FUNDAÇÃO GETULIO VARGAS
ESCOLA DE ECONOMIA DE SÃO PAULO

ANA ELISA GONÇALVES PEREIRA

ESSAYS ON COORDINATION PROBLEMS IN ECONOMICS

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Tese submetida à Escola de Economia de São Paulo da Fundação Getulio Vargas como requisito para a obtenção do título de Doutor em Economia.

Orientador: Bernardo de Vasconcellos Guimarães

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To my wonderful mother, Leila.

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ABSTRACT

There are several economic situations in which an agent's willingness to take a given action is increasing in the amount of other agents who are expected to do the same. These kind of strategic complementarities often lead to multiple equilibria. Moreover, the outcome achieved by agents' decentralized decisions may be inefficient, leaving room for policy interventions. This dissertation analyzes different environments in which coordination among individuals is a concern.

The first chapter analyzes how information manipulation and disclosure affect coordination and welfare in a bank-run model. There is a financial regulator who cannot credibly commit to reveal the situation of the banking sector truthfully. The regulator observes banks' idiosyncratic information (through a stress test, for example) and chooses whether to disclose it to the public or only to release a report on the health of the entire financial system. The aggregate report may be distorted at a cost − higher cost means higher credibility. Investors are aware of the regulator's incentives to conceal bad news from the market, but manipulation may still be effective. If the regulator's credibility is not too low, the disclosure policy is state-contingent and there is always a range of states in which there is information manipulation in equilibrium. If credibility is low enough, the regulator opts for full transparency, since opacity would trigger a systemic run no matter the state. In this case only the most solid banks survive. The level of credibility that maximizes welfare from an ex ante perspective is interior.

The second and the third chapters study coordination problems in dynamic environments. The second chapter analyzes welfare in a setting where agents receive random opportunities to switch between two competing networks. It shows that whenever the intrinsically worst one prevails, this is efficient. In fact, a central planner would be even more inclined towards the worst option. Inefficient shifts to the intrinsically best network might occur in equilibrium. When there are two competing standards or networks of different qualities, if everyone were to opt for one of them at the same time, the efficient solution would be to choose the best one. However, when there are timing frictions and agents do not switch from one option to another all at once, the efficient solution differs from conventional wisdom.

The third chapter analyzes a dynamic coordination problem with staggered decisions where agents are ex ante heterogeneous. We show there is a unique equilibrium, which is characterized by thresholds that determine the choices of each type of agent. Although payoffs are heterogeneous, the equilibrium features a lot of conformity in behavior. Equilibrium

thresholds for different types of agents partially coincide as long as there exists a set of beliefs that would make this coincidence possible. However, the equilibrium strategies never fully coincide. Moreover, we show conformity is not inefficient. In the efficient solution, agents follow others even more often than in the decentralized equilibrium.

Keywords: coordination, bank runs, global games, information disclosure, dynamic games, timing frictions, heterogeneous agents.
JEL Classification: C73, D82, D83, D84, G01.

RESUMO

No estudo da economia, há diversas situações em que a propensão de um indivíduo a tomar determinada ação é crescente na quantidade de outras pessoas que este indivíduo acredita que tomarão a mesma ação. Esse tipo de complementaridade estratégica geralmente leva à existência de múltiplos equilíbrios. Além disso, o resultado atingido pelas decisões decentralizadas dos agentes pode ser ineficiente, deixando espaço para intervenções de política econômica. Esta tese estuda diferentes ambientes em que a coordenação entre indivíduos é importante.

O primeiro capítulo analisa como a manipulação de informação e a divulgação de informação afetam a coordenação entre investidores e o bem-estar em um modelo de corridas bancárias. No modelo, há uma autoridade reguladora que não pode se comprometer a revelar a verdadeira situação do setor bancário. O regulador observa informações idiossincráticas dos bancos (através de um *stress test*, por exemplo) e escolhe se revela essa informação para o público ou se divulga somente um relatório agregado sobre a saúde do sistema financeiro como um todo. O relatório agregado pode ser distorcido a um custo – um custo mais elevado significa maior credibilidade do regulador. Os investidores estão cientes dos incentivos do regulador a esconder más notícias do mercado, mas a manipulação de informação pode, ainda assim, ser efetiva. Se a credibilidade do regulador não for muito baixa, a política de divulgação de informação é estado-contingente, e existe sempre um conjunto de estados em que há manipulação de informação em equilíbrio. Se a credibilidade for suficientemente baixa, porém, o regulador opta por transparência total dos resultados banco-específicos, caso em que somente os bancos mais sólidos sobrevivem. Uma política de opacidade levaria a uma crise bancária sistêmica, independentemente do estado. O nível de credibilidade que maximiza o bem-estar agregado do ponto de vista *ex ante* é interior.

O segundo e o terceiro capítulos estudam problemas de coordenação dinâmicos. O segundo capítulo analisa o bem-estar em um ambiente em que agentes recebem oportunidades aleatórias para migrar entre duas redes. Os resultados mostram que sempre que a rede de pior qualidade (intrínseca) prevalece, isto é eficiente. Na verdade, um planejador central estaria ainda mais inclinado a escolher a rede de pior qualidade. Em equilíbrio, pode haver mudanças ineficientes que ampliem a rede de qualidade superior. Quando indivíduos escolhem entre dois padrões ou redes com níveis de qualidade diferentes, se todos os indivíduos fizessem escolhas simultâneas, a solução eficiente seria que todos adotassem a rede de melhor qualidade. No entanto, quando há fricções e os agentes tomam decisões escalonadas, a solução eficiente difere

do senso comum.

O terceiro capítulo analisa um problema de coordenação dinâmico com decisões escalonadas em que os agentes são heterogêneos *ex ante*. No modelo, existe um único equilíbrio, caracterizado por *thresholds* que determinam as escolhas para cada tipo de agente. Apesar da heterogeneidade nos *payoffs*, há bastante conformidade nas ações individuais em equilíbrio. Os *thresholds* de diferentes tipos de agentes coincidem parcialmente contanto que exista um conjunto de crenças arbitrário que justifique esta conformidade. No entanto, as estratégias de equilíbrio de diferentes tipos nunca coincidem totalmente. Além disso, a conformidade não é ineficiente. A solução eficiente apresentaria estratégias ainda mais similares para tipos distintos em comparação com o equilíbrio decentralizado.

Palavras-chave: coordenação, corridas bancárias, jogos globais, divulgação de informação, jogos dinâmicos, timing frictions, agentes heterogêneos.
Classificação JEL: C73, D82, D83, D84, G01.

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Chapter 1

Rollover risk and the social value of credibility

Abstract

This paper studies information disclosure when a financial regulator cannot credibly commit to reveal the situation of the banking sector truthfully. I present a bank run model where a regulator observes banks' types (through a stress test, for example) and chooses whether to disclose bank-specific information or only to release a report on the health of the entire financial system, which can be biased at a cost. Investors are aware of the regulator's incentives to mask an undesirable scenario, and take that into account in their rollover decisions. The signaling in the regulator's disclosure and manipulation policies generates multiple equilibria. Still, predictions may be derived. If credibility is not too low, the regulator only discloses bank-specific information in low states (i.e., when the proportion of good banks is small); in intermediate states, the policy features opacity and information manipulation; and in high states, opacity with truth-telling. The relationship between credibility and the amount of transparency in equilibrium is non-monotonic. If credibility is low enough, the regulator loses the ability to help agents coordinate by manipulating information and must choose full transparency, in which case only the most solid banks survive. These results have implications for institutional design. The social planner would choose an interior level of credibility.

Keywords: bank runs, information disclosure, information manipulation, stress tests.
JEL classification: D82, D83, G01.

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1.1 Introduction

Stress testing has become an important regulatory tool since the 2008 financial crisis. However, the desirability of stress-tests results disclosure remains controversial. Not surprisingly, the many stress tests conducted in the last decade in different countries differed in the way supervisors handled results disclosure.[1] There is also evidence that, in some cases, regulators have tried to conceal bad news from the market, releasing information that turned out to be somewhat misleading.

This paper studies information disclosure when a financial regulator cannot commit to report the situation of the banking sector truthfully. Regulators may have incentives to mask an undesirable scenario as an attempt to avoid runs on banks that would otherwise be solvent. But if investors are aware of those incentives, would information manipulation be effective? I investigate how the regulator's credibility relates to the effectiveness of information manipulation and, especially, to the level of information disclosure. The relationship between credibility and disclosure, as well as its effects on the stability of the banking sector, raises the question of how credible we want our regulators to be. This paper proposes a rationale for the disclosure policy to depend on the regulator's credibility and discusses to which extent supervisors should be held accountable for their actions when trying to inflate the economic scenario.

I present a bank-run model where a regulator performs a stress test to learn banks' types. The regulator can choose whether to disclose bank-specific information (a policy of transparency), or to only release a report on the aggregate situation of the banking system (opacity). The regulator has the prerogative to bias the aggregate report at a cost that is increasing in the amount of distortion. The higher the cost of producing biased information, the more credible the regulator in the eyes of investors. By manipulating information, the regulator can try to avoid inefficient coordination failures, but investors anticipate the regulator's incentives and lower their expectations of the true state of the economy.

Although in a benchmark case where the regulator has full credibility the equilibrium is unique, introducing the possibility of information manipulation generates multiple equilibria in the model. However, it is still possible to derive interesting predictions. As long as the regulator has enough credibility, any equilibrium features a state-contingent disclosure

[1] For instance, the U.S. regulatory authorities disclosed very detailed bank-specific results in 2009 (SCAP stress test), and in a subsequent stress test two years later (CCAR, 2011) only the macro-scenario was published, with no bank-level result.

policy. In bad states – i.e., when the proportion of good banks is low – the regulator discloses bank-specific information; in intermediate states, the regulator opts for opacity and information manipulation; in good states, there is opacity and no bias. Information manipulation may be effective: the regulator causes a signal-jamming that boosts investors beliefs in states where they would be otherwise too pessimistic, at the cost of deteriorating beliefs in states where there is more than enough optimism to avoid any runs. When credibility is too low, though, the regulator loses the ability to use the disclosure of aggregate information as a means of generating confidence. The only option left is full transparency, which exposes low-quality banks to undesirable runs.

These results have implications for institutional design. Consider a social planner who can choose the amount of credibility regulators have in a previous stage. Increasing credibility too much reduces welfare since it precludes the financial authority from enhancing coordination in many states. However, when credibility is too low any aggregate report is meaningless to investors, so the regulator has no option but to choose transparency in all states, exposing fragile banks to runs. The paper suggests a central planner would choose an interior level of credibility, in which bank supervisors still have some room for maneuver during a financial crisis, but they are credible enough for opacity to be feasible.

The results of the paper seem consistent with evidence from the European experience in the early 2010s. The stress test conducted by the Committee of European Banking Supervisors (CEBS) in 2010 covering 91 European banks pointed to a total capital need of €3.5bn. A subsequent assessment of just the Irish banks conducted by independent consultants revealed a total capital need of €24bn.[2] European authorities came out of this episode with harmed credibility. In a new round of stress tests performed by the European Banking Authority (EBA), CEBS's successor, the degree of disclosure was much more extensive than in previous episodes, with bank-level results made available to the public. There is a widespread idea that European regulators lacked the credibility to be successful in generating confidence among investors by disclosing aggregate results, so they had to rely on full transparency. As argued by Schuermann (2014), "because credibility of European supervisors was rather low by that point, only with very detailed disclosure, bank by bank, [...] could the market do its own math and arrive at its own conclusions."

The results delivered by the paper are in line with the evidence in two dimensions: (i) one should expect there to be positive bias in aggregate reports in at least some states, and

[2] See Schuermann (2014). As another example in which supervisors seem to have misrepresented the real situation of the banking sector, there is the stress test of Spanish banks conducted in June 2012 (the Financial Sector Assessment Program). No bank-specific results were released, and the total capital need announced ranged from €17bn to €37bn. Independent assessments (by Roland Berger and Oliver Wyman) disclosed later on pointed to a total capital need of €51-62bn.

(ii) one should expect a lot more transparency, regardless of the state of the economy, when regulators lack credibility.

This paper builds on Bouvard et al. (2015), who study information disclosure in a bank-run model in the tradition of Diamond and Dybvig (1983). Their model helps explain the well documented evidence that information disclosure is usually state-contingent, with more transparency at bad times and more opacity at good times. They also suggest regulators have a commitment problem in their disclosure policy that leads to excess opacity and an increased probability of systemic crises. In a very different environment, Goldstein and Leitner (2015) reach similar results. They suggest no disclosure is needed at good times, but some disclosure may be necessary at bad times to avoid a market breakdown. Revealing too much information in their model reduces welfare by destroying risk-sharing opportunities. This paper contributes to this literature by studying how the possibility of biasing reports affects disclosure policies and the stability of the financial system.

Also related to this paper is the work by Edmond (2013), who studies a regime-change model in which a dictator engages in propaganda to try to avoid being overthrown. The dictator can add a bias to the mean of a private signal citizens receive concerning the strength of the regime. Here, I use a similar information manipulation technology, although the variable the regulator manipulates is common knowledge and the mechanism through which manipulation affect beliefs is different. In Goldstein and Huang (2016), there is also a policymaker that attempts to affect agents' beliefs, but the effectiveness of the policy requires commitment. In their regime-change game, a policymaker can commit to abandon the regime if fundamentals fall below a threshold, so the survival of the regime conveys good news and enhances coordination.

The paper is also connected to the literature on self-fulfilling runs and global games. Games with strategic complementarities and common knowledge of fundamentals usually present multiple equilibria. As first shown by Carlsson and Van Damme (1993), the introduction of private information in these environments may lead to a unique equilibrium. This approach, known as global games, has been applied to a wide variety of settings, including currency attacks (Morris and Shin, 1998), debt pricing (Morris and Shin, 2004), and bank runs (Goldstein and Pauzner, 2005). Several papers in this tradition analyze the effects on financial stability of different policy interventions, such as government guarantees, liquidity injections, capital requirements and monetary policy. Examples include Rochet and Vives (2004), Bebchuk and Goldstein (2011), Morris and Shin (2014) and Allen et al. (2015).

Here, global games techniques are used in the rollover game played among investors. However, as in Angeletos et al. (2006) and Angeletos and Pavan (2013), uniqueness is not

guaranteed. As shown by Angeletos et al. (2006), analyzing policies as comparative statics in global games models can be misleading since policies may have a signaling role. Multiplicity of equilibria may be restored when information is endogenous.

Another branch of the literature related to this paper is concerned with the social value of information. Morris and Shin (2002) examine how the precision of public signals impacts welfare in a beauty contest model (where agents aim to take an action appropriate to the underlying fundamental, but also similar to the actions of others). They show that greater precision of public information may be detrimental to welfare, since agents may place too much weight on public signals relative to their private information due to the coordination motive. In Angeletos and Pavan (2004), though, increasing the precision of public information is always welfare improving. Angeletos and Pavan (2007) shed some light on this matter by providing conditions under which one or the other result holds.[3] This paper analyzes how the credibility of public information affects welfare (the social value of credibility), since the information agents observe can be biased by an informed party.

Here, the focus is on the the study of information disclosure in a setting where there is rollover risk and strategic complementarities among investors, but the paper abstracts from other frictions potentially important for the discussion. For example, Bond and Goldstein (2015) suggest that the disclosure of information by the government might reduce the informativeness of market prices, which could be useful to guide government policies. For a comprehensive analysis of the trade-offs involved in the disclosure of sensitive information from stress tests, see Goldstein and Sapra (2013). The authors debate pros and cons of stress-test results disclosure in light of the existing literature on the subject and provide some policy recommendations.

The remaining of the paper is organized as follows. Section 1.2 presents the model. Section 1.3 describes the set of equilibria and presents comparative statics. Section 1.4 analyzes welfare and derives the optimal credibility level, and Section 1.5 concludes.

1.2 The model

This section presents the basic environment, which is based on Bouvard et al. (2015) and Morris and Shin (2000), and describes the information manipulation and disclosure technologies.

[3]For the role of information precision in regime-change games, see Iachan and Nenov (2015).

1.2.1 Environment

Consider a continuum $[0, 1] \times [0, 1]$ of investors endowed with one unit of the consumption good and a continuum of banks (or financial institutions in general) indexed by $i \in [0, 1]$. All agents are risk neutral and there is no discounting among the three periods, $t = 0, 1, 2$. Investors can either store their unit of the consumption good or invest it in a bank at $t = 0$. Banks have access to a long-term investment opportunity that yields a gross return of $1 + r_i$ at $t = 2$ for each unit invested (the maximum scale is one). The net return r_i is the sum of a common term and an idiosyncratic component. Specifically,

$$r_i = \tilde{\theta} + d_i,$$

where

$$d_i = \begin{cases} \eta & \text{with probability } p, \\ -\eta & \text{with probability } 1 - p, \end{cases}$$

i.e., banks can be of high or low type. The proportion of high-quality banks p is drawn from an uniform distribution on $[0, 1]$, so at $t = 0$, $\mathbb{E}_0 [d_i] = 0$, and the expected (net) return of the banking system is simply $\tilde{\theta}$. We can interpret the realization of p at $t = 1$ as an aggregate shock to the return of the banking sector. As long as $\tilde{\theta} > 0$, the long-term investment technology is better than storage, so all agents choose to invest their unit of the consumption good in banks, and each bank has a unit-mass continuum of investors.

Banks face rollover risk. Investors can keep their unit invested in the bank until $t = 2$ or withdraw it at the interim period. Early liquidation of the long-term project is costly: if a proportion l_i of bank i's investors withdraws their unit at $t = 1$, the per-unit return at $t = 2$ becomes $r_i - \gamma l_i$, $\gamma > 0$. Hence, there are strategic complementarities in investors decisions, in the sense that their willingness to demand early withdrawal depends on their expectation about how other investors will behave.

The expected return of the banking system $\tilde{\theta}$ is drawn from a normal distribution with mean θ and precision β_θ. At $t = 1$, investor j receives a noisy signal about the fundamental, $x_j = \tilde{\theta} + \varepsilon_j$, where ε_j is independently distributed (across investors in each bank) according to a normal distribution with mean 0 and precision β_ε. Private information is important in this setting to obtain uniqueness of equilibrium in the rollover game. Since the introduction of the noisy signal is simply an equilibrium selection device, I will focus on the limiting case with both precisions approaching infinity, satisfying $\beta_\theta^2 / \beta_\varepsilon \to 0$.

Moreover, the following parametric assumption is imposed.

Assumption 1.1.

$$0 < \theta - \eta < \theta < \frac{\gamma}{2} < \theta + \eta.$$

This assumption ensures (i) early liquidation is inefficient for both high- and low-quality banks; (ii) low-quality banks suffer run while high-quality banks do not when agents learn their bank's type[4]; and (iii) θ is low enough to trigger a run if agents act based solely on their prior information about p (which captures situations in which stress tests are a desirable regulatory tool).[5]

1.2.2 Regulation

There is a financial regulator who has access to bank-specific information at the beginning of period $t = 1$ (obtained by performing a stress test, for example). The regulator learns $\{d_i\}_{i\in[0,1]}$ and, consequently, p. The informed regulator can then choose between two disclosure regimes, *transparency* or *opacity*. A policy of transparency consists in disclosing bank-specific information to investors, while opacity consists in only releasing an aggregate report on the health of the banking system, p. If the regulator chooses to disclose idiosyncratic information, it must report it truthfully. However, the regulator cannot commit to tell the truth about the aggregate situation of the banking sector if it decides to withhold bank-specific information. It can transmit a delusive report about the aggregate state at a cost.

We can interpret full disclosure of bank-specific information as making spreadsheets containing data on the banks' financial situation available to the public. In such cases, fudging the information about the financial situation of a specific bank would be very difficult. The assumption here is that the cost of doing so is infinity. Still, manipulating aggregate information may be feasible since the public in general has no means of gathering information on all financial institutions to check if the aggregate report released by the regulator is accurate.

After observing banks' types, the regulator makes two decisions: it chooses an observable action – disclosing or withholding bank-specific information – and a hidden action – how much bias (if any) to add to the aggregate report. Formally, after observing p the regulator chooses $t \in \{0, 1\}$, where 1 represents transparency and 0 opacity, and a bias $b \in [0, 1 - p]$ in

[4] This property will follow from the result presented in Lemma 1.1.

[5] If this third part of the assumption is eliminated, that is, if $0 < \theta - \eta < \frac{\gamma}{2} < \theta$, a regulator that can choose in a previous stage whether or not to perform a stress test would choose not to do so, since there would be no runs if investors had only their prior information about p. Hence, the interesting case is when Assumption 1.1 holds.

order to form a report $z = p + b$.[6] Investors always observe z, and in case of transparency, they also observe $\{d_i\}_{i\in[0,1]}$.

The regulator's objective is to maximize the expected return of the banking system minus the cost of manipulating information. The cost of adding a bias b to the report is cb, $c \geq 0$. This cost can represent the expected punishment for the regulator in case its report is found to be false. One can interpret it as a pecuniary cost (being fired, paying a fine or losing a bonus, for example) or a reputational cost. Higher values of c are interpreted as higher *credibility*: the higher the cost of fudging aggregate information about the financial system, the more credible the aggregate report in the eyes of investors.

Timeline To summarize, the timeline of the model is as follows:

- At $t = 0$, investors invest their unit of the consumption good in the banks, and banks invest in the long-term investment project;
- At $t = 1$, $\{d_i\}_{i\in[0,1]}$ is realized and observed by the regulator (through a stress test); the regulator chooses b and makes the disclosure decision; after observing z and, in case of transparency, d_i, investors in each bank decide whether to roll over or not.
- At $t = 2$, payoffs are realized.

1.3 Information manipulation and disclosure policies

Investors' rollover decision

Before characterizing the optimal manipulation and disclosure policies for the regulator, we must find the optimal rollover decision for investors. Standard global games techniques give us the following result.

Lemma 1.1 (Morris and Shin, 2000)**.** *Investors in bank i choose to roll over at $t = 1$ whenever $\theta + \mathbb{E}_1[d_i] > \gamma/2$, and to run whenever the inequality is reversed.*

Proof. See Appendix 1.A.1. □

Lemma 1.1 implies that whenever the regulator discloses bank-specific information, investors in bank i run if $d_i = -\eta$, and do not run if $d_i = \eta$, i.e., only the high-quality banks survive. Lemma 1.1 also implies that under opacity investors' rollover decision depends on

[6] Naturally, the regulator cannot claim more than 100% of banks are of high quality, so the choice of b must satisfy the restriction that $z \leq 1$.

the information they have about the aggregate situation of the financial system. Since they cannot distinguish between good and bad banks, investors run on all banks when observing opacity if

$$\mathbb{E}\left[p|z,\right] < \bar{p} \equiv \frac{\gamma - 2(\theta-\eta)}{4\eta}, \tag{1.1}$$

and roll over if the inequality is reversed.

With this result in hand, we can then solve the regulator's problem. The regulator takes into account that if it chooses to disclose bank-specific information there is a run on low-quality banks while high-quality banks survive, and if it chooses opacity there are is a massive run on banks if condition (1.1) holds, and no runs at all if the reversed inequality holds.

I will look for Perfect Bayesian Equilibria in which investors play a cutoff strategy, that is, when observing opacity, they run if the aggregate report z is smaller than a cutoff $\hat{z}$, and rollover their investments if $z > \hat{z}$.

Regulator's problem

The financial regulator's objective is to maximize the return of the banking sector net of the costs of information manipulation. Suppose the regulator expects investors to run under opacity if $z < \hat{z}$ and to roll over otherwise. After observing the stress test results $\{d_i\}_{i\in[0,1]}$, and consequently p, the regulator chooses a bias b (in order to form a report $z = p + b \leq 1$) and also chooses $t \in \{0, 1\}$, where 1 represents transparency (i.e., disclosure of bank-specific information) and 0 represents opacity (i.e. only an aggregate report z is released). Given the agents' cutoff strategy $\hat{z}$, the regulator's problem can be written as

$$\max_{t\in\{0,1\},\, b\in[0,1-p]} U\left(p,t,b|\hat{z}\right) = t\left\{1 + p(\theta+\eta)\right\} + (1-t)\left\{1 + \left[\theta + \left(2p-1\right)\eta\right]\mathbb{I}_{\{p+b\geq\hat{z}\}}\right\} - cb \tag{1.2}$$

where $\mathbb{I}$ is an indicator function that assumes value 1 if the condition in braces is satisfied and 0 otherwise. The term multiplying t is the aggregate return of the banking sector when the regulator chooses transparency – in which case bad banks suffer a run and good banks' long-term investments mature and pay the net return $(\theta + \eta)$. The term multiplying $(1 - t)$ is the aggregate return when the regulator chooses opacity – in which case either there is a massive run and all investors get 1 (which happens if $z < \hat{z}$) or there is no run and both good- and bad-banks' investments mature (which happens if $z \geq \hat{z}$).[7] The last term is the cost of manipulating information.

[7]In any equilibrium, it must be that when $z = \hat{z}$ agents choose to roll over. Otherwise, the regulator's problem would not have a solution. When deciding the level of information manipulation, in some states the regulator would try to minimize cb subject to $b > \hat{z} - p$.

The solution to this problem yields two policy functions, $t(p)$ and $b(p)$, for a given $\hat{z}$. In equilibrium, $\hat{z}$ must be such that investors' rollover decisions are rational, taking into account the regulator's manipulation and disclosure policies. Beliefs are pinned down by Bayes rule whenever possible and off-the-equilibrium beliefs satisfy $\mu(p|z,0) < \bar{p}$ for all $z < \hat{z}$, where $\mu(p|z,t)$ is investors' expectation of p given the aggregate report and the disclosure decision observed. Furthermore, I impose the equilibrium must satisfy the *intuitive criterion* proposed by Cho and Kreps (1987), which requires off-the-equilibrium beliefs to be reasonable in the sense that investors cannot assign a positive probability to states in which the deviation is dominated in equilibrium for the regulator.[8]

1.3.1 Benchmark case: full credibility

As a benchmark, consider the case where the regulator must report aggregate information truthfully.[9] In this case there is a unique equilibrium, in which the regulator chooses to disclose bank-specific information in bad times, and not to disclose it in good times. Proposition 1.1 states this result.[10]

Proposition 1.1. *When the regulator must report aggregate information truthfully, in the unique equilibrium investors run under opacity if $z < \bar{p}$ and do not run otherwise. The regulator chooses transparency for $p < \bar{p}$ and opacity otherwise.*

Proof. See Appendix 1.A.2. □

Since manipulating information is not possible, observing z perfectly informs investors about the true state of the financial system. Whenever the aggregate state is high enough, revealing it is sufficient to avoid a run on all banks. Hence the regulator withholds banks-specific information, since disclosing it would cause an undesirable run on low-quality banks. Whenever the state is such that revealing aggregate information would not prevent a massive run on banks, the regulator chooses to disclose bank-specific information and avoid a run on high-quality banks.

[8]The intuitive criterion rules out some equilibria and allows us to derive sharp qualitative predictions. For instance, without the intuitive criterion there would always be an equilibrium with full transparency in all states in the general case presented in Section 1.3.2.

[9]This assumption is equivalent to making $c \to \infty$ in the general case in Section 1.3.2.

[10]This result is similar to the one in Bouvard et al. (2015) when the aggregate state of the banking system is common knowledge in their framework.

1.3.2 General case: limited credibility

Now, consider the case with limited credibility (i.e., when the cost of manipulating information is finite). The regulator may have incentives to disclose an aggregate report that inflates the proportion of good banks in an attempt to help agents coordinate in rolling over. However, agents are aware of those incentives and may discount the report they receive when forming their expectations about the health of the financial system.

As before, there are equilibria in which the disclosure policy is state-contingent: the regulator reveals bank-specific information when the state is low and discloses only aggregate information in higher states. However, due to the possibility of information manipulation, now agents are not certain about the true value of p when they observe opacity, so the disclosure policy will have a signaling role. By not disclosing bank-specific information the regulator conveys that the state of the financial system is not that bad (if p was very low, the regulator would have disclosed more information). But under opacity investors may also expect p to be smaller than the value reported, considering the regulator's incentives to inflate the aggregate state.

The signaling role of the disclosure policy in this setting generates multiplicity of equilibria. Proposition 1.2 below describes the set of equilibria in the general case where the regulator has limited credibility.

Proposition 1.2. *Let $p^b(z)$, p^{b*}, z^*, and $\bar{z}$ be given, respectively, by*

$$p^b(\hat{z}) = \frac{c\hat{z} - \theta + \eta}{c - \theta + \eta}, \tag{1.3}$$

$$z^* \equiv \frac{\gamma(c - \theta + \eta) - 2(\mu - \eta)(c - \mu)}{2\eta(2c - \theta + \eta)}, \tag{1.4}$$

$$p^{b*} \equiv \frac{c\gamma - 2(\theta - \eta)(c + \eta)}{2\eta(2c - \theta + \eta)}, \tag{1.5}$$

and

$$\bar{z} \equiv \frac{\gamma(c - \theta + \eta) - 2(\theta - \eta)(c - \theta - \eta)}{4c\eta}. \tag{1.6}$$

The set of Perfect Bayesian Equilibria satisfying the intuitive criterion is as follows:

1. *If $c > \theta - \eta$, for each $\left\{\hat{z}, p^b\right\}$ with $\hat{z} \in [z^*, \bar{z}]$ and $p^b = p^b(\hat{z})$ there is an equilibrium in*

which the regulator's strategy is given by

$$t(p)=\begin{cases}1 & \textit{for } p\in[0,p^b),\\ 0 & \textit{for } p\in\left[p^b,1\right],\end{cases} \tag{1.7}$$

and

$$b(p)=\begin{cases}\hat{z}-p & \textit{for } p\in\left[p^b,\hat{z}\right],\\ 0 & \textit{for } p\in[0,p^b)\cup(\hat{z},1].\end{cases} \tag{1.8}$$

2. *If $c<\theta-\eta$, the unique equilibrium features $\hat{z}>1$ (investors run under opacity for all $z\in[0,1]$) and $p^b>1$ (regulator chooses transparency for all p).*

3. *If $c=\theta-\eta$, for each $\left\{\hat{z},p^b\right\}$ with $\hat{z}=1$ and $p^b\in\left[p^{b*},1\right]$ there is an equilibrium in which the regulator's strategy is given by (1.7) and (1.8). Full transparency, as in 2., is also an equilibrium.*

Proof. See Appendix 1.A.3. □

For simplicity of the exposition, I hereafter refer to an equilibrium simply as a pair $\left\{\hat{z},p^b\right\}$, given that these two thresholds are sufficient to characterize regulator's and investors' strategies in equilibrium: investors run under opacity whenever $z<\hat{z}$ (under transparency, they run on low-quality banks and do not run on high-quality banks); the regulator chooses transparency for $p<p^b$, opacity and $b(p)=\hat{z}-p$ whenever $p\in\left[p^b,\hat{z}\right]$, and opacity with $b(p)=0$ whenever $p>\hat{z}$. An equilibrium featuring transparency for all p can be represented by $\left\{\hat{z},p^b\right\}$ with $\hat{z},p^b>1$. Notice $p^{b^*}=p^b\left(z^*\right)$ and $\bar{p}=p^b\left(\bar{z}\right)$, so $\left\{z^*,p^{b*}\right\}$ and $\{\bar{z},\bar{p}\}$ are the two extreme equilibria in item (i) (with the lowest and the highest thresholds $\hat{z}$, respectively). Also notice z^* and $\bar{z}$ are smaller than one for any $c>\theta-\eta$.

Proposition 1.2 shows that, despite the existence of multiple equilibria, some meaningful predictions arise. If the credibility level is sufficiently large, all equilibria feature transparency in low states, opacity with information manipulation in intermediate states and opacity with truth-telling in high states. In both intermediate and high states, there are no runs. Under transparency, agents run on low-quality banks, but high-quality banks survive (as shown in Lemma 1.1). The intuition for this state-contingent policy is that if the proportion of good banks is too small, a truthful aggregate report would not convince investors to roll over their investments in all banks, and biasing the aggregate report in a sufficient amount to avoid a run would be too costly. Thus the best option for the regulator is to choose transparency and at least save good banks. For intermediate levels of p, lying about the aggregate state pays

off: by inflating the report z, the regulator is able to avoid a coordination failure and save all banks. In high states, no manipulation is necessary, and there are no runs under opacity even under a truthful aggregate report.

If the credibility level is too low, though, the regulator loses the ability to help agents coordinate by manipulating information: the only equilibrium is the one with full transparency, in which case only the best banks survive. By withholding bank-specific information, a regulator with low credibility would trigger a run on all banks even if the proportion of good banks was equal to one. Investors discount too heavily the aggregate report released by a regulator that does not enjoy much credibility, so no report z would ever convey that the state is good indeed. Anticipating that, the best strategy for such regulator is to opt for full transparency, minimizing losses and avoiding runs on the most solid banks.

The next proposition presents some comparative statics on the set of equilibria with respect to the cost of information manipulation when credibility is not too low, so all equilibria feature state-contingent disclosure policies. Conditional on c being smaller than $\theta - \eta$, the equilibrium does not depend on c, since there is full transparency for all p. I check how the boundaries of the equilibrium set change as we change c.

Proposition 1.3 (Comparative statics)**.** *If $c > \theta - \eta$, we have the following:*

$$\frac{\partial z^*}{\partial c} =< 0, \ \frac{\partial p^{b*}}{\partial c} > 0,$$

$$\frac{\partial \bar{z}}{dc} < 0, \ \frac{\partial \bar{p}}{\partial c} = 0.$$

Proof. See Appendix 1.A.4. □

Analyzing the effect of an increase in c on equilibrium strategies helps build some intuition about the model. If we assume investors always play according to the equilibrium with the smallest $\hat{z}$ (i.e., $\hat{z} = z^*$), an increase in c causes a decrease in their threshold $\hat{z}$, that is, investors run less under opacity. The same comparative statics holds if we assume investors play according to the largest $\hat{z}$ (i.e., $\hat{z} = \bar{z}$), or any convex combination of the two (holding constant the weights as we change c).[11]

The effect on the equilibrium p^b's is the opposite: for $c > \theta - \eta$, the range of p for which transparency is optimal is (weakly) increasing in c (it is strictly increasing in c in the equilibrium with the smallest $\hat{z}$, $\left\{z^*, p^{b*}\right\}$, as well as in any convex combination of the two

[11]If $\tilde{z} = \alpha z^* + (1-\alpha)\bar{z}$ and $\tilde{p}^b = p^b(\tilde{z})$, $\alpha \in (0,1)$, regardless of c, then the comparative statics is qualitatively identical to the case where the equilibrium is $\left\{z^*, p^{b*}\right\}$.

extremes). It means that, as long as $c > \theta - \eta$, the more credibility the regulator has, the larger the set of states for which transparency is the optimal policy.

In other words, a decrease in the credibility level makes (i) investors run more under opacity, and (ii) the regulator rely more on opacity and information manipulation. The reason is that as c decreases, lying becomes a cheaper instrument to enhance coordination, so the regulator uses it more. However, lying also becomes less effective the smaller the cost of manipulating information. Investors discount the report more heavily, and thus require a higher z not to run under opacity.

That is why the amount of transparency there is in equilibrium only decreases as we reduce c down to some point. If credibility is too low, specifically if $c < \theta - \eta$, the aggregate report is completely disregarded by investors, so opacity would trigger a massive run on banks. Transparency is then the only option left.[12]

In short, the relationship between the level of credibility and the range of states in which there is transparency in equilibrium is non-monotonic. Figure 1.1 illustrates the comparative statics with respect to c when the equilibrium being played is the one with $\hat{z} = z^*$ (if in the equilibrium investors play some convex combination of z^* and $\bar{z}$, the qualitative results are the same).

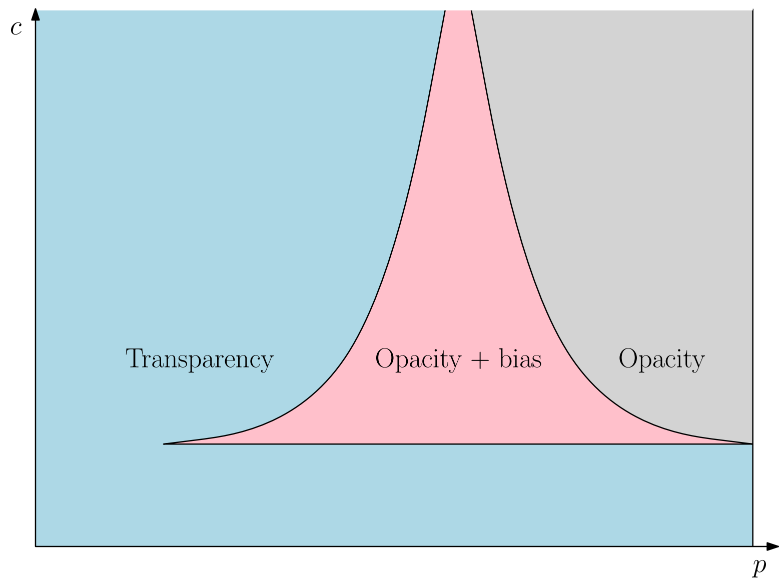

Figure 1.1: Comparative statics

As c decreases, the range of p for which the regulator's optimal strategy is to introduce a bias on aggregate information to avoid runs increases, while the transparency region decreases,

[12]When c is exactly $\theta - \eta$, as shown in Proposition 1.2 there are equilibria with full transparency for all p as well as equilibria with state-contingent disclosure policies.

as well as the region where there is opacity but no bias. However, there is a discontinuity in the relationship between credibility and the amount of transparency. If c is too low, the equilibrium features transparency in all states.

Discussion

It is interesting to understand the mechanism through which information manipulation in this setting can be effective in boosting coordination. Consider the case with $c > \theta - \eta$ (as in Proposition 1.2, item 1). In a range of intermediate states, the manipulation policy is such that for all states in that range the report issued by the regulator is the same, as can be seen in the left panel of Figure 1.2. By pulling together all those states, the regulator causes a signal-jamming that boosts investors' beliefs in a region where beliefs would be otherwise too pessimistic, and deteriorates beliefs in a region where there would be optimism to spare. The cyan area in the right panel of Figure 1.2 represents the gain in beliefs due to information manipulation, while the pink area represents the damage in beliefs.

For any $p > \bar{p}$, if information manipulation was not a possibility, there would be no runs under opacity and a truthful aggregate report. When manipulation is possible, anticipating the regulator's incentive to inflate the report, investors only rollover if the report is at least $\hat{z} > \bar{p}$. That is because they know all intermediate states are pulled together: in any state $p \in \left[p^b, \hat{z}\right]$, the issued report is $\hat{z}$. This implies a twist in beliefs in an intermediate range of p that makes it possible to avoid runs not only to the right of $\bar{p}$, but also in a range to the left of $\bar{p}$.

When $p < \bar{p}$, under a truthful report there would be a systemic run, but by manipulating information the regulator can improve confidence in the state of the economy in this region at the cost of worsening beliefs in states where there is more than enough optimism to avoid any runs. Hence, lowering investors' expectations of p in higher states down to $\bar{p}$ may pay off. The size of the range where there is information manipulation in equilibrium depends on the cost c for two reasons: the higher the cost, the less profitable it is for the regulator to use manipulation as a tool, but on the other hand, anticipating that the regulator is less prone to use it, the more effective manipulation is in increasing investors confidence.

If the cost of implementing information manipulation is too low (specifically, if $c < \theta - \eta$), investors know the regulator would have incentives to use it even when p is very low, so it becomes impossible to enhance coordination by pulling together low and high states. Even in very high states, the regulator loses the ability to convince investors the state is good indeed,

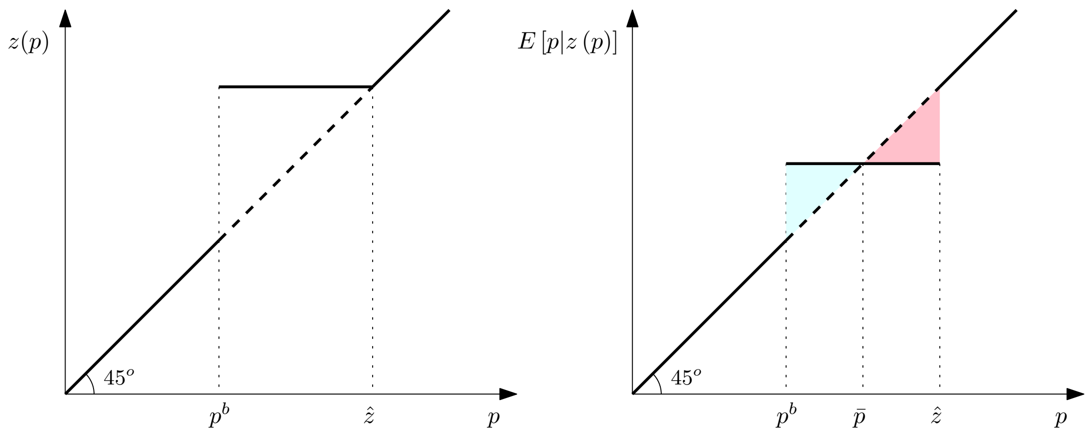

Figure 1.2: Aggregate report for a given $\hat{z} \in [0, 1]$ and investors' beliefs

because it would be too cheap to pretend this is the case if it was not. The only option left for the regulator is then to disclose bank-specific information and save the most solid banks at least.

The predictions of the model seem to be consistent with recent stress test disclosure episodes following the 2008 financial crises. First, the model says there are always states in which there is information manipulation in equilibrium as long as credibility is not too low. Second, if the regulator's credibility is too low, we should expect a lot more transparency. European authorities responsible for regulating the financial system in the early 2010s released some controversial stress test results, as discussed in the introduction. The amount of capital the banking sector had to raise in order to be able to withstand possible downturns was found to be a lot larger than the regulators had previously announced. Regulators came out of these episodes with a harmed credibility. Stress tests performed later on adopted a much more transparent disclosure policy. For instance, the EBA's stress test in 2010/2011 exhibited a degree of disclosure much more extensive then previous episodes, making bank-level results available for download in spreadsheets, so that investors could check for themselves the banks' real situation. Analyzing the European experience through the lens of the model, we could say EBA had to opt for full transparency exactly because its credibility was too low at that point. The authorities were not able to generate confidence through the disclosure of aggregate results, and the only alternative was to expose weak financial institutions to save the strong ones.

1.4 Optimal credibility

The way credibility affects optimal information disclosure policies and investors' rollover decisions may have important implications for institutional design. When deciding which authority will be responsible for regulating the financial system and what will be its incentives, a social planner should consider how the credibility level affects the outcome of the coordination game among investors. Imagine the social planner can choose c, that is, the cost the financial regulator faces to manipulate aggregate reports, what can be thought of as the expected punishment or consequences imposed to a regulator for masking the situation of the banking system. A welfare analysis can tell us what would be the socially optimal level of c from an ex ante perspective.

Assuming a given equilibrium $\left\{\hat{z}, p^b\right\}$ is being played, welfare in this economy can be written as

$$W = \int_0^{\min\left\{p^b,1\right\}} p(\theta+\eta)dp + \int_{\min\left\{p^b,1\right\}}^1 \left[\theta + (2p-1)\eta\right] dp, \tag{1.9}$$

which is the expected aggregate return of the banking system (which equals total output or total consumption).

It is easy to see that the best equilibrium in terms of efficiency is the one with smaller p^b, that is, with $\hat{z} = z^*$ and $p^b = p^{b*}$ if $c \geq \theta - \eta$ (for smaller levels of c the unique equilibrium features full transparency in all states). This equilibrium is the one where the regulator is able to avoid undesirable runs in more states.

Consider the case in which agents always play according to the best equilibrium. In the next subsection I discuss the issue of optimal credibility when this is not the case.[13] Proposition 1.4 determines the optimal credibility level from an ex ante perspective. It shows the planner would not opt for full credibility. An interior level of credibility allows the regulator to help agents coordinate by inflating the proportion of good banks, and it is good for society that the regulator has this prerogative. Hence, regulators' (expected) punishment for lying about the health of the banking sector should not be too harsh as to completely avoid information manipulation nor too low as to make opacity impractical. Figure 1.3 depicts the welfare as a function of the credibility level in this case.

Proposition 1.4 (Optimal credibility)**.** *Suppose agents always play according to the best equilibrium. The socially optimal credibility level is $c^* = \theta - \eta$.*

Proof. See Appendix 1.A.5. □

[13] For instance, as long as the worst equilibrium is not played with probability one for all $c > \theta - \eta$, there is an interior credibility level that is optimal under the assumption that c is picked from a countable set; neither full nor no credibility is optimal.

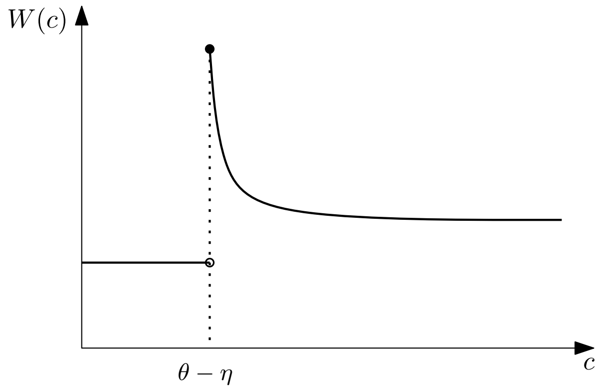

Figure 1.3: Welfare

Proposition 1.4 can help explain why some financial regulators choose to delegate the responsibility for the performance of stress tests to independent consultants. One possibility is that the regulator lacks credibility and would have to disclose very detailed bank-specific information, while a more credible party – such as an international consultancy firm that probably would not jeopardize its credibility to avoid a run in some country's banks – may be able to only release aggregate results. Furthermore, one can also interpret the recent change in financial regulation in Europe – that put the European Central Bank in charge of all stress tests procedures from 2014 on – as an attempt to raise credibility in financial regulators in the Eurozone, since national regulators as well as some authorities such as the EBA did not enjoy sufficiently credibility to be able to have discretion over how much information to disclose.

1.4.1 Optimal credibility and multiple equilibria

Now, consider the case where agents can play according to any equilibrium $\left\{\hat{z}, p^b\right\}$. Figure 1.4 depicts the range of welfare that can be achieved for each value of c depending on which equilibrium is selected.

Suppose there is a sunspot variable that determines which equilibrium will be played. Denote the implied distribution function over all possible equilibrium values of p^b for each c by $F(p^b; c)$. Also, assume c must be chosen from a set $\mathcal{C}$ that is a discretization of $\mathbb{R}^+$.[14] In this case, there is always an interior c that is optimal. Figure 1.5 exemplifies this case. Squares represent the expected welfare for each level of $c \in \mathcal{C}$, that is, $\int W\left(p^{b'}\right) dF(p^{b'}; c)$, where $W(p^{b'})$ is the welfare when the the equilibrium value of p^b is $p^{b'}$. The red square represents

[14] Without requiring $\mathcal{C}$ to be a countable set, the planner's choice of c could have no solution. For example, if the worst equilibrium happens with probability one when $c = \theta - \eta$ and the best one happens with probability one when $c > \theta - \eta$, the planner would want to set c as close as possible to $\theta - \eta$, satisfying $c > \theta - \eta$.

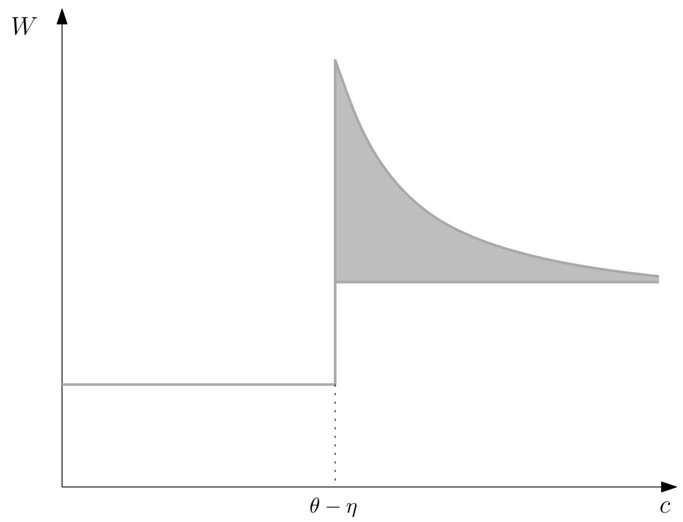

Figure 1.4: Welfare range

the maximum welfare.

Notice it can be the case that the maximum is not strict. For instance, if $F(.)$ assigns probability one to the worst equilibrium for all c, the planner would be indifferent between any $c \in \mathcal{C}$ satisfying $c > \theta - \eta$. Moreover, as long as the worst equilibrium happens with probability smaller than one for some $c > \theta - \eta$, the optimal c lies in a bounded set, i.e., full credibility is not optimal.

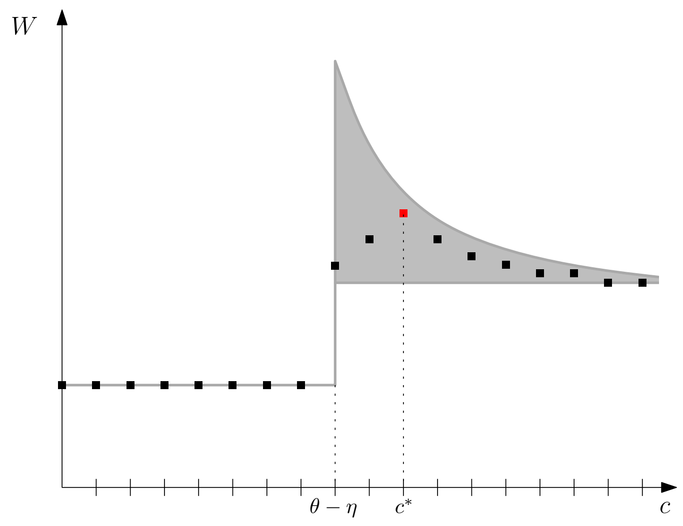

Figure 1.5: Optimal credibility

1.5 Final remarks

This paper studies information manipulation and disclosure in financial crises. It shows that a regulator that has the possibility of inflating the proportion of good banks as an attempt to avoid a systemic run will often do so. As long as the regulator has sufficient credibility, it pays off to incur in costly information manipulation, since it is very effective in boosting the stability of the financial sector. However, if the regulator's credibility is too low, it completely loses the ability to enhance confidence by releasing only aggregate information about the banking sector, so full transparency is the only option – and it causes part of the banks to fail.

These results have implications for institutional design. A social planner would choose an interior level of credibility, giving regulators room for maneuver in a crisis situation, but setting credibility in a high enough level so that opacity is still feasible. The results presented here are in line with evidence from recent stress test disclosure episodes.

1.A Proofs

1.A.1 Proof of Lemma 1.1

Conjecture a threshold equilibrium in which investors run on bank i at the end of period $t = 1$ if their expectation of $\tilde{\theta}$ is smaller than a threshold ρ_i^*, and do not run if it is larger. Investor j's expectation of $\tilde{\theta}$ given her private information is given by

$$\rho_j = \mathbb{E}\left[\tilde{\theta}|x_j\right] = \frac{\beta_\theta \theta + \beta_\varepsilon x_j}{\beta_\theta + \beta_\varepsilon}.$$

The threshold ρ_i^* is given by the indifference condition

$$\mathbb{E}\left[r_i|\rho_i^*\right] = \rho_i^* + \mathbb{E}\left[\eta_i - \gamma l_i|\rho_i^*\right] = 0,$$

which is equivalent to

$$\rho_i^* + \mathbb{E}\left[\eta_i\right] = \gamma \Phi\left(\sqrt{\alpha}\left(\rho_i^* - \theta\right)\right), \tag{1.10}$$

where $\Phi\left(.\right)$ is the standard normal distribution function and $\alpha = \frac{\beta_\theta^2(\beta_\theta+\beta_\varepsilon)}{\beta_\varepsilon(\beta_\theta+2\beta_\varepsilon)}$. Following Morris and Shin (2000), condition $\alpha\gamma^2 \leq 2\pi$ guarantees equation (1.10) to have a unique solution. Moreover, the equilibrium in threshold strategies is in fact the unique equilibrium, as can be shown by iterated elimination of strictly dominated strategies. Finally, making $\beta_\varepsilon \to 0$ we

have that $\rho_j \to \tilde{\theta}$ for all j, and since $\alpha \to 0$,

$$\rho_i^* \to \frac{\gamma}{2} - \mathbb{E}\left[\eta_i\right].$$

If we also assume $\beta_\theta \to \infty$, as long as we make $\alpha \to 0$ by assuming $\frac{\beta_\theta^2}{\beta_\varepsilon} \to 0$ to guarantee uniqueness, we have $\rho_i^* \to \theta$, so in equilibrium investors run on bank i whenever $\theta + \mathbb{E}\left[\eta_i\right] < \gamma/2$ and roll over whenever $\theta + \mathbb{E}\left[\eta_i\right] > \gamma/2$.

□

1.A.2 Proof of Proposition 1.1

Consider $z(p) = p$ for all p. Given $\hat{z}$, the optimal policy for the regulator is to set

$$t(p) = \begin{cases} 1 & \text{if } p < \hat{z}, \\ 0 & \text{if } p \geq \hat{z}. \end{cases}$$

Since the report is always truthful, Lemma 1.1 implies investors run on all banks whenever $\mathbb{E}_1\left[p|z\right] = z > \bar{p}$, and do not run otherwise. Hence, $\hat{z} = \bar{p}$.

□

1.A.3 Proof of Proposition 1.2

Regulator's decision

Suppose investors run under opacity whenever $z < \hat{z}$ and roll over otherwise. It is easy to see that $\hat{z} < 0$ is never an equilibrium. If agents never run under opacity, the optimal strategy for the regulator would be opacity (and no manipulation) for all p. But when observing any $z = p < \bar{p}$, agents would want to deviate and run (given Lemma 1.1). If $\hat{z} > 1$, that is, if agents always run under opacity, the optimal policy for the regulator is to choose transparency for all p, that is, $p^b > 1$. It is still to be determined the regulator's solution for a given $\hat{z} \in [0, 1]$. The regulator's problem in (2) boils down to choosing, in each state p, between (i) transparency and $b(p) = 0$; (ii) opacity with $b(p) = 0$; or (iii) opacity with $b(p) = \hat{z} - p$ (for $p < \hat{z}$), that is, setting a bias just large enough to avoid a run.[15] Let U^o denote the regulator's payoff under opacity and no manipulation, U^b denote its utility under opacity and manipulation ($b = \hat{z} - p$)

[15]Notice no other value of b is rationalizable. If $b \in (0, \hat{z} - p)$, the regulator pays a positive cost of lying cb and derives no benefit, since $z = p + b < \hat{z}$ and there is a massive run on banks, so this value of b is always dominated by $b = 0$. $b > \hat{z} - p$ is always dominated by $b = \hat{z} - p$, since at this level any run is avoided and further increasing b only increases the cost of information manipulation.

and U^t denote its utility under transparency (for simplicity, I subtract the constant 1 from all expressions):

$$U^o(p) = [\theta + (2p-1)\eta]\, \mathbb{I}_{\{p \geq \hat{z}\}},$$

$$U^b(p) = [\theta + (2p-1)\eta - c(\hat{z} - p)]\, \mathbb{I}_{\{p < \hat{z}\}} + U^o(p)\mathbb{I}_{\{p \geq \hat{z}\}},$$

$$U^t(p) = p\,(\theta + \eta)\,.$$

The maximized utility of the regulator will be the upper contour of these three functions, which are depicted in Figure 1.6.

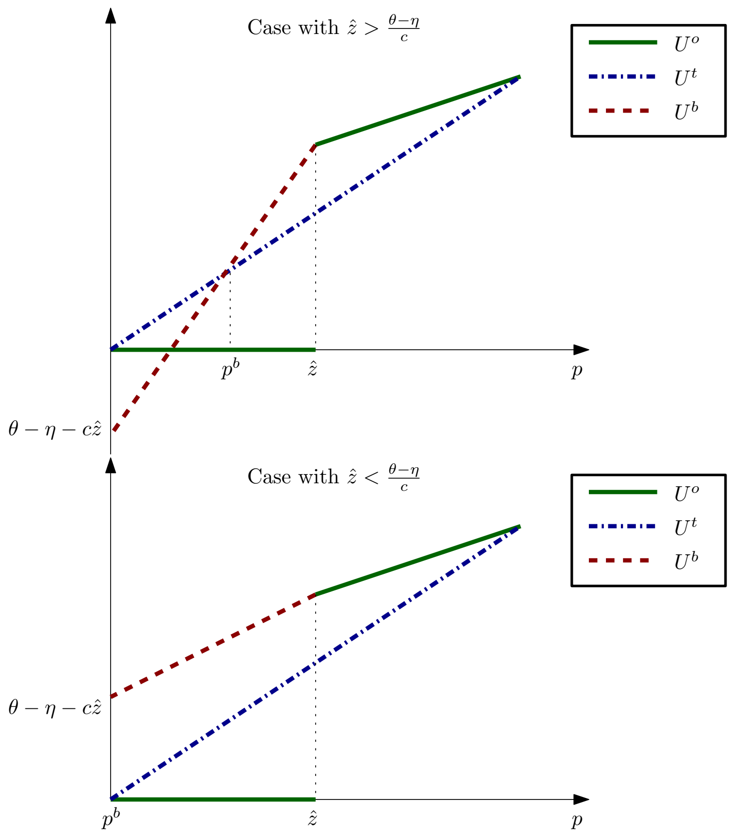

Figure 1.6: Regulator's payoffs for a given $\hat{z} \in [0, 1]$

The solution to the regulator's problem in (1.2) is as follows: if $\hat{z} > 1$, $t(p) = 1$ and

$b(p) = 0$ for all p; if $\hat{z} \in [0, 1]$,

$$t(p) = \begin{cases} 1 & \text{if } p < p^b, \\ 0 & \text{if } p \in \left[p^b, 1\right], \end{cases}$$

$$b(p) = \begin{cases} 0 & \text{if } p \in [0, p^b) \cup (\hat{z}, 1], \\ \hat{z} - p & \text{if } p \in \left[p^b, \hat{z}\right], \end{cases}$$

where

$$p^b(\hat{z}) = \begin{cases} \frac{c\hat{z} - \theta + \eta}{c - \theta + \eta} & \text{if } \hat{z} \geq \frac{\theta - \eta}{c}, \\ 0 & \text{if } \hat{z} < \frac{\theta - \eta}{c}. \end{cases} \tag{1.11}$$

Notice if $c < \theta - \eta$, $p^b(\hat{z}) = 0$ for all $\hat{z} \in [0, 1]$.

Investors' decision

In equilibrium, investors must prefer to roll over whenever $z \geq \hat{z}$, and to run when $z < \hat{z}$. Thus, given Lemma 1.1, $\hat{z}$ is such that $\mathbb{E}[p|z, t = 0] \geq \bar{p}$ for $z \geq \hat{z}$ and $\mathbb{E}[p|z, t = 0] < \bar{p}$ for $z < \hat{z}$. Given the regulator's disclosure and manipulation policies, an agent receiving the threshold message $\hat{z}$ and no bank-specific information expects p to be uniformly distributed on $\left[p^b, \hat{z}\right]$.

Consider the case with $c < \theta - \eta$. Since $p^b(\hat{z}) = 0$ for any $\hat{z} \in [0, 1]$, the posterior of an investor observing the threshold message $\hat{z}$ and opacity is that $p|\hat{z} \sim U(0, \hat{z})$. Her expectation of p is then $\hat{z}/2$, which is smaller than $\bar{p}$ for any $\hat{z} \leq 1$. Hence, there is no equilibrium with $\hat{z} \in [0, 1]$, and the unique equilibrium features $\hat{z} > 1$, transparency for all p and no information manipulation. This proves Proposition 1.2, item (ii).

Now, consider the case with $c \geq \theta - \eta$. One equilibrium can be constructed using the indifference condition:

$$\mathbb{E}[p|\hat{z}, t = 0] = \frac{p^b + \hat{z}}{2} = \bar{p},$$

which yields

$$\hat{z} = z^* \equiv \frac{\gamma(c - \theta + \eta) - 2(\theta - \eta)(c - \theta)}{2\eta(2c - \theta + \eta)} \leq 1.$$

When investors observe the report z^* and opacity, they are indifferent between running on banks or not. So for $z > z^*$, they roll over. However, receiving a signal below z^* is off the equilibrium. To sustain this equilibrium, off-the-equilibrium beliefs must satisfy $\mu(p|z, 0) < \bar{p}$ for $z < z^*$. Substituting z^* in (1.11), we have:

$$p^b(z^*) = p^{b*} \equiv \frac{c\gamma - 2(\theta - \eta)(c + \eta)}{2\eta(2c - \theta + \eta)} > 0.$$

Hence, there is an equilibrium in which investors run under opacity whenever $z < z^*$, the regulator chooses transparency for $p < p^{b*}$, opacity and information manipulation (with $b(p) = z^* - p$) for $p \in \left[p^{b*}, z^*\right]$ and opacity with $b(p) = 0$ for $p > z^*$.

This is not the only equilibrium, though. Suppose investors play according to a cutoff $\hat{z} \in (z^*, 1]$. Given that $p^b(\hat{z}) > p^{b*}$ for all $\hat{z} > z^*$, investors receiving the threshold message $\hat{z}$ and observing opacity expect p to be higher than $\bar{p}$, so this is also an equilibrium as long as off-the-equilibrium beliefs satisfy $\mu(p|z, 0) < \bar{p}$ for $z < \hat{z}$. Hence, there is an equilibrium for each cutoff $\hat{z} \in (z^*, 1]$, where the regulator chooses transparency for $p < p^b(\hat{z})$, opacity with manipulation for $p \in \left[p^b(\hat{z}), \hat{z}\right]$ and opacity with no manipulation for $p > \hat{z}$. Moreover, $\hat{z} > 1$ and $p^b > 1$, that is, transparency for all p, is also an equilibrium.

Finally, notice when $c = \theta - \eta$, we have $z^* = 1$. In this case, the curves $U^b(p)$ and $U^t(p)$ coincide, so the regulator is indifferent between transparency and opacity for all p. As long as the regulator's strategy features a $p^b \geq p^{b*}$, investors observing $z = 1$ and opacity expects $p \geq \bar{p}$. Thus, any pair $\left\{\hat{z}, p^b\right\}$ with $p^b \in \left[p^{b*}, 1\right]$ constitutes an equilibrium.

Ruling out some equilibria

In the case with $c > \theta - \eta$, there are equilibria that does not survive Cho and Kreps (1987)'s intuitive criterion.

Suppose an equilibrium in which investors play according to a cutoff $\hat{z} \in [0, 1]$ under opacity and $p^b(\hat{z}) > \bar{p}$. Consider an agent observing a report $z' = \hat{z} - \xi$, for some $\xi > 0$, and opacity. The regulator's payoff from sending a report z' under the most optimistic belief possible about investors response (i.e., investors roll over) is $U^b(p) = \theta + (2p - 1)\,\eta - c(z' - p)$, so the maximum benefit of deviating from the equilibrium strategy is

$$B(p) = \begin{cases} c\xi & \text{if } p \in \left[p^b(\hat{z}), z'\right], \\ (1 - p)\,(\theta - \eta) - c(\hat{z} - \xi - p) & \text{if } p < p^b\left(\hat{z}\right). \end{cases} \tag{1.12}$$

Since $B(p)$ is negative for all $p < p^b(z')$, the intuitive criterion imposes investors cannot assign positive probability for p being smaller than $p^b(z')$. But if $p^b(\hat{z}) > \bar{p}$, for ξ sufficiently small we have that $p^b(z') > \bar{p}$ as well. Hence, investors cannot expect p to be smaller than $\bar{p}$ when observing an off-the-equilibrium z slightly smaller than $\hat{z}$. We can then rule out any equilibria where $\hat{z}$ is such that $p^b(\hat{z}) > \bar{p}$ and restrict attention to equilibria satisfying the

restriction that $p^b(\hat{z}) \leq \bar{p}$, which implies

$$\hat{z} \leq \bar{z} \equiv \frac{\gamma(c-\theta+\eta)-2(\theta-\eta)(c-\theta-\eta)}{4c\eta} < 1.$$

Therefore, when $c > \theta - \eta$ the set of equilibria surviving the intuitive criterion reduces to all $\left\{\hat{z}, p^b\right\}$ with $\hat{z} \in [z^*, \bar{z}]$ and $p^b = p^b(\hat{z})$ as given in (1.11). When $c = \theta - \eta$, though, the maximum benefit of deviating from the equilibrium strategy given in equation (1.12) is positive for all p, so no equilibrium can be ruled out.

$\square$

1.A.4 Proof of Proposition 1.3

Consider z^*, p^{b*}, $\bar{z}$ and $\bar{p}$ as given by (1.4), (1.5), (1.6) and (1.1), respectively. Deriving each of these expressions with respect to c when $c > \theta - \eta$ we have:

$$\frac{\partial z^*}{\partial c} = \frac{(\theta-\eta)(\gamma-2(\theta+\eta))}{2\eta\left(\theta-\eta-2c\right)^2} < 0,$$

$$\frac{\partial p^{b*}}{\partial c} = -\frac{(\theta-\eta)(\gamma-2(\theta+\eta))}{2\eta\left(\theta-\eta-2c\right)^2} > 0,$$

$$\frac{\partial \bar{z}}{dc} = \frac{(\theta-\eta)(\gamma-2(\theta+\eta))}{4\eta^2} < 0,$$

$$\frac{\partial \bar{p}}{\partial c} = 0.$$

Assumption 1.1 guarantees the inequalities hold.

$\square$

1.A.5 Proof of Proposition 1.4

Assume agents always play according to the best equilibrium possible, that is, $\left\{z^*, p^{b*}\right\}$ and let $W(c)$ denote the welfare for a given c. The proof of Proposition 1.4 follows from the fact that the aggregate welfare in (1.9) is decreasing in c for all $c \geq \theta - \eta$ and the fact that the welfare when $c < \theta - \eta$ is smaller than the welfare for any $c \geq \theta - \eta$.

For $c > \theta - \eta$, we have

$$\frac{dW(c)}{dc} = -\frac{c(\theta-\eta)^2\left(\gamma-2\theta-2\eta\right)^2}{4\eta^2\left(2c-\theta+\eta\right)^3} < 0.$$

Since for all $c < \theta - \eta$,

$$W(c) = \int_0^1 p\,(\theta - \eta)\,dp$$

and

$$W(\theta - \eta) = \int_0^{p^{b*}(c)} p(\theta - \eta)dp + \int_{p^{b*}(c)}^1 [\theta + (2p-1)\,\eta]\,dp > \int_0^1 p\,(\theta - \eta)\,dp,$$

the value of c that maximizes welfare is $c^* = \theta - \eta$. Also, notice that

$$\lim_{c \to \infty} W(c) = \int_0^{\bar{p}} p(\theta + \eta)dp + \int_{\bar{p}}^1 [\theta + (2p-1)\eta]\,dp > \int_0^1 p\,(\theta - \eta)\,dp,$$

which finishes explaining Figure 1.3.

□

Chapter 2

QWERTY is efficient*

Abstract

We study a dynamic coordination problem with staggered decisions where agents choose between two competing networks. If the intrinsically worst one prevails, this is efficient. Moreover, inefficient shifts to the intrinsically best network might occur in equilibrium.

Keywords: coordination, networks, timing frictions, dynamic games.
Jel Classification: C73, D84.

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2.1 Introduction

Consider the problem of choosing between two industry standards or networks (PC or Mac, iOS or Android, Facebook or Google+, DVD or blu-ray, QWERTY or Dvorak keyboards). A consumer takes into account not only the intrinsic quality of each alternative but also the number of people in each one. Agents' choices are strategic complements: the larger the amount of people in a given network, the more each individual is willing to choose that option. Moreover, these choices are only occasionally made, typically when our current device is old or not working very well. Hence, our decisions are staggered and expectations about the future are crucial.

We observe that in many cases agents tend to follow the crowd and choose, say, Windows over Linux (even though many computer experts would recommend Linux), because it is useful to be in a large network of users.[1] Likewise, in the past century, we observed the mass adoption of the QWERTY keyboard over the Dvorak alternative because most people were used to the QWERTY standard, even though the Dvorak keyboard was arguably better in terms of its intrinsic quality.[2] This raises questions about efficiency in this problem. Is the equilibrium inefficient? Is there room for policy?

In order to answer these questions, we study efficiency in a dynamic coordination game with staggered decisions and show that the planner assigns an even lower weight to the intrinsic quality of each good than the agents. Hence the planner would be even more inclined towards QWERTY. One implication is that, if there is no other relevant externality, we should not subsidize a shift to an intrinsically better network – agents will move too early even without subsidies.

This paper builds on the model of Frankel and Pauzner (2000). They base their analysis on a model of sectorial choice (along the lines of Matsuyama (1991)), but their framework has been used to analyze location choices (Frankel and Pauzner, 2002), carry trades and speculation (Plantin and Shin, 2006), speculative attacks (Daniëls, 2009) and business cycles (Frankel and Burdzy, 2005; Guimaraes and Machado, 2015).[3] The model of currency attacks in Guimaraes (2006) and the model of debt runs in He and Xiong (2012) employ similar timing frictions.[4]

[1] In August 2015, about 85% of desktop or laptop computers worldwide used Microsoft Windows (Statcounter).

[2] See David (1985).

[3] In Guimaraes and Pereira (2015), we extend the model of Frankel and Pauzner (2000) to the case of ex-ante heterogeneous agents.

[4] This work is also related to the literature on coordination in games with strategic complementarities and asymmetric information, such as Carlsson and Van Damme (1993) and Morris and Shin (1998). The relation between this literature and that on dynamic coordination games (e.g., Frankel and Pauzner (2000) and Burdzy et al. (2001)) is discussed in Morris (2014).

The paper is also related to the literature on network externalities, in which strategic complementarities arise from consumption externalities.[5] Agents' optimal choices typically depend on what they expect others will do. However, most of this literature makes ad-hoc assumptions on how agents coordinate.[6] Argenziano (2008), an important exception, considers a static coordination game played by ex ante heterogeneous agents who must choose among two networks. In her model, agents give too much importance to their own idiosyncratic tastes, so the efficient allocation would feature a larger higher-quality network. Here, in contrast, inefficiencies result from the dynamic interaction among agents: they sometimes switch to the higher-quality network when it would be socially better for them to stay in the lower-quality one.[7]

2.2 The model

There is a continuum of infinitely-lived agents indexed by $i \in [0, 1]$. Time is continuous and agents discount the future at rate ρ. An agent can be in two possible networks. We denote by $a_{i,t} \in \{0, 1\}$ the network in which agent i is in at a given time t. Agents receive chances to revise their choice of network according to a Poisson process with arrival rate δ, and stay committed to this network until the arrival of another opportunity. This timing friction might represent a machine break-up in an environment with a choice between two technologies, an attention friction of consumers or firms, or maturity of debt in a model of debt runs.[8]

The flow utilities agent i derives from being in networks 0 or 1 are given, respectively, by:

$$u_t^0(\theta_t^0, n_t) = \theta_t^0 + \nu(1 - n_t) \text{ and } u_t^1(\theta_t^1, n_t) = \theta_t^1 + \nu n_t,$$

where θ_t^j represents the fundamentals affecting the flow-payoff of network j at time t for

[5] This literature has started with Katz and Shapiro (1985) and Katz and Shapiro (1986). See Shy (2011) for a survey.

[6] For instance, Katz and Shapiro (1986) assume that whenever there are multiple equilibria in the model, agents manage to coordinate their decisions in order to achieve the Pareto-superior outcome.

[7] In Argenziano (2008), the higher-quality network is always the largest one in equilibrium. Hence her model is not well suited to analyze situations in which the largest observed network is the one with lowest intrinsic quality. Here, owing to the dynamic aspect of our model, the economy may be in states where the lower-quality network is the largest one in equilibrium – which captures well situations like the Linux-Windows dispute or the QWERTY-Dvorak choice.

[8] This setting can be also interpreted as an overlapping generations model as in Matsuyama (1991): agents choose a network at birth and are stuck with this choice for life. They face a constant instantaneous probability of death δ throughout their lifetime. The birth rate is also δ, so total population size is constant. In the QWERTY application, it means that people learn how to type using one kind of keyboard and never switch, but new generations may adopt different standards.

$j \in \{0,1\}$, n_t is the mass of agents currently in network 1, i.e., $n_t = \int_0^1 a_{i,t} di$, and $\nu > 0$ is a parameter measuring the relative importance of strategic complementarities. The fundamental θ_t^j follows a Brownian motion with drift μ_j and variance σ_j^2.

The relative payoff function, i.e., the difference between the flow utilities in network 1 and in network 0, can be written as

$$\pi(\theta_t, n_t) = \theta_t + \gamma n_t,$$

where $\theta_t \equiv {\theta_t}^1 - {\theta_t}^0 - \nu$ and $\gamma \equiv 2\nu$. Notice θ follows a Brownian motion with drift $\mu = \mu_1 - \mu_0$ and variance $\sigma = \sigma_0^2 + \sigma_1^2$.[9]

An agent who receives an opportunity to revise her choice at time τ will choose action $a_i = 1$ (that is, will join network 1) whenever the discounted relative payoff of doing so is positive:

$$\mathbb{E} \int_\tau^\infty e^{-(\rho+\delta)(t-\tau)} (\theta_t + \gamma n_t) dt > 0. \tag{2.1}$$

If the inequality is reversed, the agent will choose $a_i = 0$.[10]

We can apply the results in Frankel and Pauzner (2000) to show that this game will present a unique equilibrium, which is characterized by a threshold in the $\mathbb{R} \times [0,1]$ space. For a given network size n, agents choose 1 if the relative quality of this network (θ) is high enough, and 0 otherwise. We focus on the case of very small shocks to fundamentals ($\mu, \sigma \to 0$) in order to be able to derive a closed-form expression for the equilibrium threshold, which is presented in Proposition 2.1.

We now look at this problem from the central planner's perspective. At every point in time, the planner decides the proportion ϕ_t of agents with an opportunity to revise their actions that will opt for network 1. At time τ, the planner maximizes the discounted sum of payoffs across agents, i.e.,

$$\mathbb{E} \int_{t=\tau}^\infty e^{-\rho(t-\tau)} \left[n_t u_t^1 + (1-n_t) u_t^0 \right] dt,$$

which is equivalent to maximizing

$$\mathbb{E} \int_{t=\tau}^\infty e^{-\rho(t-\tau)} \left[n_t \left(\theta_t - \nu\right) + \gamma n_t^2 \right] dt. \tag{2.2}$$

[9] Here we focus in the case of symmetric network effects, meaning that the benefit of a marginal increase in n for agents in network 1 is the same as the benefit of an increase in $1-n$ for agents in network 0. Section 2.3 analyzes the case of asymmetric network effects.

[10] The discounted payoff includes the factor $e^{-\delta(t-\tau)}$, which is the probability of not being drawn by the Poisson process from τ to t, i.e., the probability of still being committed to the current choice of network or, in the alternative OLG interpretation, the probability of being alive at t.

Suppose that a proportion $\phi_\tau \in [0,1)$ is optimal for the planner at time τ and consider the following deviation: the planner chooses $\phi_\tau = 1$ at time τ and future choices for any realization of shocks are kept unchanged. This deviation implies an infinitesimal increase in n_τ by dn_τ. Its effect on n_t is given by $dn_t = dn_\tau e^{-\delta(t-\tau)}$, since the initial increase in n_τ depreciates at a rate δ. A necessary condition for optimality is that such deviation is not profitable. Using (2.2), if a proportion $\phi_\tau \in [0,1)$ is optimal for the planner, it cannot be the case that

$$\mathbb{E}\int_\tau^\infty \frac{\partial\left[e^{-\rho(t-\tau)}\left(n_t(\theta_t-\nu)+\gamma n_t^2\right)\right]}{\partial n_t}\frac{dn_t}{dn_\tau}dt > 0,$$

which can be written as

$$\mathbb{E}\int_{t=\tau}^\infty e^{-(\rho+\delta)(t-\tau)}\left[\theta_t-\nu+2\gamma n_t\right]dt > 0. \tag{2.3}$$

Hence if the condition in (2.3) holds, action 1 must be optimal.

The same reasoning implies that action 0 is optimal if

$$\mathbb{E}\int_{t=\tau}^\infty e^{-(\rho+\delta)(t-\tau)}\left[\theta_t-\nu+2\gamma n_t\right]dt < 0. \tag{2.4}$$

The expression for the planner in (2.4) is very similar to the condition for an agent in (2.1). The only substantial difference is that the externality is more important for the planner (notice γ is multiplied by 2), as the planner takes into account the spillovers on others.

Mathematically, the planner's problem is very similar to the agents' problem in the decentralized equilibrium. At each moment in time, the planner chooses according to (2.3) and (2.4), considering that the future path of n must be consistent with optimality at every point, i.e., $\{n_t\}_\tau^\infty$ must result from optimal actions of the planner's future selves at all dates.[11] Hence this problem is isomorphic to a problem solved by agents with flow-payoffs given by $\theta_t - \nu + 2\gamma n_t$.

Therefore, the planner also chooses according to a downward sloping threshold and the results in Frankel and Pauzner (2000) can be applied here. Proposition 2.1 presents the planner's solution and the decentralized equilibrium.

Proposition 2.1. *Consider the case of very small shocks, $\mu, \sigma \to 0$.*

The decentralized equilibrium prescribes choosing network 1 whenever $\theta_t > Z^*(n_t)$ and 0

[11] There are no commitment issues in the planner's problem, since there is no time-inconsistency in preferences and the planner decides on everyone's actions.

otherwise, where Z^* *is given by*

$$Z^*(n) = -\frac{\gamma\delta}{\rho + 2\delta} - \frac{\gamma\rho}{\rho + 2\delta} n. \tag{2.5}$$

The planner's solution prescribes choosing network 1 *whenever* $\theta_t > Z^P(n_t)$ *and* 0 *otherwise, where* Z^P *is given by*

$$Z^P(n) = -\frac{\gamma\delta}{\rho + 2\delta} + \frac{\gamma\rho}{2(\rho + 2\delta)} - \frac{2\gamma\rho}{\rho + 2\delta} n. \tag{2.6}$$

Proof. See Appendix 2.A. □

The weight given by the planner to the current size of the network in (2.6) is twice as large as the weight given by agents in the decentralized equilibrium in (2.5). Conventional wisdom might suggest that the planner would push the agents towards the best "fundamentals" (1 when θ is high, 0 when θ is low) but the planner actually cares less about fundamentals than agents do. If agents prefer the QWERTY over the fundamentally more efficient Dvorak (i.e., if agents prefer to choose 0 even though action 1 would be the optimal choice if θ were the only relevant factor), the planner would be even more inclined to choose the fundamentally worst option. Intuitively, the planner takes into account the externality on others that agents fail to internalize, while the intrinsic quality of each good is fully taken into account by agents in the decentralized equilibrium.[12]

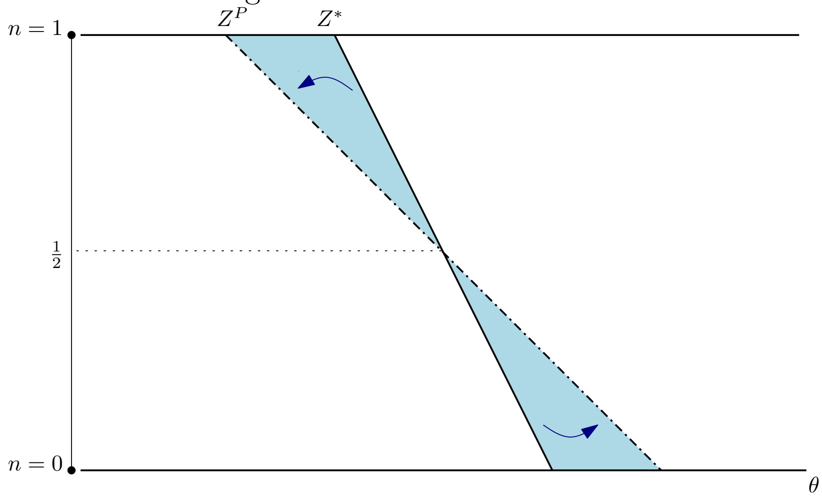
The planner would choose the more efficient Dvorak keyboard style if all agents' machines were to be replaced at a given point in time, while the agents problem in a static setting would exhibit multiple equilibria. However, this result does not apply to a dynamic environment with staggered decisions.

Figure 2.1 depicts the results in Proposition 2.1. To the right of Z^*, everyone who gets the chance to choose a network chooses 1, so n goes up, and the opposite happens to left of Z^*. The planner rotates the threshold so that its slope is half of the slope of the threshold in a decentralized equilibrium, which means n is relatively more important for the planner.[13]

[12] The efficiency results here contrast with those in models with information externalities that generate herd behavior (e.g., Bikhchandani et al. (1992)). In those models, agents follow others too much from a social point of view. Here, conformity of behavior arises because of preferences, not through learning, and they follow others too little.

[13] Here, network effects are symmetric, meaning that the benefit of a marginal increase in n for agents in network 1 is the same as the benefit of an increase in $1-n$ for agents in network 0. In an alternative setting with asymmetric network effects the planner would not only rotate the threshold but also shift it in order to enlarge the region where agents opt for the network that generates larger externalities. We analyze this case in Section 2.3.

Figure 2.1: Planner's solution



An important conclusion here is that if the worst network prevails in equilibrium, it is surely efficient, while shifts to the best network might be inefficient. The shaded area is where inefficient shifts to the intrinsically better option happen.[14] Agents start a switch to the smaller network because they take fundamentals into account but do not consider the harm this shift imposes on the large amount of agents still stuck in the (fundamentally) worst option. The planner would prevent such shifts from happening. A larger difference in fundamentals is required to make it optimal to start a shift towards the best (but smallest) network.

2.3 Extension: asymmetric network effects

Consider the following change in the model: the flow utilities agent i derives from being in networks 0 or 1 are given, respectively, by:

$$u_t^0(\theta_t^0, n_t) = \theta_t^0 + \nu^0(1 - n_t) \text{ and } u_t^1(\theta_t^1, n_t) = \theta_t^1 + \nu^1 n_t,$$

with $\nu^0, \nu^1 > 0$. The relative flow payoff is thus given by

$$\pi(\theta_t, n_t) = \theta_t + \gamma n_t,$$

[14]Notice when $n = 0.5$, $Z^* = -\gamma/2$. This is the point at which $\theta^1 = \theta^0$. We can think of a vertical line crossing $(Z^*(0.5), 0.5)$ as a dividing line between the regions where network 0 or network 1 are "intrinsically better".

where $\theta_t \equiv {\theta_t}^1 - {\theta_t}^0 - \nu^0$ and $\gamma \equiv \nu^0 + \nu^1$. Notice the main model is a special case of this one with $\nu^0 = \nu^1$.

Using the utility functions above, the results in Proposition 2.1 hold with the difference that the planner's threshold is given by

$$Z^P(n) = \nu^0 - \frac{2\gamma\delta}{\rho + 2\delta} - \frac{2\gamma\rho}{\rho + 2\delta} n.$$

When the network effect is asymmetric, that is, $\nu^0 \neq \nu^1$, the planner not only rotates the threshold around $n = 0.5$, but it also shifts the threshold in order to enlarge the region in which agents choose the action that generates more externalities.

In order to better analyze the results in this case, it is useful to define the dominance regions. Figure 2.2 depicts the two dominance region boundaries denoted by the dashed lines O and P. O is the curve along which an agent is indifferent between the two networks under the belief that everyone who gets the chance to act in the future is going to choose network 1 (O stands for *optimistic* about the path of n). To the left of O, agents have a dominant strategy of choosing 0. Analogously, to the right of P agents' dominant strategy is to choose 1, since P is the curve along which they are indifferent between the two actions under the most pessimistic belief possible (concerning the future path of n).

Figure 2.2: Planner's solution under asymmetric network effects

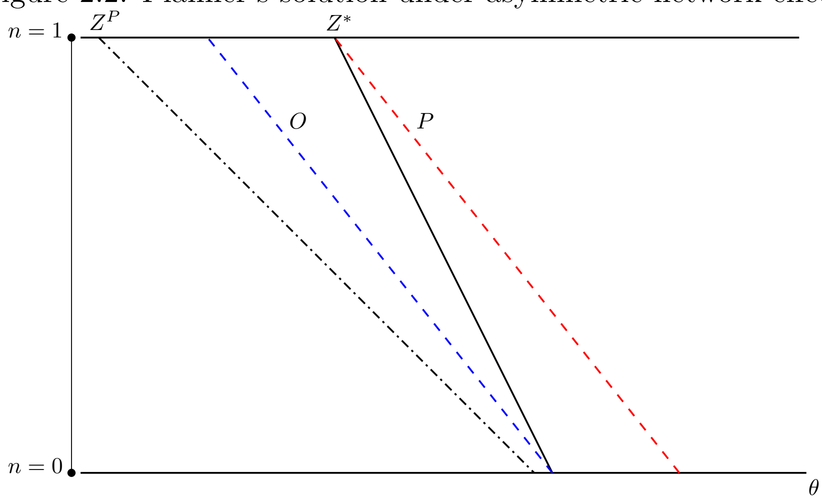


Figure 2.2 also depicts the planner's solution when externalities are larger in network 1 than in network 0 - specifically, for the case where $\nu^1/\nu^0 > (\rho + \delta)/\delta$. Since the externality in network 1 is large enough in comparison to the externality in 0, the planner's threshold lies completely on the agents' lower dominance region, so in the region between O and Z^P, the planner prescribes a strictly dominated strategy to be played by everyone. To see why,

consider for example the case in which n is large. The planner takes into account that a lot of agents are stuck in action 1 (due to timing frictions) and they all would benefit from the network effects generated by an additional increase in n.

When $(\rho+\delta)/\delta \geq \nu^1/\nu^0 > 1$, the planner does not always prescribe a strictly dominated strategy to be played, but it sometimes does. The planner's threshold crosses the agents' at some $n < 0.5$.

2.4 Final remark

We use the QWERTY vs. Dvorak keyboard case as an example of situations where there are strategic complementarities in agents decisions and the prevailing standard is not considered to be the best one. When there are two competing standards (or networks), if everyone were to make a choice at the same time (in our example, if all keyboards in the world were to be replaced at once) the efficient solution would be to choose the best one. However, since there are timing frictions and people do not switch from one network to another all at the same time, the efficient solution differs from conventional wisdom: if the worst standard prevails, this is surely efficient, and a central planner would be even more inclined towards the worst option. In fact, there are inefficient shifts to the best network in equilibrium.

2.A Proof of Proposition 2.1

Theorem 1 in Frankel and Pauzner (2000) ensures equilibrium uniqueness in our environment, and Theorem 2 helps us compute the decentralized equilibrium threshold when $\mu, \sigma \to 0$. Applying the latter, we have that the threshold must satisfy the following expression:[15]

$$(1-n_0)\int_0^\infty e^{-(\rho+\delta)t}\pi(Z^*, n_t^{\uparrow})dt + n_0\int_0^\infty e^{-(\rho+\delta)t}\pi(Z^*, n_t^{\downarrow})dt = 0, \tag{2.7}$$

[15] Burdzy et al. (1998) show that, when $\mu, \sigma \to 0$, at any point $(Z^*(n_0), n_0)$ along the threshold, it takes no time for n to start moving in one direction, and that it never comes back, i.e., n either grows continuously towards 1 or decreases towards 0. They also show that the probability of n bifurcating up is exactly $1-n_0$, and the probability of n bifurcating down is n_0. These probabilities can be used to pin down an agent's beliefs along the threshold. An agent at $(Z^*(n_0), n_0)$ assigns probability $1-n_0$ to an upward bifurcation followed by the increase of n_t at rate $\delta(1-n_t)$ – since to the right of the threshold everyone who gets the chance to revise actions is going to choose action 1 – and probability $1-n_0$ to a downward bifurcation followed by the decrease of n_t at rate $-\delta n_t$. Equation (2.7) states that an agent with such beliefs must be indifferent between actions 0 and 1 at any point along the equilibrium threshold.

where $n_t^{\uparrow} = 1 - (1 - n_0)e^{-\delta t}$ and $n_t^{\downarrow} = n_0 e^{-\delta t}$. Solving the equation above for Z^* gives us the agents' threshold in a decentralized equilibrium.

The remaining of the proof follows from the discussion in the main text. We have shown that solving the planner's problem is equivalent to solving the game agents play, but with flow-payoffs given by $\theta_t - \nu + 2\gamma n_t$. Thus the planner's solution is the threshold $Z^P(n_0)$ that solves

$$(1 - n_0) \int_0^{\infty} e^{-(\rho+\delta)t} \left(Z^P - \nu + 2\gamma n_t^{\uparrow} \right) dt + n_0 \int_0^{\infty} e^{-(\rho+\delta)t} \left(Z^P - \nu + 2\gamma n_t^{\downarrow} \right) dt = 0.$$

□

Chapter 3

Dynamic coordination among heterogeneous agents[*]

Abstract

We study a dynamic model of coordination with timing frictions and payoff heterogeneity. There is a unique equilibrium, characterized by thresholds that determine the choices of each type of agent. We characterize equilibrium for the limiting cases of vanishing timing frictions and vanishing shocks to fundamentals. A lot of conformity emerges: despite payoff heterogeneity, agents' equilibrium thresholds partially coincide as long as a set of beliefs that would make this coincidence possible exists. However, the equilibrium thresholds never fully coincide. In case of vanishing frictions, the economy behaves almost as if all agents were equal to an average type. Conformity is not inefficient. In the efficient solution, agents follow others even more often.

Keywords: coordination, conformity, timing frictions, heterogeneous agents, dynamic games.
Jel Classification: C73, D84.

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[*] This chapter is coauthored by Bernardo Guimaraes.

3.1 Introduction

Profitability of investment decisions depends on future demand for a firm's good, which, in turn, depends on whether other firms will be investing as well but also on idiosyncratic factors that affect demand for a particular product. In a problem of debt roll-over, both coordination motives and an individual's appetite for risk have to be considered. When deciding between Facebook and Google+, a consumer will take into account not only what others have been choosing but also her own tastes. Similarly, adopting a new technology may not be the best decision if others in the production chain will keep working with an old technology but heterogeneity in agents' productivity might also play an important role in this decision. In all these settings, both payoff complementarities and idiosyncratic features of preferences or technologies are important for an agent's choice.

Strategic complementarities induce players to try to do the same thing. In a dynamic setting, that means following what others are doing now and what they are likely to choose in the future. However, the idiosyncratic component of payoffs might push agents in different directions. This paper studies the interplay of complementarities and heterogeneity in payoffs in a dynamic setting.

In order to study this question we consider a dynamic environment with timing frictions as in Frankel and Pauzner (2000). Agents make a binary choice between two actions (say joining Facebook or not). Agents' instantaneous utility flow depends on an exogenously moving fundamental (which captures the intrinsic quality of Facebook), on how many others are in the network and on idiosyncratic tastes. Agents get opportunities to revise their behavior (join or leave Facebook) according to a Poisson clock, which can be seen as an attention friction modeled in a reduced-form way.

We first show there is a unique rationalizable equilibrium where agents of a given type play according to a threshold that depends on the total number of agents in a network and on the exogenous fundamental. We then obtain analytical results for the limiting cases of vanishing shocks and vanishing frictions, and provide an analytical characterization of the equilibrium thresholds in a tractable case with linear utility. Last, we solve the planner's problem to understand the inefficiencies that arise in equilibrium.

Each type of agent joins the network if the exogenous fundamental (θ) is larger than a threshold that is a function of the fraction of agents in the network (n). In the tractable limiting cases, a lot of conformity arises. Different types will always play the same strategy for some values on n unless their preferences are so heterogeneous that there is no set of (arbitrary) beliefs that would induce them to play according to the same threshold. Agents' choices are more similar for intermediate values of n, when there is more heterogeneity in their behavior

– and more dispersion of beliefs in a neighborhood around the threshold. However, from a social point of view, there is not enough conformity. The region where agents play different strategies in the planner's solution is smaller than the analogous region in the decentralized equilibrium.

In case of vanishing frictions, although agents play according to different strategies, the economy behaves almost as if all agents were identical and equal to an average type (again, unless agents' preferences are so heterogeneous that no set of beliefs could induce conformity).

This paper builds on the model of Frankel and Pauzner (2000). Their framework – a model of sectorial choice along the lines of Matsuyama (1991) – has been applied to analyze location choices (Frankel and Pauzner, 2002), carry trades and speculation (Plantin and Shin, 2006), speculative attacks (Daniëls, 2009), investment and business cycles (Frankel and Burdzy, 2005; Guimaraes and Machado, 2015) and efficiency in settings with network externalities (Guimaraes and Pereira (2016)).[1]

The paper is related to the literature on coordination in games with strategic complementarities. With complete information and no shocks, these games might exhibit multiple self-fulfilling equilibria. Carlsson and Van Damme (1993), Morris and Shin (1998) and Frankel et al. (2003) have shown that a unique equilibrium arise in a static environment in which fundamentals are not common knowledge and agents have idiosyncratic information about them (the so called global games). Frankel and Pauzner (2000) and Burdzy et al. (2001) show that a small amount of shocks in a dynamic model (with no private information) yields similar results. The relation between both literatures is discussed in Morris (2014).[2] In a related contribution, Herrendorf et al. (2000) show that if there is enough heterogeneity and a continuum of types, there is a unique equilibrium even in a dynamic setting with complete information.

Applied work employing the global games methodology has often considered heterogeneous populations in static coordination games.[3] Our results can be used in applied settings where dynamic coordination and heterogeneity are both important.

The paper is also related to literature on network externalities, in which strategic complementarities arise from consumption externalities.[4] Agents' optimal choices typically depend on what they expect others will do. However, most of this literature makes ad-hoc

[1]The model of currency attacks in Guimaraes (2006) and the model of debt runs in He and Xiong (2012) employ similar timing frictions.
[2]See also Morris and Shin (2003).
[3]Examples include heterogeneity in wealth (Goldstein and Pauzner (2004)); roles (Goldstein (2005)); risk aversion and consumption profile (Guimaraes and Morris (2007)); externalities from production (Sakovics and Steiner (2012)); and financial health (Choi (2014)).
[4]This literature has started with Katz and Shapiro (1985) and Katz and Shapiro (1986). See Shy (2011) for a survey.

assumptions on how agents coordinate.[5] One important exception is Argenziano (2008). She studies welfare in a model with differentiated networks in a static global-game model and highlights two sources of inefficiencies: agents give too much importance to their own idiosyncratic tastes and firms with the larger network charge a higher price. Both effects contribute to make the network "too balanced". This paper complements her work by studying coordination among heterogeneous agents in a dynamic setting.[6]

The efficiency results here differ from those in models where there is herd behavior due to information externalities (e.g., Bikhchandani et al. (1992)). In those models, agents follow others too much from an efficiency perspective. Here, conformity of behavior arises because of preferences, not through learning, and agents should follow others even more often.

3.2 The model

There is a continuum of infinitely-lived agents indexed by $i \in [0, 1]$. Time is continuous and agents discount the future at rate ρ. There are two possible actions $a_i \in \{0, 1\}$, but agents cannot switch from one to another at will. They receive chances to revise their actions according to a Poisson process with arrival rate δ, and stay committed to this choice until the arrival of another opportunity. This timing friction might represent an attention friction of consumers or firms, a machine break-up in an environment with a choice between two technologies or maturity of debt in a model of debt runs.

The flow payoff an agent gets from either action depends on fundamentals, on her idiosyncratic preferences and on the actions of others (there are strategic complementarities). Let n be the proportion of agents choosing action 1. Strategic complementarities can arise owing to either one-sided externalities or two-sided externalities: either the payoff of choosing action 0 is independent of the amount of agents making the same choice, but the payoff of choosing 1 is increasing in n (as in Matsuyama (1991)); or both actions become more appealing the larger is the proportion of agents taking them (as in Argenziano (2008)); or flow-payoffs from both actions can be increasing in n, but the difference in payoffs is also monotonically increasing in n (as in Guimaraes and Machado (2015)).

We denote agent i's relative flow-payoff of choosing action 1 by $\pi_{q(i)}(\theta, n)$, where $\theta \in \mathbb{R}$ denotes the fundamentals of the economy, $n \equiv \int_0^1 a_i di$ is the fraction of agents currently committed to action 1 and $q(i) \in \mathcal{Q} = \{1, ..., Q\}$ is agent i's type. All functions $\pi_q(.)$ are continuously differentiable and strictly increasing in both arguments. If we let α_q denote the

[5] For instance, Katz and Shapiro (1986) assume that whenever there are multiple equilibria in the model, agents manage to coordinate their decisions in order to achieve the Pareto-superior outcome.

[6] See also Ambrus and Argenziano (2009).

mass of type-q agents in the population and n_q the proportion of type-q agents currently playing 1, n can be written as $n = \sum_{q=1}^{Q} \alpha_q n_q$.

An agent who receives a chance to revise her choice at time τ will choose $a_i = 1$ whenever

$$\mathbb{E} \int_{\tau}^{\infty} e^{-(\rho+\delta)(t-\tau)} \pi_{q(i)}(\theta_t, n_t) dt > 0,$$

and $a_i = 0$ if the inequality is reversed. The expected discounted payoff takes into account only the states in which the agent believes she will still be committed to her action ($e^{-\delta(t-\tau)}$ expresses the probability of not receiving a revising opportunity between τ and t).

We further assume that payoff functions $\pi_q(.)$ are such that there are dominance regions for all types of agents. For each type, there is a region in the $\mathbb{R} \times [0, 1]$ space where choosing action 0 is a dominant action, and a region in which choosing action 1 is a dominant action. More specifically, for any given initial n, there is a sufficiently low level of fundamentals at which an agent prefers to play 0 even if she expects all others to play 1 when they get a chance to revise their actions, and there is a sufficiently high level of fundamentals such that it is preferable to play 1 even if no one else is expected to choose so in the future.[7]

Let P_q be the boundary of the upper dominance region of a type-q agent, i.e., the curve on which such agent is indifferent between the two actions if she believes everyone after her will choose 0 (P stands for *pessimistic* about the proportion of agents playing 1 in the future). This boundary is downward sloping: since $\pi_q(\theta, n)$ is increasing in θ and n, a higher n today means that the value of θ needed to make agents indifferent between the two actions is smaller. At the other extreme, let O_q be the boundary of the lower dominance region for a type-q player, that is, the curve on which this type of agent is indifferent between the two actions under the belief that everyone will choose 1 when they get the chance (O stands for *optimistic*). This curve is also downward sloping. Figure 3.1 shows an example of dominant regions.

[7] Formally, we can define the lower dominance region boundary for type q as a curve $O_q(n_0)$ satisfying

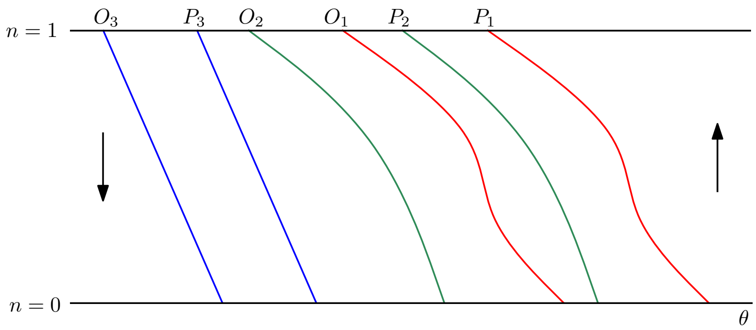
$$\mathbb{E} \left[\int_0^{\infty} e^{-(\rho+\delta)t} \pi_q(\theta_t, n_t^{\uparrow}) | \theta_0 = O_q \right] = 0,$$

where $n_t^{\uparrow} = 1 - (1 - n_0)e^{-\delta t}$. Likewise, we can define the upper dominance region boundary as the curve $P_q(n_0)$ satisfying

$$\mathbb{E} \left[\int_0^{\infty} e^{-(\rho+\delta)t} \pi_q(\theta_t, n_t^{\downarrow}) | \theta_0 = P_q \right] = 0,$$

where $n_t^{\downarrow} = n_0 e^{-\delta t}$.

Figure 3.1: Dominance regions: an example



3.2.1 Unique equilibrium

Suppose θ is constant. When θ lies either to the right of all upper dominance regions boundaries, or to the left of all lower dominance regions boundaries, there is only one rationalizable action. Nevertheless, there always exists a subset of the state space with equilibrium multiplicity.[8]

However, when there are shocks to θ, the equilibrium is unique for any amount of heterogeneity. Proposition 3.1 presents this result. The following lemma is key for the demonstration. It states that the dynamics of n_t depends on each $n_{q,t}$, $q \in \mathcal{Q}$, only through n_t.

Lemma 3.1. *For any given strategy profile, $\partial n_t/\partial t$ depends on n_t, but not on $\{n_{q,t}\}_{q\in\mathcal{Q}}$ (given n_t).*

Proof. Fix a strategy profile $\left\{s_{q(i)}\right\}_{q\in\mathcal{Q}}$. Denote by $\mathcal{I}_t$ the set of types whose strategies prescribe action 1 at time t. Notice that the path of n_q is given by the following differential equation:

$$\frac{\partial n_{q,t}}{\partial t} = \begin{cases} \delta(1 - n_{q,t}) & \text{if } q \in \mathcal{I}_t, \\ -\delta n_{q,t} & \text{if } q \notin \mathcal{I}_t. \end{cases} \tag{3.1}$$

Equation 3.1 means that a type-q agent whose strategy prescribes playing 1 and is currently playing 0 will switch to action 1 when she receives an opportunity to revise her choice (there are $1 - n_{q,t}$ such agents). Likewise, every type-q agent whose strategy prescribes playing 0 and who has previously chosen 1 will switch to action 0 at the first opportunity. Using the

[8] Herrendorf et al. (2000) shows that in a similar environment with no shocks and a continuum of types, multiplicity is ruled out if there is a sufficient amount of heterogeneity.

fact that $n = \sum_{q=1}^{Q} \alpha_q n_{q,t}$, we have that $\frac{\partial n_t}{\partial t}$ is given by:

$$\begin{aligned}
\frac{\partial n_t}{\partial t} &= \sum_{q=1}^{Q} \left(\alpha_q \frac{\partial n_{q,t}}{\partial t} \right) \\
&= \sum_{q \in \mathcal{I}_t} \alpha_q \delta (1 - n_{q,t}) + \sum_{q \notin \mathcal{I}_t} \alpha_q \left(-\delta n_{q,t} \right) \\
&= \delta \left[\sum_{q \in \mathcal{I}_t} \alpha_q - \sum_{q=1}^{Q} \alpha_q n_{q,t} \right]
\end{aligned}$$

$$\implies \frac{\partial n_t}{\partial t} = \delta \left[\sum_{q \in \mathcal{I}_t} \alpha_q - n_t \right].$$

□

Lemma 3.1 will allow us to deal with this problem in a two-dimensional space. Given a strategy profile, agents only need to consider the aggregate mass of agents currently committed to action 1 in order to understand the dynamics of the system. One could expect this dynamics to depend on the proportion of each type of agent currently choosing each option, but due to the assumption of a Poisson process for the arrival of opportunities to switch actions, that is not true. It suffices to know the aggregate n_t and each type's strategy to compute $\partial n_t / \partial t$.

Intuitively, consider that all individuals drawn by the Poisson clock at time t are automatically assigned to option 0 but have the chance to choose 1. At every instant, n_t decreases at a rate δn_t (due to the individuals assigned to option 0), but owing to the choices of agents whose strategies prescribe playing 1, n_t also increases at a rate $\delta \sum_{q \in \mathcal{I}_t} \alpha_q$ (since the law of large numbers holds and the Poisson parameter is the same across groups).

Proposition 3.1. *Suppose θ follows a Brownian motion with drift μ and variance $\sigma^2 > 0$. There is a unique equilibrium characterized by downward sloping thresholds $\left(Z_q^*\right)_{q \in \mathcal{Q}}$ in the $\mathbb{R} \times [0,1]$ space, such that*

$$a_{i,t} = \begin{cases} 1 & \text{if } \theta_t > Z_{q(i)}^*(n_t), \\ 0 & \text{if } \theta_t < Z_{q(i)}^*(n_t). \end{cases}$$

That is, each agent i called upon acting at time t plays 1 when to the right and 0 when to the left of $Z_{q(i)}^*$.

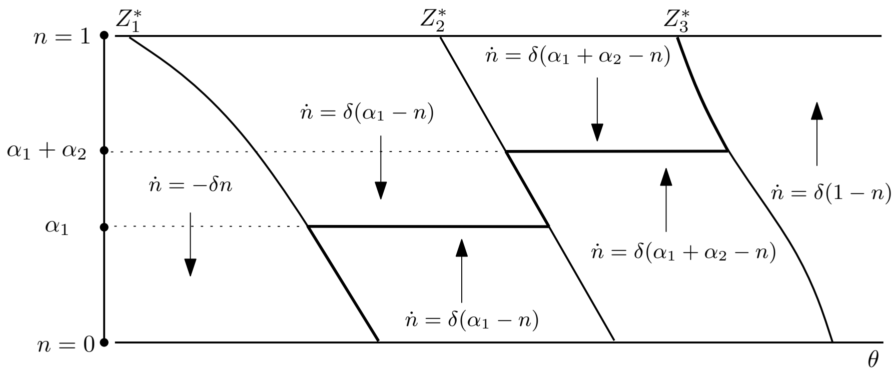
Proof. See Appendix 3.B.1. □

The proof of equilibrium uniqueness employs a strategy of iterative elimination of strictly dominated strategies, starting from the dominance regions. Even if these regions are very

remote, making it unlikely that the fundamentals will reach one of them before an agent receives another chance to revise her action, the existence of such regions triggers an iterative contagion effect until there is a single rationalizable strategy left (for each type of agent). The basic intuition is as follows: an agent at any point on the boundary of her upper dominance region is indifferent between actions 0 or 1 under the assumption that, at all future dates, all other agents will choose 0. But once shocks to fundamentals are introduced, she knows there is a positive probability that fundamentals will reach regions in which it is dominant for some agents to play 1 (while she is still committed to her choice).[9] Thus, she cannot hold the belief that others will play 0 under any circumstances. The most pessimistic belief she can hold is that agents will play 0 whenever it is not strictly dominated to do so, and under this new (a bit more optimistic) belief, there is another (smaller) level of fundamentals that makes such agent indifferent between the two actions. Extending this reasoning to all following rounds and employing an analogous procedure starting from the lower dominance regions yields a unique rationalizable equilibrium. An interesting aspect of this result – which was demonstrated by Frankel and Pauzner (2000) for the case of identical individuals – is that uniqueness of equilibrium can be achieved even with vanishing shocks to fundamentals (that is, in the limit as $\mu, \sigma \to 0$). Multiplicity of equilibrium in this environment do not survive the introduction of the smallest amount of shocks.

The unique equilibrium is characterized by thresholds for each type of agent, to the right of which these agents play 1, and to the left of which they play 0. Depending on the initial value of n, these strategies imply an upward or downward path for n_t. Figure 3.2 below exemplifies the dynamics around the equilibrium for a case with three types of agents. $\partial n_t / \partial t$ is computed as in Lemma 3.1.

Figure 3.2: Dynamics ($Q = 3$)



[9] Notice timing frictions are key for the proof. However, the uniqueness result still holds as $\delta \to \infty$.

3.3 Limiting cases

We now restrict our attention to situations with two types, $Q = 2$, and analyze two limiting cases: vanishing shocks to fundamentals and vanishing timing frictions. Assume $q(i) = \overline{q}\ \forall i \in [0, \alpha]$ and $q(i) = \underline{q}\ \forall i \in (\alpha, 1]$, i.e., there is a mass α of type-$\overline{q}$ agents in the economy and a mass $1 - \alpha$ of type-$\underline{q}$ agents. Denote their payoff functions, respectively, by $\overline{\pi}(\theta, n)$ and $\underline{\pi}(\theta, n)$. We assume that for any pair (θ, n), $\overline{\pi}(\theta, n) > \underline{\pi}(\theta, n)$, that is, type-$\overline{q}$ agents have a higher relative instantaneous payoff of choosing action 1 in every state.

The next lemma, based on Burdzy et al. (1998), characterizes agents' beliefs on the equilibrium threshold and is key for the results of the paper.

Lemma 3.2. *Suppose agents play according to two arbitrary (downward sloping and Lipschitz) thresholds $\overline{Z}(n) < \underline{Z}(n)$ for all n in some interval (n^1, n^2). Consider a point (θ, n) with $n \in (n^1, n^2)$ and either $\theta = \underline{Z}(n)$ or $\theta = \overline{Z}(n)$. As $\mu, \sigma \to 0$, the time it takes for the system to bifurcate either up or down converges to zero. Moreover, the probabilities of an upward or a downward bifurcation are computed as follows:*

(i) Consider a point (θ, n) with $\theta = \overline{Z}(n)$.

$$P(up) = \begin{cases} 0 & \text{if } n \geq \alpha, \\ 1 - \frac{n}{\alpha} & \text{if } n < \alpha, \end{cases}$$

and $P(down) = 1 - P(up)$.

(ii) Consider a point (θ, n) with $\theta = \underline{Z}(n)$.

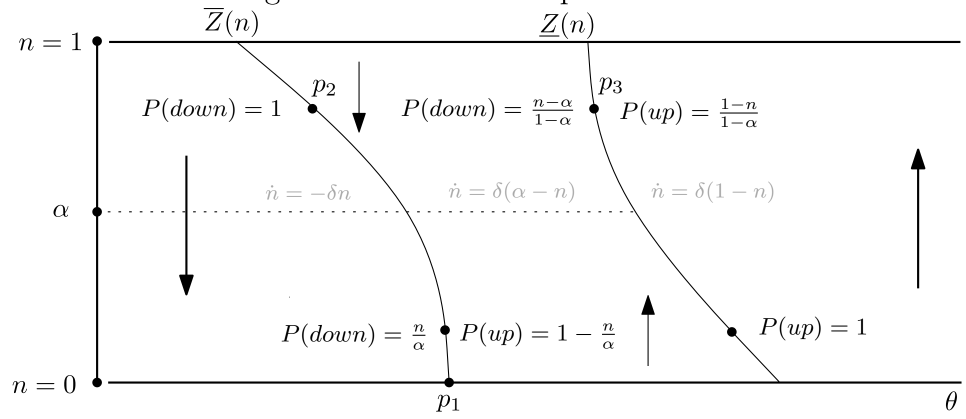
$$P(up) = \begin{cases} \frac{1-n}{1-\alpha} & \text{if } n > \alpha, \\ 1 & \text{if } n \leq \alpha, \end{cases}$$

and $P(down) = 1 - P(up)$.

Proof. See Appendix 3.B.2. □

Figure 3.3 shows the dynamics around the two types' thresholds in case they do not intersect (computed as in the proof of Lemma 3.1) and the implied bifurcation probabilities along the thresholds (computed as in Lemma 3.2). The idea behind Lemma 3.2 is that, considering an initial point exactly on an agent's threshold, the probability of the system going up or down depends on the speed of increase or decrease of n at each side of the threshold. Intuitively, once the economy has headed off in one direction, it does not revert to $\overline{Z}$ or $\underline{Z}$, since thresholds are downward sloping and shocks to fundamentals are small, but

Figure 3.3: Bifurcation probabilities



will it start going up or down? That depends on the realization of the Brownian motion in a tiny time span and on the speed of decrease and increase of n at each side of the threshold that pull the economy away from the (downward sloping) threshold.

A few examples help to illustrate the result in Lemma 3.2. Consider an agent called upon revising her action when the economy is at p_1 in Figure 3.3. As $n = 0$, a small negative shock pushing θ slightly to the left will make no difference (n cannot decrease anymore), while a small positive shock to θ will lead high type agents to choose action 1, so agents believe that n will increase with probability one. An agent at point p_2 holds the opposite belief but for a different reason: both to the left and to the right of $\overline{Z}$, n is decreasing, so the agent assigns probability one to n heading towards zero. Last, look at point p_3 in Figure 3.3. A small negative shock to θ means that all high-type agents who get the chance will play 1, but all low-types will play 0. Since there are more agents currently committed to 1 than agents willing to choose 1 (because $n > \alpha$), n decreases in that region at a rate $\delta(n - \alpha)$. A small positive shock, though, would make every agent willing to switch to action 1, hence n would increase at rate $\delta(1 - n)$. This dynamics implies that at p_3, the probability of the system bifurcating up is proportional to the relative rate at which it goes up: $\frac{\delta(1-n)}{\delta(1-n)+\delta(n-\alpha)} = \frac{1-n}{1-\alpha}$.

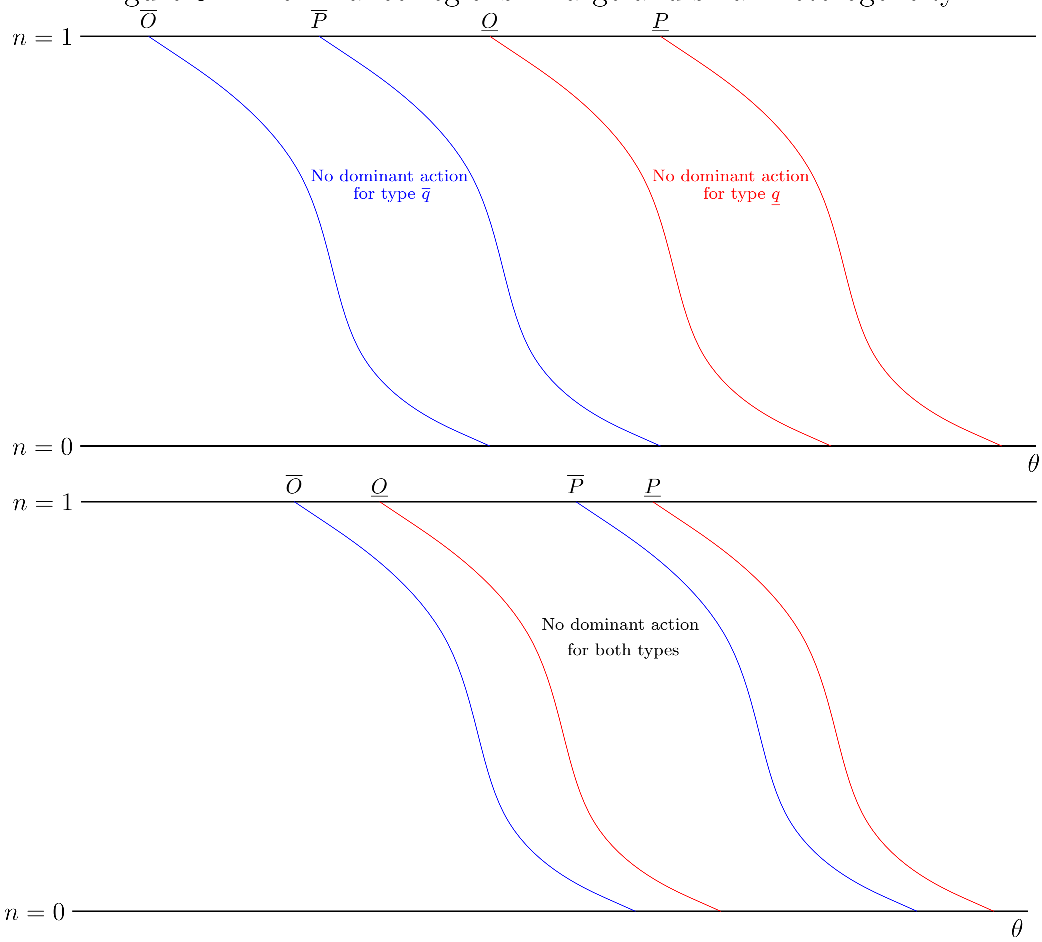
3.3.1 Vanishing shocks

Consider the limiting case in which shocks to fundamentals vanish, that is, $\mu \to 0$ and $\sigma \to 0$. Let $\overline{Z}(n)$ and $\underline{Z}(n)$ denote the two types' equilibrium thresholds. The equilibrium properties depend on the degree of payoff heterogeneity.

The relative position of the dominance regions for the two types of agents on the $\mathbb{R} \times [0, 1]$ space reflects the degree of heterogeneity in their payoff functions. For a sufficiently large degree of heterogeneity, we have that $\overline{P}(n) < \underline{Q}(n)\ \forall n$: a high-type agent that holds the worst

possible belief concerning future choices of others demands a smaller value of the fundamental to be indifferent between the two actions than a low-type agent under the most optimistic belief. This implies there is no region in the state space in which neither type has a dominant action. On the other hand, if heterogeneity is not too large, dominance regions can be such that $\underline{O}(n) < \overline{P}(n)\ \forall n$, so there is a region in which neither action is dominant for both types of agents. Figure 3.4 exemplifies those two cases.

Figure 3.4: Dominance regions - Large and small heterogeneity



In the case of vanishing shocks, the upper dominance region boundary of a high-type agent can be computed as:

$$\int_0^\infty e^{-(\rho+\delta)t}\overline{\pi}(\overline{P}, n_t^{\downarrow})dt = 0, \tag{3.2}$$

where $n_t^{\downarrow} = n_0 e^{-\delta t}$. The lower dominance region boundary of a low-type agent is given by

$$\int_0^\infty e^{-(\rho+\delta)t}\underline{\pi}(\underline{O}, n_t^{\uparrow})dt = 0, \tag{3.3}$$

where $n_t^{\uparrow} = 1 - (1 - n_0)e^{-\delta t}$.

Expressions for the equilibrium thresholds are provided in Appendix 3.A.1. Proposition 3.2 shows the main equilibrium properties for the case of vanishing shocks.

Proposition 3.2. *Suppose there are two types of agents in the economy, $\underline{q}$ and $\overline{q}$, with payoff functions given by $\underline{\pi}(\theta, n)$ and $\overline{\pi}(\theta, n)$, respectively, with $\overline{\pi}(.) > \underline{\pi}(.)\ \forall(\theta, n)$. In the limit as $\mu, \sigma \to 0$, in the unique rationalizable equilibrium:*

(i) if $\underline{O}(n) > \overline{P}(n)\ \forall n$, then $\overline{Z}(n) < \underline{Z}(n)\ \forall n$, so different types' thresholds do not intersect;

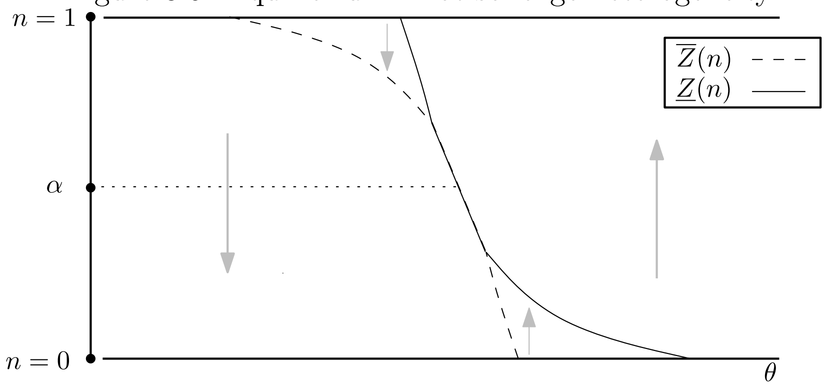
(ii) if $\underline{O}(n) < \overline{P}(n)\ \forall n$, then $\overline{Z}(n) = \underline{Z}(n)$ for all n in an interval containing α. Moreover, there are neighborhoods around 0 and 1 in which $\overline{Z}(n) < \underline{Z}(n)$.

Proof. See Appendix 3.B.3. □

The first part of Proposition 3.2 states that when there is a lot of heterogeneity so that there is no intersection between the regions in which each type does not have a dominant strategy, each type of agent will play according to a distinct threshold. Their equilibrium thresholds will never coincide, which is not surprising since that would require some agents to play a strictly dominated strategy.

The second part of Proposition 3.2 brings a surprising result: there is some conformity in agents' behavior as long as heterogeneity is not large enough to make it impossible for agents to play according to the same threshold for any (arbitrary) set of beliefs. Proposition 3.2 also states that different players will choose according to the same threshold for an intermediate range of n. Their thresholds will never fully coincide though: for extreme values of n, heterogeneity beats coordination and each type has a distinct threshold. Figure 3.5 exemplifies this result.

Figure 3.5: Equilibrium - not so large heterogeneity



In order to understand the result in Proposition 3.2, suppose agents play according to different thresholds as in Figure 3.3. For $n = \alpha$, an agent at the lower threshold (the one

at the left) holds the most pessimistic beliefs, n will surely decrease from then on. That is because all low-type agents will be choosing 0, and at $n = \alpha$ they are just enough to determine the path of the economy. Hence an agent will not choose action 1 unless it is dominant to do so. Conversely, an agent at the higher threshold (the one at the right) holds the most optimistic beliefs for exactly the same reason: high-type agents are choosing 1 and at $n = \alpha$ they are just enough to drive the economy up. Hence an agent will not choose 0 unless it is dominant to do so.

This reasoning implies that an equilibrium with two distinct thresholds at $n = \alpha$ requires (i) high-type agents being indifferent between either choice for some $\tilde{\theta}$ holding the most pessimistic beliefs; and (ii) low-type agents being indifferent between either choice for some $\theta > \tilde{\theta}$ holding the most optimistic beliefs. This can only happen in case of very large payoff heterogeneity. If that is not the case, owing to the large dispersion in beliefs offsetting idiosyncratic payoffs, both thresholds will coincide at $n = \alpha$.

This reasoning also explains why conformity fails to arise for extreme values of n. As shown in Figure 3.5, when $n = 0$, beliefs at both thresholds are not so different: both types know the economy will move up. The speed is not the same in both cases, but that is a minor difference in beliefs. Hence even a small difference in preferences leads to the existence of two distinct thresholds.

In sum, for intermediate values of n, there is huge heterogeneity in expectations about the path of n around the equilibrium threshold, which makes payoff heterogeneity less relevant. In contrast, for extreme values of n, there is less uncertainty about the path of n around the equilibrium thresholds and hence heterogeneity in preference matters for agents' optimal choice.

3.3.2 Vanishing frictions

We now consider the limiting case of $\delta \to \infty$ so that agents receive very frequent opportunities to revise their actions. Expressions for equilibrium thresholds under a general payoff function and vanishing frictions are provided in Appendix 3.A.2. Proposition 3.3 emphasizes some properties of the equilibrium when distinct types' flow payoffs differ by a constant.

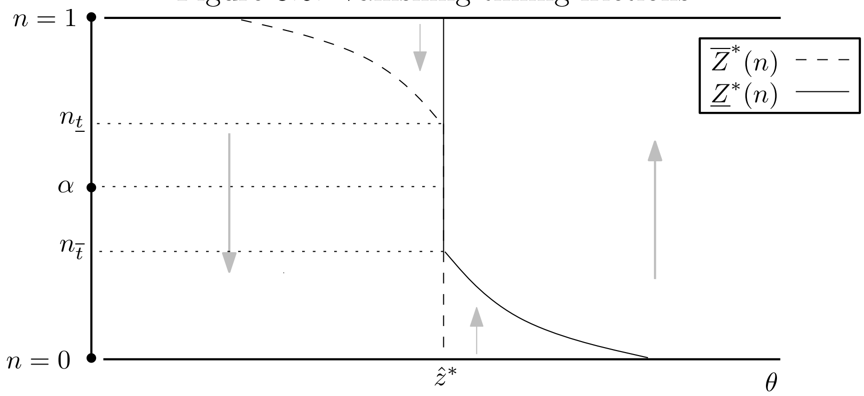
Proposition 3.3. *Let $\overline{\pi}(\theta, n) = \pi(\theta, n) + \overline{\varepsilon}$ and $\underline{\pi}(\theta, n) = \pi(\theta, n) + \underline{\varepsilon}$, where $\pi(.)$ is continuously differentiable and strictly increasing in both arguments and $\overline{\varepsilon} > \underline{\varepsilon}$. Define $\hat{\varepsilon} \equiv \alpha\overline{\varepsilon} + (1 - \alpha)\underline{\varepsilon}$ and $\overline{z}^*$, $\underline{z}^*$ and $\hat{z}^*$ as satisfying $\int_0^\alpha \pi(\overline{z}^*, n)dn = -\alpha\overline{\varepsilon}$, $\int_\alpha^1 \pi(\underline{z}^*, n)dn = -(1 - \alpha)\underline{\varepsilon}$ and $\int_0^1 \pi(\hat{z}^*, n)\, dn = -\hat{\varepsilon}$, respectively. In the limit as $\delta \to \infty$:*

(i) if $\underline{Q}(n) > \overline{P}(n)$ $\forall n$, the state space is divided in three regions: whenever $\theta_t > \underline{z}^*$, $n_t \approx 1$; whenever $\overline{z}^* < \theta_t < \underline{z}^*$, $n_t \approx \alpha$ and whenever $\theta_t < \overline{z}^*$, $n_t \approx 0$.

(ii) if $\underline{Q}(n) \leq \overline{P}(n)$ $\forall n$, the vertical line $\hat{z}^*$ divides the state space in two regions: whenever $\theta_t > \hat{z}^*$, $n_t \approx 1$ and whenever $\theta_t < \hat{z}^*$, $n_t \approx 0$.

Proof. See Appendix 3.B.4. □

Figure 3.6: Vanishing timing frictions



Proposition 3.3 states that in case of very large heterogeneity, at a given point in time, (almost) all agents of a given type will be playing the same action but different types might be playing different actions. The bounds of the region where behavior is heterogeneous (the switching point for each group) are determined by the value of θ such that, at $n = \alpha$: (i) high types with pessimistic beliefs are indifferent between either action; and (ii) low types with optimistic beliefs are indifferent between either action.

When heterogeneity is not so large, in the limiting case of vanishing frictions, the economy behaves as if agents were identical and had an intermediate preference parameter $\hat{\varepsilon}$. Although agents' strategies differ, whenever fundamentals cross the vertical division line, all agents of a certain type immediately switch actions, leading the opposite type to consider it profitable to switch actions as well. Since chances to switch arrive at a very large rate, the dynamics of the economy is basically the same as if agents were identical with preferences given by $\hat{\pi}(\theta, n) = \pi(\theta, n) + \hat{\varepsilon}$. Hence the economy behaves as in Frankel and Pauzner (2000). In the limiting case of vanishing frictions, two networks can only coexist if there is no set of (arbitrary) beliefs that would lead different agents to play according to the same threshold.

Figure 3.6 depicts the equilibrium in case of not so large heterogeneity. Note that agents' strategies differ for values of n close to 0 and 1. As explained before, that is because for high and low values of n, beliefs at both thresholds are not so different, so payoff heterogeneity

matters. This intuition is not affected when δ is large – agents at $n = 0$ at the right of $\hat{z}^*$ know n will be moving up fast, but also that they will quickly get another chance to choose.

3.4 Linear payoff function

We now present a linear example that provides intuition on the forces at play and helps us to understand the relative effects of payoff heterogeneity and complementarities in preferences. Let the relative flow-payoff of action 1 in comparison to action 0 be given by

$$\pi_i(\theta_t, n_t) = \theta_t + \gamma n_t + \varepsilon_i,$$

with

$$\varepsilon_i = \begin{cases} \overline{\varepsilon} & \forall i \in [0, \alpha], \\ \underline{\varepsilon} & \forall i \in (\alpha, 1], \end{cases}$$

that is, there are two types of agents: a proportion α with preference parameter $\overline{\varepsilon}$ and a proportion $1 - \alpha$ with preference parameter $\underline{\varepsilon}$, $\overline{\varepsilon} > \underline{\varepsilon}$.

As before, we will analyze the two limiting cases in which we can derive analytical results, starting by the case of slow-moving fundamentals.

3.4.1 Vanishing shocks

Consider again the case of $\mu, \sigma \to 0$. First, we compute the two dominance regions' boundaries that can be used to measure the degree of heterogeneity in agents' payoffs. Substituting our linear payoff function in equations (3.2) and (3.3) and solving the integrals yields the upper dominance region boundary of a high-type agent,

$$\overline{P}(n_0) = -\overline{\varepsilon} - \frac{\gamma(\rho + \delta)}{\rho + 2\delta} n_0, \tag{3.4}$$

and the lower dominance region boundary of a low-type agent,

$$\underline{O}(n_0) = -\underline{\varepsilon} - \frac{\gamma\delta}{\rho + 2\delta} - \frac{\gamma(\rho + \delta)}{\rho + 2\delta} n_0. \tag{3.5}$$

Large heterogeneity

The condition ensuring that $\overline{P}(n_0) < \underline{O}(n_0)\ \forall n_0$ is equivalent to

$$\overline{\varepsilon} - \underline{\varepsilon} > \frac{\gamma\delta}{\rho + 2\delta}. \tag{3.6}$$

If the difference between idiosyncratic preference parameters is large enough in comparison to the importance of strategic complementarities (γ), the curve on which a high-type agent with pessimistic beliefs about n is indifferent between 0 and 1 is located to the left of the curve on which a low-type agent with optimistic beliefs is indifferent between the two actions. The intersection between the region in which neither action is dominant for a high-type agent and the region with no dominant action for a low-type agent is empty (as in the top picture of Figure 3.4).

If condition (3.6) holds, then there is no set of beliefs that could induce different agents to play according to the same strategy for any value of n_0. Hence, whenever this condition is satisfied, the equilibrium in the limit as $\mu, \sigma \to 0$ will be such that type-$\overline{\varepsilon}$ and type-$\underline{\varepsilon}$ agents play according to thresholds that do not intersect, as stated in Proposition 3.2.

How can we analytically compute the thresholds in this case?[10] First note that for all $n \geq \alpha$ the high-type threshold coincides with the high-type upper dominance region. The belief a type-$\overline{\varepsilon}$ agent holds in equilibrium at some point (θ, n) with $n \geq \alpha$ is that n will fall at the maximum rate with probability one (see Figure 3.3). Under the most pessimistic belief, this agent is indifferent between the two actions. Thus, type-$\overline{\varepsilon}$ agents' threshold above α is a function $\overline{Z}(n_0) = \overline{P}(n_0)$, and thus satisfies equation (3.4). Likewise, for all $n \leq \alpha$ the low-type threshold must coincide with the low-type lower dominance region (in which agents hold the most optimistic belief), so for $n_0 \leq \alpha$, $\underline{O}(n_0)$ is given by equation (3.5).

What about the high-type threshold below α and the low-type threshold above α? Suppose, for now, that heterogeneity is large enough so that the following inequality holds:

$$\overline{\varepsilon} - \underline{\varepsilon} > \frac{\gamma(\delta + \rho\alpha)}{\rho + 2\delta}. \tag{3.7}$$

This condition ensures the equilibrium is such that $\overline{Z}(0) < \underline{Z}(\alpha)$, so that if the economy is initially at some point on $\overline{Z}(n_0)$ with $n_0 < \alpha$, it will never reach $n = 1$: it will either go down towards $n = 0$ or up towards $n = \alpha$. In other words, the system will never cross the other type's threshold. Graphically, it means that p_1 is to the left of p_2 in Figure 3.7. We can

[10] All the following derivations are equivalent to directly applying Proposition 3.4 in the Appendix 3.A.1 for the case of linear payoffs.

then compute the high-type equilibrium threshold below α as follows:

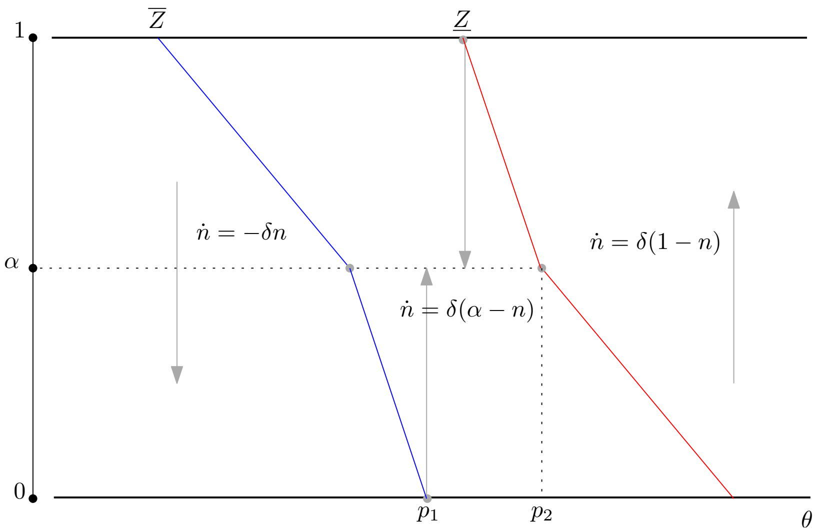
$$\underbrace{\frac{(\alpha - n_0)}{\alpha}}_{P(\text{up})} \int_0^\infty e^{-(\rho+\delta)t} \overline{\pi}(\overline{Z}, \underbrace{\alpha - (\alpha - n_0)e^{-\delta t}}_{n_t \text{ growing towards } \alpha}) dt + \underbrace{\frac{n_0}{\alpha}}_{P(\text{down})} \int_0^\infty e^{-(\rho+\delta)t} \overline{\pi}(\overline{Z}, \underbrace{n_0 e^{-\delta t}}_{n_t \text{ falling}}) dt = 0.$$

The first term of the sum is the probability of an upward bifurcation times the discounted relative payoff of action 1 when the agent expects n_t to grow until it approaches α. The second term is the probability of a downward bifurcation times the discounted payoff when the agent expects n_t to decrease towards zero. Substituting our linear functional form for $\overline{\pi}(.)$ and solving the integrals, we have that, whenever (3.7) holds, $\overline{Z}$ is given by

$$\overline{Z}(n_0) = \begin{cases} -\overline{\varepsilon} - \frac{\gamma(\rho+\delta)}{\rho+2\delta} n_0 & \text{if } \ n_0 \geq \alpha, \\ -\overline{\varepsilon} - \frac{\alpha\gamma\delta}{\rho+2\delta} - \frac{\gamma\rho}{\rho+2\delta} n_0 & \text{if } \ n_0 < \alpha. \end{cases} \tag{3.8}$$

Analogous expressions for the low-type equilibrium threshold are derived in Appendix 3.A.3. Figure 3.7 depicts the equilibrium in case $\overline{\varepsilon} - \underline{\varepsilon} > \frac{\gamma(\delta+\rho\max\{\alpha, 1-\alpha\}}{\rho+2\delta}$, that is, the case in which if the economy starts at one threshold, it will never cross the other one. [11]

Figure 3.7: Case $\overline{\varepsilon} - \underline{\varepsilon} \geq \frac{\gamma(\delta+\rho\max\{\alpha, 1-\alpha\}}{\rho+2\delta}$



Now, suppose heterogeneity is still large so that $\overline{P}$ is to the left of $\underline{Q}$, but not as large as before. Specifically, assume

$$\frac{\gamma\delta}{\rho+2\delta} < \overline{\varepsilon} - \underline{\varepsilon} \leq \frac{\gamma(\delta+\rho\alpha)}{\rho+2\delta}. \tag{3.9}$$

[11] This condition is analogous to $\overline{Z}_0 < \underline{Z}_\alpha$ and $\overline{Z}_\alpha < \underline{Z}_1$ in Proposition 3.4.

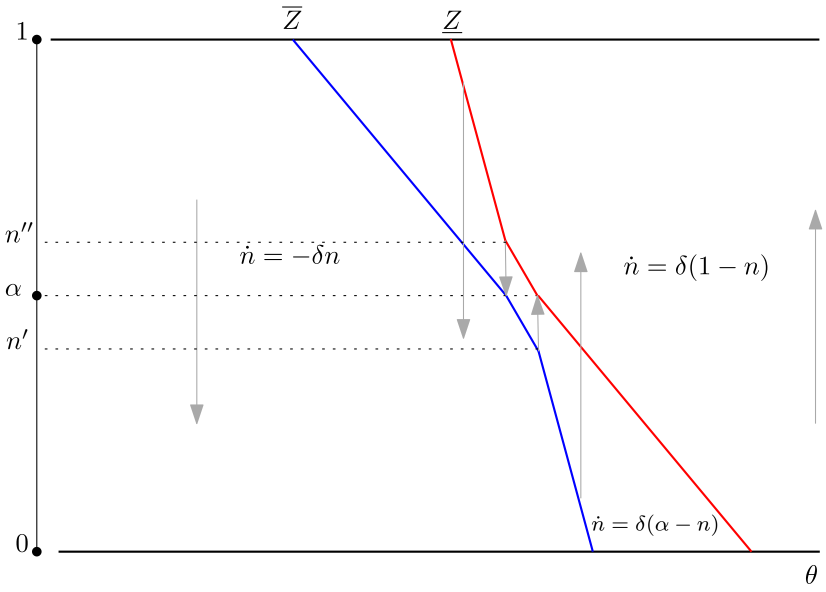
Define $n' \equiv \alpha - \frac{(\rho+2\delta)(\overline{\varepsilon}-\underline{\varepsilon})-\gamma\delta}{\gamma\rho}$.[12] The threshold $\overline{Z}$ is still given by equation (3.8) for all $n_0 \geq n'$, but for all $n_0 < n'$, it satisfies

$$\underbrace{\frac{(\alpha-n_0)}{\alpha}}_{P(\text{up})}\left\{\int_0^{\bar{t}} e^{-(\rho+\delta)t}\overline{\pi}(\overline{Z},\underbrace{\alpha-(\alpha-n_0)e^{-\delta t}}_{n_t \text{ growing towards } \alpha})dt + \int_{\bar{t}}^{\infty} e^{-(\rho+\delta)t}\overline{\pi}(\overline{Z},\underbrace{1-(1-n_{\bar{t}})e^{-\delta(t-\bar{t})}}_{n_t \text{ growing towards } 1})dt\right\}$$
$$\underbrace{\frac{n_0}{\alpha}}_{P(\text{down})}\int_0^{\infty} e^{-(\rho+\delta)t}\overline{\pi}(\overline{Z},\underbrace{n_0e^{-\delta t}}_{n_t \text{ falling}})dt = 0, \quad (3.10)$$

where $\bar{t}$ is the time at which the system reaches the other type's threshold, which is given by $\bar{t} = -\frac{1}{\delta}\ln\frac{\alpha-n_{\bar{t}}}{\alpha-n_0}$, and $n_{\bar{t}} = \underline{Z}^{-1}\left(\overline{Z}(n_0)\right) = -\left[(\rho+2\delta)\left(\overline{Z}+\underline{\varepsilon}\right)+\gamma\delta\right]/\left[(\rho+\delta)\gamma\right]$.

The expression in (3.10) can be better understood with the aid of Figure 3.8.[13] There is a range of n (n is sufficient low) such that a type-$\overline{\varepsilon}$ agent on her threshold knows that, if the system bifurcates up, it will cross the other type's threshold at some point, and thereafter n will grow at a higher rate. Then, given this more optimistic belief, the increase in the level of fundamentals an agent demands to be indifferent between the two actions for a given decrease in n_0 is smaller, i.e., the threshold is steeper.

Figure 3.8: Case $\frac{\gamma\delta}{\rho+2\delta} < \overline{\varepsilon}-\underline{\varepsilon} \leq \frac{\gamma\delta+\gamma\rho\min\{\alpha,1-\alpha\}}{\rho+2\delta}$



[12] The bound n' is the value satisfying $\overline{Z}(n') = \underline{Z}(\alpha)$.

[13] The second integral in equation (3.10) is equivalent to $\int_{\bar{t}}^{\infty} e^{-(\rho+\delta)t}(\overline{\varepsilon}-\underline{\varepsilon})dt$, because when the system arrives at $\underline{Z}$ and is expected to grow towards 1, low-type agents have zero payoff. A high-type agent holding the same belief will have a payoff larger by $\overline{\varepsilon}-\underline{\varepsilon}$ at all future dates. This substitution facilitates the algebra considerably.

Using the fact that $\underline{Z}(n) = \underline{O}(n)$ for all $n \le \alpha$ and performing a change of variables in equation (3.10), we find that, whenever (3.9) holds, $\forall n_0 < n'$, $\overline{Z}$ satisfies

$$(\rho+2\delta)(\overline{Z}+\overline{\varepsilon}) + \gamma\rho n_0 + \gamma\delta\alpha + \gamma\delta\frac{(1-\alpha)}{\alpha}\left(\frac{1}{\alpha-n_0}\right)^{\frac{\rho}{\delta}}\left[\alpha + \frac{(\overline{Z}+\underline{\varepsilon})(\rho+2\delta)+\gamma\delta}{\gamma(\rho+\delta)}\right]^{\frac{\rho+\delta}{\delta}} = 0. \tag{3.11}$$

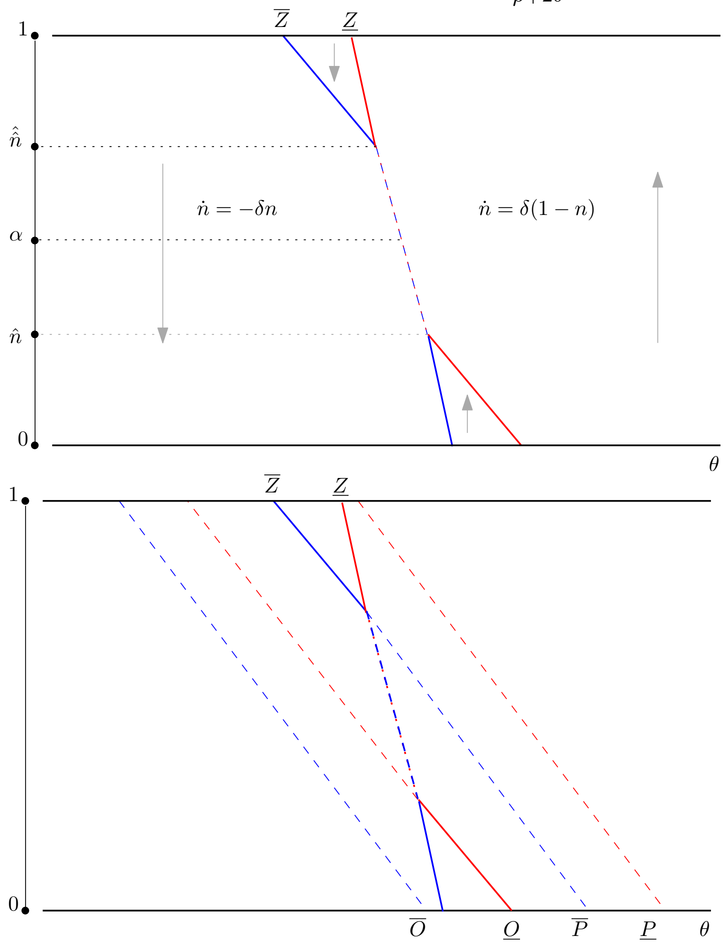
Not so large heterogeneity

Finally, consider the case in which $\overline{P}(n_0) \ge \underline{O}(n_0)\ \forall n_0$, which is equivalent to

$$\overline{\varepsilon} - \underline{\varepsilon} \le \frac{\gamma\delta}{\rho+2\delta}. \tag{3.12}$$

We know by Proposition 3.2 that different-type agents' strategies will coincide for some values of n, but never fully coincide. The equilibrium in this case is as depicted in Figure 3.9.

Figure 3.9: Case $\overline{\varepsilon} - \underline{\varepsilon} \le \frac{\gamma\delta}{\rho+2\delta}$



For all $n_0 \leq \hat{n}$, the type-$\overline{\varepsilon}$ threshold is identical to $\overline{P}(n_0)$, and for all $n_0 \geq \hat{\hat{n}}$, the type-$\overline{\varepsilon}$ threshold is given by (3.11). Analogous equations describing the low-type threshold are provided in Appendix 3.A.3.

In equilibrium, there is conformity in agents' strategies for intermediate values of n as long as the condition in (3.12) holds. In most applications, ρ (time discount rate) is much smaller than δ (frequency of opportunities to revise behavior), so the expression in (3.12) can be approximated by $\overline{\varepsilon} - \underline{\varepsilon} \leq \gamma/2$. In words, an equilibrium where agents play according to different thresholds requires payoff heterogeneity to be as important as an increase in n equal to half of the population. When (3.12) does not hold, there is no set of beliefs that would make conformity possible as $\overline{P}$ is to the left of $\underline{O}$.

The bounds $\hat{n}$ and $\hat{\hat{n}}$ in Figure 3.9 are given by

$$\hat{n} = \alpha \frac{(\overline{\varepsilon} - \underline{\varepsilon})(\rho + 2\delta)}{\gamma\delta}$$

and

$$\hat{\hat{n}} = 1 - (1 - \alpha)\frac{(\overline{\varepsilon} - \underline{\varepsilon})(\rho + 2\delta)}{\gamma\delta}.$$

In case $\alpha = 1/2$ and ρ is much smaller than δ, we get $\hat{n} \approx (\overline{\varepsilon} - \underline{\varepsilon})/\gamma$, thus $\hat{n}$ is approximately equal to the increase in n that would compensate the idiosyncratic difference in payoffs.

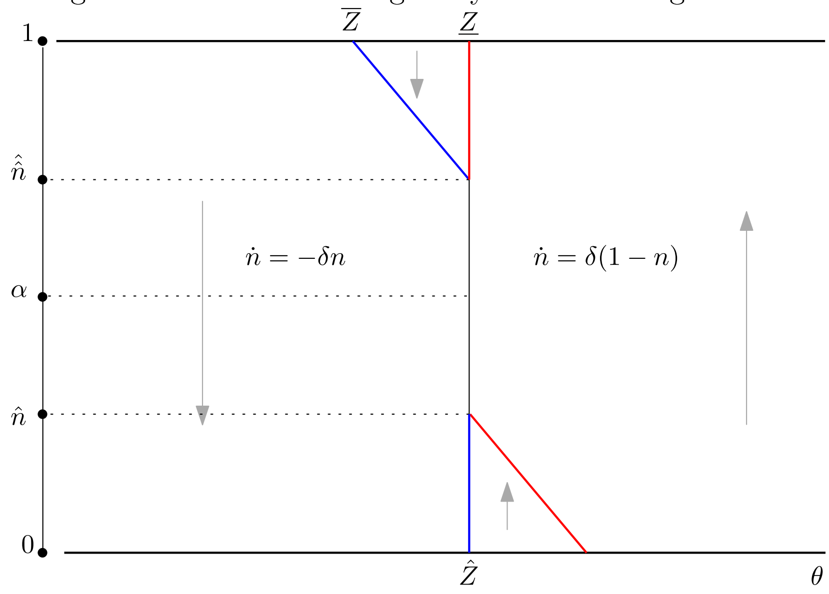
Intuitively, the existence of type-$\overline{\varepsilon}$ agents increases incentives for type-$\underline{\varepsilon}$ to choose action 1, while the existence of type-$\underline{\varepsilon}$ agents increases incentives for type-$\overline{\varepsilon}$ to opt for 0. In consequence, agents behave in a more similar way. That is particularly true when n is in an intermediate range so that the path of the economy will be decided by the actions of both groups.

3.4.2 Vanishing frictions

In case $\delta \to \infty$, we can fully characterize the equilibrium threshold. Our linear payoff function satisfies the assumptions in Proposition 3.3. In order to compute the equilibrium in the limit case of vanishing timing frictions (even if $\mu, \sigma > 0$), it suffices to apply the results in Proposition 3.5, or equivalently, take the limit as $\delta \to \infty$ of all equilibrium thresholds computed in the previous subsection. The more interesting case is when $\underline{O}$ is to the left of $\overline{P}$, which is equivalent to $\overline{\varepsilon} - \underline{\varepsilon} \leq \gamma/2$. Equilibrium is as in Figure 3.10. The line $\hat{Z}$ divides the state space in two regions: whenever $\theta_t > \hat{Z}$, $n_t \approx 1$, and whenever $\theta_t < \hat{Z}$, $n_t \approx 0$. $\hat{Z}$ coincides with the equilibrium that would be played if there was just one type of agent in the economy with preference parameter $\hat{\varepsilon} \equiv \alpha\overline{\varepsilon} + (1 - \alpha)\underline{\varepsilon}$. $\hat{Z}$ satisfies

$$\hat{Z} = -\hat{\varepsilon} - \gamma/2.$$

Figure 3.10: Small heterogeneity and vanishing frictions



The increase in n when θ crosses to the right of $\hat{Z}$ at $n \approx 0$ is triggered by high-type agents choosing action 1, as low-type agents initially keep choosing 0. However, this difference in behavior will be very short lived. One implication of this result is that an increase in the mass of high-type agents (α) will affect the behavior of everyone in the economy (it will shift the threshold to the left) but there will be virtually no difference in the behavior of low-type and high-type agents.

3.5 The planner's problem

We now solve the planner's problem for the case of linear payoffs in order to analyze efficiency in this environment. All results in this section refer to the case of very small shocks, $\mu, \sigma \to 0$.

In order to solve the planner's problem, we need to specify the flow utilities of each option, and not just the difference in payoffs. Consider that the flow utility agent i derives from being committed to action 1 is given by $u_i^1(\theta_t^1, n_t) = \theta_t^1 + \nu^1 n_t + \varepsilon_i^1$, and the flow utility from being at 0 is given by $u_i^0(\theta_t^0, n_t) = \theta_t^0 + \nu^0(1 - n_t) + \varepsilon_i^0$. n_t is the mass of agents currently playing 1, $\nu^j > 0$ is a parameter measuring the relative importance of strategic complementarities in the choice of j, θ_t^j represents the fundamentals affecting the flow-payoff of playing j at time t, and ε_i^j captures an idiosyncratic preference for action j, $j \in \{0, 1\}$. θ_t^j follows a Brownian motion with drift μ_j and variance σ_j^2. Defining $\theta_t \equiv \theta_t^1 - \theta_t^0 - \nu^0$, $\gamma \equiv \nu^1 + \nu^0$ and $\varepsilon_i \equiv \varepsilon_i^1 - \varepsilon_i^0$, we can write the relative payoff function as before: $\pi(\theta_t, n_t) = \theta_t + \gamma n_t + \varepsilon_i$. θ_t follows a

Brownian motion with drift $\mu = \mu^1 - \mu^0$ and variance $\sigma^2 = \sigma_0^2 + \sigma_1^2$.

We will refer to options 0 and 1 as networks, since the measure of agents playing each action generates externalities that can be thought of as network effects.

Consider the case $\nu^0 = \nu^1 = \nu$. At every point in time, the planner chooses the proportion of high- and low-type agents with an opportunity to revise their actions that will opt for network 1 in order to maximize aggregate welfare. Denote by $\overline{\phi}_t$ and $\underline{\phi}_t$ these proportions, respectively. At time 0, the planner maximizes the discounted sum of utilities across agents, i.e.,

$$\mathbb{E}\,\alpha \int_0^\infty e^{-\rho t} \left\{\overline{n_t}\left[{\theta_t}^1 + \nu n_t + \overline{\varepsilon}^1\right] + (1-\overline{n_t})\left[{\theta_t}^0 + \nu(1-n_t) + \overline{\varepsilon}^0\right]\right\} dt$$
$$+ (1-\alpha)\int_0^\infty e^{-\rho t}\left\{\underline{n}_t\left[{\theta_t}^1 + \nu n_t + \underline{\varepsilon}^1\right] + (1-\underline{n}_t)\left[{\theta_t}^0 + \nu(1-n_t) + \underline{\varepsilon}^0\right]\right\} dt,$$

which is equivalent to maximizing

$$\mathbb{E}\int_0^\infty e^{-\rho t}\left\{n_t\left[\theta_t - \frac{\gamma}{2} + \gamma n_t)\right] + \alpha\overline{n_t\varepsilon} + (1-\alpha)\underline{n_t\varepsilon}\right\} dt, \tag{3.13}$$

where, by definition, $n_t = \alpha\overline{n}_t + (1-\alpha)\underline{n}_t$.

Suppose the planner's optimal choice at $t = 0$ is $\left\{\overline{\phi}_0, \underline{\phi}_0\right\}$, with $\overline{\phi}_0 \in [0, 1)$. Consider a deviation in which the planner chooses $\overline{\phi}_0 = 1$ and keeps $\underline{\phi}_0$ and all future choices of $\overline{\phi}_t$ and $\underline{\phi}_t$ unchanged, for any realization of the Brownian path. Such deviation implies an infinitesimal increase in $\overline{n}_0$ by $d\overline{n}_0$, and its effect on future values of $\overline{n}_t$ is given by $d\overline{n}_t = d\overline{n}_0 e^{-\delta t}$, since the initial increase in $\overline{n}_0$ depreciates at a rate δ. A necessary condition for optimality of the planner's choice is that the deviation just described is not profitable. Using (3.13), it means that if $\overline{\phi}_0 \in [0, 1)$ is optimal, it cannot be the case that

$$\mathbb{E}\int_0^\infty \frac{\partial e^{-\rho t}\left\{n_t\left[\theta_t - \frac{\gamma}{2} + \gamma n_t)\right] + \alpha\overline{n_t\varepsilon} + (1-\alpha)\underline{n_t\varepsilon}\right\}}{\partial \overline{n}_t}\frac{d\overline{n}_t}{d\overline{n}_0} dt > 0,$$

which can be written as

$$\mathbb{E}\int_0^\infty e^{-(\rho+\delta)t}\left(\theta_t - \frac{\gamma}{2} + 2\gamma n_t + \overline{\varepsilon}\right) dt > 0. \tag{3.14}$$

Hence, if the condition in (3.14) holds, it must be optimal for the planner to set $\overline{\phi}_0 = 1$, that is, the planner recommends action 1 for all high-type agents with a revision opportunity in hand. If the inequality in (3.14) is reversed, action 0 is optimal for high-type agents at $t = 0$. An analogous reasoning implies that whenever

$$\mathbb{E}\int_0^\infty e^{-(\rho+\delta)t}\left(\theta_t - \frac{\gamma}{2} + 2\gamma n_t + \underline{\varepsilon}\right)dt > 0, \tag{3.15}$$

action 1 is optimal for low-type agents, and if the inequality is reversed, action 0 is optimal for low-type agents, at $t = 0$. These expressions are similar to the indifference condition in the decentralized equilibrium: an agent is indifferent between 0 and 1 when

$$\mathbb{E}\int_{t=0}^\infty e^{(\rho+\delta)t}\left[\theta + \gamma n_t + \varepsilon_i\right]dt = 0,$$

$\varepsilon_i \in \{\overline{\varepsilon}, \underline{\varepsilon}\}$. One important difference is that the externality is more important for the planner (γ is multiplied by 2). Intuitively, the planner takes into account the externality on others that agents fail to internalize, while the intrinsic quality of each option and idiosyncratic tastes are fully taken into account by agents in the decentralized equilibrium.

Mathematically, the planner's problem and the one solved by agents in the decentralized equilibrium are very similar: at every time t, the planner chooses according to the conditions in (3.14) and (3.15), knowing that its future selves will act optimally at all future dates, so the path of n must be consistent with optimality.[14] Hence the planner's problem is isomorphic to a game played by agents, only with different payoffs.

Proposition 3.6 in Appendix 3.A.4 characterizes the planner's solution analytically. The main result in this section is that the region in which the planner prescribes the same strategy for different types is always larger than in the decentralized equilibrium. Hence, from the planner's point of view, there is not enough conformity. That is because the planner internalizes the externalities from agents' choices and thus puts a higher weight on coordination.[15] Moreover, the planner's threshold is always flatter, meaning that the planner sacrifices gains stemming from good fundamentals in order to explore strategic complementarities.[16]

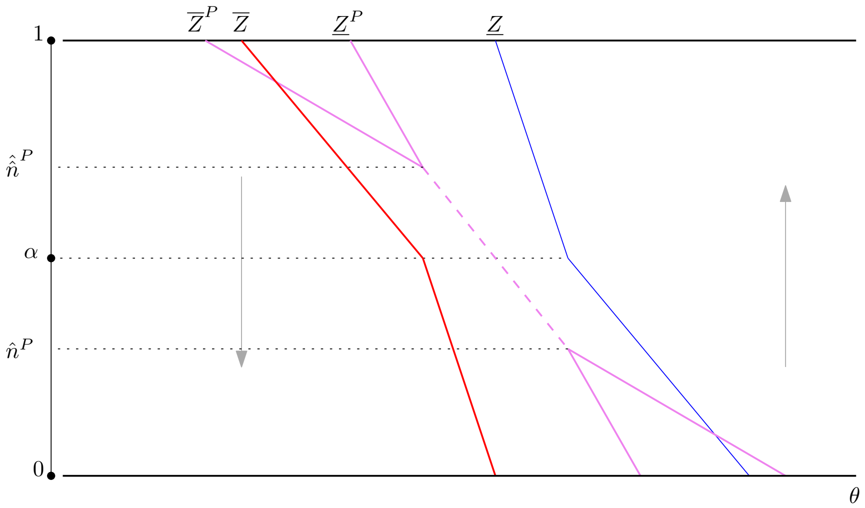
Figure 3.11 considers a case with large heterogeneity. In the decentralized equilibrium, for some values of θ, two networks will coexist for long periods of time, with some agents choosing 1 and others going for 0. However, the efficient outcome would feature a single network (except for brief transition periods). The strategies prescribed by the planner imply that n would almost always be very close to 0 or 1.

[14] There are no commitment concerns here since the planner dictates everyone's actions and preferences exhibit no time inconsistency.

[15] A similar reasoning implies that in equilibrium, networks are 'too balanced' in the static model of Argenziano (2008).

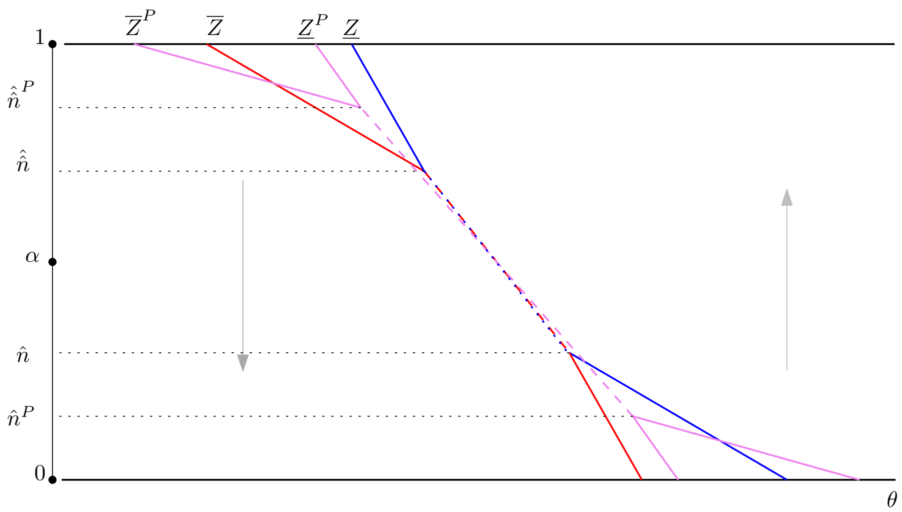
[16] This effect is unrelated to heterogeneity and is analyzed in Guimaraes and Pereira (2016). Intuitively, agents do not internalize the effects of their choices on others' payoffs, hence they give relatively more importance to the intrinsic quality of each option.

Figure 3.11: Planner's solution when $\frac{\gamma(\delta+\rho\max\{\alpha,(1-\alpha)\})}{\rho+2\delta} < \bar{\varepsilon} - \underline{\varepsilon} \leq \frac{2\gamma\delta}{\rho+2\delta}$



In the example in Figure 3.12, heterogeneity is not so large and the equilibrium threshold of both types coincide for some values of n. In this case, the range of values of n for which agents choose different actions is (exactly) twice as large as the analogous range for the planner.

Figure 3.12: Planner's solution when $\bar{\varepsilon} - \underline{\varepsilon} \leq \frac{\gamma\delta}{\rho+2\delta}$



So far, we have focused on the case $\nu^0 = \nu^1$. However, the main results of this section hold when the network effect is asymmetric, that is, $\nu^0 \neq \nu^1$. In this case, the planner also shifts the threshold in order to enlarge the region in which agents choose the action that generates more externalities. But a parallel shift of the threshold is the only difference.[17]

[17]The characterization of the planner's solution under asymmetric network effects follows the same steps

3.6 Final remarks

This paper shows that in a dynamic coordination model with timing frictions, heterogeneous agents will often play similar strategies. Agents predisposed to a certain action will be less willing to take it anticipating the behavior of agents less inclined to choose that action, and vice versa. That is particularly true when there is an intermediate number of people in a network, in which case there is more uncertainty about the path of the economy and coordination motives dominate idiosyncratic tastes.

Network effects are key for internet companies, for example. Therefore, according to this paper, we should expect a lot of conformity in people's choices, hence a lot of concentration, but occasional large (positive or negative) shifts in the market share of these firms – which seems consistent with the stylized facts. Future research might build on this model to quantitatively analyze this kind of dynamic process.

3.A Expressions for the equilibrium thresholds

3.A.1 Vanishing shocks

Characterization of equilibrium in the limiting case of vanishing shocks is summarized in Proposition 3.4. Define $\overline{Z}_0$, $\underline{Z}_\alpha$, $\overline{Z}_\alpha$ and $\underline{Z}_1$ as satisfying, respectively,

$$\int_0^\alpha (\alpha - n)^{\frac{\varrho}{\delta}} \,\overline{\pi}\left(\overline{Z}_0, n\right) dn = 0, \tag{3.16}$$

$$\int_\alpha^1 (1 - n)^{\frac{\varrho}{\delta}} \underline{\pi}\left(\underline{Z}_\alpha, n\right) dn = 0, \tag{3.17}$$

$$\int_0^\alpha n^{\frac{\varrho}{\delta}} \overline{\pi}\left(\overline{Z}_\alpha, n\right) dn = 0 \tag{3.18}$$

and

$$\int_\alpha^1 (n - \alpha)^{\frac{\varrho}{\delta}} \underline{\pi}\left(\underline{Z}_1, n\right) dn = 0. \tag{3.19}$$

Proposition 3.4. *In the limiting case in which $\mu, \sigma \to 0$, thresholds are computed as follows:*

1. *(Large heterogeneity) Case $\overline{P}(n) < \underline{O}(n) \forall n$*

   (a) *Type-$\overline{q}$ agents' threshold:*

      i. *If $\overline{Z}_0 < \underline{Z}_\alpha$, then*

presented in this section and is available upon request.

- $\forall n_0 \geq \alpha$, $\overline{Z}(n_0) = \overline{P}(n_0)$ *and satisfies*

$$\int_0^{n_0} n^{\frac{\rho}{\delta}} \overline{\pi}\left(\overline{Z}, n\right) dn = 0; \tag{3.20}$$

- $\forall n_0 < \alpha$, $\overline{Z}(n_0)$ *satisfies*

$$\int_0^{n_0} \left(\frac{n}{n_0}\right)^{\frac{\rho}{\delta}} \overline{\pi}(\overline{Z}, n) dn + \int_{n_0}^{\alpha} \left(\frac{\alpha - n}{\alpha - n_0}\right)^{\frac{\rho}{\delta}} \overline{\pi}(\overline{Z}, n) dn = 0. \tag{3.21}$$

ii. If $\overline{Z}_0 \geq \underline{Z}_\alpha$, *then*

- $\forall n_0 \geq \alpha$, $\overline{Z}(n_0) = \overline{P}(n_0)$ *and satisfies (3.20);*
- $\forall n_0 \in (n', \alpha)$, $\overline{Z}(n_0)$ *satisfies (3.21);*
- $\forall n_0 \leq n'$, $\overline{Z}(n_0)$ *is the solution to the system*

$$\begin{aligned}\int_0^{n_0} \left(\frac{n}{n_0}\right)^{\frac{\rho}{\delta}} \overline{\pi}\left(\overline{Z}, n\right) dn + \int_{n_0}^{n_{\bar{t}}} \left(\frac{\alpha - n}{\alpha - n_0}\right)^{\frac{\rho}{\delta}} \overline{\pi}\left(\overline{Z}, n\right) dn \\ + \left(\frac{\alpha - n_{\bar{t}}}{1 - n_{\bar{t}}}\right)^{\frac{\rho+\delta}{\delta}} \int_{n_{\bar{t}}}^{1} \left(\frac{1 - n}{\alpha - n_0}\right)^{\frac{\rho}{\delta}} \overline{\pi}\left(\overline{Z}, n\right) dn = 0,\end{aligned} \tag{3.22}$$

$$\int_{n_{\bar{t}}}^{1} (1-n)^{\frac{\rho}{\delta}} \underline{\pi}(\overline{Z}, n) dn = 0;$$

- n' *is the value satisfying* $\overline{Z}(n') = \underline{Z}(\alpha)$.

(b) Type-$\underline{q}$ *agents' threshold:*

i. If $\overline{Z}_\alpha < \underline{Z}_1$, *then*

- $\forall n_0 \leq \alpha$, $\underline{Z}(n_0) = \underline{O}(n_0)$ *and satisfies*

$$\int_{n_0}^{1} (1-n)^{\frac{\rho}{\delta}} \underline{\pi}\left(\underline{Z}, n\right) dn = 0; \tag{3.23}$$

- $\forall n_0 > \alpha$, $\underline{Z}(n_0)$ *satisfies*

$$\int_{\alpha}^{n_0} \left(\frac{n - \alpha}{n_0 - \alpha}\right)^{\frac{\rho}{\delta}} \underline{\pi}\left(\underline{Z}, n\right) dn + \int_{n_0}^{1} \left(\frac{1 - n}{1 - n_0}\right)^{\frac{\rho}{\delta}} \underline{\pi}\left(\underline{Z}, n\right) dn = 0. \tag{3.24}$$

ii. If $\overline{Z}_\alpha \geq \underline{Z}_1$, *then*

- $\forall n_0 \leq \alpha$, $\underline{Z}(n_0) = \underline{O}(n_0)$ *and satisfies (3.23);*
- $\forall n_0 \in (\alpha, n'')$, $\underline{Z}(n_0)$ *satisfies (3.24);*

- $\forall n_0 \geq n''$, $\underline{Z}(n_0)$ *is the solution to the system*

$$\int_0^{n_{\underline{t}}} \left(\frac{n}{n_0-\alpha}\right)^{\frac{\rho}{\delta}} \left(\frac{n_{\underline{t}}-\alpha}{n_{\underline{t}}}\right)^{\frac{\rho+\delta}{\delta}} \underline{\pi}\left(\underline{Z}, n\right) dn + \int_{n_{\underline{t}}}^{n_0} \left(\frac{n-\alpha}{n_0-\alpha}\right)^{\frac{\rho}{\delta}} \underline{\pi}\left(\underline{Z}, n\right) dn + \int_{n_0}^{1} \left(\frac{1-n}{1-n_0}\right)^{\frac{\rho}{\delta}} \underline{\pi}\left(\underline{Z}, n\right) dn = 0, \quad (3.25)$$

$$\int_0^{n_{\underline{t}}} n^{\frac{\rho}{\delta}} \overline{\pi}(\underline{Z}, n) dn = 0;$$

- n'' *is the value satisfying* $\underline{Z}(n'') = \overline{Z}(\alpha)$.

2. *(Not so large heterogeneity) Case* $\overline{P}(n) \geq \underline{O}(n) \forall n$

   (a) *Type-$\overline{q}$ agents' threshold: For each n_0 such that $\overline{Z}(n_0) \neq \underline{Z}(n_0)$,*
   - *if $n_0 > \alpha$, $\overline{Z}(n_0) = \overline{P}(n_0)$ and satisfies (3.20);*
   - *if $n_0 < \alpha$, $\overline{Z}(n_0)$ is the solution to (3.22).*

   (b) *Type-$\underline{q}$ agents' threshold: For each n_0 such that $\underline{Z}(n_0) \neq \overline{Z}(n_0)$,*
   - *if $n_0 > \alpha$, $\underline{Z}(n_0)$ is the solution to (3.25);*
   - *if $n_0 < \alpha$, $\underline{Z}(n_0) = \underline{O}(n_0)$ and satisfies (3.23).*

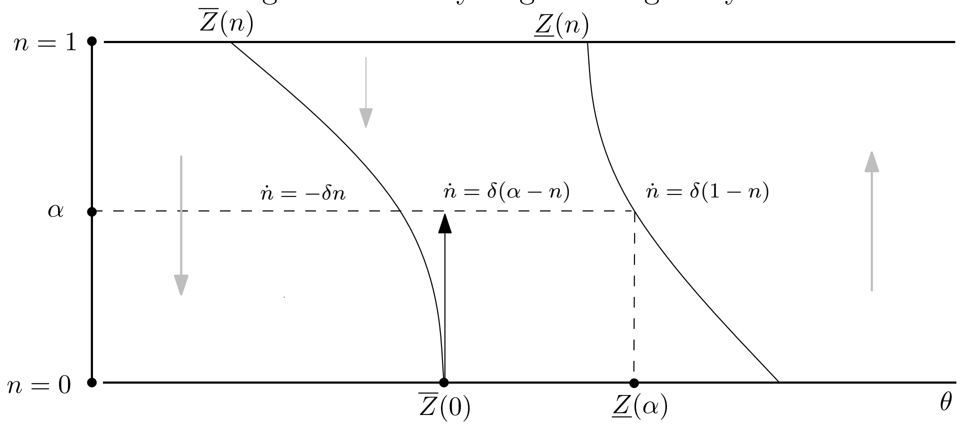
Proof. Suppose $\mu, \sigma \to 0$.

Large heterogeneity Suppose $\overline{P}(n) < \underline{O}(n) \forall n$. By Proposition 3.2, we know that $\overline{Z}(n) < \underline{Z}(n) \forall n$. So, we can apply Lemma 3.2 to compute the bifurcation probabilities at all points along the thresholds. For all $n_0 \geq \alpha$, a type-$\overline{q}$ agent's belief over n_t in equilibrium is exactly the belief we assume to compute her upper dominance region boundary, i.e., she assigns probability one to n going down at the maximum rate, $\dot{n}_t = -\delta n_t$. Thus, type-$\overline{q}$ agents' threshold $\forall n_0 \geq \alpha$ is given by equation (3.2). Performing a change of variables such that $n = n_t^{\downarrow} = n_0 e^{-\delta t}$, we obtain equation (3.20) in 1.(a)i. Likewise, for all $n_0 \leq \alpha$, a type-$\underline{q}$ agent's belief over n_t in equilibrium is the more optimistic as possible, so her threshold is given by equation (3.3) $\forall n \leq \alpha$. A change of variables such that $n = n_t^{\uparrow} = 1 - (1-n_0)e^{-\delta t}$ gives us equation (3.23) in 1.(b)i.

We still have to compute type-$\overline{q}$ threshold below α and type-$\underline{q}$ threshold above α in the case of large heterogeneity ($\overline{P}$ to the left of $\underline{O}$). Consider a high-type agent. Lets assume, for now, that the equilibrium is such that the distance between the thresholds of the two types

of agents is big enough so that $\overline{Z}(0) < \underline{Z}(\alpha)$ as in Figure 3.13. We will show that this is the case whenever $\overline{Z}_0 < \underline{Z}_\alpha$.

Figure 3.13: Very large heterogeneity



Notice that at any point on $\overline{Z}$ below α, if the system bifurcates up, n_t will grow towards α and it will never reach the low-type threshold. Consider an agent $i \in [0, \alpha]$ at some point (θ_0, n_0) with $\theta_0 = \overline{Z}(n_0)$ and $n_0 < \alpha$, i.e., at some point on her threshold below α. Equating her expected payoff to zero, we have

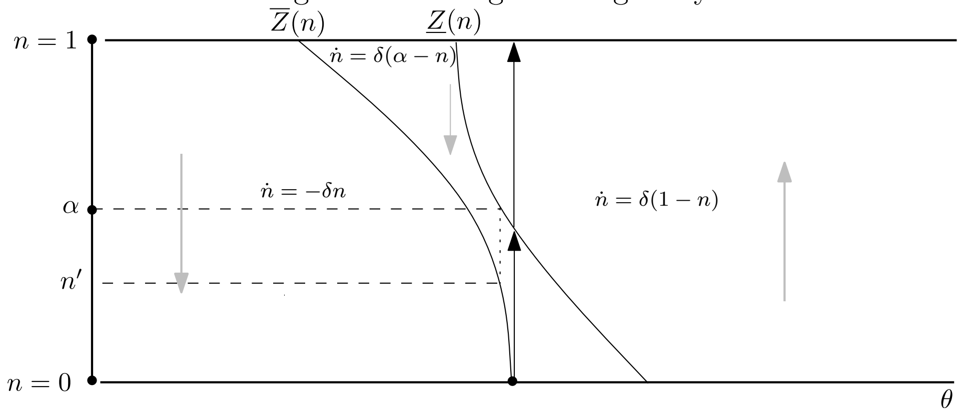
$$\underbrace{\frac{(\alpha - n_0)}{\alpha}}_{P(\text{up})} \int_0^\infty e^{-(\rho+\delta)t} \pi(\overline{Z}, \underbrace{\alpha - (\alpha - n_0)e^{-\delta t}}_{n_t \text{ growing towards } \alpha})dt + \underbrace{\frac{n_0}{\alpha}}_{P(\text{down})} \int_0^\infty e^{-(\rho+\delta)t} \pi(\overline{Z}, \underbrace{n_0 e^{-\delta t}}_{n_t \text{ falling}})dt = 0. \tag{3.26}$$

The first term of the sum is the probability of an upward bifurcation times the discounted payoff when the agent expects n_t to grow until it approaches α. The second one is the probability of a downward bifurcation times the discounted payoff when the agent expects n_t to decrease towards zero. Integrating by substitution the two terms in the equation above (letting $n = \alpha - (\alpha - n_0)e^{-\delta t}$ in the first and $n = n_0 e^{-\delta t}$ in the second integral), we have equation (3.21) in 1.(a)i. Remember we have computed this threshold assuming $\overline{Z}(0) < \underline{Z}(\alpha)$, which is only the case when the value of $\overline{Z}(0)$ obtained using the expression above is smaller than $\underline{Z}(\alpha)$. Evaluating the equation above at $n_0 = 0$ and operating a change of variables such that $n = \alpha - \alpha e^{-\delta t}$, we have the expression for $\overline{Z}_0$ in equation (3.16). Thus, whenever $\overline{Z}_0 < \underline{Z}_\alpha$, where $\underline{Z}_\alpha \equiv \underline{Z}(\alpha)$, the high-type threshold is in fact given by equation (3.21) for all $n < \alpha$.

Now, assume instead that $\overline{Z}_0 \geq \underline{Z}_\alpha$. In that case, a high-type agent making a choice at some point on her threshold needs to take into account that, depending on the initial state (θ_0, n_0), the system may bifurcate up but not only towards $n_t = \alpha$. For some (low) values of

n_0, following an upward bifurcation, n_t will grow at a lower rate (all high-type agents play 1 but all low types play 0) until the system crosses the low-type threshold, and thereafter everyone who gets the chance to revise theirs actions will choose 1. Figure 3.14 illustrates this case.

Figure 3.14: Large heterogeneity



Define n' as satisfying $\overline{Z}(n') = \underline{Z}(\alpha)$. For all $n_0 \in (n', \alpha)$, $\overline{Z}(n_0)$ is still given by equation (3.21). But consider now an initial point (θ_0, n_0) with $\theta_0 = \overline{Z}(n_0)$ and $n_0 \le n'$. In this case, the high-type threshold can be computed as

$$\underbrace{\frac{(\alpha - n_0)}{\alpha}}_{P(\text{up})} \left\{ \int_0^{\bar{t}} e^{-(\rho+\delta)t} \overline{\pi}(\overline{Z}, \underbrace{\alpha - (\alpha - n_0)e^{-\delta t}}_{n_t \text{ growing towards } \alpha}) dt + \int_{\bar{t}}^{\infty} e^{-(\rho+\delta)t} \overline{\pi}(\overline{Z}. \underbrace{1 - (1 - n_{\bar{t}})e^{-\delta(t-\bar{t})}}_{n_t \text{ growing towards } 1}) dt \right\}$$
$$\underbrace{\frac{n_0}{\alpha}}_{P(\text{down})} \int_0^{\infty} e^{-(\rho+\delta)t} \overline{\pi}(\overline{Z}, \underbrace{n_0 e^{-\delta t}}_{n_t \text{ falling}}) dt = 0, \quad (3.27)$$

where $\bar{t}$ denote the time at which the economy reaches $\underline{Z}$ in case an upward bifurcation occurs and $n_{\bar{t}} = \underline{Z}^{-1}(\overline{Z}(n_0))$. Since $n_{\bar{t}} = \alpha - (\alpha - n_0)e^{-\delta \bar{t}}$, we have that $\bar{t} = -\frac{1}{\delta} \ln \frac{\alpha - n_{\bar{t}}}{\alpha - n_0}$. Performing a change of variables in each one of the integrals in the equation above, we get to the first line of the system in (3.22). The second line is equivalent to $n_{\bar{t}} = \underline{Z}^{-1}(\overline{Z}(n_0))$, using the fact that, for all $n \le \alpha$, $\underline{Z}$ is given by equation (3.23). The low-type threshold above α when either $\overline{Z}_\alpha < \underline{Z}_1$ or $\overline{Z}_\alpha \ge \underline{Z}_1$ is computed following analogous steps.

Small heterogeneity Now, suppose $\overline{P}(n) \ge \underline{O}(n) \forall n$. By Proposition 3.2, we know that there is a neighborhood of α such that agents play according to the same strategy $Z(n)$ whenever n is in this neighborhood, but thresholds never coincide for all $n \in [0, 1]$. Parts of thresholds that coincide cannot be analytically computed, since Lemma 3.2 cannot be used

to pin down agents' beliefs. Yet, for every value of n such that $\overline{Z}(n) \neq \underline{Z}(n)$, it is possible to compute the thresholds using the bifurcation probabilities in Lemma 3.2, as in the case of large heterogeneity. If $n_0 \geq \alpha$, $\overline{Z}(n_0) = \overline{P}(n_0)$ and thus satisfies (3.20); $\underline{Z}(n_0)$ is the solution to (3.25), given that whenever the system bifurcates down, it eventually crosses $\overline{Z}$ and decreases towards $n = 0$. If $n_0 \leq \alpha$, $\overline{Z}(n_0)$ is the solution to (3.22) and $\underline{Z}(n_0) = \underline{O}(n_0)$ and thus satisfies (3.23). Computing the interval(s) of n such that thresholds coincide requires knowing the specific functional form of payoffs (see linear example.)

□

3.A.2 Vanishing frictions

The next proposition characterize the equilibrium in the limit as timing frictions shrink.

Proposition 3.5. *In the limit as frictions vanish ($\delta \to \infty$), the equilibrium is characterized by thresholds $\overline{Z}^*(n_0)$ and $\underline{Z}^*(n_0)$ computed as follows. The dominance regions' boundaries of interest now satisfy*

$$\int_0^{n_0} \overline{\pi}(\overline{P}^*, n)dn = 0$$

and

$$\int_{n_0}^1 \underline{\pi}(\underline{O}^*, n)dn = 0.$$

1. *(Large heterogeneity) Case $\overline{P}^*(n) < \underline{O}^*(n) \forall n$*

   (a) *Type-$\overline{q}$ agents' threshold:*

   - *$\forall n_0 \geq \alpha$, $\overline{Z}^*(n_0)$ satisfies*

   $$\int_0^{n_0} \overline{\pi}\left(\overline{Z}^*, n\right) dn = 0; \tag{3.28}$$

   - *$\forall n < \alpha$, $\overline{Z}^*(n_0)$ satisfies*

   $$\int_0^{\alpha} \overline{\pi}(\overline{Z}^*, n)dn = 0, \tag{3.29}$$

   which is independent of n_0.

   (b) *Type-$\underline{q}$ agents' threshold:*

   - *$\forall n_0 \leq \alpha$, $\underline{Z}^*(n_0)$ satisfies*

   $$\int_{n_0}^1 \underline{\pi}\left(\underline{Z}^*, n\right) dn = 0; \tag{3.30}$$

- $\forall n_0 > \alpha$, $\underline{Z}^*(n_0)$ *satisfies*

$$\int_{\alpha}^{1} \underline{\pi}\left(\underline{Z}^*, n\right) dn = 0, \tag{3.31}$$

which is independent of n_0.

2. *(Not so large heterogeneity) Case* $\overline{P}^*(n) \geq \underline{O}^*(n) \forall n$

   (a) *Type-*$\overline{q}$ *agents' threshold: For each* n_0 *such that* $\overline{Z}^*(n_0) \neq \underline{Z}^*(n_0)$,

   - *if* $n_0 > \alpha$, $\overline{Z}^*(n_0) = \overline{P}^*(n_0)$ *and satisfies (3.28);*
   - *if* $n_0 < \alpha$, $\overline{Z}^*(n_0)$ *is the solution to the system*

$$\int_{0}^{n_{\bar{t}}} \overline{\pi}\left(\overline{Z}^*, n\right) dn + \left(\frac{\alpha - n_{\bar{t}}}{1 - n_{\bar{t}}}\right)^{\frac{\rho+\delta}{\delta}} \int_{n_{\bar{t}}}^{1} \overline{\pi}\left(\overline{Z}^*, n\right) dn = 0, \tag{3.32}$$

$$\int_{n_{\bar{t}}}^{1} \underline{\pi}(\overline{Z}^*, n) dn = 0.$$

   Notice that $\overline{Z}^*$ *is a vertical line for* $n_0 < \alpha$.

   (b) *Type-*$\underline{q}$ *agents' threshold: For each* n_0 *such that* $\underline{Z}^*(n_0) \neq \overline{Z}^*(n_0)$,

   - *if* $n_0 < \alpha$, $\underline{Z}^*(n_0) = \underline{O}^*(n_0)$ *and satisfies (3.30);*
   - *if* $n_0 > \alpha$, $\underline{Z}^*(n_0)$ *is the solution to the system*

$$\left(\frac{n_{\underline{t}} - \alpha}{n_{\underline{t}}}\right)^{\frac{\rho+\delta}{\delta}} \int_{0}^{n_{\underline{t}}} \underline{\pi}\left(\underline{Z}^*, n\right) dn + \int_{n_{\underline{t}}}^{1} \underline{\pi}\left(\underline{Z}^*, n\right) dn = 0, \tag{3.33}$$

$$\int_{0}^{n_{\underline{t}}} \overline{\pi}(\underline{Z}^*, n) dn = 0.$$

   Notice that $\underline{Z}^*$*is a vertical line for* $n_0 > \alpha$.

If $\overline{\pi}(\theta, n) = \pi(\theta, n) + \overline{\varepsilon}$ *and* $\underline{\pi}(\theta, n) = \pi(\theta, n) + \underline{\varepsilon}$, *with* $\overline{\varepsilon} > \underline{\varepsilon}$, *the equilibrium is fully characterized as follows: Define* $\hat{\varepsilon} \equiv \alpha\overline{\varepsilon} + (1-\alpha)\underline{\varepsilon}$ *and* $\hat{z}^*$ *as satisfying* $\int_0^1 \pi\left(\hat{z}^*, n\right) dn = -\hat{\varepsilon}$. $\forall n_0 \geq n_{\bar{t}}$, $\underline{Z}^*(n_0) = \hat{z}^*$ *and* $\forall n_0 < n_{\bar{t}}$, $\underline{Z}^*(n_0)$ *is given by equation (3.30).* $\forall n_0 \leq n_{\underline{t}}$, $\overline{Z}^*(n_0) = \hat{z}^*$ *and* $\forall n_0 > n_{\underline{t}}$, $\overline{Z}^*(n_0)$ *is given by equation (3.28).* $n_{\underline{t}}$ *and* $n_{\bar{t}}$ *satisfy* $\int_0^{n_{\underline{t}}} \pi(\hat{z}^*, n) dn = -n_{\underline{t}}\overline{\varepsilon}$ *and* $\int_{n_{\bar{t}}}^1 \pi(\hat{z}^*, n) dn = -(1 - n_{\bar{t}})\underline{\varepsilon}$, *respectively.*

Proof. We know that, for any t, $(\theta_t - \theta_0) \sim \mathcal{N}\left(\mu t, \sigma^2 t\right)$. If we rescale time as $\tilde{t} = t/\delta$, as in Theorem 3 in Frankel and Pauzner (2000), we can apply Proposition 3.4 in order to compute

the equilibrium, given that taking the limit as $\delta \to \infty$ is analogous to assuming $\mu, \sigma \to 0$. Proposition 3.4 and the fact that $\lim_{\delta\to\infty} \frac{\rho}{\delta} = 0$ and $\lim_{\delta\to\infty} \frac{\rho+\delta}{\delta} = 1$ give us the desired result. Notice that it is not necessary to divide the analysis of the large heterogeneity case in two: since the lower part of the high-type threshold and the upper part of the low-type threshold are vertical, whenever $\underline{O}$ is to the left of $\overline{P}$, conditions $\overline{Z}_0 < \underline{Z}_\alpha$ and $\overline{Z}_\alpha < \underline{Z}_1$ (with $\rho/\delta \to 0$) are automatically satisfied.

Finally, suppose $\underline{\pi}(\theta, n) = \pi(\theta, n) + \underline{\varepsilon}$, $\overline{\pi}(\theta, n) = \pi(\theta, n) + \overline{\varepsilon}$ and not too large heterogeneity. Let $\hat{\overline{Z}}$ be the solution to system (3.32) (which gives us the high-type threshold for low values of n) and $\hat{\underline{Z}}$ be the solution to system (3.33) (which gives us the low-type threshold for high values of n). Solving the two systems, we find $\int_0^1 \pi\left(\hat{\overline{Z}}, n\right) dn = -\left[\alpha\overline{\varepsilon} + (1-\alpha)\underline{\varepsilon}\right] \equiv -\hat{\varepsilon}$ and $\int_0^1 \pi\left(\hat{\underline{Z}}, n\right) dn = -\hat{\varepsilon}$, which implies $\hat{\overline{Z}} = \hat{\underline{Z}} = \hat{z}^*$. Also, we know that $\overline{Z}(n) = \underline{Z}(n)$ for n is some neighborhood of α. Since thresholds cannot be upward sloping, whenever agents play the same strategy, their threshold is also given by $\hat{z}^*$. Hence, we can fully characterize the equilibrium in this particular case, which is depicted in Figure 3.6.

□

3.A.3 Low-type equilibrium threshold with linear payoffs

- If $\overline{\varepsilon} - \underline{\varepsilon} > \frac{\gamma[\delta+\rho(1-\alpha)]}{\rho+2\delta}$,

$$\underline{Z} = \begin{cases} -\underline{\varepsilon} - \frac{\gamma\delta(1+\alpha)}{\rho+2\delta} - \frac{\gamma\rho}{\rho+2\delta} n & \text{if } n > \alpha, \\ -\underline{\varepsilon} - \frac{\gamma\delta}{\rho+2\delta} - \frac{\gamma(\rho+\delta)}{\rho+2\delta} n & \text{if } n \leq \alpha. \end{cases} \tag{3.34}$$

- If $\frac{\gamma[\delta+\rho(1-\alpha)]}{\rho+2\delta} \leq \overline{\varepsilon} - \underline{\varepsilon} < \frac{\gamma\delta}{\rho+2\delta}$, $\underline{Z}$ is given by (3.34) $\forall n \leq n''$ and otherwise it satisfies

$$\begin{aligned} \frac{(1-n)}{1-\alpha} \int_0^\infty & e^{-(\rho+\delta)t} \left[\underline{Z} + \underline{\varepsilon} + \gamma\left(1 - (1-n)e^{-\delta t}\right)\right] dt \\ & + \frac{(n-\alpha)}{1-\alpha} \left\{ \int_0^{\underline{t}} e^{-(\rho+\delta)t} \left[\underline{Z} + \underline{\varepsilon} + \gamma\left(\alpha + (n-\alpha)e^{-\delta t}\right)\right] \right. \\ & \left. + \int_{\underline{t}}^\infty e^{-(\rho+\delta)t} \left[\underline{Z} + \underline{\varepsilon} + \gamma\left(n_{\underline{t}} e^{-\delta(t-\underline{t})}\right)\right] dt \right\} = 0, \end{aligned} \tag{3.35}$$

where $\underline{t} = -\frac{1}{\delta} \ln \frac{n_{\underline{t}}-\alpha}{n_0-\alpha}$ and $n_{\underline{t}} = -\left(\underline{Z} + \overline{\varepsilon}\right)(\rho + 2\delta)/\gamma(\rho + \delta)$. Integrating by substitution, we can express $\underline{Z}(n) \forall n \leq n''$ as satisfying

$$(\rho+2\delta)(\underline{Z}+\underline{\varepsilon})+\gamma\rho n_0+\gamma\delta(1+\alpha)-\frac{\alpha}{(1-\alpha)}\gamma\delta\left(\frac{1}{n_0-\alpha}\right)^{\frac{\rho}{\delta}}\left[-\frac{(\underline{Z}+\overline{\varepsilon})(\rho+2\delta)}{\gamma(\rho+\delta)}-\alpha\right]^{\frac{\rho+\delta}{\delta}}=0.$$

n'' is the value satisfying $\underline{Z}(n'')=\overline{Z}(\alpha)$, which results in $n''=\alpha+\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)-\gamma\delta}{\gamma\rho}$.

- If $\overline{\varepsilon}-\underline{\varepsilon}\leq\frac{\gamma\delta}{\rho+2\delta}$, for all $n\in\left[\hat{n},\hat{\hat{n}}\right]$, the two types of agents play according to the same (downward sloping) threshold, which cannot be computed analytically. For all $n\leq\hat{n}$,

$$\underline{Z}=-\underline{\varepsilon}-\frac{\gamma\delta}{\rho+2\delta}-\frac{\gamma(\rho+\delta)}{\rho+2\delta}n,$$

and for all $n\geq\hat{\hat{n}}$, $\underline{Z}$ satisfies (3.35). Since we know the equations describing the two types' thresholds whenever they play distinct strategies, we can compute the values of n at which these thresholds intersect. By doing so, we find that

$$\hat{n}=\alpha\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)}{\gamma\delta}<\alpha$$

and

$$\hat{\hat{n}}=1-(1-\alpha)\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)}{\gamma\delta}>\alpha.$$

3.A.4 Planner's solution with two types of agents

Proposition 3.6. *Consider the model with two types of agents and linear payoff functions with $\nu^0=\nu^1$. The planner's solution is characterized by thresholds $\overline{Z}^P$ and $\underline{Z}^P$ as follows:*

1. *Planner's type-$\overline{\varepsilon}$ threshold:*

   (a) If $\overline{\varepsilon}-\underline{\varepsilon}>\frac{2\gamma(\delta+\rho\alpha)}{\rho+2\delta}$,

$$\overline{Z}^P=\begin{cases}-\overline{\varepsilon}+\frac{\gamma}{2}-\frac{2\gamma(\rho+\delta)}{\rho+2\delta}n & \textit{if } n\geq\alpha,\\ -\overline{\varepsilon}+\frac{\gamma}{2}-\frac{2\alpha\gamma\delta}{\rho+2\delta}-\frac{2\gamma\rho}{\rho+2\delta}n & \textit{if } n<\alpha.\end{cases}\tag{3.36}$$

   (b) If $\frac{2\gamma\delta}{\rho+2\delta}<\overline{\varepsilon}-\underline{\varepsilon}\leq\frac{2\gamma(\delta+\alpha\rho)}{\rho+2\delta}$,

   - $\forall n\geq n'^P\equiv\alpha-\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)-2\gamma\delta}{2\gamma\rho}$, $\overline{Z}^P$ *is given by equation (3.36);*

- $\forall n < n'^P$, *it satisfies*

$$(\rho+2\delta)\left(\overline{Z}^P+\overline{\varepsilon}-\gamma/2\right)+2\gamma\delta\alpha+2\gamma\rho n_0 \\ +2\gamma\delta\frac{(1-\alpha)}{\alpha}\left(\frac{1}{\alpha-n_0}\right)^{\frac{\rho}{\delta}}\left(\alpha+\frac{(\overline{Z}^P+\underline{\varepsilon}-\gamma/2)(\rho+2\delta)+2\gamma\delta}{2\gamma(\rho+\delta)}\right)^{\frac{\rho+\delta}{\delta}}=0; \tag{3.37}$$

(c) *If* $\overline{\varepsilon}-\underline{\varepsilon}\leq\frac{2\gamma\delta}{\rho+2\delta}$,

- $\forall n \leq \hat{n}^P \equiv \alpha\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)}{2\gamma\delta}$, $\overline{Z}^P$ *satisfies (3.37);*
- $\forall n \geq \hat{\hat{n}}^P \equiv 1-(1-\alpha)\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)}{2\gamma\delta}$,

$$\overline{Z}^P=-\overline{\varepsilon}+\frac{\gamma}{2}-\frac{2\gamma(\rho+\delta)}{\rho+2\delta}n.$$

2. *Planner's type-$\underline{\varepsilon}$ threshold:*

(a) *If* $\overline{\varepsilon}-\underline{\varepsilon}>\frac{2\gamma(\delta+\rho(1-\alpha))}{\rho+2\delta}$,

$$\underline{Z}^P=\begin{cases}-\underline{\varepsilon}+\frac{\gamma}{2}-\frac{2\gamma\delta(1+\alpha)}{\rho+2\delta}-\frac{2\gamma\rho}{\rho+2\delta}n & \text{if } n>\alpha,\\ -\underline{\varepsilon}+\frac{\gamma}{2}-\frac{2\gamma\delta}{\rho+2\delta}-\frac{2\gamma(\rho+\delta)}{\rho+2\delta}n & \text{if } n\leq\alpha.\end{cases} \tag{3.38}$$

(b) *If* $\frac{2\gamma\delta}{\rho+2\delta}<\overline{\varepsilon}-\underline{\varepsilon}\leq\frac{2\gamma(\delta+\rho(1-\alpha))}{\rho+2\delta}$,

- $\forall n \leq n''^P=\alpha+\frac{(\overline{\varepsilon}-\underline{\varepsilon})(\rho+2\delta)-2\gamma\delta}{2\gamma\rho}$, $\underline{Z}^P$ *is given by equation (3.38);*
- $\forall n > n''^P$, $\underline{Z}^P$ *satisfies*

$$(\rho+2\delta)(\underline{Z}^P+\underline{\varepsilon}-\gamma/2)+2\gamma\rho n_0+2\gamma\delta(1+\alpha) \\ +2\gamma\delta\frac{\alpha}{(1-\alpha)}\left(\frac{1}{n_0-\alpha}\right)^{\frac{\rho}{\delta}}\left(\alpha+\frac{(\underline{Z}^P+\overline{\varepsilon}-\gamma/2)(\rho+2\delta)}{2\gamma(\rho+\delta)}\right)^{\frac{\rho+\delta}{\delta}}=0. \tag{3.39}$$

(c) *If* $\overline{\varepsilon}-\underline{\varepsilon}\leq\frac{2\gamma\delta}{\rho+2\delta}$,

- $\forall n \geq \hat{\hat{n}}^P$, $\underline{Z}^P$ *satisfies (3.39);*
- $\forall n \leq \hat{n}^P$,

$$\underline{Z}^P=-\overline{\varepsilon}+\frac{\gamma}{2}-\frac{2\gamma\delta}{\rho+2\delta}-\frac{2\gamma(\rho+\delta)}{\rho+2\delta}n.$$

Proof. The proof of Proposition 3.6 follows from the discussion in section 3.5. Since solving the planner's problem is equivalent to solving a game with different payoffs, this proof is

equivalent to the proof of Proposition 3.4 if we substitute agents' flow-playoffs, $\underline{\pi}(.)$ and $\overline{\pi}(.)$, by the expressions arising from the optimality conditions: $\underline{Z_t}^P + \underline{\varepsilon} - \frac{\gamma}{2} + 2\gamma n_t$ and $\overline{Z_t}^P + \overline{\varepsilon} - \frac{\gamma}{2} + 2\gamma n_t$, respectively.

□

3.B Proofs

3.B.1 Proof of Proposition 3.1

The proof of equilibrium uniqueness follows an analogous reasoning as in the case of identical individuals (Frankel and Pauzner (2000)). Consider a type-q agent at some point on P_q. She is indifferent between actions 0 and 1 under the belief that everyone called upon choosing an action while she is committed to her choice will pick 0 under any circumstances. But when θ moves stochastically, there is always the possibility that it will spend some time to the right of some players' dominance region boundaries. Notice that even if q is such that P_q is the leftmost upper boundary (say P_3 in Figure 3.1), she cannot expect every other player to choose 0 under any circumstances while she is committed to her choice. If θ moves slightly to the right, it will be strictly dominant for type-q agents to pick 1, and thus a fraction α_q of the agents that get the chance will not choose 0. The most pessimistic (regarding the path of n) belief that agents can hold consistent with the dominance regions is that each type-q agent plays 1 when to the right of P_q , and 0 when to the left of it. In other words, agents do not play strictly dominated strategies. Under this (more optimistic) new belief, the agent on P_q is not indifferent anymore, but strictly preferring to play 1. To make her indifferent, we must lower θ. We can then construct for each type q a new boundary P_q^2 (to the left of P_q), to the right of which a type-q player chooses 1 when she expects all other agents to play according to $(P_q)_{q\in\mathcal{Q}}$. This procedure can be repeated *ad infinitum*. At each round, we look for the curve P_q^k on which a type-q player has zero discounted payoff when assuming that other agents play according to $\left(P_q^{k-1}\right)_{q\in\mathcal{Q}}$. Denote the limit of this sequence by $\left(P_q^{\infty}\right)_{q\in\mathcal{Q}}$. Notice that each agent i playing according to $P_{q(i)}^{\infty}$ is, in fact, an equilibrium: if she expects others to play according to $\left(P_q^{\infty}\right)_{q\in\mathcal{Q}}$, her best response is to play according to $P_{q(i)}^{\infty}$.

We now turn to a different iterative process starting from the lower dominance regions. Let $\left(P_q^{\lambda_0}\right)_{q\in\mathcal{Q}}$ be translations of the curves P_q^{∞} to the left by an amount λ_0. Fix λ_0 as the smallest distance such that all translations lie completely on the lower dominance region of each corresponding type. Figure 3.16 below exemplifies this step.

Now, construct for each type a new curve $P_q^{\lambda_1}$ as the rightmost translation of $P_q^{\lambda_0}$ to the left of which each type-q agent must play 0 if they expect others to play according to

Figure 3.15: Iterative deletion of strictly dominated strategies from the upper dominance region

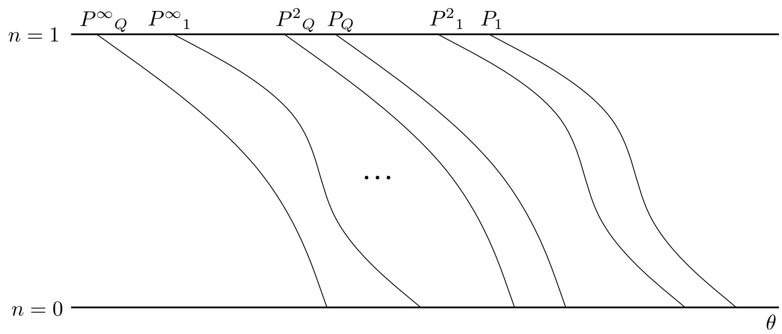
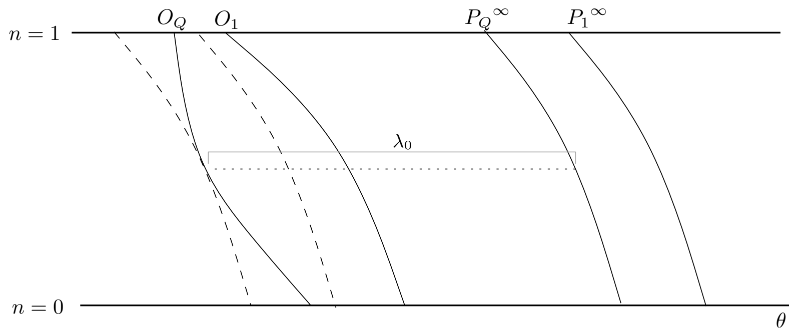


Figure 3.16: Translations of P_q^{∞}



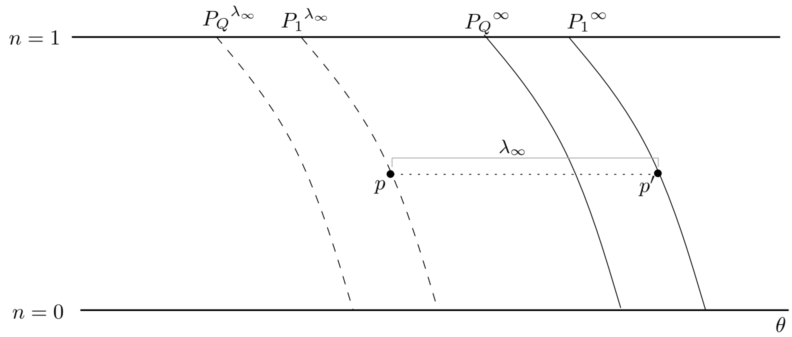
$\left(P_q^{\lambda_0}\right)_{q\in\mathcal{Q}}$.[18] Let $P_q^{\lambda_\infty}$ be the limit of this sequence, for each q. There is at least one point in some $P_q^{\lambda_\infty}$ curve on which a type-q agent is indifferent between the two networks, otherwise iterations would not have stopped. Without loss of generality, suppose there is a point of indifference in $P_1^{\lambda_\infty}$ and name it p. Let p' denote the point on P_1^{∞} at the same height as p. If we establish that p and p' coincide, we show that the whole curves coincide and, since we have translated all curves by the same λ's, $P_q^{\lambda_\infty} = P_q^{\infty} \forall q$, that is, the equilibrium is unique.

Lets compare two type-1 players, one receiving an opportunity to choose an action on p (expecting others to play according to the limit translations), and the other on p' (expecting others to play according to $\left(P_q^{\infty}\right)_{q\in\mathcal{Q}}$). Lets name those players p and p', respectively. We know that both players expect changes in the fundamentals relative to its starting point to have the same distribution. Also, since the original curves and their translations have the same shape and the pairwise distances between $P_q^{\lambda_\infty}$'s are the same as the distances between P_q^{∞}'s (each round, we have translated all curves by the same λ), for a given path of the fundamental, they both expect the same dynamics for n_t (computed as in Lemma 3.1).[19]

[18]Note that what we are doing is eliminating strictly dominated strategies once again, but we are not necessarily eliminating *all* dominated strategies each round.

[19]Lemma 3.3 below proves that the dynamical system for $\partial n_t/\partial t$ presented in Lemma 3.1 has an unique

Figure 3.17: Equilibrium uniqueness



If $\lambda_\infty > 0$, we get a contradiction: the two players expect the same relative dynamics for the (θ_t, n_t) system and the θ that p' expects at all times exceeds the θ that the agent on p expects, thus they cannot both have zero payoff. Then, $\lambda_\infty = 0$, that is, the points p and p' must coincide. The equilibrium is unique and it is characterized by thresholds $\left(Z_q^*\right)_{q\in\mathcal{Q}}$, where $Z_q^* \equiv P_q^\infty$.

We can show these thresholds are downward sloping by induction. Under the most pessimistic beliefs possible, a type-q agent's incentives to choose action 1 is increasing in the initial values of both n and θ, meaning that P_q is downward sloping (for all q). Under the assumption that all other agents are choosing according to downward sloping thresholds $\left(P_q^{k-1}\right)_{q\in\mathcal{Q}}$, the relative payoff of a given agent must be again increasing in both initial values of θ and n, since increases in either of these values would make the system spend more time to the right of other agents' thresholds for any given path of the Brownian motion, and thus $P_q{}^k$ must also be downward sloping.

□

Lemma 3.3 (used in the proof of Proposition 3.1)**.** *Suppose agents play according to thresholds* $\left\{P_q^\infty\right\}_{q\in\mathcal{Q}}$ *or* $\left\{P_q^{\lambda_\infty}\right\}_{q\in\mathcal{Q}}$*, as defined in the proof of Proposition 3.1. For almost every path of* θ*, there is an unique path for* n*.*

Proof. The proof follows from Theorem 1 in Burdzy et al. (1998). In order to apply their result here, we must prove that all P_q^∞'s are Lipschitz continuous (and so are $P_q^{\lambda_\infty}$'s), that is, there exists a constant c such that $|P_q^\infty(n) - P_q^\infty(n') \leq c|n - n'|$ for all n, n' and for all $q \in \mathcal{Q}$. Notice every curve P_q^k is contained in a compact set $[0,1] \times [O_q(0), P_q(1)]$. Thus, for each type q there are constants b_q and d_q such that, at all points (θ, n) on each P_q^k, $\partial\pi_q/\partial n < b_q$ and $\partial\pi_q/\partial\theta > d_q$, given our assumption that all functions $\pi_q(\theta, n)$ are continuously differentiable in both arguments.

solution.

First, we show that the upper dominance region boundaries $\{P_q\}_{q\in\mathcal{Q}}$ are Lipschitz. Fix an arbitrary q and consider two points along P_q, $(\tilde{\theta},\tilde{n})$ and (θ',n'), with $\tilde{\theta}>\theta'$ and $\tilde{n}<n'$. Lets compare, for any path $(\theta_t)_{t\geq 0}$ of the Brownian motion starting at $\theta_0=\tilde{\theta}$, the payoff of a type-q agent at $(\tilde{\theta},\tilde{n})$ to her payoff at (θ',n') when the Brownian motion is $(\theta_t+\theta'-\tilde{\theta})_{t\geq 0}$ (that is, under the same realizaion of shocks to θ). At all future dates, the difference in fundamentals is constant at $\tilde{\theta}-\theta'$. Since we compute P_q assuming the worst belief possible, the difference in future values of n is $(n'-n)e^{-\delta(\tau-t)}<(n'-n)$. This implies that the difference in (relative) flow payoffs at any future date when the system starts at $(\tilde{\theta},\tilde{n})$ in comparison to when it starts at (θ',n') is always greater than $d_q(\tilde{\theta}-\theta')-b_q(n'-\tilde{n})$. For type-$q$ agents to be indifferent at both points, it must be that $d_q(\tilde{\theta}-\theta')-b_q(n'-\tilde{n})\leq 0$,[20] or $(\tilde{\theta}-\theta')/(n'-\tilde{n})\leq b_q/d_q=c_q$, which means P_q is Lipschitz with constant c_q. Thus, defining $c\equiv\max\{c_q\}_{q\in\mathcal{Q}}$, we have that all P_q's are Lipschitz with constant c.

We can now show by induction that all $\left\{P_q^k\right\}_{q\in\mathcal{Q}}$ curves are Lipschitz with constant c. Let $\left\{P_q^{k-1}\right\}_{q\in\mathcal{Q}}$ be Lipschitz with constant c and suppose by contradiction that $P_{q'}^k$ is not (for an arbitrary q'). Then, there must be points $A=(\theta',n')$ and $B=(\tilde{\theta},\tilde{n})$ along $P_{q'}^k$ with $\tilde{\theta}>\theta'$ and $\tilde{n}<n'$ such that $(\tilde{\theta}-\theta')/(n'-\tilde{n})>c$. As before, lets compare how the state (θ,n) evolves when starting at each of these points for any given realization of shocks. The difference in the fundamentals is constant at $\tilde{\theta}-\theta'$. Lets check how the difference in n evolves. Remember $P_{q'}^k$ is constructed assuming agents expect others to play according to $\left\{P_q^{k-1}\right\}_{q\in\mathcal{Q}}$. Using Lemma 3.1, we have that, at a given point (θ_t,n_t),

$$\frac{\partial n_t}{\partial t}=\delta\left(\sum_{q\in I_t}\alpha_q-n_t\right), \tag{3.40}$$

where $I_t=\left\{q\in\mathcal{Q}:\theta_t>P_q^{k-1}(n_t)\right\}$. Notice if A is to the right of some threshold P_q^{k-1} (meaning $\theta'>P_q^{k-1}(n')$), so is B (i.e., $\tilde{\theta}>P_q^{k-1}(\tilde{n})$), given that P_q^{k-1} is Lipschitz with constant c while $(\tilde{\theta}-\theta')/(n'-\tilde{n})>c$. Likewise, whenever B is to the left of some threshold P_q^{k-1}, so is A. It means that at point B the first term in parenthesis in equation (3.40) is always larger than or equal to the same term at point A (i.e., there are at least as many types playing 1 at B than at A). This, plus the fact that $\tilde{n}<n'$, imply $\partial n_0/\partial t$ at A is always smaller than at B, so at $t=0$, the difference in n is shrinking. Hence, at the earliest future date at which the path that started at B reaches a threshold to the right of $P_{q'}^k$ (if there is any), the path that started at A will still be to the left of this given threshold, meaning the difference in n continues to shrink. Also, the path starting at A will certainly reach a threshold to the

[20] If differences in flow payoffs were positive at all future dates, they would not integrate to zero.

left of $P_{q'}^k$ earlier (if there is any), making the difference in n shrink as well. It means that, at any future date, the difference in n between the two paths is always smaller than $n' - \tilde{n}$, so the same calculations used before apply: the difference in terms of flow-payoffs at any future date between the paths starting at B and at A is larger than $d_{q'}(\tilde{\theta} - \theta') - b_{q'}(n' - \tilde{n})$, so we must have $(\tilde{\theta} - \theta')/(n' - \tilde{n}) \leq b_{q'}/d_{q'} = c_{q'} \leq c$, which is a contradiction. Thus, $\left\{P_q^k\right\}_{q\in\mathcal{Q}}$ are Lipschitz with constant c, and so are the limits $\left\{P_q^\infty\right\}_{q\in\mathcal{Q}}$ (and also $\left\{P_q^{\lambda\infty}\right\}_{q\in\mathcal{Q}}$, since they are translations of $\left\{P_q^\infty\right\}_{q\in\mathcal{Q}}$).

□

3.B.2 Proof of Lemma 3.2

Suppose $\overline{Z}(n) < \underline{Z}(n)$ for all n in some interval. By Lemma 3.1, we know that the dynamics of n (whenever in that interval) is given by the following dynamical system:

$$\dot{n}_t = \begin{cases} -\delta n_t & \text{if } \theta_t < \overline{Z}(n_t), \\ \delta(\alpha - n_t) & \text{if } \overline{Z}(n_t) < \theta_t < \underline{Z}(n_t), \\ \delta(1 - n_t) & \text{if } \theta_t > \underline{Z}(n_t). \end{cases}$$

(i) Consider a starting point (θ_0, n_0) such that $\theta_0 = \overline{Z}(n_0)$. With vanishing shocks, that is, $\mu, \sigma \to 0$, since $\underline{Z}(n) > \overline{Z}(n)$, we can focus on the behavior of the system only around $\overline{Z}(n_0)$ to compute $\dot{n}_t$ at $t = 0$ (shocks are not large enough to push θ to the right of $\underline{Z}$). So, in a neighborhood of the threshold $\overline{Z}$, we can write the dynamics of n as:

$$\dot{n}_t = \begin{cases} \delta(\alpha - n_t) & \text{if } \theta_t > \overline{Z}(n_t), \\ -\delta n_t & \text{if } \theta_t < \overline{Z}(n_t). \end{cases}$$

Defining $x_t \equiv n_t/\alpha$, we have that $\dot{x}_t = \dot{n}_t/\alpha$, that is,

$$\dot{x}_t = \begin{cases} \delta(1 - x_t) & \text{if } \theta_t > \overline{Z}(\alpha x_t), \\ -\delta x_t & \text{if } \theta_t < \overline{Z}(\alpha x_t). \end{cases}$$

We can, then, directly apply Theorem 2 in Burdzy et al. (1998), which gives us the desired result. The probability of the system bifurcating up at some point (θ_0, n_0) with $\theta_0 = \overline{Z}(\alpha x_0) \equiv \overline{Z}(n_0)$ and $n_0 < \alpha$ is given by $P(\text{up}) = \frac{\delta(1-x_0)}{\delta(1-x_0)+\delta x_0} = 1 - x_0 = 1 - \frac{n_0}{\alpha}$, $P(\text{down}) = \frac{n_0}{\alpha}$, and the time it takes for the system to bifurcate either up or down converges to zero. If $n_0 > \alpha$, $\dot{n}_0 < 0$ both to the left and to the right of $\overline{Z}(n_0)$, so the system bifurcates down with probability one

at time zero.[21]

(ii) As $\mu, \sigma \to 0$, the dynamics around $\underline{Z}$ can be written as:

$$\dot{n}_t = \begin{cases} \delta(1 - n_t) & \text{if } \theta_t > \underline{Z}(n_t), \\ \delta(\alpha - n_t) & \text{if } \theta_t < \underline{Z}(n_t). \end{cases}$$

Define $y_t \equiv \frac{n-\alpha}{1-\alpha}$. $\dot{y}_t = \dot{n}_t/(1-\alpha)$, that is,

$$\dot{y}_t = \begin{cases} \delta(1 - y_t) & \text{if } \theta_t > \underline{Z}((1-\alpha)y_t + \alpha), \\ -\delta(y_t) & \text{if } \theta_t < \underline{Z}((1-\alpha)y_t + \alpha). \end{cases}$$

Applying Theorem 2 in Burdzy et al. (1998), we find that at (θ_0, n_0) with $\theta_0 = \underline{Z}((1-\alpha)y_t + \alpha) \equiv \underline{Z}(n_0)$ and $n_0 > \alpha$, $P(\text{up}) = 1 - y_0 = \frac{1-n_0}{1-\alpha}$, $P(\text{down}) = \frac{n_0-\alpha}{1-\alpha}$ and the time it takes for the system to bifurcate either direction converges to zero. If $n_0 \leq \alpha$, then $\dot{n}_0 \geq 0$ both to the right and to the left of $\underline{Z}$, so the system bifurcates up with probability one at time zero.[22]

$\square$

3.B.3 Proof of Proposition 3.2

(i) Let $\underline{O}(n) > \overline{P}(n)\ \forall n$. On the equilibrium, agents cannot play strictly dominated strategies, then $\underline{Z}(n) \in [\underline{O}(n), \underline{P}(n)]\, \forall n$ and $\overline{Z}(n) \in \left[\overline{O}(n), \overline{P}(n)\right] \forall n$. Since $[\underline{O}(n), \underline{P}(n)] \cap \left[\overline{O}(n), \overline{P}(n)\right] = \emptyset\ \forall n$, there is no n such that $\underline{Z}(n) = \overline{Z}(n)$.

(ii) Let the dominance regions be such that $\underline{O}(n) \leq \overline{P}(n)\ \forall n \in [0, 1]$. First, notice that it is never the case that $\overline{Z}(n) > \underline{Z}(n)$, for any n. Otherwise, at any point in $(\overline{Z}(n), \underline{Z}(n))$, both types of players would face the same θ, have the same expected path for n_t, and type-$\overline{q}$ would have a higher preference for action 1. Yet, such player would have negative payoff, while a type-$\underline{q}$ player would have positive payoff of playing 1, a contradiction.

Suppose the equilibrium is such that $\overline{Z}(\alpha) < \underline{Z}(\alpha)$. Lemma 3.2 implies that a type-$\overline{q}$ agent at $\overline{Z}(\alpha)$ expects n_t to decrease with probability one at the maximum rate, while a type-$\underline{q}$ player at $\underline{Z}(\alpha)$ expects n_t to increase with probability one at the maximum rate, which implies $\overline{Z}(\alpha) = \overline{P}(\alpha)$ and $\underline{Z}(\alpha) = \underline{O}(\alpha)$. Thus, $\overline{P}(\alpha) < \underline{O}(\alpha)$, contradiction. We must have that $\overline{Z}(\alpha) = \underline{Z}(\alpha)$. Moreover, this point must be somewhere in the interval $\left[\underline{O}(\alpha), \overline{P}(\alpha)\right]$, so that no agent plays strictly dominated strategies in equilibrium.

We also need to show that agents never play the same strategy for every $n \in [0, 1]$.

[21]Exactly at $n_0 = \alpha$, $\dot{n} = 0$ to the right of $\overline{Z}(n_0)$ and $\dot{n} < 0$ to the left of it, so $P(\text{down})$ is also equal to one.
[22]Exactly at $n_0 = \alpha$, $\dot{n} = 0$ to the left of $\underline{Z}(n_0)$ and $\dot{n} > 0$ to the right of it, so $P(\text{up})$ is also equal to one.

Consider a type-$\underline{q}$ agent at her threshold at $n = 0$. Regardless of the position of the other type's threshold ($\overline{Z}(0) = \underline{Z}(0)$ or $\overline{Z}(0) < \underline{Z}(0)$), the beliefs over n_t such agent hold are the most optimistic as possible, by Lemma 3.2, and thus we know she is indifferent between both actions exactly at $\underline{O}(0)$. Hence, $\underline{Z}(0) = \underline{O}(0)$. Now, consider the case of type-$\overline{q}$ agents. If they play according to $\underline{O}(0)$, their gains from choosing 1 at this point must be strictly positive, since they have the same expected path for the fundamentals as the low-type agents, the same expected beliefs over the path of n_t and $\overline{\pi}(\theta, n) > \underline{\pi}(\theta, n)\ \forall(\theta, n)$ by assumption. As we move the candidate to the high-type threshold at $n = 0$ to the left along the θ axis starting at $\underline{O}(0)$, the relative payoff of action 1 decreases for two reasons: the initial θ is smaller, and also the beliefs over n_t become worse. Notice Lemma 3.2 implies that whenever $\overline{Z}(n) < \underline{Z}(n)$ for $n < \alpha$ we have that $\underline{Z}(n) = \underline{O}(n)$. Then, since thresholds are downward sloping (by Proposition 3.1), if $\overline{Z}(0) < \underline{Z}(0) = \underline{O}(0)$, it must be that $\underline{Z}(n) = \underline{O}(n)$ for all $n \leq \underline{O}^{-1}\left(\overline{Z}(0)\right)$. It implies that a type-$\overline{q}$ agent at a threshold $\overline{Z}(0) < \underline{Z}(0)$ holds the belief that n will bifurcate up with probability 1, but it will increase at a smaller rate (specifically $\dot{n}_t = \delta(\alpha - n_t)$) until it crosses $\underline{O}^{-1}\left(\overline{Z}(0)\right)$, and thereafter it will go up at the maximum rate towards one.[23] We also know that if high-type agents play according to $\overline{O}(0)$, their payoff must be strictly negative, since it would be zero under the most optimistic beliefs possible, which do not hold anymore. Hence, there must be a threshold $\overline{Z}(0)$ in $\left(\overline{O}(0), \underline{O}(0)\right)$ at which the relative payoff of type-$\overline{q}$ agents is zero, i.e., they are indifferent between both actions. Thus, $\overline{Z}(0) < \underline{Z}(0)$. This and the fact that thresholds are downward sloping imply that $\overline{Z}(n) < \underline{Z}(n)$ for all n in some interval $[0, n_1)$. An analogous reasoning implies that $\overline{Z}(n) < \underline{Z}(n)$ for all n in some interval $(n_2, 1]$ as well.

Last, we must show that when the condition on the dominance regions holds with strict inequality, i.e. $\underline{O}(n) < \overline{P}(n)\ \forall n$, $\overline{Z}(n) = \underline{Z}(n)$ for all n in some interval $C \supset \alpha$. Suppose there is no such C. Then, for every arbitrary interval $\tilde{C} \supset \alpha$, $\exists n \in \tilde{C}$ such that $\overline{Z}(n) \neq \underline{Z}(n)$. Fix $\varepsilon > 0$ and let $\hat{C} = [\alpha - \varepsilon, \alpha + \varepsilon]$. There must exist a point $d \in \hat{C}$ such that $\overline{Z}(d) \neq \underline{Z}(d)$. Without loss of generality, assume $d > \alpha$. We can choose an appropriate $\hat{\varepsilon} \leq \varepsilon$ in order to write $d = \alpha + \hat{\varepsilon}$. Lemma 3.2 implies that $\overline{Z}(\alpha + \hat{\varepsilon}) = \overline{P}(\alpha + \hat{\varepsilon})$ and since thresholds are downward sloping we must have that $\overline{Z}(\alpha) \in \left[\overline{P}(\alpha + \hat{\varepsilon}), \overline{P}(\alpha)\right] \equiv A$.[24] Now, consider a $b \in [\alpha - \hat{\varepsilon}, \alpha]$ and suppose by contradiction that $\overline{Z}(b) \neq \underline{Z}(b)$. Using Lemma 3.2 once again, we have that $\underline{Z}(b) = \underline{O}(b)$ and, since $\underline{Z}$ is downward sloping, $\underline{Z}(\alpha) \in [\underline{O}(\alpha), \underline{O}(b)] \equiv B$. Notice that $\underline{Z}(\alpha) = \overline{Z}(\alpha)$ must lie in $A \cap B$. However, if ε is small enough, $A \cap B = \emptyset$,

[23]Notice that in the limiting case with $\mu, \sigma \to 0$, this analysis can be done regardless of the position of equilibrium thresholds for higher values of n, since thresholds are downward sloping and θ remains almost constant.

[24]Remember we had before that $\overline{Z}(\alpha) = \underline{Z}(\alpha) \in \left[\underline{O}(\alpha), \overline{P}(\alpha)\right]$.

given that $\underline{Q}(n) < \overline{P}(n)\ \forall n$, and hence we reach a contradiction. At $n = b$, agents must play according to the same strategy. Finally, since we have fixed an arbitrary b, it must be the case that $\overline{Z}(n) = \underline{Z}(n)$ for all $n \in [\alpha - \hat{\varepsilon}, \alpha]$, contradicting the fact that $\nexists C \supset \alpha$ such that $\overline{Z}(n) = \underline{Z}(n)\ \forall n \in C$. This concludes the proof. $\square$

3.B.4 Proof of Proposition 3.3

Proposition 3.3 follows from the results in Proposition 3.5 and the dynamics of n_t presented in the proof of Lemma 3.1 when $\delta \to \infty$. $\square$

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