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# Competition in Cascades\*

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## Abstract

Hydroelectric generation is the main source of energy production in many countries. When firms operate in the same river, or in cascades, the output of an upstream firm is the input of its downstream rival. We build a dynamic stochastic duopoly model of competition in cascades and show that the decentralized market is efficient at the critical times when rain is infrequent, but inefficient when rain is more frequent. Market power is an issue when peak prices are sufficiently higher than off-peak prices: Upstream firms delay production in off-peak times, limiting their rival downstream generators' production in peak times.

*JEL classification* : D24; L13; L94

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## 1 Introduction

In many countries, the electricity sector is highly regulated and often centralized. Market design in this sector is complicated because of the specificities of the electricity market. In part, this is due to the specific characteristics of hydro-technology, mainly the uncertainty related to input availability and production coordination. Efficient hydroelectric production calls for coordination of power plants located in cascades on the same river. Since they all use water from the river as an input, they must coordinate to maximize production. A commonly held belief is that a decentralized market fails to achieve these coordination gains. Worse, upstream plants could exercise unilateral market power by withholding production and forcing downstream plants not to produce as well.

This paper provides a model of a decentralized market in which generators compete in cascades; and we address the issues of market failure and inefficiency in hydroelectric markets. Our model provides a benchmark with which we can understand some of the mechanisms in this market and gain intuition about when the market failures are mild and when they are not. For example, we

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address the coordination issue and show that in critical times, when water is scarce, generators have the correct incentives and behave as a central planner would prescribe. However, frequent rain results in an efficiency loss. Second, we address the issue of market power in these markets and show that the exercise of unilateral market power occurs only when peak prices are sufficiently higher than off-peak prices.

Electricity production usually relies on natural resources such as water, oil, natural gas, sunlight, wind, and uranium. Different endowments of such resources lead countries to use different mixes of production technologies, which defines the power industry configuration and structure. Despite these differences, with slightly varying intensity across locations, the electricity industry is heavily regulated. Even countries that have undertaken broad liberalization reforms regulate their industries. Countries such as Brazil, Russia, China, India, and Venezuela, among others, with a large share of hydroelectric production, have centralized dispatch of generators. In some specific cases, like Brazil, Canada, and Norway, hydro-generation is the most important source of energy production. Indeed, in provinces such as British Columbia, Manitoba, and Quebec, more than 90% of the electricity comes from hydro-generation. The Nordpool market is characterized by the importance of hydroelectric generation, coming mostly from Norway, which competes with other technologies more prevalent in the other Nordic countries.

We can highlight three important features of hydro-production here. First, the input (water) of hydro-production depends on rain and on each firm's reservoir, and the cost of transport is prohibitively large, making water essentially a non-marketable input. Second, in many cases, generators operate in the same river, leading to a production in cascades. In Quebec, for example, the La Grande River has nine generators located in it and British Columbia's Peace River and Columbia River also have important generators located in cascades. In Southeast Brazil, up to 19 large-scale generators are in cascades in the Rio Grande and Parana River. In such cases, the output of an upstream generator is (or will be) the input of a downstream generator located in the same river. Thus, a decentralized market might imply large market power to upstream firms. Third, some generators have large reservoirs and their production decisions in a free market are part of a long-term strategy, while other generators have very small or no reservoirs so they cannot retain water for future use. These generators are known as run-of-river generators.<sup>1</sup>

A closer inspection of some markets where hydro power is a major source, Brazil, Canada's Quebec and New Zealand shows that there is no situation where generators located on the same river compete to sell energy. In Brazil there is centralized dispatch, so generators have no say in

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<sup>1</sup>In Norway, for example, run-of-river hydro generation accounts for 30% of annual productions (Førsund [6]).

production decisions.(see Brazil’s ONS).<sup>2</sup> In Quebec’s La Grande river, all power plants are state owned (see Hydro-Quebec [11]). In New Zealand, the set of plants located in any river belongs to the same owner (see NZ Electricity Authority).<sup>3</sup> We believe these market arrangements are a consequence of the belief that competition is not possible, or at least inefficient, among plants located in cascade on the same river.

In this paper, we construct a simplified theoretical model of generators competing in cascades to address the argument that decentralized markets are inherently inefficient in hydroelectric production. Specifically, we build a dynamic stochastic duopoly model using two firms operating in the same river. Our model is an adaptation of Garcia, Reitzes, and Stachetti [7] to the context of competition in cascades. We start with two firms, each with a unit capacity reservoir. Water follows a simple stationary stochastic process in which at every period there is a constant probability that it rains at the upstream generator. Energy is modelled as an indivisible unit and can be sold or stored for the following period if the reservoir is not yet full. In our benchmark model, each firm can hold only one unit of energy at a time.<sup>4</sup>

Demand is assumed to be unitary and constant over time. Thus, a firm with a full reservoir can sell at a monopoly price when its competitor has an empty reservoir, or they can compete in prices, a la Bertrand, when both have a full reservoir. Despite having a zero marginal cost, there is a future value in saving water, so prices never drop to zero. Because of the inelastic demand assumption, efficiency simply means that supply meets demand. Our model does not account for deadweight loss due to higher prices.<sup>5</sup>

Our model is meant to capture the operation of hydroelectric generators in cascade, but with a focus on the fact that the upstream generator’s output will become the downstream firm’s input. We remark here that some important features of actual hydroelectric operation are not incorporated in our model, such as joint-ownership, state-ownership, and environmental regulation mandating river flows. Additionally, there is a long discussion on productivity: any generator is more productive if it has more water stored (the downfall is higher and this generates more energy). Thus, while our theoretical exercise is meant to capture relevant features of hydroelectric production, our results

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<sup>2</sup><http://www.ons.org.br/>

<sup>3</sup><http://www.emi.ea.govt.nz/>

<sup>4</sup>This is, of course, a great simplification, but it allows us to focus on the strategic issues involved in the output-input feature of competition in cascades. For rivers with generators with large reservoirs, the cascade problem is less interesting, since firms have water most of the time, but the analysis in this paper provides intuition for the moments in which these reservoirs are nearly full.

<sup>5</sup>A discrete demand is a simplification of our analysis, just like in Garcia et al. [7]. One could think about relaxing the discrete assumption by using the approach by von de Fehr and Harbord [19], but this is beyond the scope of our analysis.

must be taken with caution.

We derive policy implications from the analysis. First, efficiency implies that the downstream firm should deploy its reservoir in the competitive state. Second, we show that the downstream firm's reservoirs might impair efficiency. This is due to the downstream firm's strategic use of the reservoir, forcing the upstream firm to sell in competitive states when the optimal is for the downstream firm to sell. This provides an argument in favor of constructing run-of-river generators downstream.

Finally, a widespread argument against a decentralized market is that upstream firms would exercise market power and hold water while waiting for periods with higher prices. To study this question, we extend our benchmark model to include peak and off-peak periods. We show that this is a valid concern: When peak prices are sufficiently higher than off-peak prices, it might be optimal for the upstream firm to hold water and wait for higher energy prices.

The setting presented in this paper, where the output of one firm is the input of its rival and with non-marketable and stochastic inputs, is special to the problem at hand, that is, hydro-competitors in cascades. However, this rather special environment shares certain features with other markets. Markets with important secondhand markets, like the housing market where existing houses compete with newly built houses, can benefit from insights offered in this paper. Markets in which recycling (e.g., of paper) is relevant also fall into this setup. A market in which information is the final product also shares this feature since the output of a producer (information) is also the input of a seller.

There is a large literature that analyzes electricity markets. Centralized electricity production has received many criticisms for quite some time now. Joskow and Schmalensee's [12] pioneering work pointed to efficiency problems in centralized systems and advocated for full liberalization and competition. Several important contributions such as Green and Newbery [10] and Wilson [20], among many others, followed, dealing mainly with electricity market design issues. Castro et al. [4] discusses the Illinois experience, while Castro et al. [5] the Colombian one.

Another part of the literature focuses on hydroelectric generation. Scott and Read [18] analyze a model of the New Zealand electricity market where there are hydro and thermal generators competing a la Cournot. Crampes and Moreaux [3] compute the first best, monopoly and duopoly allocation of a market with a thermal and a hydro plant. Genc and Thille [9] analyze a dynamic game between a hydro producer and a thermal one in a decentralized market. Garcia, Reitzes and Stacchetti [7] analyze competition between two hydroelectric generators. In this paper, we build on

their set up to analyze competition when plants are located on the same river. Ambec and Doucet [2] analyze decentralization of a hydroelectric industry and show that while a monopoly brings market power concerns, a decentralized market may have suboptimal use of water resources. Gene and Thille [8] look at investment behavior when there both hydro and thermal plants competing. Ambec and Crampes [1] studies the decentralization of electricity production in the presence of intermittent sources. They also discuss policy instruments to achieve the first and second best outcomes in these environments.

Moita [15] analyzes the entry problem of a hydro generator and empirically shows that the cascade effect matters on entry decisions in rivers with many plants.

Rangel [17] is the first theory paper to focus on the cascade problem. He builds a duopoly competition in cascades in which firms compete in quantity, Cournot and Stackelberg, and shows that the upstream firm produces less than if its output did not supply the rival's input. He was the first to show that market power is more likely on off-peak periods, a result that we were also able to obtain in our dynamic horizon cascade game.

This topic has been studied in the engineering literature. Marques, Cicogna and Soares [14] numerically estimate the benefits of production coordination when plants operate in cascade. They compare a centralized maximization of production with a decentralized one, where each plant maximizes profit in a market. They find that the benefits of coordination are small. Barroso et al. [13] also compare centralized versus decentralized production, but do not explicitly model plants located in cascade.

The paper proceeds as follows. The next section lays out the model and derive the main results concerning equilibrium and efficiency of this type of electricity market. Section 3 modifies the model to account for a more extreme form of market power that may arise. The last section concludes.

## 2 Model

Consider an infinite horizon duopoly  $\{A, B\}$ , in which firms discount profits at rate  $\beta < 1$ . Assume that firm  $A$  is upstream, whereas  $B$  is downstream. At each period  $t = 1, 2, \dots$ , a hydro generator might have full capacity or empty capacity, which we will denote by the binary variable  $\{0, 1\}$ . We assume an inelastic unitary demand at each period and assume that the reservation price of consumers is given by  $p^*$ . At every period, if a seller has a full reservoir and is the only seller with a full reservoir, it might sell it and earn  $p^*$ . On the other hand, if both firms have a full reservoir, they will enter a Bertrand competition and only one of them will sell its unit of water, converted

to energy. Thus, energy is modelled as an indivisible unit. If a firm sells its unit for a given price, it earns the price but will finish the period with empty reservoirs. If it does not sell energy, it will enter the following period with full capacity. With these assumptions, we can focus on the extensive margin of the energy production.

When it rains, it rains one unit only at  $A$ . Rain happens every period with probability  $\pi$ . The set of states of the world,  $S$ , is based on reservoirs being empty (0) or full (1):  $S = \{(0,0), (0,1), (1,0), (1,1)\}$ . Let  $s$  be a typical element of  $S$  in which its first element represents  $A$ 's reservoir, while  $B$ 's reservoir is represented by the second element. An important feature of this upstream-downstream model is that whenever firm  $A$  sells energy, it fills  $B$ 's reservoir. For simplicity, we assume that each firm can hold at most 1 unit of water and any extra water is wasted at zero cost (and zero benefit).

Therefore, we can summarize the timing of each period as follows: (1) one of the two firms sells energy (as a monopolist if  $s \in \{(1,0), (0,1)\}$  or they enter a Bertrand competition if  $s = (1,1)$ ); (2) reservoirs are updated in two steps: first, the firm that sold energy has an empty reservoir. Second, if  $A$  sold energy, its water goes to hydro producer  $B$ . If  $B$ 's reservoir was already full, the water from  $A$  is lost, but if  $B$ 's reservoir was empty, it now becomes full; (3) rain happens with probability  $\pi$ : if  $A$  had an empty reservoir, it becomes full with the rain, if  $A$  had a full reservoir, the water from the rain goes to  $B$ , which will retain it if it was empty, but will spill it if it was already full.

## 2.1 Central Planner

The planner incurs a welfare cost of deficit which we will label by  $c$ . This cost might be interpreted as the deadweight loss of not serving the consumers in a given period. The optimal policy is such that the planner serves the market whenever there is water available. That is, at states  $(0,1)$  and  $(1,0)$ , the monopolist serves the market and at state  $(1,1)$ , the market is served by one of the two firms. Let us look at each one the two possible options, when  $A$  serves at the competitive state and when  $B$  serves at that state.

Note that due to the Markovian nature of the stochastic process, the problem can be written in a simple recursive way. It will be convenient to write the present-value of the total cost of the central planner as a continuation value function for each of the states. That is, if  $A$  serves the market in the competitive state  $(1,1)$ , we have the following expected continuation Bellman cost functions



for the states where (i) no one has water, (ii)  $A$  is a monopolist, and (iii)  $B$  is a monopolist:

$$\begin{aligned} C(0,0) &= c + \beta(\pi C(1,0) + (1-\pi)C(0,0)) \\ C(1,0) &= \beta(\pi C(1,1) + (1-\pi)C(0,1)) \\ C(0,1) &= \beta(\pi C(1,0) + (1-\pi)C(0,0)), \end{aligned} \tag{1}$$

for the competitive state we have:

$$C^A(1,1) = \beta(\pi C(1,1) + (1-\pi)C(0,1)), \tag{2}$$

where, with slight abuse of notation, we write  $C^A(1,1)$  as the cost function at state  $(1,1)$  in the policy where  $A$  sells at that state. Similarly, for the policy in which  $B$  sells at the competitive state, we have the first same three equations given by (1) and the following cost function for the competitive state

$$C^B(1,1) = \beta(\pi C(1,1) + (1-\pi)C(1,0)), \tag{3}$$

where  $C^B(1,1)$  is the cost function at state  $(1,1)$  in this policy where  $B$  sells at that state.

**Proposition 1 (Planner's Solution)** *The optimal solution for the central planner is to deploy  $B$ 's reservoir in the competitive state.*

**Proof.** The system of equations (1) and (2) implies that  $C(0,0)$  is the highest expected continuation cost among the four different continuation cost functions. That is:  $C(0,0) > C(s)$ ,  $\forall s \in S \setminus (0,0)$ .

Fix the policy in the monopoly states to be such that the monopolist sells at some price  $p$ . Let us show that if the policy is such that the planner deploys  $A$ 's reservoir in the competitive state, then the planner will have an incentive to deviate. If the planner follows the prescribed policy, it will have an expected cost at the competitive state of

$$\beta(\pi C(1,1) + (1-\pi)C(0,1)),$$

whereas if it deviates, it will have an expected cost given by

$$\beta(\pi C(1,1) + (1-\pi)C(1,0))$$

We will prove the claim that deviating in the competitive state is better for the planner, that is:

$$\pi C(1,1) + (1-\pi)C(1,0) \leq \pi C(1,1) + (1-\pi)C(0,1) \tag{4}$$

Thus, in order to show that this claim is true it suffices to show that

$$C(1, 0) \leq C(0, 1) \quad (5)$$

Given that we have fixed the policy at all states, the relation (5) holds if and only if:

$$\pi C(1, 1) + (1 - \pi) C(0, 1) \leq \pi C(1, 0) + (1 - \pi) C(0, 0)$$

We know that  $C(0, 0) > C(0, 1)$ , so in order to prove our claim (4) it suffices to show that  $C(1, 1) \leq C(1, 0)$ , which is a very intuitive result since in one case there is water in both reservoirs, whereas in the other case there is water only in  $A$ 's reservoir. Since we are analyzing the policy in which  $A$ 's reservoir is deployed in the competitive state,  $A$  will have its reservoir empty and the water discarded will not be useful to  $B$ , since  $B$  is already full. Thus, under this policy, both states have the same expected continuation costs. For completeness, let us show the result. Since  $C(1, 1) = \beta(\pi C(1, 1) + (1 - \pi) C(0, 1))$  and  $C(1, 0) = \beta(\pi C(1, 1) + (1 - \pi) C(0, 1))$ , we have that  $C(1, 1) = C(1, 0)$ , as we wanted to show. Thus, we know that the policy in which  $A$  sells in the competitive state cannot be optimal. By the one-step-deviation property, the policy in which  $B$  sells at the competitive state is the optimal one. ■

## 2.2 Decentralized Market

Consider the same market as above, but in which the two firms face a Bertrand competition every period. The game has complete and public information, so a history of the game at period  $t$  is defined as  $h_t$  where  $h_t$  specifies the sequence of prices, states of the world and the identity of the seller for each period  $\tau \leq t - 1$ . The set  $\mathcal{H}_t$  is the set of all histories at period  $t$  and the set of histories in the game is  $\mathcal{H} = \cup_{t=1}^{\infty} \mathcal{H}_t$ . A strategy for each player is a map from the history  $h \in \mathcal{H}$  to the set of prices. Denote by  $\Sigma_i$  the set of all possible strategies for firm  $i$ .

We will concentrate on equilibria with Markov strategies in which whenever a firm is a monopolist, it sells energy at the maximum possible price, which we will denote by  $p^*$ . Thus, a Markov strategy is a map from the state  $s \in S$  to a price  $p \in [0, \infty)$ , that is,

$$\sigma_i : S \rightarrow [0, \infty), i = A, B.$$

The payoff for each firm in a given period is either zero, when it does not sell, or the price for which its energy is sold. Firms discount their payoffs using a discount rate  $\beta < 1$ . The expected continuation payoff for firm  $i$  starting at some history  $h$  and given that firms are using the strategy profile  $(\sigma_i, \sigma_{-i})$  is denoted by  $V^i(\sigma_i, \sigma_{-i}|h)$ .

**Definition 1 (Markov Perfect Equilibrium)** *A Markov Perfect Equilibrium in this game is a Markov strategy profile  $\sigma = (\sigma_A, \sigma_B)$  such that  $V^i(\sigma_A, \sigma_B|h) \geq V^i(\sigma'_i, \sigma_{-i}|h)$ ,  $\forall \sigma'_i \in \Sigma_i, i = A, B$ ,  $\forall h \in \mathcal{H}$ .*

We abuse notation and write the value function as a function of the states only, but keep in mind that the players are using equilibrium strategies. Thus, we write  $V^i(s)$  to be generator  $i$ 's expected continuation payoff at any history  $h$  where the state of the world is  $s$ . The expected continuation values of firm A for each state of the economy can be written as:

$$\begin{aligned} V^A(0,0) &= \beta (\pi V^A(1,0) + (1-\pi) V^A(0,0)) \\ V^A(1,0) &= p^* + \beta (\pi V^A(1,1) + (1-\pi) V^A(0,1)) \\ V^A(0,1) &= \beta (\pi V^A(1,0) + (1-\pi) V^A(0,0)) \\ V^A(1,1) &= \max \left\{ \begin{array}{l} p_{1,1} + \beta (\pi V^A(1,1) + (1-\pi) V^A(0,1)) \\ \beta (\pi V^A(1,1) + (1-\pi) V^A(1,0)) \end{array} \right\} \end{aligned}$$

Similarly, firm B, has the following value functions:

$$\begin{aligned} V^B(0,0) &= \beta (\pi V^B(1,0) + (1-\pi) V^B(0,0)) \\ V^B(0,1) &= p^* + \beta (\pi V^B(1,0) + (1-\pi) V^B(0,0)) \\ V^B(1,0) &= \beta (\pi V^B(1,1) + (1-\pi) V^B(0,1)) \\ V^B(1,1) &= \max \left\{ \begin{array}{l} p_{1,1} + \beta (\pi V^B(1,1) + (1-\pi) V^B(1,0)) \\ \beta (\pi V^B(1,1) + (1-\pi) V^B(0,1)) \end{array} \right\} \end{aligned}$$

In this economy, the maximum that a firm might receive in any given period is the monopoly price  $p^*$ . In any equilibrium it must be the case that whenever a firm is a monopolist in the period, it will find it optimal to sell and receive the price  $p^*$ . Thus, equilibria can only differ in how players behave in the competitive states. At that state, they play a Bertrand game, but unlike a static Bertrand where the equilibrium price would be given by the highest marginal cost between the two firms, here the price at the competitive state, which we denote by  $p_{1,1}$ , is given by the highest reservation value of waiting between the two firms. Therefore, unless both firms have identical continuation values for waiting, there should be a clear winner in the competitive state. This is what we show in the following proposition.

**Proposition 2 (Generically Unique Equilibrium)** *For any given set of parameters, with  $\pi \in (0,1)$ , the Markov Perfect Equilibrium set is generically unique. This equilibrium is such that A sells when it is a monopolist,  $s = (1,0)$ , B sells when it is a monopolist,  $s = (0,1)$ , and depending on the parameters, either A sells or B sells at state  $s = (1,1)$ .*

**Proof.** Solving the system of equations under each of the two proposed equilibria, we have the following. When  $A$  sells at the competitive state, the equilibrium price is:

$$p_{1,1} = \beta p^* (1 - \pi) (1 - \beta(1 - \pi)) \quad (6)$$

With some algebra, it can be shown that the no-deviation conditions of the problem imply that this equilibrium only holds if:

$$-\beta^2 p^* (\pi - 1) (\beta^2 \pi^3 - 2 \beta^2 \pi^2 + \beta^2 \pi + \beta \pi^2 - \beta \pi + 2 \pi - 1) \geq 0 \quad (7)$$

For the equilibrium in which  $B$  sells, the equilibrium price is:

$$p_{1,1} = \frac{\beta p^* (1 - \beta \pi) (1 - \pi)}{1 - \beta^2 \pi (1 - \pi)} \quad (8)$$

The condition for this equilibrium is:

$$\beta^2 p^* (\pi - 1) (\beta^2 \pi^3 - 2 \beta^2 \pi^2 + \beta^2 \pi + \beta \pi^2 - \beta \pi + 2 \pi - 1) \geq 0 \quad (9)$$

Given that  $\pi \in (0, 1)$  and  $\beta > 0, p^* > 0$ , conditions (7) and (9) can only hold both at the same time if the polynomial expression  $\beta^2 \pi^3 - 2 \beta^2 \pi^2 + \beta^2 \pi + \beta \pi^2 - \beta \pi + 2 \pi - 1 = 0$ . For any given  $\beta$ , there will only be at most three values of  $\pi$  such that both conditions hold. Thus, we have that there is a generically unique equilibrium in Markov strategies. In this equilibrium, either  $A$  sells at the competitive state or  $B$  sells. In the special cases where  $\beta$  and  $\pi$  are such that expressions (7) and (9) are both zero, we have both equilibria. ■

As we discussed above, at the competitive state  $s = (1, 1)$ , there is a Bertrand competition between the two firms. While the marginal cost of water is zero, there is a shadow value given by the expected future price that a firm can get by saving water. Thus, depending on the parameters one firm or the other will be more willing to sell at the current competitive state rather than waiting.

**Proposition 3** *There exists  $\bar{\pi}$  and  $\underline{\pi}$  with  $0 < \underline{\pi} < \bar{\pi} < 1$ , such that  $\forall \pi \geq \bar{\pi}$  the only equilibrium in Markov strategies is such that firm  $A$  sells at state  $s = (1, 1)$  and  $\forall \pi \leq \underline{\pi}$  the only equilibrium in Markov strategies is such that firm  $B$  sells at state  $s = (1, 1)$ .<sup>6</sup>*

**Proof.** As we have shown in the previous proposition the equilibrium depends on which of the

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<sup>6</sup>For  $\pi \in (\underline{\pi}, \bar{\pi})$ , the equilibrium depends on the  $\beta$  as well.

two conditions (7) and (9) hold. Thus, the equilibrium in which  $A$  sells happens when

$$\begin{aligned}
\beta^2 \pi^3 - 2 \beta^2 \pi^2 + \beta^2 \pi + \beta \pi^2 - \beta \pi + 2 \pi - 1 &\geq 0 \\
\beta^2 (\pi^3 - 2 \pi^2 + \pi) + \beta (\pi^2 - \pi) + 2 \pi - 1 &\geq 0 \\
\beta^2 \pi (\pi^2 - 2 \pi + 1) + \beta \pi (\pi - 1) + 2 \pi - 1 &\geq 0 \\
\beta^2 \pi (\pi - 1)^2 + \beta \pi (\pi - 1) + 2 \pi - 1 &\geq 0
\end{aligned}$$

This condition is satisfied whenever  $\pi > \frac{5}{8}$ , regardless of  $p^*$  and  $\beta$ , as we can see below.

$$\begin{aligned}
\beta^2 \pi (\pi - 1)^2 + \beta \pi (\pi - 1) + 2 \pi - 1 &\geq 0 \\
&\geq \beta \pi (\pi - 1) + 2 \pi - 1 \\
&\geq -\frac{1}{4} + 2 \pi - 1 \\
&= 2 \pi - \frac{5}{4}
\end{aligned}$$

Where the first inequality comes from the fact that  $\beta^2 \pi (\pi - 1)^2 > 0$  and the second inequality follows from the fact that  $\beta \pi (\pi - 1) \geq -\frac{1}{4}, \forall \beta$  and  $\forall \pi$ .

On the other hand, the opposite holds whenever  $\pi < \frac{1}{2}$ , regardless of the other parameters of the model.

$$\begin{aligned}
\beta^2 \pi (\pi - 1)^2 + \beta \pi (\pi - 1) + 2 \pi - 1 &\leq \beta^2 \pi (\pi - 1)^2 + \beta \pi (\pi - 1) \\
&= \beta \pi (\pi - 1) (\beta (\pi - 1) + 1) \\
&= \beta \pi (\pi - 1) (1 - \beta + \beta \pi) \\
&\leq 0.
\end{aligned}$$

■

We summarize our result: when the economy has a low probability of raining, the cascade problem is more significant, since the downstream firm is more dependent on the water coming from the upstream rival. In this case, we have shown that the market equilibrium coincides with the Planner's solution. However, when there is a high probability of raining, the equilibrium is inefficient. That is, the upstream firm deploys its reservoir when both have their reservoirs full. In our model, this implies that water is wasted with certainty. Given that this is a case where it rains with high probability, wasting water is not as dramatic as it would be in a world where water is scarce.

In the remainder of this section, we extend the benchmark model and look at the case where  $B$  is a run-of-river generator. That is, it does not have a reservoir. This version of the model is

meant to capture two features of the hydro producers: first, it is fairly common that downstream generators are indeed run-of-river, so this model helps us understand this situation, and second, with this model we add an insight about the optimal configuration of the electric system.

Formally, we will assume that  $B$  will produce whenever it has a reservoir of 1 and can't store water for the subsequent period. In our game this implies that  $B$  is a myopic player. Moreover, it also means that whenever both firms  $A$  and  $B$  have their reservoirs full, it is  $B$  who sells the water (at a price that makes  $A$  indifferent between selling and waiting). Using the proof of proposition 3, it is immediate that we now have only one equilibrium regardless of the parameters: one in which the monopolist sells water and at the competitive state  $B$  sells. We state this result below.

**Proposition 4 (Run-of-River: Efficiency)** *When  $A$  has a reservoir that stores water from one period to another, but  $B$  does not ( $B$  is run-of-river), the outcome of the decentralized market is efficient.*

This result tell us that run-of-river hydro producers might improve efficiency in a decentralized market. This result is stronger than the result in Moita and Monte [16] in which the authors showed that reservoirs might be unnecessary and a fraction of hydro generators may be replaced by run-of-river generators with no loss in welfare.

Of course, this result does not mean that it is necessarily better to sacrifice any reservoir that  $B$  might have. Instead, it highlights a mechanism that is present in competition is cascades that might otherwise be masqueraded by the overall result. In other words: on one hand, it is intuitive that a bigger reservoir improves efficiency (ignoring costs), but on the other hand, a downstream firm might hurt efficiency through the strategic use of its reservoir. Indeed, the latter effect (negative) might be the dominant one, as it is in our simple model illustrated here.

### 3 Market Power

One of the main concerns in a hypothetical decentralized market for hydroelectric energy is the possibility that the upstream firm might take advantage of the fact that it can save water so that the downstream firm cannot produce. The fear that an extreme version of this might take place in an open market is a cause of concern in discussions of market design in the sector. In our benchmark model this “excess market power” is not present. The reason is that, with a reservoir of one unit only, the upstream firm can gain nothing by waiting an extra period when it is a monopolist: if it rains, it loses monopoly and if it does not rain it is back to the same situation as before, where

it was a monopoly. Given that being a monopoly is the best outcome to a firm in the model, the firm should sell energy once it is in that state.

Thus, in order to capture this feature of “excess market power” we extend the benchmark model in two fronts: the downstream firm  $B$ , is run-of river and the upstream firm,  $A$ , has a reservoir capacity of 2. We also extend the model to include off-peak and peak periods, where prices are high (peak)  $\bar{p}$  or low (off-peak)  $\underline{p}$ . While these might be stochastic in reality, there is a clear seasonal and predictive component in the variation of peak and off-peak periods. We assume that this alternation is deterministic and that a peak period follows an off-peak period.

The model now has one strategic agent (the upstream firm, while the downstream firm sells whenever it has water) and twelve possible reservoir states. We summarize in the table below all the twelve states and the actions that are observed in any Markovian equilibrium. Two features are more straightforward:  $B$  sells whenever it has water and  $A$  sells whenever it is a monopolist in a peak period.

State of Reservoirs	Action Observed
$(2, 0)^{off}$	?
$(2, 0)^{peak}$	$A$ sells
$(2, 1)^{off}$	$B$ sells
$(2, 1)^{peak}$	$B$ sells
$(1, 0)^{off}$	?
$(1, 0)^{peak}$	$A$ sells
$(1, 1)^{off}$	$B$ sells
$(1, 1)^{peak}$	$B$ sells
$(0, 0)^{off}$	both wait
$(0, 0)^{peak}$	both wait
$(0, 1)^{off}$	$B$ sells
$(0, 1)^{peak}$	$B$ sells

Recall that both firms play a Bertrand game every period, so when  $B$  sells, the equilibrium price at the period is the one that makes  $A$  indifferent between selling and waiting. Given that  $B$ 's value of waiting is zero, it will always outbid  $A$ . The only two states that we are interested in are  $(2, 0)^{off}$  and  $(1, 0)^{off}$ . Let us look at the value functions at these states:

$$V(2, 0)^{off} = \max \left\{ \underline{p} + \beta \left( \pi V(2, 1)^{peak} + (1 - \pi) V(1, 1)^{peak} \right), \beta \left( \pi V(2, 1)^{peak} + (1 - \pi) V(2, 0)^{peak} \right) \right\}$$

Thus,  $A$  **will sell in state**  $(2, 0)^{off}$  if and only if:

$$\begin{aligned}\underline{p} + \beta \left( \pi V(2, 1)^{peak} + (1 - \pi) V(1, 1)^{peak} \right) &\geq \beta \left( \pi V(2, 1)^{peak} + (1 - \pi) V(2, 0)^{peak} \right) \\ \underline{p} + \beta (1 - \pi) V(1, 1)^{peak} &\geq \beta (1 - \pi) V(2, 0)^{peak}\end{aligned}$$

Which gives us the following equation:

$$\underline{p} \geq \beta (1 - \pi) \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) \quad (10)$$

The other important value function is:

$$V(1, 0)^{off} = \max \left\{ \underline{p} + \beta \left( \pi V(1, 1)^{peak} + (1 - \pi) V(0, 1)^{peak} \right), \beta \left( \pi V(2, 0)^{peak} + (1 - \pi) V(1, 0)^{peak} \right) \right\}$$

and  $A$  **will sell in state**  $(1, 0)^{off}$  if and only if:

$$\underline{p} + \beta \left( \pi V(1, 1)^{peak} + (1 - \pi) V(0, 1)^{peak} \right) \geq \beta \left( \pi V(2, 0)^{peak} + (1 - \pi) V(1, 0)^{peak} \right)$$

Which is equivalent to:

$$\underline{p} \geq \beta \pi \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) + \beta (1 - \pi) \left( V(1, 0)^{peak} - V(0, 1)^{peak} \right) \quad (11)$$

We will use conditions (10) and (11) to prove the following result.

**Proposition 5 (Extreme Strategic Market Power)** *An equilibrium in Markov strategies in which firm  $A$  does not sell energy in states  $(1, 0)^{off}$  and  $(2, 0)^{off}$  exists if and only if  $\underline{p} \leq \beta (1 - \pi) \bar{p}$ .*

**Proof.** First, let us write the value function for the states  $(2, 0)^{peak}$  and  $(1, 1)^{peak}$

$$\begin{aligned}V(2, 0)^{peak} &\geq \bar{p} + \beta \left( \pi V(2, 1)^{off} + (1 - \pi) V(1, 1)^{off} \right) \\ V(1, 1)^{peak} &= \beta \left( \pi V(2, 0)^{off} + (1 - \pi) V(1, 0)^{off} \right)\end{aligned}$$

We can re-write condition (10) as follows:

$$\begin{aligned}\underline{p} &\geq \beta (1 - \pi) \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) \\ &\geq \beta (1 - \pi) \left( \bar{p} + \beta \pi \left( V(2, 1)^{off} - V(2, 0)^{off} \right) + \beta (1 - \pi) \left( V(1, 1)^{off} - V(1, 0)^{off} \right) \right)\end{aligned}$$

Rearranging, we have that

$$\underline{p} \geq \beta (1 - \pi) \bar{p} + \beta^2 (1 - \pi) \left( \pi \left( V(2, 1)^{off} - V(2, 0)^{off} \right) + (1 - \pi) \left( V(1, 1)^{off} - V(1, 0)^{off} \right) \right) \quad (12)$$



( $\implies$ ) If such an equilibrium exists,  $V(2, 1)^{off} = V(2, 0)^{off}$  and  $V(1, 1)^{off} = V(1, 0)^{off}$ . Thus, condition (12), which is a necessary condition for  $A$  to sell in state  $(2, 0)^{off}$ , becomes:

$$\underline{p} \geq \beta(1 - \pi)\bar{p}$$

Given that  $A$  does not sell in state  $(2, 0)^{off}$ , it must be the case that  $\underline{p} \leq \beta(1 - \pi)\bar{p}$ .

( $\impliedby$ ) Suppose that  $\underline{p} < \beta(1 - \pi)\bar{p}$ . Let us show that an equilibrium in Markov strategies in which firm  $A$  does not sell energy in states  $(1, 0)^{off}$  and  $(2, 0)^{off}$  exists. Firm  $A$  does not sell in state  $(2, 0)^{off}$  if

$$\underline{p} \leq \beta(1 - \pi) \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) \quad (13)$$

However, we have that:

$$\begin{aligned} V(2, 0)^{peak} - V(1, 1)^{peak} &= \bar{p} + \beta \left( \pi V(2, 1)^{off} + (1 - \pi) V(1, 1)^{off} \right) - \beta \left( \pi V(2, 0)^{off} + (1 - \pi) V(1, 0)^{off} \right) \\ &= \bar{p} + \beta \pi \left( V(2, 1)^{off} - V(2, 0)^{off} \right) + \beta(1 - \pi) \left( V(1, 1)^{off} - V(1, 0)^{off} \right) \\ &\leq \bar{p} \end{aligned}$$

Therefore (13) becomes

$$\underline{p} \leq \beta(1 - \pi) \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) \leq \beta(1 - \pi)\bar{p} \quad (14)$$

which is true by assumption. Moreover, Firm  $A$  does not sell in state  $(1, 0)^{off}$  if

$$\underline{p} \leq \beta \pi \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) + \beta(1 - \pi) \left( V(1, 0)^{peak} - V(0, 1)^{peak} \right) \quad (15)$$

Also:

$$\begin{aligned} V(1, 0)^{peak} - V(0, 1)^{peak} &= \bar{p} + \beta \left( \pi V(1, 1)^{off} + (1 - \pi) V(0, 1)^{off} \right) - \beta \left( \pi V(1, 0)^{off} + (1 - \pi) V(0, 0)^{off} \right) \\ &\leq \bar{p} \end{aligned}$$

Thus, we have that

$$\begin{aligned} \underline{p} &\leq \beta \pi \left( V(2, 0)^{peak} - V(1, 1)^{peak} \right) + \beta(1 - \pi) \left( V(1, 0)^{peak} - V(0, 1)^{peak} \right) \\ &\leq \beta \pi \bar{p} + \beta(1 - \pi)\bar{p} = \beta \bar{p} \end{aligned}$$

and given that, by assumption,  $\underline{p} \leq \beta(1 - \pi)\bar{p} < \beta \bar{p}$ , we have the result. ■

When the upstream firm has a full reservoir on an off-peak period, when deciding to wait it must trade-off the fact that waiting might lead to monopoly in a peak period if it doesn't rain,

but might also lead to excess water that drains to the downstream firm, so that the upstream firm loses market power in the following peak period. On the other hand, if the upstream firm does not have a full reservoir in the off-peak period then waiting necessarily leads to a monopoly situation. Informally, this suggests that market power is more likely when the upstream firm's reservoir is not yet full (since there is no risk of spillage). Indeed, we prove the following result: in any equilibrium in which there is exercise of market power, it must be the case that the firm exerts this market power when its reservoir is not full.

**Remark 1** *Note that if the condition for market power is present (for example, a very high peak price or very low off-peak price) then it might be the case that the upstream firm always exerts market power. What we rule out is a situation in which the upstream firm sells energy in the off-peak period with full reservoir but not in the off-peak period with a reservoir only partially filled. Formally, there is no equilibrium in which firm A sells energy in state  $(2, 0)^{off}$ , but does not sell in state  $(1, 0)^{off}$ .<sup>7</sup>*

## 4 Conclusion

This paper develops and analyzes a model of generators competing in cascade.

The solution of the central planner problem shows that the way to minimize water loss is to produce with the downstream plant first. We show that the market equilibrium can be distinct from the social optimum. Two results are important here. First, there are situations where the upstream plant outbids its rival and produces energy. Second, and perhaps more importantly, this non optimal situation happens only when water is abundant. Thus, we have a positive result here: market competition leads to an efficient situation in critical times, when water is more scarce.

We next turned to the possibility that the upstream plant held production to wait for higher prices in the future. We constructed a model where there are peak and off-peak periods. We showed that it is indeed an equilibrium for the upstream plant to hold production. However, it can only happen in environments in which off-peak periods prices are much lower than peak prices. In such cases, the upstream generator has a high incentive to be a monopolist in peak periods.

In summary, the outcome of firms competing in cascade can differ from the social optimum and it is important to understand how demand varies over time, since this variance may or may not induce market power abuse.

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<sup>7</sup>The proof, which we omit, is by contradiction and involves solving a tentative equilibrium in which the firm sells in  $(2, 0)^{off}$  but not in  $(1, 0)^{off}$ .

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