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A Bidimensional Model of Matching in the Marriage Market with Women Labor Decision

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Dissertação submetida a Escola de Pós-Graduação em Economia como requisito parcial para a obtenção do grau de Mestre em Economia.

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LUIZA GUELLER ZARDIN

A BIDIMENSIONAL MODEL OF MATCHING IN THE MARRIAGE MARKET WITH WOMEN LABOR DECISION.

Dissertação apresentada ao Curso de Mestrado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Mestre em Economia.

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Abstract

We construct a frictionless matching model of the marriage market where women have bidimensional attributes, one continuous (income) and the other dichotomous (home ability). Equilibrium in the marriage market determines intrahousehold allocation of resources and female labor participation. Our model is able to predict partial non-assortative matching, with rich men marrying women with low income but high home ability. We then perform numerical exercises to evaluate the impacts of income taxes in individual welfare and find that there is considerable divergence in the female labor participation response to taxes between the short run and the long run.

KEYWORDS: *Marriage market, multidimensional matching, labor supply, household behavior.*

Contents

List of Figures

1 Introduction

An increasing concern in economics and policy making is inequality. Nevertheless, the unitary approach often adopted in economic models treats the household as if consisting of a single decision maker, despite being indisputable that marriage plays an important role in the generation of welfare and its distribution both within and across households.

Furthermore, it is difficult to disentangle the interaction of the labor and the marriage markets, specially because whom one marries might determine how time is allocated in different tasks within the couple and also because the ability to perform certain tasks might lead to different choices of spouse.

This paper explores labor decisions and policy implications when female labor market participation and individual welfare are equilibrium outcomes of the marriage market. We build on a two-sided matching model of the marriage market as in Becker (1973), where agents in the same household have transferable utility (TU).

Here couples make a decisions in the labor and marriage markets concomitantly and based on multidimensional traits. More specifically, we consider bidimensional attributes and allow women to allocate time in two different activities based on their bidimensional types and on whom they marry with.

Our hypotheses allow the simple characterization of the possible equilibria and a distinctive matching configuration arises in equilibrium. We are able to predict partial non-assortative matching: the men with the highest income renouncing the women in the top of the income distribution and marrying poorer high-ability women. However, if we consider only the households where the women join the labor market, matching is assortative.

The model also serves well our specific purpose of demonstrating some of the potential welfare implications of taxation in a multidimensional context with labor decisions. For a particular specification we perform numerical exercises and are able to illustrate impacts of policies for the welfare at the individual level. We find that the introduction of taxes benefits the housewives the most, mostly by a shift in the welfare division in such couples, with utility flowing from the husband to the wife.

Finally, we also find that there are major differences in the response to taxes of the

female labor participation in the short and the long run, indicating that the interaction of the marriage and labor markets might play an important role in the effect of policies in the long run.

The outline of the rest of the paper is as follows. Section 2 reviews related literature and Section 3 describes the model. Section 4 characterize the possible matching equilibria and specializes the model to the setting which we adopt in the numerical exercises of Section 5. Finally, Section 6 concludes.

2 Literature Review

In the last couple of decades the literature has taken important steps in the direction of addressing questions such as who marries whom and how couples make decisions, and two main strategies have been developed to attack these problems.

The collective approach of Chiappori (1988) and Apps and Rees (1988) makes solely one assumption: within each marriage Pareto efficient allocations are attained. Frequently, however, this model is further specialized in order to consider a Nash bargain game. The relative bargain power of each person can be represented either by his or her Pareto weight or by the assumed threat points. These are allowed to vary with prices, total income, and distribution factors, the latter being any variable that does not change the budget constraint or preferences but affect the weights. This framework provides insights into how the resources are allocated in a particular household by assuming that the Pareto weights are exogenously-given functions.

Alternatively, since Becker (1973) matching models have been employed in order to characterize the intra-household allocation of resources as an equilibrium outcome of the marriage market. As a result, it is possible to endogenously determine who marries whom, the division of marital output in every marriage, and the variables that are able to shift the allocation of power between spouses. Since one of the goals of this work is to understand what are the policies that can change the allocation of marital output (and how they change it), it seems more appropriate to address this question with the use of matching models and this is the direction we follow here.

Becker's theory is built on a two-sided assignment model with transferable utility

(TU) and agents on each side of the market characterized by a set of characteristics on which the matching takes place. Agents on one side of the market choose whom to match with on the other side of the market in order to maximize their own individual payoff. Due to the assumption of transferable utility, it is possible to condense the relevant information of a potential marriage into a surplus function, which is defined as the total utility generated in the marriage minus the sum of the utilities of each spouse when single.

Matching models in which individual traits are unidimensional have been largely studied, with the well-known result of positive assortative matching when the surplus function is supermodular, that is, the partners' traits complement each other in the production of marital surplus.[1] For instance, Chiappori and Oreffice (2008) study the impact of innovations in birth control technology on intrahousehold allocation of resources by allowing men and women to differ in their preferences for children. Another interesting application of the unidimensional model is Chiappori et al. (2015), where an equilibrium lifecycle model of education, marriage and labor supply and consumption is considered and individuals match based on human capital.

Nonetheless, observed marriage patterns are much more complex and it is natural to assume that there are several traits one takes into account when contemplating potential spouses. To bring the restrictive outcomes derived by the nonfrictional unidimensional matching model more close to reality, there are two options. First, as was first done in Shimer and Smith (2000), it is possible to introduce frictions in the meeting process, which is then assumed to be sequential and random. Since agents discount their time, this implies that one does not wait for his or her perfect assortative mate anymore and also that who marries whom has now a random component, features that resemble more those of the real world. The alternative to this is to keep a frictionless setting and allow multidimensionality.

Multidimensional settings, however, are difficult to theoretically characterize. In order to simplify the problem, some papers have attempted to condense several traits into a one-dimensional index. This approach was used in Chiappori et al. (2012) and allowed the authors to recover the marginal rates of substitution between

[1]More precisely, a function $f : \mathbb{R}^2 \to \mathbb{R}$ is supermodular if, for every (x, y) and (x', y') such that $x \geq x'$ and $y \geq y'$, $f(x, y) + f(x', y') \geq f(x', y) + f(x, y')$.

characteristics such as body mass index and wages or education. However, this strategy might not represent well how multidimensional traits affect marriage decisions, a point that is emphasized in Dupuy and Galichon (2012). By rejecting the hypothesis that sorting occurs on a single dimension they show that individuals face important trade offs between the attributes of their spouses which are not amenable to a one-dimensional index combining the various characteristics.

Fortunately, beginning with Choo and Siow (2006), a large body of the literature[2] has assumed that each individual can be characterized by a set of observable attributes and also by an unobserved heterogeneity, defined by characteristics that the researcher cannot observe but are relevant to the matching process.

This new branch of the literature builds on discrete choice theory and is therefore restricted to the case of discrete characteristics. Under some assumptions about the distribution of the unobserved heterogeneity, if there is a relatively small number of observable attributes, and if marital output is restricted to be independent of the interaction of the unobservables, it is possible to estimate the surplus generated by each couple and its division for an observed matching.

Testability would then come from the observation of several isolated marriage markets with the same surplus function but with different population characteristics. In practice, however, it is difficult to verify whether the considered markets indeed have the same surplus function.

Also, this approach is clearly a powerful tool from an empirical perspective, but since multidimensional models have not been greatly explored in order to generate clear predictions from the theory, it is hard to disentangle the empirical results and understand their meaning.

Another point of concern is that currently there is no approach to handle sorting on multiple attributes if one of them is continuous, or, even if all of them are discrete, it might be the case that some of the unobservable traits interact with each other in the composition of the marital surplus, breaking a crucial hypothesis in Choo and Siow (2006). The present work sheds some light on both problems.

In this work we deviate from the current mainstream literature (e.g., Choo and Siow (2006)) and take a more theoretical approach as we look at a model for which

[2]Recent papers are Chiappori et al. (2009) and Salanié and Galichon (2015).

the primitives are the preferences and the household production function. We consider bidimensional attributes and the structure of the marriage market is very similar to that of Quintana-Domeque et al. (2014). In their work, traits are also bidimensional: one continuous, representing socioeconomic status and the other dichotomous, indicating whether the individual is a smoker or not. Their starting point is a surplus function, which is supermodular in the socioeconomic traits of the couple and is affected by the spouses' smoking status in a multiplicative way so that if at least one of the members of the household is a smoker, the surplus is diminished by a constant factor.

An important departure from their work is our formulation of how one's own attributes interact in the generation of marital surplus. Since we further restrict the model so that a woman's decision to allocate time in one type of occupation precludes the participation in the other type and each activity requires a specific trait – income if the choice is for the labor market and home productivity for the alternative of home production – a person who is endowed with high levels of both attributes will not be able to apply them at the same time, eliminating any complementarity of traits.

In this aspect, our model resembles a sector model as in Roy (1951), whereas in Quintana-Domeque et al. (2014) there is the possibility of complementarity between a person's own characteristics. For example, in their paper a nonsmoking man in a marriage with a nonsmoking woman generates strictly higher surplus than what would be generated in the marriage of a smoking man with the same socioeconomic status and the same woman. This leads to significant different matching patterns as the ones found in this paper, which are in turn similar to the patterns found in Low (2015), where matching is based in income and women's "reproductive capital". A major difference between the two papers is that in her work there are no labor decisions but there is a prior stage in which women decide whether to invest in human capital facing a trade-off between higher income and reduced fertility.

Perhaps one of the most important features of this model, here couples make a decisions in the labor and marriage markets concomitantly and based on multidimensional traits, a feature that, to our knowledge, has not appeared in the literature before, with the exception of Gayle and Shephard (2015). In this preliminary paper

they develop a multidimensional matching model of the marriage market together with labor decisions in order to study the impact of taxes. Still, their work is based on Choo and Siow (2006) and is, therefore, empirical, facing limitations which will be discussed in the literature review section.

3 The Model

This section introduces a relatively general specification of the model, for which we are able to prove equilibrium existence and characterize it. Later, we specialize some of the described functions (utilities and distributions) in order to achieve further characterization and provide numerical examples.

This paper builds on a two-sided assignment model. We assume that individuals are characterized by their potential labor incomes, denoted $x \in X = [\underline{x}, \bar{x}]$ for women and $y \in Y = [\underline{y}, \bar{y}]$ for men. Additionally, women can differ in an extra discrete dimension $\alpha \in \{\underline{\alpha}, \bar{\alpha}\}$, which we denote home ability. Based on these traits, a matching game takes place, in which agents on one side of the market choose whom to match with on the other side in order to maximize their own individual payoff.

Transferable utility is commonly assumed in the literature of family economics since it implies a one-to-one utility exchange rate within the household which allows us to summarize the marital output from any match in a single number. This greatly simplifies equilibrium characterization. Here, we condense all the relevant information of a potential marriage into the total utility function.

With the purpose of improving tractability, we consider that couples make labor decisions in a very simplistic way. Since women have both home ability and income characteristics, they face a trade-off between boosting home production or joining the labor force and increasing the household total income. We allow them to decide where to allocate their time, but the choice of one activity precludes the other, leaving them with the binary decision of whether to work (t=1) or not (t=0).

The total utility generated within any particular household can be decomposed in two parts that interact in a multiplicative way:

$$U(x, \alpha, y, t) = f(t\underline{\alpha} + (1-t)\alpha)g(y + tx).$$

In the first part, the increasing function $f : \mathbb{R}_+ \to \mathbb{R}_{++}$ accounts for how the female home ability dimension might increase (or not) the total household utility. The second term, $g : \mathbb{R}_+ \to \mathbb{R}_+$, depends on the total earned income of the household and is assumed to be three times differentiable. We further request that $g(0) = 0, g' > 0, g'' > 0$ and $g''' \leq 0$.

The assumption that $g'' > 0$ implies that the total household utility is supermodular in the spouses' income. Moreover, $g''' < 0$ implies that the surplus function is not "more" supermodular the higher the couple's total income. For example, if we consider three women (x,$x + \delta$ and $x + 2\delta$) and three men (y,$y + \sigma$ and $y + 2\sigma$), the gains from the assortative matching (versus nonassortative matching) of the pair (x,$x + \delta$) with the pair (y,$y + \sigma$) are higher than the gains that arise from the assortative matching (versus nonassortative matching) of the pair ($x + \delta$,$x + 2\delta$) with the pair ($y + \sigma$,$y + 2\sigma$).[3] We need this hypothesis for later proof of the equilibrium characteristics, more specifically to show that once a man y^* marries a woman who stays at home in equilibrium, every men with higher income also does so.

In order to account for the possibility of singles, we introduce fictional types in each side of the market. If a woman is single, we represent her partner as man $\varnothing_y$ and assume their union generates no surplus relatively to the woman being single. Clearly a single woman will always join the workforce since otherwise her utility would be the lowest possible. Analogously, if a man is single, we represent his wife as $\varnothing_x$ and in the same way assume there is no surplus from their fictional marriage.

To simplify the notation, we introduce the superscripts S, W and H to denote whether the function refers to a household composed, respectively, of a single person, a couple in which the woman works or a couple in which the woman contributes to home production.

Giving the so far exposed, singles' utility only depends on labor income ($z \in X \cup Y$) and can be written as

$$U^S(z) = f(\underline{\alpha})g(z).$$

[3] $g(x+y) + g(x+\delta+y+\sigma) - g(x+\delta+y) - g(x+y+\sigma) > g(x+\delta+y+\delta) + g(x+2\delta+y+2\sigma) - g(x+2\delta+y+\sigma) - g(x+\delta+y+2\sigma)$

Differently, a couple's total utility will depend on labor decisions according to

$$U^H(x,y,\alpha) = f(\alpha)g(y)$$
$$U^W(x,y,\alpha) = f(\underline{\alpha})g(x+y),$$

which results in a total utility of

$$U(x,y,\alpha) = \max\{U^H(x,y,\alpha), U^W(x,y,\alpha)\}.$$

The hypotheses made so far imply that there are no gains from the interactions between the ability dimension of spouses (since men here have no ability) and between the ability and wage dimensions of the same individual (since women cannot split their time between activities). What persists is a potential synergy between the spouses' wages if both of them work and between the wage of one of them and the ability of the other otherwise. This allows the simple characterization of the possible equilibria, which are not totally assortative in income, and also serves well our specific purpose of demonstrating some of the potential welfare implications of taxation in a multidimensional context with labor decisions.

It is customary in the TU matching literature to represent a match by the total surplus it generates, instead of the total utility as we have done so far. In accordance, we can write the surplus for each case as

$$S^H(x,y,\alpha) = U^H(x,y,\alpha) - U^S(x) - U^S(y)$$
$$S^W(x,y,\alpha) = U^W(x,y,\alpha) - U^S(x) - U^S(y)$$

which results in a final surplus of

$$S(x,y,\alpha) = \max\{S^H(x,y,\alpha), S^W(x,y,\alpha)\}.$$

Indeed, since the utility of someone as single is independent of whom he or she chooses to marry, the total surplus in one marriage is greater than in some other if and only if the total utility goes in the same direction. Accordingly, total surplus is also supermodular in the income dimensions. These acknowledgments indicate that

it is equivalent to formulate the matching problem in terms of any of these functions.

Because some of the utilities presented so far depend only on a subset of the types, we will simplify the notation and omit the irrelevant variables whenever it is convenient.

Given these functions, women and men take part in a one-to-one matching game where a woman's type is characterized by $(x,\alpha) \in \tilde{X} = X \times \{\underline{\alpha}, \bar{\alpha}\} \cup \{\varnothing_x\}$ and a man's type is solely his income $y \in \tilde{Y} = Y \cup \{\varnothing_y\}$. The income measures of low ability women, high ability women and men are respectively denoted $\underline{\gamma}_x : X \to \mathbb{R}_+$, $\bar{\gamma}_x : X \to \mathbb{R}_+$, and $\gamma_y : Y \to \mathbb{R}_+$ and are assumed to be absolutely continuous with respect to the Lebesgue measures on each set.[4]

In this context, a matching is a measure γ on the product space $\tilde{X} \times \tilde{Y}$, the projections of which coincide with the initial distributions on each set.

A matching is said to be pure if almost all agents are matched with probability 1 to exactly one other agent, that is, if γ takes values in $\{0, 1\}$ almost everywhere in $\tilde{X} \times \tilde{Y}$. If this is the case, then there is a bijection between $\tilde{X}$ and $\tilde{Y}$ which defines for any woman (any man) who she (he) is married to. As we shall later see, this does not need to be the case in the equilibrium defined below.

The stable matching concept of Shapley and Shubik (1971) is the relevant equilibrium definition in this framework. A matching is said to be stable if (i) there is no man and woman who would both prefer to be married to each other rather than remain in their current situation, and (ii) there is no married person who would rather be single.

The mass of the types $\varnothing_x$ and $\varnothing_y$ will be an equilibrium outcome. Since any marriage in which both spouses have strictly positive labor incomes generates a strictly positive surplus, the definition of stable matching implies that in equilibrium there will only be singles if there is an excess mass of people in one of the sides of the market, in which case the side in excess supply will have singles. It is without loss of generality to assume that the male population has measure $\gamma_y(Y) = 1$ and that the total mass of women is $\underline{\gamma}_x(X) + \bar{\gamma}_x(X) = \mu > 1$, since this would not change the equilibrium matching patterns except for which side of the market bares the singles. Therefore, there will be a total of $\mu - 1$ men of type $\varnothing_y$ and no woman of type $\varnothing_x$ in

[4]The associated probability distributions are denoted $\underline{F}_x$, $\bar{F}_x$ and F_y.

equilibrium.

Additionally, we define $p = \frac{\bar{\gamma}_x(X)}{\mu}$ as the fraction of high ability women in the total population and assume that $p\mu < 1$ and $(1-p)\mu < 1$, so that at least some women of both ability types are married.

Underlying any particular stable matching, there are payoff functions that support it as an equilibrium outcome. We denote these functions $u(x,\alpha)$ for women and $v(y)$ for men. Since the utility U^W generated in a marriage in which the woman works does not depend on the woman's home ability, it follows from the definition of a stable matching that if $(x,\underline{\alpha})$ and $(x,\bar{\alpha})$ work with positive probability in equilibrium, then $u(x,\underline{\alpha}) = u(x,\bar{\alpha}) \equiv u^W(x)$, since no woman would accept to be in a marriage if she can get a higher payoff in another. Analogously, if two women $(x',\bar{\alpha})$ and $(x'',\bar{\alpha})$ stay at home in equilibrium with a positive probability, then $u(x',\bar{\alpha}) = u(x'',\bar{\alpha})$, since no man married to a woman who stays at home would accept offering her more than the payoff that any other woman who stays at home gets.

Bearing this in mind, we simplify the notation as follows:[5]

$$u(x,\alpha) = \begin{cases} u^W(x) & \text{if the woman } (x,\alpha) \text{ works} \\ u^H & \text{if the woman } (x,\bar{\alpha}) \text{ stays at home.} \end{cases}$$

For what follows, we assume that at least some high ability women join the workforce.[6] As a consequence, the payoff u^H of the women who stay at home is pinned down by the greater outside option that a woman who makes this decision has, that is

$$u^H = \sup_{x \in X^H} u^W(x),$$

where $X^H \subseteq X$ is the subset of women who have a positive probability of being in a marriage in which they stay at home in equilibrium.

It is a direct consequence of the stable matching definition that the resulting util-

[5]Note that no woman with the low ability type will stay at home in equilibrium.

[6]This will be the case if either $\bar{\alpha}$ is not too high or if the maximum income in the support of $\bar{\gamma}$ is large enough.

ities $u^W(x)$ of any woman who work and the utilities $v(y)$ of any man satisfy

$$u^W(x) + v(y) \geq U^W(x, y)\ \forall x \in X\ \forall y \in Y \tag{1}$$

$$u^H + v(y) \geq U^H(y)\ \forall y \in Y, \tag{2}$$

since otherwise it would be possible for a man and a woman to marry and get payoffs strictly higher than what they would get in equilibrium. If woman (x, α) and man y are married and woman (x, α) works in equilibrium, then equation (1) holds if equality. In the same way, if woman (x, α) and man y are married and woman (x, α) stays at home in equilibrium, then equation (2) holds with equality.

This in turn implies two separate maximization problems. One for women,

$$u^W(x) = \max_{y \in Y} U^W(x, y) - v(y) \tag{3}$$

and the other for men

$$v(y) = \max\{\max_{x \in X} U^W(x, y) - u^W(y), U^H(y) - u^H\}. \tag{4}$$

If we consider only couples where the woman works in equilibrium, then matching becomes unidimensional and, because $U^W(x, y)$ is supermodular, the matching must be positively assortative in income (Becker (1973)). Therefore, spouses in marriages where the woman works are assigned to each other with probability one in equilibrium.[7] This allows us to describe who marries whom in marriages where the woman works by bijective functions. We let $\psi(x)$ denote the husband of woman x and $\phi(y)$ the wife of man y. Obviously we have that $\psi(\phi(y)) = y$.

Hence, the solution to problem (3) can be obtained by integration of the following envelope condition

$$\frac{\partial u^W(x)}{\partial x} = \frac{\partial U^W(x, y)}{\partial x}|_{y=\psi(x)}.$$

Because the utility of single women is pinned down by equation (3), we can use it as an initial condition, yielding

[7]This need not be true for the cases where one of the spouses is indifferent between a marriage where the woman works and one in which she stays at home, but as we will demonstrate, this cases have measure zero.

$$u^W(x) = U^S(x_S) + \int_{x_S}^{x} \frac{\partial U^W(x, \psi(x))}{\partial x} dx \; \forall x > x_S,$$

where x_S is the single woman with the highest income.

4 Equilibrium

a Existence

The existence of a stable matching is guaranteed by a very general result that states the equivalence between a particular stable matching in a multidimensional transferable utility context and a particular solution to an optimal transportation problem. More precisely, a matching γ is stable if and only if it is also a solution to the maximization of aggregate surplus over all possible measures on the product space $\tilde{X} \times \tilde{Y}$. Since the set $\tilde{X} \times \tilde{Y}$ is a compact subset of a compact metric space, the set of measures on $\tilde{X} \times \tilde{Y}$ is also compact. Combining this with the fact that the surplus function is continuous, there is a solution to the above mentioned transportation problem. The payoffs that support a measure as a stable matching can be obtained from the solution of a dual problem.[8]

b Characterization

Proposition 1 (Characterization of the set of stable matchings)**.** In any stable matching there exists cutoffs $y^* \in Y$ and $x_T, x_S \in X$ such that:

(i) Women of type $(x, \underline{\alpha})$, with $x < x_S$ are single with probability one:
$\gamma(B, \underline{\alpha}, \varnothing_y) = \underline{\gamma}_x(B) \; \forall B \in \mathcal{B}(\mathbb{R}) \cap [\underline{x}, x_S)$.

(ii) Women of type $(x, \bar{\alpha})$, with $x < x_T$ do not work with probability one:
$\gamma(B^H, \bar{\alpha}, \tilde{Y}) = \bar{\gamma}_x(B) \; \forall B \in \mathcal{B}(\mathbb{R}) \cap [\underline{x}, x_T)$, where $B^H = \{x \in B | u^H > u(x)\}$.

(iii) Women of type $(x, \bar{\alpha})$, with $x > x_T$ work with probability one:
$\gamma(B^W, \bar{\alpha}, \tilde{Y}) = \bar{\gamma}_x(B) \; \forall B \in \mathcal{B}(\mathbb{R}) \cap (x_T, \bar{x}]$, where $B^W = \{x \in B | u(x) > u^H\}$.

[8] For a detailed formulation and proof see Ekeland (2010) and Chiappori et al. (2010).

(iv) Men of type y, with $y > y^*$ marry women of type $(x, \bar{\alpha})$, with $x \leq x_T$ with probability one:

$\gamma(X^H, \bar{\alpha}, B) = \gamma_y(B)\ \forall B \in \mathcal{B}(\mathbb{R}) \cap [y^*, \bar{y}]$, where $X^H = \{x \in X | x \leq x_T\}$.

(v) Men of type y, with $y < y^*$ match with women who work positive-assortatively in income:

$\gamma(\phi^W(B), \bar{\alpha}, B) + \gamma(\phi(B), \underline{\alpha}, B) = \gamma_y(B)\ \forall B \in \mathcal{B}(\mathbb{R}) \cap [\underline{y}, y^*)$, where

$\phi^W(B) = \{x \in \phi(B) | x > x_T\}$.

Proof. In the Appendix. □

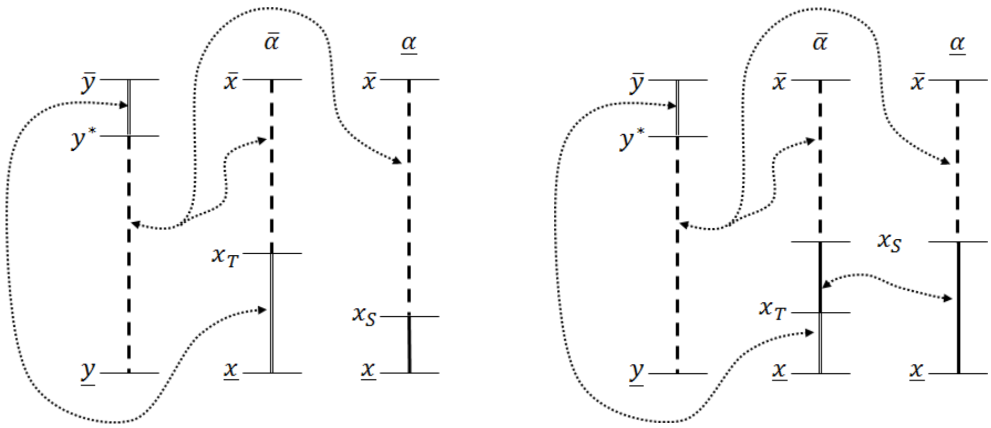
A direct way of grasping Proposition (1) is through Figure (1). The structure outlined in the proposition gives rise to two classes of equilibria, depending on whether $x_T > x_S$ or otherwise. The first diagram in Figure (1) represents the Type-A equilibria, where $x_T > x_S$. In this case, the bijective functions that represent the matches where the woman works can be determined from:

$$
\begin{aligned}
&1 - F_y(y^*) = \bar{F}_x(x_T) \\
&\underline{F}_x(x_S) = \mu - 1 \\
&F_y(\psi(x)) = \underline{F}_x(x) - \underline{F}_x(x_S) \text{ if } x \in (x_S, x_T] \\
&F_y(\psi(x)) = \underline{F}_x(x) - \underline{F}_x(x_S) + \bar{F}_x(x) - \bar{F}_x(x_T) \text{ if } x > x_T.
\end{aligned}
$$

Analogously, the second diagram in Figure (1) represents the Type-B equilibria, where $x_T \leq x_S$. In this case, the high ability woman $x \in (x_T, x_S)$ are also single and the corresponding matching functions are obtained from:

$$
\begin{aligned}
&1 - F_y(y^*) = \bar{F}_x(x_T) \\
&\underline{F}_x(x_S) + \bar{F}_x(x_S) - \bar{F}_x(x_T) = \mu - 1 \\
&F_y(\psi(x)) = \underline{F}_x(x) - \underline{F}_x(x_S) + \bar{F}_x(x) - \bar{F}_x(x_S).
\end{aligned}
$$

Everything else held constant, the higher the ratio $\bar{\alpha}/\underline{\alpha}$, the largest the cutoff x_T and, therefore, the more likely it is that the equilibrium will be of Type-A. In the same way, the higher the mass of high ability women with low income, the higher the chance that the equilibrium is of Type-B.



(a) A Type-A Equilibrium. (b) A Type-B Equilibrium.

Figure 1: The two types of equilibrium. Same patterns in the two sides of the market indicate which group of men marry which group of women. For the dotted line group, matching is assortative in income and therefore unique. Couples of the double line group might be formed by any two individuals, one of each side, and the equilibrium definition even allows for randomization. Single women are indicated by the fainted line, and in the equilibria of Type-B they are of both low and high ability.

It is noteworthy that in the Type-B Equilibrium the women of the high home ability type who are single have higher income than the ones of the same ability type who are married and stay at home. This has to be the case since if there were a single woman of high ability with income x' and another woman with high ability who marries and stays at home with income x'', such that $x' < x''$, then $(x'', \bar{\alpha})$'s husband could offer $(x', \bar{\alpha})$ a payoff U for her to stay at home such that $u(x'', \bar{\alpha}) > U > u(x', \bar{\alpha})$. In this case both him and woman $(x', \bar{\alpha})$ would be better off, breaking the equilibrium.

A feature that stands out is that the described equilibria need not be pure, since spouses in couples (x, α, y) where $x \in [0, x_T]$, $\alpha = \bar{\alpha}$ and $y > y^*$ are actually indifferent between any other partner in the same subspace of types. It follows that there are multiple equilibria. Yet, it might be the case that the cutoffs x_T, x_S and y^* are unique, but so far we haven't been able to proof this for a general surplus function.

Both classes of equilibria have the characteristic that there is assortative matching in the income dimension, with the exception of the very top of the men's income distribution, who end up marrying high ability women with low potential income. This negative correlation between spouses' incomes arises from the gains of specialization due to the interaction of the men's income dimension and the women's home production ability dimension.

c A Particular Specification

Here we introduce the primitives of an economy where the total utilities of the households are a particular case of the model we have explored so far. This results in a quadratic g, which is also considered in Quintana-Domeque et al. (2014).

Agents derive utility from the consumption of a marketable private good and also from a public good which is produced within the household and exclusively consumed by its members.

Each person cares only about his or her own consumption of the two types of goods in the economy and individual utility takes the following form:[9]

$$V_i(q_i, P) = q_i P$$

where q_i is the private consumption of agent i and P is the household public good.

This utility is a particular case of the generalized quasi-linear (GQL) form of Bergstrom (1989) and accomplishes two goals.[10] First, utility transfers between spouses can be achieved by shifting the division of the private good and holding public good consumption fixed. Second, there is always a positive surplus from marriage due to the presence of the public good.

The public good can be produced using the private good and time at home $t \in \{0, 1\}$ according to the following production function:

$$P(Q, t, \alpha) = [t\underline{\alpha} + (1-t)\alpha]Q$$

where, Q is the amount of private good used in the production and, as before, $t = 0$ if the woman contributes to home production and $t = 1$ otherwise.

First we consider the decision faced by a single woman with wage dimension x. If a single woman decides to contribute to the home production, she will have no earnings and, therefore, no consumption. As a result, irrespective of their ability dimension, singles always join the workforce. Therefore, their problem can be written as

[9] The same utility is used in Chiappori et al. (2009) and Browning et al. (2014).

[10] The GQL preferences are represented by $u_i(q_i, P) = F(P)q_i + G_i(P)$ and in the case of only one public good and one private good, this form provides necessary and sufficient conditions for transferable utility.

$$\max_{q,Q} \underline{\alpha} q Q$$
$$\text{subject to } q + Q \leq z$$

with resulting utility

$$U^S(z) = \underline{\alpha} z^2/4. \tag{5}$$

Couples, on the other hand, have the possibility of specialization, so labor decisions are now part of the problem. As before, some women have high home ability and, therefore, are capable of increasing the home productivity from $\underline{\alpha}$ to $\bar{\alpha}$.

If either the woman's ability is low or it is efficient for her to join the workforce, then any efficient allocation of resources within the couple where both spouses consume positive amounts must solve

$$\max_{q_F, q_M, Q} \underline{\alpha}(q_F + q_M)Q$$
$$\text{subject to } q_M + q_F + Q \leq x + y$$

generating a total utility for the couple of

$$U^W(x, y) = \underline{\alpha}(x + y)^2/4.$$

Otherwise, if the woman joins the workforce the household problem becomes

$$\max_{q_F, q_M, Q} \bar{\alpha}(q_F + q_M)Q$$
$$\text{subject to } q_M + q_F + Q \leq y,$$

in which case the total output is given by

$$U^H(x, y) = \bar{\alpha} y^2/4.$$

We have previously assumed that the couples' labor and consumption decisions were efficient and this indeed has to be the case in any stable equilibrium. This is true since otherwise a close substitute to one's partner would be able to offer more utility in an efficient union and both would be better off together.

As also noted before, in any equilibrium all the low ability women join the work-

force. Also, because there is a public good, any marriage generates a positive surplus so that in equilibrium only a measure $\mu - 1$ of women are single and every man is married.

Comparing this setting with our more general model, we can see that this is a specific case if we consider the function f as the identity and $g(z) = z^2/4$. These functions satisfy all the restrictions we imposed before and therefore Proposition (1) applies here.

Since single women's utilities are pinned down by equation (5), the whole utility profile of working women is related to $u^W(x_S)$. If both sides had the same mass of individuals (if we allowed $\mu = 1$), then there would be no single person at the stable equilibrium. In this case, the division of marital output between men and women would be pinned down only up to a constant determined by how much surplus each side would get at the worst marriage. In turn, if we considered more men than women, there would be single men in equilibrium and thus the utility of the single man with the highest income would shape the utility profiles.

Finally, we consider uniform income distributions with support in $[0, 1]$. A more complex distribution would play a role in determining the utilities in equilibrium, since the local scarcity of any gender is a well-known distribution factor, but would not change the more general equilibrium patterns.

Since income distributions are uniform, in any Type-A Matching x_T and x_S are given by

$$x_S^a = \frac{\mu - 1}{\mu(1-p)} \qquad\qquad x_T^a = \frac{1-y^*}{\mu p}. \tag{6}$$

The indifferent man y^* is determined by the equality in equation (A.1), together with the definitions in equation (6).

Therefore, we can determine the matching function $\psi(x)$ that describes the husband of woman x as

$$\psi^a(x) = \begin{cases} \mu(1-p)(x - x_S) & \text{if } x \in [x_S, x_T] \quad \text{(7a)} \\ \mu(x - x_T) + \mu(1-p)(x_T - x_S) & \text{if } x \in [x_T, 1] \quad \text{(7b)} \end{cases}$$

Likewise, $\phi(y)$ describes the wife of man y as

$$\phi^a(y) = \begin{cases} x_S + \dfrac{y}{\mu(1-p)} & \text{if } y \in [0, y_0) \quad (8a) \\ \dfrac{y}{\mu} + px_T + (1-p)x_S & \text{if } y \in [y_0, y^*]. \quad (8b) \end{cases}$$

Therefore, the solution to problem (3) can be written as:

$$u^W(\phi^a(y)) = \begin{cases} u^W(x_S^a) + \dfrac{\alpha}{4(1+\mu(1-p))}[(y+\phi^a(y))^2 - (x_S^a)^2] \ \forall y \in [0, y_0) & (9a) \\ u^W(x_T^a) + \dfrac{\alpha}{4(1+\mu)}[(y+\phi^a(y))^2 - (y_0 + x_T^a)^2] \ \forall y \in [y_0, y^*]. & (9b) \end{cases}$$

The same reasoning can be applied to Type-B equilibria, resulting in the following cutoffs:

$$x_S^b = \frac{\mu - y^*}{\mu} \qquad\qquad x_T^b = \frac{1 - y^*}{\mu p}. \tag{10}$$

Therefore, we can define the matching function $\psi(x)$ that describes x's husband if she works as

$$\psi^b(x) = \mu(x - x_S) \text{ if } y \in [x_S, 1] \tag{11}$$

Likewise, $\phi(y)$ describes the y's wife as

$$\phi(y)^b = x_S + \frac{y}{\mu} \text{ if } y \in [0, y^*] \tag{12}$$

Therefore, the solution to problem (3) can be written as:

$$u^W(\phi^b(y)) = u^W(x_S^b) + \frac{\alpha}{4(1+\mu)}[(y+\phi^b(y))^2 - (x_S^b)^2] \ \forall y \in [0, y^*]. \tag{13}$$

In the appendix we provide a proof of Proposition (1) for this specific setting, which also guarantees the uniqueness of the cutoffs.

Proposition 2. If the surplus function is quadratic and the income distributions are uniform, the cutoffs x_T, x_S, and y^* are unique.

Proof. Proved in the appendix. □

5 Taxation and equilibrium outcomes

An advantage of modeling labor and marriage decisions together is the possibility to evaluate the impact of policies in both markets. With the next proposition and the numerical exercise we intend to clarify some of the potential impacts of taxation in the equilibrium outcomes. Before all else, it is important to observe that the proof of Proposition (1) still holds if we introduce a linear income tax for individuals.

The following proposition illustrates how a policy supposedly intended to benefit married women might have the opposite consequence due to equilibrium adjustments.

Proposition 3. Consider a small proportional tax τ on men's income used to increase the potential income of every married woman by Γ and suppose the original equilibrium is of the Type-B Matching with $y^* \in (0,1)$. Then, the cutoff x_T is reduced, and, consequently, the proportion of married women in the labor market is increased. Also, every nonworking married woman is made worse off.

This simple result is a direct consequence of the fact that the utility of single women is not changed in the new equilibrium and that the crowding out of the home activity pushes downwards the utilities of all the women who stay at home.[11] Yet, this type of conclusion cannot be reached in a perfectly assortative unidimensional model.

Now, in a numerical exercise we intend to show how the utility profiles and the welfare division within and across couples change in response to linear taxation. With this example we are also able to expose major differences between the long and short run income elasticities of the marginal high ability woman. In the short run, we expect marriage market equilibrium to be sustained given any small change in the parameters of the model if we assume that there is a strictly positive cost from divorce. On the other hand, as time passes, new marriages are formed and, for reasons exogenous to the model, some marriages are broken, implying a long-run tendency towards a new equilibrium. We find that the long run elasticity is much higher than the short run.

[11] An increase in the potential income of every married women without changing the income profile of men would result in the same type of effects.

For this, we assume that income is taxed at a linear rate t. In order to keep the government budget balance at a constant level, we assume that each individual receives a lump-sum transfer T, which is independent of his or her marital status and sex, and in such a manner that there is no tax revenue.

Also, we choose a specific parametrization such that there is 5% more women than men (μ=1.05), the high ability women have a home productivity three times that of the low ability ($\frac{\bar{\alpha}}{\underline{\alpha}}$=3) and correspond to half of the female population (p=0.5).

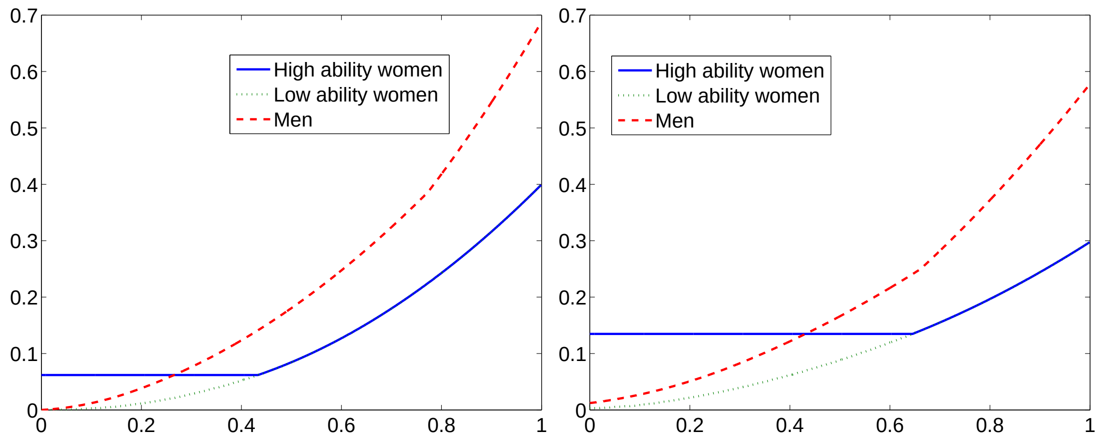
Without any taxes, the equilibrium results in 43% of the high ability women staying at home, which corresponds to 23% of all the marriages.

Next, we assume a welfare function of the form

$$W = \int_{x\in X} w(u(\bar{\alpha}, x))d\bar{\gamma}_x(x) + \int_{x\in X} w(u(\underline{\alpha}, x))d\underline{\gamma}_x(x) + \int_{y\in Y} w(v(y))d\gamma_y(y),$$

where $w(z) = z^{0.5}$.

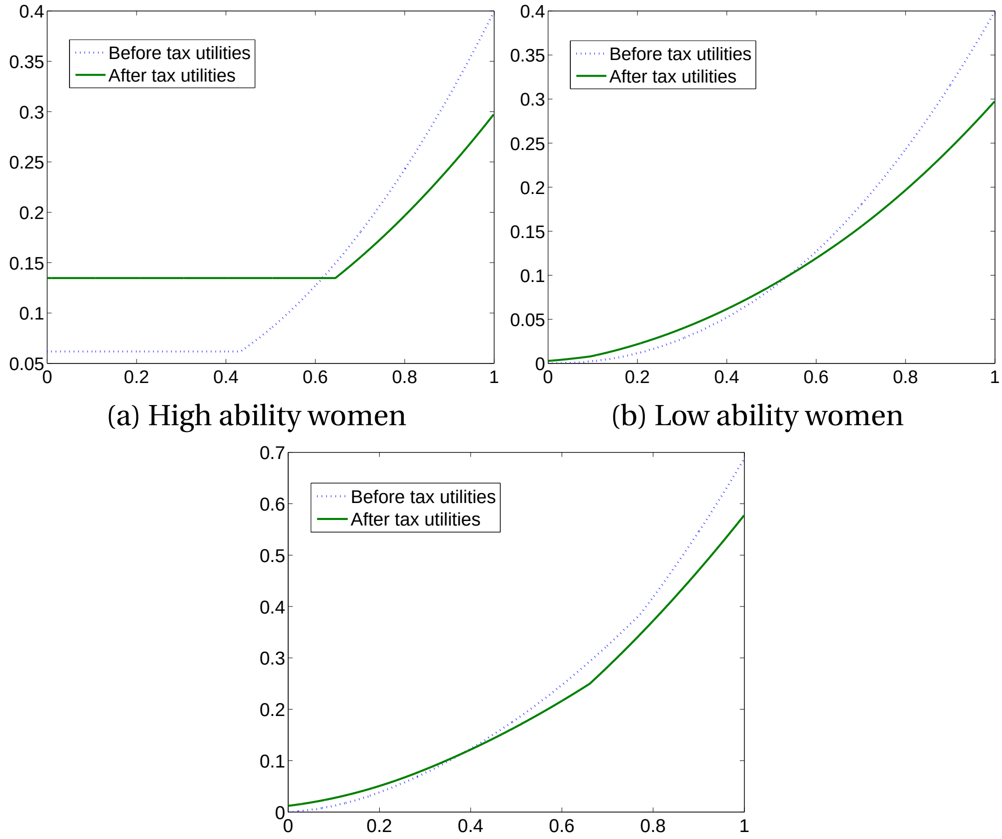
Our taxation design problem is based on an individualistic social welfare function, with inequality both within and across households adversely affecting social welfare. Note that it is necessary to introduce some curvature in the welfare function because otherwise the supermodularity of the couples' total utilities would imply a maximum income inequality in the optimal. With this specification, we obtain an optimal linear tax rate of $t = 23.6\%$ and a corresponding lump-sum transfer of $T = 0.1056$. In this optimal setting total welfare is increased by 2% and we illustrate the variations that occur in the utility profiles of the individuals in Figures (2) and (3).



(a) Without taxes

(b) Under optimal linear tax rate

Figure 2: Utility profiles without taxes and under the optimal linear tax rate.



(a) High ability women (b) Low ability women

(c) Men

Figure 3: Utility profiles for each discrete type.

Clearly, the introduction of taxes benefits the housewives the most. This is mostly due to the mechanical change in the equilibrium cutoff x_T, since with higher taxes more women will decide not to work and the equilibrium payoff of all such women is pinned down by the utility of woman $u^W(x_T)$, which is increasing in x_T.

The introduction of taxes raises the cutoff x_T since it increases the comparative advantage of higher home productivity. The new percentage of high ability women staying at home is 64%, which corresponds to 34% of all the marriages.

From here on, the subscripts 1 and 2 will represent the equilibrium variables in the short and long run, respectively.

In order to draw a parallel, we evaluate the impact of a 1% decrease in the tax rate starting from the optimal taxation (from t=23.6% to t=23.36%), which induces a change in x_T from $x_{T,1} = 0.6449$ to $x_{T,2} = 0.6426$, pushing all women $[x_{T,2}, x_{T,1}]$ into the labor market and leading woman $x_{T,2}$ to be indifferent between marrying man $y_{T,2}$ in $[y_2^*, 1]$ and staying at home or marrying man $\psi_2(x_{T,2})$ and working. Hence, a 1% decrease in the tax rate, keeping tax revenues constant, results in a 0.115% increase

in female labor market participation in the long run.

Now, if we fix the matchings to be as in the first equilibrium, what percentage change in the tax rate would be needed for the woman $x_{T,2}$ to join the workforce? In this case, we should compare the utility generated in the marriage between her and her husband $y_{T,1}$ in $[y_1^*, 1]$ in each situation. This means finding t such that $U^W(x_{T,2}, y_{T,1})|_{t+\delta} > U^H(y_{T,1})|_{t+\delta}$ and $U^W(x_{T,2}, y_{T,1})|_{t-\delta} < U^H(y_{T,1})|_{t-\delta}$ for any $\delta > 0$. Since $y_{T,1}$ might be any men in $[y_1^*, 1]$, there is actually an interval of possible tax rates. If $x_{T,2}$ is married to y_1^*, the indifference tax rate would be t=19.46%, whereas if $x_{T,2}$ is married to $y = 1$, there is no positive tax rate that would make the couple better of if $x_{T,2}$ decided to work.

Consequently, we see that in the short run, the minimum percentage decrease in the tax rate starting from the optimal that would prompt woman $x_{T,2}$ to work is 17.6% (from t=23.6% to t=19.46%), which is much higher than the 1% needed in the long run. Since similar results are obtained for different parameter specifications, these numbers illustrate the potential relevance of modeling the labor and marriage markets together, specially for a better understanding of the long run elasticities of labor supply.

In this example, the labor market participation elasticity in the short run would be zero, since small changes in wages would have no effect in female labor market participation because of the wedge between the husband woman x_T has in equilibrium if she works ($\psi(x_T) < y^*$) and the man she marries if she stays at home ($y \in [y^*, 1]$). Still, the contrast of the 1% needed in the long run vs. the 17.6% needed in the short run can be interpreted as a comparison between some kind of inverse female labor participation elasticities.

Of course this expressive difference of one order of magnitude might be related to some of the stylized characteristics of the model. For example, since we only model labor participation decisions and do not allow individuals to adjust hours of work, all the labor supply responses to changes in the income tax policy are in the extensive margin. If we allowed intensive margin responses, possibly the adjustments in the marriage market would be less drastic, with women maintaining the same spouses (or close substitutes to them) and making fine adjustments of their time allocations.

In this case, the introduction of an extra degree of freedom in labor choices would

possibly make participation decisions less sensitive to tax changes and perhaps the differences seen in the long and short run ”participation elasticities” would be less pronounced. Nevertheless, this example indicates an important and not explored mechanism affecting labor supply responses, at least in the long run.

6 Conclusion

In this paper we have presented an integrated model of the marriage market and female labor supply. The model main features are the bidimensional traits of women, our specification of how those traits are able to interact in the generation of marital surplus, and, of course, the inclusion of women labor decisions. These characteristics are relevant since multidimensional attributes have not been greatly explored in the literature and the papers that have ventured in this direction lack at least one of the other two cited features.

Our model is able to generate non-monotonic matching on income on the marriage market, since the men at the top of the income distribution forgo marrying their female counterparts in order to mate with high home ability women.

For a particular setting, where the surplus function is quadratic and the parameters of the model are chosen, we illustrate the impacts of policies for the welfare at the individual level. We see that a great share of welfare gains is due to an increment in the utility of housewifes, essentially because with higher taxes more women choose not to join the labor market and the equilibrium payoff of all housewifes is determined by the outside option of the one with the highest income.

Moreover, in a numerical exercise, we show that there are major differences between short and long run labor participation responses to taxes. In our specific example, we find that a 1% decrease in the tax rate is how much it would take for a marginal woman to join the labor market in the long run, that is, if we allow marriage market equilibrium adjustments. On the other hand, if we fix the matchings, it would take at least a 17.56% decrease in the tax rate in order for the same woman to change her labor participation decision. We believe this indicates that the interaction of these two markets is indeed important for the understanding of policy responses in the long run, and therefore should be explored more deeply.

Appendices

a Proof of Proposition 1

First note that it must be the case that every high productivity woman who does not work in equilibrium gets the same payoff u^H. Suppose this is not the case, so there exists two women $(x_1, \bar{\alpha})$ and $(x_2, \bar{\alpha})$ who do not work in equilibrium and such that $u(x_1, \bar{\alpha}) > u(x_2, \bar{\alpha})$. Their husbands are denoted respectively y_1 and y_2, with payoffs $v(y_1)$ and $v(y_2)$. Then, since $U^W(x_1, y_1) = U^W(x_2, y_2) = U^W(x_2, y_1)$, it would be possible for x_2 to offer man y_1 payoff $v(y_1) + \delta$ and get herself a payoff $u(x_1, \bar{\alpha}) - \delta$, which is strictly higher than $u(x_2, \bar{\alpha})$ for some $\delta > 0$ close enough to zero.

Second, suppose the woman $(x, \bar{\alpha})$ works, marries the man $\psi(x)$, and get payoff $u^W(x)$ in equilibrium. Then, since any woman $(x', \bar{\alpha})$ with $x' > x$ can also marry $\psi(x)$, work and get payoff $U^W(x', \psi(x)) - v(\psi(x)) > U^W(x, \psi(x)) - v(\psi(x)) > u^H$, therefore in equilibrium $(x', \bar{\alpha})$ also works. In the same way, if the woman $(x, \bar{\alpha})$ marries and stays at home in the equilibrium, then any woman $(x', \bar{\alpha})$ with $x' < x$ also stays at home, since $u^H > U^W(x, y) - v(y) > U^W(x', y) - v(y) \, \forall \, y$.

These reasoning implies that if there is an equilibrium, there must be a cutoff x_T such that every woman $(x, \bar{\alpha})$ with $x > x_T$ works and every woman $(x, \bar{\alpha})$ with $x < x_T$ stays at home.

Now we consider men. For this, we define the following function

$$Z(x, y) \equiv U^W(x, y) - u^W(x) - U^H(y) + u^H.$$

We can write its derivative with respect to y as

$$Z'(x, y) \equiv \frac{\partial Z(x, y)}{\partial y} = f(\underline{\alpha})g'(x + y) - f(\bar{\alpha})g'(y)$$

and the second derivative is

$$Z''(x, y) \equiv \frac{\partial^2 Z(x, y)}{\partial y^2} = f(\underline{\alpha})g''(x + y) - f(\bar{\alpha})g''(y) < 0$$

where the inequality follows from the assumption that $g'''(.) \leq 0$ and $\bar{\alpha} > \underline{\alpha}$.

Suppose by contradiction there exists $\hat{y} > \underline{y}$ such that every man $y < \hat{y}$ marries a

woman who does not work in equilibrium and, with abuse of notation, let $\hat{y}$ denote the man with the highest income among those. Because $\mu p < 1$, $\hat{y} < \bar{y}$. Since man $\hat{y}+\delta$ marries a woman who works in equilibrium, by the continuity of the equilibrium utilities, it must be that $\hat{y}$ is indifferent between marrying a woman x' who works or any woman who stays at home. Also, because the matching is assortative among the marriages where both spouses work and the income measures are absolutely continuous, it must be that $x' = x_S$.

Then $Z(x_S, \hat{y}) = 0$ and $Z(x_S, 0) = f(\underline{\alpha})g(x_S) + u^H - u^W(x_S)$.

If $x_S < x_T$, $Z(0) > 0$. Else, $u^W(x_S) = f(\underline{\alpha})g(x_S)$, and $Z(0) > 0$. Since $Z'' < 0$ and $Z(x_S, 0) > Z(x_S, \hat{y})$, it must be that $Z'(x_S, \hat{y}) < 0$. But then $Z(x_S, \hat{y} - \delta) > 0$ for $\delta > 0$ small enough, a contradiction.

Therefore, there exists y' such that every man $y \leq y'$ marries a woman who works in equilibrium. With abuse of notation, let y' be the man with the highest income among them. Then either $y' = \bar{y}$ and all men marry women who work, or $y' < \bar{y}$ and y' is indifferent between marrying a woman who stays at home and a woman $x' = \phi(y')$ who works, since for any $\delta > 0$ small enough we have that $U^W(\phi(y' - \delta), y' - \delta) - u(\phi(y' - \delta), \bar{\alpha}) \geq U^H(y' - \delta) - u^H$ and $U^W(\phi(y' + \delta), y' + \delta) - u(\phi(y' + \delta), \bar{\alpha}) \leq U^H(y' + \delta) - u^H$ and all functions are continuous. Suppose by contradiction that there is a man $y'' > y'$ who marries a woman x'' who works in equilibrium. Then, by the same argument as before, there must be a man $y''' \in (y', y'')$ such that he is also indifferent between marrying a woman x''' who works and another woman who stays at home, and such that every men in (y', y''') prefer to marry women who stay at home. Note that it must be the case that $x' = x'''$, since for couples where both spouses work marriage is positive assortative in the income dimension and the measures are absolutely continuous. This implies that $Z(x', y') = Z(x', y''') = 0$.

Because we assumed that every man y in $y \in (y', y''')$ marries a woman who stays at home in equilibrium, it must be that $Z'(x', y') \leq 0$, a contradiction with $Z(x', y''') = 0$, since $Z'' < 0$.

Finally note that if we applied a transformation I, such that $I' > 0$ and $I'' = 0$, to the income of all couples, the signs of the derivatives of Z would be unchanged and the same analysis would hold. Therefore, for the linear tax schedule $I(z) = (1 - t)z + 2T$, the proof still holds. For a progressive tax schedule we could give con-

ditions involving up to the third derivative of I so that the proposition would also hold. In this case, more restrictions are necessary, since otherwise the progressivity of the tax schedule could also offset the supermodularity of the surplus function, and undermine the remaining assortativeness.

b Proof of Proposition 2

We provide a constructive proof of the existence of the equilibrium for the specific case of quadratic surplus and uniform income distributions. In this particular case, we show that there is only one possible candidate for y^* for each parameter vector, and that therefore the cutoffs are unique.

In order to prove that there exists a stable matching of the proposed form, it is sufficient to show that we can find $y^* \in [0, 1]$ such that the following three equations hold:

$$v(y) = U^H(y) - u^H \geq U^W(x, y) - u^W(x) \; \forall y \geq y^* \; \forall x \tag{A.1}$$

$$v(y) = U^W(\phi(y), y) - u^W(\phi(y)) > U^H(y) - u^H \; \forall y < y^* \; \forall x. \tag{A.2}$$

$$v(y) = U^W(\phi(y), y) - u^W(\phi(y)) > U^W(x, y) - u^W(x) \; \forall y < y^* \; \forall x. \tag{A.3}$$

If this is the case, no man will be able to provide the equilibrium payoff to any woman and get himself a payoff higher than what he gets with his own wife in any equilibrium.

Because of the definition of a stable matching, we were able to define the individual payoffs as in (4). Therefore, we have that

$$U^W(\phi(y), y) - u(\phi(y)) \geq U^W(x, y) - u^W(x) \; \forall x \; \forall y \in [0, y^*]. \tag{A.4}$$

This allows us to omit equation (A.3).

Also, because of the supermodularity of $U^W(x, y)$, we have that

$$U^W(1, y) + U^W(x, y^*) \geq U^W(x, y) + U^W(1, y^*) \; \forall y \geq y^* \; \forall x \in [0, 1]. \tag{A.5}$$

Combining (A.4) and (A.5), we have that

$$U^W(1,y) - u(1) \geq U^W(x,y) - u^W(x) \; \forall x \; \forall y \geq y^*.$$

Given this, it is possible to substitute equations (A.1) to (A.3) by the following pair:

$$U^H(y) - u^W(x_T) \geq U^W(1,y) - u^W(1) \text{ for } y \geq y^* \tag{A.6}$$

$$U^H(y) - u^W(x_T) < U^W(\phi(y),y) - u^W(\phi(y)) \text{ for } y < y^*. \tag{A.7}$$

Which is equivalent to

$$\frac{\bar{\alpha}}{\underline{\alpha}} y^2 - \frac{4}{\underline{\alpha}} u^W(x_T) - (y+1)^2 + \frac{4}{\underline{\alpha}} u^W(1) \geq 0 \text{ for } y \geq y^* \tag{A.8}$$

$$\frac{\bar{\alpha}}{\underline{\alpha}} y^2 - \frac{4}{\underline{\alpha}} u^W(x_T) - (y+\phi(y))^2 + \frac{4}{\underline{\alpha}} u^W(\phi(y)) < 0 \text{ for } y < y^* \tag{A.9}$$

First we analyze the polynomials of the Type-A Matching and prove some lemmas. Next we do the same for equilibria of the Type-B Matching .

Using equation (9b) and equation (A.8) with equality we can construct a polynomial in order to determine the candidates for y^*.

$$\begin{aligned} P_a(y) &= ay^2 + by + c \\ a &= \frac{\bar{\alpha}}{\underline{\alpha}} - \frac{\mu}{1+\mu} - \frac{1}{1+\mu}\frac{(\mu(1-p)+1)^2}{\mu^2 p^2} \\ b &= -2\frac{\mu}{1+\mu} - \frac{1}{1+\mu}\left[-2\frac{(\mu(1-p)+1)^2}{\mu^2 p^2} + 2(\mu-1)\frac{(\mu(1-p)+1)}{\mu p}\right] \\ c &= -\frac{\mu}{1+\mu} - \frac{1}{1+\mu}\left[(\mu-1)^2 + \frac{(\mu(1-p)+1)^2}{\mu^2 p^2} - 2(\mu-1)\frac{(\mu(1-p)+1)}{\mu p}\right] < 0. \end{aligned} \tag{A.10}$$

Lemma 1. If $\frac{\bar{\alpha}}{\underline{\alpha}} \geq 1+\mu$ then equation (A.10) has a unique root $r_a \in [0,1]$. Else, the polynomial (A.10) has no root in $[0,1]$ and it is always negative in this region.

Proof. All the following analysis is based on the fact that equation (A.10) is a second degree polynomial.
If $a > 0$, since $c < 0$, there exists a root $\in [0,1]$ if and only if $P_a(1) = a+b+c = \frac{\bar{\alpha}}{\underline{\alpha}} - (1+\mu) \geq 0$, in which case this is the unique root in this interval.

In the event that $a < 0$ and $P_a(1) = a+b+c = \frac{\bar{\alpha}}{\underline{\alpha}} - (1+\mu) > 0$, because $P_a(0) = c < 0$, there exists a root $\in [0,1]$ and it is unique in this interval. Now, we consider the event

that $P_a(1) = a+b+c = \frac{\bar{\alpha}}{\underline{\alpha}} - (1+\mu) \leq 0$ and we have two possible cases. If $P_a'(0) = b < 0$ then there are no roots $\in [0,1]$. Else, if $P_a'(0) = b > 0$ and $a > c$, then there are either two roots or no root $\in [0,1]$. Suppose by contradiction that there are two roots $\in [0,1]$. Then $P_a'(1) = 2a + b < 0$ so that $b^2 < 4a^2 < 4ac$, a contradiction. So there are no roots in this case. On the other hand, if $P_a'(0) = b > 0$ and $a < c$, then

$$\begin{aligned} b + 2c &= \frac{2}{1+\mu}\left[\frac{\mu-1}{\mu p}(\mu(1-p)+1) - (\mu-1)^2 - 2\mu\right] \\ &\leq \frac{2}{1+\mu}\left[\mu(1-p) + 1 - (\mu-1)^2 - 2\mu\right] \\ &\leq \frac{2}{1+\mu}\left[\mu + 1 - (\mu-1)^2 - 2\mu\right] \\ &= \frac{-2(\mu-1)}{1+\mu}\mu < 0, \end{aligned}$$

implying that $b^2 < 4c^2 < 4ac$, so there is also no roots $\in [0,1]$ in this case. $\square$

Lemma 2. Suppose $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \mu + 1$ and $r_a \in [1 - \mu p, \frac{1-\mu p}{1-p}]$. Then equations (A.8) and (A.9) hold with $y^* = r_a$, x_T and x_S as in (6) and, $\phi^a(y)$ as in (8b).

Proof. If $y \in [r_a, 1]$, then the LHS of (A.8) can be represented by

$$R_a(y) = \ddot{a}y^2 + \dddot{b}y + \ddot{c}$$

$$\ddot{a} = \frac{\bar{\alpha}}{\underline{\alpha}} - 1 > 0$$

$$\dddot{b} = -2 < 0$$

$$\ddot{c} = \frac{1}{1+\mu}[(r_a+1)^2 - (y_0 + x_T)^2] - 1.$$

Numerically, we are able to see that for every possible values of the parameters, we have that

$$R_a'(r_b) = 2(r_a(\frac{\bar{\alpha}}{\underline{\alpha}} - 1) - 1) > 0.$$

Hence, r_a must be the second root of polynomial R_a, and therefore $R_a(y) \geq 0$ for every $y \geq r_a$.

Now we suppose $y < r_a$. Then the function $\phi^a(y)$ that determines the wife of man y in (8b) has two distinct functional forms, therefore in order to represent the LHS of (A.9), we break the analysis into two regions.

(i) $y \in [0, y_0)$

$$Q_a(y) = \ddot{a}y^2 + \ddot{b}y + \ddot{c}$$
$$\ddot{a} = \frac{\bar{\alpha}}{\underline{\alpha}} - \frac{\mu}{1+\mu}\left[\frac{(1+\mu(1-p))^2}{\mu^2(1-p)^2}\right]$$
$$\ddot{b} = -2\frac{\mu(\mu-1)}{1+\mu}\frac{(\mu(1-p)+1)}{\mu^2(1-p)^2} < 0$$
$$\ddot{c} = \frac{-1}{1+\mu}[\mu(x_S^a)^2 + (y_0 + x_T^a)^2] < 0.$$

If $\ddot{a} > 0$, since $\ddot{c} < 0$ and $Q_a(y_0) = Q_a(y_0) < 0$, then $Q_a(y) < 0\, \forall y \in [0, y_0]$.

If $\ddot{a} < 0$, since $Q_a'(0) = \ddot{b} < 0$ and $\ddot{c} < 0$, then $Q_a(y) < 0\, \forall y \in [0, \infty)$.

(ii) $y \in [y_0, r_a]$

$$Q_a(y) = \dot{a}y^2 + \dot{b}y + \dot{c}$$
$$\dot{a} = \frac{\bar{\alpha}}{\underline{\alpha}} - \frac{1+\mu}{\mu} > 0$$
$$\dot{b} = -2\frac{\mu - r_a}{\mu} < 0$$
$$\dot{c} = \frac{-1}{1+\mu}[\mu(px_T^a + (1-p)x_S^a)^2 + (y_0 + x_T^a)^2] < 0.$$

Since $\dot{a} > 0$, $\dot{c} < 0$, and $Q_a(r_a) = P_a(r_a) = 0$, we have that $Q_a(y) \leq 0\, \forall y \in [0, r_a]$.

$\square$

Now we focus on the polynomial that arises when we consider Type-B Matching equilibria.

Using equation (13) and equation (A.8) with equality we can construct a polynomial in order to determine the candidates for y^*.

$$\begin{aligned} P_b(y) &= ay^2 + by + c \\ a &= \frac{\bar{\alpha}}{\underline{\alpha}} - \frac{\mu}{1+\mu} + \frac{1}{\mu(1+\mu)} - \frac{1}{\mu^2p^2} \\ b &= 2[\frac{1}{\mu^2p^2} - 1] > 0 \\ c &= -\frac{1}{\mu^2p^2} < 0 \end{aligned} \tag{A.11}$$

Lemma 3. If $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \frac{3\mu-1}{\mu}$ then equation (A.11) has a unique root $r_b \in [0, 1]$. Else, the polynomial (A.11) has no root $\in [0, 1]$ and it is always negative in this region.

Proof. All the following analysis is based on the fact that equation (A.11) is a second degree polynomial.

If $a > 0$, since $c < 0$, there exists a root $\in [0,1]$ if and only if $P_b(1) = a+b+c = \frac{\bar{\alpha}}{\underline{\alpha}} + \frac{1-3\mu}{\mu} \geq 0$, in which case this is the unique root in this interval.

In the event that $a < 0$ and $P_b(1) = a+b+c = \frac{\bar{\alpha}}{\underline{\alpha}} + \frac{1-3\mu}{\mu} \geq 0$, because $P_b(0) = c < 0$, there exists a root $\in [0,1]$ and it is unique in this interval. Now, we consider the event that $P_b(1) = a+b+c = \frac{\bar{\alpha}}{\underline{\alpha}} + \frac{1-3\mu}{\mu} < 0$ and we have two possible cases. Note that $P_b'(0) = b > 0$. If $a > c$, then there are either two roots or no root $\in [0,1]$. Suppose by contradiction that there are two roots $\in [0,1]$. Then $P_b'(1) = 2a + b < 0$ so that $b^2 < 4a^2 < 4ac$, a contradiction. So there are no roots in this case. On the other hand, if $a < c$, then

$$b + 2c = -2\frac{1}{1+\mu} - 2\frac{\mu}{1+\mu} < 0,$$

implying that $b^2 < 4c^2 < 4ac$, so there is also no roots $\in [0,1]$ in this case. □

Lemma 4. Suppose that $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \frac{3\mu-1}{\mu}$ and $r_b > \frac{1-\mu p}{1-p}$. Then equations (A.8) and (A.9) hold with $y^* = r_b$, x_T and x_S as in (10) and, $\phi^b(y)$ as in (12).

Proof. If $y \in [r_b, 1]$ the LHS of (A.8) can be represented by

$$R_b(y) = \ddot{a}y^2 + \ddot{b}y + \ddot{c}$$
$$\ddot{a} = \frac{\bar{\alpha}}{\underline{\alpha}} - 1 > 0$$
$$\ddot{b} = -2 < 0$$
$$\ddot{c} = \frac{\mu}{1+\mu}(x_S^2 - 1) - x_T^2.$$

Numerically, we are able to see that for every possible values of the parameters, we have that

$$R_b'(r_b) = 2(r_b(\frac{\bar{\alpha}}{\underline{\alpha}} - 1) - 1) > 0.$$

Hence, r_b must be the second root of polynomial R_b, and therefore $R_b(y) \geq 0$ for every $y \geq r_b$.

Now, if $y < r_b$, we can use the function $\phi^b(y)$ that determines the wife of man y in

(12) to represent the LHS of (A.9).

$$Q_b(y) = \dot{a}y^2 + \dot{b}y + \dot{c}$$
$$\dot{a} = \frac{\bar{\alpha}}{\underline{\alpha}} - \frac{1+\mu}{\mu}$$
$$\dot{b} = -2x_S^b < 0$$
$$\dot{c} = -(x_T^b)^2 < 0.$$

If $\dot{a} > 0$, since $\dot{c} < 0$ and $Q_b(r_b) = Q_b(r_b) < 0$, then $Q_b(y) < 0\,\forall y \in [0, r_b]$.

If $\dot{a} < 0$, since $Q_b'(0) = \dot{b} < 0$ and $\dot{c} < 0$, then $Q_b(y) < 0\,\forall y \in [0, \infty)$. $\square$

Lemma 5. $r_a < \frac{1-\mu p}{1-p}$ iff $r_b < \frac{1-\mu p}{1-p}$

Proof. From equation (A.8) we have that

$$r_a^2[\frac{\bar{\alpha}}{\underline{\alpha}} - \frac{\mu}{1+\mu}] - 2\frac{\mu}{1+\mu}r_a - \frac{\mu}{1+\mu} = \frac{1}{1+\mu}[x_T^a + \psi(x_T^a)]^2 > 0 \qquad \text{(A.12)}$$

and

$$r_b^2[\frac{\bar{\alpha}}{\underline{\alpha}} - \frac{\mu}{1+\mu}] - 2\frac{\mu}{1+\mu}r_b - \frac{\mu}{1+\mu} = -\frac{\mu}{1+\mu}(x_S^b)^2 + (x_T^b)^2. \qquad \text{(A.13)}$$

First note that the LHS of equations (A.12) and (A.13) can be written as a quadratic equation $V(r) = ar^2 + br + c$ with $a > 0$ and $c < 0$. Since $V(r_a) = ar_a^2 + br_a + c > 0$ and $r_a > 0$ it must be that $V'(r_a) > 0$.

Suppose by contradiction $\exists(\mu, p, \bar{\alpha})$ st. $r_a < \frac{1-\mu p}{1-p}$ but $r_b > \frac{1-\mu p}{1-p}$. Because $r_b > r_a$, we also have that $V'(r_b) > 0$. Again, because $r_b > r_a$, this implies that $V(r_b) > V(r_a) > 0$. Therefore,

$$-\frac{\mu}{1+\mu}(x_S^b)^2 + (x_T^b)^2 > \frac{1}{1+\mu}[x_T^a + \psi(x_T^a)]^2.$$

Rearranging, we have that

$$0 \geq \frac{\mu}{1+\mu}[(x_T^b)^2 - (x_S^b)^2] > \frac{1}{1+\mu}[(x_T^a)^2 - (x_T^b)^2] + \frac{1}{1+\mu}[\psi(x_T^a)^2 + 2x_T^a\psi(x_T^a)] > 0,$$

where the first inequality follows from $r_b > \frac{1-\mu p}{1-p}$ and the third from $0 < r_a < r_b$.

Now suppose by contradiction $\exists(\mu, p, \bar{\alpha})$ st. $r_a > \frac{1-\mu p}{1-p}$ but $r_b < \frac{1-\mu p}{1-p}$. In this case, we have that $x_T^a < x_S^a$ and $x_S^b < x_T^b$. Since $r_a > r_b$ we have that $x_T^b > x_T^a$, which implies

that $x_S^b > x_S^a$.

Because $r_a > r_b$, we have that $V(r_a) > V(r_b)$, and, therefore,

$$\frac{1}{1+\mu}[x_T^a + \psi(x_T^a)]^2 > \frac{1}{1+\mu}(x_S^b)^2 - [(x_S^b)^2 - (x_T^b)^2] > \frac{1}{1+\mu}(x_S^b)^2 > \frac{1}{1+\mu}(x_S^a)^2,$$

But from (7b) we have that

$$x_T^a + \psi(x_T^a) = -\mu(1-p)x_S^a + (1+\mu(1-p))x_T^a > -x_S^a$$
$$x_T^a + \psi(x_T^a) = x_S^a - (1+\mu(1-p))(x_S^a - x_T^a) < x_S^a.$$

because $r_b < \frac{1-\mu p}{1-p} < 1$ implies that $x_S^b > 0$ and $x_T^b > 0$. □

Lemma 6. First suppose $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \mu + 1$. Then both equations (A.10) and (A.11) have unique solutions $\in [0,1]$, respectively r_a and r_b. If $r_a \in [1-\mu p, \frac{1-\mu p}{1-p}]$, then $1 > x_T \geq x_S > 0$ are as calculated in (6), and equations (A.6) and (A.7) hold with $y^* = r_a$. If $r_a < 1-\mu p$, this equations hold with $y^* = 1-\mu p$. If $r_a > \frac{1-\mu p}{1-p}$, then $x_T < x_S$ are as calculated in (10) and equations (A.6) and (A.7) hold with $y^* = r_b$.
Now consider that $\frac{\bar{\alpha}}{\underline{\alpha}} \in [\frac{3\mu-1}{\mu}, \mu+1]$. Then (A.11) still has a unique solution $\in [0,1]$, r_b, and equations (A.6) and (A.7) hold with $y^* = r_b$.
Finally, if $\frac{\bar{\alpha}}{\underline{\alpha}} < \frac{3\mu-1}{\mu}$, then equations (A.6) and (A.7) hold with $y^* = 1$.

Proof. From Lemma (1), we know that if $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \mu+1$, then equation (A.10) has a unique solution $r_a \in [0,1]$ and if $r_a \in [1-\mu p, \frac{1-\mu p}{1-p}]$, then $1 > x_T \geq x_S > 0$ are as calculated in (6), and equations (A.6) and (A.7) hold with $y^* = r_a$.

If $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \mu+1$ and $r_a < 1-\mu p$, then equations (A.6) and (A.7) hold with $y^* = 1-\mu p$.

If $\frac{\bar{\alpha}}{\underline{\alpha}} \geq \mu+1$ and $r_a > \frac{1-\mu p}{1-p}$, we know from Lemma (3) that equation (A.11) has a unique solution $r_b \in [0,1]$ and from Lemma (5) that $r_b > \frac{1-\mu p}{1-p}$. Then, from Lemma (4), equations (A.6) and (A.7) hold with $y^* = r_b$ and x_T and x_S as calculated in (10).

Now consider that $\frac{\bar{\alpha}}{\underline{\alpha}} \in [\frac{3\mu-1}{\mu}, \mu+1]$. Then, from Lemma (3) (A.11) still has a unique solution $r_b \in [0,1]$ and equations (A.6) and (A.7) hold with $y^* = r_b$.

Finally, if $\frac{\bar{\alpha}}{\underline{\alpha}} < \frac{3\mu-1}{\mu}$, then equations (A.6) and (A.7) hold with $y^* = 1$. □

c Proof of Proposition 3

For the proposed transfer system, the government budget balance implies that $T = \frac{\tau}{2y^*}$.

First, note that since in equilibrium $P_b(1) > 0$ and y^* is the only root in the interval $(0, 1)$, P_b is increasing at y^*.

It suffices to prove that the introduction of a differential tax reduces the proportion of women who stay at home. For this, we look at how y^* changes.

The function to be considered in this case is

$$f_0(y,\tau) = \frac{\bar{\alpha}}{\underline{\alpha}}[(1-\tau)y]^2 - \frac{\mu}{1+\mu}[(1-\tau)y+1+\Gamma]^2 - \frac{1}{1+\mu}[x_S+\Gamma]^2 + x_T^2 - x_S^2$$

$$\left.\frac{\partial f_0(y,\tau)}{\partial y}\right|_{(y=y^*,\tau=0)} = 2\frac{\bar{\alpha}}{\underline{\alpha}}y^* - 2(1+y^*)\frac{\mu}{1+\mu} - 2\frac{x_S}{1+\mu} - 2\frac{x_T}{\mu p}$$

$$\left.\frac{\partial f_0(y,\tau)}{\partial \tau}\right|_{(y=y^*,\tau=0)} = -2\frac{\bar{\alpha}}{\underline{\alpha}}y^{*2} + 2y^*(1+y^*)\frac{\mu}{1+\mu}.$$

Therefore,

$$\left.\frac{\partial f_0(y,\tau)}{\partial \tau}\right|_{(y=y^*,\tau=0)} = -y^*\left.\frac{\partial f_0(y,\tau)}{\partial y}\right|_{(y=y^*,\tau=0)} - 2y^*\frac{x_S}{1+\mu} - 2y^*\frac{x_T}{\mu p} < 0,$$

where the last inequality follows from $\left.\frac{\partial f_0(y,\tau)}{\partial y}\right|_{(y=y^*,\tau=0)} = \left.\frac{\partial P_b(y)}{\partial y}\right|_{(y=y^*)} > 0$.

If y^* changes as to maintain an equilibrium, then

$$\left.\mathrm{d}f_0\right|_{(y=y^*,\tau=0)} = \left.\frac{\partial f_0}{\partial \tau}\right|_{(y=y^*,\tau=0)}\mathrm{d}\tau + \left.\frac{\partial f_0}{\partial y}\right|_{(y=y^*,\tau=0)}\mathrm{d}y = 0,$$

which implies that $\mathrm{d}y > 0$ for $\mathrm{d}\tau > 0$. Therefore the introduction of a small tax changes the threshold of women who stay at home to $x_T' < x_T$. Since $U^S(x_T') < U^S(x_T)$, every non-working women $x \in [0, x_T']$ is made worse-off.

References

Apps, P. F. and R. Rees (1988). Taxation and the household. *Journal of Public Economics 35*(3), 355–369.

Becker, G. S. (1973). A theory of marriage: Part i. *Journal of Political Economy 81*(4), 813–46.

Bergstrom, T. C. (1989). A Fresh Look at the Rotten Kid Theorem–and Other Household Mysteries. *Journal of Political Economy 97*(5), 1138–59.

Browning, M., P.-A. Chiappori, and Weiss (2014). *Economics of the Family*. Number 9780521795395 in Cambridge Books. Cambridge University Press.

Chiappori, P.-A. (1988). Rational Household Labor Supply. *Econometrica 56*(1), 63–90.

Chiappori, P.-A., R. Blundell, and C. Meghir (2002). Collective labour supply with children. IFS Working Papers W02/08, Institute for Fiscal Studies.

Chiappori, P.-A., M. C. Dias, and C. Meghir (2015). The Marriage Market, Labor Supply and Education Choice. NBER Working Papers 21004, National Bureau of Economic Research, Inc.

Chiappori, P.-A., M. Iyigun, and Y. Weiss (2009). Investment in schooling and the marriage market. *American Economic Review 99*(5), 1689–1713.

Chiappori, P.-A., R. J. McCann, and L. P. Nesheim (2010). Hedonic price equilibria, stable matching, and optimal transport: Equivalence, topology, and uniqueness. *Economic Theory 42*(2), 317–354.

Chiappori, P.-A. and S. Oreffice (2008). Birth control and female empowerment: An equilibrium analysis. *Journal of Political Economy 116*(1), 113–140.

Chiappori, P.-A., S. Oreffice, and C. Quintana-Domeque (2012). Fatter attraction: Anthropometric and socioeconomic matching on the marriage market. *Journal of Political Economy 120*(4), 659–695.

Choo, E. and A. Siow (2006). Who Marries Whom and Why. *Journal of Political Economy 114*(1), 175–201.

Dupuy, A. and A. Galichon (2012, October). Personality Traits and the Marriage Market. IZA Discussion Papers 6943, Institute for the Study of Labor (IZA).

Ekeland, I. (2010). Existence, uniqueness and efficiency of equilibrium in hedonic markets with multidimensional types. *Economic Theory 42*(2), 275–315.

Gayle, G.-L. and A. Shephard (2015, Feb). Optimal taxation, marriage, home production, and family labor supply. Available online at `https://www.economicdynamics.org/meetpapers/2015/paper_882.pdf`.

Lise, J. and S. Seitz (2011). Consumption inequality and intra-household allocations. *Review of Economic Studies 78*(1), 328–355.

Low, C. (2015, Apr). Pricing the biological clock: Reproductive capital on the us marriage market. Available online at `http://assets.wharton.upenn.edu/~corlow/Low_RepCap_latest.pdf`.

Quintana-Domeque, C., P.-A. Chiappori, and S. Oreffice (2014). Bidimensional Matching with Heterogeneous Preferences: Smoking in the Marriage Market. Economics Series Working Papers 696, University of Oxford, Department of Economics.

Roy, A. D. (1951). Some thoughts on the distribution of earnings. *Oxford Economic Papers 3*(2), 135–146.

Salanié, B. and A. Galichon (2015, Feb). Cupid's invisible hand: Social surplus and identification in matching models. Available online at `http://ssrn.com/abstract=1804623`.

Shapley, L. and M. Shubik (1971). The assignment game i: The core. *International Journal of Game Theory 1*(1), 111–130.

Shimer, R. and L. Smith (2000). Assortative matching and search. *Econometrica 68*(2), 343–369.