FUNDAÇÃO GETULIO VARGAS
ESCOLA DE ECONOMIA DE SÃO PAULO

MARIA FERNANDA PETRI BETTO

DYNAMIC MARKETS FOR LEMONS

São Paulo
2017

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Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para obtenção do título de Mestre em Economia de Empresas

Campo de Conhecimento:
Microeconomia

Orientador: Prof. Dr. Braz Camargo

São Paulo

2017

Betto, Maria Fernanda Petri.
Dynamic markets for lemons / Maria Fernanda Petri Betto. - 2017.
23 f.

Orientador: Bráz Ministério de Camargo
Dissertação (mestrado) - Escola de Economia de São Paulo.

1. Limão. 2. Seleção adversa. 3. Microeconomia. 4. Equilíbrio econômico. I. Camargo, Bráz Ministério de. II. Dissertação (mestrado) - Escola de Economia de São Paulo. III. Título.

CDU 330.101.52

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Data de Aprovação:
___/___/______

Banca examinadora:

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AGRADECIMENTOS

Agradecimentos especiais ao Braz Camargo, pela orientação; aos demais professores da EESP, pelo apoio e sugestões; e aos membros integrantes da banca, pelos comentários valiosos.

Agradeço também à minha família e em especial à minha irmã, pelo incentivo e pela inspiração.

ABSTRACT

This paper investigates the role of signaling in a dynamic, decentralized market for lemons. I derive general properties of equilibria, and in particular of fully separating equilibria. The most efficient separating equilibrium allows for the trade of every type in every period, a feature that remains even when all agents are infinitely patient – unlike the market freezing result obtained by Moreno e Wooders (2016) in a similar context but without the possibility for signaling through prices.

Keywords: adverse selection. signaling. decentralized markets.

RESUMO

Este trabalho investiga o papel da sinalização em um mercado dinâmico de limões. Nós derivamos propriedades gerais dos equilíbrios, e em particular dos equilíbrios totalmente separantes. O equilíbrio separante mais eficiente permite que trocas ocorram em todos os períodos para tos os tipos, uma característica que permanece mesmo quado os agentes são infinitamente pacientes – ao contrário do resultado de congelamento de mercados obtido por Moreno e Wooders (2016) num contexto similiar, porém sem a possibilidade de sinalização via preços.

Palavras-chave: seleção adversa. sinalização. mercados descentralizados.

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1 Introduction

There exists by now a vast literature dedicated to extending Akerlof's model to a dynamic setting. Generally, these models comprise buyers and sellers meeting over time to trade a differentiated commodity whose quality is known only to its seller. It is typically assumed that the buyer – that is, the uninformed party – makes a take-it-or-leave-it offer (CAMARGO; LESTER, 2014; MORENO; WOODERS, 2010; MORENO; WOODERS, 2016), an assumption which precludes the potentiality that prices may act as signals of a goods' underlying quality. This feature, while frequently left unexplored, seems quite natural in the general context of adverse selection.

This paper aims to explore that possibility. Specifically, I analyze a decentralized lemons market which lasts for finitely many periods. I assume sellers can produce one unit of a commodity which can be of one of finitely many qualities. The information regarding a commodity's quality is privately held by the seller who produces it. In each period, agents are randomly and anonymously matched in pairs of a buyer and a seller each. Once matched, the seller makes a take-it-or-leave-it price offer to the buyer; if the price is accepted, both agents leave the market. Otherwise, the match is dissolved and buyer and seller stay for the next period (as long as there still is a next period). Agents who do not trade by the last period leave the market with zero gains.

This environment is very similar to the one found in Moreno e Wooders (2016). The main difference is that the authors follow the bulk of the literature in assuming the buyers are responsible for making the offers. I investigate what changes when sellers yield the bargaining power and use prices to convey information regarding quality.

In particular, when signaling is introduced into the dynamic model, an additional mechanism for distinguishing between types, other than time, is introduced. This becomes evident when we let frictions become small (discount factors approaching one), rendering ineffectual the use of time for separation purposes. With signaling, it remains possible for high-quality items to trade even when adverse selection is severe, while in the context of Moreno e Wooders (2016) the market freezes in (almost) every period.

The remainder of this paper is organized as follows. Section 1 contains a detailed description of the environment. Equilibria are subsequently defined in section 1. In Section 1, we derive basic properties of equilibria; as we shall see, a full characterization is difficult since in the context under study equilibria can come in many different flavors. Section 1 discusses briefly how pooling offers may arise in equilibrium in certain periods, and when they may not. In Section 1, the focus is on separating equilibria – those where signals are perfectly informative. We describe some of their properties and identify the most efficient among them. Subsection 6.1, contains an analysis of the effects of small frictions on the efficient separating equilibrium. A

discount factor close to one eliminates the possibility of using time as a screening device, but the signaling channel remains operable. Subsection 6.2 presents the effect of a large horizon on the probabilities of trade for the efficient separating equilibrium. Finally, Section 6.2 has some final remarks.

2 Environment

Time is discrete and the market lasts for $T < \infty$ periods. The discount factor between two periods is $\delta \in (0, 1)$.

The setting is a dynamic, decentralized market for an indivisible good, where in each period a continuum of buyers and sellers of equal mass meet for the purpose of trading. Each seller is typified by the quality θ of the unit of the good she can produce, and the set of seller types is given by $\Theta := \{1, \ldots, K\}$.

Entry occurs only once, before the market opens in $t = 1$. In the entering mass, buyers and sellers each have measure normalized to one. We let $(\beta_\theta^{T-t})_{t=1}^T$ and $(\gamma_\theta^{T-t})_{t=1}^T$ be sequences where β_θ^{T-t} and γ_θ^{T-t}, $t \in \{1, \ldots, T\}$, denote the measure and proportion, respectively, of type-θ sellers in the beginning of period t. The entering proportions are given by $\beta_\theta = \gamma_\theta := \beta_\theta^{T-1} = \gamma_\theta^{T-1}$.

Preferences. Whenever a unit of quality θ is traded, the buyer gains utility v_θ, whereas the seller incurs a production cost of c_θ. It is assumed that (i) v_θ and c_θ are both increasing in type, and (ii) for all types $\theta \in \Theta$, $v_\theta > c_\theta \geq 0$. A buyer or seller who does not trade after T periods earns zero utility.

We make an additional assumption which states that the average initial quality of the goods available for production in the market is "low" (adverse selection is severe). Formally,

$$\sum_{\theta \in \Theta} \gamma_\theta^{T-1} v_\theta < c_K. \tag{2.1}$$

Matching and Trade. In the beginning of each period t, agents are randomly and anonymously matched, with probability $\alpha \in (0, 1)$, in pairs consisting of one buyer and one seller. The quality of the good produced by the seller is her private information.

Once matched, the seller makes a take-it-or-leave-it offer. If accepted, the proposed exchange is realized and both agents leave the market. An agent that is either unmatched or in a match where the offer turns out to be rejected remains in the market and waits for the next period, unless $t = T$, in which case trade is forsaken.

Strategies. A history for a type-θ seller is a list specifying, for each of the preceding periods, whether she was in a match and, if so, what price was offered (and rejected). It is purposeless, however, for a seller to condition her behavior on past matches and prices. Since there is

a continuum of agents in each period, and since histories are private, previous interactions experienced in the market provide no new information a seller could possibly benefit from. Thus, a type-θ seller's strategy can be suitably expressed as a sequence

$$\lambda_\theta = (\lambda_\theta^{T-t})_{t=1}^T, \tag{2.2}$$

where $\lambda_\theta^s \in [0,1]^{\mathbb{R}_+}$ denotes a probability distribution over possible price offers in period $T-s$. To simplify notation later on, for all $s \in \{0,\ldots,T-1\}$ and all $\theta \in \Theta$, define

$$\Lambda_\theta^s := \text{supp}\lambda_\theta^s.$$

Through congruous reasoning, the buyer's strategy may also be represented by a sequence

$$\sigma_B = (\sigma_B^{T-t})_{t=1}^T, \tag{2.3}$$

where $\sigma_B^s \in [0,1]^{\mathbb{R}_+}$ is a function which outputs, for each offer $p \in \mathbb{R}_+$, a probability $\sigma_B^s(p)$ of it being accepted in period $T-s$.

Beliefs. A buyer's beliefs are given by

$$\pi_\theta = (\pi_\theta^{T-t})_{t=1}^T, \tag{2.4}$$

where $\pi_\theta^s \in [0,1]^{\mathbb{R}_+}$ is a function which outputs the probability $\pi_\theta^s(p)$ attributed by the buyer in period $T-s$ that a seller offering p is of type θ.

Payoffs. Define the following expected payoff sequences

$$V_\theta := (V_\theta^{T-t})_{t=1}^T \text{ and } V_B := (V_B^{T-t})_{t=1}^T, \tag{2.5}$$

where $V_\theta^s \in \mathbb{R}$ and $V_B^s \in \mathbb{R}$ denote, respectively, a type-θ seller's and a buyer's expected payoffs in period $T-s$.

It is also useful to define the sequences of buyers' and sellers' reservation values

$$(w_\theta^{T-t})_{t=1}^T \text{ and } (r_\theta^{T-t})_{t=1}^T, \tag{2.6}$$

where

$$w_\theta^s := v_\theta - \delta V_B^{s-1}$$

corresponds to the maximum price a buyer is willing to pay for the consumption of a type-θ good in $T-s$ and

$$r_\theta^s := c_\theta + \delta V_\theta^{s-1}$$

is the minimum value a type-θ seller will accept in period $T-s$ for the exchange of one unit of her good.

3 Equilibrium

We are ready to define an equilibrium in the context described above.

Definition. A symmetric **equilibrium** is a list

$$((\lambda_\theta)_{\theta\in\Theta}, \sigma_B, (V_\theta)_{\theta\in\Theta}, V_B, (\pi_\theta)_{\theta\in\Theta}, (\gamma_\theta)_{\theta\in\Theta})$$

satisfying

Seller optimality. Sellers maximize expected payoffs, i.e., for each $\theta \in \Theta$, $s \in \{0, \ldots, T-1\}$ and $p^s_\theta \in \Lambda^s_\theta$,

$$p^s_\theta \in \arg\max_{p\in\mathbb{R}_+} \{\sigma^s_B(p)(p - r^s_\theta)\} \tag{3.1}$$

Buyer optimality. Buyers maximize expected payoffs given their beliefs, i.e., for each $s \in \{0, \ldots, T-1\}$ and $p \in \mathbb{R}_+$,

$$\sigma^s_B(p) \in \arg\max_{\sigma\in[0,1]} \left\{ \sigma \sum_{\theta\in\Theta} \pi^s_\theta(p)(w^s_\theta - p) \right\} \tag{3.2}$$

Consistency of beliefs. [1] Beliefs π_θ follow Bayes' rule on the equilibrium path.

Consistency of payoffs and proportions. Expected payoffs and the proportion of types in the population are consistent with sellers and buyers optimal strategies, i.e., for each $\theta \in \Theta$ and $s \in \{0, \ldots, T-1\}$,

$$V^s_\theta = \max_{p\in\mathbb{R}_+}\{\alpha\sigma^s_B(p)(p - c_\theta) + (1 - \alpha\sigma^s_B(p))\delta V^{s-1}_\theta\}, \tag{3.3}$$

$$V^s_B = \alpha \sum_{\theta\in\Theta} \gamma^s_\theta \int \max \left\{ \sum_{\hat\theta\in\Theta} \pi_{\hat\theta}(p)(v_{\hat\theta} - p), 0 \right\} d\lambda^s_\theta(p) + (1-\alpha)\delta V^{s-1}_B, \tag{3.4}$$

with $V^{-1}_\theta = V^{-1}_B = 0$, and

$$\gamma^s_\theta = \frac{\beta^s_\theta}{\sum_{\hat\theta} \beta^s_{\hat\theta}}, \tag{3.5}$$

where

$$\beta^{s-1}_\theta = \beta^s_\theta \left(1 - \alpha \int \sigma^{s+1}_B(p) d\lambda^{s+1}_\theta(p)\right). \tag{3.6}$$

Equilibria naturally come in many different flavors in this environment. It is useful to categorize some of them according to usual nomenclature.

[1] Note that out-of-equilibrium beliefs are left unrestricted.

Definition (Separating Equilibrium)**.** An equilibrium is **separating** in period $T-s$ if, for all $\theta, \theta' \in \Theta$ with $\theta \neq \theta'$

$$\Lambda_\theta^s \cap \Lambda_{\theta'}^s = \emptyset. \tag{3.7}$$

Definition (Pooling Equilibrium)**.** An equilibrium is **pooling** in period $T-s$ if there exists $p \in \mathbb{R}_+$ such that, for all $\theta \in \Theta$,

$$\Lambda_\theta^s = \{p\}. \tag{3.8}$$

Of course, these two categories cannot possibly exhaust all patterns of price offerings that can be observed in equilibria. For instance, in each period, a proper subset of seller types might pool at some price, while the rest chooses separating prices or even a different pool. Sellers can also randomize between pooling and separating offers, or between more than one pool. There are many possibilities to consider that could be sustained in equilibrium by appropriate beliefs.

4 Basic Properties of Equilibria

In this section we derive a few properties of equilibria without any further assumptions. This provides a general characterization which paves the way for the discussion of specificities later on[1].

We begin by fixing an arbitrary equilibrium as defined in Section 1.

Lemma 1. *Let $\theta \in \Theta$, $s \in \{0, \dots, T-1\}$ and $p \in \mathbb{R}_+$. Then,*

(i) $V_B^s \geq \delta V_B^{s-1}, V_\theta^s \geq \delta V_\theta^{s-1}$.

(ii) *If $p < w_1^s$, then $\sigma_B^s(p) = 1$.*

(iii) $V_1^s > 0$.

Item (i) is a straightforward consequence of the fact that all agents can, in every period, ensure at least a zero (instantaneous) payoff: a seller can offer her own reservation value, for instance, and a buyer can reject every price. Recursiveness ensures that $V_B^s \geq 0$ and $V_\theta^s \geq 0$. It follows that $r_\theta^s \geq c_\theta$ and $w_\theta^s \leq v_\theta$.

Item (ii) just says that very cheap offers must be accepted for sure. This is useful for showing (iii). Since $r_1^0 = c_1 < v_1 = w_1^s$, at least in the last period, the lowest type can potentially profit from making a very cheap offer $p \in (c_1, v_1)$. Since such an offer would be accepted with certainty, $V_1^0 \geq \alpha(p - c_1) > 0$. By (i), $V_1^s > 0$ for all s.

Lemma 2. *Let $\theta, \theta' \in \Theta$ with $\theta > \theta'$, $s \in \{0, \dots, T-1\}$, $p_\theta^s \in \Lambda_\theta^s$ and $p_{\theta'}^s \in \Lambda_{\theta'}^s$. Then,*

[1] We will not bother with existence right now. As we shall see in section 1, a separating equilibrium is very easily constructed.

(i) $r^s_\theta > r^s_{\theta'}$.

(ii) $0 \leq \sigma^s_B(p^s_\theta) \leq \sigma^s_B(p^s_{\theta'})$.

(iii) $V^s_\theta \leq V^s_{\theta'}$.

The lemma says that higher types require higher offers to accept trading at any given period (item (i)). This higher selectiveness implies trading at reduced rates (item (ii)). Finally, lower types get higher payoffs, reflecting the additional informational rent required to discourage them from emulating higher types (item (iii)).

The property in item (ii) is interesting because it implies that if a type θ trades with positive probability in $T-s$, then every type lower than θ will necessarily trade with positive probability in that period. Correspondingly, if θ makes an offer that is rejected for sure, then no type higher than θ will trade in that period. Because of this property, we find it useful to define a "threshold" type θ^s_{max} for every $s \in \{0,\ldots,T-1\}$:

$$\theta^s_{max} := \max\{\theta \in \Theta : \sigma^s(p) > 0 \text{ for some } p \in \Lambda^s_\theta\}. \tag{4.1}$$

Another consequence of item (ii) is that the average quality of goods traded increases with time:

Corollary 1. *Let $s \in \{1,\ldots,T-1\}$ and $p^s_\theta \in \Lambda^s_\theta$. Then,*

$$\sum_{\theta\in\Theta} \gamma^s_\theta v^\theta \leq \sum_{\theta\in\Theta} \gamma^{s-1}_\theta v^\theta. \tag{4.2}$$

In every period, the highest qualities are always the least likely to trade. As time goes by, higher-type sellers end up accumulating in the market. This shows that the severe adverse selection present when the market opens may be overturned if trade goes on for long enough.

Lemma 3. *Let $\theta,\theta' \in \Theta$ with $\theta^s_{max} \geq \theta > \theta'$, $s \in \{0,\ldots,T-1\}$, $p^s_\theta \in \Lambda^s_\theta$ and $p^s_{\theta'} \in \Lambda^s_{\theta'}$. Then,*

(i) $V^s_{\theta'} > V^s_\theta \geq 0$.

(ii) $p^s_\theta \geq p^s_{\theta'}$.

(iii) *If $\sigma^s_B(p^s_\theta) = \sigma^s_B(p^s_{\theta'})$, then $p^s_\theta = p^s_{\theta'}$*

The strict inequality in item (i) is a consequence of reservation values being increasing in type. To see this, observe that, for some not-always-rejected offer p^s_θ, $\sigma^s_B(p^s_\theta)(p^s_\theta - r^s_{\theta'}) > \sigma^s_B(p^s_\theta)(p^s_\theta - r^s_\theta) \geq 0$. That means a type-$\theta'$ can get a (strictly) higher instantaneous payoff than θ even if she offers exactly the same price as θ.

Item (ii) implies that accepted price offers are non-decreasing in type. It also implies that $\Lambda^s_\theta \cap \Lambda^s_{\theta'}$ is either empty or unitary. A straightforward corollary follows:

Corollary 2. *Let $\theta, \theta' \in \Theta$ with $\theta^s_{max} \geq \theta > \theta'$ and $s \in \{0, \dots, T-1\}$. If $p \in \Lambda^s_{\theta'} \cap \Lambda^s_{\theta}$, then*

$$\Lambda^s_k = \{p\} \text{ for all } k \in \Theta \text{ with } \theta' < k < \theta.$$

In words, if two types make a pooling offer that is accepted with positive probability, then every intermediate type must pool at that offer.

Finally, item (iii) says that you cannot have two different types making different offers which are accepted with the same probability – otherwise, no one would make the lesser offer.

Lemma 4. *Let $s \in \{0, \dots, T-1\}$. The total amount of offers that are accepted with positive probability in $T - s$ is finite.*

Lemma 4 results from lemma 3 item (ii) and from the fact that each type can make at most one separating offer that is accepted with positive probability[2]. In particular, a type-1 and a a type-θ^s_{max} seller each has at most two offers that accepted with positive probability; each $k \in \Theta$ with $1 < k < \overline{\theta}^s$ have at most three offers each that are accepted with positive probability.

The next lemma says that, in order to prevent deviations among accepted offers on the path of equilibrium, it is necessary and sufficient to ensure incentive compatibility holds for consecutive types. Though this is a standard result, stating it precisely in this context requires some additional (and cumbrsome) notation. To improve readability, and since there is no ambiguity, I will drop the superscript s for the remainder of the section. It should be noted, however, that all objects are dependent on the time period.

Fix a period $T-s$. Let J denote the finite number of offers that are accepted with positive probability in that period. Let $P := \{p(j)\}_{j \in \{1,\dots,J\}}$ denote the set of offers that are accepted with positive probability and let $\Theta(j)$ be the set of types that offer $p(j)$.

For all $j, l \leq J$ and all $\theta \in \Theta^s(j)$, incentive compatibility among accepted offers is given by

$$\sigma_B(p(j))(p(j) - r_\theta) \geq \sigma^s_B(p(l))(p(l) - r_\theta) \tag{4.3}$$

Define $\overline{\theta}(j) := \max \Theta(j)$ and $\underline{\theta}(j) := \min \Theta(j)$.

Lemma 5. *Fix a period $T-s$, with $s \in \{0, \dots, T-1\}$. For all $j < J$, a necessary and sufficient condition for 4.3 is*

$$\sigma_B\left(p(j)\right)\left[p(j) - r_{\overline{\theta}(j)}\right] \geq \sigma_B\left(p(j+1)\right)\left[p(j+1) - r_{\overline{\theta}(j)}\right] \tag{4.4}$$

$$\sigma_B\left(p(j+1)\right)\left[p(j+1) - r_{\underline{\theta}(j)}\right] \geq \sigma_B\left(p(j)\right)\left[p(j) - r_{\underline{\theta}(j+1)}\right] \tag{4.5}$$

[2] If there were more than one, at least one of them would be strictly lower than the buyer's valuation. This is clearly suboptimal for type 1. For higher types, the buyer would accept such an offer with probability one, violating lemma 2(ii) and lemma 3(iii)

Apart from these basic properties, it is very difficult to further detail the characterization of the set of equilibria. Equilibria with separation in every period are easier to characterize and a natural class to consider whenever adverse selection remains severe throughout the duration of the game. As lower-quality goods trade at a quicker pace than the higher types, however, average good quality increases with time, and may well rise high enough as to allow for pooling equilibria.

5 Pooling

The purpose of this section is to rule out pooling as a viable outcome in some special cases.

First off, an equilibrium with pooling in $T-s$ is possible if, and only if,

$$\sum_{\theta\in\Theta}\gamma_\theta^s w_\theta^s \geq r_K^s, \tag{5.1}$$

and the pooling offer can be anything in the $\left[r_K^s, \sum_{\theta\in\Theta}\gamma_\theta^s w_\theta^s\right]$ interval.

Clearly, pooling is not an option right when the market opens, as the absence of severe adverse selection is necessary for eq. (5.1). However, as we have noted before, average quality tends to increase as time goes by, so that severe adverse selection may eventually be overturn. Ultimately, eq. (5.1) may be satisfied at some point. A pooling offer which is accepted with probability one "clears the market" (in the loose sense that every matched agent trades).

In some cases, however, pooling cannot be observed in any period in equilibrium.

Lemma 6. *If α is small enough, a pooling offer which is accepted with positive probability will not be possible in any period.*

The intuition here is simple. If the matching probability is small enough, the trade of low-quality items does not occur fast enough to reverse severe adverse selection before the market closes in period T.

Even if α is close to one, pooling offers might still not be possible, as lemma 7 shows.

Lemma 7. *Let α be close to 1. Then, for δ sufficiently close to 1, a pooling offer which is accepted with probability 1 will not be possible in any period.*

The intuition is as follows. If α and δ are both sufficiently high, then a pooling offer which is accepted with probability one sometime in the future will be attractive in particular to lowest-type sellers. Since there is no cost involved in waiting, the lowest type will refrain from making an offer in each of the previous periods. This implies that every higher type will also not trade. Since no types are trading, lower types cannot leave the market at higher rates. Severe adverse selection persists, and the foreseen pooling offer can never actually manifest.

6 Separating Equilibria

Separation is interesting in its own right since it represents the case where signaling through prices perfectly informs the buyer which type of good is to be traded. In this section, I provide additional characterization for equilibria which are separating in every period. I also identify the most efficient separating equilibrium by considering the case where the local incentive compatibility constraints bind in every period for every type. The subsections 6.1 and 6.2 consider the behavior of the most efficient separating equilibrium under, respectively, small frictions (δ and α close to one) and a long horizon (T approaching infinity). These cases are compared to results found in Moreno e Wooders (2016).

We begin the discussion with lemma 8, which identifies the price offers that are accepted with positive probability in a separating equilibrium.

Lemma 8. *In a separating equilibrium, for all $s \in \{0, 1, \ldots, T-1\}$,*

(i) If $\theta \in \Theta$ with $\theta \leq \theta^s_{max}$, then $\Lambda^s_\theta = \{v_\theta\}$.

(ii) $V^s_B \equiv 0$.[1]

Each seller type can make at most one separating price offer which is accepted with positive probability. A type-1 seller must offer w^s_1, which is accepted for sure[2]. Incentive compatibility compels every other accepted offer to be rejected with positive probability, implying the buyer must be indifferent between acceptance and rejection. As a result, buyers never yield a positive surplus, and every accepted price offered by a type $\theta \in \Theta$ seller is exactly the buyers' valuation $w^s_\theta = v_\theta$.

This does not say much of offers that are rejected for sure, but these offers are of little importance. In fact, the only thing that matters is that it is optimal for the buyer to reject them. Thus, there is no loss in assuming $\Lambda^s_\theta = \{v_\theta\}$ for all θ, since the buyer is indifferent between accepting and rejecting an offer equal to her valuation.

Local incentive compatibility is necessary and sufficient for seller optimality on the equilibrium path, as in lemma 5. Under separation in every period, this condition is given by

Local Incentive Compatibility under Separation. For all $s \in \{0, 1, \ldots, T-1\}$, $\sigma^s_B(v_1) = 1$ and, for all $\theta \in \Theta \backslash \{1\}$,

$$\sigma^s_B(v_{\theta-1}) \frac{v_{\theta-1} - r^s_\theta}{v_\theta - r^s_\theta} \leq \sigma^s_B(v_\theta) \leq \sigma^s_B(v_{\theta-1}) \frac{v_{\theta-1} - r^s_{\theta-1}}{v_\theta - r^s_{\theta-1}}. \tag{6.1}$$

[1] This is not, of course, an exclusive property to separating equilibria. A sufficient condition for $V^s_B \equiv 0$ is separation of the lowest type in every period.

[2] Otherwise she could lower her price offer by an $\epsilon > 0$ small enough, ensuring acceptance regardless of beliefs.

Since each type makes only one offer in equilibrium, we can define without ambiguity the probabilities and discounted probabilities of trade associated with each type for a given separating equilibrium.

Definition. Let $s \in \{0, 1, \ldots, T-1\}$ and $\theta \in \Theta$. Let ϕ_θ^s and ψ_θ^s denote, respectively, the discounted probability of trade and the probability of trade from period $T-s$ to T.

$$\phi_\theta^s = \sum_{u=0}^{s} \alpha \sigma_B^{s-u}(v_\theta) \delta^u \prod_{v=0}^{u-1} (1 - \alpha \sigma_B^{s-v}(v_\theta)),$$
$$\psi_\theta^s = \sum_{u=0}^{s} \alpha \sigma_B^{s-u}(v_\theta) \prod_{v=0}^{u-1} (1 - \alpha \sigma_B^{s-v}(v_\theta)).$$

The probability of trade ψ_θ^s gives the probability that a type-θ will be sold in periods from $T-s$ to T, given that the good has not been traded before $T-s$. The discounted probability of trade is an analogous concept, but discounts future probabilities for delayed trading.

A type-θ seller that manages to trade gets an instantaneous payoff equal to $(v_\theta - c_\theta)$, which is independent of the time period in which trade occurred. Thus, for all s and all θ, $V_\theta^s = (v_\theta - c_\theta)\phi_\theta^s$, which is a convenient way of representing seller payoffs and, ultimately, welfare.

The following lemma fixes the most efficient separating equilibrium. As it turns out, fixing instantaneous probabilities of trade at the right-hand bounds in eq. (6.1) maximizes welfare, which is given by $W := \sum_{\theta \in \Theta} \beta_\theta (v_\theta - c_\theta) \phi_\theta$.

Lemma 9. *The most efficient separating equilibrium occurs when, for all $s \in \{0, 1, \ldots, T-1\}$, and all $\theta \in \Theta \backslash \{1\}$,*

$$\sigma_B^s(v_\theta) = \sigma_B^s(v_{\theta-1}) \frac{v_{\theta-1} - r_{\theta-1}^s}{v_\theta - r_{\theta-1}^s}. \tag{6.2}$$

For each s and each θ, we shall let the instantaneous probabilities of trade for the most efficient separating equilibrium be denoted by $\sigma_{B,\theta}^s := \sigma_B^s(v_\theta)$.

Lemma 10. *In the most efficient separating equilibrium, for all $\theta \in \Theta$, ϕ_θ^s and ψ_θ^s are increasing in s.*

As time goes by, the probabilities of trade (discounted or not) get progressively smaller. This means the lower types feel progressively less inclined to emulate the higher types, since by doing so they would be facing a higher risk of reaching period T without trading. As incentive compatibility becomes easier to impose, instantaneous probabilities of trade rise for the higher types as we get closer to the end of the game:

Corollary 3. *In the most efficient separating equilibrium, for all $\theta \in \Theta$, $\sigma_{B,\theta}^s$ is decreasing in s.*

Our next step is to analyze the behavior of the most-efficient separating equilibrium under special conditions.

6.1 Small Frictions

When agents become infinitely patient, the screening mechanism supplied through the time dimension ceases to exist. The impact over equilibria is not dramatic in my model, however, since signaling through prices remains available. Once we let α approach one as well, however, we shut down the signaling channel too[3]. As matching becomes a certain event, the lowest-type seller, offering v_1, trades for sure in every period. Consider, however, what would happen if in any period $T-s$, $s \in \{1, \ldots, T-1\}$, an offer larger than v_1 were made and accepted with a positive probability. Such an offer is incompatible with equilibrium, since clearly a type-1 seller would prefer to (costlessly) wait until $T-s$ and try to capture the larger surplus. If unsuccessful, losses would be nonexistent: there is zero cost in waiting for the next period, where trade would again be certain. This implies that in any separating equilibrium with δ and α close to one, only the lowest type can trade in every period, and these exchanges would be largely concentrated in $t = 1$ (due to high α). Every other type would only be able to trade in the last period, when agents play essentially the static version of the game and dynamic considerations no longer matter.

We formalize the argument above in lemma 11:

Lemma 11. *For all $s \in \{0, 1, \ldots, T-1\}$, and all $\theta \in \Theta \backslash \{1\}$,*

$$\lim_{\delta \to 1} \sigma^s_{B,\theta} > 0$$

and, for all $s \in \{1, \ldots, T-1\}$, and all $\theta \in \Theta \backslash \{1\}$

$$\lim_{\alpha \to 1} \lim_{\delta \to 1} \sigma^s_{B,\theta} = 0.$$

In Moreno e Wooders (2016), where the buyer is responsible for making take-it-or-leave-it offers, time is the only way to screen between seller types. It is thus unsurprising that, as δ approaches one, the authors find that markets "freeze". Neutralizing the temporal dimension is enough, in that context, to completely bring markets to a halt.

6.2 Long Horizon

As the horizon becomes larger in the efficient separating equilibrium, sellers trade with the same instantaneous probability in every period. This convergence result arises from the monotonicity property derived in corollary 3.

Lemma 12. *For all $\theta \in \Theta$,*

[3] This is also true for any equilibrium where the lowest type separates in every period.

(i) $\lim_{T\to\infty} \sigma_{B,\theta}^{T-t} = \sigma_\theta^\infty > 0$, *for all* t.

(ii) $\lim_{T\to\infty} \psi_\theta^{T-t} = 1$, *for all* t.

The probability of trade approaches one, so that every good is eventually sold. This is not true of every separating equilibrium, however, only the most efficient one. We can quite easily construct separating equilibria where one or more types never trade (or never trade after a finite number of periods), and extending the horizon will not matter to the total probability of trade.

In Moreno e Wooders (2016), an infinite horizon will also imply every type trades for sure.

7 Conclusion

In this paper I analyzed the role of prices as signaling devices in a decentralized, dynamic market for lemons. The nonstationary environment make it difficult to fully characterize the set of equilibria, but properties regarding how signaling works in this context could be derived. The most efficient separating equilibrium allows for the trade of every type in all periods, a feature that remains even when all agents are infinitely patient and time is no longer useful as a screening device. The signaling mechanism which aids trade under severe adverse selection is only dissolved if both the discount factor and the matching probability approach one, resulting in a market freeze.

Appendix

his appendix includes the proofs of the important results mentioned in the main text. The proofs not included here are either trivial or standard in the literature.

Before proving lemma 2, it helps to prove the following claim first:

Claim 1. $V_{\theta'}^s - V_\theta^s < \delta^{-1}(c_\theta - c_{\theta'})$

Proof of claim 1. In equilibrium, seller optimality implies a type-θ seller should not wish to behave as a type-θ' seller would, and vice-versa. So:

$$\sigma_B^s(p_\theta^s)(p_\theta^s - r_\theta^s) \geq \sigma_B^s(p_{\theta'}^s)(p_{\theta'}^s - r_\theta^s) \tag{7.1}$$

$$\sigma_B^s(p_{\theta'}^s)(p_{\theta'}^s - r_{\theta'}^s) \geq \sigma_B^s(p_\theta^s)(p_\theta^s - r_{\theta'}^s) \tag{7.2}$$

The argument is by induction on s.

Fix $s = 0$. Using eq. (7.1) and $c_\theta < c_{\theta'}$, we get $V_{\theta'}^0 - V_\theta^0 < \delta^{-1}(c_\theta - c_{\theta'})$.

Now fix $s \in \{1, \dots, T-1\}$ and assume $V_{\theta'}^{s-1} - V_\theta^{s-1} < \delta^{-1}(c_\theta - c_{\theta'})$. Using this assumption and eq. (7.1) yields $V_{\theta'}^s - V_\theta^s < \delta^{-1}(c_\theta - c_{\theta'})$ □

Proof of lemma 2.

$$r_\theta^s - r_{\theta'}^s = (c_\theta - c_{\theta'}) - \delta(V_{\theta'}^{s-1} - V_\theta^{s-1}) > 0 \tag{7.3}$$

Adding the restrictions in eq. (7.1) and eq. (7.2) and using eq. (7.3) yields $\sigma_B^s(p_\theta^s) \leq \sigma_B^s(p_{\theta'}^s)$.

So the only thing left is to show that $V_{\theta'}^s - V_\theta^s \geq 0$. This is another induction on s.

Fix $s = 0$. From eq. (7.1) and since $c_\theta > c_{\theta'}$,

$$V_{\theta'}^0 = \alpha\sigma_B^0(p_{\theta'}^0)(p_{\theta'}^0 - c_{\theta'}) \geq \alpha\sigma_B^0(p_\theta^0)(p_\theta^0 - c_{\theta'}) \geq \alpha\sigma_B^0(p_\theta^0)(p_\theta^0 - c_\theta) = V_\theta^0. \tag{7.4}$$

Now fix $s > 1$ and assume $V_{\theta'}^{s-1} \geq V_\theta^{s-1}$. From eq. (7.1) and eq. (7.3),

$$\sigma_B^s(p_{\theta'}^s)(p_{\theta'}^s - r_{\theta'}^s) \geq \sigma_B^s(p_\theta^s)(p_\theta^s - r_{\theta'}^s) \geq \sigma_B^s(p_\theta^s)(p_\theta^s - r_\theta^s),$$

and, therefore,

$$V_{\theta'}^s - V_\theta^s = \alpha\sigma_B^s(p_{\theta'}^s)(p_{\theta'}^s - r_{\theta'}^s) - \alpha\sigma_B^s(p_\theta^s)(p_\theta^s - r_\theta^s) + \delta(V_{\theta'}^{s-1} - V_\theta^{s-1}) \geq 0. \tag{7.5}$$

□

Proof of lemma 3. We begin with item (i). Since $0 < \sigma_B^s(p) \leq \sigma_B^s(p_{\theta'}^s)$, then

$$\sigma_B^s(p_{\theta'}^s)(p_{\theta'}^s - r_{\theta'}^s) \geq \sigma_B^s(p)(p - r_{\theta'}^s) > \sigma_B^s(p)(p - r_\theta^s),$$

so that $V_{\theta'}^s - V_\theta^s > \delta[V_{\theta'}^{s-1} - V_\theta^{s-1}] \geq 0$.

For item (ii), assume first that $0 < \sigma_B^s(p_\theta^s) \leq \sigma_B^s(p_{\theta'}^s)$. Then eq. (7.1) implies $p_\theta^s \geq p_{\theta'}^s$.

If, on the other hand, $0 = \sigma_B^s(p_\theta^s) < \sigma_B^s(p_{\theta'}^s)$, then $w_\theta^s \leq \sum_{k\in\Theta} \pi_k^s(p_\theta^s)w_k^s \leq p_\theta^s$. Moreover, $p_{\theta'}^s \leq \sum_{k\in\Theta} \pi_k^s(p_{\theta'}^s)w_k^s < w_\theta^s$. Thus, $p_{\theta'}^s < p_\theta^s$.

Now for item (iii). Given the assumptions, $\sigma_B^s(p_\theta^s) = \sigma_B^s(p_{\theta'}^s) > 0$. Using eq. (7.1) yields the desired result. □

Proof of corollary 1. Let $s \in \{1, \ldots, T-1\}$. Then,

$$\sum_{k\in\Theta} \gamma_k^{s-1} v_k = \frac{\sum_{k\in\Theta} \beta_k^{s-1} v_k}{\sum_{k\in\Theta} \beta_k^{s-1}} = \frac{\sum_{k\in\Theta} \beta_k^s (1 - \alpha\sigma^s(p_k^s))v_k}{\sum_{k\in\Theta} \beta_k^s (1 - \alpha\sigma^s(p_k^s))} \geq \frac{\sum_{k\in\Theta} \beta_k^s v_k}{\sum_{k\in\Theta} \beta_k^s} = \sum_{k\in\Theta} \gamma_k^s v_k \tag{7.6}$$

□

Proof of lemma 6. Assume p_θ^s denotes any element of Λ_θ^s in equilibrium. Then,

$$\sum_{k\in\Theta} \gamma_k^s v_k = \frac{\sum_{k\in\Theta} \beta_k^s v_k}{\sum_{k\in\Theta} \beta_k^s} = \frac{\sum_{k\in\Theta} \beta_k^{T-1} v_k \prod_{u=1}^{T-s-1}(1 - \alpha\sigma_B^{T-u}(p_k^{T-u}))}{\sum_{k\in\Theta} \beta_k^{T-1} \prod_{u=1}^{T-s-1}(1 - \alpha\sigma_B^{T-u}(p_k^{T-u}))} \tag{7.7}$$

□

If $\alpha \to 0$, then $\sum_{k\in\Theta} \gamma_k^s v_k \to \sum_{k\in\Theta} \gamma_k^{T-1} v_k < c_K$.

Proof of lemma 7. Consider $T - s$ as the last period where adverse selection is severe. Since there cannot be a pooling offer in this particular period, the maximum offer a type-1 seller could make is some $p < \sum_{k\in\Theta} \gamma_k^s w_k^s \leq \sum_{k\in\Theta} \gamma_k^s v_k < c_K$.

Assume the equilibrium encompasses a pooling offer in $T - u$. The pooling price is, therefore, some $p^* \in [r_K^u, \sum_{k\in\Theta} \gamma_\theta^u w_\theta^u]$. Observe that $p^* \geq r_K^u \geq c_K > p$.

A paired type-1 seller in $T - s$ can either make an offer or wait (make an offer that is rejected). In particular, she can wait for the pooling offer to come around in a couple of periods. If she makes an offer, her payoff is $p - c_1$. If she waits, her payoff will be at least $\delta^{u-s}\{\alpha[p^* - c_1] + (1 - \alpha\delta)V_1^{u-1}\}$. If α and δ are large enough, then waiting will be advantageous. If a type-1 seller, however, then decides to make an offer that is rejected, then every other type in that period will not trade. Then, adverse selection will never be overturn in equilibrium. □

Proof of eq. (6.2). We wish to maximize welfare subject to

$$\sigma^s_{B,\theta} \leq \sigma^s_{B,\theta-1}\frac{v_{\theta-1} - r^s_{\theta-1}}{v_\theta - r^s_{\theta-1}} \text{ for each } s \in \{0, \dots, T-1\}.$$

Observe that, for all s, $\sigma^s_{B,K}$ only impacts positively welfare, so we may promptly assume the restriction binds. The same occurs with $\sigma^{T-1}_{B,\theta}$ for every θ.

Now we use an argument by induction. Assume the restriction above binds for every $\sigma^{s+1}_{B,\theta+1}$, $\sigma^{s+1}_{B,\theta}$ and $\sigma^s_{B,\theta+1}$. It is enough to show that an increase in $\sigma^s_{B,\theta}$ impacts type $\theta+1$'s welfare positively. To determine whether or not this is true, we calculate:

$$\frac{\partial \alpha\sigma^{s+1}_{B,\theta+1}}{\partial \phi^s_\theta}\frac{\partial \phi^s_\theta}{\partial \alpha\sigma^s_{B,\theta}} + \delta\frac{\partial(1-\alpha\sigma^{s+1}_{B,\theta+1})\alpha\sigma^s_{B,\theta+1}}{\partial \alpha\sigma^s_{B,\theta}}$$

$$= \frac{-\delta}{A_{\theta+1} - \delta\phi^s_\theta}\left[\alpha\sigma^{s+1}_{B,\theta} - \alpha\sigma^{s+1}_{B,\theta+1}\right]\left[1-\delta\phi^{s-1}_\theta\right] + \delta\frac{1-\delta\phi^{s-1}_\theta}{A_{\theta+1}-\delta\phi^{s-1}_\theta}(1-\alpha\sigma^{s+1}_{B,\theta+1}) +$$

$$+ \frac{\delta}{A_{\theta+1} - \delta\phi^s_\theta}\left[\alpha\sigma^{s+1}_{B,\theta} - \alpha\sigma^{s+1}_{B,\theta+1}\right][1-\delta\phi^{s-1}_\theta]\delta\alpha\sigma^s_{B,\theta+1}$$

$$= \frac{-\delta}{A_{\theta+1} - \delta\phi^s_\theta}\left[\alpha\sigma^{s+1}_{B,\theta} - \alpha\sigma^{s+1}_{B,\theta+1}\right]\left[1-\delta\phi^{s-1}_\theta\right]\left[1-\delta\alpha\sigma^s_{B,\theta+1}\right] + \delta\frac{1-\delta\phi^{s-1}_\theta}{A_{\theta+1}-\delta\phi^{s-1}_\theta}(1-\alpha\sigma^{s+1}_{B,\theta+1})$$

$$\geq \delta[1-\delta\phi^{s-1}_\theta]\left[1-\alpha\sigma^{s+1}_{B,\theta+1}\right]\left\{\frac{1}{A_{\theta+1}-\delta\phi^{s-1}_\theta} - \frac{\left[1-\delta\alpha\sigma^s_{B,\theta+1}\right]}{A_{\theta+1}-\delta\phi^s_\theta}\right\} \geq 0.$$

□

Proof of lemma 10. We can write ψ^s_θ recursively, where $\psi^{-1}_\theta = 0$:

$$\psi^s_\theta = \alpha\sigma^s_{B,\theta} + (1-\alpha\sigma^s_{B,\theta})\psi^{s-1}_\theta$$

Clearly $\psi^s_\theta \geq \psi^{s-1}_\theta$.

The proof for ϕ^s_θ is by induction on θ.

First, observe that $\phi^s_1 = \sum_{u=0}^{s}\alpha\delta^s(1-\alpha)^s$ is increasing in s.

Assume that

$$\phi^s_\theta = \alpha\sigma^s_{B,\theta} + (1-\alpha\sigma^s_{B,\theta})\delta\phi^{s-1}_\theta \geq \phi^{s-1}_\theta \Leftrightarrow \alpha\sigma^s_{B,\theta} \geq \frac{(1-\delta)\phi^{s-1}_\theta}{1-\delta\phi^{s-1}_\theta}.$$

Using the expression for $\sigma^s_{B,\theta+1}$:

$$\alpha\sigma^s_{\theta+1} \geq \frac{(1-\delta)\phi^{s-1}_\theta}{\frac{v_{\theta+1}-c_\theta}{v_\theta - c_\theta} - \delta\phi^{s-1}_\theta}.$$

Since $\phi_{\theta}^{s-1}(v_\theta - c_\theta) \geq \phi_{\theta+1}^{s-1}(v_{\theta+1} - c_\theta)$,

$$\alpha\sigma_{B,\theta+1}^{s} \geq \frac{(1-\delta)\phi_{\theta+1}^{s-1}}{1-\delta\phi_{\theta+1}^{s-1}} \Leftrightarrow \phi_{\theta+1}^{s} \geq \phi_{\theta+1}^{s-1}.$$

□

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