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Carlos E. da Costa, Lucas A. de Lima

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Taxation of Couples: a Mirrleesian Approach for Non-Unitary Households*

Carlos E. da Costa — *EPGE-FGV*

Lucas A. de Lima — *Harvard*

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Abstract

Optimal tax theory in the Mirrlees' (1971) tradition implicitly relies on the assumption that all agents are single or that couples may be treated as individuals, despite accumulating evidence against this view of household behavior. We consider an economy where agents may either be single or married, in which case choices result from Nash bargaining between spouses. In such an environment, tax schedules must play the double role of: *i)* defining households' objective functions through their impact on threat points, and; *ii)* inducing the desired allocations as optimal choices for households given these objectives. We find that the *taxation principle*, which asserts that there is no loss in relying on tax schedules is not valid here: there are constrained efficient allocations which cannot be implemented via taxes. More sophisticated mechanisms expand the set of implementable allocations by: *i)* aligning the households' and planner's objectives; *ii)* manipulating taxable income elasticities, and; *iii)* freeing the design of singles' tax schedules from its consequences on households' objectives. ***Keywords:*** *Mechanism Design; Collective Households; Nash-bargain.* ***JEL Codes:*** *D13; H21; H31.*

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I Introduction

The use of labor income tax schedules to advance societies' distributive goals is grounded on sound theoretical basis guaranteeing that relying on such instruments is without loss. Whereas the revelation principle — Dasgupta et al. (1979); Myerson (1979); Harris and Townsend (1981) — proves that the set of allocations that can be implemented is not constrained by the use of a direct mechanism, the taxation principle — Hammond (1979, 1987) — guarantees that there is always a tax schedule that induces the allocations implemented by a direct mechanism. Put together these results imply that one can do no better than using a tax schedule.

In the context of optimal tax theory both the revelation and the taxation principles have been proved under the assumption that either everyone is single or couples that can be treated as if they were individuals.[1] Of course we all recognize that most adults are not single. Moreover, recent advances in family economics suggest that the conditions under which couples may be treated as individuals are very stringent and not likely to be verified in practice. The purpose of this paper is to assess whether the extension of results from optimal tax theory from singles to couples is granted when these conditions are not met.

We modify Mirrlees' (1971) economy by taking the non-unitary nature of couples seriously. The informational structure in Mirrlees (1971) is modified by the assumption that spouses know each other's productivities, which are otherwise private information. Finally, married agents decide through a bargain procedure that satisfy Nash's (1950) axioms. In agreement, spouses maximize a Nash product which threat points are determined by the equilibrium of a non-cooperative disagreement game that depends on the very institutional setting under which household choices are made.

The first question we ask is whether one may still rely on the revelation principle to characterize the set of implementable allocations. Our answer is yes: any implementable allocation may be truthfully implemented by a Direct Mechanism – DM. For singles, nothing new: agents reports their productivity and an outcome function maps announcements into transactions. As for couples, at the moment a married agent 'communicates with the mechanism' he (she)

[1]The expression 'be treated as if' is used to represent both the positive assessment regarding how couples choose and the normative idea that there is no 'dissonance', as defined in Apps and Rees (1988), between the couples' and the planner's objective.

knows his (her) own productivity, his (her) spouses' productivity and whether the household has reached an agreement or not. This informational structure imposes the definition of a married agent's type, a vector comprised of: *i)* his (her) productivity; *ii)* his (her) spouses' productivity, and; *iii)* whether the couple is in agreement or disagreement.[2] An outcome maps both spouses reports about their types into transactions.

If households' objectives were invariant to policy, all the results in Hammond (1979, 1987); Guesnerie (1998) regarding the taxation principle would still be valid here. In fact, very simple mechanisms for which each agent is only asked his (her) productivity would suffice. What makes the non-unitary setting special is the fact that threat points are as much determined by the tax schedules as the choices made under the ordering represented by the Nash product. Hence, the equivalence between two institutions depends not only on their ability to replicate choices *conditional on threat points*, but also on their capacity to generate the same threat points as equilibrium utilities for households in disagreement.

The taxation principle fails, i.e., tax schedules cannot in general implement an arbitrary incentive-feasible allocation, because a single schedule must play the double role of implementing the desired allocation for given household objectives and inducing those objectives. By contrast, by allowing different choice sets for couples whether in agreement or disagreement alternative institutions/mechanism generate some separation between the two instruments.

The logic of our results can be grasped from an optimal taxation perspective as follows. When one perturbs a tax schedule, the welfare impact is very closely related to the mechanical impact on tax revenues due to a direct application of the envelope theorem. With dissonance, this needs not to be true: behavioral responses in the form of redistribution across spouses have first order effect on welfare when the marginal value of income differ between spouses from the planner's perspective. This is dissonance, as defined by Apps and Rees (1988), in a nutshell. If threat points can be manipulated, this may be useful to align the planner's objective with that of households.[3]

[2]Throughout the paper we use the terms agreement and disagreement to define, respectively, the state under which an agreement to cooperate and split the surplus of marriage has been reached, and the one for which no such agreement has yet been reached and decisions are made in non-cooperatively.

[3]This is most apparent when utility is transferable, the case studied by da Costa and Diniz (2015), since threat points only affect the distribution of households surpluses.

With regards to behavioral responses, it is important to recall that policy elasticities, e.g., Hendren (2016), are not structural parameters. In the context of household taxation, elasticities are bound to depend on threat points therefore being choice parameters. As in Kopczuk and Slemrod (2002), elasticities should be optimally chosen.

The rest of the paper is organized as follows. After a brief related literature overview, Section II presents the environment. In Section VI we define a Direct Mechanism and prove that the revelation principle is valid in this non-unitary setting. Section VII studies tax systems here taken as collections of tax schedules. Finally, in Section VIII.1 we explains how the manipulation of threat points made possible by a DM expands the set of allocations which may be implemented by a tax system. Section IX concludes.

I.1 Literature Review

In its essence the paper addresses the use of mechanism design to a group decisions problem motivated by a modern view of household decisions. It belongs in both the optimal taxation and the family economics literature. To understand the current state of optimal taxation of households, one must start with the pioneering work of Boskin and Sheshinski (1983) who invited the profession to take seriously the fact that most individuals in society interact directly with their spouses in ways that have potentially important consequences for tax design. They proceeded by adopting what is nowadays referred to as the unitary approach to household behavior. Under this approach couples are modeled essentially as a single agent with multiple, perfectly assignable, choices of effort.

If couples may be treated as single agents, then the optimal taxation problem can be solved by noting that: *i)* a direct incentive feasible mechanism is without loss (i.e., the revelation principle applies), and; *ii)* the allocations implemented by the mechanism are the same as the allocations induced by a suitably defined tax schedule (i.e., the taxation principle applies).

The problem is that, however tractable one may find the unitary approach, its poor adherence to the data — e.g., Browning and Chiappori (1998) — has led to a recurring questioning of its usefulness to policy design. Indeed, this failure to describe actual choices made by couples has already crossed the boundaries of academic to influence actual policy design.[4]

[4] Oportunidades in Mexico and Bolsa Família in Brazil are prominent examples of social pro-

The decline of this unitary view of the household is followed by the emergence of an alternative, collective view of household behavior. It agrees with the unitary approach with respect to the incentives spouses have to choose efficiently but recognizes the conflicting opinions regarding which efficient choices to make.

The goal of our paper is to bring optimal tax theory to terms with the novel findings in Family Economics by providing analogues of (i) and (ii) for households which behavior is better approximated by the collective model.

In a broader sense, we refer to collective models as those, axiomatic in nature, which only assume that households can achieve efficient outcomes without specifying the process leading to such outcomes — see the discussion in Chiappori and Mazzocco (2015). At this level of generality, however, not very much can be said from a normative perspective. We adopt the early formulation of Manser and Brown (1980); McElroy and Horney (1981) which assumes that couples decisions are made through Nash bargaining. Central to our analysis is, therefore, the impact of policies on threat points.[5]

The literature has considered two types of threat points: i) external, usually modeled as the utility of being single, and; ii) internal, given by the utility attained within marriage if cooperation ceases. 'External' threat points were used in the pioneering works of Manser and Brown (1980); McElroy and Horney (1981). Internal threat points are important if one is to account for changes in household behavior that result from factors that only affect spouses while married, as in the 'separate spheres' model of Lundberg and Pollak (1993).

From a purely theoretical perspective, Binmore (1985); Binmore et al. (1989) make the point that outside options do not define threat points. They, instead, provide bounds for utilities that can arise in any bargain. Application of this idea to household economics – Bergstrom (1996) – leads to the idea that it is the utility attained when cooperation ceases and choice are made non-cooperatively that ultimately defines threat points.

Our work also has implications for the perturbation methods (or elasticities approach) developed by Piketty (1997); Dahlby (1998); Saez (2001) which has

grams which design explicitly acknowledges the failure of one of the well known properties of unitary models: income pooling.

[5]Since the axiomatic solution to a Nash bargain imposes efficiency, these models are collective in the sense just described. Moreover, without a specification about how threat points are determined, Nash-bargain has no additional testable implications when compared to general collective models — Chiappori et al. (2012).

become the standard for optimal labor income taxation. This approach for the taxation of couples is very useful if the unitary approach may be used given the multi-dimensional nature of the screening problem — e.g., Kleven et al. (2009).[6] When (i) and (ii) apply one may either solve the mechanism design problem and use the resulting allocations to build the optimal tax schedule, or start from a candidate optimum schedule and show use local changes to derive necessary conditions for an optimum.[7] Under the latter approach one needs not to rely on the simplifying assumptions adopted by Mirrlees (1971), which are very hard to hold in practice when we consider the underlying structure of the household problem. More specifically, the uni-dimensional nature of the problem that allowed single-crossing to arise under very weak assumptions on preferences in the case of singles is lost in the case of couples.

Although our results show that an optimal tax schedule need not implement the optimal allocation, we may still ask whether the elasticities approach can be used to study the somewhat less ambitious goal of deriving optimal tax systems. By using carefully chosen filing options the perturbations and elasticities remain meaningful, which will not be the case if one relies on simple tax systems. Moreover, policy elasticities are not structural parameters.[8] Instead they are determined by the hole structure of the economy and should be optimally chosen as in Slemrod and Kopczuk (2002).

II Environment

The economy we describe is an extension of Mirrlees' (1971) to a setting where we allow a subset of the agents to be married.

A continuum of individuals, of two different genders, $i = f, m$, have preferences defined over consumption, $\mathfrak{c}$, and leisure, $\mathfrak{l}$, which may depend on their gender, but are otherwise identical across individuals. Preferences for a gender i agent may be represented by the utility function $u_i(\mathfrak{c}, \mathfrak{l})$, increasing in both variables and strictly quasi-concave.[9]

[6] Such an application of variational methods is found in Golosov et al. (2014).

[7] The main drawback is, of course, that it does not offer a method for actually finding the optimum.

[8] An account of the nature of policy elasticities is explained in Hendren (2016). Our results show that they vary for the same principal tax schedule, if we vary the filing option.

[9] $u_i(\cdot)$ is also the cardinal representation that will be relevant for the Nash bargaining problem.

As in Mirrlees (1971), agents differ with respect to their labor market productivity, $\theta_i \in \Theta$, $i = f, m$.

Agents may be single or married. For notation simplicity only we shall restrict the analysis to couples comprised of agents of different genders. A couple is, therefore, identified by a pair of productivities: θ_f for the wife and θ_m for the husband. We use $\eta = (\theta_f, \theta_f)$ for brevity.

The distribution of singles of gender i is given by a measure μ_{s}^i in Θ. The distribution of couples is given by a measure $\boldsymbol{\mu}_{\mathrm{c}}$ in Θ^2. We assume that these are known (by the planner at least) distributions which are held fixed as we change policies.

Finally, technology is represented by a transformation function, $G : Z^2 \mapsto R$ that assigns a non-positive value for feasible allocations and a positive value for unfeasible ones.

Informational Structure The information structure is as follows. An agent's type is his or her private information. For singles, this means that a gender i agent is the only one to know its type, θ_i.

For married agents we assume that each spouse i — for $i = f, m$ — in a couple $\eta = (\theta_f, \theta_m)$ knows not only his or her productivity θ_i, but also his or her spouses' productivity, θ_j — for $i, j = f, m$. I.e., both the husband and the wife and noone else knows η. This assumption about the informational structure of an economy with households is consistent with the type of efficient decisions implied by the collective model in general and the Nash bargaining model in particular.[10]

Transactions The consumption bundle $(\mathfrak{c}, \mathfrak{l})$ of an agent is observed by noone but the agent him or herself, if the agent is single. It is observed by both spouses when the agent is married. What is observed by everyone are the transactions made by single agents and household. A tax schedule may only, as a consequence, depend on transactions.[11]

[10]The idea that households can efficiently bargain for instance may not be reasonable if spouses have private information with respect to each other.

[11]For a direct mechanism, the inclusion of obedience constraints allows one to write in terms of final consumption. Yet, by embedding these constraints in households' conditional preferences for transaction we make the comparison of institutions simpler.

Singles A transaction is a vector $z \in \mathcal{Z} \subset \mathbb{R}^2$, where $\mathcal{Z}$ is a compact subset of $\mathbb{R}^2$, which describes everything an agents buys as positive entries and everything he or she sells as negative entries. All agents are assumed to have the same endowment, $(0,1)$, of consumption good and time. A transaction vector $z = (c, -n)$, describes the amount of hours, n, the agent supplies and the amount of consumption goods he or she acquires in the market, c. Given the endowment $(0,1)$ the bundles attainable by a type θ_i agent are $(\mathfrak{c}, \mathfrak{l}) \le (c, 1 - n/\theta_i)$.

Because utility is strictly increasing in both $\mathfrak{c}$ and $\mathfrak{l}$, a single agent will always choose $(\mathfrak{c}, \mathfrak{l}) = (c, 1 - n/\theta_i)$. Hence, there is one to one mapping between transactions, z, and consumption bundles, $(\mathfrak{c}, \mathfrak{l})$, which allows us to define the induced preferences over the set $\mathcal{Z}$, $\succeq_{\theta_i}$. We write $\underline{z} \in \mathcal{Z}$ to define transactions such that $\underline{z} \preceq_{\theta_i} z$ for all $z \in \mathcal{Z}$, all θ_i, $i = f, m$, which we assume to exist.

Couples For couples, define $\boldsymbol{z} = (z_f, z_m) \in \mathcal{Z}^2$, where z_f are the transactions made by the wife and z_m those made by the husband.

The mapping from transactions to consumption bundles for couples is more involved than those for singles. We define a function $F^a : \mathcal{Z}^2 \times \Theta^2 \to 2^X$ which maps the transaction realized by a couple into sets of feasible pairs of bundles that can be consumed by this couple. Embedded in this function are not only the material gains from marriage but also all relevant transferability restrictions. For example, if a $\eta = (\theta_f, \theta_m)$ chooses transaction $\boldsymbol{z} = (c_f, -n_f, c_m, -n_m)$, then the wife must be enjoying leisure $\mathfrak{l}_f \ge 1 - n_f/\theta_f$ and the husband, $\mathfrak{l}_m \ge 1 - n_m/\theta_m$. As for consumption goods, we may allow for gains of scale by assuming that all consumption pairs $(\mathfrak{c}_f, \mathfrak{c}_m)$ such that $\mathfrak{c}_f + \mathfrak{c}_m \le \alpha(c_f + c_m)$ for $\alpha > 1$ are attainable. We write

$$F^a(\boldsymbol{z}, \eta) := \left\{ \left((\mathfrak{c}_f, \mathfrak{l}_f), (\mathfrak{c}_m, \mathfrak{l}_m) \right); \mathfrak{c}_f + \mathfrak{c}_m \le \alpha \left[c_f + c_m \right]; \mathfrak{l}_f \le 1 - \frac{n_f}{\theta_f}; \mathfrak{l}_m \le 1 - \frac{n_m}{\theta_m} \right\}.$$

Note that consumption is transferable across agents, hence non-assignable, while leisure is not. Transactions are not, therefore, directly mapped into utilities. Instead, for any given $\boldsymbol{z} = (z_f, z_m)$ and any η, a (conditional) utility possibility set $\mathcal{U}_{\boldsymbol{z}}(\eta)$ is defined.

II.1 Households' Decision Process

Couples' decisions are the outcome of a bargain which solution satisfies a variation of Nash's (1950) axioms advanced by Zambrano (2016) to account for possible non-convexities of utility possibility sets.[12] The Nash bargaining solution is formulated in terms of a set o feasible utility pairs that households can attain by cooperating and a disagreement pair $(\bar{u}_f, \bar{u}_m)$ that represents the utility that spouses obtain if an agreement is not reached.

Of course, a crucial question that needs to be addressed in any application is what these disagreement utilities are. There are potentially many candidates depending on the application we are considering, and in family economics different possibilities have been considered. The early works of Manser and Brown (1980); McElroy and Horney (1981) took divorce to be the relevant threat points whereas Lundberg and Pollak (1993) pioneered the used of 'internal' threat points as the utility attained as spouses cannot reach a disagreement and choose non-cooperatively. This is also the view sponsored byBergstrom (1996) who uses the results in Binmore (1985) to argue in favor of 'burning toasts' threat points, i.e., utilities attained by spouses as agreement is not reached and cooperation ceases. Outside options, here interpreted as the utilities attained at divorce, define lower bounds on utilities that agents can be assigned, but otherwise play no role in the definition of the outcome. In Binmore et al. (1989), where laboratory experiments are used to assess which of these threat points are more relevant, outside options are referred to as 'breakdown points' whereas the utilities attained if bargaining continues without an agreement are called 'impasse point'. It is important however to mention that in Binmore (1985), Binmore et al. (1989) and Bergstrom (1996) both the breakdown points and the impasse points are givens, whereas a central concern in household economics is how changes in the environment affect these threat points.

Understanding how the institutional design affect threat points is the essence of our analysis. Toward this end, we shall define two different states in which a marriage can be: agreement and disagreement. Note that this, too is private information. Moreover, this disagreement state does not happen in equilibrium. Spouses do however understand and anticipate clearly what would hap-

[12] Zambrano's (2016) 'preference for symmetry' substitutes for Nash's (1950) symmetry axiom. The pair of utility chosen is then a selection from the argmax of Nash's product, which coincided with Nash's solution whenever the utility possibility set is convex.

pen were they not able to reach an agreement.

A couple is in agreement if spouses have been able to settle their differences, having thus agreed on how to rank transactions and on how to allocate the product of such transactions between themselves. In this case, spouses maximize the Nash product,

$$(u_f(\mathfrak{c}_f, \mathfrak{l}_f) - \bar{u}_f)\,(u_m(\mathfrak{c}_m, \mathfrak{l}_m) - \bar{u}_m)\,, \tag{1}$$

where $\bar{u}_f$ and $\bar{u}_m$ are the threat points with respect to which choices are made.

In disagreement spouses play a non-cooperative game that defines these threat points. Contrary to Bergstrom (1996) and Binmore (1985), for that matter, outside options play in our setting a role in disagreement. We assume that reversion to singlehood is possible at any moment. Hence, these breakdown points define lower bounds on threat points. As in Bergstrom (1996); Binmore et al. (1989); Binmore (1985) outside options do not per se define the threat points. Yet, because we provide each spouse with the ability to unilaterally force the dissolution of marriage at any stage, they define lower bounds for the disagreement game. A straightforward consequence is that outside options never bind in agreement.

III Heuristics for Main Results

Final consumption of goods and leisure are not observed. Hence, it cannot be the base for taxation. Instead, tax schedules are imposed on transactions. In the case of singles, this is of little consequence, since strong monotonicity in preferences guarantee a one to one mapping from transactions to final consumption. For couples matters are not so simple.

Our departure point is an optimal tax system which defines for a single agent of gender i, $i = f, m$ the set of feasible transactions $\{z_i \in Z | \psi_i(z_i) \leq 0\}$, in the form of a budget set induced by the optimal tax schedule. Similarly for couples, $\{\boldsymbol{z} \in Z^2 | \boldsymbol{\psi}(\boldsymbol{z}) \leq 0\}$, where $\boldsymbol{\psi}(\cdot)$ plays for couples the role that $\psi_i(\cdot)$ play for gender i singles.

In the case of multi-persons households, not all goods are assignable. In particular, the planner does not observe how the consumption goods, acquired by a couple, $c_f + c_m$, are split between spouses. As a consequence, transactions are not mapped directly into utilities, as in the case of singles. Instead, transactions are mapped into conditional utility possibility sets, which we denote $\mathcal{U}_{\boldsymbol{z}}(\eta)$.

Which point in the set is chosen, then depends on a bargain which solution we assume to satisfy the axioms proposed by Nash (1950).

Threat points must be defined for the household objective, the Nash product, to be determined. This happens as follows. Couples maybe in one of two alternative states: agreement or disagreement. In agreement, spouses act cooperatively and maximize the Nash product. In disagreement, i.e., if they have failed to reach common grounds to make a choice, spouses play a non-cooperative game which defines the threat points. We model this latter stage as follows.

For each couple η let Φ be a mapping from transactions $\boldsymbol{z}$ to a pair of utilities $(\bar{u}_f, \bar{u}_m)$, in the interior of set $\mathcal{U}_{\boldsymbol{z}}(\eta)$, $\Phi(\boldsymbol{z}, \eta) = (\bar{u}_f, \bar{u}_m) \in \mathring{\mathcal{U}}_{\boldsymbol{z}(\eta)}$. We shall have more to say about this mapping later. For now, one need only recognize that it is through this function that threat points are determined.

It is important to note that threat points depend only on transactions that are chosen at the Nash equilibrium for the disagreement game. They do not depend on the specific way under which these transactions are reached. It is this feature that allows us to compare threat points across different institutional settings.

Let us, then compare outcomes for two different institutions. First, is the optimal tax system previously described.

Second, is a general mechanism

$$\mathcal{M} \equiv \left(\{\Sigma_i^s, \Sigma_i^c\}_{i=f,m}, \{g_i^s(\cdot)\}_{i=f,m}, \boldsymbol{g}^c(\cdot,\cdot) \right),$$

where $(\Sigma_i^s)_{i=f,m}$ are message spaces for single agents, Σ_i^c message spaces for married agents, $(g_i^s(\cdot))_{i=f,m}$ outcome functions for singles and $\boldsymbol{g}^c(\cdot,\cdot)$ outcome function for couples.

Under a tax system agents choose transactions directly, and we let $\mathcal{Z}_\Psi^c$ denote the set of all transactions that are feasible for couples under the tax system, i.e., $\mathcal{Z}_\Psi^c := \{\boldsymbol{z} \in Z^2 | \boldsymbol{\psi}(\boldsymbol{z}) \leq 0\}$. Similar sets are defined for singles of either gender.

Under a mechanism, transactions are chosen indirectly through the messages sent to the center. We define the set of transactions available to any household under $\mathcal{M}$ as $\mathcal{Z}_\mathcal{M}^c := \{\boldsymbol{z} \in Z^2 | \exists (\sigma_f, \sigma_m) \in \Sigma_f \times \Sigma_m \text{ such that } \boldsymbol{g}^c(\sigma_f, \sigma_m) = \boldsymbol{z}\}$.

For households are in agreement, if $\mathcal{Z}_\mathcal{M}^c = \mathcal{Z}_\Psi^c$, the same choices will be made provided that threat points are the same. Moreover, because optimal choices are necessarily in the frontier of these feasible transactions sets all that is needed for the frontier of the two sets to coincide.

The question is whether threat points coincided for the two institutions. The answer is, not necessarily. In the non-cooperative game, different mechanisms will be associated with different strategy spaces. Under a tax schedule, a husband chooses his transactions taking his wife's transactions as given, whereas under a mechanism he chooses his announcement taking his wife's announcements as given. Hence, even if $\mathcal{Z}^c_{\mathcal{M}} = \mathcal{Z}^c_{\Psi}$, different equilibrium transactions will, in general result. Different threat points may, therefore, arise in the disagreement game.

Hence, given an optimal tax system, it may still be possible to improve upon the resulting allocation if one is able to change threat points in useful directions.[13] Of course we have no reason to restrict reforms to those that preserve the set of attainable transactions. In fact we next present two examples of how threat points may improve upon a given allocation. In our second example, threat points are used to break a couple' indifference between two transactions. A typical reform would use the fact that the household is no longer indifferent between the two transactions to redesign the set of available transactions. For our first example, however, we consider a reform that preserves the set of available transactions.

Consider an economy in which, for all households, utility is transferable across spouses. The convenient aspect of such example is that households choose transactions that maximize their surplus regardless of how it will later be split — see Lemma 2. Let $\mathfrak{l}_i = 1 - n_i/\theta_i$ for $i = f, m.$, denote the amount of leisure that spouse i whose productivity is θ_i consumes. Then, given transactions $\boldsymbol{z} = (c_f, -n_f, c_m, -n_m)$, the household solves

$$\max_{\mathfrak{c}_f} \left[\mathfrak{c}_f + h\left(1 - \frac{n_f}{\theta_f}\right) - \bar{u}_f \right] \left[c_f + c_m - \mathfrak{c}_f + h\left(1 - \frac{n_m}{\theta_m}\right) - \bar{u}_m \right],$$

where $(\bar{u}_f, \bar{u}_m)$, are the relevant threat points and $h(\cdot)$ is an increasing concave function that captures the utility of leisure.

Consider a planner whose objective is to maximize the average logarithm of agents utilities. The concave social welfare function is needed for this quasi-linear example. In this case, the household and the planner objectives are aligned if and only if $\bar{u}_f - \bar{u}_m = 0$. If under the optimal tax schedule $\boldsymbol{z}$ maximizes

[13] A simple example of such reforms is found in da Costa and Diniz (2015), in the context of transferable utilities. There, filing options are used to change the threat points.

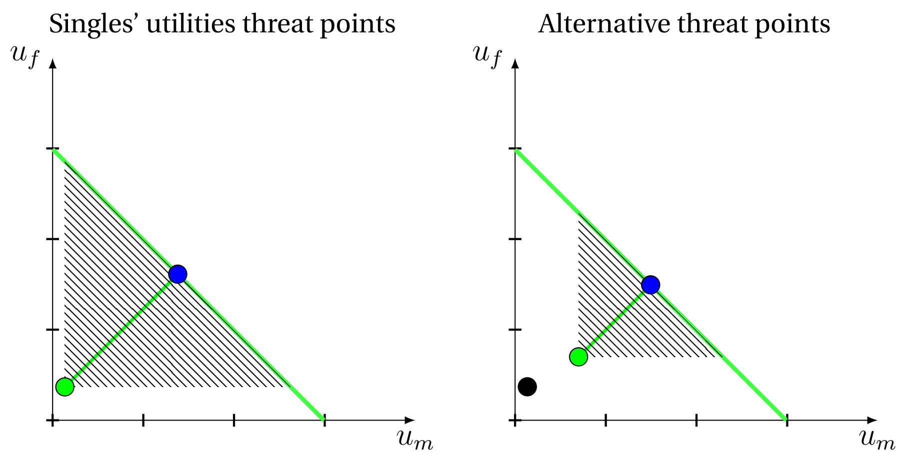

Figure 1: **Aligning Objectives** The panel in the left displays the set of individually rational utility pairs, when the threat point — green dot — is induced by the tax schedule in place. The blue dot denotes optimal choice for the couple. If the planner is able to induce a different threat point — green dot in the right panel — a new choice distribution of utilities is induced for the same transactions.

the household's surplus, therefore, inducing this transaction as the household's choice, then we can represent the utility possibility set for as in Figure 1. The optimal choice is given by a straight 45 degrees line from $(\bar{u}_f, \bar{u}_m)$ to the frontier of the utility possibility set; the downward sloping green line. Given its objective, the planner desires more symmetric utility allocations, but threat point differences induce utility differences. If the planner could use some instrument to reduce threat point utility differences without changing couples' optimal transactions, then the planner's objective would have its value increased for the same revenue raised. The right panel in Figure 1 illustrates such point.

Transferable utilities are nice for they provide a simple example where transactions are independent of threat points. For the exact same reason they are not a good choice if our goal is to show how to induce more desirable transactions. So, as a second example consider the general case for which utility is only partially transferable. For a given $\boldsymbol{z} = (c_f, -n_f, c_m, -n_m)$, the frontier of the utility possibility set is no longer a straight line of slope -1. Instead, we represent the utility possibility set for the couple who has chosen $\boldsymbol{z}$ as the convex set bounded by the green curve in Figure 2. Note that this is a conditional utility possibility set, since it is built holding $\boldsymbol{z}$ fixed. The set bounded by the blue curve is the utility possibility set for the same couple it it chose, instead, $\boldsymbol{z}'$. Point A, in the left panel denotes the optimal choice of utilities for the household given its

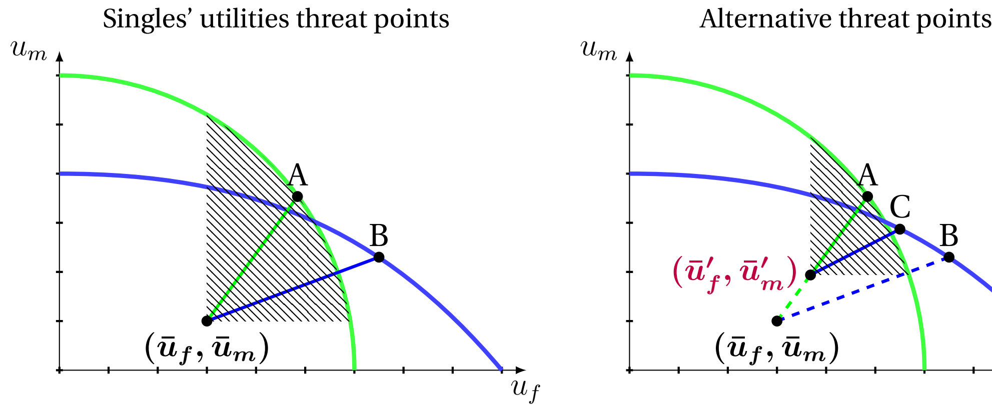

Figure 2: The blue curve denotes the utility frontier for the same couple for the case in which a false report $\eta' \neq \eta$ is made. Point B denotes the utility pair the couple would attain in case of a lie. If the threat points changes from $(\bar{u}_f, \bar{u}_m)$ to $(\bar{u}'_f, \bar{u}'_m)$ equilibrium choices are not changed, yet the deviation utility pair changes from B to C.

choice of $\boldsymbol{z}$, whereas point B denotes the preferred utility pair were the couple to choose transactions $\boldsymbol{z}'$. Finally, the green line connecting $(\bar{u}_f, \bar{u}_m)$ to point A defines the set of all potential threat points (greater than $(\bar{u}_f, \bar{u}_m)$) that induce A as the optimal choice for this couple, given $\boldsymbol{z}$. If the couple were instead choosing $\boldsymbol{z}'$, then the blue line connecting $(\bar{u}_f, \bar{u}_m)$ to B would represent the analogous set of threat points that make B the preferred utility pair.

Assume that the couple is just indifferent between A and B. It is therefore just indifferent between transactions $\boldsymbol{z}$ and $\boldsymbol{z}'$, both assumed to be feasible given the tax schedule in place.[14] For concreteness, let $\boldsymbol{z}$ be the couple's choice, which we assume to be the one which raises more tax revenues and generate at least as much welfare as judged by the planners' metric. The planner cannot, in this case, increase this couple's tax liabilities since this would lead the couple to strictly prefer $\boldsymbol{z}'$.

Now, if the planner were somehow able to induce another threat point along the green curve, e.g., the one denoted $(\bar{u}'_f, \bar{u}'_m)$ in Figure 2, then Point A remains the optimal choice for the couple if it chooses $\boldsymbol{z}$, while point B is no longer the optimal choice for the couple if it chooses $\boldsymbol{z}'$. It is, in fact, possible to show that A is now strictly better than B; hence, $\boldsymbol{z}$ strictly preferred to $\boldsymbol{z}'$. Some space for raising more taxes is thus created.

The rationale is, of course, that by moving along the curve we are in fact

[14]Axiom 'Preference for symmetry', as defined by Zambrano (2016), substitutes for 'symmetry' to deal with any possible non-convexity in the utility possibility sets.

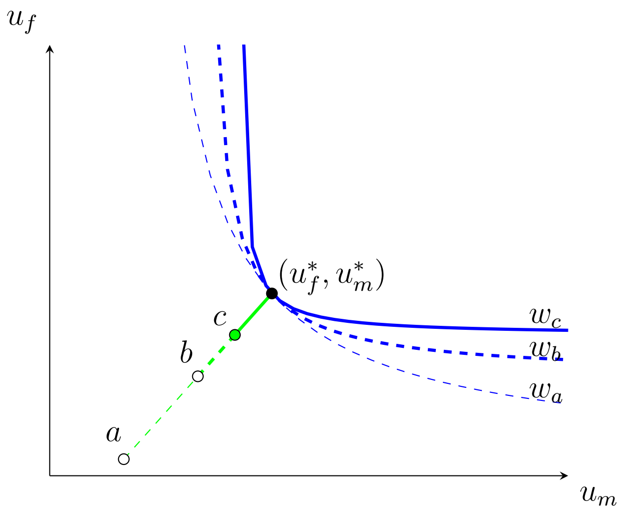

Figure 3: **Changing Elasticities.** The figure shows how the set of utility pairs which are preferred to (u_f^*, u_m^*) shrinks as we move along the curve connecting point a to point (u_f^*, u_m^*).

reducing the flexibility that the couple has to substitute the utility of one spouse for the other, as shown in Figure 3. We are in practice changing the relevant elasticities.

A welfare improving reform will typically combine elements of the two examples we have offered.

IV Household Choices

Different institutions – tax systems, game forms, etc. – will define the set of transactions that are available to all agents, directly, in the case of tax schedules or indirectly for more general mechanisms. To further describe the feasible transactions sets we appeal to the notion of an *institutional setting*, for which we use a generic domination, $\mathcal{E}$.

IV.1 Institutional Settings

An institutional setting $\mathcal{E}$ is comprised of sets $S^i_{\mathcal{E}}$, with typical element $s^i_{\mathcal{E}}$, representing the choice set for a gender i single agent, and $S^{c,i}_{\mathcal{E}}$, with typical element $s^{c,i}_{\mathcal{E}}$, the choice set for a gender i married agent. Completing the description of $\mathcal{E}$, are functions $\zeta^i_{\mathcal{E}} : S^i_{\mathcal{E}} \mapsto \mathcal{Z}$, mapping choices, $s^i_{\mathcal{E}}$, into transactions for gender i single agents, and $\boldsymbol{\zeta}_{\mathcal{E}} : S^{c,f}_{\mathcal{E}} \times S^{c,m}_{\mathcal{E}} \mapsto \mathcal{Z} \times \mathcal{Z}$ for married agents.

Now $\mathcal{Z}^i_{\mathcal{E}} := \zeta^i_{\mathcal{E}}(S^i_{\mathcal{E}}) \subset \mathbb{R}^2$, $i = f, m$, and $\mathcal{Z}^c_{\mathcal{E}} := \boldsymbol{\zeta}_{\mathcal{E}}(S^{c,f}_{\mathcal{E}} \times S^{c,m}_{\mathcal{E}}) \subset \mathbb{R}^4$ define the transactions which are attainable for singles and couples. That is, for a gender

i single agent to realize a transaction z under $\mathcal{E}$ it must choose $s \in S^i_{\mathcal{E}}$ such that $z = \zeta^i_{\mathcal{E}}(s)$, provided that such s exists. If no such s exists then, $z \notin \mathcal{Z}^i_{\mathcal{E}}$. Similarly $\boldsymbol{z}$ may be chosen by a couple if $s_f \in S^{c,f}_{\mathcal{E}}$ and $s_m \in S^{c,m}_{\mathcal{E}}$ exist such that $\boldsymbol{z} = \boldsymbol{\zeta}_{\mathcal{E}}(s_f, s_m)$. Recalling that $\mathcal{Z}^c_{\mathcal{E}}$ is the set of transactions, $\boldsymbol{z} \in \mathcal{Z}^2$, which are feasible under the institutional setting $\mathcal{E}$ we define for any couple η, $\mathcal{U}_{\mathcal{E}}(\eta) := \bigcup_{\boldsymbol{z}' \in \mathcal{Z}^c_{\mathcal{E}}} \mathcal{U}_{\boldsymbol{z}'}(\eta) \subset R^2$ be the set of all attainable utilities under $\mathcal{E}$.

The next two examples of institutional environments illustrate the definitions we are using.

A Simple Tax System — STS A simple tax system is comprised of (possibly) gender based tax schedules for singles and a tax schedule for couples.

We represent a tax schedule by the budget constraint it induces. For a single agent of gender i let $T_i : R_+ \mapsto R$ be the associated tax schedule. Then, we define the budget set, $\mathbb{B}_i$ as

$$\mathbb{B}_i := \{z; \psi_i(c, n) = \psi_i(z) = c - n - T_i(n) \leq 0\}. \tag{2}$$

In this case, $S^i_{\Psi} = \mathbb{B}_i$, and ζ^i_{Ψ} is the identity mapping. We also define $\partial\mathbb{B}_i := \{z \in \mathbb{B}_i; z' > z \Rightarrow z' \notin \mathbb{B}_i\}$, the frontier of $\mathbb{B}_i$.

A single agent of type θ_i solves

$$\max_{(\mathfrak{c},\mathfrak{l}) \leq (c, 1-n), z \in \mathbb{B}_i} u(\mathfrak{c}, \mathfrak{l}; \theta_i).$$

We let $z^i_{\Psi}(\theta_i)$ denote the equilibrium transactions and $v^i_{\Psi}(\theta_i)$, the utility attained by a gender i single agent, $i = f, m$, under the tax schedule defined in (2).

For couples, let $\boldsymbol{z} = (z_f, z_m)$. In this case,

$$\mathbb{B}^c := \left\{\boldsymbol{z} \in \mathcal{Z}^2; \boldsymbol{\psi}(\boldsymbol{z}) = c_f + c_m - n_f - n_m - T^c(n_f, n_m) = 0\right\}. \tag{3}$$

Analogously to $\partial\mathbb{B}_i$, define $\partial\mathbb{B}^c := \{\boldsymbol{z} \in \mathbb{B}^c; \boldsymbol{z}' > \boldsymbol{z} \Rightarrow \boldsymbol{z}' \notin \mathbb{B}^c\}$.

The choice set $S^{c,i}_{\Psi}$ is simply $S^{c,f}_{\Psi} := Z$, whereas $\boldsymbol{\zeta}_{\Psi}$ is the identity mapping for $\boldsymbol{\psi}(\boldsymbol{z}) \leq 0$, and $\boldsymbol{z} = 0$, otherwise.

A General Mechanism A *game form* or *mechanism*, $\mathcal{M}$, is a collection of message sets Σ^s_i for gender i, $i = f, m$, singles; message sets, Σ^c_i, for gender i, $i = f, m$,

married agents; outcome functions $g_i^s : \Sigma_i^s \to \mathcal{Z}$ for gender i singles, and; outcome functions $\boldsymbol{g} : \Sigma_f^c \times \Sigma_m^c \to \mathcal{Z}^2$, for couples.

That is,

$$\mathcal{M} \equiv \left(\{\Sigma_i^s, \Sigma_i^c\}_{i=f,m}, \{g_i^s(\cdot)\}_{i=f,m}, \boldsymbol{g}^c(\cdot,\cdot) \right). \tag{4}$$

This definition of a mechanism is mapped into our general notion of an institutional setting by noting that the choice sets are the message spaces, $S^i_{\mathcal{M}} = \Sigma_i^s$, $S^{c,i}_{\mathcal{M}} = \Sigma_i^c$, $i = f, m$, whereas the outcome functions are $\zeta^i_{\mathcal{M}}(\cdot) := g_i^s(\cdot)$, for singles, and $\boldsymbol{\zeta}_{\mathcal{M}}(\cdot,\cdot) := \boldsymbol{g}^c(\cdot,\cdot)$, for couples.[15]

IV.2 Choices in Agreement

Then, for any $(\bar{u}_f, \bar{u}_m)$ in the interior of $\mathcal{U}_{\boldsymbol{z}}(\eta)$, let

$$W(\boldsymbol{z}; \eta, \bar{u}_f, \bar{u}_m) := \begin{cases} \max & \left(u_f\left(\mathfrak{c}_f, \mathfrak{l}_f\right) - \bar{u}_f\right)\left(u_m\left(\mathfrak{c}_m, \mathfrak{l}_m\right) - \bar{u}_m\right) \\ \text{s.t.} & \left(\left(\mathfrak{c}_f, \mathfrak{l}_f\right), \left(\mathfrak{c}_m, \mathfrak{l}_m\right)\right) \in F^a(\boldsymbol{z}, \eta) \end{cases}. \tag{5}$$

For each couple η, equation (5) defines a utility function, $W : \mathcal{Z}^2 \to \mathbb{R}$ parametrized by $(\bar{u}_f, \bar{u}_m)$. $W(\cdot; \eta, \bar{u}_f, \bar{u}_m)$, therefore, represents a complete preorder in the space of transactions, $\mathcal{Z}^2$, for a couple η which threat points are $\bar{u}_f$ and $\bar{u}_m$. We shall henceforth refer to this ordering as 'household preferences', $\succeq_{\eta|\bar{u}_f,\bar{u}_m}$, recognizing its dependence on the threat points.

We assume that there is $\underline{z} \in \mathcal{Z}^2$ such that for any η, any $((\mathfrak{c}_f, \mathfrak{l}_f), (\mathfrak{c}_m, \mathfrak{l}_m)) \in F^a(\underline{z}, \eta)$, either $u_f(\mathfrak{c}_f, \mathfrak{l}_f) \leq u_f(\hat{\mathfrak{c}}_f, \hat{\mathfrak{l}}_f)$ for all $\hat{\mathfrak{c}}_f, \hat{\mathfrak{l}}_f \in R^2_+$ or $u_m(\mathfrak{c}_m, \mathfrak{l}_m) \leq u_m(\hat{\mathfrak{c}}_m, \hat{\mathfrak{l}}_m)$ for all $\hat{\mathfrak{c}}_m, \hat{\mathfrak{l}}_m \in R^2_+$.

IV.3 The Disagreement Game

A type η couple maximizes the objective (1), which is only fully specified once threat points are determined. How this determination occurs for each environment is what we discuss now.

[15] In principle, the outcome function could depend on messages from all agents. Because there is a continuum of agents and because the distribution of types is known, assuming the outcome functions to depend only on what each agent (or couple) announces is without loss.

Define for any $\boldsymbol{z} \in \mathcal{Z}^2$,

$$\Phi\left(z_f, z_m; \eta\right) \equiv \left(\begin{array}{c} \phi_f\left(z_f, z_m; \eta\right) \\ \phi_m\left(z_f, z_m; \eta\right) \end{array}\right) = \left(\begin{array}{c} v_f \\ v_m \end{array}\right) \tag{6}$$

a function which maps transactions $\boldsymbol{z} \in \mathcal{Z}^2$ into a pair of utilities, $(v_f, v_m) \in R^2$. This function will play the role of defining the disagreement utilities that result from any transactions realized by the spouses. We endow $\Phi(\cdot, \cdot; \eta)$ with the following properties: *i)* for all $\boldsymbol{z}$, and all η, $\Phi(\boldsymbol{z}, \eta) \in \mathring{\mathcal{U}}_{\boldsymbol{z}}(\eta)$, where $\mathring{\mathcal{U}}_{\boldsymbol{z}}(\eta)$ is the interior of $\mathcal{U}_{\boldsymbol{z}}(\eta)$; *ii)* $\phi_f\left(\cdot, z_m; \eta\right)$ is increasing in z_f for all z_m, and; *iii)* $\phi_m\left(z_f, \cdot; \eta\right)$ is increasing in z_m for all z_f.

To make sense of these properties, starting with *(ii)* and *(iii)*, note that z_i is increased either by making more consumption goods available without an increase in spouse i's effort, n_i or by reducing i's effort without a reduction in resources available for consumption of both spouses. We assume that in either case this increases spouse i's utility. It may or may not increase his or her spouse's utility. Because, leisure is not transferable across spouses, a lower n_i can only reduce i's utility if his or her consumption is substantially reduced. Similarly, provided that a higher c_i does not lead to a lower $\mathfrak{c}_i$ the assumption is valid for this case as well.

As for *(i)*, agents act non-cooperatively when they are in disagreement. They will not, in general, be able to reach all points in $\mathcal{U}_{\boldsymbol{z}}(\eta)$. For example, they need not be able to attain all material gains from cohabitation. We take this into account by assuming that the set of available allocations for spouses in disagreement is given by a function $F^d(\boldsymbol{z}, \eta) \subset F^a(\boldsymbol{z}, \eta)$.[16]

For a concrete example of such a function, assume that in disagreement, $\mathfrak{c}_i = c_i/(c_f + c_m)$, i.e., consumption of each spouse is proportional to what each one contributes to the set of available consumption goods. In this case, provided that $F^d(\boldsymbol{z}, \eta) \subset F^a(\boldsymbol{z}, \eta)$, Φ has properties *(i)* to *(iii)*.

Equilibrium of the Disagreement Game Now to understand how threat points are defined, one must describe how $\boldsymbol{z}$ is determined. Spouse i chooses an ele-

[16] For example, we may consider

$$F^d(\boldsymbol{z}, \eta) := \left\{\left(\left(\mathfrak{c}_f, \mathfrak{l}_f\right), \left(\mathfrak{c}_m, \mathfrak{l}_m\right)\right);\ \mathfrak{c}_f + \mathfrak{c}_m \leq \beta\left[c_f + c_m\right]; \mathfrak{l}_f \leq 1 - n_f/\theta_f; \mathfrak{l}_m \leq 1 - n_m/\theta_m\right\},$$

where $1 < \beta < \alpha$ incorporates the fact that, despite some inefficiencies that arise due to disagreement, spouses might still enjoy some benefits from cohabitation.

ment of $S_{\mathcal{E}}^{c,i}$ which, along with i's spouses' choice, $s_{\mathcal{E}}^{c,j}$ is mapped into transactions by $\boldsymbol{\zeta}_E$ and finally into i's utility by ϕ^i. Note how the strategy space available to each spouse depends on the specific institutional arrangements, $\mathcal{E}$, that we are considering. The crucial assumption regarding choices in disagreement is that, in contrast with choices made in agreement, the pair (z_f, z_m) is selected non-cooperatively by the couple. That is, $\boldsymbol{z}$ is the (outcome of) a Nash equilibrium of a non-cooperative game played by spouses under $\mathcal{E}$.

A key underlying assumption of our approach is that spouses are *not* able to commit to a strategy to be used in disagreement.[17] We further require spouses to behave rationally if in disagreement in the sense that each spouse still maximizes his or her own utility.

A final important assumption we make is that no agent can be made worse off than what he or she can attain as a single agent. At any moment and under all institutions that we consider, a spouse may unilaterally call off marriage and attain a utility which is increasing in the utility that someone with her or his productivity attains as a single. One cannot, however, fake one's marital status. Moreover, a spouses' decision to become single imposes the same choice on the other spouse.

In most of what follows, to economize on notation, we consider the case in which a single agent and a divorced agent of identical productivities attain the same utility. This is but one possibility for the sensible view that the utility that one may reach upon divorce is an increasing function of the utility that a single agent of the same type can attain.[18]

Let

$$\chi_{\mathcal{E}}^{f}(s_{\mathcal{E}}^{c,m}, \eta) := \operatorname*{argmax}_{s \in S_{\mathcal{E}}^{c,f}} \phi_f \left(\boldsymbol{\zeta}_{\mathcal{E}}(s, s_{\mathcal{E}}^{c,m}); \eta\right)$$

define the wife's reaction function for the game played under $\mathcal{E}$ with analogous definition, $\chi_{\mathcal{E}}^{m}(s_{\mathcal{E}}^{c,f}, \eta)$, for the husband. An equilibrium, $s_f^* = \chi_{\mathcal{E}}^{f}(s_m^*, \eta), s_m^* = \chi_{\mathcal{E}}^{m}(s_f^*, \eta)$, for the disagreement game defines $(\bar{z}_{\mathcal{E}}^{c,f}(\eta), \bar{z}_{\mathcal{E}}^{c,m}(\eta)) = \boldsymbol{\zeta}_{\mathcal{E}}(s_f^*, s_m^*)$.

Whether a pure strategy equilibrium exists and, when it exists, whether it is unique depends not only on the properties of Φ but also on those of $\mathcal{E}$. Thus, for each institution, we make assumptions directly on the game effectively played. Provided an equilibrium does exist, $\bar{\boldsymbol{z}}_{\mathcal{E}}(\eta) = (\bar{z}_{\mathcal{E}}^{c,f}(\eta), \bar{z}_{\mathcal{E}}^{c,m}(\eta))$ are the transactions

[17] This rules out many different approaches for determining the threat points — see Myerson (1997) for a discussion.

[18] We may also allow the utility attained upon divorce to depend on η.

that would be conducted by a couple η if they were not able to reach an agreement under the institutional setting $\mathcal{E}$. We finally use $(\bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{\mathcal{E}}(\eta))$ to denote the threat points that arise under $\mathcal{E}$.

IV.4 Equilibrium Allocations Under $\mathcal{E}$

Define for each environment $\mathcal{E}$

$$\hat{\boldsymbol{s}}_{\mathcal{E}}(\eta, \bar{u}_f, \bar{u}_m) := \underset{s_f, s_m \in (S^{c,f}_{\mathcal{E}} \times S^{c,m}_{\mathcal{E}})}{\text{argmax}} W(\boldsymbol{\zeta}_{\mathcal{E}}(s_f, s_m); \eta, \bar{u}_f, \bar{u}_m), \tag{7}$$

the *conditional choices* by a couple η when threat points are $(\bar{u}_f, \bar{u}_m)$. We write, $\hat{\boldsymbol{z}}_{\mathcal{E}}(\eta, \bar{u}_f, \bar{u}_m) := \boldsymbol{\zeta}_{\mathcal{E}}(\hat{\boldsymbol{s}}_{\mathcal{E}}(\eta, \bar{u}_f, \bar{u}_m))$ to denote conditional transactions.

As we have just seen, the institutional setting, $\mathcal{E}$, also determines threat points through the disagreement game it induces. For $(\bar{u}^f, \bar{u}^m) = (\bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{\mathcal{E}}(\eta))$, i.e., when threat points are those that arise in the equilibrium of the disagreement game, we arrive at

$$\boldsymbol{s}_{\mathcal{E}}(\eta) := \underset{s_f, s_m \in (S^{c,f}_{\mathcal{E}} \times S^{c,m}_{\mathcal{E}})}{\text{argmax}} W(\boldsymbol{\zeta}_{\mathcal{E}}(s_f, s_m); \eta, \bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{E}(\eta),$$

the equilibrium choices for spouses in a couple η under institutions $\mathcal{E}$.[19] Equilibrium transactions $\boldsymbol{z}_{\mathcal{E}}(\eta)$, are, then $\boldsymbol{z}_{\mathcal{E}}(\eta) := \boldsymbol{\zeta}_{E}(s^{c,f}_{\mathcal{E}}(\eta), s^{c,m}_{\mathcal{E}}(\eta))$. We may therefore the *allocation implemented by* $\mathcal{E}$,

$$\left\{ \left(z^i_{\mathcal{E}}(\theta_i) \right)_{\theta_i, i=f,m}, \left(\boldsymbol{z}_{\mathcal{E}}(\eta) \right)_{\eta} \right\}, \tag{8}$$

We let $\{(v^i_{\mathcal{E}}(\theta_i))_{\theta_i}, \left(v^{c,i}_{\mathcal{E}}(\eta) \right)_{\eta}\}_{i=f,m}$, represent the associated utility profile.

Aggregate transactions under $\mathcal{E}$ are

$$Z_{\mathcal{E}} = \sum_{i=f,m} \left\{ \int z^i_{\mathcal{E}}(\theta_i) d\mu^i_{\text{s}}(\theta_i) + \int z^{c,i}_{\mathcal{E}}(\eta) d\mu_{\text{c}}(\eta) \right\}$$

An allocation is *feasible* if $G(Z_{\mathcal{E}}) \leq 0$, where $G(\cdot)$ is the transformation function that represents the economy's technology.

[19] Noting that $\boldsymbol{s}_{\mathcal{E}}(\eta) = (s^{c,f}_{\mathcal{E}}(\eta), s^{c,m}_{\mathcal{E}}(\eta))'$ and $\boldsymbol{z}_{\mathcal{E}}(\eta) = (z^{c,f}_{\mathcal{E}}(\eta), z^{c,m}_{\mathcal{E}}(\eta))'$, we have, for all $\mathcal{E}$ and all η, $(s^{c,f}_{\mathcal{E}}(\eta), s^{c,m}_{\mathcal{E}}(\eta)) = \hat{\boldsymbol{s}}_{\mathcal{E}}(\eta, \bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{\mathcal{E}}(\eta))$, $(z^{c,f}_{\mathcal{E}}(\eta), z^{c,m}_{\mathcal{E}}(\eta)) = \hat{\boldsymbol{z}}_{\mathcal{E}}(\eta, \bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{\mathcal{E}}(\eta))$.

Threat points and commitment Unless otherwise stated we will retain the assumption that couples cannot commit to a given behavior in disagreement.

If such commitments were however possible, the set of implementable allocations would be significantly restricted.

Proposition 1. *For an environment* $\mathcal{E}$*, let* $(\underline{v}^f_{\mathcal{E}}(\eta), \underline{v}^m_{\mathcal{E}}(\eta))$ *be the pair of equilibrium utilities that arises for a couple* $\eta = (\theta_f, \theta_m)$ *at the maximization (in* (s_f, s_m)*) of* $W(\boldsymbol{\zeta}_{\mathcal{E}}(s_f, s_m); \eta, v^f_{\mathcal{E}}(\theta_f), v^m_{\mathcal{E}}(\theta_m))$*. Let*

$$V^{\alpha}_{\mathcal{E}} := \left\{(u_f, u_m) | \exists \alpha \in [0,1]; (u_f, u_m) = \alpha(\underline{v}^f_{\mathcal{E}}(\eta), \underline{v}^m_{\mathcal{E}}(\eta)) + (1-\alpha)(v^f_{\mathcal{E}}(\theta_f), v^m_{\mathcal{E}}(\theta_m))\right\}.$$

If spouses can commit on disagreement choices, then $(\bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^f_{\mathcal{E}}(\eta)) \in V^{\alpha}_{\mathcal{E}}$*.*

Proof. See Appendix A.1. □

Assume that $(\bar{u}_f, \bar{u}_m)$ in figure 2 is the utilities that spouses would attain if they divorced, which we are assuming to be the utilities that single agents with identical productivities attain under $\mathcal{E}$, $(v^f_{\mathcal{E}}(\theta_f), v^m_{\mathcal{E}}(\theta_m))$. In this case, $(\underline{\mathrm{v}}^f_{\mathcal{E}}(\eta), \underline{\mathrm{v}}^m_{\mathcal{E}}(\eta))$ is point A in the same figure, and the set $V^{\alpha}_{\mathcal{E}}$ is the green line connecting $(\bar{u}_f, \bar{u}_m)$ to point A.

V Agreement Randomizations

Thus far we have allowed for non-convex utility possibility sets by replacing Nash's (1950) symmetry by Zambrano's (2016) preference for symmetry. An alternative approach would be to rule out non-convexity altogether by assuming that households may randomize across different transactions.

To allow for randomization we extend the utility representation to a expected utility representation, and apply the Nash-Bargaining approach over this utility.

Let's consider an example. Suppose that a couple has access to both the frontier $\mathcal{U}_{\boldsymbol{z}}(\eta)$ and $\mathcal{U}_{\boldsymbol{z}'}(\eta)$ in Figure 4, generated by transactions $\boldsymbol{z}$ and $\boldsymbol{z}'$, respectively. Suppose that the planner wants the couple to choose the bundle $\boldsymbol{z}$.

If the couple cannot randomize, then the relevant Pareto frontier is the envelope of $\mathcal{U}_{\boldsymbol{z}}(\eta)$ and $\mathcal{U}_{\boldsymbol{z}'}(\eta)$. To implement $\boldsymbol{z}$, the planner only needs to make sure that there is a point in $\mathcal{U}_{\boldsymbol{z}}(\eta)$ that is better, in the sense of yielding a higher value for the Nash product, than any point on $\mathcal{U}_{\boldsymbol{z}'}(\eta)$, for the couple's preference.

That is illustrated as point $\boldsymbol{u}_1 := (u_1^f, u_1^m)$ in Figure 4. Point $\boldsymbol{u}_2$ is the point that maximizes the Nash product if $\boldsymbol{z}'$ is chosen, instead.

If spouses are, however, able to randomize, then, in our example, they can attain a better utility pair. This is shown in figure 5. The relevant Pareto Frontier is the convex hull of $\mathcal{U}_{\boldsymbol{z}}(\eta)$ and $\mathcal{U}_{\boldsymbol{z}'}(\eta)$. Notice that the particular randomization presented in the picture implies that one of the spouses maybe worse in the marriage than his/her disagreement utility.

In this case, to implement $\boldsymbol{z}$, the planner needs to guarantee no point in $\mathcal{U}_{\boldsymbol{z}'}(\eta)$ is above the tangent of $\mathcal{U}_{\boldsymbol{z}}(\eta)$ and the relevant indifference curve. An example where it happens is given on 6, whereas Figure 5 provides an example where it does not.

Whether this randomization is a reasonable description about household choices or not crucially depends on how we interpret randomization in this setting.

As in Mirrlees (1971) the natural interpretation of our model is that it is a static representation of life-long choices made by individuals. In this sense randomization could be seen as an ex-ante commitment to a one of the two allocations $\boldsymbol{u}_a$ or $\boldsymbol{u}_b$. For this reason, this case is called *Randomization with strong commitment*.

Although possible this seems to contradict the very idea of a collective model. Indeed, one of the underlying justifications for ruling out the unitary model as a valid description of household behavior relies on the inability to commit as explained in **?**. Hence, if we only assume some form of 'spot' commitment, then the idea that households decide once and for all to choose an allocation such as $\boldsymbol{u}_a$ or $\boldsymbol{u}_b$ does not seem plausible.[20]

Alternatively we may be assuming non-stationarity of allocations. Spouses alternate between $\boldsymbol{u}_a$ and $\boldsymbol{u}_b$ with the right frequency. Again, some form of commitment beyond spot commitment is needed. So, the only interpretation of randomized choices compatible with 'spot commitment' is literal lottery used in the beginning of each period to decide whether $\boldsymbol{u}_a$ or $\boldsymbol{u}_b$ will be chosen. If this is the case, then all utility possibility sets must be convex and threat points will have no baring on incentive compatibility in the convexified range.

A weaker assumption regarding commitment is that couples choose between

[20]Spouses must commit at least on how to share the consumption goods that are brought home and on the total amount of these goods that will be brought home at each moment in time.

$\boldsymbol{z}$ and $\boldsymbol{z}'$, and given this choice decide how to share consumption in order to maximize the Nash product. This leads to the following restriction on the planner's ability to implement the allocations it may desire. Assume that the planner aims at implementing $\boldsymbol{z}$, in which case $\boldsymbol{u}_1$ would be chosen. If the couple were instead to choose $\boldsymbol{z}'$ then $\boldsymbol{u}_2$ would be chosen instead. Moreover, any lottery involving $\boldsymbol{z}$ and $\boldsymbol{z}'$ would lead to a lower value for the household then $\boldsymbol{u}_1$. In this case, we would say that $\boldsymbol{u}_1$ was implementable. Otherwise, it would not be.

In other words the couple can only commit to utility pairs that maximize the Nash product for each transaction. Randomization still restricts the set of implementable allocations, but less so than in the case of full commitment. The relevant Pareto Frontier is the convex combination of those. By construction, these points are preferred to the disagreement utility, and are consistent with a new bargaining process after uncertainty is resolved.

To implement $\boldsymbol{z}$, the planner needs to keep f_2 below the tangent to $\mathcal{U}_{\boldsymbol{z}}(\eta)$ at f_1. We show this on figure 7, notice that there are $\mathcal{U}_{\boldsymbol{z}'}(\eta)$ points above the tangent.

Up to now we have assumed that disagreement utilities are given. We identified two disagreement effects on the non-random case. Apparently, the disagreement can be used on all cases to affect which point on $\mathcal{U}_{\boldsymbol{z}}(\eta)$ is chosen, this was one of the channels.

The other one was based on changes that kept $\boldsymbol{u}_1$ fixed, but made other points less desirable, from the couple's perspective. This channel is ineffective on the Randomization with commitment case, because the disagreement has no effect on the points chosen to randomize. We get the same results with transferable utilities, for a different reason.

This elasticity channel still effective on the Randomization without commitment case, as changes on the disagreement that don't change $\boldsymbol{u}_1$ will affect the point chosen on $\mathcal{U}_{\boldsymbol{z}'}(\eta)$.

VI Implementable Allocations

To characterize the set of incentive feasible allocations in this economy we introduce an appropriate definition of mechanism for our setting. We retain the assumption that the marital status of an agent is public information. That is, no agent can claim to be single when he or she is in fact married or claim to be

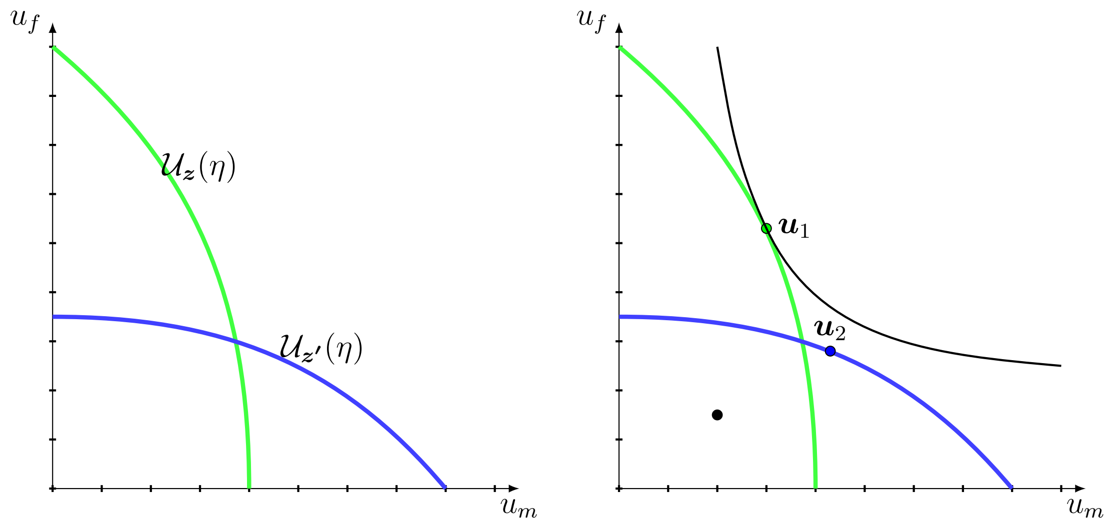

Figure 4

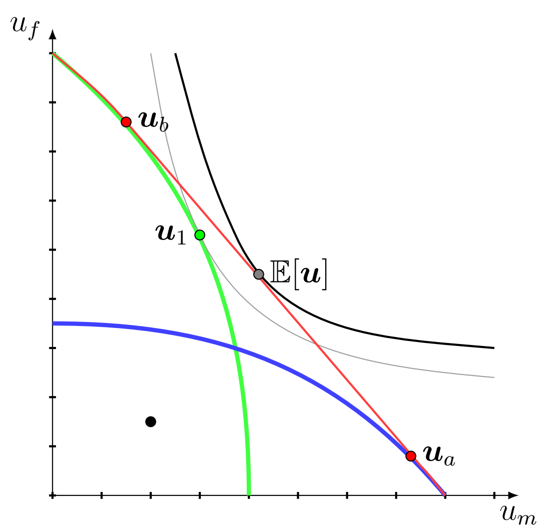

Figure 5: Randomization between transactions $\boldsymbol{z}$ and $\boldsymbol{z}'$ and commitment to an allocation of resources leading to utility pairs $\boldsymbol{u}_a$ and $\boldsymbol{u}_b$ leads to an expected utility pair $\mathbb{E}[\boldsymbol{u}] := \mathbb{E}[(u^f, u^m)]$.

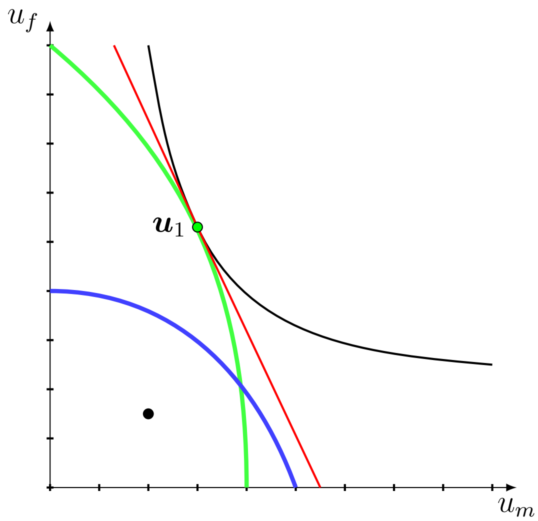

Figure 6: No choices or lotteries involving $\boldsymbol{z}'$ lead to a higher Nash product than $\boldsymbol{u}_1$. $\boldsymbol{z}$ is therefore implementable even if we allow for the strongest possible form of commitment between spouses.

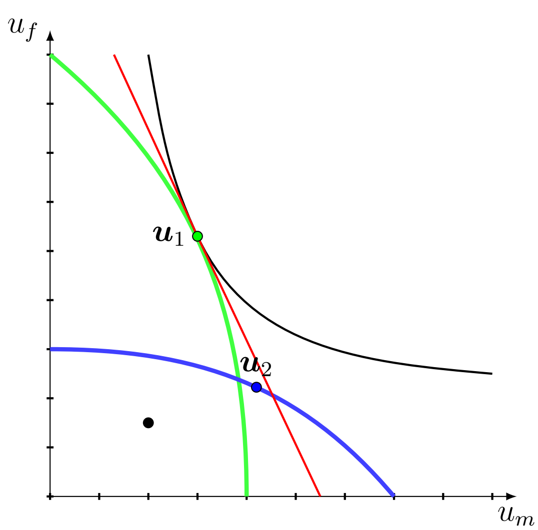

Figure 7: Points $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ are the Nash product maximizing choices for transactions $\boldsymbol{z}$ and $\boldsymbol{z}'$, respectively. Lotteries involving $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ yield a lower Nash product than $\boldsymbol{u}_1$.

married when in fact single.

VI.1 A Direct Mechanism - DM

In a direct mechanism, Γ, the government asks each agent his or type. Given the economy's informational structure, we define a single agent's type as his or her productivity, θ_i. For couples, a θ_f woman married to a θ_m man is said to have type $(\theta_f, \theta_m, \mathsf{a})$ if the couple is in agreement. If the couple is, instead, in disagreement the same agent is said to have a type $(\theta_f, \theta_m, \mathsf{d})$. That is, a married agent's type, $(\eta, \iota) \in \Theta^2 \times \mathcal{I}$, $\mathcal{I} = \{a, d\}$, is the same as his or her spouses'. Yet, since gender is public information, a (η, ι) woman can be distinguished from her husband, a (η, ι) man, for all purposes.

For singles, upon a report $\hat{\theta}_i$, an outcome functions assigns transactions $z = \gamma_i^s(\hat{\theta}_i)$. Married agents must also report their type, which, as we have seen, is the combination of their productivity, θ_i, their spouses' productivity, θ_{-i}, for $i, -i = f, m$, and whether they are in agreement or disagreement $\iota \in \mathcal{I} := \{\mathsf{a}, \mathsf{d}\}$. The outcome function $\boldsymbol{\gamma} : (\Theta^2 \times \mathcal{I}) \times (\Theta^2 \times \mathcal{I})$ maps announcements of types into transactions for both spouses. Hence, $S_\Gamma^i = \Theta$, $S_\Gamma^{c,i} = \Theta^2 \times \mathcal{I}$, $\zeta_\Gamma^i(\cdot) := \gamma_i^s(\cdot)$, and, $\boldsymbol{\zeta}_\Gamma(\cdot, \cdot) := \boldsymbol{\gamma}^c(\cdot, \cdot)$.

Before we formally define incentive-feasibility it is worth providing further details about the disagreement game under the DM. An announcement $(\hat{\eta}, \iota) \in \Theta^2 \times \mathcal{I}$ by a husband and an announcement $(\hat{\eta}', \iota') \in \Theta^2 \times \mathcal{I}$ by his wife induce transactions $\boldsymbol{\gamma}^c(\hat{\eta}', \iota', \hat{\eta}, \iota)$. For couples in disagreement, these transactions map into a pair of utilities (v_f, v_m) through Φ, therefore defining a game in the space of announcements, $\Theta^2 \times \mathcal{I}$, through the impact of announcements on transactions.

The reaction function for f in a couple η is, in this case,

$$\chi_\Gamma^f(\hat{\eta}_m, \hat{\iota}_m, \eta) := \underset{(\hat{\eta}, \hat{\iota}) \in \Theta^2 \times \mathcal{I}}{\operatorname{argmax}} \phi_f \left(\gamma_f^c(\hat{\eta}, \hat{\iota}, \hat{\eta}_m, \hat{\iota}_m), \gamma_m^c(\hat{\eta}, \hat{\iota}, \hat{\eta}_m, \hat{\iota}_m); \eta \right), \tag{9}$$

with analogous definition for $\chi_\Gamma^m(\hat{\eta}_f, \hat{\iota}_f, \eta)$.

An equilibrium for this game,

$$(\eta_f^*, \iota_f^*, \eta_m^*, \iota_m^*) = \left(\chi_\Gamma^f(\hat{\eta}_m^*, \iota_m^*, \eta), \chi_\Gamma^m(\hat{\eta}_f^*, \iota_f^*, \eta) \right)$$

defines a pair of threat points, $\left(\bar{u}_\Gamma^f(\eta), \bar{u}_\Gamma^m(\eta) \right) \in \mathring{\mathcal{U}}_\Gamma(\eta)$.

Note that lies are detected whenever conflicting announcements $(\eta', \iota', \eta'', \iota'')$ such that $\eta' \neq \eta''$ or $\iota' \neq \iota''$ are made. One of the spouses must be lying when this happens for they are making two reports over the same information which is common knowledge that they know.

Note that if any spouse chooses to report as single, the couple must split. That is, the condition for using a single agent's tax schedule is to be single.

We say that a Direct Mechanism - DM - is incentive-feasible if

1. For every single agent θ_i,

$$u(\gamma_i^s(\theta_i), \theta_i) \geq u(\gamma_i^s(\hat{\theta}_i), \theta_i) \; \forall \hat{\theta}_i, i = f, m. \tag{10}$$

2. For every couple η, and all $(\hat{\eta}, \hat{\hat{\eta}}, \iota, \hat{\iota}) \in \Theta^2 \times \mathcal{I}^2$,

$$W(\boldsymbol{\gamma}(\eta, a, \eta, a), \eta; \bar{u}_f, \bar{u}_m) \geq W(\boldsymbol{\gamma}(\hat{\eta}, \iota, \hat{\hat{\eta}}, \hat{\iota}), \eta; \bar{u}_f, \bar{u}_m), , \tag{11}$$

where,

$$\begin{pmatrix} \bar{u}_f \\ \bar{u}_m \end{pmatrix} = \begin{pmatrix} \phi_f \left(\boldsymbol{\gamma}(\eta, d, \eta, d); \eta\right) \\ \phi_m \left(\boldsymbol{\gamma}(\eta, d, \eta, d); \eta\right) \end{pmatrix} \tag{12}$$

3. For all η,

$$(\eta, d) \in \arg \max_{\hat{\eta} \in \Theta^2, \iota \in \mathcal{I}} \phi_f \left(\boldsymbol{\gamma}(\hat{\eta}, \iota, \eta, d); \eta\right) \tag{13}$$

and

$$(\eta, d) \in \arg \max_{\hat{\eta} \in \Theta^2, \iota \in \mathcal{I}} \phi_m \left(\boldsymbol{\gamma}(\eta, d, \hat{\eta}, \iota); \eta\right) \tag{14}$$

4. For any $\eta = (\theta_f, \theta_m)$,

$$v_\Gamma^{c,i}(\eta) \geq u(\gamma_i^s(\hat{\theta}_i), \theta_i) \; \forall \hat{\theta}_i, i = f, m. \tag{15}$$

5. If the distribution of singles of gender i is given by a measure μ_{s}^i and the distribution of couples is given by a measure μ_{c} then,

$$G\left(\sum_{i=f,m} \left\{\int \gamma_i^s(\theta_i) d\mu_{\mathrm{s}}^i(\theta_i) + \int \gamma_i^c(\eta, a, \eta, a) d\mu_{\mathrm{c}}(\eta)\right\}\right) \leq 0. \tag{16}$$

The first constraint is simply an incentive compatibility constraint for a single agent which implicitly assumes that agents cannot misreport their gender.

The second constraint guarantees that a type η couple in agreement does not misreport its type for threat points which are defined by the pair of utilities that arise when both decide to announce truthfully in case of disagreement. The third constraint is that truth-telling be an equilibrium announcement for spouses in disagreement. The forth constraint imposes the condition that no married agent would rather become single. Constraint five is the feasibility constraint. Although we do not require feasibility under disagreement, we do impose incentive-compatibility.

VI.2 The Revelation Principle

The main result from this Section is that the Revelation Principle is valid in this setting.

Proposition 2. *Let*

$$\left\{ \left(z^i_{\mathcal{M}}(\theta_i) \right)_{\theta_i, i=f,m}, \left(\boldsymbol{z}_{\mathcal{M}}(\eta) \right)_{\eta} \right\} \tag{17}$$

be an implementable allocation, i.e., an allocation for which there is a mechanism $\mathcal{M}$ for which (17) is the equilibrium allocation. There, there exists a Direct Mechanism - DM - which implements the same allocation.

Proof. See Appendix A.1. □

Implementation should be taken here in the weak sense. There is an equilibrium for the direct mechanism that generates the outcome corresponding to the target allocation. We cannot rule out the existence of other equilibria for the direct mechanism generating different outcomes. Nor can we rule out the introduction of equilibrium outcomes which were not equilibria in the original mechanism, $\mathcal{M}$.

As we have noted, an important aspect of implementation in the environment considered here is the ability of manipulating threat points under each institutional setting, an issue we return to next.

VI.3 'Choosing' threat points

Under our assumptions, each $\mathcal{E}$ induces a unique $(\bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{\mathcal{E}}(\eta)) \in \Phi(Z^2, \eta)$, for a couple η. Therefore, starting from any institutional setting $\mathcal{E}$ and associated

allocation, $(\{z^i_{\mathcal{E}}(\theta_i)\}_{\theta_i, i=f,m}, \{\boldsymbol{z}_{\mathcal{E}}(\eta)\}_\eta)$, an improvement can only occur through the replacement of $\mathcal{E}$ by an alternative $\mathcal{E}'$.

By replacing one institutional setting by another one typically changes not only $\succeq_{\eta|\bar{u}^f_{\mathcal{E}}, \bar{u}^m_{\mathcal{E}}}$ but also $\mathcal{Z}^c_{\mathcal{E}}$. Therefore, even choices may change even if we were able to hold $\succeq_{\eta|\bar{u}^f_{\mathcal{E}}, \bar{u}^m_{\mathcal{E}}}$ fixed.

To isolate the role of different institutional settings in affecting allocations through their impact on threat points it will be useful to consider $\mathcal{E}$ and $\mathcal{E}'$ such that the boundary of the set of available transactions is not altered.

An example of such reform occurs when choice sets and outcome functions change but feasible transactions remain the same, i.e., $S^{c,i}_{\mathcal{E}'} \neq S^{c,i}_{\mathcal{E}}$, but $\mathcal{Z}^c_{\mathcal{E}'} = \mathcal{Z}^c_{\mathcal{E}}$. Let $\partial \mathcal{Z}^c_{\mathcal{E}} := \{\boldsymbol{z} \in \mathcal{Z}^c_{\mathcal{E}} | \nexists \boldsymbol{z}' \in \mathcal{Z}^c_{\mathcal{E}}$ such that $\boldsymbol{z}' > \boldsymbol{z}\}$, we consider replacing $\mathcal{E}$ by $\mathcal{E}'$ such that $\partial \mathcal{Z}^c_{\mathcal{E}'} = \partial \mathcal{Z}^c_{\mathcal{E}}$.

What is interesting about this latter possibility is that households in agreement face exactly the same relevant portions of their budget sets. If $\boldsymbol{z}_{\mathcal{E}'}(\eta) \neq \boldsymbol{z}_{\mathcal{E}}(\eta)$, for some η, it must be due to changes in the family's objective. That is, such reforms only change the 'distribution factors', e.g., Chiappori and Bourguignon (1992).

By considering $\mathcal{E}'$ such that $\boldsymbol{z}_{\mathcal{E}'}(\eta) \neq \boldsymbol{z}_{\mathcal{E}}(\eta)$ we may be describing a complete reform, or a first step in a full reform to a new institutional setting $\mathcal{E}''$ for which the set of induced preferences is such that $\mathcal{Z}^c_{\mathcal{E}''} \neq \mathcal{Z}^c_{\mathcal{E}}$ induces a more desirable allocation.

The first step in the reform does not change the (conditional) utility possibility sets, $\mathcal{U}_{\boldsymbol{z}(\eta)}(\eta)$. Note also that, for all η, inequality (11) remains valid if we hold $(\bar{u}_f, \bar{u}_m) = (\bar{u}^f_{\mathcal{E}}(\eta), \bar{u}^m_{\mathcal{E}}(\eta))$ fixed. Of course, this need not imply incentive compatibility: inequality (11) need not be satisfied at the new threat points $(\bar{u}'_f, \bar{u}'_m) = (\bar{u}^f_{\mathcal{E}'}(\eta), \bar{u}^m_{\mathcal{E}'}(\eta))$. It is good that it does not, for it is this very possibility that motivates an important part of our analysis.[21]

VII Tax Systems

Following Mirrlees' (1971) seminal work, the characterization of optimal tax schedules relied on a two steps approach. First, constrained efficient alloca-

[21] Examples of reforms which respect these restrictions are, in the case of a DM, replacing $\boldsymbol{\gamma}$ by some $\hat{\boldsymbol{\gamma}}$ such that $\boldsymbol{\gamma}(\eta, a, \eta, a) = \hat{\boldsymbol{\gamma}}(\eta, a, \eta, a)$ for all η, and for all $(\eta', \iota, \eta'', \iota')$, $\hat{\boldsymbol{\gamma}}(\eta', \iota, \eta'', \iota') \leq \boldsymbol{\gamma}(\eta, a, \eta, a)$ for some η.

tions were first derived using a direct mechanism. Second, budget sets that supported these allocations were derived.

It is our goal in this Section to assess whether tax implementation is without loss when couples are non-unitary. In Section VI we have seen that a DM implements any incentive feasible allocation. We need now ask whether any incentive-feasible allocation can be implemented via tax systems.

We make a distinction between a tax schedule and a tax system, which is how we call a collection of tax schedules. A Simple Tax System - STS, is comprised of a tax schedule for single females, another one for single males and, finally, one for couples. We shall also consider a Dual Tax System - DTS, which, adds to these schedules an alternative schedule for couples which can be chosen upon the manifestation of any one of the spouses. An example of such dual systems is provided by filing options found in countries like the United States.

VII.1 STS and DM: Conditional Equivalence

We have already described an STS in Section II. We shall first show where the difficulties for implementation do *not* lie by comparing implementable allocations when households' objective functions are given, i.e., holding threat points fixed.

VII.1.1 Conditional allocations under the STS

The tax schedule of which the STS is comprised defines budget sets for singles of each gender and for couples under which choices are made. In the case of married agents choices may be made in agreement, through the maximization of a function $W(\boldsymbol{z}, \eta; \bar{u}^f, \bar{u}^m)$, and in disagreement as an equilibrium for the non-cooperative game played under the tax system.

For any given $(\bar{u}^f, \bar{u}^m)$ the solution of the household maximization program under the restriction that $\boldsymbol{\psi}(\boldsymbol{z}) \leq 0$, is $\hat{\boldsymbol{z}}(\eta, \bar{u}^f, \bar{u}^m))$. We shall use the term conditional allocations when we consider changes in the tax system holding $(\bar{u}^f, \bar{u}^m)$ fixed.

If a couple η is in disagreement threat points, $(\bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta))$ are determined as an equilibrium for the non-cooperative game played under the the tax system. In Section VII.2 we provide further details about this game played under the tax system. For now, just note that these equilibrium threat points define

the ordering of transactions effectively used by household η under the tax systems. The solution to the household problem under this ordering captured by the objective $W(\boldsymbol{z}, \eta; \bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta))$, finally determines the equilibrium transactions $\boldsymbol{z}_\Psi(\eta) \equiv \hat{\boldsymbol{z}}(\eta, \bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta))$ for the tax system.

A STS is feasible if $G(Z_\Psi) \leq 0$, with

$$Z_\Psi := \sum_{i=f,m} \left\{ \int z^i_\Psi(\theta_i) d\mu^i_{\mathrm{s}}(\theta_i) + \int z^{c,i}_\Psi(\eta) d\mu_{\mathrm{c}}(\eta) \right\}.$$

VII.1.2 Conditional allocations under the DM

A DM defines a set of feasible transactions, $\{z; \exists \theta_i; \gamma_i(\theta_i) = z\}$ for gender i, $i = f, m$, singles and $\{\boldsymbol{z}; \exists (\eta, i, \hat{\eta}, j) \in \Theta^2 \times \mathcal{I} \times \Theta^2 \times \mathcal{I}; \boldsymbol{\gamma}^c(\eta, i, \hat{\eta}, j) = \boldsymbol{z}\}$, for couples. Using (7), we recall that the conditional (on $(\bar{u}_f, \bar{u}_m)$) choices by a couple η under the DM are

$$\hat{\boldsymbol{s}}_\Gamma(\eta, \bar{u}_f, \bar{u}_m) := \underset{(\eta', \iota, \eta'', \hat{\iota}) \in \Theta^2 \times \mathcal{I} \times \Theta^2 \times \mathcal{I}}{\operatorname{argmax}} W(\boldsymbol{\gamma}^c(\eta', \iota, \eta'', \hat{\iota}), \eta; \bar{u}_f, \bar{u}_m), \tag{18}$$

The associated transactions are $\hat{\boldsymbol{z}}_\Gamma(\eta, \bar{u}_f, \bar{u}_m) \equiv \boldsymbol{\gamma}^c(\hat{\boldsymbol{s}}_\Gamma(\eta, \bar{u}_f, \bar{u}_m))$.

For each couple η, $(\bar{u}^f_\Gamma(\eta), \bar{u}^m_\Gamma(\eta))$ is the pair of threat points which results from the game described in Section IV.3 under Γ. We say that the allocation is incentive compatible if for every η, $\hat{\boldsymbol{s}}_\Gamma(\eta, \bar{u}_f, \bar{u}_m) = (\eta, a, \eta, a)$ for $(\bar{u}^f, \bar{u}^f) = (\bar{u}^f_\Gamma(\eta), \bar{u}^m_\Gamma(\eta))$. Naturally, $\boldsymbol{z}_\Gamma(\eta) = \boldsymbol{\gamma}^c(\hat{\boldsymbol{s}}_\Gamma(\eta, \bar{u}^f_\Gamma(\eta), \bar{u}^m_\Gamma(\eta)))$ are the associated transactions.

Our main interest in this Section is to compare the allocations that can be implemented by an STS with those that can be implemented by a DM, *when threat points are held fixed.* Hence, it will also be important to define conditional (on $(\bar{u}_f, \bar{u}_m)$) incentive compatibility as $\hat{\boldsymbol{s}}_\Gamma(\eta, \bar{u}_f, \bar{u}_m) = (\eta, a, \eta, a)$ for a given arbitrary $(\bar{u}_f, \bar{u}_m)$.

We say that an allocation is implementable by a tax system ψ if there is a tax system for which such an allocation is an equilibrium induced by ψ. We say that an allocation is implementable by a direct mechanism Γ if there is a truthful equilibrium for the game played under the mechanism, which is mapped into this allocation by the associated outcome function.

What we show next is that if threat points are held fixed, an allocation is implementable by an STS if an only if it is implementable by a DM.

Proposition 3. *Let*

$$\left(\left\{z^i_\Psi(\theta_i)\right\}_{\theta_i,i=f,m}, \left\{\boldsymbol{z}_\Psi(\eta)\right\}_\eta\right)$$

be the allocation induced by an STS. For every couple, η, let $(\bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta)) \in \mathring{\mathcal{U}}_\Psi(\eta)$ be the equilibrium threat points under the STS. Then, there is a DM, Γ, such that $z^i_\Gamma(\theta_i) = z^i_\Psi(\theta_i)$ for all θ_i, $i = f, m.$, $\boldsymbol{\gamma}^c(\hat{\boldsymbol{s}}_\Gamma(\eta, \bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta))) = \boldsymbol{z}_\Psi(\eta)$, for all η.

Proof. Immediate consequence of Proposition 2. □

Proposition 3 is, of course, a direct consequence of Proposition 2, which states that a direct mechanism can implement any incentive feasible allocation. Still, in Appendix A.1 we offer an alternative direct proof that parallels those found in Hammond (1979); Guesnerie (1998) to highlight the fact that it is only in its impact on threat points that the equivalence may fail.

Proposition 4. *Let*

$$\left(\left\{z^i_\Gamma(\theta_i)\right\}_{\theta_i,i=f,m}, \left\{\boldsymbol{z}_\Gamma(\eta)\right\}_\eta\right)$$

be the allocation implemented by a DM. For every couple, η, let $(\bar{u}^f_\Gamma(\eta), \bar{u}^m_\Gamma(\eta)) \in \mathring{\mathcal{U}}_\Gamma(\eta)$ be the equilibrium threat points under the DM. Then, there is a STS, ψ, such that $z^i_\Psi(\theta_i) = z^i_\Gamma(\theta_i)$ for all θ_i, $i = f, m.$, $\hat{\boldsymbol{z}}_\Psi(\eta, \bar{u}^f_\Gamma(\eta), \bar{u}^m_\Gamma(\eta)) = \boldsymbol{z}_\Gamma(\eta)$, for all η.

Proof. See Appendix A.1. □

The logic of both proofs is the same used by Hammond (1979) and further explained by Guesnerie (1998). The reason why the result remains true here is because couples have well defined preferences and make choices that respect those preferences, *when we hold the threat points fixed.*

Hence, we may refine the implementation problem by focusing on the following two more specific questions. For an arbitrary STS, is it possible to find a direct mechanism which, at the same time, generates the threat points that arise under the tax system and the household choices that lead to the transactions that arise under the STS? Conversely, for any DM, is it possible to find a STS which induces the same threat points that arise as the DM equilibrium threat points while inducing the household transactions that arise under the DM.

Each institution plays a dual role of defining household preferences and establishing the space in which choices by these households are to be made. We

have seen that, for given household preferences, the second task is possible. We shall now consider the role each institution plays in determining these household preferences.

VII.2 STS vs NDM: non-equivalence

We start this section noting that Proposition 2 implies, in particular that anything that may be implemented by an STS may also be implemented by a DM. How about the converse? Is it possible to find for any mechanism a tax schedule that implements the associated allocation?

The answer is no. One can always choose desired transactions as the message space of a mechanism, and restrict the space of messages to transactions that were available under the tax system. Alternative mechanisms will define different strategy spaces for the agents which will typically induce different equilibrium transactions for the disagreement game.

To make this point clear we further explain the disagreement game under a simple tax system.

Disagreement Game under a STS Spouses in disagreement play a game non-cooperative game in the space of transactions, with payoff function Φ, defined in (6) under the budget set induced by ψ.

We assume that there is a feasible $\underline{z}$ such that

$$\Phi\left(\underline{z}, z_m; \eta\right)' = \Phi\left(z_f, \underline{z}; \eta\right)' = \left(v_\Psi^f(\theta_f), v_\Psi^m(\theta_m)\right),\ \forall z_f, z_m, \theta_f, \theta_m, \tag{19}$$

where $\underline{z} \in \mathcal{Z}$ is such that there is no $z' \in \mathcal{Z}$ such that $\underline{z} \geq z'$, $\underline{z} \neq z'$.

Equation (19) incorporates the fact that it is always possible for a married agent to divorce and attain the utility of a single agent of his or her type, hence, we can focus on the outcome function $\bar{\Phi}$. If an agent is at the lowest possible transaction, (19) implies that there is no way the other spouse can compensate him or her in such a way as to make remaining married better than divorcing.

For a given tax schedule, $\boldsymbol{\psi}$, let

$$\chi_\Psi^f(z_m, \eta) := \underset{z | \boldsymbol{\psi}(z, z_m) \leq 0}{\operatorname{argmax}}\ \phi_f\left(z, z_m; \eta\right)$$

define the reaction function for the wife, with analogous definition, $\chi_\Psi^m(z_f, \eta)$,

for the husband.

As we previously commented, whether a pure strategy equilibrium exists and, when it exists, whether it is unique depends not only on the properties of Φ but also on those of $\boldsymbol{\psi}$. Assumption VII.1, next is then used for the case of a simple tax system.

Assumption VII.1. *There exists a unique Nash equilibrium,*

$$(z_f^*, z_m^*) = \left(\chi_\Psi^f(z_m^*, \eta), \chi_\Psi^m(z_f^*, \eta)\right),$$

for the disagreement game played under the tax system.

In the representation above we have implicitly assumed a Pure strategy Nash equilibrium. This is for notation convenience only. If only mixed strategy equilibrium exists, then threat points are the expected utilities attained by each spouse. All results remain valid, but the presentation becomes very cumbersome.

Non-equivalence Applying the notation defined in Section II, threat points for a couple η under the tax system Ψ are $(\bar{u}_\Psi^f(\eta), (\bar{u}_\Psi^m(\eta))$. They are the utilities attained by the two spouses as the value of $\Phi(\cdot, \eta)$ at the non-cooperative equilibrium transactions, $\bar{\boldsymbol{z}}_\Psi(\eta) = (\bar{z}_\Psi^{c,f}(\eta), \bar{z}_\Psi^{c,m}(\eta))$. For these threat points, transactions chosen in agreement are $\boldsymbol{z}_\Psi(\eta) = \hat{\boldsymbol{z}}_\Psi\left(\eta, \Phi(\bar{\boldsymbol{z}}_\Psi(\eta); \eta)\right)$, for $\hat{\boldsymbol{z}}_\Psi$ defined as in (7). If we replace Ψ by Γ we have the analogous definitions for the DM.

Next we offer an example of an incentive-feasible allocation which cannot be implemented by a STS.

Example VII.1. *Starting from a mechanism Γ one must make sure that all transactions chosen by the couples are available. At a minimum, let $\boldsymbol{\psi}(\boldsymbol{z}) \leq 0$ for all z such that there is η such that $\boldsymbol{z} < \boldsymbol{z}_\Gamma(\eta)$ and $\boldsymbol{\psi}(\boldsymbol{z}) > 0$, otherwise. If for each couple $\bar{\boldsymbol{z}}_\Gamma(\eta)$ denotes the equilibrium choices for the disagreement game under Γ. If for any couple η a transaction $\boldsymbol{z}$ exists such that $\boldsymbol{\psi}\left(z, \bar{z}_\Gamma^{c,m}(\eta)\right) \leq 0$, and*

$$\phi_f\left(\left(z_f, \bar{z}_\Gamma^{c,m}(\eta)\right), \eta\right) > \phi_f\left(\left(\bar{z}_\Gamma^{c,f}(\eta), \bar{z}_\Gamma^{c,m}(\eta)\right), \eta\right).$$

or

$$\phi_m\left(\left(\bar{z}_\Gamma^{c,f}(\eta), z_m\right), \eta\right) > \phi_m\left(\left(\bar{z}_\Gamma^{c,f}(\eta), \bar{z}_\Gamma^{c,m}(\eta)\right), \eta\right).$$

then, clearly, the allocation $\boldsymbol{z}_\Gamma(\eta)$ is not an equilibrium allocation for the tax system Ψ. Note that this may still be compatible with

$$\phi_f\left(\boldsymbol{\gamma}^c(\eta_f^*, d, \eta_m^*, d), \eta\right) > \phi_f\left(\boldsymbol{\gamma}^c(\hat{\eta}, d, \eta_m^*, d), \eta\right) \forall\hat{\eta}$$

and

$$\phi_m\left(\boldsymbol{\gamma}^c(\eta_f^*, d, \eta_m^*, d), \eta\right) > \phi_m\left(\boldsymbol{\gamma}^c(\eta_f^*, d, \hat{\eta}, d), \eta\right) \forall\hat{\eta}$$

A final important point we make relates to the role played by the informational structure we have assumed. We do so in the context of a very simple mechanism that does not exploit the fact that both spouses know each other's productivities.

Richness and a Very Simple Mechanism - VSM The main assumption we have made regarding the informational structure of the problem is that spouses' know each other's productivities which is otherwise private information.

When agents choose independently and non-cooperatively, this type of informational structure may be exploited by the planner.[22] Here, in contrast, spouses in agreement choose cooperatively, meaning that such use of correlated information is not possible in our setting. The planner does exploit the correlation in the information set of spouses but only when spouses are in disagreement. Hence, only through the manipulation of threat points.

We consider a very simple mechanism – VSM – that allows us to make this point in a stark manner. Define the VSM as follows. The message space for all agents is $\Sigma_i = \Theta$, and the outcome function, $\tilde{\Gamma}$, is

$$\tilde{\Gamma} = \left\{\{\tilde{\gamma}_i^s : \Theta \to \mathcal{Z}\}_{i=f,m} ; \tilde{\boldsymbol{\gamma}}^c : \Theta^2 \to \mathcal{Z}^2\right\},$$

where $\tilde{\gamma}_i^s(.)$ is the outcome function for singles and $\tilde{\boldsymbol{\gamma}}^c(\cdot) = \left(\tilde{\gamma}_f^c(.), \tilde{\gamma}_m^c(.)\right)$ is the outcome function for couples. Each agent is, therefore, asked his or her productivity, θ_i, independently of his or her marital status.

First, note that conditional equivalence holds.

Proposition 5. *Let*

$$\left(\left\{z_\Psi^i(\theta_i)\right\}_{\theta_i, i=f,m}, \left\{\boldsymbol{z}_\Psi(\eta)\right\}_\eta\right)$$

[22] An extreme well known version of such possibility is found in Crémer and McLean (1988). There, belief correlations allows the planner to implement first best allocations in an auctions context.

be the allocation induced by a STS. For every couple, η, let $(\bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta)) \in \mathring{\mathcal{U}}_\Psi(\eta)$ be the equilibrium threat points under the STS. Then, there is a VSM, $\tilde{\Gamma}$, such that $z^i_{\tilde{\Gamma}}(\theta_i) = z^i_\Psi(\theta_i)$ for all θ_i, $i = f, m.$, $\tilde{\boldsymbol{\gamma}}^c(\hat{\boldsymbol{s}}_{\tilde{\Gamma}}(\eta, \bar{u}^f_\Psi(\eta), \bar{u}^m_\Psi(\eta))) = \boldsymbol{z}_\Psi(\eta)$, for all η.

The proof of Proposition 5 is very similar to that of Proposition 3. Indeed, the problem for singles is standard, thus choosing $\tilde{\gamma}^i(\theta_i) = z^i_\Psi(\theta_i)$ for all θ_i does the trick. As for couples, let $\tilde{\boldsymbol{\gamma}}^c(\theta_f, \theta_m) = \boldsymbol{z}_\Psi(\theta_f, \theta_m)$.

It is not hard to see that the proof of Proposition 3 goes through if $\tilde{\boldsymbol{\gamma}}^c(\eta)$ substitutes for $\boldsymbol{\gamma}^c(\eta, a, \eta, a)$ for all η.

From Proposition 5 it is clear that the planner gains nothing from the informational structure if threat points are held fixed. The problem with this simpler mechanism lies only on the disagreement game. We shall now show that a VSM need not be capable of implementing the allocation induced by an arbitrary STS.

Let $\bar{\boldsymbol{z}}_\Psi(\eta)$ be the equilibrium transactions from the disagreement game. Assume that for every $\boldsymbol{z}$ such that there is η for which $\boldsymbol{z} = \bar{\boldsymbol{z}}_\Psi(\eta)$ there is also $\hat{\eta}$ such that $\boldsymbol{z} = \boldsymbol{z}_\Psi(\hat{\eta})$. The question we must ask is whether $(\hat{\theta}_f, \hat{\theta}_m) = \hat{\eta}$ is an equilibrium for the announcement game played by spouses in disagreement.

Recall that $\boldsymbol{\gamma}^c(\hat{\theta}_f, \hat{\theta}_m) = (z^{c,f}_\Psi(\hat{\theta}_f, \hat{\theta}_m), z^{c,m}_\Psi(\hat{\theta}_f, \hat{\theta}_m))$. We also know that there is no z^f such that $\phi^f(z^f, z^{c,m}_\Psi(\hat{\theta}_f, \hat{\theta}_m)), \eta) > \phi^f(z^{c,f}_\Psi(\hat{\theta}_f, \hat{\theta}_m), z^{c,m}_\Psi(\hat{\theta}_f, \hat{\theta}_m), \eta)$, and no z^m such that $\phi^m(z^{c,f}_\Psi(\hat{\theta}_f, \hat{\theta}_m), z^m, \eta) > \phi^m(z^{c,f}_\Psi(\hat{\theta}_f, \hat{\theta}_m), z^{c,m}_\Psi(\hat{\theta}_f, \hat{\theta}_m), \eta)$.

However, a deviation by f in the space of messages leads to transactions $\boldsymbol{\gamma}^c(\hat{\hat{\theta}}_f, \hat{\theta}_m) = (z^{c,f}_\Psi(\hat{\hat{\theta}}_f, \hat{\theta}_m), z^{c,m}_\Psi(\hat{\hat{\theta}}_f, \hat{\theta}_m))$. It is not $z^{c,m}_\Psi(\hat{\theta}_f, \hat{\theta}_m)$ which is held fixed when f deviates. Hence, we cannot a priori rule out an advantageous deviation in the space of messages even if no such deviations exists in the space of transactions.

VII.3 Dual Tax Systems

An important limitation of STS's when compared to a DM is that it does not use information regarding whether households are in agreement or disagreement.

Fortunately, this is easy to remedy. Define a Dual Tax System – DTS – as follows. For single agents tax schedules are as before, $\mathbb{B}^i_\Psi := \{z \in \mathcal{Z}; \psi_i(z) \leq 0\}$.

For couples two schedules are offered: *i)* a schedule that induces a budget set represented by the restriction $\boldsymbol{\psi}(\boldsymbol{z}) \leq 0$, and; *ii)* a fall-back schedule captured by the restriction $\bar{\boldsymbol{\psi}}(\boldsymbol{z}) \leq 0$, which must apply to both spouses if any of the spouses

opts to use it. That is, while both spouses must choose to use $\boldsymbol{\psi}$, if any spouse decides to use $\bar{\boldsymbol{\psi}}$, instead, the other spouse must abide by his or her decision. For any $\boldsymbol{z} \in Z^2$ such that $\bar{\boldsymbol{\psi}}(\boldsymbol{z}) \leq 0$ we have $\boldsymbol{\psi}(\boldsymbol{z}) \leq 0$. We use $(\boldsymbol{\psi}, \bar{\boldsymbol{\psi}})$ to denote such tax system.

The first question we want to answer is whether the set of allocations that is implementable by a DTS is the set of implementable allocations (those implementable by a DM).

Unfortunately, this is not the case. Although a DTS will, in general, expand the set of allocations that may be implemented by a STS, they cannot mimic all that may be accomplished by an arbitrary DM.[23] In disagreement, spouse play a non-cooperative game. The strategy space defined by each institutional setting matters for determining which transactions arise in the equilibrium of these disagreement games. Under a dual tax system, $(\psi, \bar{\psi})$, then

$$\chi_f^{\bar{\Psi}}(z_m, \eta) := \underset{z \mid \bar{\boldsymbol{\psi}}(z, z_m) \leq 0}{\operatorname{argmax}} \ \phi_f\left(z, z_m; \eta\right)$$

defines a reaction function for the wife. An analogous expression defines a reaction function, $\chi_m^{\bar{\Psi}}(z_f, \eta)$, for the husband. Spouses are directly choosing transactions holding the other spouses' transactions fixed.

The set of admissible deviations is different from that of a STS only in that $\bar{\boldsymbol{\psi}}$ substitutes for $\boldsymbol{\psi}$. These deviations are very different from those allowed under a DM, as is made apparent by the reaction functions in (9).

Again, Proposition 2 implies that any allocation induced by a DTS may be implemented by a DM. The converse need not be true, of course.

Assume that we restrict ourselves to a subset of incentive feasible DM's with disagreement outcome functions of the form $\boldsymbol{\gamma}(\eta', d, \eta'', d) = (g_f^d(\eta'), g_m^d(\eta''))$. Let $\bar{\boldsymbol{z}}_{\bar{\Gamma}}(\eta) = (\bar{z}_{\bar{\Gamma}}^{c,f}(\eta), \bar{z}_{\bar{\Gamma}}^{c,m}(\eta))$ be the transactions that result from the equilibrium of this disagreement game. If $\bar{\mathcal{Z}}_{\Gamma} := \bigcup_\eta \bar{\boldsymbol{z}}_{\Gamma}(\eta)$, then, for any $\boldsymbol{z} \in \mathcal{Z}^2$ one may let $\bar{\boldsymbol{z}}_{\bar{\Psi}}(\boldsymbol{z}) \leq 0$ if and only if $\boldsymbol{z} \leq \boldsymbol{z}'$ for any $z' \in \bar{\mathcal{Z}}_{\Gamma}$. All we need to note is that, for all η, $(\bar{z}_{\Gamma}^{c,f}(\eta), \bar{z}_{\Gamma}^{c,m}(\eta))$ is a Nash equilibrium for the disagreement game under the shadow schedule $\bar{\boldsymbol{\psi}}$.[24]

[23] This is not to say that DTS are not useful. We have already argued that they expand the set of implementable allocations. da Costa and Diniz (2015) provide an example of such use of a DTS which is a simplified version of the current US code.

[24] Assume that it is not. Then, there must be a deviation in the $\mathcal{Z}$ space by agent i, say, $i = f$ such that the utility attained by such deviation is greater than that attained in the proposed equilibrium. Let $\hat{z}_f$ be such deviation. For the deviation to be feasible it must be such that

The logic behind this result is that households in disagreement are, in practice, playing a game over $\mathcal{Z} \times \mathcal{Z}$. Of course there is no reason for the planner to be restricted to a DM of the previously mentioned form. In the next section we explain how a DM can expand the set of allocations that are implementable by a DTS.

Note also the advantage a DTS has over a STS. A specific schedule is designed to deal with the disagreement game, whereas a STS must make available all allocations chosen by a couple in agreement at the same time that it guarantees that the same equilibrium arise in the disagreement games.

da Costa and Diniz (2015) consider the following real world example of a DTS. Start from a tax system for which a single tax schedule is at households' disposal, that is a system for which married agents are required to file jointly. If spouses are unable to reach an agreement, they choose their transactions non-cooperatively on the budget set generated by the joint tax schedule. Threat points are the equilibrium utilities that arise from this game. Assume now that the planner offers the possibility of individual filing. As in da Costa and Diniz (2015), assume also that the household's budget set under individual filing is a subset of the budget set under joint filing. Hence, in agreement, joint filing is always chosen. Yet, the filing option may be used by any one of the spouses in disagreement if the equilibrium utility he or she attains in the non-cooperative game played under such budget sets is higher than that attained under the joint budget set. Although the introduction of such filing options does not change household's transactions it has the potential of changing the distribution of utility through its effect on the threat points.

This is, of course, only a simple example of additional instruments that the planner may use to enlarge the set of implementable allocation. In what follows we formalize those ideas and ask, in particular, whether such augmented tax systems can implement all constrained efficient allocations.

$\psi(\hat{z}, \bar{z}_m(\eta)) \leq 0$, in which case there is η' such that $(\hat{z}, \bar{z}_m(\eta)) \leq (\bar{z}_f(\eta'), \bar{z}_m(\eta'))$. In any case, since $\bar{z}_m(\eta))$ is held fixed and we cannot have $(\bar{z}_f(\eta), \bar{z}_m(\eta)) > (\bar{z}_f(\eta'), \bar{z}_m(\eta'))$ by incentive compatibility, it must be the case that $\hat{z} \leq \bar{z}_f(\eta)$. Hence the utility attained by f is lower. A contradiction with our assumption that there is a profitable deviation.

VII.4 The Variational Approach

In recent years, an alternative one step approach to optimal taxation has became predominant: the variational approach.

This variational approach consists in perturbing a candidate optimum schedule and relies on taxable income elasticities as sufficient statistics — Piketty (1997); Dahlby (1998); Saez (2001) — for welfare analysis. An important advantage of this approach in the context of household taxation is that it does not rely on stringent assumptions which are unlikely to hold for the taxation of couples, e.g., single crossing. Hence, in its full generality — Golosov et al. (2014) — this method has the potential to characterize optimal taxation in an environment for which the solution through the traditional approach has proven elusive. More important is the fact that optimal tax formulae are all based on statistics that can be recovered from the data. The main drawback is, of course, that it does not provide a procedure for finding optimal taxes but simply for checking whether a candidate schedule is optimal.

What we try now is to give a brief account of the consequence of our findings for this variational approach.

Taking the two examples in the beginning of this paper as a departure point, we can split the discussion into: i) the consequences for welfare impact evaluation, and; the consequences for the behavioral responses to tax reforms.

One property which facilitates the welfare analysis in the case of single agents is that the utility impact of any perturbation of the tax system is a simple transformation of the mechanical revenue impact. The rationale is a straightforward consequence of the envelope theorem. For couples, this need not be the case, even if a unitary view of the household can be relied upon. The point here is that dissonance leads to first order welfare impacts from the planner's perspective of re-optimization.[25] What the first example in Section III teaches us is that these first order welfare impacts will vary depending on which threat points are relevant for the tax system. Moreover, the very idea of holding the threat points fixed as one perturbs the tax schedule will depend on other non-local aspects of the tax system.

When it comes to the behavioral responses, policy elasticities, e.g., Hendren

[25]From the household perspective there are still no first order welfare effects. However, since the marginal value of income in the hands of each spouse may differ from the planner's perspective, any redistribution from one spouse to the other may affect welfare.

(2016), are not structural parameters. They depend on the details of the institutional environment under consideration. The implication of our results is that they should be optimally chosen, along the lines of what was done in the second example in Section III.[26]

VII.5 Generalizing Preferences and Technology

With regards to preferences and technology, our model is exactly that of Mirrlees (1971). This is for ease of communication only. It is not hard to see how our framework can be extended to allow for preferences defined over more goods, multidimensional heterogeneity, etc. More importantly, for married agents we may allow for externalities, public goods and other forms of interdependence in utility.

Let $\mathfrak{x} = (\mathfrak{c}, \mathfrak{l})' \in \mathbb{R}^2_+$ denote the bundle consumed by an agent. It is apparent that all we have shown so far applies more generally to $\mathfrak{x} \in \mathbb{R}^n_+$, and $\nu_i(\mathfrak{x}_i, \mathfrak{x}_{-i}, \theta_i)$, $i, -i = f, m$. As in the Mirrlees' case, consumption goods may be split into assignable $\mathfrak{x}_A$ and non-assignable goods $\mathfrak{x}_{NA}$ with $\mathfrak{x} = (\mathfrak{x}_A, \mathfrak{x}_{NA})\ \mathbb{R}^{n_A+n_{NA}}_+$. Generalizing Θ, which could be any subset of $\mathbb{R}^m$ is also elementary. Finally, more complex technologies mapping $\mathcal{Z}^2$ with $\mathcal{Z} \subset R^m$ into 2^X, $X \subset R^n_+$ may be considered. Of course, which properties one would want to impose on Φ under each different specification is something one needs to carefully consider.

Some of these generalizations are of great practical relevance. For instance, central to any discussion involving household choices are the investments made on children, usually thought of as a public good from spouses' perspective. We can easily accommodate the presence of public goods in our description. Let g be the amount of public good produced, we can simply add to the description of $F^a(z, \eta)$, the pair of constraints $g_f \leq g$, $g_m \leq g$, and modify the time constraints to $\mathfrak{l}_f \leq 1 - n_f - t_f$, and $\mathfrak{l}_m \leq 1 - n_m - t_m$, $g = f(t_f, t_m)$.[27]

VIII Optimal Threat Points

In this section we explain how threat points can be optimally 'chosen' in order to attain the best possible allocation for this economy.

[26] Optimally choosing elasticities, in a very different context, is also what **?** explore.

[27] From a theoretical perspective, it is the presence of public goods one of the main motivations for the use of a 'separate spheres' view of the household by Lundberg and Pollak (1993).

We first discuss the conceptual issues – section VIII.1 –, then provide some numeric examples that illustrate their workings in specific cases – section VIII.2.

VIII.1 Threat Points and Equilibrium Allocations

Household objective, to which we have referred as household preferences, $\succsim_{\eta|\bar{u}^f,\bar{u}^m}$, depend on threat points $(\bar{u}^f, \bar{u}^m)$, which are determined through a non-cooperative game which details are determined by the institution $\mathcal{E}$ under which choices are made. The special feature of this environment with collective households is, therefore, the possibility of manipulating 'preferences'. In the case of a direct mechanism this is possible through a careful choice of the mapping from all announcements announcements which are not of the form (η, a, η, a) to outcomes.

Let us start with an optimal tax system and its associated equilibrium allocation,

$$\left(\left\{z^i_{\Psi}(\theta_i)\right\}_{\theta_i, i=f,m}, \left\{\boldsymbol{z}_{\Psi}(\eta)\right\}_{\eta}\right),$$

Threat points are, in this case $(\bar{u}^f_{\Psi}(\eta), \bar{u}^m_{\Psi}(\eta))$. How can we improve upon such allocation?

Propositions 4 and 2 guarantee that there is no other institution $\mathcal{E}$ and associated allocation,

$$\left(\left\{z^i_{\mathcal{E}}(\theta_i)\right\}_{\theta_i, i=f,m}, \left\{\boldsymbol{z}_{\mathcal{E}}(\eta)\right\}_{\eta}\right)$$

such that $z^i_{\mathcal{E}}(\theta_i) \succeq_{\theta_i} z^i_{\Psi}(\theta_i)$ for all θ_i, $i = f, m$ and $\boldsymbol{z}_{\mathcal{E}}(\eta) \succeq_{\eta|\bar{u}^f_{\Psi},\bar{u}^m_{\Psi}} \boldsymbol{z}_{\Psi}(\eta)$ for all η with strict preference for some subset of positive measure of θ_i or η. In words, these are Pareto efficient allocations *if we treat couples as individuals with fixed preferences,* $\succeq_{\eta|\bar{u}^f_{\Psi},\bar{u}^m_{\Psi}}$.

Moreover, there is no other incentive feasible allocation which the planner prefers, if it must respect these 'preferences'. Because no welfare gains conditional on the induced preferences, $\succeq_{\eta|\bar{u}^f_{\Psi},\bar{u}^m_{\Psi}}$, are possible, potential improvements must arise from the 'manipulation' of these preferences. In what follows we discuss how this can be done.

Dealing with 'dissonance' The allocation of resources between spouses need not be in agreement with the planner's objective. That is, given the utilities that are attainable by the household under Ψ, the household may be choosing a

distribution which is not the one preferred by the planner. Since the household's objective depends on $(\bar{u}^f_\Psi, \bar{u}^m_\Psi)$, it is clear that if the planner is able to change $(\bar{u}^f, \bar{u}^m)$ it may try to align the household's objective with its own. In other words, the planner may want to consider an alternative environment $\mathcal{E}'$ such that the new threat points induces a more desirable utility split between spouses.

Figure 1 displays an example of such possibility. The focus on transferable utilities allows this effect to be isolated from the other ones, since $\boldsymbol{z}_\mathcal{E}(\eta)$ does not depend on the threat points for this case.

Relaxing Incentive Constraints In general, redistribution matters not only between spouses but across households as well. The possibility of redistributing across households is however limited by incentive compatibility. A first issue that is particular to the problem we examine is how intertwined the design of allocations for singles is with the allocation for couples.

If threat points are utilities attained by agents as singles this imposes an extra restriction on the planner: it must take into account its effect on married agents' preferences. Being able to dissociate the utility attained by single agents from the fall back utilities that define the threat points has the potential of freeing the planner from this constraint.

Finally, a more surprising role for manipulation of household preference arises. If at the equilibrium allocations incentive constraints bind, the ability to manipulate household preferences, $\succeq_{\eta|\bar{u}_f,\bar{u}_m}$, by changing $(\bar{u}_f, \bar{u}_m)$ can reduce inefficiencies. When incentive constraints bind, inefficiencies are optimally introduced to make envied bundles less desirable. By manipulating household preferences it may be possible for the planner to make an allocation cease to be envied by a household even if no equilibrium allocations are changed.

Figure 2 displays an example of an allocation (A) which is not incentive compatible when the threat points are the utilities of being single, $(\bar{u}_f, \bar{u}_m)$, since allocation (B) is preferred, but is incentive compatible under the alternative threat points $(\bar{u}'_f, \bar{u}'_m)$, (C) is not better than (A). Given an agreement bundle, $\boldsymbol{z}$, the green curve represents the frontier of the set $\mathcal{U}_{\boldsymbol{z}}(\eta)$ of possible utilities attainable by a couple η if they report truthfully. That is, we hold the transactions conducted by the couple, $\boldsymbol{z}$, fixed at the value prescribed to η by the mechanism to build the set. The black dot labeled $(\bar{u}_f, \bar{u}_m)$ represents the disagreement util-

ities. Utility pairs (u_f, u_m) within the utility possibility set to the northeast of $(\bar{u}_f, \bar{u}_m)$ are all individually rational feasible utility pairs for the couple η when they announce their types truthfully. Given the disagreement point $(\bar{u}_f, \bar{u}_m)$, the couple will choose point A, in the utility frontier through a Nash Bargaining Process. That point represents the utility division in agreement $(u^f_{\mathcal{E}}(\eta), u^m_{\mathcal{E}}(\eta))$.

A couple need not report truthfully, however. Were spouse to coordinate and lie about their types, they would make different transactions $\hat{\boldsymbol{z}}$ and access a different utility set — blue curve in Figure 2. Under this set they would choose a different division of utility in agreement, say point B. Let us assume that the couple prefers B to A. By varying the institutional settings among those for which $\partial\mathcal{Z}^c_{\mathcal{E}'} = \partial\mathcal{Z}^c_{\mathcal{E}}$ we define the set

$$\Lambda_{\mathcal{E}}(\eta) := \{(\bar{u}_f, \bar{u}_m) | \exists E' \text{ such that } \partial\mathcal{Z}^c_{\mathcal{E}'} = \partial\mathcal{Z}^c_{\mathcal{E}} \\ \text{and } (\bar{u}_f, \bar{u}_m) = \Phi(\bar{z}^{c,f}_{\mathcal{E}}(\eta), \bar{z}^{c,m}_{\mathcal{E}}(\eta), \eta) \forall \eta\}$$

Assume that $(\bar{u}'_f, \bar{u}'_m) \in \Lambda_{\mathcal{E}}(\eta)$, and the planner replaces $\mathcal{E}$ by the environment $\mathcal{E}'$ that has $(\bar{u}'_f, \bar{u}'_m)$ as a threat point for η. Now, the upper contour set containing A shrinks and the previously preferable allocations B ceases to be so.

Proposition 6. *Assume that under $\mathcal{E}$, for some couple $\hat{\eta}$ there is another couple η which is indifferent between its own allocation and that of $\hat{\eta}$. If using the DM can lead to change couple η's threat point as in Lemma 1, then it is possible to make $\hat{\eta}$ strictly prefer its own transaction.*

Proof. See Appendix A.1. □

VIII.2 Numeric Examples

Our first example aims at exploring the issue of dissonance. The next examples allow us to consider the possibility of reducing the inefficiency of an allocation by making them less attractive to families which are not choosing them.

Dealing with 'dissonance' In this example, a type θ_i agents have preferences represented by $u_i(\mathfrak{c}, \mathfrak{l}) = \mathfrak{c} + \mathfrak{l}$. Given transactions $\boldsymbol{z} = (c_1, c_2, -n_1, -n_2)$, a couple $\eta = (\theta_1, \theta_2)$ solves

$$\max_{\mathfrak{c}_1, \mathfrak{c}_2} (\mathfrak{c}_1 + \mathfrak{l}_1 - \bar{u}_1)(\mathfrak{c}_2 + \mathfrak{l}_2 - \bar{u}_2),$$

subject to

$$\mathfrak{c}_1 + \mathfrak{c}_2 \leq \alpha(c_1 + c_2),\ \mathfrak{l}_i \leq 1 - n_i/\theta_i,\ \text{and}\ \mathfrak{c}_i + \mathfrak{l}_i \geq \bar{u}_i, i =, f, m.,$$

where we recall that α measures the material gains from marriage.

We assume, that $\Theta = \{\mathrm{H}, \mathrm{L}\}$, and that each type has the same frequency in the population. Half the population is male, half female. Half is married, half single. Finally, the spouses' types are independent from one another.

The planner's objective is

$$W = \sum_{\theta, i=f,m} \log(U_{i,\theta})\lambda_{i,\theta} + \sum_{\eta, i=f,m.} \log(U_{i,\eta})\lambda_{i,\eta},$$

where $\lambda_{i,\theta}$ and $\lambda_{i,\eta}$ are the weights that the planner assigns to the utility of each type of individual. In our example μ_i^s is the fraction of the population that is single, has type θ and gender i, in our example, it is equal to $\frac{1}{8}$. Similarly, $\mu_{i,\eta}$ is the proportion of married agent, with type η and gender i, in our example. It is equal to $\frac{1}{16}$. We consider a production function for the economy of the form

$$f_1(N) = \left(\prod_{i,\theta} n_{i,\theta}\right)^{\frac{1}{16}} \left(\prod_{i,\eta} n_{i,\eta}\right)^{\frac{1}{32}},$$

where L is the vector of labor supplies.[28]

Start with $\lambda_{s,\theta} = \frac{1}{16}$ and $\lambda_{s,\eta} = \frac{1}{32}$, that is, the weights on the planner utility are equal to the actual proportions on the population.

Table 1 displays the results. L_f indicates that the wife has low productivity, whereas H_f indicates a high productivity wife. Subscript m represents the same for the husband. The first line (Agreement) indicates the utility attained by each type at the optimum. The second line (Disagreement) is the threat point utility, and the third (Singles) is the utility attained by each type if single. In the top panel of Table 1 we consider the outcomes for the case in which the threat points are the utilities attained as singles.

The logic underlying the numeric findings is displayed in Figure 8 where one can see different conditional (on $\boldsymbol{z}$) utility possibility sets for different house-

[28]The nonlinearities are useful to guarantee the existence of solution (and to rule out trivial ones), but the general ideas should be robust to other functional forms. We use a Cobb-Douglas specification on each type of labor to guarantee an interior solution, again, for convenience.

hold types. Threat points, represented as green circles, are the utilities that spouses would obtain as singles. The dashed area, therefore, represents the set of feasible individually rational utility pairs that each couple may attain. Efficient allocations are displayed as blue dots, and they are at the intersection of the utility possibility set frontier and a 45 degrees line starting at the threat points. The asymmetric outcome for asymmetric couples is a consequence of differences in spouses utilities as singles: the benchmark threat point for the example.

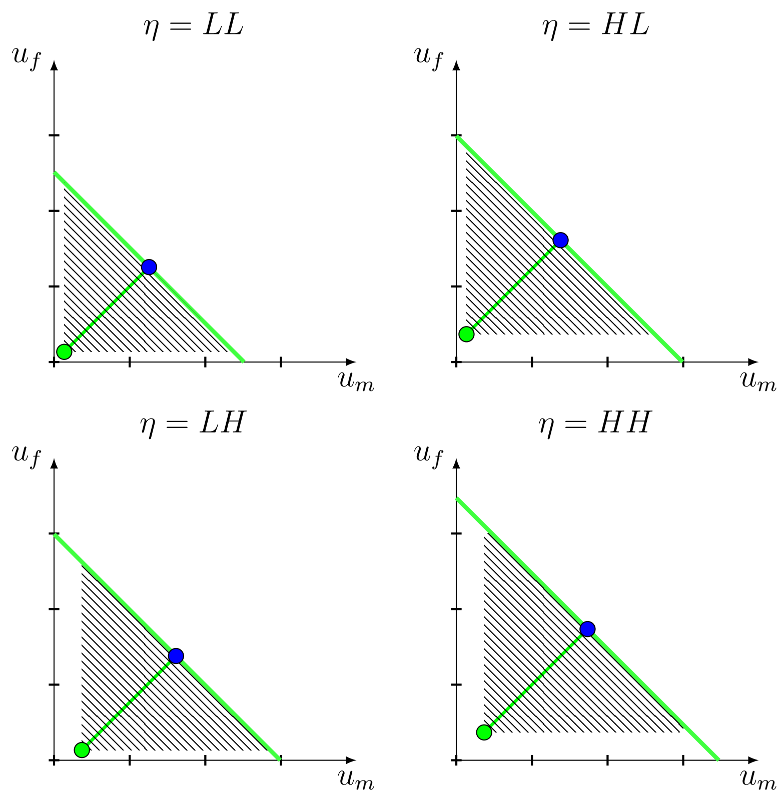

Figure 8: Utility possibility sets for couples $\eta = LL, LH, HK, HH$. Green dots are disagreement utilities, which are the utilities attained by spouses as singles. Blue lines are equilibrium utilities.

Singles of different productivities attain different utilities. As a consequence, for asymmetric couples, LH and HL, utility differences will arise in equilibrium must we insist on having the utilities as single as threat points. Assume, however, that another environment $\mathcal{E}'$ exists for which $\partial \mathcal{Z}^c_{\mathcal{E}'} = \partial \mathcal{Z}^c_{\mathcal{E}}$ but such that new threat points arise. In figure 9, threat points are again represented by green circles. This time, however, utilities attained as singles, black dots, do not coincide with the threat points. The new allocation is given by figure 9 and the

	L_m	L_f	L_m	H_f	H_m	L_f	H_m	H_f
Singles' Utilities as Threat Points								
Agreement	1.26	1.26	1.38	1.61	1.61	1.38	1.74	1.74
Disagreement	0.14	0.14	0.14	0.37	0.37	0.14	0.37	0.37
Singles	0.14	0.14	0.14	0.37	0.37	0.14	0.37	0.37
Alternative Threat Points								
	L_m	L_f	L_m	H_f	H_m	L_f	H_m	H_f
Agreement	1.25	1.25	1.49	1.49	1.49	1.49	1.73	1.73
Disagreement	0.7	0.7	0.7	0.7	0.7	0.7	0.7	0.7
Singles	0.14	0.14	0.14	0.37	0.37	0.14	0.37	0.37

Table 1: **Transferable Utility.** In this table L_f indicates a low productivity female, H_f indicates a high productivity female, L_m a low productivity male, and H_m a high productivity male. The first line (Agreement) indicates the utility attained by each type at the optimum if they are married. The second line (Disagreement) is the threat point utility for the same type. The third line (Singles) is the utility attained by each type if single. In the top panel we consider the outcomes for the case in which the threat points are the utilities attained as singles, and in the bottom, an alternative threat point.

bottom panel of Table 1.[29] Note, in particular, how the planner is able to produce symmetric outcomes for asymmetric couples if it is able to chose symmetric threat points. Because this produces no effect on household equilibrium transactions, the reform is feasible and aligns the household objective with the planner's. Taking again the singles' perspective, the planner may optimize with respect to their allocations knowing that it will not affect the distribution of utilities across spouses.[30]

Although we have assumed the utilities as singles to be the benchmark threat points, the point is more general. Take any institution $\mathcal{E}$, and consider the associated threat points $(u^f_{\mathcal{E}}(\eta), u^m_{\mathcal{E}}(\eta))$, to assess whether this is the best possible outcome, the planner must check whether it is not able to induce another $(\bar{u}^f, \bar{u}^m) \in \Phi(Z^2, \eta)$ which is a 'more desirable' threat point than the one initially chosen.[31]

[29]Any symmetric disagreement utility that is above the singles utility and below the agreement utility would work. We picked those numbers just to be easier to see on the figure.

[30]This is not to say that the allocations assigned to singles are optimally dissociated from those assigned to couples. In our last example we shall see how the possibility of gender based policies may lead the planner to distort the singles allocations to improve upon the allocations assigned to couples. Asymmetries may arise even when the fundamentals are completely symmetric.

[31]In da Costa and Diniz (2015), for example, the departure point is the utility attained as an equilibrium for a disagreement game played under a joint tax schedule. By allowing the possibility of individual filling, the planer/policy maker is able to affect the threat points.

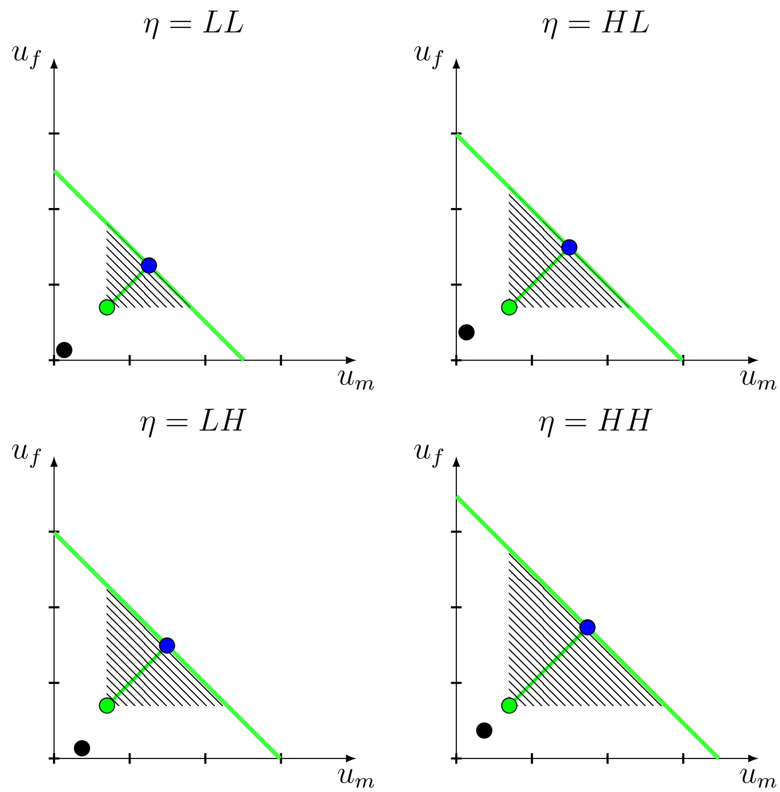

Figure 9: Utility possibility sets for couples $\eta = LL, LH, HK, HH$ under the individual rationality restriction. Black dots represent the utilities that spouses attain as singles. Green dots are the disagreement utilities, whereas blue dots are the equilibrium utilities.

Relaxing IC constraints With transferable utility, changes in threat points do not affect incentive compatibility. When utility is only partially transferable, however, this needs no longer to be the case. To study the importance of threat points on incentive constraints we consider utility functions of the form

$$u_i(\mathfrak{c}, \mathfrak{l} \mid \theta) = \frac{\mathfrak{c}^\sigma}{\sigma} - \frac{\mathfrak{l}}{\theta}.$$

For the numeric exercises, we use $\sigma = 0.5$.

Non-linearities on the welfare function or the production function are no longer needed. Hence we define the welfare and production functions,

$$W_2(U) = \sum_{\theta, i=f,m.} U_{i,\theta}\lambda_{i,\theta} + \sum_{\eta, i=f,m.} U_{i,\eta}\lambda_{i,\eta},$$

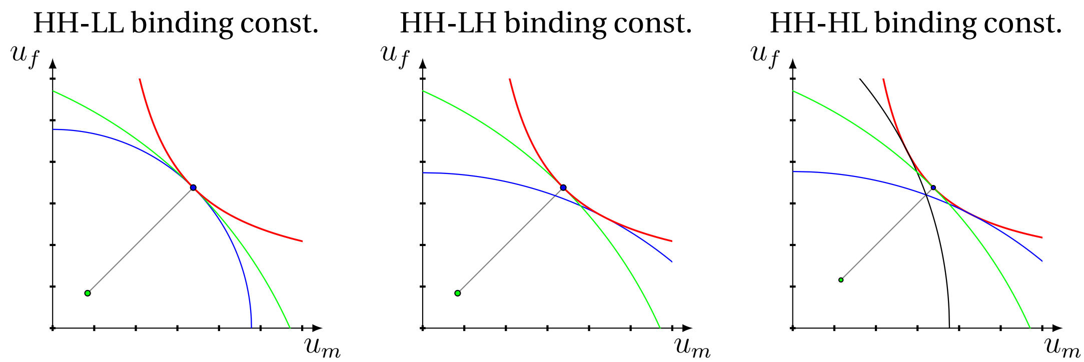

Figure 10: **Binding Incentive Constraints** The Figure displays the utility possibility set for a couple HH (green curve) along with the indifference curve associated with its optimal choice (red curve). For the left panel it is assumed that the couple is indifferent between its bundle and that of a LL couple. The blue curve represents the utility possibility set for the HH couple who chose the $\boldsymbol{z}(L, L)$ bundle. The two other panels represent case in which the binding constrains are with respect to the $\boldsymbol{z}(L, H)$ bundle (middle panel) and to both the $\boldsymbol{z}(L, H)$ and the $\boldsymbol{z}(H, L)$ bundles (right panel).

and

$$f_2(N) = \frac{1}{16} \sum_{\theta, i=f,m.} n_{i,\theta} + \frac{1}{32} \sum_{\eta, i=f,m.} n_{\eta, i=f,m.},$$

respectively.

Once again the benchmark threat points for our exercises are the spouses' utilities were they single.

The green curve in all panels of Figure 10 represents the frontier of the conditional utility possibility set, $\mathcal{U}_{E|\boldsymbol{z}}(\eta)$, for an (H, H) couple if spouses choose the bundle $\boldsymbol{z}(H, H)$ assigned to them. The indifference curve associated with transaction $\boldsymbol{z}(H, H)$ is the red curve convex to the origin. The assumption underlying the left panel is that an (H, H) couple is indifferent between its own transactions $\boldsymbol{z}(H, H)$ and those of a (L, L) couple, $\boldsymbol{z}(L, L)$. The utility frontier for the (H, H) couple when they use the bundle assigned to them is displayed in green. In blue is the utility frontier for the case in which they choose, instead, the bundle assigned to an (L, L) couple. Any threat point along the gray line would induce the same choices both if spouses choose $\boldsymbol{z}(H, H)$ or if they choose $\boldsymbol{z}(L, L)$. Since the exact same utility assignment is chosen, nothing is gained from moving along the gray line.

Things are different if we consider the middle panel of Figure 10. There, the (H, H) couple is indifferent between $\boldsymbol{z}(H, H)$ and the bundle $\boldsymbol{z}(L, H)$ assigned to the (L, H) couple. Were they to choose $\boldsymbol{z}(L, H)$, they would face the utility pos-

sibility set $\mathcal{U}_{E,\boldsymbol{z}(L,H)}(HH)$ and choose the utility pair corresponding to the point where the black and the blue curves are tangent, and attain the same value for the Nash product. The couple is, therefore, indifferent between bundles $\boldsymbol{z}(H,H)$ and $\boldsymbol{z}(L,H)$. Finally, in the rightmost panel, the couple is indifferent between its own transactions $\boldsymbol{z}(H,H)$ and both $\boldsymbol{z}(L,H)$ and $\boldsymbol{z}(H,L)$. If the planner is able to move the threat point along the gray line it makes the indifference curve (in red) more convex: couples dislike inequality within the couple even more.The indifference is broken in favor of $\boldsymbol{z}(H,H)$.[32]

The results are displayed in the top panel of Table 2. We have dropped LL couples for simplicity. One might at first be puzzled by the fact that singles have the same utility independently of their productivities. This is due to the fact that, at the optimum for all problems, low productivity single agents do not work and attain zero disutility from work. A high productivity agent mimicking a low productivity agent does not work either, hence attaining the same utility. The fact that the incentive compatibility constraint binds at the optimum is the final step to understanding the result.

Figure 11 displays the forces at play. The black dot represents the singles' utilities. In purple is the point chosen by Φ if the single's utilities were at the origin of the graphs. The green dot is the actual disagreement (we still need to give in disagreement at least the utility as single). Finally the blue dot is the point chosen in agreement. We also draw the couples' indifference curve at the optimal agreement utility level. The line connecting the disagreement utilities $(\bar{u}^f \bar{u}^f)$, represented as a green dot in all panels of Figure 11, to the agreement utilities (u^f, u^m), represented as a blue dot, define the locus of disagreement utilities for which (u^f, u^m) are optimal agreement utilities.

As we have explained in Section VI, the planner induces threat points by the disagreement games that take place under the environment $\mathcal{E}$. Of course, not all utility pairs are equilibrium outcomes for environments that induce $(\boldsymbol{z}_E(\eta))$ as equilibrium choices for a couple η. Moreover, even if we allow equilibrium choices to change, a meaningful discussion must focus on pairs $(\bar{u}^f, \bar{u}^m)$ for which an environment exists for which this is the equilibrium disagreement outcome. I.e., for each η we focus on the set $\Lambda(\eta)$ defined in Section VI, represented in Figure 11 as an orange line.

[32]If the agreement bundle is not completely symmetric, then moving along the green line has two effects, to dislike inequality and to bias the in bargaining in favor of the member with more utility in agreement, those effects exactly cancel out to keep the agreement utility pair the same.

Singles' Utilities as Threat Points						
	L_m	H_f	H_m	L_f	H_m	H_f
Agreement	4.27	2.74	2.74	4.27	3.38	3.38
Disagreement	1.16	1.16	1.16	1.16	1.16	1.16
Singles	1.16	1.16	1.16	1.16	1.16	1.16
Alternative Threat Points						
	L_m	H_f	H_m	L_f	H_m	H_f
Agreement	4.33	2.79	2.79	4.33	3.26	3.26
Disagreement	1.15	1.15	1.15	1.15	3.26	3.26
Singles	1.15	1.15	1.15	1.15	1.15	1.15

Table 2: **Partially Transferable Utility.** In this table L_f indicates a low productivity female, H_f indicates a high productivity female, L_m a low productivity male, and H_m a high productivity male. The first line (Agreement) indicates the utility attained by each type at the optimum if they are married. The second line (Disagreement) is the threat point utility for the same type. The third line (Singles) is the utility attained by each type if single. In the top panel we consider the outcomes for the case in which the threat points are the utilities attained as singles, and in the bottom, an alternative threat point.

We obtain figure **??** and table 8 . Now that the IC is relaxed, one may transfer more resources from the HH couples to the asymmetric couples.

Consider, now, the southwest panel in Figure 11. The point where the orange curve representing the set $\Lambda(\eta)$ represents a feasible choice for the planner. Indeed, we know that the equilibrium transaction for the couple given this new threat point is still $\boldsymbol{z}_{\mathcal{E}}(\eta)$ and the associated utility pair is still $(u^f_{\mathcal{E}}(\eta), u^m_{\mathcal{E}}(\eta))$. Since the set $\Lambda(\eta)$ is restricted to threat points for which the associated transactions respect incentive compatibility (11), then this threat point is a possible 'choice' for the planner. This new threat point is interesting if one notices the couple's indifference curve becomes more convex as the disagreement utilities approach the agreement utilities frontier. If the couple were initially indifferent between its own bundle and that of another couple then the replacement of the former threat point by the new one would have relaxed incentive constraints with no other impact on the allocation.

At the limit if it were possible to choose a disagreement threat point that coincided with the agreement, a graph like the one in the southeast panel of Figure 11 would result. This is the case exemplified in the bottom panel of Table 2. The problem is that if there are transactions $\boldsymbol{z}$ that lead the couple to attain a pair of utilities (u^f, u^m) despite a non-cooperative behavior, then it would do even better by cooperating. Such $\boldsymbol{z}$ would lead to a violation of (11).

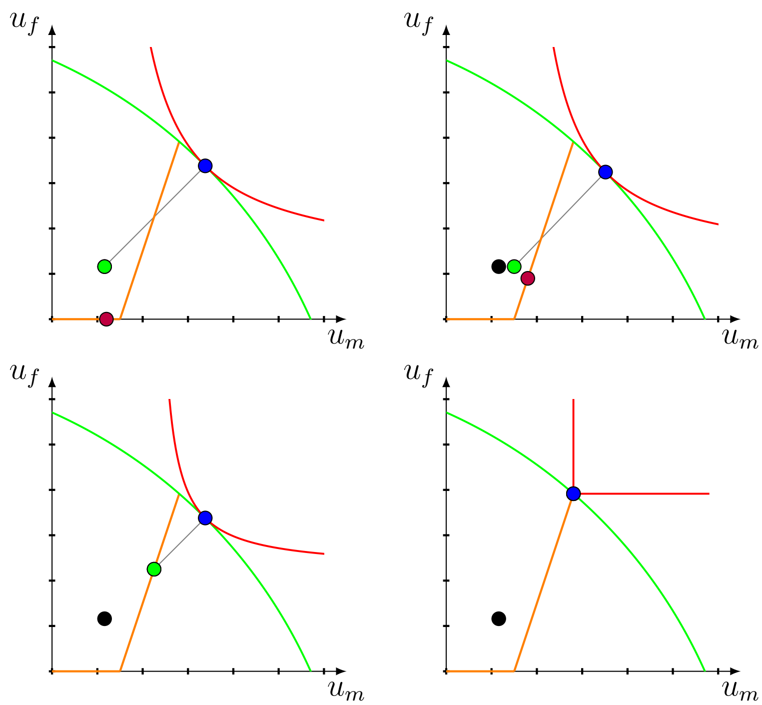

Figure 11: In all panels, the green curve is the frontier of the utility possibility set, and the red curve, the household indifference curve, $\{(u_f, u_m)|[(u_f - \bar{u}_f)][u_m - \bar{u}_m] = w\}$. The orange curve represents all the threat points that the planner can induce by some mechanism which does not violate IC for the agreement transactions that generate the same utility possibility. Black dots are the utilities attained by spouses as singles, red dots are utility pairs that do not respect individual rationality and green dots are threat points.

Objectives Alignment vs IC Relaxation Thus far we have tried to isolate the IC 'channel'. Yet, we have already seen how changes in threat points may increase or reduce the alignment of household and planner's preferences. This "preference" channel, which is the attempt to align couple's objectives with those of singles, arises even when types are known. The issue on which we shall focus is on how is interacts with the IC channel.

If there are couples for which spouses have different productivities, then there is a source of welfare loss countering the IC gains from moving along the gray line. A consequence of endowing the planner with a utilitarian objective is that, for all asymmetric couples, the planner and the couple do not agree on how to allocate utility between family members. In our example, the optimal division within a couple, from the planner point of view, is to equalize the marginal utilities of consumption. There is no reason to believe that the couple will do that. The Nash product punishes utility differences. Moreover, the closer

the threat point is from the equilibrium choice, $(u^f_{\mathcal{E}}(\eta), u^m_{\mathcal{E}}(\eta))$, the stronger is the aversion to inequality in utilities. The Utilitarian metric, on the other hand, does not punish utility differences *per se*. Assume, for example, that the threat point is such that $\bar{u}^f = \bar{u}^m$. If one of the spouses suffers more disutility from effort, from a household perspective, then he or she must be compensated by getting more utility from consumption. It then becomes costly to move along the gray line in an attempt to relax IC constraints.

The planner would like, in this case, to increase the disagreement utility for the low type and decrease for the high type. If the disagreement game can be used to do that, then it will, as the disagreement game provides the cheapest way of doing it (no feasibility constraint and only indirect effect on IC). The problem is that this conflicts with the goal of relaxing IC constraints. Indeed, the disagreement utility tends to close the gap between the agreement utilities, so it goes in the wrong direction. The desire to reduce dissonance dampens the relaxation effect on incentive compatibility constraints.

Gender based policies Another possibility the planer may entertain is to use gender based policies.

In a couple for which the wife has low productivity and the husband has high productivity, assume that the man is compensated for higher effort by getting higher consumption. Assume also that the distribution of utilities for singles is optimal among all gender blind policies under the utilitarian metric and taking into account its effect on household threat points. The planner may then increase the inequality in utilities between single low productivity men and single low productivity women to bias the decision in favor of married men in LH couples thus increasing the sum of utilities.

The problem with this policy is that the planner must also take into account the impact that it will produce on couples with a high-type woman and a low-type man. If the disagreement utility remains the single's utility as the reform is implemented, then the bias would favor the men member and the sum of utilities for HL couples would reduce. This is all assuming that the threat points are utilities attained as singles. If one is not restricted on this aspect, there is a way out of this dilemma. The planner can use the disagreement utilities to isolate the couple's decision from their utilities as single. The allocation associated with such policy is shown in Table 3. Women are better off than men in this case

— see also Figure 12. The other case is completely symmetric, and would yield the same utility to the planner.

The surprising aspect of this example is that an asymmetric allocation arises in a completely symmetric environment. There is gender-based taxation even though genders are inconsequential in all possible ways.[33]

In the general asymmetric case, the distribution of productivities for each gender will matter. For instance, assume that there are more low-type men than women, but everything else is symmetric. Then the planner would prefer to bias the allocation towards men, because there would be more (low-type man,high-type woman) couples and the gains from biasing in that direction would be larger than targeting women.

If the disagreement is not symmetric gender-wise, e.g., if low-type women in a (HL) couple have a better disagreement than the low-type man inside a (HL) couple, then the planner may bias the allocation towards the women.

In general those two things interact with the "preference" channel.

If the planner could use only the disagreement, then he would, as it is the cheapest way to allocate bundles. So, in our example, if the disagreement favors heavily the low-type inside a couple, then he would set symmetric allocations for singles and use only the disagreement.

On the other extreme of the spectrum, if the disagreement favors heavily the high-type, then the planner would also set a symmetric allocation for the singles. Now, exactly because the disagreement is heavily biased in the "wrong" direction, the planner can not isolate the effect of gender asymmetries. So we have a region where this asymmetry shows up as we vary the disagreement allocation on the asymmetric couples from favoring low-types to favoring high-types.

IX Conclusion

We have asked whether tax implementation is without loss when couples do not behave as a single individual, but instead make decisions through a bargain which outcome we assume to satisfy Nash's (1950) axioms.

We find that tax schedules are poor instruments to play the dual role of in-

[33] The fact that the planner uses gender to produce this asymmetry is irrelevant; any other meaningless observable exogenous variable could be used as a 'tag'.

	L_m	H_f	H_m	L_f	H_m	H_f
Agreement	4.19	2.80	2.62	4.54	3.21	3.21
Disagreement	1.45	1.45	0.95	1.45	3.21	3.2
Singles	0.95	1.45	0.95	1.45	0.95	1.45

Table 3: **Asymmetric allocations with symmetric fundamentals** In this table L_f indicates a low productivity female, H_f indicates a high productivity female, L_m a low productivity male, and H_m a high productivity male. The first line (Agreement) indicates the utility attained by each type at the optimum if they are married. The second line (Disagreement) is the threat point utility for the same type. The third line (Singles) is the utility attained by each type if single.

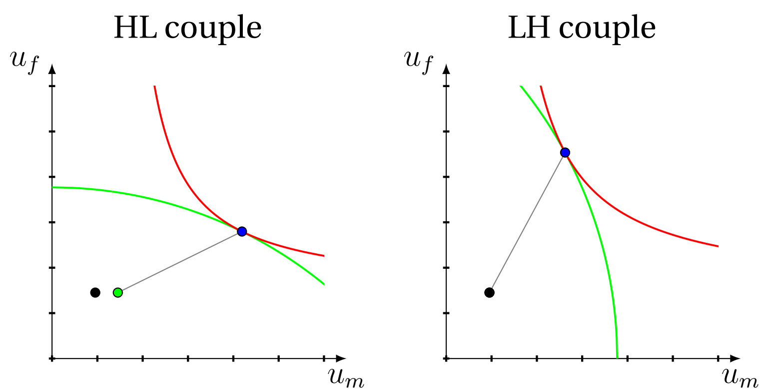

Figure 12: **Gender Based Policies.** In this figure we show how women and men may optimally have different allocations, even though the fundamentals of the economy and the planners objective is completely symmetric with respect to gender. For LH couples, threat points are the utilities attained by spouses as singles, whereas for HL couples the two points differ.

ducing allocations conditional on households' objectives and defining these objectives through its impact on threat points. Tax induced allocations can be improved upon by more general mechanism through the inducement of convenient threat points that allow for: the alignment of household and the planner's objectives; the relaxation of incentive constraints.

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A Appendix

A.1 Proofs

Proof of Proposition 1. One of the key axioms in Nash's (1950) solution concept is Pareto efficiency. Therefore, a spouse can only utility by reducing the other spouses'. This would be possible if he or she were able to increase his or her threat point with respect to his or her spouses'. The 'divorce' utility is always attainable by a unilateral choice of either one of the spouses. Hence, a utility choice based on threat point utility differences that differ from the ones attainable with divorce would be blocked by one of the spouses through a commitment to split. Threat points which do not violate this condition are those in the set V_E^α. □

Proof of Proposition 2. Consider a general mechanism as defined in (4). The original mechanism $\mathcal{M}$ is a set of message spaces for single females, Σ^f, single males, Σ^m, married females, $\Sigma^{c,f}$, and married males, $\Sigma^{c,m}$, and outcome functions for single females, $g^f : \Sigma^f \to \mathcal{Z}$, single males, $g^m : \Sigma^m \to \mathcal{Z}$, and couples $\boldsymbol{g}^c : \Sigma^{c,f} \times \Sigma^{c,m} \to \mathcal{Z}^2$. Let $\{\sigma_i^s(\theta_i)\}_{\theta_i, i=f,m}$, $\{\sigma^{c,i}(\eta, a)\}_{\eta, i=f,m}$, and $\{\sigma^{c,i}(\eta, d)\}_{\eta, i=f,m}$ denote, respectively, equilibrium announcements for singles, married agents in couples in agreement and married agents in couples in disagreement.

An equilibrium allocation for the mechanism is

$$\left\{(z_{\mathcal{M}}^f(\theta_f))_{\theta_f}, (z_{\mathcal{M}}^m(\theta_m))_{\theta_m} (\boldsymbol{z}_{\mathcal{M}}(\eta))_\eta\right\},$$

where

$$z_{\mathcal{M}}^f(\theta_f) = g^f(\sigma^f(\theta_f)),$$
$$z_{\mathcal{M}}^m(\theta_m) = g^m(\sigma^m(\theta_m)),$$

and

$$\boldsymbol{z}_{\mathcal{M}}(\eta) = \boldsymbol{g}^c\left(\sigma^{c,f}(\eta, a), \sigma^{c,m}(\eta, a)\right).$$

We consider the case in which the game induces a Pure Strategy Nash equilibrium. The case in which mixed strategy equilibrium are allowed follows the same step but requires the introduction of additional notation.

Here,

$$\sigma^f(\theta_f) = \underset{m\in\Sigma^f}{\operatorname{argmax}}\, u(g^f(m), \theta_f), \tag{20}$$

$$\sigma^m(\theta_m) = \underset{m\in\Sigma^f}{\operatorname{argmax}}\, u(g^m(m), \theta_m), \tag{21}$$

and

$$\left(\sigma^{c,f}(\eta,a),\sigma^{c,m}(\eta,a)\right)=\underset{m\in\Sigma^f,m'\in\Sigma^m}{\operatorname{argmax}} W(\boldsymbol{g}^c\left(m,m'\right),\eta,\bar{u}^f_{\mathcal{M}}(\eta),\bar{u}^m_{\mathcal{M}}(\eta)). \tag{22}$$

The threat points in (22) are

$$\left(\bar{u}^f_{\mathcal{M}}(\eta),\bar{u}^m_{\mathcal{M}}(\eta)\right)=\Phi\left(\sigma^{c,f}(\eta,d),\sigma^{c,m}(\eta,d),\eta\right) \tag{23}$$

for

$$\sigma^{c,f}(\eta,d)=\underset{m\in\Sigma^f}{\operatorname{argmax}}\,\phi^f\left(\boldsymbol{g}^c\left(m,\sigma^{c,m}(\eta,d)\right),\eta\right), \tag{24}$$

and

$$\sigma^{c,m}(\eta,d)=\underset{m'\in\Sigma^m}{\operatorname{argmax}}\,\phi^m\left(\boldsymbol{g}^c\left(\sigma^{c,f}(\eta,d),m'\right),\eta\right). \tag{25}$$

Define the outcome for the direct mechanism as follows. For singles,

$$\gamma^f(\theta_f)=g^f(\sigma^f(\theta_f)),$$

and

$$\gamma^m(\theta_m)=g^m(\sigma^m(\theta_m)).$$

For couples,

$$\boldsymbol{\gamma}^c(\eta,\iota,\eta',\iota')=\boldsymbol{g}^c\left(\sigma^{c,f}(\eta,\iota),\sigma^{c,m}(\eta',\iota')\right)$$

for all η, η', $\iota,\iota'\in\{a,d\}$.

For singles the proof is standard and we omit for brevity. For couples, assume that threat points, $\bar{u}^i_{\mathcal{M}}(\eta)=\bar{u}^i_{\Gamma}(\eta)\ \forall\eta$, $i=f,m$, are given by (23). Then, by (22) and $\boldsymbol{\gamma}^c(\eta,\iota,\eta',\iota')=\boldsymbol{g}^c\left(\sigma^{c,f}(\eta,\iota),\sigma^{c,m}(\eta',\iota')\right)$ for all η, η', $\iota,\iota'\in\{a,d\}$ for any $(\hat{\eta},\hat{\iota},\hat{\eta}',\hat{\iota}')$ it must be the case that

$$W(\boldsymbol{\gamma}^c(\hat{\eta},\hat{\iota},\hat{\eta}',\hat{\iota}'),\eta,\bar{u}^f_{\mathcal{M}}(\eta),\bar{u}^m_{\mathcal{M}}(\eta))\leq W(\boldsymbol{\gamma}^c(\eta,a,\eta,a),\eta,\bar{u}^f_{\mathcal{M}}(\eta),\bar{u}^m_{\mathcal{M}}(\eta)).$$

So, we only need to show that $\bar{u}^i_{\mathcal{M}}(\eta)=\bar{u}^i_{\Gamma}(\eta)\ \forall\eta$, $i=f,m$. We do it by showing that truth telling is an equilibrium for the disagreement game.

Assume that spouse f of couple η announces (η,d), and let (η',i) be an announcement that her husband finds better than (η,d), i.e., $\phi^m\left(\boldsymbol{\gamma}^c(\eta,d,\eta',i),\eta\right)>$

$\phi^m\left(\boldsymbol{\gamma}^c(\eta, d, \eta, d), \eta\right)$. Then,

$$\begin{aligned}\phi^m\left(\boldsymbol{g}^c\left(\sigma^{c,f}(\eta, d), \sigma^{c,m}(\eta', i)\right), \eta\right) &= \phi^m\left(\boldsymbol{\gamma}^c(\eta, d, \eta', i), \eta\right) \\ &> \phi^m\left(\boldsymbol{\gamma}^c(\eta, d, \eta, d), \eta\right) = \phi^m\left(\boldsymbol{g}^c\left(\sigma^{c,f}(\eta, d), \sigma^{c,m}(\eta, d)\right), \eta\right).\end{aligned}$$

This is a contradiction with (25). □

Proof of Proposition 3. For a given equilibrium allocation

$$\left(\left\{z^i_\Psi(\theta_i)\right\}_{\theta_i, i=f,m}, \left\{\boldsymbol{z}_\Psi(\eta)\right\}_\eta\right)$$

induced by the tax system ψ consider a naive direct mechanism which outcome function is defined as follows. Let $\mathcal{Z}_\Psi := \bigcup_\eta z_\Psi(\eta)$.

(i) For singles $\gamma^s_i(\theta_i) = z^i_\Psi(\theta_i)$.

(ii) For couples

(a) $\boldsymbol{\gamma}(\eta, a, \eta, a) = \left(z^{c,f}_\Psi(\eta), z^{c,m}_\Psi(\eta)\right)$

(b) $\boldsymbol{\gamma}(\eta, d, \eta, d) = \left(\bar{z}^{c,f}_\Psi(\eta), \bar{z}^{c,m}_\Psi(\eta)\right)$

(c) $\boldsymbol{\gamma}(\eta, \iota, \eta', \iota') = (\underline{z}, \underline{z})$ if $\eta \neq \eta'$ or $\iota \neq \iota'$

Note that the allocation is feasible. Hence, if the allocation is incentive compatible then it is incentive-feasible.

We start by considering single agents. Let us assume that there is an agent θ_i and an announcement $\hat{\theta}_i$, such that the agent prefers to announce $\hat{\theta}_i$ to announcing the truth, θ_i. In this case, $z^i_\Psi(\hat{\theta}_i) \succeq_{\theta_i} z^i_\Psi(\theta_i)$, and $\psi(z^i_\Psi(\hat{\theta}_i)) \leq 0$. But in this case the agent could have chosen $z^i_\Psi(\hat{\theta}_i)$ under the tax system. A contradiction.

As for couples, assume there is a couple η and an announcement $(\hat{\eta}, \iota, \hat{\eta}', \iota')$ such that $\boldsymbol{\gamma}^c(\hat{\eta}, \iota, \hat{\eta}', \iota') \succ_{\eta|\bar{u}_f, \bar{u}_m} \boldsymbol{\gamma}^c(\eta, a, \eta, a)$.

It cannot be $\hat{\eta} \neq \hat{\eta}'$ or $\iota \neq \iota'$ since, for all $\boldsymbol{z} \in \mathcal{Z}^2$, $\boldsymbol{z} \succ_{\eta|\bar{u}_f, \bar{u}_m} \underline{\boldsymbol{z}}$. So it is either $\boldsymbol{\gamma}^c(\hat{\eta}, \iota, \hat{\eta}', \iota') = \boldsymbol{\gamma}^c(\hat{\eta}, a, \hat{\eta}, a) = \boldsymbol{z}_\Psi(\hat{\eta})$ or $\boldsymbol{\gamma}^c(\hat{\eta}, \iota, \hat{\eta}', \iota') = \boldsymbol{\gamma}^c(\hat{\eta}', d, \hat{\eta}', d) = \bar{\boldsymbol{z}}_\Psi(\hat{\eta})$.

Because these choices were made either by households in agreement or by households in disagreement, both $\psi(\boldsymbol{z}_\Psi(\hat{\eta})) \leq 0$ and $\psi(\bar{\boldsymbol{z}}_\Psi(\hat{\eta}')) \leq 0$ must be true. These choices were therefore available to the couple under ψ. This is in contradiction with $\boldsymbol{z}_\Psi(\eta) \succeq_{\eta|\bar{u}_f, \bar{u}_m} \boldsymbol{z}_\Psi(\eta')$. □

Proof of Proposition 4. Start from an allocation,

$$\left(\left\{z^i_\Gamma(\theta_i)\right\}_{\theta_i, i=f,m}, \{\boldsymbol{z}_\Gamma(\eta)\}_\eta\right)$$

implemented by a DM. For singles of gender i, define the tax function using $\psi_i(z) = 0$ if there is θ_i such that $z = z^i_\Gamma(\theta_i)$, $\psi_i(z') < 0$ for all $z' < z$ and $\psi_i(z') > 0$ for all $z' > 0$. For couples, define $\mathcal{Z}^2_\Gamma = \bigcup_\eta \boldsymbol{z}_\Gamma(\eta)$. $\boldsymbol{\psi}(\boldsymbol{z}) \leq 0$ if and only if $\exists \boldsymbol{z}' \in Z^2_\Gamma$ such that $\boldsymbol{z} \leq \boldsymbol{z}'$.

We start by considering single agents. Let us assume that there is an agent θ_i and a choice z such that $\psi_i(z) \leq 0$ such $z \succeq_{\theta_i} z^i_\Gamma(\theta_i)$. Note that either there is $\hat{\theta}_i$ such that $z = z^i_\Gamma(\hat{\theta}_i)$, or $z < z^i_\Gamma(\hat{\theta}_i)$. Monotonicity guarantees that we may restrict attention to the former. But if this is the case, then, announcing $\hat{\theta}_i$ dominates truth telling. A contradiction.

For couples, assume that there is η and $\boldsymbol{z}'$ such that $\boldsymbol{\psi}(\boldsymbol{z}') \leq 0$ and $\boldsymbol{z}' \succ_{\eta|\bar{u}_f,\bar{u}_m} \boldsymbol{\gamma}^c(\eta, a, \eta, a)$. Monotonicity implies that we can restrict our attention to $\boldsymbol{z}'$ such that there is no $\boldsymbol{z}'' > \boldsymbol{z}'$ such that $\boldsymbol{\psi}(\boldsymbol{z}'') \leq 0$, i.e., $\boldsymbol{z}' \in \mathcal{Z}^2_\Gamma$. In this case, there is $(\hat{\eta}', \iota, \hat{\eta}, \iota)$ such that $z' = \boldsymbol{\gamma}^c(\hat{\eta}', \iota, \hat{\eta}, \iota)$. But then $\boldsymbol{\gamma}^c(\hat{\eta}', \iota, \hat{\eta}, \iota) = \boldsymbol{z}' \succ_{\eta|\bar{u}_f,\bar{u}_m} \boldsymbol{\gamma}^c(\eta, a, \eta, a)$. A contradiction. Feasibility of STS is due to feasibility of DM. □

Lemma 1. *Let*

$$W(\boldsymbol{z}_\mathcal{E}(\eta), \eta, \bar{u}^f_\mathcal{E}(\eta), \bar{u}^m_\mathcal{E}(\eta)) = W(\boldsymbol{z}_\mathcal{E}(\eta'), \eta, \bar{u}^f_\mathcal{E}(\eta), \bar{u}^m_\mathcal{E}(\eta))$$

for some $\eta' \neq \eta$.

Assume that there is $\mathcal{E}'$ *such that*

$$(\bar{u}^f_{\mathcal{E}'}(\eta), \bar{u}^m_{\mathcal{E}'}(\eta)) = \alpha(\bar{u}^f_\mathcal{E}(\eta), \bar{u}^m_\mathcal{E}(\eta)) + (1-\alpha)(u^f_\mathcal{E}(\eta), u^m_\mathcal{E}(\eta))$$

for $\alpha \in (0, 1)$.

Then,

$$W(\boldsymbol{z}_\mathcal{E}(\eta), \eta, \bar{u}^f_{\mathcal{E}'}(\eta), \bar{u}^m_{\mathcal{E}'}(\eta)) > W(\boldsymbol{z}_\mathcal{E}(\eta'), \eta, \bar{u}^f_{\mathcal{E}'}(\eta), \bar{u}^m_{\mathcal{E}'}(\eta)).$$

Proof. Without loss, let the spouses utilities at the original threat point be $(0, 0)$. Let (u_f, u_m) be the corresponding agreement solution. We know, in this case, that $(u_f, u_m) \geq (u'_f, u'_m)$ for any feasible utility pair (u'_f, u'_m).

Suppose that there is $\alpha \in (0, \frac{1}{2})$ such that the planner can induce disagree-

ment utilities $(\alpha u_f, \alpha u_m)$. In this case, we have

$$u'_f u'_m \leq (u'_f u'_m)^{\alpha}(u_f u_m)^{1-\alpha} = (u'_f u_m)^{\alpha}(u'_f u'_m)^{\alpha}(u_f u_m)^{1-2\alpha} \\ < \alpha u'_f u_m + \alpha u_f u'_m + (1-2\alpha)u_f u_m$$

Adding $\alpha^2 u_f u_m$ in both sides and rearranging we get

$$u'_f u'_m - \alpha u'_f u_m + \alpha u_f u'_m + \alpha^2 u_f u_m < (1 - 2\alpha + \alpha^2) u_f u_m$$

Which we can rewrite as

$$(u'_f - \alpha u_f)(u'_m - \alpha u_m) < (u_f - \alpha u_f)(u_m - \alpha u_m)$$

If we can only find $\alpha > \frac{1}{2}$, then we just go until $\frac{1}{2}$, then until $\frac{3}{4}$, and so forth until we reach α and apply the same argument as above. □

Lemma 2. *Assume that utilities that represent agents preferences are of the form* $u_i(\mathfrak{c}, \mathfrak{l}) = \mathfrak{c} + h(\mathfrak{l})$ *for* $h(\cdot)$ *strictly increasing and concave. Then, optimal transactions are independent of threat points.*

Proof. Let $\mathfrak{h}^i = h(n^i/\theta^i)$, and $\hat{\mathfrak{h}}^i = h(\hat{n}^i/\theta^i)$, $i = f, m$. The household prefers transactions $(c^f, -n^f, c^m, -n^m)$ to $(\hat{c}^f, -\hat{n}^f, \hat{c}^m, -\hat{n}^m)$ if

$$\max_{\mathfrak{c}^f} \left[\mathfrak{c}^f - \mathfrak{h}^f - \bar{u}^f\right] \left[x - \mathfrak{c}^f - \mathfrak{h}^m - \bar{u}^m\right] \geq \\ \max_{\mathfrak{c}^f} \left[\mathfrak{c}^f - \hat{\mathfrak{h}}^f - \bar{u}^f\right] \left[\hat{x} - \mathfrak{c}^f - \hat{\mathfrak{h}}^m - \bar{u}^m\right], \quad (26)$$

for $x = c^f + c^m$, $\hat{x} = \hat{c}^f + \hat{c}^m$.

At the optimum, for the maximization problems above we have

$$\left[x - \mathfrak{c}^f + \mathfrak{h}^m - \bar{u}^m\right] = \left[\mathfrak{c}^f + \mathfrak{h}^f - \bar{u}^f,\right]$$

hence,

$$\frac{x + (\upsilon^f + \bar{u}^f) - (\upsilon^m + \bar{u}^m)}{2} = \mathfrak{c}^f$$

with analogous expressions for $(\hat{c}^f, -\hat{n}^f, \hat{c}^m, -\hat{n}^m)$.

The value of the program, for a given $\boldsymbol{z} = (c^f, -n^f, c^m, -n^m)$ is, therefore,

$$\left[\frac{x + (\mathfrak{h}^f + \bar{u}^f) - (\mathfrak{h}^m + \bar{u}^m)}{2} + \mathfrak{h}^f - \bar{u}^f\right]\left[x - \frac{x + (\mathfrak{h}^f + \bar{u}^f) + (\mathfrak{h}^m - \bar{u}^m)}{2} + \mathfrak{h}^m - \bar{u}^m\right],$$

or

$$\frac{1}{4}\left[x + (\mathfrak{h}^f - \bar{u}^f) + (\mathfrak{h}^m - \bar{u}^m)\right]^2.$$

Equation (26) is therefore equivalent to

$$\Big[\underbrace{x - (\mathfrak{h}^f + \mathfrak{h}^m)}_{y} - \underbrace{(\bar{u}^f + \bar{u}^m)}_{\bar{v}}\Big]^2 \geq \Big[\underbrace{\hat{x} - (\hat{\mathfrak{h}}^f + \hat{\mathfrak{h}}^m)}_{\hat{y}} - \underbrace{(\bar{u}^f + \bar{u}^m)}_{\bar{v}}\Big]^2$$

Finally, using $y - \bar{v} > 0$, and $\hat{y} - \bar{v} > 0$, (26) is shown to be equivalent to

$$y - \bar{v} \geq \hat{y} - \bar{v} \Leftrightarrow y \geq \hat{y}.$$

□