results

For the rest of the paper, we will work with $u(y) = y^\kappa$ and $c(y) = y$. The variable κ stands for a risk aversion parameter. Keeping the cost function constant, it is expected that higher risk aversion (i.e., lower κ) is related to higher utility gains from trade.

We follow Deviatov (2006) examples and find the optimal allocation for $\kappa = .2$, $.4$ and $.6$. For each κ value, we calculate the optimal allocation (core-on) using both approaches to monetary policy transition. Also for each κ value, using again both approaches, we solve for the relaxed (core-off) problem. Thus, for each κ value, we have four tables describing optimal allocations.

Following Deviatov (2006), each table presents the results for economies with different values of β. A column shows the optimal allocation main features of a given economy. It contains the output, relative to first-best output,[24] and payments[25] for each four meetings, money distribution summarized by the proportion of people with 0 and 1 unit of money, inflation and transfers. When using alternative policy transition, we also provide the welfare gain relative to the economy solved with the usual approach.

For a clearer presentation, we will present, compare and comment each four tables block in sequence.

[24] The first-best output is the one that would be achieved if we have no frictions. The number reported in tables are as proportion to the first-best output.

[25] Since in all examples agents never pay two units of money, the value of λ_{ij} stands for the probability of payment of one unit of money in meeting (i, j).

First we take $\kappa = .2$. Table 3.1 replicates Deviatov (2006) work using usual policy transition and active core constraint. Note in this case, that monetary policy is active only for low discount factor values. More, note that, keeping monetary policy inactive, money distribution is getting worse (μ_1 decreases)[26] as β is decreasing. Then, when $\beta = .50$ is reached, transfers takes place and money distribution improves.

This result seems direct related to the intuition raised in section 3.3. Since money is well distributed for high β levels, inflation cannot improve this margin anymore. However, as impatience increases, money distribution gets more dispersed, and, an active monetary policy can lessen this dispersion.

Also, note that bad distribution for lower β is supported by higher spending levels for meeting $(1, 1)$. Indeed, the spending level in this meeting is key to maintain a good distribution.[27] That said, also note that inflation only takes place when this meeting spending is on a corner. The intuition is that an active monetary policy has, through inflation, an indirect effect on distribution besides its direct one.

It is a hot potato effect. As inflation increases, people save less, inducing a more dispersed money distribution. Thus, the actual trade-off of an active monetary policy is: a direct improvement in extensive margin due to redistribution, against its harmful effect on intensive margin and its indirect effect on savings behavior that harms extensive margin. Note that inflation only takes place when the second harmful effect is off.

TABLE 3.1: Usual matrix and core on: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	1.0000*	1.0000*	0.4353*	0.2109*
y_{02}	3.7614	2.4951	1.7038*	1.0486*	0.4353*	0.2109*
y_{11}	0.2296*	0.2356*	0.2124*	0.1593*	0.1204*	0.0531*
y_{12}	1.0000*	1.0000*	0.4928*	0.2415*	0.1204*	0.0531*
λ_{01}	0.0915	0.2565	0.5872	0.9538	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0682	0.1921	0.4306	0.6607	1	1
λ_{12}	0.2972	0.8155	1	1	1	1
μ_0	0.1932	0.2600	0.3232	0.3679	0.3686	0.3810
μ_1	0.6553	0.5314	0.4302	0.3769	0.3970	0.4016
inflation	0	0	0	0	0.1763	0.2018
transfers	0	0	0	0	0.2498	0.2795

* Producer's incentive constraint is binding.

[26] As it was discussed, in this economy a good money distribution is one with high μ_1 value.

[27] Since in this meeting, with high spending, we begin with two agents with 1 u.m, but finish with one agent with 0 and the other with 2.

Now let's look at the alternative approach. Table 3.2 shows a striking difference for table 3.1: monetary policy is active for all discount factor values. Associated with it, we have a substantial welfare improvement. This is achieved by the improvement in money distribution obtained with redistribution caused by monetary policy. Note, comparing μ_0 and μ_1 from both tables, that the improvement in money dispersion is significant, even for the cases where monetary policy was already active.

Also, note that the harmful effect of inflation on intensive margin is translated in lower output in comparison with the usual approach.

For examples with $\beta \leq .83$, when savings in meeting $(1, 1)$ is in a corner and the hot potato effect is off, the money transfer hits the upper bound.[28] This reinforce the importance of savings to money distribution.

Finally, using tables 3.1 and 3.2, compare the meeting $(0, 2)$ output for $\beta = .95$. Note that in both approaches, there is an inefficient excess of production. Nevertheless, note that for alternative approach, as inflation takes place, this excess is reduced contributing to welfare improvement.[29]

TABLE 3.2: Alternative matrix and core on: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	0.6213*	0.4255*	0.3190*	0.2715*	0.1515*
y_{02}	2.7432*	0.6213*	0.4255*	0.3190*	0.2916*	0.2124*
y_{11}	0.2425*	0.2336*	0.1608*	0.1214*	0.1013*	0.0609*
y_{12}	0.8832*	0.2336*	0.1608*	0.1214*	0.1013*	0.0609*
λ_{01}	0.3645	1	1	1	1	1
λ_{02}	1	1	1	1	1.1986	2
λ_{11}	0.2746	1	1	1	1	1
λ_{12}	1	1	1	1	1	1
μ_0	0.1457	0.2508	0.2702	0.2819	0.2839	0.3077
μ_1	0.7086	0.6690	0.6605	0.6549	0.6533	0.6390
inflation	0.2107	0.5466	0.5558	0.5614	0.5621	0.5729
transfer	0.2669	1.0000	1.0000	1.0000	1.0000	1.0000
welfare gain	7.30%	14.56%	22.53%	28.06%	29.01%	29.97%

* Producer's incentive constraint is binding.

Now, consider the economy without the core requirement. Comparing tables 3.1 and 3.3, for the usual approach, we note that money distribution achieved in the economy with core off is much better than distribution with core on. This is supported by high savings level for

[28] This bound is set due to the lack of appeal for policies with $\tau > 1$ in the model with usual policy transition.
[29] As output in tables are relative to optimal level, $y_{ij} > 1$ is associated with inefficiency.

the meeting $(1,1)$. Therefore, for the usual approach, there is no role for inflation when core requirement is off. Spending control seems to be more effective than redistribution.

However, comparing tables 3.2 and 3.4, we see that, for the alternative approach, this is not the case. Monetary policy is active for all β values. Moreover, the welfare improvement is still significant. Thus, spending control and redistribution seem to be complementary.

For examples with $\beta \geq .55$, comparing tables 3.2 and 3.4, we can see that monetary policy leads to an improvement in extensive margin (higher μ_1) at the cost of a deterioration in intensive margin (lower y_{ij}).[30] However, for $\beta = .50$ and $.33$, money distribution is actually worsened. In these cases, the welfare gain is achieved by a substantial improvement in the intensive margin (i.e. output increase). But this is only achievable because money redistribution maintains a good distribution, despite the increase in spending for meeting $(1,1)$.

TABLE 3.3: Usual matrix and core off: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000	1.0000	1.0000	0.9558*	0.7443*	0.3329*
y_{02}	3.6649*	2.3707*	1.3539*	0.9236*	0.7443*	0.3329*
y_{11}	0.1279*	0.0793*	0.0423*	0.0255*	0.0196*	0.0089*
y_{12}	1.0000*	1.0000*	0.7499*	0.4580*	0.3572*	0.1624*
λ_{01}	1	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0282	0.0458	0.0564	0.0556	0.0550	0.0546
λ_{12}	0.2212	0.5784	1	1	1	1
μ_0	0.1299	0.1619	0.1736	0.1774	0.1797	0.1880
μ_1	0.7484	0.6996	0.6774	0.6783	0.6792	0.6784
inflation	0	0	0	0	0	0
transfer	0	0	0	0	0	0

* Producer's incentive constraint is binding.

Now, for $\kappa = .4$, we can see in table 3.5, that inflation is not optimal for any β value. On the other hand, in table 3.6, we can note that monetary policy is active for all instances. Since using the alternative approach the two extra costs of inflations are vanished, monetary policy can improve the welfare. Note that for all β values, the welfare gain comes from an improvement in extensive margin (μ_1 increases).

In addition, note the U-shape form of welfare gains: for $\beta = .66$ we have the lowest welfare gain. This may be related to two phenomena. First, lower β leads to worse money distribution, and then, an active monetary policy has more room to improvement. Second, high discount factor values leads to very high output in meeting $(0,2)$. The possible excess of

[30]When $y_{ij} > 1$ this cost may be actually a benefit. See footnote 29.

TABLE 3.4: Alternative matrix and core off: $\kappa = .2$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	0.8676*	0.5769*	0.2712*	0.1514*
y_{02}	2.5985*	1.4046*	0.8609*	0.5769*	0.2712*	0.1750*
y_{11}	0.1252*	0.0822*	0.0448*	0.0297*	0.0968*	0.0596*
y_{12}	0.9068*	0.5988*	0.3395*	0.2334*	0.1027*	0.0596*
λ_{01}	0.3848	1	1	1	1	1
λ_{02}	1	1	1	1	1	1.3966
λ_{11}	0.1381	0.1373	0.1320	0.1273	0.9421	1
λ_{12}	1	1	1	1	1	1
μ_0	0.0943	0.0942	0.0902	0.0874	0.2820	0.3047
μ_1	0.8115	0.8116	0.8197	0.8252	0.6587	0.6420
inflation	0.2249	0.2240	0.2295	0.2316	0.5627	0.5719
transfer	0.2901	0.2887	0.2979	0.3014	1.0000	1.0000
welfare gain	5.86%	9.70%	9.35%	8.82%	9.08%	10.63%

* Producer's incentive constraint is binding.

production in this meeting is made to maintain money valuable and, thus, be able to increase savings in meeting $(1, 1)$ which is crucial to sustain a good money distribution. Therefore, an active monetary policy would preserve a good money distribution while inflation would reduce this excess of production, bringing it closer to its optimal level. In the middle, these two benefits are meager.

Next, tables 3.7 and 3.8 show the optimal allocation for an economy without core constraint using, respectively, usual and alternative transitions. As expected, using the usual approach, there is no room for inflation in a economy without core requirement. Spending control is sufficient to improve extensive margin. Notwithstanding, as we can see in table 8, even with core off, monetary policy is active for high β values using the alternative approach. As we can notice, redistribution improves money distribution. Moreover, the inefficient excces of output in meeting $(0, 2)$ is reduced leading to a significant welfare gain.

Finally, we take $\kappa = .6$. Again, as expected, monetary policy is inactive using usual transition matrix as we can note in table 3.9. Nevertheless, using the alternative approach, we have a surprising result. As can be notice in table 3.10, active monetary policy takes place for $\beta = .95$. Again, this result seems to be supported by the reduction in the excessive production of meeting $(0, 2)$. Inflation would bring its output closer to its optimal level while preserving a similar money distribution.

Looking at tables 3.11 and 3.12, it can be noticed that the same occurs for an economy without core restriction. Monetary policy is active, money distribution is only slightly deteriorated, but, output in meeting $(0, 2)$ is reduced, getting closer to its efficient level. This

TABLE 3.5: Usual matrix and core on: $\kappa = .4$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	0.5783*	0.3249*	0.2439*	0.0921*
y_{02}	2.5289	1.4809*	0.5783*	0.3249*	0.2439*	0.0921*
y_{11}	0.3318*	0.3173*	0.1649*	0.0801*	0.0512*	0.0111*
y_{12}	1.0000*	0.7438*	0.1961*	0.0801*	0.0512*	0.0111*
λ_{01}	0.2495	0.6753	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.1605	0.4267	0.8408	1	1	1
λ_{12}	0.4837	1	1	1	1	1
μ_0	0.2437	0.3196	0.4327	0.4678	0.4834	0.5229
μ_1	0.5541	0.4317	0.3412	0.3172	0.3134	0.3023
inflation	0	0	0	0	0	0
transfers	0	0	0	0	0	0

* Producer's incentive constraint is binding.

TABLE 3.6: Alternative matrix and core on: $\kappa = .4$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	0.5542*	0.2806*	0.2061*	0.0312*
y_{02}	1.8940*	1.0866*	0.5542*	0.2806*	0.2061*	0.0464*
y_{11}	0.3414*	0.2853*	0.1624*	0.0703*	0.0458*	0.0152*
y_{12}	0.9940*	0.5120*	0.1863*	0.0703*	0.0458*	0.0152*
λ_{01}	0.5280	0.9203	1	1	1	1
λ_{02}	1	1	1	1	1	2
λ_{11}	0.3435	0.5572	0.8722	1	1	1
λ_{12}	1	1	1	1	1	1
μ_0	0.2240	0.3196	0.4314	0.4571	0.4700	0.4115
μ_1	0.5684	0.4580	0.3484	0.3592	0.3633	0.5464
inflation	0.0923	0.0586	0.0134	0.0593	0.0743	0.5131
transfer	0.1000	0.0562	0.0107	0.0458	0.0559	0.6645
welfare gain	3.53%	1.52%	0.05%	0.46%	0.51%	5.69%

* Producer's incentive constraint is binding.

leads again to small improvement in welfare.

3.7 Conclusion

In monetary theory literature in which money is essential, the problem of find out the optimal monetary policy is, usually, not trivial. Wallace (2014b) conjectures that optimal monetary policy is almost always active. Besides, it is suggested that it may be the case that, for some instances, it must require taking a form of a regressive policy. Nevertheless, we show that this seems not be the case for the Deviatov (2006) economy. The inaction of monetary policy for a large set of parameters in this work seems to be related to a modeling peculiarity. The inconveniences associated to the modeling approach to monetary policy were driving

TABLE 3.7: Usual matrix and core off: $\kappa = .4$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000	0.9998*	0.5278*	0.2871*	0.2094*	0.0723*
y_{02}	2.3222*	1.4354*	0.5278*	0.2871*	0.2094*	0.0723*
y_{11}	0.2343*	0.1397*	0.0497*	0.0233*	0.0156*	0.0044*
y_{12}	1.0000*	0.7945*	0.2432*	0.1194*	0.0847*	0.0294*
λ_{01}	0.7816	0.6965	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.1054	0.1759	0.2045	0.1950	0.1839	0.1488
λ_{12}	0.4499	1	1	1	1	1
μ_0	0.2084	0.2541	0.3289	0.3550	0.3612	0.3613
μ_1	0.6059	0.5423	0.5096	0.5049	0.5076	0.5252
inflation	0	0	0	0	0	0
transfer	0	0	0	0	0	0

* Producer's incentive constraint is binding.

TABLE 3.8: Alternative matrix and core off: $\kappa = .4$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	1.0000*	0.5278*	0.2871*	0.2095*	0.0724*
y_{02}	1.8436*	1.1027*	0.5278*	0.2871*	0.2095*	0.0724*
y_{11}	0.2235*	0.1332*	0.0497*	0.0233*	0.0156*	0.0044*
y_{12}	0.9909*	0.5843*	0.2432*	0.1194*	0.0846*	0.0293*
λ_{01}	0.5424	0.9069	1	1	1	1
λ_{02}	1	1	1	1	1	2
λ_{11}	0.2255	0.2279	0.2046	0.1950	0.1848	0.1500
λ_{12}	1	1	1	1	1	1
μ_0	0.1851	0.2462	0.3289	0.3550	0.3615	0.3622
μ_1	0.6339	0.5718	0.5096	0.5049	0.5071	0.5240
inflation	0.0947	0.0485	0	0	0	0
transfer	0.1042	0.0477	0	0	0	0
welfare gain	2.85%	1.24%	0%	0%	0%	0%

* Producer's incentive constraint is binding.

the results.

In this paper we present the issues raised by this usual approach to monetary policy. It raises an excessive risk of holding money and may not even be able to shift money distribution towards its mean. Therefore, an active policy would have very harmful effect on intensive margin and low or no beneficial effect on extensive margin, becoming very unattractive. This explain why an active monetary policy seems to be suboptimal for a great set of parameters in Deviatov (2006).

We suggest an alternative modeling approach to monetary policy. It gets rid of such inconveniences and maintains the spirit of such policies. This way, monetary policy becomes much more attractive and, as we verify by numerical simulations, is optimal for a much larger set of parameters.

TABLE 3.9: Usual matrix and core on: $\kappa = .6$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	0.6207*	0.1786*	0.0800*	0.0526*	0.0126*
y_{02}	2.0690	0.6207*	0.1786*	0.0800*	0.0526*	0.0126*
y_{11}	0.4073*	0.2370*	0.0641*	0.0212*	0.0119*	0.0016*
y_{12}	1.0000*	0.3489*	0.0641*	0.0212*	0.0119*	0.0016*
λ_{01}	0.4492	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.2621	0.6792	1	1	1	1
λ_{12}	0.6434	1	1	1	1	1
μ_0	0.2689	0.4284	0.5454	0.5875	0.6043	0.6451
μ_1	0.4936	0.3629	0.2950	0.2795	0.2727	0.2545
inflation	0	0	0	0	0	0
transfers	0	0	0	0	0	0

* Producer's incentive constraint is binding.

TABLE 3.10: Alternative matrix and core on: $\kappa = .6$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	0.6207*	0.1786*	0.0800*	0.0526*	0.0126*
y_{02}	1.7455*	0.6207*	0.1786*	0.0800*	0.0526*	0.0126*
y_{11}	0.4054*	0.2370*	0.0641*	0.0212*	0.0119*	0.0016*
y_{12}	1.0000*	0.3489*	0.0641*	0.0212*	0.0119*	0.0016*
λ_{01}	0.5729	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.3333	0.6792	1	1	1	1
λ_{12}	0.8221	1	1	1	1	1
μ_0	0.2720	0.4284	0.5454	0.5876	0.6043	0.6451
μ_1	0.4890	0.3629	0.2950	0.2795	0.2727	0.2545
inflation	0.0195	0	0	0	0	0
transfer	0.0192	0	0	0	0	0
welfare gain	0.79%	0%	0%	0%	0%	0%

* Producer's incentive constraint is binding.

This result seems important since these issues can be contained in similar economies with larger set of money holdings.[31] It also shows that the direct effect of monetary policy on extensive margin is dominant and, so, progressive policies seems to be more attractive.

[31] In a series of exercise in line with section 3.3, we find that monetary policy modeled by the two independent matrices can have the same effects for larger set of money holdings. These effects may be turned off as money upper bound increases, but only as long only deterministic policies would be considered.

TABLE 3.11: Usual matrix and core off: $\kappa = .6$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	0.9999	0.6122*	0.1770*	0.0794*	0.0523*	0.0126*
y_{02}	1.9942*	0.6122*	0.1770*	0.0794*	0.0523*	0.0126*
y_{11}	0.3191*	0.1570*	0.0455*	0.0178*	0.0106*	0.0016*
y_{12}	1.0000*	0.3536*	0.0660*	0.0216*	0.0121*	0.0017*
λ_{01}	0.5618	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.2061	0.4442	0.6892	0.8245	0.8788	0.9554
λ_{12}	0.6459	1	1	1	1	1
μ_0	0.2519	0.4028	0.5329	0.5821	0.6009	0.6435
μ_1	0.5237	0.4110	0.3280	0.2948	0.2824	0.2578
inflation	0	0	0	0	0	0
transfer	0	0	0	0	0	0

* Producer's incentive constraint is binding.

TABLE 3.12: Alternative matrix and core off: $\kappa = .6$

β	0.95	0.83	0.66	0.55	0.50	0.33
y_{01}	1.0000*	0.6122*	0.1770*	0.0793*	0.0523*	0.0126*
y_{02}	1.7449*	0.6122*	0.1770*	0.0793*	0.0523*	0.0126*
y_{11}	0.3156*	0.1570*	0.0456*	0.0176*	0.0106*	0.0016*
y_{12}	1.0000*	0.3536*	0.0660*	0.0217*	0.0121*	0.0017*
λ_{01}	0.5731	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.2579	0.4442	0.6921	0.8111	0.8788	0.9453
λ_{12}	0.8172	1	1	1	1	1
μ_0	0.2537	0.4028	0.5331	0.5816	0.6009	0.6433
μ_1	0.5225	0.4110	0.3276	0.2961	0.2824	0.2585
inflation	0.0193	0	0	0	0	0
transfer	0.0191	0	0	0	0	0
welfare gain	0.61%	0%	0%	0%	0%	0%

* Producer's incentive constraint is binding.

Appendix A

Proof of the propositions

Proposition 1 For m in the support of distributions of meetings, output is $\hat{y}(m) = m_3$ when intermediation is relaxed, and $\tilde{y}(m) = \min\{m_2, m_3\}$ when savings need not be incentive compatible. In these relaxed problems, moreover, welfare satisfies $w(\hat{s}, \hat{y}) \geq w(\tilde{s}, \tilde{y}) \geq w(s^*, y^*)$, with inequalities replaced by equalities when there is a single type of trader.

Proof. That optimal welfare $w(s^*, y^*)$ is bounded above by $w(\tilde{s}, \tilde{y})$ is trivial. Consider now optimal allocations with pairwise trades described by Cavalcanti and Puzzello (2010). They show that consumers should spend all their holdings and keep all surplus from trade. Such allocations are implementable in our setting when $x(m) \leq m_2$ is relaxed and intermediaries can make loans matching holdings of money by consumers, without profits. Since profits do not affect aggregate welfare (1.1), we conclude that such allocations solve the relaxed problem with $\hat{y}(m) = m_3$. Cavalcanti and Puzzello (2010) also show that in their economy, since holdings of money by producers do not matter for output, then individuals do not care about the distribution of money when making saving choices: if they become producers their surplus is zero, and if they become consumers they do not care whether they meet with rich or poor producers. This means that savings are decided on the basis of realizations of idiosyncratic shocks and on output obtained when consumers, without consideration of how others are choosing money holdings. As a result, the distribution of money can be computed residually, after an incentive-compatible savings function associated to $\hat{y}$ is found. This proves the inequality $w(\tilde{s}, \tilde{y}) \leq w(\hat{s}, \hat{y})$. More generally, Cavalcanti and Puzzello (2010) show that the pairwise optimum solves a relaxed problem with no wedge between private and social savings. Although allocation $(\tilde{s}, \tilde{x}, \tilde{y}, \tilde{z})$ is obtained by ignoring private incentives for saving, these ignored constraints do not bind in the associated pairwise problem. In addition,

if there is a single type then in all meetings consumers and intermediaries have exactly the same money holdings. In this case, the cash-in-advance requirement $x(m) \leq m_2$ is irrelevant and $w(s^*, y^*) = w(\tilde{s}, \tilde{y}) = w(\hat{s}, \hat{y})$ must hold. Finally, $\tilde{y}(m) = \min\{m_2, m_3\}$ follows from an application of the upper-bound construction of Cavalcanti and Puzzello (2010): if saving incentives can be ignored, the arrangement that maximizes $w(s, \cdot)$ for a given savings function has all trade surplus going to consumers, and has consumers spending all their holdings up to the bound dictated by intermediation. □

Proposition 2 If there is more than one type of trader then welfare is increasing in the profit rate r in a neighborhood of zero, so that it is not optimal to give all surpluses to consumers.

Proof. For sufficiently small r, incentive-compatible savings s_i for type i satisfies

$$\theta_i - \beta = \alpha\beta[u'(s_i) - 1]\frac{i}{n} + \alpha\beta r\frac{i-1}{n}.$$

The term $u'(s_i) - 1$ reflects the utility gain of consumption from bringing an extra unit of money when the consumer receives all the surplus from trade, net of the expected opportunity cost of carrying money, which has been assumed equal to the unity. The expression $\theta_i - \beta = \alpha g(s_i)$, for $g(k) = \beta(u'(k) - 1)$, was obtained by Cavalcanti and Puzzello (2010). With intermediation, $\alpha g(s_i)$ must be adjusted by the probability that the consumer is paired with an intermediary carrying at least the same quantity of money, $\frac{i}{n}$ (otherwise there is no marginal effect of an extra unit saved). The term $\alpha r\frac{i-1}{n}$ represents the marginal, expected payment of profits from richer consumers. Now the derivative of the welfare function with respect to r, w', satisfies

$$w' = \sum_{i=1}^{n}\left[-\frac{\theta_i - \beta}{n}s_i' + \frac{\alpha}{n^2}\left(\sum_{j\geq i} g(s_j)s_j' + \sum_{j<i} g(s_i)s_i'\right)\right]$$

where s_i' denotes the derivative of s_i with respect to r. Using now that for $r = 0$

$$g(s_j) = \frac{(\theta_j - \beta)n}{\alpha j}$$

then

$$\begin{aligned} w'\big|_{r=0} &= \sum_{i=1}^{n}\left[-\frac{\theta_i-\beta}{n}s_i'+\frac{\alpha}{n^2}\left(g(s_i)s_i'+\sum_{j>i}g(s_j)s_j'+\sum_{j<i}g_i(s_i)s_i'\right)\right]= \\ & \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j>i}\frac{\theta_j-\beta}{j}s_j'+\sum_{j<i}\frac{\theta_i-\beta}{i}s_i'\right). \end{aligned}$$

Now, since u is concave then s_i' is positive if $i > 1$ and r is sufficiently small. Therefore, under the assumption that $n > 1$, w' is positive for such r. □

Appendix B

Test case

In this appendix we present some facts about economies without intermediation, like those studied by Deviatov (2006). Reproducing his numerical experiments is useful as a test case (see more on our numerical approach in the second part of the appendix), and for showing that inflationary interventions can appear just to repair effects of lack of commitment in corner situations. In order to do that, we need to change the timing of monetary policy in the model presented above. In Deviatov (2006) money transfers occur first, followed by the inflationary process that keeps the quantity of money constant. We should remark, by the way, that we tried other configurations to make sure the relationship between positive inflation and corner outcomes is robust to timing specifications (there are small changes in allocations overall).

The first table below is produced with Deviatov's specification (and the same utility function as in the study of intermediation). It turns out that in his economy the consumer never spends more than one unit of money. Hence we can just report λ_{ij}, defined as the probability that one unit is transferred in meeting (i, j) — henceforth a meeting in which the producer has i and the consumer has j units of money — in addition to reporting output relative to first-best output y^*.

In Table B.1 we can notice two effects taking place as the discount factor falls: money spent in meeting $(1, 1)$ increases and, consequently, holdings are scattered as the set of people holding one unit loses mass (μ_1 falls). This is relevant because people holding one unit can be both producers and consumers. When discount factors β are very low, expansionary policies are needed, but these are also corner cases in which velocity effects are absent. To see that interventions would not be necessary if spending is sufficiently controlled we turn off the core requirement preventing group defections, as in Table B.2.

TABLE B.1: Pairwise meetings and core on

β	.95	.83	.66	.55	.50	.33
y_{01}	1.0000*	1.0000*	1.0000*	1.0000*	0.4353*	0.2109*
y_{02}	3.7614	2.4951	1.7038*	1.0486*	0.4353*	0.2109*
y_{11}	0.2296*	0.2356*	0.2124*	0.1593*	0.1204*	0.0531*
y_{12}	1.0000*	1.0000*	0.4928*	0.2415*	0.1204*	0.0531*
λ_{01}	0.0915†	0.2565†	0.5872†	0.9538†	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0682†	0.1921†	0.4306†	0.6607†	1†	1†
λ_{12}	0.2972†	0.8155†	1	1	1	1
μ_0	0.1932	0.2600	0.3232	0.3679	0.3686	0.3810
μ_1	0.6553	0.5314	0.4302	0.3769	0.3970	0.4016
v_0	1.0169	0.0783	0.0000	0.0000	0.0217	0.0088
v_1	2.5514	0.7040	0.3417	0.2524	0.1890	0.1348
v_1	3.0240	0.9008	0.4405	0.3105	0.2174	0.1476
Inflation	0	0	0	0	0.1763	0.2018
Transfers	0	0	0	0	0.2498	0.2795

* Producer's incentive constraint is binding.
† The core constraint is binding.

When the core is off, the planner manages to implement a better distribution of money. In this case, the average payment in meeting $(1, 1)$ is low even when a low β tightens up the producer constraint and reduces output.

TABLE B.2: Pairwise meetings and core off

β	.95	.83	.66	.55	.50	.33
y_{01}	0.9634	0.9132	0.8751	0.8265	0.7449*	0.3328*
y_{02}	3.6649*	2.3717*	1.3552*	0.9237*	0.7449*	0.3328*
y_{11}	0.1279*	0.0792*	0.0426*	0.0254*	0.0194*	0.0089*
y_{12}	1.0000*	1.0000*	0.7501*	0.4577*	0.3567*	0.1623*
λ_{01}	1	1	1	1	1	1
λ_{02}	1	1	1	1	1	1
λ_{11}	0.0282	0.0458	0.0563	0.0557	0.0551	0.0545
λ_{12}	0.2212	0.5778	1	1	1	1
μ_0	0.1299	0.1619	0.1738	0.1774	0.1800	0.1882
μ_1	0.7484	0.6997	0.6774	0.6782	0.6789	0.6785
v_0	1.9821	0.2728	0.0435	0.0067	0.0000	0.0000
v_1	2.4065	0.6532	0.3154	0.2290	0.1992	0.1336
v_1	3.0414	0.9310	0.4658	0.3392	0.2946	0.1988
Inflation	0	0	0	0	0	0
Transfer	0	0	0	0	0	0

* Producer's incentive constraint is binding.

Appendix C

Auxiliary objects and numerical approach

We describe below in more detail objects used in our simulations. The probability distribution of after-trade holdings, λ, is in fact a 3×3 matrix, $\lambda(m) = (\lambda_1(m); \lambda_2(m); \lambda_3(m))$. In particular, $\lambda_i(m) = (\lambda_i^0(m), \lambda_i^1(m), \lambda_i^2(m))$ is a line vector for $i = 1, 2, 3$, where $\lambda_i^j(m)$ denotes the (marginal) probability that 'person i'(the person starting with m_i) leaves the meeting holding $j \in \{0, 1, 2\}$ units of money. For example, $\lambda_1^j(m)$ denotes the probability that the producer leaves the meeting holding j units of money.

The state space can be written as $\{n, b\} \times \{0, 1, 2\} = \{(n, 0), (n, 1) \ldots, (b, 1), (b, 2)\}$. The value function can be written in vector notation as $v = (v_0^n, v_1^n, v_2^n, v_0^b, v_1^b, v_2^b)'$. For this configuration of states, monetary policy implies two transition matrices. The inflation matrix P is

$$P = \begin{bmatrix} \Pi & \mathbf{0}_3 \\ \mathbf{0}_3 & \Pi \end{bmatrix}, \tag{C.1}$$

where

$$\Pi = \begin{pmatrix} 1 & 0 & 0 \\ \pi & 1-\pi & 0 \\ \pi^2 & 2\pi(1-\pi) & (1-\pi)^2 \end{pmatrix} \tag{C.2}$$

and $\mathbf{0}_3$ is a 3×3 matrix of zeros. Money transfers imply the following matrix

$$T = \begin{bmatrix} \Psi^n & \mathbf{0}_3 \\ \mathbf{0}_3 & \Psi^b \end{bmatrix} \tag{C.3}$$

where

$$\Psi^k = \begin{pmatrix} 1-\tau^k & \tau^k & 0 \\ 0 & 1-\tau^k & \tau^k \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{for } k \in \{b, n\}. \tag{C.4}$$

For occupation shocks, let

$$\Lambda = \begin{pmatrix} \frac{1+\rho}{2} & \frac{1-\rho}{2} \\ 1-\rho & \rho \end{pmatrix}. \tag{C.5}$$

The transition matrix generated by occupation shocks can be written as $S = \Lambda \otimes \mathbf{I}_3$, where $\mathbf{I}_3$ is the 3×3 identity matrix and $\otimes$ represents the Kronecker product.

Now, let $\mathcal{I}^n = [\mathbf{I}_3, \mathbf{0}_3]$ and $\mathcal{I}^b = [\mathbf{0}_3, \mathbf{I}_3]$, then we can write the matrices A^n and A^b described in the text as:

$$A^n = \mathcal{I}^n PTS \tag{C.6}$$
$$A^b = \mathcal{I}^b PTS. \tag{C.7}$$

Finally, let $\mathbf{e}_k$ be a canonical vector in direction k of $\mathbb{R}^3$, $\mathbf{f}_i^n = [\mathbf{e}_{i+1}, (0,0,0)]$ and $\mathbf{f}_i^b = [(0,0,0), \mathbf{e}_{i+1}]$. Then,

$$A_{0i}^n = \mathbf{f}_i^n PTS \tag{C.8}$$
$$A_{0i}^b = \mathbf{f}_i^b PTS. \tag{C.9}$$

If we denote by $\sigma^m(i,j)$ the joint probability that after-meeting holdings of the producer-consumer pair is precisely (i,j) then

$$\lambda_1^i(m) = \sum_j \sigma^m(i,j) \tag{C.10}$$
$$\lambda_3^j(m) = \sum_i \sigma^m(i,j) \tag{C.11}$$
$$\lambda_2^k(m) = \sum_{(i,j):i+j+k=\bar{m}} \sigma^m(i,j) \tag{C.12}$$

where $\bar{m} = \sum_\ell m_\ell$.

Now we rewrite participation constraints as

$$\Pi_1(m) = -y(m) + \beta \sum_i \lambda_1^i(m)(\mathbf{f}_i^n - \mathbf{f}_{m_1}^n) PTSv \geq 0 \tag{C.13}$$

$$\Pi_2(m) = \beta \sum_k \lambda_2^k(m)(\mathbf{f}_k^b - \mathbf{f}_{m_2}^b) PTSv \geq 0 \tag{C.14}$$

$$\Pi_3(m) = u(y(m)) + \beta \sum_j \lambda_3^j(m)(\mathbf{f}_j^n - \mathbf{f}_{m_3}^n) PTSv \geq 0 \tag{C.15}$$

and, using σ,

$$\Pi_1(m) = -y(m) + \beta \sum_i \sum_j \sigma^m(i,j)(\mathbf{f}_j^n - \mathbf{f}_{m_1}^n) PTSv \geq 0 \tag{C.16}$$

$$\Pi_2(m) = \beta \sum_k \sum_{(i,j):i+j+k=\bar{m}} \sigma^m(i,j)(\mathbf{f}_k^b - \mathbf{f}_{m_2}^b) PTSv \geq 0 \tag{C.17}$$

$$\Pi_3(m) = u(y(m)) + \beta \sum_j \sum_i \sigma^m(i,j)(\mathbf{f}_j^n - \mathbf{f}_{m_3}^n) PTSv \geq 0. \tag{C.18}$$

As a result, the problem defining the core in meeting m is

Problem 1.

$$\underset{y(m),\sigma^m}{\text{Max}} \ \Pi_3(m)$$

$$\textit{s.t. } \Pi_1(m) \geq \gamma_1(m) \textit{ and } \Pi_2(m) \geq \gamma_2(m).$$

for some (meeting-specific) $\gamma_1(m)$ *and* $\gamma_2(m)$ *consistent with participation constraints.*

Let $\zeta_1(m)$ and $\zeta_2(m)$ be Lagrange multipliers associated to the restrictions in the problem above. It is easy to see that $\zeta_1(m) = u'(y(m))$. Also, let $L_{ij}(m)$ denote the derivative of the Lagrangian with respect to $\sigma^m(i,j)$. As a consequence of the linearity of Π's in σ's, the solution must satisfy

$$\sigma^m(i,j)\left(\underset{i',j'}{\text{Max}}\, L_{i'j'}(m) - L_{ij}(m)\right) = 0, \qquad \forall i,j. \tag{C.19}$$

The numerical problem is to find allocations maximizing (1.4) subject to constraints dictated by rationality, stationarity, core and feasibility, given value-function definitions and bounds on money holdings. In particular, there are bounds necessary to guarantee that measures of people across states add up to one, and transition probabilities defined by lotteries

also add up to one, so that in the outside-money case money is not created nor destroyed in meetings.

Our approach is to guess and verify that value functions are increasing and concave (that is, $0 \leq v_0^k < v_1^k < v_2^k$ and $v_2^k - v_1^k < v_1^k - v_0^k$ for $k = n, b$). We also restrict lotteries associated to people with intermediation occupations, due to incentive constraints, and transform (C.19) into inequality constraints. This way the numerical problem fits into conventional non-linear maximization routines. We then resort to the *KNITRO* solver. Issues related to local optima are handled by considering of many alternative initial conditions.

We now resort to an example of how (C.19) is handled, and how some lotteries can be eliminated in the outside-money case. Let us fix $\tilde{m} = (0, 2, 1)$. Since the consumer never ends with two units of money, we put $\sigma^{\tilde{m}}(i, 2) = 0$ for $i \in \{0, 1, 2\}$. Also, we can impose $\sigma^{\tilde{m}}(0, 0) = 0$, since money cannot be destroyed. In addition, the intermediary would not entertain an allocation with less than two units after trade, so that $\sigma^{\tilde{m}}(2, 0) = \sigma^{\tilde{m}}(1, 1) = \sigma^{\tilde{m}}(2, 1) = 0$. It remains to be determined just two transition probabilities for this meeting, that is, choices of $\sigma^{\tilde{m}}(1, 0)$ and $\sigma^{\tilde{m}}(0, 1)$. Hence we can write

$$\begin{aligned} L_{01}(\tilde{m}) &= (\mathbf{f}_1^n - \mathbf{f}_1^n)PTSv + (\mathbf{f}_1^n - \mathbf{f}_1^n)PTSv = 0. \\ L_{10}(\tilde{m}) &= (\mathbf{f}_0^n - \mathbf{f}_1^n)PTSv + \zeta_1(\tilde{m})(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSv \\ &= (\mathbf{f}_0^n - \mathbf{f}_1^n)PTSv + u'(y(\tilde{m}))(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSv. \end{aligned} \tag{C.20}$$

Given that $\sigma^{\tilde{m}}(1, 0) + \sigma^{\tilde{m}}(0, 1) = 1$, the core constraint for meeting $\tilde{m}$ becomes

$$\begin{aligned} \left(u'(y(\tilde{m})) - \frac{(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSv}{(\mathbf{f}_1^n - \mathbf{f}_0^n)PTSv} \right) \sigma^{\tilde{m}}(1, 0) &\geq 0 \\ \Leftrightarrow \left(u'(y(\tilde{m})) - 1 \right) \sigma^{\tilde{m}}(1, 0) &\geq 0. \end{aligned} \tag{C.21}$$

Appendix D

Kuhn-Tucker

We will work, as in numerical simulation, with $c(y) = y$. Also, the subscript (ij) for output, y, and payments, λ, are omitted.

Kuhn-Tucker theorem can give us necessary and sufficient conditions to problem 3.5.

It is easy to verify that problem 3.5 satisfies Kuhn-Tucker conditions.[1]

Then, the finite version of Kuhn-Tucker theorem asserts that for the maximum of the constrained problem occurs in (y_0, λ_0), is necessary and sufficient that exist a non-negative Lagrangian multiplier, μ, such that:

$$(y_0, \lambda_0) \quad \in \quad \underset{(y,\lambda)\in R\times\Delta}{argmax} \quad \mathcal{L}(y, \lambda; \mu)$$

and

$$\mu \left[-y_0 + \beta \sum_k \lambda_0^k (e_{i+k} - e_i) DV' \right] = 0$$

where, the Lagrangian is given by:

$$\mathcal{L}(y, \lambda; \mu) = u(y) + \beta \sum_k \lambda^k (e_j - e_{j-k}) DV' + \mu \left[-y + \beta \sum_k \lambda^k (e_{i+k} - e_i) DV' \right]$$

Then, as the Lagrangian is concave in y, we must have:

$$u'(y_0) = \mu \tag{D.1}$$

[1] Concave objective function, convex constraints and Slater's condition.

Besides, note that the Lagrangian is linear in λ^k. Therefore, as λ is in a simplex, in order to maximize the Lagrangian, we must put positive mass only in the direction that has higher derivative. In other words, using D.1, we must have:

$$\lambda^k(\max_s\{\mathcal{L}_s\} - \mathcal{L}_k) = 0 \qquad \text{for all} \quad k \tag{D.2}$$

where, $\mathcal{L}_k = \frac{d\mathcal{L}}{d\lambda^k} = \beta\left[(e_{i+k} - e_i)u'(y_0) - (e_j - e_{j-k})\right]DV'$.

Therefore, a necessary and sufficient condition to problem 3.5 is given by equation D.2.

Appendix E

Kuhn-Tucker with liquidity constraint

As in appendix D, the necessary and sufficient condition comes from Kuhn-Tucker.

$$\lambda^k(\max_s\{\mathcal{L}_s\} - \mathcal{L}_k) = 0 \qquad \text{for all} \quad k > 0 \tag{E.1}$$

$$\text{and} \tag{E.2}$$

$$\left(\xi - \sum_{s\neq 0} \lambda^s\right)(\max_s\{\mathcal{L}_s\} - 0) = 0 \tag{E.3}$$

The difference is because only the liquidy portion of λ lies in a simplex.

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