FUNDAÇÃO GETULIO VARGAS
ESCOLA DE ECONOMIA DE SÃO PAULO

MARCOS SOUZA FERREIRA

BUBBLE DETECTION IN BRAZIL'S STOCK MARKET:
Application of the generalized superior augmented Dickey-Fuller test

SÃO PAULO
2016

MARCOS SOUZA FERREIRA

BUBBLE DETECTION IN BRAZIL'S STOCK MARKET:
Application of the generalized superior augmented Dickey-Fuller test

Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas, como requisito para obtenção do título de Mestre em Finanças e Economia

Campo do conhecimento:
Finanças Quantitativas

Orientador: Prof. PhD João de Mendonça Mergulhão

SÃO PAULO
2016

Ferreira, Marcos Souza.
Bubble detection in Brazil’s stock market / Marcos Souza Ferreira. - 2016.
43 f.

Orientador: João de Mendonça Mergulhão
Dissertação (MPFE) - Escola de Economia de São Paulo.

1. Cointegração. 2. Mercado financeiro. 3. Mercado de ações - Previsão. 4. Bolsa de Valores de São Paulo. I. Mergulhão, João de Mendonça. II. Dissertação (MPFE) - Escola de Economia de São Paulo. III. Título.

CDU 336.76(81)

MARCOS SOUZA FERREIRA

BUBBLE DETECTION IN BRAZIL’S STOCK MARKET:
Application of the generalized superior augmented Dickey-Fuller test

Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas, como requisito para obtenção do título de Mestre em Finanças e Economia

Campo do conhecimento:
Finanças Quantitativas

Data de Aprovação

__/__/____

Banca Examinadora

____
Prof. PhD João de Mendonça Mergulhão (orientador)
FGV-EESP

____
Prof Dr Emerson Marçal
FGV-EESP

____
Prof Dr Marco Lyrio
Insper

“It is not knowledge, but the act of learning, not possession but the act of getting there, which grants the greatest enjoyment.”

Carl Friedrich Gauss

ACKNOWLEDGEMENTS

I would like to express my gratitude to my family, first of all, for all the support I have received over the years. They've always had my back and kept me going even through the toughest times. I know that no matter what happens, no matter how complicated things get, they will always be there, and I can always count on them for guidance, patience and comfort, even when I made bad decisions.

I would like to thank my teachers for taking the time to share their immense knowledge. Without it, none of this would be possible.

Finally, I would like to thank my friends. For many years we have been close, even though we may physically be far. I hope they will remain being my other family for the rest of my time.

ABSTRACT

Considering the importance of the proper detection of bubbles in financial markets for policymakers and market agents, we used two techniques described in Diba and Grossman (1988b) and in Phillips, Shi, and Yu (2015) to detect periods of exuberance in the recent history of the Brazillian stock market. First, a simple cointegration test is applied. Secondly, we conducted several augmented, right-tailed Dickey-Fuller tests on rolling windows of data to determine the point in which there’s a structural break and the series loses its stationarity.

Keywords: Bubbles, Bubble detection, ADF test, Stationarity, Integration, Cointegration.

LIST OF FIGURES

LIST OF TABLES

CONTENTS

1. INTRODUCTION

The subject of the correct pricing of assets has influenced publications in several areas. An important aspect on this subject is the detection of bubbles. By definition, a bubble refers to the case when the asset price consistently exceeds its fundamental value. Fundamental value is the value implied by the current and future cash flows generated by the asset.

Policymakers can gain from the correct assessment of bubbles. It is important to know when market prices diverge significantly from the assets fundamental values because such prices are not sustainable. A bubble always precedes a burst, and, if big enough, the burst can trigger subsequent losses across the economy.

The subject of bubbles has generated extensive literature on its types, causes, and methods for detection. The focus on this paper is rational bubbles, that is, bubbles that are formed when all investors behave rationally.

Several methods for detecting rational bubbles have been created. Many of which only indicate the presence of bubbles, but do not pinpoint its initiation and later collapse. Using a series of right tailed rolling window ADF tests, Phillips, Shi, and Yu (2015) propose a technique to datestamp the beginning and end of periods of high exuberance in the market. The authors use this to detect several well-known episodes of bubbles and collapses in the S&P 500 historical data.

This paper sought to apply the same technique, along with a classical cointegration test, to detect bubbles in the IBOVESPA Index, from São Paulo's stock exchange. Using data for monthly prices and dividends from November 1996 to January 2016, the Generalized Sup ADF test (GSADF) shows evidence of a bubble in the final months of 2003, which is consistent with the optimism that Brazil was experiencing by that time.

This paper is divided in four sections. In Section 2 the theoretical background is presented. The types of bubbles are described with some of the models that generated these occurrences and the tests that have been developed for each case. In section 3 the tests proposed by Phillips et al. (2015) and by Diba and Grossman (1988b) are defined and in

section 4 the empirical results from the tests are shown. Section 5 contains the conclusion and final remarks.

2. THEORETICAL BACKGROUND

A bubble occurs when an asset's price exceeds by a material amount the price implied by its fundamental value. This means that the cash flows generated by the asset cannot justify its current valuation. There is extensive literature on the subject and it is known that there are different types of bubbles depending on how they are created. For each type of bubble there have been developed different classes of tests for detection.

In this section, we will start by explaining the different classes of bubbles that have already been studied and the scenarios under which each can occur. We will later explain the asset pricing model by Blanchard (1979) which is the basis for the tests conducted in this paper. As we progress on the evolutions of the asset pricing and bubble value models we will explain the different tests that have been developed for detecting rational bubbles.

The types of bubbles fall into two major classes: rational and irrational bubbles. This paper's objective is to identify rational bubbles in assets. The irrational ones (sometimes referred to as behavioral bubbles) fall outside the scope of this work, however we will briefly explain them in the next section.

2.1. Irrational Bubbles

We start with a short description of irrational bubbles and the associated literature before moving on to the focus of this paper.

Irrational bubbles arise when not all players on a given market make decisions solely on rational aspects. There could, in fact, be a significant number of rational investors and still a bubble can be formed. The main point of this class of bubbles is having interaction between traders with heterogeneous beliefs.

The difference in investor's prior distributions can arise from psychological biases. If investors are overconfident in their ability to read market signals they can have different

opinions on return patterns. The mismatch in beliefs can persist even after all information is shared.

Heterogeneous beliefs in a scenario with short sale restrictions can lead to overpricing, since optimists push up the price and pessimist cannot counterbalance it, according to Miller (1977). Ofek and Richardson (2003) use this framework to describe the internet bubble in the 1990s.

Harrison and Kreps (1978) show that an asset's price can exceed even the fundamental value of the most optimistic player. As long as that player believes the other investors will be more optimistic in the future, he can sustain the valuation believing he can sell the asset for a higher price in the future.

Delong et al. (1990) show that noise traders can create discrepancies on the market that can persist for some time. Rational investors can experience losses if they attack the bubble at the wrong time. That way the noise traders are rewarded for the situation they created.

Abreu and Brunnemeier (2003) argue that sometimes rational traders cannot drive prices back to fundamental value because they cannot properly synchronize their actions. Also, it can be profitable to ride the bubble instead of attacking it or sitting out. They claim this is more likely in the presence of short horizon traders, such as hedge funds that may be required to present results periodically.

The authors show that a triggering event, for example a pessimistic news release, could be the signal for traders to burst the bubble.

Scheinkman and Xiong (2003) explore the relationship between bubbles and the amount of float for a given security. They show that an asset with little float acts as if it faced short sale restrictions. In their model investors have asymmetrical access to information and less informed investors show overconfidence.

The optimistic investors drive the price up. When more informed insiders are allowed to sell their positions (for example on the end of a lock up period) the float increases and the

bubble is more likely to burst. That is consistent with the fall of the dotcom stocks during the 1990s.

In all cases cited here it is essential that rational or pessimistic investors have limited power to bring down asset prices. That can be accomplished if pessimistic investors are too few to influence prices, or if there are restrictions on short sale.

2.2. Rational Bubbles

This class of bubbles is the focus of this paper. All models that will be cited here consider that the entire universe of agents is rational. However agents can differ on the amount of information they possess. Here we separate bubbles into two categories. On the first there is free flow of information and on the second one we have the interaction between more and less informed agents.

2.2.1. Rational Bubbles under Symmetrical Information

Under certain conditions bubbles can emerge even considering rational investors and total information symmetry. A condition associated with many of the models found in literature is the short sale restriction. Tirole (1982) argues that without restrictions a bubble cannot exist in a world with infinite lived agents, if the agents know that the initial general equilibrium allocation is Pareto efficient. The agent that holds the overpriced asset will be likely to sell it. However, since the initial allocation is Pareto efficient, the agents that do not hold the asset will not be willing to buy it, since it will reduce their utility. The asset's price will be naturally adjusted to a no-bubble price.

However, Tirole (1985) shows that bubbles can exist if agents have finite lives and overlapping generations. The agents are willing to buy an overpriced asset if they believe they can resell it for even a higher price in the future. The author also shows that, under no arbitrage conditions, a bubble must grow at the same rate as the risk free rate of the economy.

2.2.2. Rational Bubbles under Asymmetrical Information

Tirole (1982) also creates a model with infinite lived agents under which a bubble can be formed. However certain assumptions need to be made. The investors cannot know that the initial allocation is Pareto efficient, there has to be restrictions on short selling, and prices cannot reveal all the available information.

Another possibility for bubbles is described in Allen and Gale (2000). In their model, banking intermediation can create an agency problem which can lead to bubbles. There are two types of assets, a risky one and a safe one. There is asymmetry of information because banks do not know how to invest in risky assets.

In this scenario the best course of action for the investor is to borrow and invest on the risky asset. If the investment loses value, the investor can declare bankruptcy and will not need to repay the debt. That drives asset prices up and creates bubbles. Variations in the supply of credit determine the size of the bubble and when it bursts.

2.2.3. Pricing Model and Bubble Detection Tests

Rational bubbles under symmetrical information are the focus of this paper. We will now describe the evolution of the pricing models, starting by the pioneer model by Blanchard (1979). As the models got more sophisticated so did the bubble detecting procedures.

In Blanchard's model all agents are rational, which means they do not commit systematic errors. The bubbles are formed by self-fulfilling expectations. That means investors expecting to sell the asset for a higher value in the future will be likely to buy the asset. With a sufficient number of optimistic investors a bubble is formed.

In this model, agents seek to maximize the expected utility. Utility depends on the level of consumption by the agent on any given time. In this model time is discrete and the agent makes decisions based on:

$$\max E_t \left[\sum_{i=0}^{\infty} \beta^{i} u(c_{t+i}) \right] \tag{1}$$

Subject to the budget constraint:

$$c_{t+i} = y_{t+i} + (P_{t+i} + D_{t+i})x_{t+i} - P_{t+i}x_{t+i+1} \qquad (2)$$

Where:

y_t is the initial endowment;
β_t is the discount factor for future utility;
x_t is the stock of financial assets (in this case shares);
P_t is the share price after dividends have been distributed;
D_t is the value of the dividend at time t.
c_t is the real amount consumed at time t.

The first order condition is:

$$E_t[\beta u'(c_{t+i})(P_{t+i} + D_{t+i})] = E_t[u'(c_{t+i+1})P_{t+i+1}] \qquad (3)$$

If utility is linear, i.e. marginal utility is constant, and the agents are risk neutral, equation (3) becomes:

$$E_t[\beta(P_{t+i} + D_{t+i})] = E_t[P_{t+i+1}] \qquad (4)$$

If a risk free asset exists with a given return r for each period, the no arbitrage condition implies:

$$\frac{1}{1+r}E_t[(P_{t+i+1} + D_{t+i+1})] = E_t[P_{t+i}] \qquad (5)$$

Iterating this equation into the future, considering its first differences leads us to the solution:

$$P_t = \sum_{i=1}^{\infty}\left(\frac{1}{1+r}\right)^i E_t[D_{t+i}] + B_t \qquad (6)$$

$$E_t[B_{t+1}] = (1+r)B_t \qquad (7)$$

That solution is the base of many of the asset pricing models used across the literature. The asset price has two components. The first is dependent upon the cash flows generated by the asset. This is the asset's fundamental value. The second component is the bubble.

In this set up a bubble appears even though all agents are rational. It is not a mispricing effect. There is, however, no opportunity for arbitrage. Equation (7) prevents it, since r is the risk free rate.

The assumption that dividends grow slower than r, guarantees that the market fundamental part of the asset price converges. The bubble component, however, is non-stationary. The asset price may exceed its fundamentals if agents believe they can sell the asset for a greater price in the future.

Equation (7) restricts the motion of the bubble's value, however it creates a different path for each initial bubble value. The asset price's path isn't unique. An additional assumption of the initial value is necessary.

A special case is when $B_0 = 0$. In this scenario the bubble value never exceeds zero and the price remains equal to its fundamental value. The assumptions embedded in this formulation are:

1. There are no information asymmetries, which means that price movements aren't driven by uninformed traders;
2. The agents are risk neutral;
3. The discount rate r is constant.

The no bubbles case is justified by a transversality condition. The price of the asset today is the sum of the discounted dividends plus the discounted expected resale price.

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i E_t[D_{t+i}] + \lim_{i=\infty} \left(\frac{1}{1+r}\right)^i P_{t+i} \tag{8}$$

The transversality condition ensures that the term on the right will be zero. If a bubble is present the infinite lived agent can sell the asset and the loss utility given by the discounted dividends will be lower than the selling price. If all agents are well informed and rational they will all wish to sell and drive the price back to its fundamental value. Tirole (1982) rules out bubbles in the presence of infinitely lived agents, but the same author (Tirole, 1985) proves there can be bubbles in a model with overlapping generations.

The empirical tests for bubbles usually start from equations (6) and (7).Shiller (1981) describe a variance bound test. The null hypothesis is that the “market fundamental” is the solution to equation (6):

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i E_t[D_{t+i}] \tag{9}$$

We can define P_t^*, the ex-post rational price , as the present value of actual dividends:

$$P_t^* = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i D_{t+i}$$

When all agents are rational, the difference between actual and expected dividends is an unforecastable, mean zero variable ε_t.

$$P_t^* = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i [E(d_{t+i}) + \varepsilon_{\mathrm{i}}] = P_t + \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i \varepsilon_{\mathrm{i}} \tag{10}$$

As ε_t is uncorrelated with all information at time t, the variance of P_t^* can be determined by:

$$V(P_t^*) = V(P_t) + \varphi V(\varepsilon_t) \geq V(P_t) \tag{11}$$

where φ is equal to $\left[\frac{1}{1+r}\right]^2 / \left[1 - \left(\frac{1}{1+r}\right)^2\right]$

This equation shows that there is an upper bound on the variance of the observed prices as long as the price equals fundamental value (equation (9)). If the variance bound is violated by the observed data it indicates that prices do not follow equation (9).

The implementation of this test is somewhat complicated since P_t^* is never observed.

In his paper, Shiller used S&P 500 prices going back to 1871, at an annual frequency. The test showed that actual price volatility exceeds the bound imposed by the model. Shiller (1981) and Grossman and Shiller (1981) used this evidence to critique the present value of dividends pricing model. However Tirole (1985) and Blanchard and Watson (1982) suggest that the excess volatility should be attributed to bubbles.

Determining the terminal ex-post rational price is an issue with this kind of test and has led to criticisms among the literature**.** Flavin (1983) showed that using the mean price as the terminal price creates a bias toward rejection for small samples. To get around that problem, Mankiw, Romer, and Shapiro (1985) used the last observed price as the terminal price. However, the authors have noted that in this case the test is not well suited for bubble detection.

This assumption defines the observable part of the ex-post rational price as:

$$P_t^* = \sum_{i=t+1}^{T} \left(\frac{1}{1+r}\right)^{i-t} d_i + \left(\frac{1}{1+r}\right)^{T-t} P_T \qquad (12)$$

Under the assumption that there is a rational bubble in the data, we have:

$$P_t = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^{i} E_t[d_i] + B_t \neq 0 \qquad (13)$$

P_t^*can be rewritten as:

$$P_t^* = P_t + \sum_{i=t+1}^{T} \left(\frac{1}{1+r}\right)^{i-t} \varepsilon_i + \sum_{i=t+1}^{\infty} \left(\frac{1}{1+r}\right)^{i-t} \left[E_T(d_i) - E_{t(d_i)}\right] + \left[\left(\frac{1}{1+r}\right)^{T-t} B_T - B_t\right]$$

The last three terms on the right-hand side are all uncorrelated with P_t, therefore they add a nonnegative amount to the variance of P_t^*. So, the variance bound is:

$$P_t^* \geq P_t \quad (14)$$

This inequality was derived under the assumption that a rational bubble exists. Therefore, even if the variance bound is violated, it cannot be attributed to the presence of a bubble in the data. Because of that variance bound tests are more appropriate to test the present value model instead of the bubble itself.

Cochrane (1992) proposes a variance test, but this time using the dividend/price ratio. Essentially, he tests if there is a discount rate that can properly “explain” the dividend/price volatility. The base model claims that the present value of dividends must converge, therefore there must be a nonnegative rate that justifies the current price. If an appropriate rate cannot be determined then there must be a bubble driving prices.

Variance bound tests can be used for testing bubbles or the model itself. These two are related, but different endeavors and testing for bubbles can be contaminated by misspecifications of the model itself. For a conclusive test for bubbles, the bubbles themselves should be at least in the set of the alternatives. The test proposed by West (1987) specifically places the bubble as the alternative hypothesis.

West observed that under the assumption of no bubbles, it is possible to estimate the equation for no arbitrage pricing. If dividends can be represented as an autoregressive process, knowing the discount rate and the parameters of the AR process, the relationship between dividends and market fundamental price can be discovered. The actual relationship between stock prices and dividends can be estimated by regressing price on dividends. Considering the absence of bubbles the estimation on actual prices and the derivation of the market fundamental should not differ.

The tests so far have not addressed in depth the structure of the bubbles. Basically they have tried to identify bubbles by eliminating all other possibilities. Bubbles however have theoretical properties that can be exploited for detection.

Diba and Grossman (1987, 1988a) observe that, according to equation (7) a bubble cannot start. If it exists now, it must have always existed. In this case the bubble will follow an integrated autoregressive process. From equation (7):

$$B_{t+1} - (1+r)B_t = z_{t+1} \tag{15}$$

With: $$E[z_{t+i}] = 0 \qquad \forall i \geq 1$$

Diba and Grossman specify the market fundamental price as:

$$P_t^f = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i E_t[d_{t+i} + o_t] \tag{16}$$

Where o_t are the unobservable market fundamentals. They assume that o_t is of an equal or lower autoregressive order than d_t. The market fundamental price, therefore, will be as stationary as dividends, under the assumption of no bubbles.

A way to test for bubbles, then, is to check whether prices are stationary when differenced the number of times required to make dividends stationary. Considering the null hypothesis of no bubbles dividends and prices should be cointegrated.

The test consists in determining the order if integration of prices and dividends and testing for cointegration between the series.

A milestone in the bubble literature is Evans' (1991) collapsing bubble model. Evans argues that a bubble can collapse randomly to a small nonzero value and then continue to increase. That bubble would violate Diba and Grossman's claim of a bubble not collapsing and still not violate arbitrage rules.

His example of periodically collapsing bubble is:

$$B_{t+1} = (1 + r)B_t v_{t+1} \qquad\qquad if\ B_t \le \alpha \qquad (17)$$

$$B_{t+1} = \{\delta + \pi^{-1}(1 + r)\theta_{t+1}[B_t - (1 + r)^{-1}\delta]\}v_{t+1} \qquad\qquad if\ B_t > \alpha \qquad (18)$$

Where $E_t[v_{t+1}] = 1$, and θ_{t+1} takes the value of 1 with probability π and 0 with probability $(1 - \pi)$.

This formulation satisfies equation (7), since the expected return on the bubble is r. For small values of B_t the bubble increases slowly. Once the bubble is larger than α, it expands faster but may collapse at each step with probability $(1 - \pi)$. During a collapse the bubble doesn't fall to zero. Instead it shrinks to a small positive value δ, so it can restart the process. This process has a constant expected growth rate r.

Evans showed that Diba and Grossman's test performed poorly on this kind of bubble for values of π below 0.95. This kind of bubble behaves more like a stationary process than an explosive one, because of the periodic collapses.

Evans' criticism led to a number of authors trying to tackle the problem of detecting periodically collapsing bubbles. The most used method was to treat the expanding and collapsing periods as two different regimes and using a Markov switching process. One example can be found on Hall, Psaradakis, and Sola (1999) who conduct an ADF test.

Recently, Philips and Yu (2009) propose a new date stamping strategy for bubbles using Sup ADF test. Later Philips, Shi, and Yu (2015) show that the original test generates poor results when exposed to multiple bubbles in the data. The authors then propose a generalized version of the Sup ADF test and show that it identifies several of the recorded bubbles in the S&P stock market data from January 1871 to December 2010. These tests are the basis of the work conducted on this paper.

2.2.4. Tests for bubbles in Brazil's stock market

Across the literature it is possible to find evidence of rational bubbles in Brazil using different detection techniques.

Nunes and Silva (2009) use monthly price and dividend data from July 1994 to December 2006 to test for three different types of bubbles already described in this paper: explosive bubbles, periodically collapsing bubbles, and intrinsic bubbles. Using cointegration tests, threshold autoregressive cointegration (TAR), momentum threshold autoregressive cointegration (M-TAR), and regressing the price against the dividends and a constant the authors found evidence of explosive and periodically collapsing bubbles in the data. They did not find significant evidence for intrinsic bubbles, however.

Kimura and Nakamura (2004) used Markov switching regression techniques to identify four regimes, which the authors named early bull market, late bull market, early bear market and late bear market. They used price return data for the Ibovespa index from July 1994 to March 2004. The authors found strong evidence of regimes of accelerated growth and heavy price decrease. Considering that under the assumption of an existing bubble price returns would exhibit nonlinear relationship with fundamentals, the authors indicated that a high growth regime is likely to contain a bubble.

Using Kalman filtering technique Queiroz, Medeiros and Neto (2004) analyzed stock market data from 1994 to 2004. In their model the bubble is considered a non-observable state vector. They found evidence of bubble, but could not datestamp it.

3. SELECTED TESTS FOR BRAZL'S STOCK MARKET

In this paper, two different techniques were applied to assess the presence of bubbles. In this section we describe the two tests.

3.1. Cointegration Test

Diba and Grossman (1988b) describe a series of tests to determine the presence of a bubble on price and dividend time series. Their model assumes a constant discount rate

and allows for unobserved market variables that affect fundamental prices. Their model starts with equations (15) and (16).

Under the assumption of no bubbles, and considering the unobserved fundamentals have an order of integration lower than price and dividends, then the dividends should be as integrated as prices are and both series should be cointegrated.

Their procedure is to run the two step Engle-Granger test for cointeration between real stock prices and dividends. Under the assumption that the two series are nonstationary and cointegrated, a combination of the two values must be integrated, as shown in equation (19).

$$\alpha P_t + \beta D_t = u_t \tag{19}$$

Where:

P_t is the price at time t;
D_t is the dividend at time t;
α and β are parameters to be estimated;
u_t is the stationary series formed by a linear combination of two unstationary cointegrated ones;

The test consists on regressing one series on the other, such as equation (20), and then testing the regression errors for stationarity.

$$P_t = \beta_0 + \beta_1 D_t + u_t \tag{20}$$

The Dickey-Fuller test is the most common choice for the stationarity of errors.

The authors emphasize that, while a positive result for cointegration between series can indicate that a bubble is not present, the rejection of the cointegration assumption does not necessarily indicate a bubble. The result may be caused by misspecification of the unobserved market fundamental factors or the entire model.

3.2. The Sup ADF and Generalized Sup ADF Tests

The bubble detection test used on this paper was first described in Phillips and Yu (2009) and later improved in Phillips, Wu and Yu (2011), and Phillips, Shi, and Yu (2015). The basic goal of this test is to detect explosive behavior on asset price returns, which, as shown on the previous section, is governed by an autoregressive process.

The previous section showed that in the presence of bubbles, the asset price distribution shifts from a martingale to an explosive series. Phillips, Wu and Yu (2011) test consists in determining if the series exhibits a unit root. In order to date stamp the moment of the bubbles initiation, the authors break down the data into subgroups and perform recursively calculated right side unit root tests, such as the Dickey-Fuller test.

3.2.1. Pricing Model and Test Statistic

Starting from equations (15) and (16):

$$P_t = P_t^f + B_t$$

$$P_t^f = \sum_{i=1}^{\infty} \left(\frac{1}{1+r}\right)^i E_t[d_{t+i} + o_t]$$

$$B_{t+1} - (1+r)B_t = z_{t+1}$$

With: $$E[z_{t+i}] = 0 \qquad \forall i \geq 1$$

In the absence of bubbles the price is as stationary as the dividend and the market fundamental series. For example, if the dividend series is integrated I(1) and the market fundamental is I(0) or I(1), then the price should be at most I(1). If dividends are stationary once differentiated, then any explosive behavior on differentiated price should be attributed to bubbles.

The sup ADF test is conducted by getting subsamples from de data set and conducting the right sided Dickey-Fuller test. The used subsample r_w is taken by dT, where T is the entire dataset, and $d \epsilon [0,1]$. The sample size is increased from $d = r_0$ to 1, where r_0 is the smallest possible sample considering estimation efficiency. The ADF test is conducted on each iteration.

For each subsample, an autoregressive series is specified as:

$$X_t = \mu + \delta X_{t-1} + \varepsilon_t \,, \;\; \varepsilon_t \sim iid\ (0, \sigma^2) \tag{21}$$

The null hypothesis is $H_0\colon \delta = 1$ and the right side alternative hypothesis is $H_1\colon \delta > 1$.

For each new regression the subsample is formed by combining the subsample from the previous iteration with new observations. If each subsample has size $\tau = dT$, the coefficient test statistic and the Dickey-Fuller t statistic (DF_r^{δ} and DF_r^{t}, respectively) are:

$$DF_r^{\delta} := \tau\big(\hat{\delta}_\tau(\tau) - 1\big) \,, \qquad DF_r^{t} \coloneqq \left(\frac{\sum_{j=1}^{\tau} \tilde{X}_{j-1}^2}{\hat{\sigma}_\tau}\right)^{\frac{1}{2}} \big(\hat{\delta}_\tau(\tau) - 1\big) \tag{22}$$

where $\hat{\delta}_\tau$ is the least squares estimate of δ, based on the first $\tau = dT$ observations, $\hat{\sigma}_\tau^2$ is the estimate of σ^2, and $\tilde{X}_{j-1} = X_{j-1} - \tau^{-1} \sum_{j-1}^{\tau} X_{j-1}$.The values are compared to the DF test critical values.

However Phillips, Shi, and Yu (2015) argue that the SADF test performs poorly when there are multiple bubbles in the data. The authors propose a variation of the procedure called the Generalized Sup ADF test (GSADF). Basically they use the same idea of implementing the Dickey-Fuller test on a series of subsamples of the dataset, however they vary not only the final point of each subset, but also the beginning. The diagram bellow illustrates the subset concept of each test.

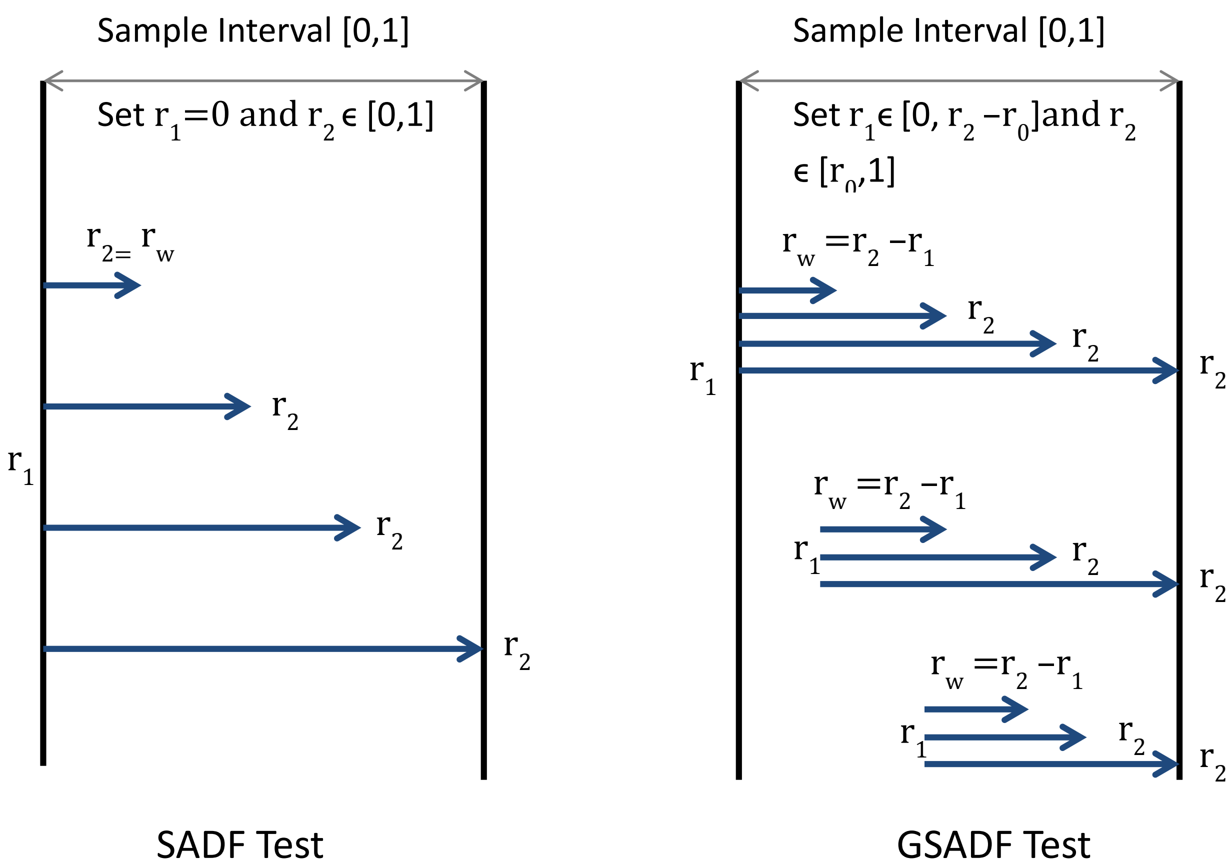


Figure 1 – SADF and GSADF algorithms

3.2.2. Critical Values

Once the test statistic is calculated it is necessary to compare it with the test's critical values. Since this is a right tailed test it is not possible to simply use ADF tables. But before it can be possible to discuss critical values, it is important to cover all assumption regarding the null hypothesis.

Phillips et al. (2015) allowed the null to be a martingale with an asymptotically negligible drift. The prototypical model of this type has the following weak intercept form:

$$y_t = dT^{-\eta} + \theta y_{t-1} + \varepsilon_t, \varepsilon_t \sim^{iid} (0, \sigma^2), \theta = 1 \tag{23}$$

where d is a constant, T is the sample size, and the parameter η is a localizing coefficient that controls the magnitude of the intercept and drift. Solving (20) gives:

$$y_t = d\left(\frac{t}{T^{\eta}}\right) + \sum_{j=1}^{t} \varepsilon_j + y_0 \tag{24}$$

Revealing the deterministic drift $\frac{dt}{T^{\eta}}$.

Given the null hypothesis described in (20), Phillips, Shi, and Yu (2015) show that when the model includes an intercept, the limit distribution for the GSADF statistic is:

$$\sup_{\substack{r_2 \epsilon [r_0, 1] \\ r_1 \epsilon [0, r_2 - r_0]}} \left\{ \frac{\left(\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_{r_1}^{r_2} W(r)dr \;\; [W(r_2) - W(r_1)]\right)}{r_w^{\frac{1}{2}} \left\{ r_w \int_{r_1}^{r_2} W(r)^2 dr - \left[\int_{r_1}^{r_2} W(r)dr\right]^2 \right\}} \right\}$$

Where $r_w = r_2 - r_1$and W is a standard Wiener process.

In practice given a finite sample the authors find the 90, 95 and 99% critical values by conducting a Monte Carlo simulation with 2,000 interactions of a random walk given by the null hypothesis described above.

4. DATA AND EMPIRICAL TESTS

Diba and Grossman's (1988b) test, described on the previous section was used to assess bubbles in USA's stock market. The price series used by the authors was the Standard and Poor's composite Stock Price index for January of each year from 1871 to 1996 divided by the wholesale price index for that month. The dividend series is total dividends accrued on each year divided by the average wholesale price for that year.

The GSADF technique described on the previous section was used by Phillips, Shi, and Yu (2015) to detect bubbles in the S&P 500 index. The authors collected monthly real prices and dividends per share from January 1871 to December 2010. Data were obtained from Robert Shiller's website. In total there were 1,680 observations.

4.1. Tests on the S&P 500

The cointegration test showed evidence of nonstationarity of prices and dividends. It also showed that both series seem to be stationary in first differences and cointegrated. The authors however argue that the power of the test is not significant for the sample size they used. They conclude that there is weak evidence against the presence of bubbles.

Phillips, Shi, and Yu (2015) also use Standard and Poor's Composite price and dividend series on their test. The authors conduct the test on the price to dividend series, to determine price deviation from its fundamental value. The series is normalized to 100 at its beginning.

The authors mention that after several trials they determined that a rule of thumb for the minimum window size (r_0) should be $r_0 = 0.01 + 1.8/\sqrt{T}$, where T is the sample size. This rule is used for the bubble detection tests on the S&P 500 index, and will be used in this paper later on for the Ibovespa tests.

Given these specifications the authors conduct both de GSADF and the SADF tests. While the SADF test only detected the DotCom bubble in the data, the GSADF test was able to pinpoint several well known cases of market exuberance.

Using simulated data, the authors showed that the SADF test has inferior bubble detecting capabilities when there is more than on bubble on the series. Given the results, we will give more focus to the GSADF test.

4.2. Tests on the IBOVESPA index

Using the Bloomberg terminal we extracted data for prices and dividends for the index. Just like the Phillips, Shi, and Yu (2015), for this paper monthly data was used extending from November 1996 to January 2016 totaling 231 observations. The price to dividend ratio was normalized to 100 at the beginning. The following graph shows the price, dividend and ratio series:

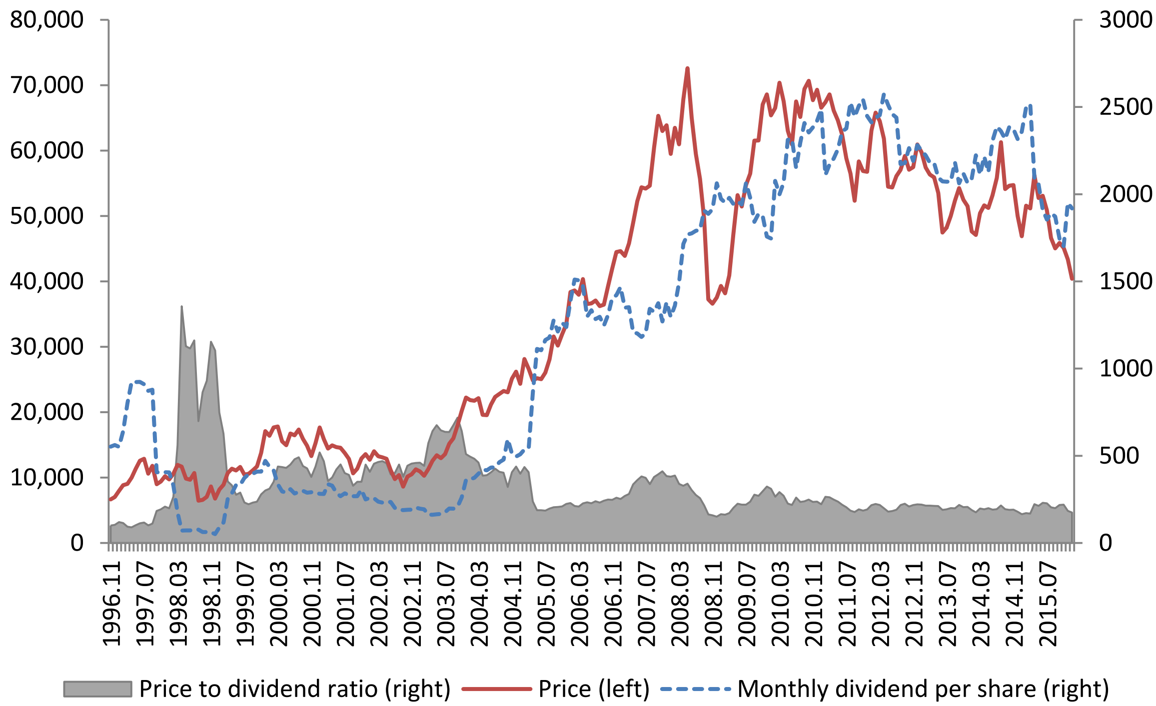
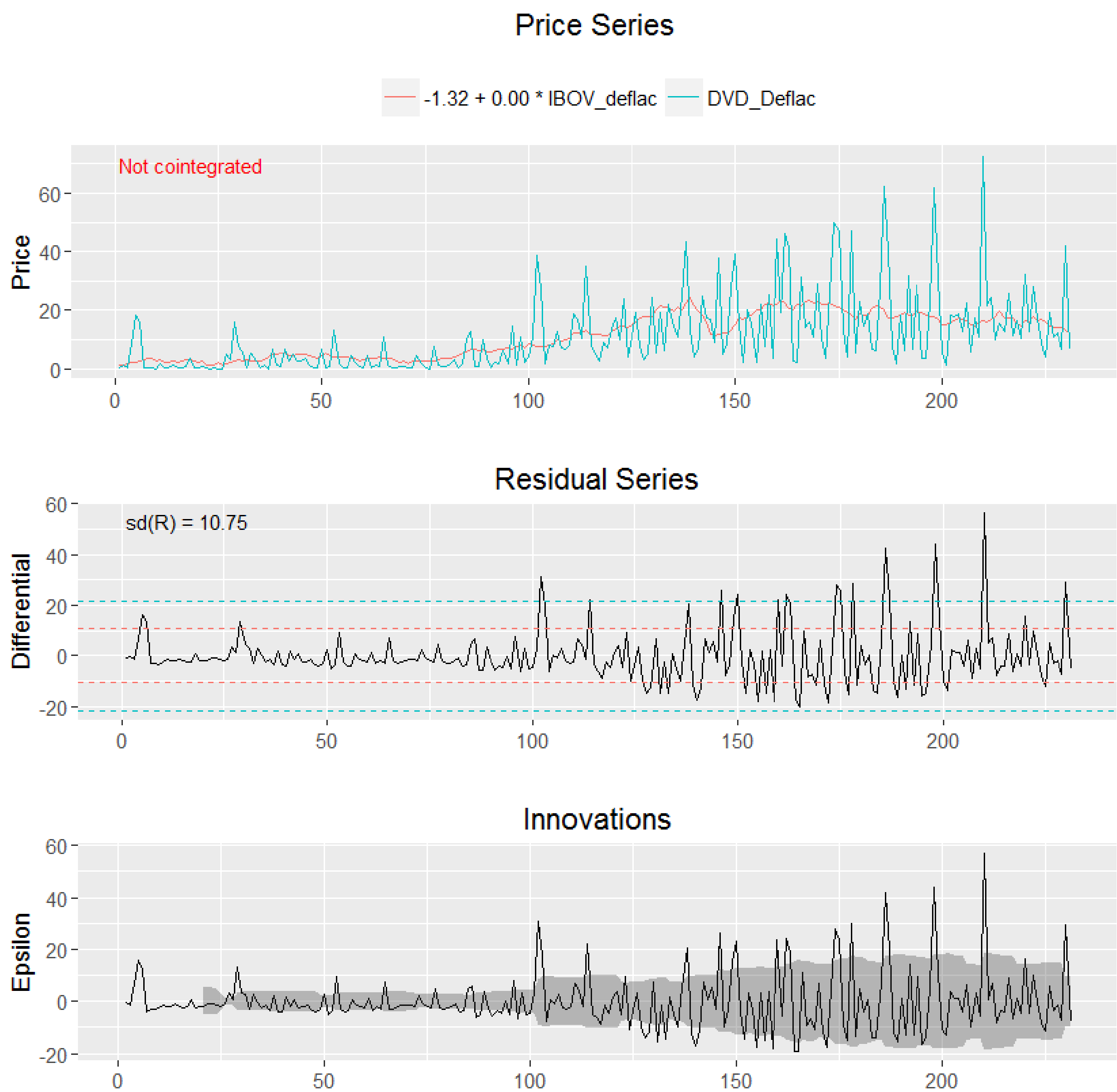


Figure 2 – Price and dividend historical data for Ibovespa index

Before running the very sophisticated GSADF test, we will run a more basic test described on this paper. We will use the procedure described in Diba and Grossman (1988b) and run a cointegration test on the price and dividend series.

First, we will discount the inflation from both series using the IPCA index extracted from IPEA's website. For this test we used the statistic software R and the EGCM package. After running the test we've obtained the following results:



```
DVD_Deflac[i] =   0.0032 IBOV_deflac[i] -   1.3200 + R[i], R[i] =   0.0780 R[i-1] + eps[i], eps ~ N(0, 10.7484^2)
                 (0.0003)                  (5.5562)                (0.0660)

R[231] = -5.4195 (t = -0.504)

WARNING: DVD_Deflac does not seem to be integrated. IBOV_deflac and DVD_Deflac do not appear to be cointegrated.

Unit Root Tests of Residuals
                                                    Statistic    p-value
  Augmented Dickey Fuller (ADF)                        -6.708    0.00010
  Phillips-Perron (PP)                               -187.963    0.00010
  Pantula, Gonzales-Farias and Fuller (PGFF)            0.065    0.00010
  Elliott, Rothenberg and Stock DF-GLS (ERSD)          -8.145    0.00010
  Johansen's Trace Test (JOT)                        -109.714    0.00010
  Schmidt and Phillips Rho (SPR)                      133.203    0.99990

Variances
  SD(diff(IBOV_deflac)) = 288.938403
  SD(diff(DVD_Deflac)) =  14.774249
  SD(diff(residuals))  =  14.697292
  SD(residuals)        =  10.750335
  SD(innovations)      =  10.748391

Half life       =   0.271778
R[last]         =  -5.419499 (t=-0.50)
Warning message:
In pp.test(X) : p-value smaller than printed p-value
```

Figure 3 – Results from the Engle-Granger cointegration test on Ibovespa prices and dividends

There are two things to consider. First, the dividend series does not appear to be integrated. Second, when applying the Dickey-Fuller test to the residuals there is a small p-value, which indicates we should reject the null hypothesis of a unit root.

If we only looked at the Dickey-Fuller test it would seem that the regression errors are stationary and the series are cointegrated. However, from equation (16) it is possible to see that, under the assumption of no bubbles and that the unobservable market fundamental value o_t is of a lower integration order than price and dividends, the series tested should have the same order of integration.

The fact that the dividend series is not integrated indicates that the series do not follow the model proposed by Diba and Grossman (1988). Therefore we should reject the null hypothesis of a pricing model described by equation (16). This does not necessarily indicate the presence of a bubble. It could be a misspecification of the model itself, or a misspecification of the integration order of o_t. To seek a more definitive answer we will apply the GSADF test.

The same rule of thumb for the minimum window of the GSADF test ($r_0 = 0.01 + 1.8/\sqrt{T}$) was used and for the critical values a Monte Carlo was conducted with 2,000 iterations simulating a random walk with negligible drift. Matlab was the software used to replicate the work of Phillips, Shi, and Yu (2015).

The results for the test statistic are shown on table 1.

Table 1 - Results from SADF and GSADF tests on the Ibovespa historical series

		Finite Sample Critical Values		
	Test Statistic	90%	95%	99%
SADF	-2.79	1.06	1.32	1.96
GSADF	0.41	1.72	1.94	2.53

We can see that the test statistics are lower than the critical value even for 90% significance. However this simple yes or no test is extremely conservative. To determine critical values, 2000 scenarios are simulated and a distribution is made using the highest values in each of the paths simulated. We will still conduct the datestamping algorithm to see what it can reveal. As explained above this paper will focus on the GSADF datestamping methodology.

Finally we ran the 90%, 95% and 99% critical values for the GSADF and the test statistic series for the Ibovespa index. The graph below shows the time series, the critical values and the price to dividend series.

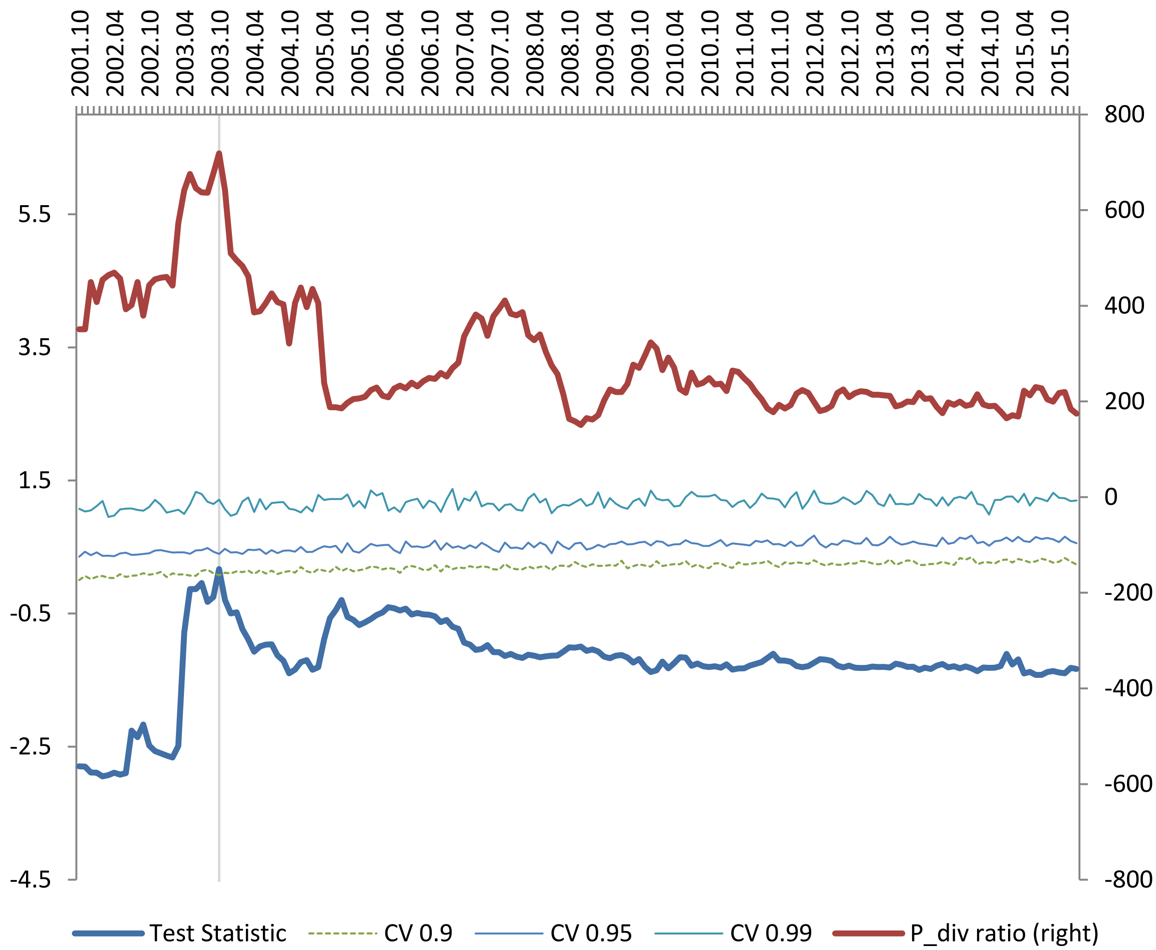


Figure 4 – GSADF datestamping technique applied to historical Ibovespa price to dividend ratio

As shown in the graph, the test indicated a structural break and explosive behavior for the end of 2003. That was the only bubble identified by the data. In other words, at the end of 2003 the test indicated a surge in prices not explained by the behavior of dividends.

To check this result we conducted the same test except using only price information. The GSADF is basically an ADF test for stationarity, and so, given the model described by equation (3) we can use it to determine explosive behavior in price return.

First, the effects of inflation had to be removed. Using the IPCA, the General Price index historical data extracted from the IPEA website, we obtained real monthly prices for

Ibovespa on the same range as the price to dividend ratio. To only capture the abnormal returns, the test was conducted on the natural logarithmic of real prices.

The 90%, 95% and 99% critical values were determined using the same procedure described above, a Monte Carlo with 2,000 iterations. The final results are presented below.

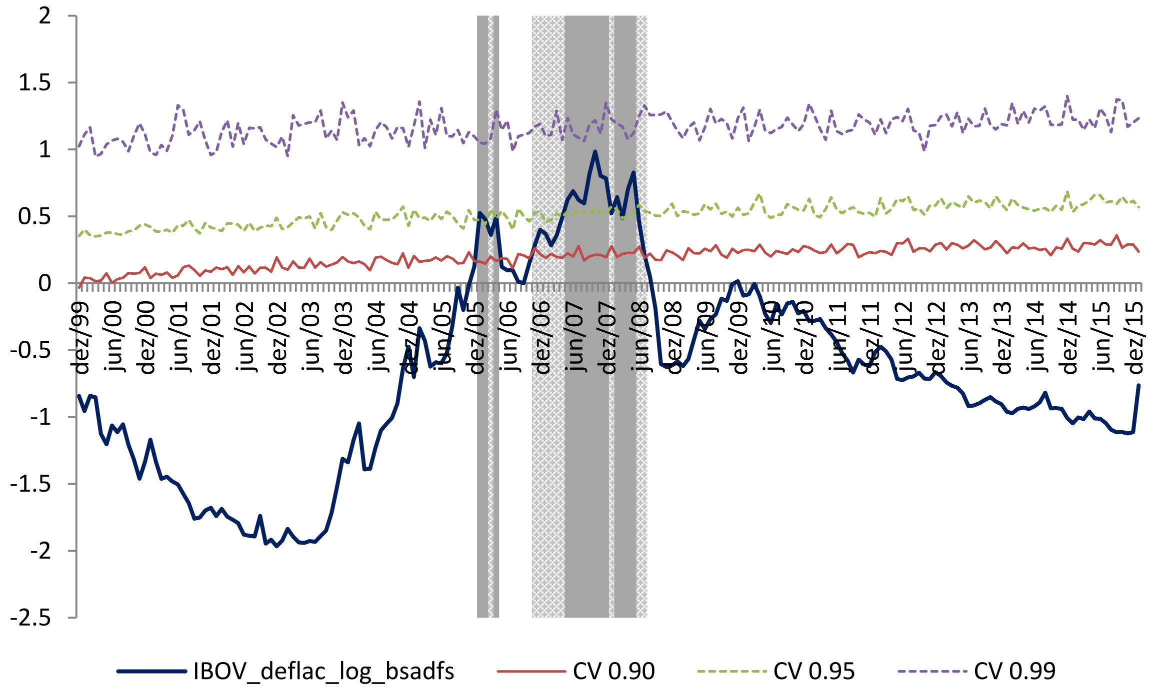


Figure 5 – GSADF datestamping technique applied to historical Ibovespa stock price returns

The hashed areas represent bubbles at 10% significance and the solid areas bubbles with 5% significance. Either way the test showed non stationary price returns in the end of 2005 and especially in the second half of 2007 up to the beginning of 2008.

When analyzed combined, the tests wield some very interesting results. First the test conducted on price return speaks directly to the trend shown in figure 4 and in recent history.

Like many other emerging markets, Brazil went through a cycle of very fast growth from 2003 to early 2008, when the subprime crisis started disturb international markets. Many were the factors that contributed to this growth.

Central banks in developed markets had lowered interest rates in the beginning of the decade. That combined with the better perception of emerging markets by international investors led to a heavy flow of capital and liquidity into countries like Brazil, China, India and Russia. Historically, periods of excess liquidity across global markets have been strongly correlated with growth in emerging markets, and Brazil is no different.

China was also a main factor for the high and steady growth of the Brazilian economy. Brazil has always depended heavily on commodity exports. China experienced more than a decade of very heavy growth, which caused the country to increase its demand for commodities, both soft and hard. Iron ore and oil for example experienced a rally in prices that lasted for several years and contributed for the high growth rate for Brazil's economy and its capital markets.

Finally 2007 was the peak of this growth, not only for Brazil but for markets in general, which culminated on the collapse caused by the subprime crisis in 2008. For that last period of high growth the test showed non stationary behavior at a 5% significance. In fact 2007 was, and still is up to this day the best year for Ibovespa since its inception.

However, the interesting thing is that the test conducted using the price to dividend ratio did not show a bubble for that same period. If we take another look at the price and dividend data for that specific period in figure 7, it is possible to see that although the index increased its value very fast, the growth was accompanied by a similar growth in dividends.

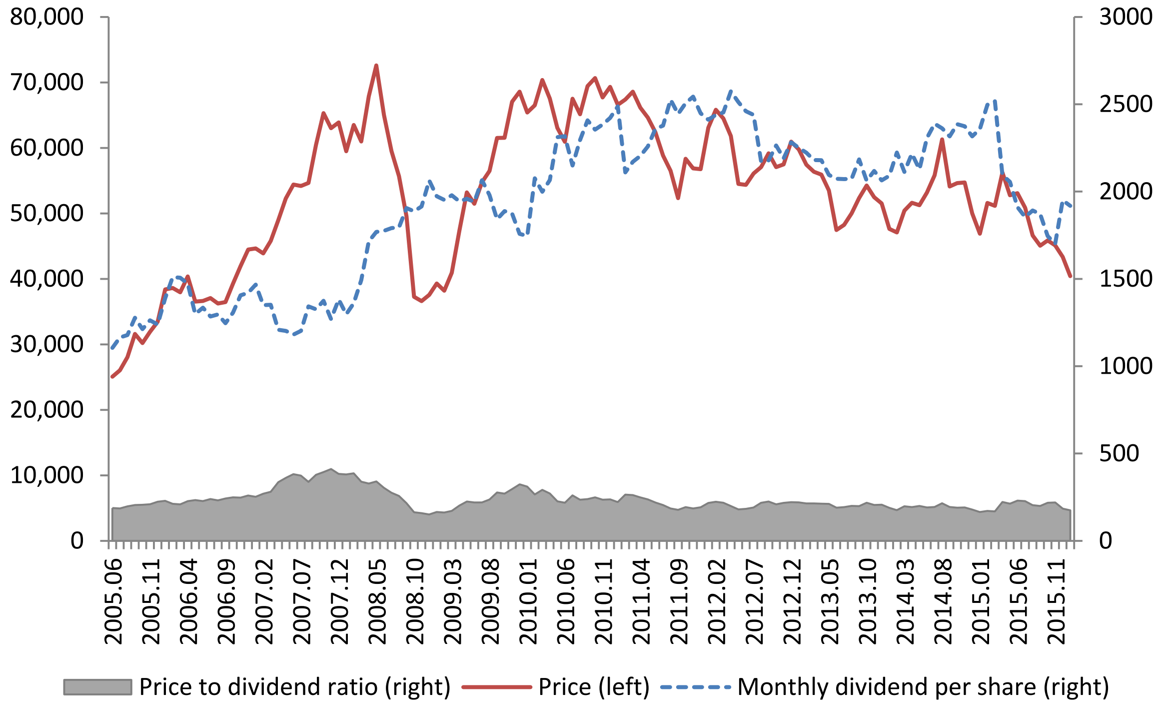


Figure 6 – Price and dividend data for Ibovespa – most recent 10 years.

The price to dividend ratio remains fairly stable during that time. So even considering that we have found explosive behavior in price returns, it is not possible to attribute it to a bubble, since price is still responding to changes in stocks fundamental value.

So now the focus should be on the actual bubble found by testing the price dividend ratio at the end of 2003. In fact 2003 was the best performing year for Ibovespa until the record was broken in 2007. With a closer look at the price and dividend data it is possible to see that, unlike 2007, the surge in price was not immediately accompanied by an increase in dividends.

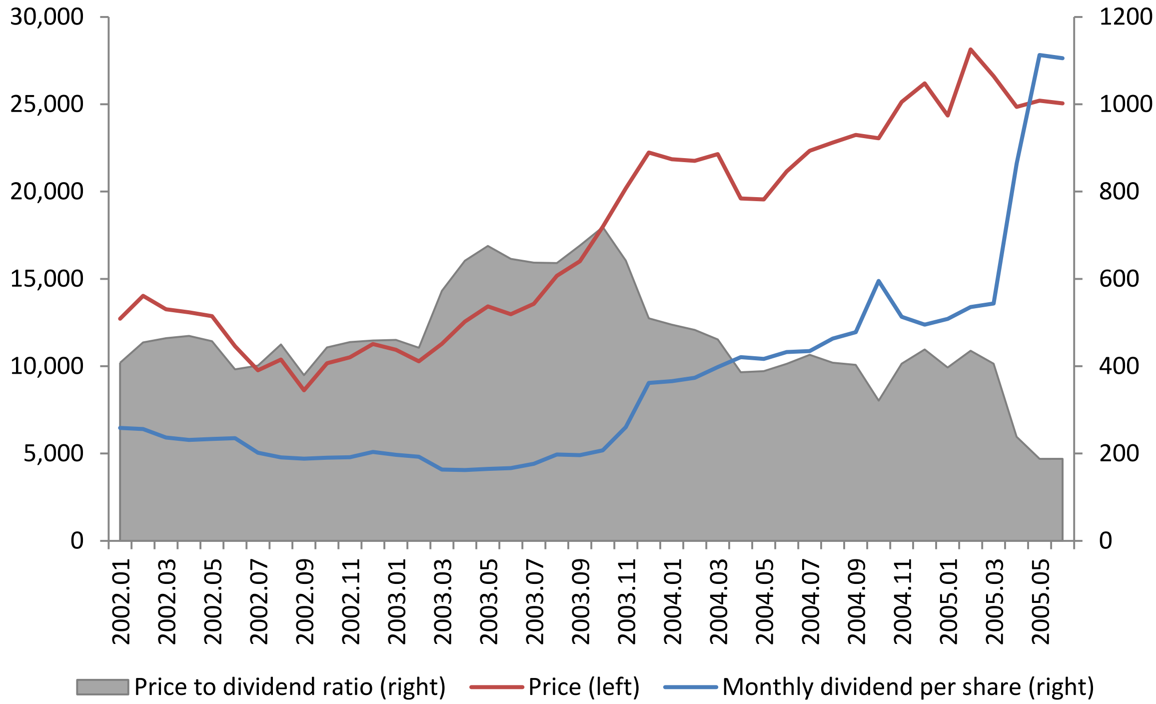


Figure 7 – Price and dividend series for Ibovespa – 2002 to 2005

In fact, unlike 2007, the surge in 2003 was more related to market sentiment than fundamental factors. The low point in the third quarter of 2002 happened mainly because of Brazil's presidential elections. The stock market reacted negatively to the prospect of a government led by Luis Inacio Lula da Silva. The exchange rate soared to almost 4.00 BRL/USD and the Central Bank was forced to increase short term interest rates to contain inflation. The graph below shows the exchange rate taken from the Central Bank's website, the overnight interbank rate, taken from Cetip's website, and the IPCA inflation index gathered from IPEA's website. The inflation data was taken monthly and accumulated for the previous 12 months

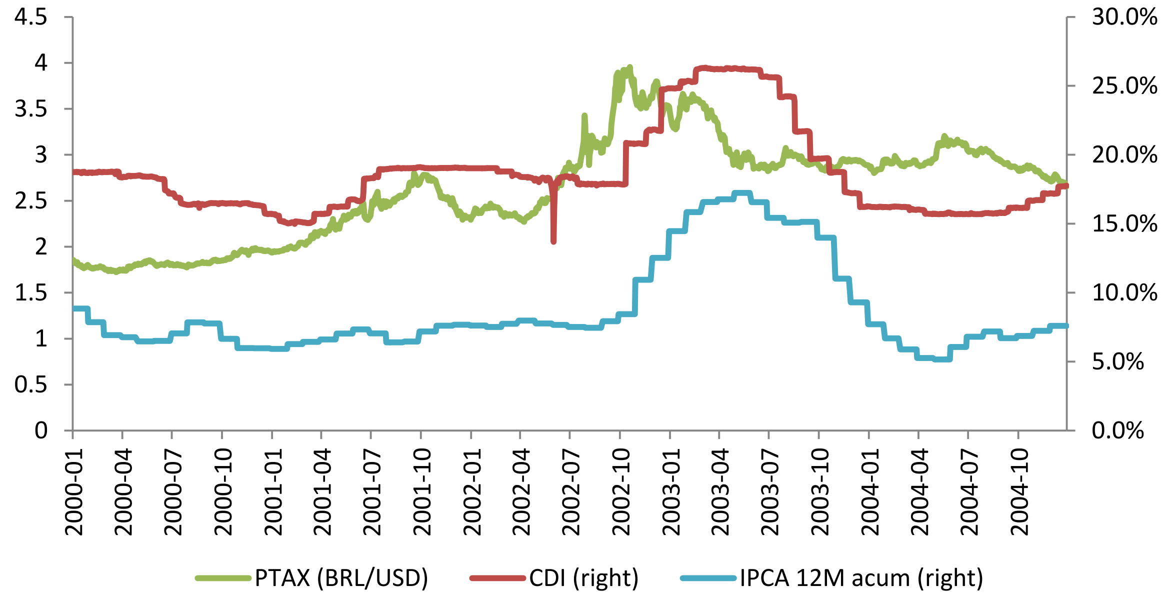


Figure 8 – Exchange rate (PTAX), inflation and nominal interest rate for Brazil

2003 marked the recovery of the Brazilian stock market after very poor returns experienced in previous years. After the elections, the political uncertainty was mitigated. The exchange rate fell, which influenced inflation, and so, the central bank was able to bring down interest rates, as shown on figure 10.

5. CONCLUSIONS

There is extensive literature on the subject of bubbles. The correct determination of fundamental of asset prices is of great importance for market analysts, investors and policymakers. To this extent there is extensive literature on types of bubbles, their causes and detection techniques. This paper used both a classical cointegration test and a fairly recently developed method, that has presented strong results and sought to find evidence of bubbles in the IBOVESPA index. The technique used was to conduct recursive right tailed ADF unit root tests as described by Phillips, Shi, and Yu (2015).

We used monthly prices and dividends for the IBOVESPA Index ranging from November 1996 to January 2016. Unfortunately, in Brazil it is not possible to use long range data, such as in the S&P 500, because the country has experienced several structural breaks in past decades.

The cointegration test did not rule out the presence of bubbles. However it is not possible to affirm that the results were not caused by misspecifications in the model. We then applied the GSADF test to possibly obtain stronger evidence.

The test indicated some evidence of a bubble in the third quarter of 2003. It coincided with a period of strong market recovery after the poor results in 2001 and 2002. The stabilization of the political scene in Brazil combined with excess liquidity in developed markets led to a surge in stock prices that was not immediatly accompanied by dividends.

Interestingly, evidence of nonstationary returns have also been found during late 2006 and 2007, which was the best year for IBOVESPA since inception. However, the results are misleading as far as bubble detection goes. Although the prices exhibit explosive behavior, the rally in index prices was closely accompanied by an increase in dividends.

6. REFERENCES

Abreu, D., Brunnermeier, M.K., 2003, “Bubbles and Crashes”, Econometrica, Vol. 71, No 1, p. 173-204

Allen, Frank and Gale, Douglas (2001). “Bubbles and Crises”, The Economic Journal 110, pp. 236-255

Blanchard, Olivier. (1979). “Speculative Bubbles, Crashes and Rational Expectations” Economic Letters 3, pp. 387-389

Blanchard, Olivier and Mark Watson.(1982). “Bubbles, Rational Expectations, and Financial Markets,” in Paul Wachter (ed.) Crises in the Economic and Financial Structure. Lexington, MA: Lexington Books, pp. 295-315.

Cochrane, John. (1992). “Explaining the Variance of Price Dividend Ratios” Review of Financial Studies 5(2), pp. 243-80.

De Long, J., B., Shleifer, A., Summers, L.H., Waldmann, R.J., 1990, “Noise Trader Risk in Financial Markets”, The Journal of Political Economy, vol. 98, No. 4, p. 703-738

Diba, Behzad and Grossman, Herschel.(1987). “On the Inception of Rational Bubbles,” Quarterly Journal of Economics 87(August) pp.697-700.

Diba, Behzad and Grossman, Herschel. (1988). “The Theory of Rational Bubbles in Stock Prices,” The Economic Journal 98(September), pp. 746-54

Diba, Behzad and Grossman, Herschel. (1998). “Explosive Rational Bubbles in stock Prices?”, The American Economic Review, vol. 78, No. 3, pp. 520-530

Evans, George. (1991). “Pitfalls in Testing for Explosive Bubbles in Asset Prices,” American Economic Review 31(September), pp.922-30.

Flavin, Marjorie. (1983). “Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence,” Journal of Political Economy91(December), pp. 929-956.

Grossman, Sanford, and Robert Shiller. (1981) “The Determinants of the Variability of Stock Market Prices,” American Economic Review 71(May),pp. 222-227.

Hall, Stephen, Zacharias Psaradakis, and Martin Sola. (1999). “Detecting Periodically Collapsing Bubbles: A Markov-Switching Unit Root Test,”Journal of Applied Econometrics 14: 143-154.

Harrison, J., M., Kreps, D., M. 1978 “Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations”, The Quarterly Journal of Economics, Vol. 92, No. 2, p. 323-336

Kimura, H., Nakamura, W. T.,Kayo, E., Martin, D.M.L., 2004 “Identificando Bolhas Especulativas Racionais no IBOVESPA (Pós-Plano Real), a partir de Regimes Markovianos de Conversão”, Revista EconomiA, v. 5, n. 3, p.219-252

Mankiw, N. Gregory, David Romer, and Matthew Shapiro. (1985). “An Unbiased Reexamination of Stock Market Volatility,” Journal of Finance40(July), pp. 677-687.

Miller, E. M. 1977. “Risk, Uncertainty, and Divergence of Opinion”.The Journal of Finance, Vol. 32, No. 4, 1151-1168.

Nunes, M.S., Silva, S., 2009, “Bolhas Racionais no Índice Bovespa”, RBE, c. 63, n. 2, p. 119-134

Ofek, E., Richardson, M., 2003 “DotCom Mania: Rise and Fall of Internet Stock Prices” The Journal of Finance, vol.63, No. 3, p. 1113-1137

Phillips, P.C.B, Shi, S., Yu, J. 2011, “Testing for multiple bubbles” Cowles Foundation Discussion Paper

Phillips, P.C.B., Shi, S., Yu, J., 2015 “Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P 500”, International Economic Review, vol. 56, No. 4, p. 1043-1077

Phillips, P.C.B., Wu, Y., and Yu, J., 2011, Explosive behavior in the 1990s Nasdaq: When did exuberance escalate asset values? International Economic Review, 52, 201-226.

Phillips, P.C.B., and Yu, J., 2009, “Limit theory for dating the origination and collapse of mildly explosive periods in time series data”, Singapore Management University, Unpublished Manuscript.

Queiroz, T., Medeiros, O., Neto, J. 2011 “Evidence of Speculative Bubbles on the BOVESPA: Na Application of the Kalman Filter” Rev. Bras. Finanças, Vol. 9, No. 2

Scheinkman, J.A., Xiong, W., 2003, “Overconfidence and Speculative Bubbles”, Journal of Political Economy, Vol. 111, No. 6, p. 1183-1219

Shi, S., Phillips, P.C.B., and Yu, J., 2010, “Hypothesis and model specification in the right-tail unit root test”, Working Paper.

Shiller, Robert. (1981). “Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends? ”American Economic Review 71(June),pp. 421-436.

Tirole, Jean. (1982). “On the Possibility of Speculation under Rational Expectations,” Econometrica 50, pp. 1163-1182.

Tirole, Jean. (1985). “Asset Bubbles and Overlapping Generations,” Econometrica 53, pp. 1499-1528.

Vissing-Jorgensen, Annette. (2004). “Perspectives on Behavioural Finance: Does “Irrationality” Disappear with Wealth? Evidence from Expectations and Actions,” working paper, Northwestern University.

West, Kenneth. (1987). “A Specification Test for Speculative Bubbles,” The Quarterly Journal of Economics 102(August), pp. 553-80.