FUNDAÇÃO GETULIO VARGAS
ESCOLA de PÓS-GRADUAÇÃO em ECONOMIA

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The Long Term Effects of Bolsa Família on Child Labour and School Enrollment

Rio de Janeiro
2015

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Dissertação para obtenção do grau de mestre apresentada à Escola de Pós-Grauação em Economia

Área de concentração: Macroeconomia do Crescimento

Orientador: Pedro Ferreira Cavalcanti

Rio de Janeiro
2015

Ficha catalográfica elaborada pela Biblioteca Mario Henrique Simonsen/FGV

Peruffo, Marcel Cortes
The long term effects of Bolsa Família on child labour and school enrollment / Marcel Cortes Peruffo. – 2015.
42 f.

Dissertação (mestrado) - Fundação Getulio Vargas, Escola de Pós-Graduação em Economia.
Orientador: Pedro Cavalcanti Ferreira.
Inclui bibliografia.

1. Macroeconomia. 2. Programa Bolsa Família (Brasil). 3. Avaliação de pesquisa (Programas de ação social). 4. Programas de sustentação de renda. 5. Transferências de recursos financeiros governamentais. 6. Trabalho infantil. 7. Educação. I. Ferreira, Pedro Cavalcanti. II. Fundação Getulio Vargas. Escola de Pós-Graduação em Economia. III. Título.

CDD – 339

MARCEL CORTES PERUFFO

THE LONG TERM EFFECTS OF BOLSA FAMÍLIA ON CHILD LABOUR AND SCHOOL ENROLLMENT.

Dissertação apresentada ao Curso de Mestrado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Mestre em Economia.

Data da defesa: 30/04/2015

ASSINATURA DOS MEMBROS DA BANCA EXAMINADORA

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Não tenho como estimar o tamanho da contribuição dos que estiveram ao meu lado durante todo o período de pesquisa, incluindo Vinícius Barcelos, Luiz Gustavo Moza, Tomás Milanez e Felipe Flores. Por fim, gostaria de dedicar este trabalho a Vanir Peruffo, a Lenita Cortes, a Leticia Peruffo e a Mariana Araújo, por me acompanharem ao longo de todo o mestrado

Agradecimentos

Gostaria de agradecer aos três integrantes da banca pelas suas valiosas contribuições a esse trabalho. Também gostaria de agradecer a Vinícius Barcelos, Felipe Iachan, Murillo Esteves e aos demais participantes do grupo de pesquisa de Macroeconomia da EPGE.

Resumo

Neste trabalho, estudamos os impactos de transferências condicionais de renda sobre o trabalho e a educação infantis. Para tanto, desenvolvemos modelo dinâmico de equilíbrio geral com agentes heterogêneos, onde as famílias enfrentam tradeoffs com relação à alocação de tempo das crianças em atividades de lazer, em escolaridade e em trabalhar. O modelo é calibrado usando dados da *Pesquisa Nacional por Amostra em Domicílios*, de modo que podemos quantificar os efeitos de uma política de transferência de renda. Finalmente, avaliamos o impacto de um política semelhante ao atual Bolsa Família. Nossos resultados sugerem que o programa, no longo prazo, é capaz de induzir um aumento substancial na escolaridade, além de ser efetivo na redução do trabalho infantil e da pobreza. Além disso, mostramos que um programa progressivo de transferência condicional de renda resulta em benefícios ainda maiores.

Palavras-chave: Macroecnomia, Desenvolvimento, Crescimento, Transferência Condicional de Renda, Educação, Trabalho Infantil

Abstract

In this paper we study the effects of conditional cash transfers in school enrolment and tackling child labour. We develop a dynamic heterogeneous agent general equilibrium model, where households face a set of tradeoffs while allocating their children's time in leisure activities, schooling and working. We calibrate the model using data from the Brazilian survey PNAD in order to quantify the effects of a conditional transfer. We then evaluate the results of a policy experiment that implements a conditional cash transfer scheme similar to the Brazilian Bolsa Familia. Our results suggest that the program, in the long term, is able to substantially increase school registration and reduce child labour and poverty. In addition, we find out that a progressive conditional cash transfer results in even more benefits.

Keywords: Macroeconomics, Development, Growth, Conditional Cash Transfers, Education, Child Labour

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Chapter 1

Introduction

Malala Yousafzai and Kailash Satyarthi won the 2014 Nobel Peace Prize "for their struggle against the suppression of children and young people and for the right of all children to education". The girl, the youngest person ever to win the Nobel prize, became famous after sending a letter to BBC explaining the situation of education where she lived, in northwest Pakistan, where the Taliban banned girls from schooling. Kailash's efforts largely influenced the International Labour Organization to adopt the Convention no. 182, which actually consists of the organization's main guidelines concerning the worst forms of child labour[1].

Child Labour concerns are appealing. They motivate people and governments to gather resources or to provide specific policies, aimed at reducing child labour and raising school enrolment. The concerns range from child health and safety to human capital accumulation and economic development. For instance, in the Brazilian context, the PETI (Program for the Eradication of Child Labour) offers a cash transfer to families whose children are in working situations. In addition, Brazilian Bolsa Família could also have an effect on reducing child labour and inducing school enrolment as, fortunately, many other poverty-reducing policies ([de Hoop and Rosati, 2013] covers an extensive recent literature on cash transfers and their effects on child labour).

The efforts to mitigate child labour appear to be succeeding, since the [ILO-IPEC, 2013] report suggests that governments, workers and employers organisations, and civil society are on the right track and moving in the right directions. Data corroborate this statement: according to the same report, from 2000 to 2012, the number of children working has fallen from 255 million to 168 million[2], which accounts for barely 11% of world's child (5-17 year-old) population[3].

In this work, we provide a dynamic heterogeneous agent model that includes the most important determinants of child labour and enrolment and then assess the effects of the Brazilian Bolsa Familia, an extensive conditional cash transfer program adopted after 2002. We model a household as a unit consisting of not only a working adult, but also a child, where the trade-off lies in the time allocation of the child, either in leisure, working or investing in human capital.

Our goal is to compare the outcomes generated by different policy incentives in stationary equilibria and then assess the groups who are better off and worse off with respect to their previous situations. In addition, we are able to understand differences in aggregate outputs, such as the total human capital accumulation, total child labour, inequality measures and also whether the policy-targeted groups react as the policy-maker expected.

[1] In October 2014, a total of 179 countries have ratified the Convention no.182.

[2] Statistics may differ from institution from institution due the the fact that there are differences in the understood definition of child labour. For instance, UNICEF claims that there are actually 215 million of children working worldwide, with more than half of them being exposed to serious hazards.

[3] The standard definition of *child* labour, by ILO, differs from *children in employment*. The second one stands as any form of child work, paid or not, including light and harmless forms of work. The first one, which is narrower, includes children working below the acceptable age (in Brazil, 14 years-old at any circumstance, between 14 and 16 if not in the condition of apprentice and under 18 in the case of night work or potentially harmful activities)

We calibrate the model using Brazilian data. Our results suggest that, in the long-term, the program is successful in increasing school matriculation and reducing child labour. Specifically, with a flat transfer schedule, the new policy raises secondary school enrolment by 30% while the number of working children decrease by more than one third. Also, the model suggests that the program effectively reduces poverty in the whole economy - the introduction of a transfer roughly eradicates poverty among its beneficiaries.

We also perform a counterfactual experiment with a progressive transfer scheme. In this case, the policy is able to reach the most impoverished families. In comparison, with a flat transfer scheme, many families chose not to take the transfer because they would have to accept the conditionality terms. The counterfactual experiment suggests that a progressive transfer scheme that gives more to the poorest is a much more powerful poverty-reducing policy.

The rest of this paper is organized as follows: chapter 2 introduces the Bolsa Familia program, chapter 3 looks at the related literature, chapter 4 describes some aspects of the data, chapter 5 presents the model itself, chapter 6 explains the calibration strategy, chapter 7 implements the Bolsa Familia and the counterfactual experiment and evaluates the results and chapter 8 concludes.

Chapter 2

The Bolsa Família Program

In 2004, the ex-president Lula da Silva united four previous transfer programs in order to create the Bolsa Famila, the biggest Brazilian conditional cash transfer program. In 2006, the program had already reached its initial coverage goal - 11 million families ([Soares et al., 2009]). In 2009, the government authorized its expansion to 12.6 million families. As a result of the program, school registration increased and labour offer did not decrease at all ([Soares et al., 2010]). Also, [Soares et al., 2006] estimate that the Bolsa Familia was responsible for one fifth of the reduction of the Gini index from 1995 to 2004 (the total decrease was 4.7 points).

Actually, the Bolsa Familia conditionalities rely basically on family per capita income, school registration, and regular health check-ups. Beneficiaries are divided into two categories: extremely poor families, which have a per capita income below R$77.00, and the poor families, whose per capita income range from R$77.00 to R$154.00 [1]. The extremely poor families received a monthly "basic benefit" fixed at R$77.00. Also, there's a monthly variable benefit of R$35.00 for each child from 0 to 15 years old[2] (up to 5 children), which is conditional on school registration for children older than 6. There's also another variable benefit per up to two adolescents (16-17 years old), which is worth R$42.00. Finally, there's a discretionary benefit, paid to those families who are in the range of extreme poverty. All in all, the average benefit paid in June 2014 was R$167.00 per family per month[3].

In december 2012, according to IPEA, the benefit was paid to 13.9 million out of almost 67 million families, with a total of roughly two billion Brazilian reais (an average of R$144.74 per family). Using the data for nominal GDP obtained from the same source (IPEA), we find out that the Bolsa Familia accounted for 0.55% of the GDP in that year.

[1] One Brazilian real was worth roughly $0.3 at the beginning of March/2015.

[2] Also given to the families with pregnant mothers. There is also an extra benefit for children ranging from 0-6 months, due to nutritional concerns, which is worth R$35.00.

[3] Source: Ministerio do Desenvolvimento Social

Chapter 3

Related Works

■ Child labour incidence is directly related to income level. Household income directly impacts child labour offer, so, not surprisingly, the poorest countries are those where child labour is more persisting. For instance, [ILO-IPEC, 2013] reports that 30% of Sub-Saharian African children work, a third of which under hazardous conditions. The high incidence of child labour among poor households not only is worrying by itself, but also due to its dynamic effects - a working child has a lower performance at school, as extensively reviewed by [Orazem and Gunnarsson, 2003] - which could ultimately translate into lower future productivity and, thus, wages. As the time invested in acquiring human capital lowers, its accumulation also lowers, inducing a kind of intergenerational child labour trap, as emphasized by [Baland and Robinson, 2000].

Concerns over child labour rely on health and its effects over education, as covered by [Dorman, 2008]. This study provides many examples of works showing that child education and thus human capital accumulation is negatively correlated not only with the incidence of child labour, but also with working hours. In addition, school achievement could also be affected by child labour, which ultimately translates into lower wages in the future ([Ilahi et al., 2005]).

On the other hand, [Dorman, 2008] provides broad evidence that child health is negatively affected by working hours. Although very hard to empirically assess, the influence of working on child health attracted the attention of the International Labour Organization, who then produced the Convention no. 182.

The incidence of child labour may reflect the inability of households to optimally invest in human capital. This fact is explored in a simple framework provided by [Lochner and Caucutt, 2012] where, when credit constraints problems arise, parents are not able to borrow against their children future earnings. However, empirical works that show the linking between child labour and credit constraints are relatively rare ([Edmonds, 2008]), due to the unavailability of adequate data. In spite of this difficulty, [Beegle et al., 2005], who find out that the availability of collateralizable assets offsets the increase of child labour due to a productivity shock, is an example of such evidence.

Borrowing constraints not only restrict households from investing in schooling but also from consuming. Hence, child labour may arise as a response to a very high benefit on the marginal consumption, so poverty itself is theoretically enough to explain the incidence of child labour and low levels of school attainment, even in the absence of credit constraints.

Some works try to explicitly model child labour and draw some conclusions over welfare and the desirability of its existence. Much of this literature builds on [Basu and Van, 1998], who also model the household composed of parents and children as the relevant agent in the economy. On the same line, [Baland and Robinson, 2000] explain that even if parents are fully altruistic, as in our model, (Pareto) inefficient child labour may arise due to market imperfections. Also, [Doepke and Krueger, 2006] explore child labour not only under a positive perspective, but also from a normative political-economy approach. They argue that, when market imperfections exist, even though a total ban may be (Pareto) inefficient, it could arise due to political-economic

reasons.

Our model elaborates on [Krueger and Donohue, 2004] and [Restuccia and Urrutia, 2004], who also construct dynastic overlapping generation models. The first authors calibrate the parameters using US data from 1870, evaluating policies such as a child labour ban, free education and free and mandatory education[1]. On the other hand, another work by [de la Croix and Doepke, 2002] explores, in a framework without physical capital accumulation, how child labour and human capital accumulation interact with fertility and evaluates benefits of adopting public schooling.

When it comes to evaluating the effects of conditional cash transfers, two works are particularly worth remarking. First, [Cespedes, 2010] develops a life-cycle model that includes human capital accumulation to evaluate the effects of Mexican PROGRESA, finding that this policy induces human capital accumulation and decreases poverty as a whole. On the other hand, [Zilberman and Berriel, 2012] also models the Bolsa Familia transfers in Brazil, though abstracting from human capital and focusing on the financial side, also suggesting that a conditional cash transfer appears to increase welfare.

As for the empirical works, there is no lack of this kind of analysis. For instance, a study that reviews the effect of Bolsa Familia in Brazil is [Cardoso and Souza, 2004]. Using data from census they find that the program increased child school matriculation, from 92% to 95%. Moreover, it also changes the work-schooling distribution of children's endowment of time. Children who previously only worked and children who were previously idle seem to be enrolling after the treatment. On the other hand, child labour remained almost stable as there was an increase in the children that go to school and work both.

A very influential work in this field is [Skoufias and Parker, 2001], who also explore the effects of Mexican PROGRESA (now renamed *Oportunidades*). They find evidence that, in response to the new policy, children reallocate their leisure time, for the increase in school registration is lower than the decrease in child labour. This fact is also shown by [Ravallion and Wodon, 1999] in Bangladesh[2]. Another research conducted in Costa Rica by [Duryea and Morrison, 2004] obtained the same (qualitative) conclusions.

Finally, the last kind of work that evaluates conditional transfers involves structural econometrics. Two works are worth citing. First, [Todd and Wolpin, 2002] construct a dynamic structural model and validate it, fitting it to real data obtained also from Mexican *Oportunidades*. Although, as reported by the authors, the analysis does not include general equilibrium effects, on the short term the model was able to provide a policy outcome forecast accurately[3]. In the Brazilian context, [Bourguignon et al., 2003] has a humbler structural estimation, though seeking the perform the same analysis.

3.1 Conditional Cash Transfers around the World

■ Conditional cash transfers (CCTs) have become widely popular amongst developing countries throughout the nineties and, perhaps surprisingly, even in developed countries. The first CCT program that jumped to the world's eyes was the Mexican *Oportunidades*, but, in fact, the first experience of this kind took place in Campinas, a city in Brazil, two years prior to PROGRESA implementation ([Sedlacek et al., 2000]).

Nowadays, examples of such policies in Latin America are the Costa Rican *Avancemos*, Ecuadorian *Beca Escolar*, Nicaraguan extinct *Red de Proteccion Social*, Bengali Food-for-Education, and Chilean *Subsidio Unitario Escolar*. In Africa, CCTs were introduced in countries such as

[1] [Krueger and Donohue, 2004] find out that a free education policy is more effective than banning child labour as a whole.

[2] The Bengali program, the Food-for-Education, provided a non-negligible amount of rice (worth about half the mean income of a poor family), the most important compound of Bengali diet, for children who regularly attended school.

[3] Of course the relative accuracy of the model is a subjective measure. However, the model almost unambiguously make qualitative correct predictions, which are consistent with the empirical studies cited above.

Burundi, Burkina Faso, Côte d'Ivoire, Cameroon, Ethiopia, Ghana, Guinea, Gambia, Kenya, Madagascar, Mozambique, Malawi, Nigeria, Uganda, and Zambia ([Nanak Kakwani, 2005]). In Asia, the Cambodian Education Sector Support Project, the 4Ps in Philippines, along with the Food-For-Education can be cited. In Europe we can find the Macedonian Conditional Cash Transfer Program and even in the United States a transfer program was launched in New York in 2007, being extinct in 2010[4].

Such policies, as mentioned, are primarily focused on reducing poverty as a whole. In the context of a dynamic general equilibrium model they relax the non-negativity constraint either on bequests or on asset accumulation. However, only a cash transfer may not be enough to smooth consumption the way the families desire, so instead of allocating the new resource on, say, education (which is arguably desirable by authorities), people use it to alleviate hunger. The result is an educational trap, with high intergenerational correlation of human capital.

On the other hand, when the targeted households are required to send their children to school, the whole economy may dynamically push into an equilibrium (whose policy is part of) where the aggregate human capital and the marginal returns on capital are high, so the equilibrium output is higher than before. In fact, the greatest virtue of a general equilibrium model is allowing us to understand the interaction of apparently unlinked variables.

[4]The Opportunity was particularly well-designed in terms of prizes for goal achievements. For instance, transfers were granted not only for grade and enrolment, but also for each teacher-parent conference attended. Moreover, the program incentivized employment instead of only serving as an insurance, paying $150 dollars per month if the beneficiaries held a full time job.

Chapter 4

Available Data

■ The *Pesquisa Nacional por Amostra de Domicílios* (PNAD), a Brazilian national household sample survey, provides detailed micro-data on each household member occupation. We selected two specific years in order to inspect data on school enrolment and on child labour[1]: 1997 and 2011, before and after the wide implementation of Bolsa Familia.

Before presenting these data, it is important to understand what was happening to the Brazilian economy at the time. The Workers Party, after being elected in 2002, strengthened the existing social programs, and created the Bolsa Familia. Also, the country experienced an impressive economic growth. Between 1997 and 2011, *per capita* income increased by 30%[2].

Meanwhile, the human capital index[3] increased by 18%, while school enrolment and child labour have moved towards the desired directions. We can see, from table 4.1, that more children are enrolled in 2011 in comparison to 1997, and less are working. However, this table calls the attention for other correlations. First, children who work are, consistently for all ages and both periods, less likely to go to school. Second, we can see that there is a higher increase in school enrolment among children who also work, which suggests that families could be reallocating their children's time (mostly) from leisure into schooling.

Table 4.1: Percentage of Children Enrolled in School, by Age and Year

	age	10	11	12	13	14	15	16	17	10-17
1997	|w=1	89,6%	88,5%	88,8%	83,9%	76,7%	69,2%	61,6%	55,2%	70,0%
	|w=0	95,9%	96,0%	95,2%	94,1%	90,2%	86,1%	80,3%	72,5%	89,9%
2011	|w=1	98,5%	98,3%	98,2%	95,3%	94,7%	86,2%	77,8%	64,0%	80,2%
	|w=0	99,3%	98,8%	98,9%	98,2%	96,8%	92,9%	87,8%	78,2%	94,6%

Note: $w = 1$ means that the child was working at the same month the survey was made, while $w = 0$ means that she wasn't.

Table 4.2 shows another interesting correlation. First, it makes clear that there has been an increase in enrolment as well as a decrease in working children and idleness[4]. In addition, we can see that there is a positive correlation between family income and school enrolment. In fact, children who go to school have, on average, a higher family income (excluding their own) when compared to children that work, and this relation holds for both 1997 and 2011[5].

[1] Unfortunately, there is no data on child labour for children below 10 years-old

[2] According to IPEA.

[3] From PWT 8.0

[4] An idle child is defined as a child that neither goes to school nor works.

[5] Interestingly, the average income of families whose children only went to school decreased in this period, a

Table 4.2: Distribution of children between working and schooling and their family income

	work+school	only school	only work	idle	work
1997	15,5% R$ 1.515,45	70,1% R$ 2.241,64	6,6% R$ 915,66	7,8% R$ 953,60	22,1% R$ 1.335,26
2011	10,1% R$ 1.706	82,6% R$ 2.067	2,5% R$ 1.367	4,7% R$ 1.344	12,6% R$ 1.639

Note: Family include excludes the surveyed child's income. Incomes are measured in 2011 Brazilian reais.

Both tables 4.1 and 4.2 show that there has been, between 1997 and 2011, a negative correlation between *per capita* income and child labour, a positive correlation between *per capita* income and school enrolment, and a negative correlation between child labour and school enrolment, all of which consistent with the empirical literature.

In the next chapter, we provide a dynamic heterogeneous agent general equilibrium model containing the main determinants of school enrolment and child labour. Our ultimate goal is to evaluate policy experiments, and we focus on the introduction of a conditional cash transfer scheme.

fact that suggests that schools have become more accessible for poor families.

Chapter 5

Model

5.1 Economic Environment

■ The economic environment consists of overlapping generations, with time being discrete, and no population growth. Individuals live for four periods, one as a child, one as a young adult and two as adult parents. At the beginning of their third period of life, individuals give birth to a child. Thus, each household consists of an adult parent and a child or a young adult. Hence, the dynasty is the relevant agent in this economy. All decisions are jointly taken within the dynasty, so we assume that there is no conflict between its members. Also, an individual dies at the end of the fourth period.

A model period consists of 17 years. However, during his first 10 years of life, an individual can only spend his time in leisure. During the other seven years of childhood he can either work, go to school or stay idle. It is possible to choose between three levels of schooling at this period: no schooling at all, only primary schooling, and secondary schooling. Schooling has not only an explicit cost but also a cost in terms of foregone earnings, which also depends on the educational level chosen. On the other hand, at the same time, a young parent of the same dynasty provides all of his endowment of time to the representative firm, in exchange for his wages.

At the end of the first period of his life, a child becomes a young adult and a young parent becomes an old parent. A young adult can either go to college or not (as in [Restuccia and Urrutia, 2004]), which takes 4.5 years of the individual's full time and costs a certain amount of resources. The remaining of a young adult's time is fully devoted to work. The old parent also devotes his full time to work. At the end of this period, a young adult becomes a young parent as the old parent dies. This way, the dynasty continues as generations overlap.

A dynasty can spend its resources either in consumption, schooling or in precautionary savings (which, in the second period, take the form of bequests). In the first period, a dynasty chooses the child's level of schooling and the child's labour offer to the market representative firm. Also, the dynasty chooses the level of physical capital to be taken to the second period. In the second period, beside the asset holdings decision, a young adult that attended secondary schooling is able to choose whether to go to college or not.

In this economy, there's is only one way to directly transfer resources across time, which is by lending to the representative firm. The representative firm faces no uncertainty, and thus, in the following period, surely pays an interest rate to the lenders.

5.2 Preferences

■ Here, we divide the dynasty's life-cycle into two periods. In the first of them, the dynasty consists of a child and a parent living through its third period of life. In the second, it consists of a young adult and of an adult who's living through its fourth period of life.

Each dynasty seeks to maximize its discounted expected utility. In the first period, dynasties

wish to consume but do not wish to send their children to work. Hence, its utility function is:

$$u_1(c, m) = \left(\frac{c^{1-\gamma}}{1-\gamma}\right) - (\phi + \mathbb{I}[h' > h_0]\phi_{ws})m \tag{5.1}$$

The first term of the r.h.s represents the perceived utility from household consumption, while m represents the amount of time a children works.. Also ϕ represents the marginal disutility of child labour, while ϕ_{ws} represents the incremental marginal disutility of working and studying both.

In the second period, there's no child labour decision. Hence, the dynasty utility function is only represented by:

$$u_2(c) = \left(\frac{c^{1-\gamma}}{1-\gamma}\right) \tag{5.2}$$

5.3 Age Earning Profile

■ The effective units of labour an individual can offer to the market is given by his productivity, which depends on his (relative) human capital and on an exogenous shock.

There are four levels of human capital, so $h_i \in \{h_0, h_1, h_2, h_3\}$, which refer respectively to no schooling, primary schooling, secondary schooling and college. Also, an individual can be affected by his work experience, which is given by $\xi_i(t_i, h_i)$, where $t_i \in \{1, 2, 3, 4\}$ refers to the individual's period of life.

Productivity shocks only affect adult individuals[1]. Let z_i be the shock that affects the adult individual of a household i. I assume, as in [Seshadri and Lee, 2014], that z_i follows a log-normal distribution.

$$\log(z_i') = -\frac{(1-\rho)\sigma^2}{2} + \rho \log(z_i) + \epsilon_i, \qquad \epsilon_i \sim N(0, \sigma^2), \tag{5.3}$$

where $\rho \in (0, 1)$. Note that we normalize the unconditional average of this AR(1) process to be $-\frac{\sigma^2}{2}$, so we have $\mathbb{E}(z) = 1$. Also, the unconditional variance equals $\frac{\sigma^2}{1-\rho^2}$. There are two underlying assumptions behind this process. First, the persistence of a the productivity is considered *within* household, not within individuals. That is, the productivity of the parent who's living through his fourth period of life affects the productivity his son will have in the next period the same way the productivity of a parent living through his third period of life affects his productivity during his last period of life. Second, the productivity variance is the same regardless of an individual's age or education.

We will return to this issue in the calibration section.

5.4 Production Technology

■ The representative firm has the usual constant returns to scale technology, denoted by:

$$F(K_t, L_t) = AK_t^{\alpha} L_t^{1-\alpha} \tag{5.4}$$

Where K_t denotes the aggregate level of capital, L_t the aggregate level of labour efficiency units and A is the TFP. In this work I consider any type of human capital level (including child labour) as perfect substitutes. The firm pays an interest rate r and wages w for its inputs.

Since capital depreciates at rate δ, the firm's problem becomes:

$$\max_{\{K_t, L_t\} \geq 0} AK_t^{\alpha} L_t^{1-\alpha} - wL_t - (r+\delta)K_t \tag{5.5}$$

[1] In this paper we abstract from idiosyncratic skill or ability.

The firm's problem yields the competitive labour and capital prices:

$$w_t = A(1-\alpha)\left(\frac{K_t}{L_t}\right)^{\alpha} \tag{5.6}$$

and

$$r_t = A\alpha\left(\frac{K_t}{L_t}\right)^{\alpha-1} - \delta \tag{5.7}$$

5.5 Schooling Technology

■ In order to invest in human capital accumulation, dynasties are able to send their children and young adults to school. We denote by $\kappa(h_i)$ the cost of obtaining h_i units of human capital.

Attending school also takes a fraction of an individual time endowment. During the first period, this fraction is strictly increasing in the level of education. During the second period, young adults that attend college also forgo a fixed fraction of their time endowment. We will use the function $\varsigma(\cdot)$ and the parameter ϖ to denote the time required to attend a certain level of schooling. More details on this issue will be explored in the calibration section.

5.6 The Government

■ The government runs a budget balance in each period. Its revenues consist of total income taxes, which I denote (τ). It rebates its revenues either with a conditional transfer (*ctr*) or with school subsidies. The conditional transfers may be conditioned on (i) school matriculation, which I denote *sthr*, and (ii) an income threshold, which I denote by *ithr*. The government is able to define a subsidy schedule, which I denote by $\mathcal{S}(h_i)$.

We assume throughout this paper that the government is fully credible and its policies are fully observable. Hence, in a steady state equilibrium, which is the type of equilibrium we will be interested in, the policies are constant over time. We also assume that the government policies are fully enforceable at no cost.

5.7 The State Space

■ The state space of a dynasty who's going through the first period consists of the bequest it received, denoted by a, the human capital accumulated by the young adult h, and the productivity shock that affects him, z. Let the state space of period one be denoted by $x^1 = \{z, h, a\}$.

The families who are living through the second period have to take into consideration not only the parent's level of human capital (which I keep denoting by h) but also the education the young adult has taken in the first period, which I will denote by h_c. Also, the parent's observable shock and their initial level of asset holdings are considered. We can hence define the state space of the second period dynasty as $x^2 = \{z, h, h_c, a\}$.

5.8 The Dynasty Recursive Problem

5.8.1 Period 1

■ In the first period, the household's problem can be defined as:

$$V_1(x^1) = \max_{c,m,h',a'}\left(\frac{c^{1-\gamma}}{1-\gamma}\right) - (\phi + \mathbb{I}[h' > h_0]\phi_{ws})m + \beta\sum_{z'}\text{Prob}(z'|z)V_2(z', h, h', a') \tag{5.8}$$

subject to:

$$0 \leq m + \varsigma(h') \leq \bar{m} \tag{5.9}$$

$$\begin{aligned} c + (1 - \mathcal{S}(h'))\kappa(h') + a' = a + (1 - \tau)[(z\xi(3, h) + \xi(1, h')m)w + ra] + \\ ctr \times \mathbb{I}\left\{[(z\xi(3, h) + \xi(1, h')m)w + ra] \leq ithr\right\} \times \mathbb{I}[h' \geq sth] \end{aligned} \tag{5.10}$$

$$h' \in \{h_0, h_1, h_2\} \tag{5.11}$$

$$a' \geq \underline{a} \tag{5.12}$$

$$m \in [0, \bar{m}]\ ,\ c \geq 0, \tag{5.13}$$

where β is discount rate and $\mathbb{I}(\cdot)$ is the indicator function. Note that the argument h reappears in the continuation value of the dynasty, since the adult parent keeps his actual level of human capital.

Constraint (5.9) represents the fraction of endowment of time of a child that can be used for attending school and working (the rest is implicitly allocated into leisure). Equation (5.10) represents the dynasty's resource constraint. In the right-hand-side, the expression $[(z\xi(3, h) + \xi(1, h')m)w]$ represents the gross labour income of the family. The first term in the parenthesis, $z\xi(3, h)$, represents the efficiency units of labour provided by the adult parent. On the other hand, $\xi(1, h')$ represents the efficiency units of labour provided by the child. A further explanation of why $\xi(\cdot)$ depends on h' will be provided in the calibration section.

Constraint (5.11) represents the human capital investment possibilities in period 1, which are no schooling at all, primary, or secondary schooling, while equation (5.12) represents the credit constraint. Finally, equations (5.13) represent time feasibility and consumption non-negativity .

As one can predict, depending on the first period's level of schooling, the child will have different choice possibilities in the second period. If the child hasn't attended secondary school, the dynasty cannot invest in his education any more. On the other hand, attending secondary school gives the young adult the possibility to attend college and thus increase his future earnings. For these reasons, two value functions will be defined for the household's problem in period two.

5.8.2 Period 2 - Dynasties whose children attended secondary schooling

■ In the second period, the household whose young adult can still go to college solves:

$$V_{2,s}(x_s^2) = \max_{c,g,a'} \left(\frac{c^{1-\gamma}}{1-\gamma}\right) + \beta \sum_{z'} \text{Prob}(z'|z)V_1(z', h', a') \tag{5.14}$$

subject to:

$$g \in \{0, 1\} \tag{5.15}$$

$$h' = h_3 g + h_2(1 - g) \tag{5.16}$$

$$c + g(1 - \mathcal{S}(h_3))\kappa(h_3) + a' = a + (1 - \tau)[(z\xi(4, h) + \xi(2, h_2)(1 - g) + g(1 - \varpi)\xi(2, h_3))w + ra] \tag{5.17}$$

$$a' \geq \underline{a} \tag{5.18}$$

$$c \geq 0, \tag{5.19}$$

where $x_s^2 = (z, h, h_2, a)$. Moreover, ϖ represents the time required to attend college, and g equals one if the young adult attends college and 0 otherwise. As before, restrictions (5.18) and (5.19) refer respectively to the credit constraint and to the non-negativity of consumption.

Equation (5.17) represents the household's resource constraint. In the left-hand-side, $(1 - \mathcal{S}(h_3))g\kappa(h_3)$ represents the net cost of attending college (h_3). In the right-hand-side, $[(z\xi(4,h) + \xi(2,h_2)(1-g)\varpi + g(1-\varpi)\xi(2,h_3))w]$ represents the total gross labour income of the dynasty. The term $z\xi(4,h)$ represents the efficiency units of labour provided by the adult parent, $\xi(2,h_2)$ represents the total labour efficiency units provided by the young adult who does not attend college, and $\xi(2,h_3)$ represents the efficiency units provided by the young adult who has attended college. Notice that $\xi(2,h_3)$ will be provided only during a fraction $(1-\varpi)$ of a period.

5.8.3 Period 2 - Dynasties whose children did not to attend secondary schooling

■ In the second period, the household whose young adult can't go to college is:

$$V_{2,n}(x_n^2) = \max_{c,a'} \left(\frac{c^{1-\gamma}}{1-\gamma}\right) + \beta \sum_{z'} \text{Prob}(z'|z) V_1(z', h', a') \tag{5.20}$$

subject to:

$$c + a' = a + (1-\tau)[(z\xi(4,h) + \xi(2,h_c))w + ra] \tag{5.21}$$

$$h' = h_c \tag{5.22}$$

$$a' \geq \underline{a} \tag{5.23}$$

$$c \geq 0, \tag{5.24}$$

where $x_n^2 = (z, h, h_c, a)$ and $h_c \in \{h_0, h_1\}$, which, along with (5.22), states that the young adult cannot invest any further in human capital. Notice that, in this case, the household's problem is summarized by only an asset holdings choice, with the actual level of the young adult's human capital being carried on to the next period.

5.9 Definition of Equilibrium

■ In this work, we are only interested in stationary recursive competitive equilibria. A stationary recursive competitive equilibrium consists of:

D1 Three groups of functions: (i) $\{V_1, g_1^c, g_1^h, g_1^m, g_1^a\}$, (ii) $\{V_{2,n}, g_{2,n}^c, g_{2,n}^a\}$, and (iii) $\{V_{2,s}, g_{2,s}^c, g_{2,s}^h, g_{2,s}^a\}$, defined by, respectively:

- i The value function, the consumption, the human capital, the child's labour offer, and the asset holdings policy functions, both corresponding to the first period.
- ii The value function, the consumption, and the asset holdings policy functions, both corresponding to the second period when the young adult cannot enter college.

iii The value function, the consumption, the human capital (college), and the asset holdings policy functions, both corresponding to the second period when the young adult has previously attended the secondary school.

D2 Factor prices $\{w, r\}$.

D3 A government policy $\{\tau, \mathcal{S}(h), ctr, sthr, ithr\}$.

D4 A pair of time-invariant measures over states λ_1 and λ_2, respectively referring to periods one and two, whose laws of motion are:

$$\lambda_2(z', h, h', a') = \sum_{\{(z,h,a):h'=g_1^h, a'=g_1^a\}} Prob(z'|z)\lambda_1(z, h, a) \tag{5.25}$$

and

$$\begin{aligned} \lambda_1(z', h', a') = \mathbb{I}(h_c < h_2) \left\{ \sum_{\{(z,h,a):h'=h_c, a'=g_{2,n}^a\}} Prob(z'|z)\lambda_2(z, h, h_c, a) \right\} + \\ \mathbb{I}(h_c = h_2) \left\{ \sum_{\{(z,h,a):h'=g_{2,s}^h \cdot h_3 + (1-g_{2,s}^h)\cdot h_2, a'=g_{2,s}^a\}} Prob(z'|z)\lambda_2(z, h, h_c, a) \right\} \end{aligned} \tag{5.26}$$

where,

1 Given [D2] and [D3], the groups of functions [D1] maximize the household's problems defined in the previous section.

2 Factor prices are given by (5.6) and (5.7).

3 The government runs a budget balance:

$$\begin{aligned} \tau(wL + rK) = \sum_{z,h,a} \lambda_1(z, h, a)\kappa(g_1^h) \cdot \mathcal{S}(g_1^h) + \sum_{z,h,h_c,a} \lambda_2(z, h, h_c, a) g_{2,s}^h \kappa(h_3)\mathcal{S}(h_3) \\ + ctr \cdot \sum_{z,h,a} \mathbb{I}(g_1^h \geq sthr) \cdot \mathbb{I}\left\{ [(z\xi(3, h)h + \xi(1, g_1^h) g_1^m) w + ra] \leq ithr \right\} \end{aligned} \tag{5.27}$$

4 Market clearing and consistency conditions hold:

$$\sum_{z,h,a} \lambda_1 g_1^c + \left[\mathbb{I}(h_c < h_2) \sum_{z,h,h_c,a} \lambda_2 g_{2,n}^c \right] + \left[\mathbb{I}(h_c = h_2) \sum_{z,h,h_c,a} \lambda_2 g_{2,s}^c \right] = C, \tag{5.28}$$

which is the total consumption,

$$\sum_{z,h,a} \lambda_1 \kappa(g_1^h) + \left[\mathbb{I}(h_c = h_2) \sum_{z,h,h_c,a} \lambda_2 \kappa(h_3) \cdot g_{2,s}^h \right] = \mathcal{E}, \tag{5.29}$$

which represents the total expenditure in education,

$$\sum_{z,h,a} \lambda_1 g_1^a + \left[\mathbb{I}(h_c < h_2) \sum_{z,h,h_c,a} \lambda_2 g_{2,n}^a \right] + \left[\mathbb{I}(h_c = h_2) \sum_{z,h,h_c,a} \lambda_2 g_{2,s}^a \right] = K, \tag{5.30}$$

the aggregate level of capital, and

$$\sum_{z,h,a} \lambda_1(\xi(3,h)zh + \xi(1,g_1^h)g_1^m) + \left[\mathbb{I}(h_c < h_2) \sum_{z,h,h_c,a} \lambda_2(\xi(4,h)zh + \xi(2,h_c)h_c)\right]$$
$$+ \left\{\mathbb{I}(h_c = h_2) \sum_{z,h,h_c,a} \left[\lambda_2(\xi(4,h)zh + (1-g_{2,s}^h)\xi(2,h_2)h_2 + g_{2,s}^h(1-\varpi)\xi(2,h_3)h_3)\right]\right\} = L, \tag{5.31}$$

the aggregate level of labour offer, measured in efficiency units. Thus,

$$C + \delta K + \mathcal{E} = F(K, L) \tag{5.32}$$

which is the economy resource constraint.

Chapter 6

Calibration Strategy

■ This chapter describes how we choose the model parameters, which include A, $\underline{a}$, γ, β, α, δ, $\xi(\cdot)$, $\kappa(\cdot)$, σ^2, ρ, $\mathcal{S}(\cdot)$, ϕ, ϕ_{ws}, and h.

Some parameters have a direct empirical counterpart. First, we normalize $A = 1$. We also set the annual rate of depreciation to 0.06, so that $\delta = 0.6507$, as standard in the business cycles literature, and the borrowing constraint to $\underline{a} = 0$.

In the first period, the time required to attend schooling is chosen as following. First, we suppose that schooling takes half of the child non-sleeping daytime. Moreover, we assume that, according with Brazilian legislation at the time[1], the minimum number of school days is 200. Dividing this number by the number of non-weekend days means that a child must attend schooling during roughly 75% of an year's week days[2].

In Brazil, secondary schooling requires three years while primary required eight at 1997[3]. We assume, in this model, that every child completes at least the fourth grade of primary school. Hence, in order to complete primary schooling, a child spends $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{4}{17} = 8.8\%$ of a period's time endowment. Remember that the child's effective endowment of time in the first period is only $\frac{7}{17}$. The same way, attending primary plus secondary school takes $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{7}{17} = 15.5\%$ of a period's time.

We assume that college and working are mutually exclusive and that attending college requires 4.5 years[4]. Hence, the young adult who chooses to go to college works for the remaining period time, which amount to $\frac{12.5}{17} = 73.5\%$ of a period time endowment.

6.0.1 The Wage Efficiency Profile

Our data source for estimation of $\xi(t, h)$, the labour efficiency profile, is the *Pesquisa Nacional por Amostra de Domicílios* of 1997. In order to calculate the age efficiency profile, we restrict the data for the people who work 40-48 hours per week.

First, in order to obtain the values of $\xi(1, h')$, we need to take in account the dynasty's decision for the child's education. For example, if a child only goes to primary schooling and if he works the remaining time, at the end of the period he will have worked four years part-time with no investment in human capital and more three years full time, with the level of human capital being h_1.

With this fact in mind, our hypothesis is that the (potentially) working child divides his labour hours proportionately to his available time in each condition. In the example above,

[1] Law 9394/96.

[2] The same legislation states that these days should incorporate **at least** 800 hours of classes, excluding examinations. Since, in the time required to attend schooling, we also incorporate the other transaction costs such as transportation and exams time, we found it reasonable to assume that schooling requires 50% of a child non-sleeping hours.

[3] Actually it has changed to nine years.

[4] [Restuccia and Urrutia, 2004] assume that college takes 4 years while [Bohacek and Kapicka, 2012] assume it takes 5 years.

hence, the child's labour efficiency corresponds to an weighted average of four years working part time plus three years working full time, no matter how much of the available time is devoted to working. Finally, we normalize $\xi(2, h_0) = 1$, obtaining from the data the average relative productivities $\xi(1, h_0) = 0.37$, $\xi(1, h_1) = 0.48$ and $\xi(1, h_2) = 0.46$.

In the baseline model, we restrict $\xi(2, h) = \xi(3, h)$ for each h and also $\xi(4, h) = a\xi(3, h)$, where a is obtained dividing the average wages of individuals who are 52-68 by the average wages of individuals from 18 to 51 years old. We then obtain $a = 1.11$. The relative wage premia, obtained using data for individuals from 18 to 68 years old, are set to $\xi(2, h_1) = \xi(3, h_1) = 1.5$, $\xi(2, h_2) = \xi(3, h_2) = 2.5$, and $\xi(2, h_3) = \xi(3, h_3) = 6.3$.

6.0.2 Fiscal Policy

■ In Brazil, schooling is heavily subsidized, at all levels. For instance, in 2000, public expenditures on education, including federal, state and municipalities, accounted for 3.9% of GDP[5]. In order to estimate the fraction of public expenditures in education per student, we use information from two databases: The National Institute for Educational Research and Policy (INEP) and the Brazilian household expenditures survey[6].

First, the INEP provides an estimate for public expenditures as a fraction of per capita GDP. We use the estimates for 2002, in order to match our data from POF. In 2002, *for each* student ranging from the fifth to the eighth level of primary schooling (10-14 years old), the expenditures per students accounted for 12.3% of the per capita GDP. The same ratio for secondary students was 8.9%, while for tertiary students it was 121%.

Given these ratios, we try to estimate the private expenditures in education per student. We use the 2002-2003 POF. For each family in this survey, we have information on expenditures in education, divided in six categories: (i) expenses in regular courses (any schooling up to secondary), (ii) expenses in tertiary courses[7], (iii) total expenses in other courses, (iv) total expenditures in books and scientific publications, (v) total expenditures in school articles, and (vi) others.

We then separate the families that spend only either on superior courses or regular courses. Therefore, we assume that a family that spends only on one of the categories devotes all other education expenditures in activities or goods linked to that category, which, in our model, are also considered expenditures in education. Finally, to assess the average expenditure per student, we suppose that every family has two heads and hence we divide the average expenditure for each category by the average household size minus two. Finally, since we do not have separate data for primary or secondary education, we suppose that the average private expenditures on both is the same.

Finally, we divide the obtained values by the 2002 per capita GDP. The result is that each tertiary education students privately costs 32.9% of the per capita income, while a student of a regular course costs on average 11.9%[8]. We divide the government by the total expenditures to obtain the fraction of expenditures that the government subsidizes.

However, the way we define the subsidizing ($\mathcal{S}(\cdot)$) function requires a bit of modification of the previous obtained values. To start with, $\mathcal{S}(h_2)$ equals the value obtained with the method above, because every resource spent in secondary education is embedded in $\kappa(h_2)$. However, as the model states, $\kappa(h_2)$ involves the cost of attending not only to secondary but also to primary school. So the true value of subsidy on primary schooling does not equal $\mathcal{S}(h_1)$[9], but instead

[5]Source: INEP/MEC

[6]*Pesquisa do Orçamento Familiar*, POF

[7]In Portuguese, superior courses.

[8]The average family size for those who are enrolled in a superior course is 3.61, while for regular is 4.13. Samples sizes are respectively 4338 and 1555 observations.

[9]Remember that our data for private expenditures in private and secondary schooling is indistinguishable, but the government expenditures data is identifiable, so the value for the subsidy differs.

$\mathcal{S}(h_1)$ is a function of the value obtained and the quantity of children that attend each level of schooling.

We know that the (true) subsidy level equals $s_{1,true}\frac{government\ expenditures}{total\ expenditures}$, for any given level of schooling. In our model, when it comes to the primary education, the true subsidy equals:

$$s_{1,true} = \frac{\kappa(h_1)\mathcal{S}(h_1)d_1 + \kappa(h_1)\mathcal{S}(h_2)d_2}{\kappa(h_1)d_1 + \kappa(h_1)d_2}, \tag{6.1}$$

where $d_1 = \sum_{z,a} \lambda_1(z, h_1, a)$ and $d_2 = \sum_{z,a,h_c} \lambda_2(z, h_1, h_c, a)$. By isolating $\mathcal{S}(h_1)$ in equation (6.1) we obtain the value of $\mathcal{S}(h_1)$.

In sum, the value obtained is $\mathcal{S}(h_1) = 46.5\%$. Moreover, we directly obtain the other subsidy values from the data, which are $\mathcal{S}(h_2) = 42.78\%$ and $\mathcal{S}(h_3) = 78.63\%$[10].

6.1 Exogenous Process for productivity

■ In order to approximate a continuous AR(1) process, we discretize it in the following way. First, in order to estimate the static variance of wages, we run the following regression:

$$\log(wage_i) = \alpha + \beta_1 \mathbb{I}_{[h=h_1]} + \beta_2 \mathbb{I}_{[h=h_2]} + \beta_3 \mathbb{I}_{[h=h_3]} + \varepsilon_i, \quad \varepsilon_i \sim N(0, \sigma^2), \tag{6.2}$$

from which we take the error variance and assume it is the unconditional variance of the exogenous AR(1) process (equation (5.3)) - our underlying hypothesis here is that the unconditional variance is the same regardless of the productivity level. To run the regression, we use data from PNAD for individuals within the third period of the model, thus from 35 to 51 years old, normalizing the $wage_i$ relative to the average wage of an uneducated adult from 35 to 51 years old.

Then, we use Tauchen's method to discretize the AR(1). From the obtained data we can pin down σ, given a specific value of ρ[11]. Finally, ρ is jointly set along with $\kappa(h_1)$, $\kappa(h_2)$, $\kappa(h_3)$, ϕ, ϕ_{ws}, and β, in order to match several data features, which are explained below.

Table 6.1: Literature and Data Obtained Parameters.

Parameter	Value	Description
A	1	Production Function TFP
γ	1.5	CRRA coefficient, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$
δ	0.6507	Physical capital depreciation
α	$\frac{1}{3}$	Physical Capital share in the production function
ϖ	0.26	Fraction of period required to attend college
$\varsigma(2)$, $\varsigma(3)$	0.088, 0.155	Fraction of period required to attend respectively primary and secondary school
$\mathcal{S}(2)$, $\mathcal{S}(3)$, $\mathcal{S}(4)$	0.51, 0.43, 0.79	Government Subsidy Schedule
a	1.11	Experience Bonus
σ	0.572	Variance of the exogenous process for productivity in the third period of life[12].
$\underline{a}$	0	Borrowing constraint.

6.2 Targeted Moments

In sum, 27 parameters (some of which are part of the same functions) are taken from the literature or using hypotheses, given the data. The parameters are summarized in Tables 6.1

[10] The true value of the primary schooling subsidy is 50.79%.

[11] The unconditional variance, $\frac{\sigma^2}{1-\rho^2}$, equals 0.624.

Table 6.2: Values for $\xi(t,h)$.

ξ	h_0	h_1	h_2	h_3
$t=1$	0.37	0.48	0.46	-
$t=2$	1	1.5	2.5	6.3
$t=3$	1	1.5	2.5	6.3
$t=4$	1.11	1.665	2.775	6.99

and 6.2. The remaining seven parameters are jointly chosen using the model to simulate a set of seven exactly identified empirical moments[13]. Let the parameter vector be:

$$\Theta = [\phi_{ws}\ \phi\ \kappa(h_1)\ \kappa(h_2)\ \kappa(h_3)\ \beta\ \rho]' \tag{6.3}$$

The model generates seven moments, which we denote by $M(\Theta)$. The vector of empirical moments is denoted by M_s and summarized in Table 6.3. We find the estimate for $\hat{\Theta}$ by:

$$\hat{\Theta} = \arg\min_{\Theta} \left\{[M(\Theta)] - M_s]'W[M(\Theta) - M_s]\right\} \tag{6.4}$$

where W is a weighting matrix. Here, we set $W = \mathcal{I}$, since there's no clear choice of W.
The calibration results are summarized in table 6.4.

Table 6.3: Empirical Moments.

$M_{s,i}$	Data	Model	Description
i=1.	15.5%	12.7%	Fraction of students who both work and go to school (PNAD 1997).
2.	6.6%	9.2%	Fraction of students who only work (PNAD 1997).
3.	55.3%	55.1%	Fraction of adults who only completed primary education (PNAD 1997).
4.	21.3%	24.7%	Fraction of adults with secondary education, but not tertiary (PNAD 1997).
5.	5.15%	5.3%	Fraction of adults with complete tertiary education (PNAD 1997).
6.	17.4%	17.3%	Gross Capital Formation (IBGE 1997)
7.	0.69	0.70	Intergenerational Elasticity of Earnings ([?])

Table 6.4: Parameters Obtained through the Simulated Method of Moments.

Parameter	Value	Description
ϕ	0.32	Marginal disutility of child labour
ϕ_{ws}	0.02	Incremental marginal disutility of child labour when working and schooling
$\kappa(h_1)$	0.13	Gross cost of primary schooling.
$\kappa(h_2)$	0.53	Gross costs of attending both primary and secondary schooling.
$\kappa(h_3)$	4.3	Gross cost of college education.
β	0.945	Preference (annual) rate of discount.
ρ	0.69	Persistence of the exogenous productivity shock process.

[13] As explained, given ρ, we directly obtain σ.

6.3 Model Outcomes

■ In this chapter, I briefly introduce some outcomes related to poverty and inequality, which will be important in our future policy evaluation.

When it comes to wealth distribution, our model displays just one fundamental source of heterogeneity, so it is expected that it will not generate the data income or wealth distribution ([Cagetti and Nardi, 2005]). However, despite its limitations on the issue, it performs reasonably well. The calibrated model generates an income inequality, measured as the Gini coefficient on household per capita income (which, in our model, since all households have the same size, equals household income distribution), of 0.3965 - the true value for 1997, is instead 0.60 (according to the IPEA, who uses data from PNAD). On the other hand, the model's wealth Gini equals 0.6502, while the estimated value in 2000 was 0.78.

We are also able to identify the poverty headcount ratio line, defined as the proportion of the population who earns less than two dollars (PPP 2005) a day, or 730 dollars a year. According to the world bank, 20.5% of the Brazilian population at 1997 earned less than two dollars a day. Using data for Brazilian PPP GDP, we see that the poverty line annual income represents 9.23% of per capita GDP[14]. In our policy experiments, we fix the poverty headcount at the income level of the first 20.5 percentile of the population in the baseline specification.

[14] The per capita GDP in 2005 PPP dollars was $7.903,11.

Chapter 7

Introducing the Bolsa Família

■ In this chapter, we introduce a conditional cash transfer and evaluate the new stationary equilibrium. We prefer not to face our results against existing literature (though they do point to the same direction) - we prefer, as previously stated, to interpret our results as a potential long-term achievement of a continuous conditional cash transfer policy.

We set the transfer such that it represents 0.055% of the total income and is paid to 20.74% of the households, thus 41.5% of the households living through the first period, which fits true data in 2012. Notice that the proposed design benefits roughly the same share of the population as the share of households below the poverty line. Finally, transfers are only paid to families who send their children to secondary school (so families who are living through their second period of the model do not receive the transfer).

We also perform a counterfactual experiment, trying a different design. We provide a progressive transfer scheme, using the same income threshold as before. The families whose income are below half of the threshold earn a transfer 70% higher than the transfer paid in the previous experiment (henceforth baseline CCT), while the families whose earnings lie between half the threshold and the threshold itself are given only 85% of the previous transfer. All in all, in the steady state equilibrium, the transfers account for the same share of the GDP and cover 17.1% of the population.

In the counterfactual experiment, less than 2% of the population were given the higher transfer, while the vast majority of the covered households, 16.5% of the total population, only received the smaller transfer. However, as we shall see, the cash transfer effects are reasonably stronger than in the baseline CCT, which suggests that progressiveness is a very important feature of such a program.

It is also worth noticing the share of the population that chose not to get the transfer - in the baseline CCT formulation, this share accounted for 11.7% of the population, while in the progressive specification the share is only 6.8%. This fact also stresses the importance of a progressive transfer schedule. As in our model, there is evidence of undercoverage and mistargeting in the actual Bolsa Familia design[1].

Finally, we can also notice that, if we take into account the no transfer specification's steady state distribution, we see that, in the baseline CCT, 56.3% of the covered households had already decided to sent their children to secondary school, while the same ratio for the progressive scheme was 34.4%. While the transfer is not designed only to induce a higher education, a progressive schedule seems to be best choice when it comes to this specific output.

[1] [Soares et al., 2010] classify mistargeting as remarkably high. Also, [Soares et al., 2009] present data with respect to mistargeting: according to this work, the undercoverage rate (which represents the ratio of the non-beneficiary poor to the total poor population) was 59% in 2004, while the inclusion error (which is is the number of beneficiary non-poor divided by the total beneficiary population) was 41%.

Table 7.1: Tax and Transfers summary.

	No Transfer	Baseline	Progressive
Tax rate	9.58%	10.95%	10.20%
(Potencially) Covered Families	19.83%	20.70%	17.12%
Transfer Importance (% GDP)	0%	0.055%	0.058%

Note: The potentially covered families involve the same states at the non-CCT specification that, after the new policy is introduced and a new steady state equilibrium is reached, receive the transfer.

7.1 Long Term Macroeconomic Effects

■ The results for macroeconomic variables are shown it Table 7.2. First of all, we have to notice that the CCT serves as an insurance for the households, weakening precautionary motives, especially for those households who are at risk of being borrowing constrained ([Zilberman and Berriel, 2012]). This fact will play a crucial role in the program's outcomes. On the other hand, we should expect the program's incentives to education to increase the marginal return on physical capital, and thus mitigate the insurance effect. Hence physical capital accumulation output will be defined by these two counteracting forces.

In the baseline CCT, it turns out that absolute capital level increases by 15%, but, as expected, the investment rate remains roughly constant, as shown in table (7.2). The partial elimination of the risk was not able to completely counterbalance the strong rise in human capital efficiency units, which, by itself, was responsible to an increase of 11% on the marginal productivity of capital. Insurance and education effects combined raised the interest rate by 0.4 percentage points. Finally, as expected due to the decreased marginal return on labour, the (base) wages had a 4% decrease.

On the other hand, the progressive specification displays surprisingly good macroeconomic outputs.. Both output, human capital (in efficiency units) and absolute capital are even higher than in the baseline CCT.

When it comes to education, it seems quite clear that the Bolsa Familia, as designed in this paper, encourages schooling (unless the families choose not to take the transfer). By construction, at least 32% of the adults, in a steady state equilibrium, have attended at least secondary schooling. In fact, secondary schooling matriculation raised by 33 percentage points, and was greatly responsible for the sharp increase in education in the economy as a whole. On the other hand, the cash transfer program was not fully able to encourage tertiary schooling, which raised by only 0.54 percentage points (roughly a 10% increase).

On the other hand, the progressive program exhibited roughly no increase in tertiary school enrolment, but a sharper increase in secondary schooling, 13% higher than in the previous specification.

7.2 Long Term effects on Child Labour

■In the long-term, there are several general equilibrium effects that appear to be driving children out of the labour market, namely (i) children could reduce labour offer to be eligible for the program, (ii) reduced precautionary motives, (iii) reduced basic wages, (iv) increased taxation, (v) more demand for leisure, due to alleviated poverty, and (vi) school enrolment requisites, which consumes children's time endowment. It turns out that (iv) is probably not true, for tax rates are roughly the same, as we shall see in the next sections.

The conditional cash transfer is indeed efficient in reducing child labour for the whole population, as shown in table 7.2. In both CCT specifications, the share of children who both work

Table 7.2: Macroeconomic, Educational and Child Labour Outcomes.

Macroeconomic			
	No Transfer	Baseline	Progressive
GDP	-	+16.62%	+21.75%
Basic Wages	-	-4.22%	-4.17%
Annual Interest Rate*	2.64%	3.05%	3.04%
HC(Efficiency Units)	-	+17.21%	+22.21%
Investment Rate	-	-0.17%	-0.13%
Capital Level	-	+15.45%	+20.84%
Education			
	No Transfer	Baseline	Progressive
Uneducated*	14.82%	10.69%	6.61%
Only Primary Schooling*	55.10%	27.59%	19.36%
Secondary Schooling*	24.74%	55.84%	68.47%
Tertiary*	5.34%	5.88%	5.56%
Child Labour			
	No Transfer	Baseline	Progressive
OS*	78%	85.4%	90.3%
WS*	12.7%	8%	5.8%
OW*	9.2%	6.5%	3.9%
AVG Income if working	-	+7.4%	+16.6%
AVG Income if ws	-	+9.3%	+18.5%
AVG Wealth if working	-	-0.32%	-3.8%
Avg wealth if WS	-	+0.70%	+9.3%

Note: WS means both working and attending school, OS means only schooling, and OS means only working.
* Measured in levels.

and attend school or only work falls - again, in the progressive specification, the results are more substantial. In the baseline CCT specification, as we will see in the "Targeted Families" section, the reduction does not seem to be a direct impact of the transfer. In the progressive specification, we will see that those families who receive the conditional transfer do not drastically change their children's allocation of time.

In the progressive specification, we also see that, on average, the income of the families whose children work have raised. The income rise is consistent with data presented in table 4.2. In the baseline CCT specification, we see no changes in the wealth and incomes of the households whose children work. We'll take a closer look at reasons for that in the "Targeted Families" section, where we will stress the importance of a progressive cash transfer scheme.

7.3 Long Term Effects on Poverty and Inequality

One of the main goals of the Bolsa Familia program is to reduce extreme poverty. Even the benefit scheme is designed with some degree of discretionary with regard to extremely poor families. To evaluate extreme poverty reduction, we consider the poverty line as being the income earn by the family in the first 20.5 percentile of the income distribution[2]. Also, in the period between 1997 to 2012, income inequality in Brazil, measures as the Gini coefficient, fell from 0.60 to 0.52. Despite our model's limitations, it is possible to obtain useful insights on how the Bolsa Familia is able to reduce poverty. The results in this section are displayed in table 7.3.

The baseline specification has some impact on alleviating poverty - it reduces the (total) income Gini coefficient by 1.7 percentage points, while the wealth Gini fell by 3.2 percentage points. We also see a slight reduction of the wealth and income share of the richest - for instance, the fraction of the labour income captured by the top 1 percentile falls from 7.04% to 6.33%. The reduction on the wealth share of the top 1 percentile is stronger, representing 4 percentage points out of 15% in the no transfer specification.

Table 7.3: Poverty and Inequality Outcomes

Poverty and Inequality			
	No Transfer	Baseline	Progressive
Gini on Labor	0.40	0.37	0.34
Gini Wealth	0.65	0.62	0.59
Gini Total Income	0.43	0.41	0.40
Top 1% Wealth	15.0%	11.0%	12.5%
Top 5% wealth	39.9%	30.7%	29.5%
Top 1% Labour Income	7.0%	6.3%	6.4%
Top 5% Labour Income	20.6%	18.2%	18.4%
Poverty Line (% of the population)	20.5%	11.5%	5.5%
IE of (labour)Earnings	0.70	0.72	0.69

On the other hand, the progressive cash transfer scheme is a powerful poverty-reduction policy. It outranks the baseline specification in every presented aspect concerning poverty, except for the labour income share of the top percentiles. Despite this fact, since the progressive scheme is able to sharply reduce the Gini coefficient of labour income, we cannot say that it is not effective in reducing labour income inequality.

The channel through which both transfer schemes attack labour income inequality is clearly

[2] According to the world bank, 20.5% of the Brazilian population at 1997 earned less than two dollars a day.

by bringing a large share of the population to secondary schooling, *without* significantly affecting tertiary school enrolment. Since the productivity shock is widely responsible for heterogeneity in this economy, we believe a 5.2 pp reduction in the Gini coefficient of labour income to be significant.

Nevertheless, the most impressive impacts of the conditional cash transfer programs come from reducing poverty directly, instead of reducing inequality. The share of households who are below the poverty line (as defined in the section "Model Outcomes") fall by 9.0 percentage points in the baseline CCT specification and by an astonishing 15.0 percentage points in the progressive specification. Remember that all these results should be interpreted as a potential long-term poverty reducing power - so it is clear that it not the transfer itself that is able to bring households out of poverty, but, instead, the key here is the incentive to go to school.

Lastly, we see that social mobility, measured as the elasticity of the offspring's (labour) earnings with respect to parents' earnings did not respond to any of the cash transfer programs proposed. We believe that this result is driven by the way we construct the model whose only source of heterogeneity is the productivity shock. Models who also embed innate ability (or talent) in its framework are more successful in capturing variations in the intergenerational elasticity of earnings[3].

7.4 Targeted Families

In this section we evaluate how the targeted families reacted to the new policy. In the context of this model, the targeted families includes all the states x^1 who receive the transfer. We evaluate those states before and after the introduction of the transfer, in their respective steady-state equilibria. The outcomes for the targeted families are listed in table 7.4.

Table 7.4: Baseline CCT - Targeted Families

	No transfer	Baseline
WS	0%	0%
OS	100%	100%
Average Income	26%	34%
Average Wealth	8%	29%
Below Poverty	16%	0%

Note: WS means working and attending school and OS means only schooling.

The first fact here that jumps to one's eyes is that the baseline CCT program is *not able* to reach the poorest. In fact, it does not reach families whose kids work, but only families whose kids were not planning to go to secondary school. Moreover, only 15.65% of the covered families are below the poverty line. However, at least in the sense of stimulating school registration, the program is efficient. The fact that the transfer did not reach the poorest but the overall effects of the program were fairly beneficial gives a hint about the indirect power of such a policy.

In the baseline CCT program, the targeted households had their average income increased by more than 26%, as well as a 8% increase in their wealth[4].Also, the fraction of households below

[3] [Soares et al., 2010] also report that the Bolsa Familia fails to affect the intergenerational persistence of earnings.

[4] The way we compute these statistics are the following: we obtain the set of states that are eligible and choose to get the get the transfer. We then integrate the household income (which is a function of the state) over this

poverty in the targeted group falls almost to 0%[5], which is particularly surprising if we take into consideration that the transfer accounts for only 5.6% of the average income of targeted households. Hence, it is clear that the transfer plays an important role at the margin, taking many households who were previously "almost" indifferent between what level of education to choose to take their children to secondary school - in the long term, a virtuous circle is created, shifting the steady state equilibrium to one where there are more educated and wealthy families.

A more interesting and insightful case comes from the progressive cash transfer scheme, which is able to cover a larger part of the poorest families in the whole economy. We see that only by comparing the average income of the targeted families in this setup against the baseline CCT. Also, in this setup, there's a group of families whose children are working and attending school both. This policy is not able to fully eliminate child labour in the targeted families - contrarily, it has a strong child labour reducing effect in the economy as a whole. Also, targeted families display a sharp increase in their wealth and income, and an almost total decrease in poverty - from 42.7 [6] to less than 2% of the targeted families.

Table 7.5: Progressive CCT

	No Transfer	Progressive
WS	6.65%	4.92%
OS	93.35%	95.08%
Below Poverty	42.69%	1.78%

Note: WS means working and attending school and OS means only working.

7.5 Welfare

■ Our last analysis concerns the welfare evaluations between steady state equilibria. To start with, let's define the welfare, in consumption equivalent units, for a given state, as being:

$$W_1(x^1) = \frac{1}{1-\beta}\left[\left(\frac{g_1^{c(1-\gamma)}}{1-\gamma}\right) - (\phi + \mathbb{I}[h' > h_0]\phi_{ws})g_1^m\right], \tag{7.1}$$

for the first period and:

$$W_2(x^2) = \frac{1}{1-\beta}\left[\left(\frac{g_2^{c(1-\gamma)}}{1-\gamma}\right)\right], \tag{7.2}$$

for the second period. To obtain the total economy's welfare we just integrate over the possible states, using the invariant measure *before* the introduction of the new policy.

We only execute comparisons of steady state equilibria in our welfare analysis. Thus, contrary to what [Zilberman and Berriel, 2012] does, we do not claim anything about political support, neither we perform definite welfare analysis, since we do not compute the transition path in this

set, both in the no transfer specification and in the baseline CCT specification. To conclude, what an increase in the average income of the targeted families tells us is that there was an increase on the relative measure (in the model, the function $\lambda_1(x^1)$) of those states with relatively higher income within the targeted set.

[5] The average income of those families whose children are not going to school but only working is below 40% the average income of the targeted families. Those families are in such a poverty situation that they choose not to accept the transfer.

[6] [Soares et al., 2010] reports that 41% of the targeted families are in poverty situation.

model. Nevertheless, the steady state comparison is, if anything, suggestive.

To measure the equivalent variation of welfare in terms of consumption, we solve the following implicit equations for Δ:

$$\left\{\left(\frac{(g_1^c+\Delta)^{(1-\gamma)}}{1-\gamma}\right) - (\phi + \mathbb{I}[h' > h_0]\phi_{ws})g_1^m\right\}_{before} = \left\{\left(\frac{(g_1^c)^{(1-\gamma)}}{1-\gamma}\right) - (\phi + \mathbb{I}[h' > h_0]\phi_{ws})g_1^m\right\}_{after}, \tag{7.3}$$

in the first period and

$$\left[\left(\frac{(g_2^c+\Delta)^{(1-\gamma)}}{1-\gamma}\right)\right]_{before} = \left[\left(\frac{(g_2^c)^{(1-\gamma)}}{1-\gamma}\right)\right]_{after}, \tag{7.4}$$

in the second period.

Table 7.6 displays the results for the overall population and for only the targeted families. This table shows that the increased welfare is higher for the targeted families than for the economy as a whole, in both specifications. Also the ratio of households that are better off after reaching the new equilibrium is impressive, especially in the progressive specification.

Table 7.6: Welfare Comparison

Whole Economy		
	Baseline	Progressive
% Change in welfare	+0.48%	+2.24%
% of the Population with higher welfare	46.51%	100.00%
Difference between steady-state total welfare*	+16.37%	24.51%
Targeted Households		
	Baseline	Progressive
%Change in welfare	2.15%	3.51%
% with higher welfare	99.88%	100%

Note: Welfare is measured in consumption equivalent units (CEV). A possible interpretation of this table could be " a comparison between whether an unborn person would choose to live in a pre-CCT world or in a post-CCT world, after the new steady state equilibrium is achieved."
* Takes into consideration the differences across steady equilibria measures of each state (the number is higher than, for instance, % change in welfare because, in both CCT specifications, there are more families in "wealthier" states.

Finally, comparing the difference between steady-states in the total economy's welfare, we are able to answer the main question proposed by this paper. In the long-term, a conditional cash transfer as the Bolsa Familia, progressive or not, is able to substantially increase the economy's total welfare.

Chapter 8

Conclusion

■ In this paper, we evaluated the long-term quantitative effects of the Brazilian conditional cash transfer program, Bolsa Familia. We have proposed two transfer schedules: (i) transferring an equal amount to every family whose income is below a specific threshold, and (ii) providing a progressive transfer, with the lowest income families receiving a higher transfer. Both proposed policies are comparable, in the sense that the total amount of transfers as a ratio of the GDP (in the steady state equilibrium) are roughly the same, and the number of covered families is similar.

We find out that, no matter the schedule, the policy is able to significantly raise school registration and is fairly able to reduce child labour. The flat transfer scheme is able to raise the ratio of children who complete secondary school by 33 percentage points, lowering the ratio of children who do not attend school at all by 4 percentage points. It is also able to reduce the ratio working children from 22% to 15%. The transfer is also able to raise total output by almost 17%.

The progressive transfer scheme is more efficient in many aspects. It raises school enrolment relatively more than the flat transfer and has a stronger impact in child labour. Even so, it is not able to reach the most impoverished households, who prefer not to pay the secondary schooling costs and hence are not allowed the transfer.

All in all, the Bolsa Familia proves to be a strong policy instrument, not only benefiting the targeted families but also the households who aren't eligible. We believe that the most important aspect of such a policy is that it is almost does not distort the household optimal decisions - the increased taxation required to fund it is, in equilibrium, offset by the sharp increase in productivity and thus income - the result is that the tax rates can even *decrease* in equilibrium. For instance, it is feasible that the raise in productivity is so high that the government can lower the tax rate for a given expenditure. However, this was not the case in our economy: in fact, the government has to increase the tax rate to subsidize the secondary, which can be seen in table 7.1. Even so, the increase is slight, and the impacts of the conditional cash transfer are impressive.

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Appendix

Computing the Stationary Competitive Equilibrium

Steps to compute the Stationary Equilibrium:

1 Start with a lower and an upper bound for the interest rate. Use their average as the guess for the rate and pin down the wage rate after the aggregate capital-labour ratio.

2 Solve for the policy function in each state: guess an initial value function, maximize over the choice variables and reiterate the value function until convergence.

3 Guess an initial distribution, simulate the economy using the policy functions obtained until the distribution converges to a stationary one.

4 Using the stationary distribution and using the definition of equilibrium provided, compute the economy-wide aggregate capital and labour efficiency units.

5 Pin down the demand-side interest rate. If the new rate is above the guess, make the lower bound equal it. If the new rate is below the guess, make the upper bound equal it. Reiterate from step (1) until the absolute distance between the guess and the demand-side rate is close to zero (below a set tolerance).

Computing the Government Policy

Steps to compute the Stationary Equilibrium with Government:

1 Propose a policy $ctr, ithr, sthr, \mathcal{S}(\cdot)$.

1 Start with a lower and an upper bound for the tax. Use their average as the guess for the taxation. Guess starting values for the aggregate variables.

2 Solve for the Stationary Equilibrium as in appendix 5.1.

3 Using the economy-wide new aggregates, check if the government balances its budget, by checking the tax necessary to finance its policies.

4 Verify if the tax converges. If not, update it as the following: if the new rate is above the guess, make the lower bound equal it. If the new rate is below the guess, make the upper bound equal it. Restart from step (2).