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PRICE DISCOVERY USING A REGIME-SENSITIVE COINTEGRATION APPROACH

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Dissertação apresentada à Escola de Economia de São Paulo da Fundação Getulio Vargas, como requisito para obtenção do título de Mestre em Economia

Campo de Conhecimento:
Finanças Empíricas

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ABSTRACT

This work proposes a method to examine variations in the cointegration relation between preferred and common stocks in the Brazilian stock market via Markovian regime switches. It aims on contributing for future works in "pairs trading" and, more specifically, to price discovery, given that, conditional on the state, the system is assumed stationary. This implies there exists a (conditional) moving average representation from which measures of "information share" (IS) could be extracted. For identification purposes, the Markov error correction model is estimated within a Bayesian MCMC framework. Inference and capability of detecting regime changes are shown using a Montecarlo experiment. I also highlight the necessity of modeling financial effects of high frequency data for reliable inference.

Keywords: Markov Switching, High Frequency, Gibbs Sampling, Cointegration, Nonlinearity

RESUMO

Este trabalho propõe um método para examinar variações na relação cointegração de preços de ações preferenciais e ordinárias da bolsa brasileira através de mudanças de regime no sentido de Markov. Este modelo tem como objetivo contribuir tanto para futuros trabalhos em negociações de pares ("pairs trading") quanto, principalmente, para aplicação em descoberta de preços visto que, condicional nos estados, é pressuposta estacionariedade no sistema. Desta maneira seria possível a extração de medidas de "parcela de informação" (IS) baseadas na representação de médias móveis de um modelo de correção de erros Markoviano, estimado através de um ferramental bayesiano do tipo MCMC por questões de identificação. A validade do modelo no sentido de capturar as variações de regime é demonstrada através de experimento de Montecarlo, bem como é evidenciada a necessidade da modelar não normalidades na distribuição dos dados de alta frequência visando inferência.

Palavras-Chave: Mudança de Regime Markoviano, Alta Frequência, Amostragem de Gibbs, Cointegração, Não Linearidade

Contents

1 Introduction

The modeling of long run stable relations has always been key to the economics literature. The theorem of representation by Engle e Granger (1987) allowed the study of non-stationary time series built on this very principle. In particular, a large part of the academia interested in finance (and financial econometrics) has been devoting great attention to understand the joint evolution of asset prices.
In recent years there has been a significant increase of available data and high frequency traders have been pivotal to a more rigorous assessment of the dynamics of information dissemination among markets.

Price discovery relies on the fundamental value of the firms. For illustration, if we look at the prices of two different classes of the same stock (say PETR4 and PETR3, for Brazilian oil company Petrobras preferred and common shares respectively) no arbitrage tells us we must not have strong divergences on their patterns. This implies that they eventually cointegrate. The question that arises is then which share class reacts first to the arrival of relevant information.

The methodological contribution in Fernandes e Scherrer (2012) propose a measure of Information Share (IS) based on the relative variance of the innovations vector of the moving average (VMA) representation of the error correction process. The authors, however, suggest that this measure is not constant over the whole period considered in the paper.
This opens a window of possibilities for considering non-linear models. In particular, the results in Granger e Siklos (1997), who argue that there's no reason to expect the error correction process to be symmetric and linear, are explored by allowing the equilibrium relation between the price series to be temporarily "switched off". If this is the case, there exists two VMA representations of the system, which can give rise to two IS measures.
In their paper, however, the exact moment when the structural break point occurs is completely exogenous. Psaradakis, Sola e Spagnolo (2004) extend the idea by introducing a Markov error correction (MEC) model where the changes of regime are produced endogenously. To detect this behavior they use a multi-step test procedure as in Balke e Fomby (1997), which looks at differences between local and global evolution of the system [1].

[1] A thorough appraisal on detecting periods of short run deviations is offered by Kim (2003)

Although valid, threshold cointegration as in Balke e Fomby (1997) is potentially misleading either in small samples or if the regime where cointegration is present is not the dominant over the period, as pointed out by Sugita (2008). To circumvent these issues, I follow the methodology in Sugita (2006) by employing a Bayesian technique, which circumvents identification and inference issues in estimating the VECM.

This setup is then applied to high frequency financial data in the Brazilian stock market. Particularly, I'm interested in the dual-class effects of information dissemination as investigated in Fernandes e Scherrer (2012). I restrict attention to the two companies with most liquid stocks in the local market and asses the capability of the model in identifying a Markovian error correction model.

The rest of the work is organized as follows: Chapter 2 describes the data. Chapter 3 presents the model and estimation procedure. Results and diagnostics are reported in Chapter 4. Chapter 5 concludes.

2 Data

The database is constructed using tick by tick transaction information from B&MFBOVESPA (the Brazilian stock exchange) for Petrobras (Petróleo Brasileiro S.A.) and Vale from January 2011 through December 2011.

In the Brazilian market, Petrobras and Vale issue two types of shares: common (or ON - PETR3 and VALE3 respectively) and preferred (or PN - PETR4 and VALE5 respectively). Due to regulatory issues, until January 2001 all national companies listed in the local stock exchange were allowed a 2-by-1 issuance rate for PN:ON, leaving a ratio of shares outstanding of about 3-to-1 as of today. This has great impact on liquidity differences, which will force gathering data at a high frequency in order not to distort any lead-lag effect[1].

Regular trading (excluding pre and after-market) hours in Bovespa range from 10:00:00 AM to 5:00:00 PM from mid February until mid November. The rest of the year accounts for "daylight saving time", in which trading hours are delayed by one hour. Despite the vast amount of information on each tick, only transaction prices (in logarithm) are kept.

2.1 Data filtering

Dealing with tick data raises concerns about the quality of the information contained. As stated by Brownlees e Gallo (2006), the faster the trades occur the greater the chances of noise[2] in data.

In what follows, I adopt the methodology in Brownlees e Gallo (2006) which is, for convenience, described below:

1. Set i=1 and let p_i denote the "ith" available price of a day in sample

2. Construct $\bar{p}_i(k,\delta)$ and $s_i(k,\delta)$, the δ trimmed mean and standard deviation in the vicinity of k observations[3] around p_i

[1] Suppose stocks A and B are traded and A is more liquid. If we aggregate in a sufficiently low frequency we can't observe stock B only responding seconds later to a variation in A

[2] Bid-ask bounce, wrong reporting and market inconsistent quotes are examples

[3] Note that the first k/2 prices of the day will remain inside the first k prices window. Same happens for the last k/2 prices of the day

3. If $|p_i - \bar{p}_i(k,\delta)| < 3s_i(k,\delta) + \gamma$, where γ is a granularity parameter, we keep p_i. Else, p_i is discarded.

4. Set i=i+1 and repeat items 2-4 until i=N, the number of transactions for each security.

Before running the above steps I also discard the first and last 15 minutes of trades of the day. This is done to disregard opening and closing market activities which, in general, account for greater trading intensity.
Table 2.1.1 reports raw data count and quantity of trades after filtering, for each security. Percentage of outliers is also informed.

According to the literature, the filtering procedure is quite sensitive to the choice of k and γ. It is good practice to choose k according to the securities' liquidity, so that the time span covered by the window is not too large, which could cause outliers to be misidentified. In this work k is allowed to vary with trade intensity, ranging from 30 (for less liquid periods) to 60. The granularity parameter is fixed at 0.01, the smallest possible price variation, guaranteeing $|p_i - \bar{p}_i(k,\delta)| = 0 < \gamma$ whenever $s_i(k,\delta) = 0$.

Table 2.1.1 shows the number of transactions before and after filtering, as well as the percentage of outliers detected by the algorithm.

Table 2.1.1 – Raw and filtered data for $\delta = 10\%$ reported in millions of observations

Company	Ticker	# of Raw Observations	# of Filtered Observations	Outliers (%)
Petrobras	PETR4	5.664	5.216	7.91
Petrobras	PETR3	2.084	1.947	6.60
Vale	VALE5	5.237	4.810	8.15
Vale	VALE3	2.047	1.906	6.92

2.2 Data aggregation

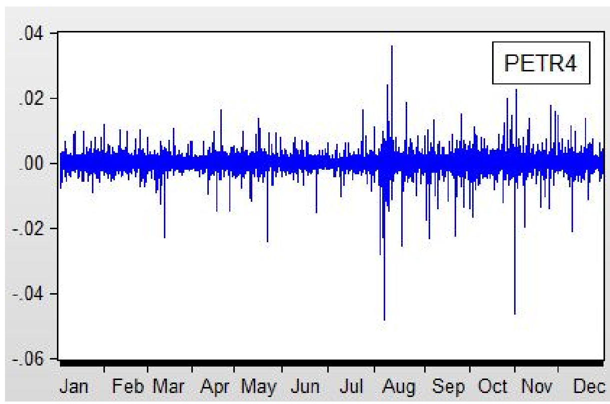
In order to adopt the methodology in Fernandes e Scherrer (2012) one must have equally spaced observations. To do so I consider time windows of 5 and 15 minutes (giving rise to roughly 78 and 26 observations respectively per day). The "window price" is chosen such that synchronicity is respected. This goal is achieved by minimizing the difference in time when transactions occur[4] which also attenuates the effects of the lack of liquidity of the local

[4] See Barndorff-Nielsen et al. (2011) for reference, who do so for covariance estimation purposes.

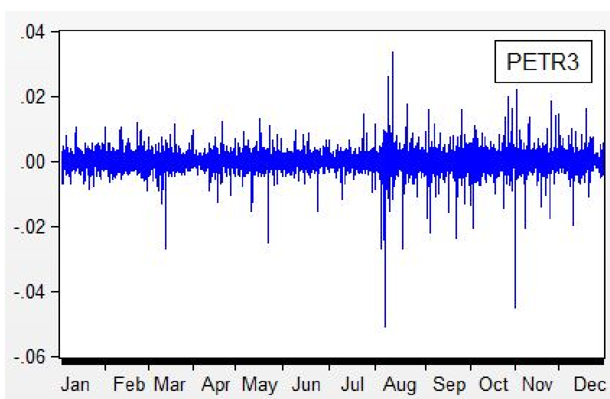
market.
Empty windows have its price repeated from the previous observation[5].
Aggregation plays a role since it represents an important trade-off: the lower the frequency, the lower the contemporaneous correlation but also the lower the microstructure effects.

Figures 1 and 2 plot the returns for Petrobras in 5 and 15 aggregation intervals. The plots suggest non-constant oscillatory behavior through the year, with periods of higher variation. The shock in the beginning of august appears to be of major importance and persistence.

Figure 1 – Returns for Petrobras: 5 minutes aggregation intervals. y axis represents returns in decimals



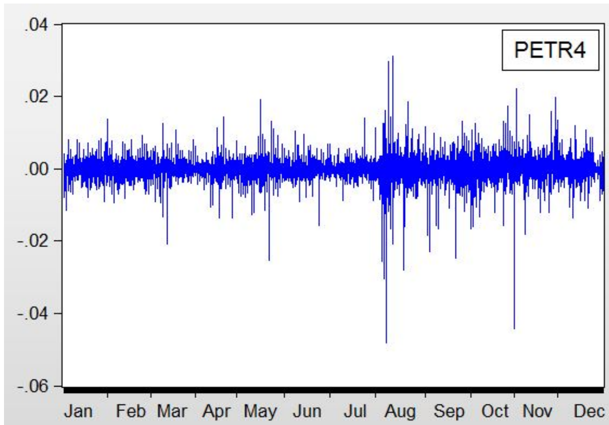
(a) 5 minutes aggregation for PETR4



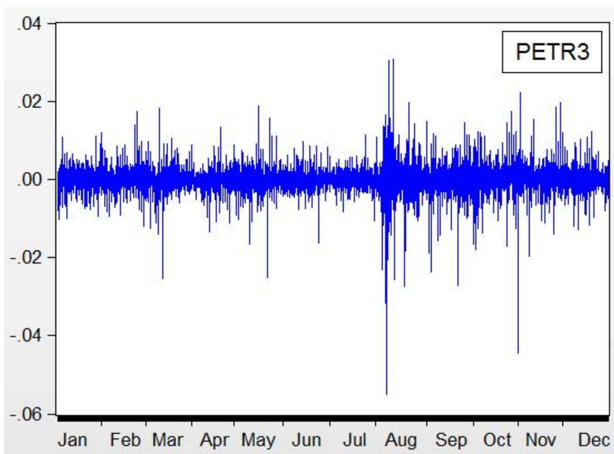
(b) 5 minutes aggregation for PETR3

5 For the given sample, aggregation frame of 5 minutes gives rise to 16 "zero return prices" for PETR3 and 6 for VALE3. Fifteen minute windows present no such events

Figure 2 – Returns for Petrobras: 15 minutes aggregation intervals. y axis represents returns in decimals



(a) 15 minutes aggregation for PETR4

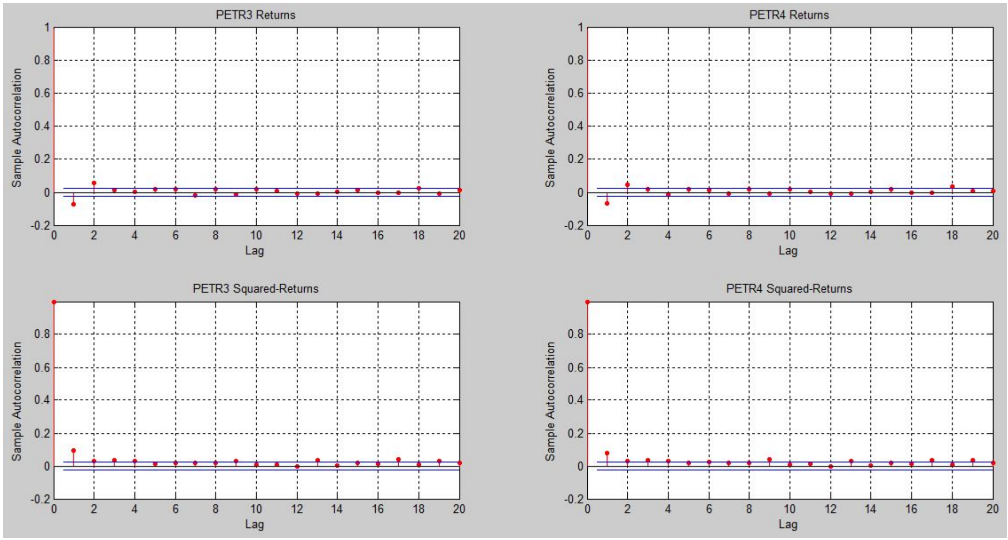


(b) 15 minutes aggregation

To examine the persistent behavior I look at the autocorrelation of the returns and squared-returns in figure[6] 3. For both PETR3 and PETR4 the effects of the lagged returns on their contemporary counterpart appear statistically insignificant after 3 lags. The same inference, however, can't be drawn if looking at the squared returns, which can be an indicative of strong variance persistence.

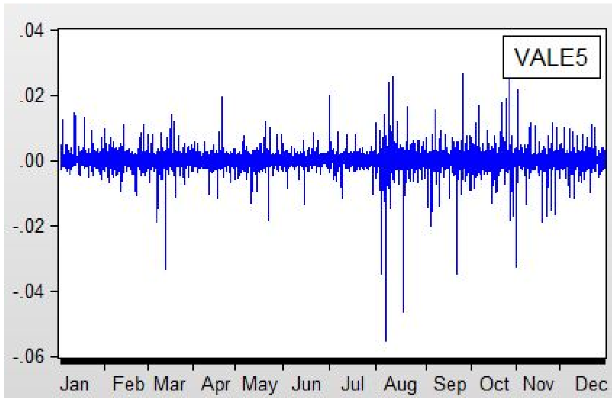
6 The plots correspond to the series with 15 minutes aggregation. Although not quantitatively equal, the same qualitative information can be drawn with 5 minutes aggregation

Figure 3 – Autocorrelations for Petrobras

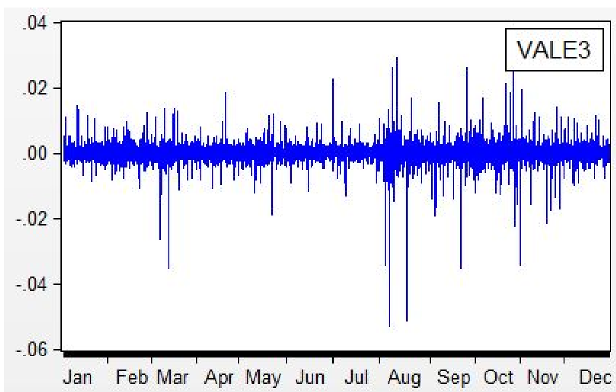


Figures 4 and 5 plot the returns for Vale: the oscillatory behavior is similar to the one presented by the returns of Petrobras.

Figure 4 – Returns for Vale: 5 minutes aggregation intervals. y axis represents returns in decimals

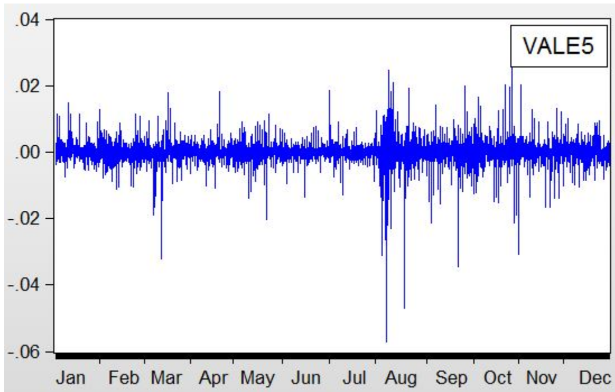


(a) 5 minutes aggregation for VALE5

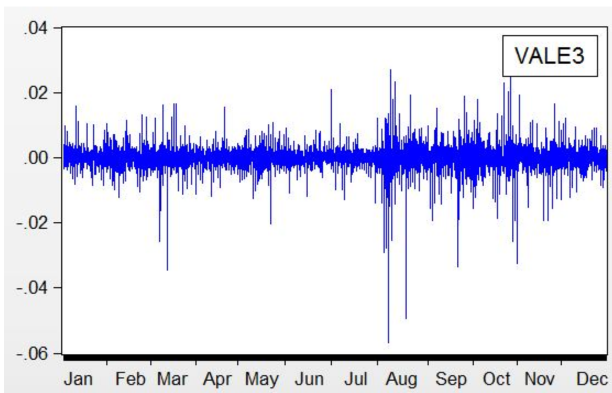


(b) 5 minutes aggregation for VALE3

Figure 5 – Returns for Vale: 15 minutes aggregation intervals. y axis represents returns in decimals



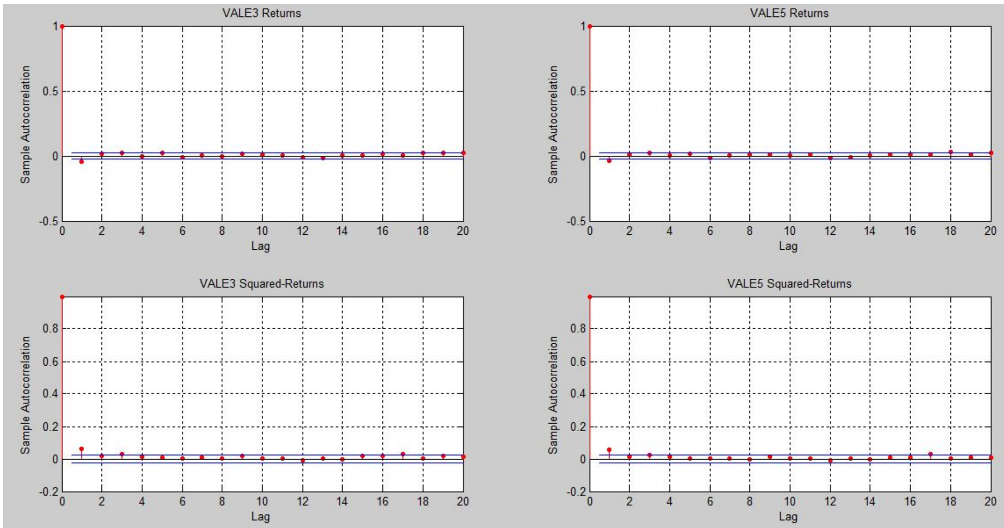
(a) 15 minutes aggregation for VALE5



(b) 15 minutes aggregation for VALE3

The same exercise is performed for the autocorrelations of the mining firm, which is presented in figure 6 below.

Figure 6 – Autocorrelations for Vale



A quick visual inspection would suggest that the returns and squared-returns for Vale are more short-lived if compared to the returns of the oil company.

In general, 2011 can be considered turbulent in the financial markets, being not much favorable to emerging markets.
In early march, a nuclear catastrophe in Japan plummeted commodities prices, adding more volatility to asset prices. In particular Vale was mostly affected, having Japan as an important commercial partner, with exports to that country representing around 11% of whole exports in 2010.
By the end of may of 2011 the European Union offers financial support for Portugal and Ireland, while Greece struggles to repay its debts.
August is marked as critical month due to contagion risk in Europe. The lack of resolution of this sovereign situation causes a lack of liquidity of the global financial system on the second semester. Combined with a weak demand of the G3 countries, slowing Chinese growth and a drop in the business confidence in Brazil, the second semester is also marked as very volatile in the financial market.

Table 2.2.1 reports descriptive statistics of the returns of the stocks for the two different aggregation windows, namely 5 and 15 minutes. In general, we can notice a negative skewness and large positive excess kurtosis. The latter was highly expected and has a positive relation with aggregation window size: greater windows tend to diminish the tails of the distribution due to the smaller presence of intraday noise whereas the opposite occurs as we augment the frequency, as noticed below.

Table 2.2.1 – Descriptive statistics of returns for aggregation intervals

5 minutes

Ticker	Mean ($\times 10^{-6}$)	Std. Dev.	Skewness	Kurtosis	Sample Size
PETR4	-11.66	0.0019	-1.810	70.5522	19396
PETR3	-13.98	0.0021	-1.562	53.0815	19396
VALE5	-10.47	0.0018	-3.130	115.202	19396
VALE3	-15.14	0.0020	-2.535	88.4551	19396

15 minutes

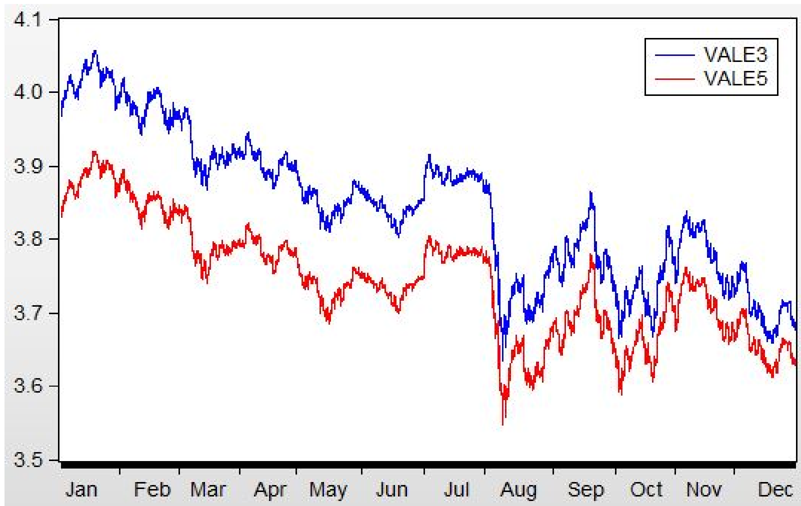
Ticker	Mean($\times 10^{-6}$)	Std. Dev.	Skewness	Kurtosis	Sample Size
PETR4	-33.46	0.0032	-1.073	27.4656	6459
PETR3	-40.88	0.0035	-1.156	24.9896	6459
VALE5	-31.17	0.0031	-2.100	43.7211	6464
VALE3	-45.33	0.0034	-1.738	34.4982	6464

Figure 7 plots the logarithm of the prices for Petrobras and Vale with a 5 minute aggregation window[7]. From January until the beginning of August there's a clear downward trend which may be explained by internally by weak industrial production and a tightening monetary cicle and globally by uncertainties regarding the contagion risk of the European debt crisis[8]. For the rest of the year there exists a noticeable rebound by the end of August that lost force, which can be related to the lack of global liquidity and slow economic recovery of Europe, China and USA, making the performance of the preferred shares of Petrobras and Vale close the year with a negative return of approximately 20.2% and 25.8% respectively and the common stocks with losses around 23.1% and 28.4% in the period.

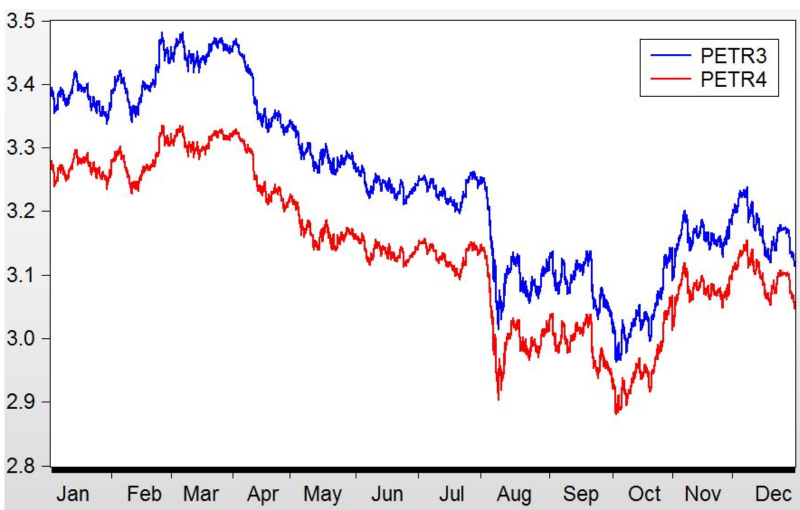
[7] 15 minute aggregation won't be plotted due to the great similarity of the series in level.

[8] Greece's been bailed out by the European Union in July 2011, raising concerns over the situations of other peripheral countries. There's also been a downgrade, by Standard and Poors, of the US credit rating, from AAA to AA+ in early August.

Figure 7 – Log prices in 5 minutes aggregation



(a) Log prices of Vale



(b) Log prices of Petrobras

3 The Model

3.1 Markov Switching Error Correction Model

Consider $\{x_t^i\}_{t=1}^T$ to be, for each $i \in \mathbb{I} \equiv \{1, ..., N\}$, an integrated of order 1 time series and, for sake of notation, define $X_t \equiv [x_t^1 \quad ... \quad x_t^N]'$. Assume there exists a set of r vectors $\beta \in \mathbb{R}^N \times \mathbb{R}^r$ for which $\beta' X_t \quad \text{with} \quad t \in \mathbb{T} \subset \{1, 2, ..., T\}$ is a vector with stationary time series (in any other subset of times they do not cointegrate).

Let S_t follow a first-order two states Markov chain with transition probabilities (p_{ii}, p_{jj}) with $i, j \in \mathbb{S} \equiv \{0, 1\} \quad \text{and} \quad i \neq j$, i.e., $S_t \in \mathbb{S} \quad \forall t \in 1, 2, ...T$ and $\forall i, j \in \mathbb{S}$,
$\text{Prob}(S_t = j | S_{t-1} = i) = p_{ij}$.

If we assume $t \in \mathbb{T} \quad \text{iff} \quad S_t = 1$ we might want to jointly model these time series using a vector error correction representation with Markov switching behavior:

$$\Delta X_t = \mu + \alpha(S_t)\beta' X_{t-1} + \sum_{i=1}^{p-1} \Phi_i \Delta X_{t-i} + \varepsilon_t \tag{3.1}$$

with $\varepsilon_t \sim N(0, \Sigma(S_t))$, μ being a constant parameter and $\alpha(S_t)$ an Nxr matrix of loadings that evolves according to the Markovian process above.
To cope with the setup in Sugita (2006) I also allow the constant term μ and the lagged variable coefficients Φ to be regime-dependent. Hence, equation 3.1 could be conveniently rewritten as:

$$\Delta X_t = \mu(S_t) + \alpha(S_t)\beta' X_{t-1} + \sum_{i=1}^{p-1} \Phi_i(S_t) \Delta X_{t-i} + \varepsilon_t \tag{3.2}$$

Notice that equation 3.2 has a natural matrix representation of the form:

$$Y = X\Gamma + Z\beta\alpha' + E \equiv WB + E \tag{3.3}$$

with $\quad W = [X \quad I_0 Z\beta \quad I_1 Z\beta] \quad$ and $\quad B = [\Gamma' \quad \alpha]'$, $\alpha = [\alpha_1, \alpha_0]$,
$\Gamma = [\mu_1, \mu_0, \Phi_{1,1}, \ldots, \Phi_{p-1,1}, \Phi_{1,0}, \ldots, \Phi_{p-1,0}]'$ and

$$Y = \begin{bmatrix} \Delta X'_p \\ \Delta X'_{p+1} \\ \vdots \\ \Delta X_T \end{bmatrix}, \quad Z = \begin{bmatrix} X'_{p-1} \\ X'_p \\ \vdots \\ X_{T-1} \end{bmatrix}, \quad E = \begin{bmatrix} \varepsilon'_p \\ \varepsilon'_{p+1} \\ \vdots \\ \varepsilon'_T \end{bmatrix}$$

$$X = \begin{bmatrix} s_{1,p} & s_{0,p} & s_{1,p}\Delta X'_{p-1} & \dots & s_{1,p}\Delta X'_1 & s_{0,p}\Delta X'_{p-1} & \dots & s_{0,p}\Delta X'_1 \\ s_{1,p+1} & s_{0,p+1} & s_{1,p+1}\Delta X'_p & \dots & s_{1,p+1}\Delta X'_2 & s_{0,p+1}\Delta X'_p & \dots & s_{1,p+1}\Delta X'_2 \\ \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \vdots \\ s_{1,T} & s_{0,T} & s_{1,T}\Delta X'_{T-1} & \dots & s_{1,T}\Delta X'_{T-p+1} & s_{0,T}\Delta X'_{T-1} & \dots & s_{0,T}\Delta X'_{T-p+1} \end{bmatrix}$$

As in Sugita (2006) $s_{m,t}$ is an indicator variable $\mathbf{1}\{S_t = m\}$ and I_m is a square matrix where the off-diagonal elements are equal to 0 and the t-th diagonal entry is $s_{m,t}$.

3.2 Estimation

Testing for breaks in the cointegration relation using classical methods is an intricate problem. In particular, consider the hypothesis test (in usual notation) on equation 3.2:

$$T : \begin{cases} H_0 : \ \alpha_{(S_t)} = 0 \\ H_1 : \ \alpha_{(S_t)} \neq 0 \end{cases}$$

It is clear that the model is not identified under the null. Andrews e Ploberger (1994), for example, offer a theoretical framework for testing for breakpoints under non identifiability of nuisance parameters. Some ex ante structure, however, must be assumed for detection. Balke e Fomby (1997) propose a multi-step algorithm in which they detect global cointegration and test locally for parameter instability and non-linearities, building thresholds for the long run relation. Given the lack of power of Johansen's tests under non-normality of the residual, this alternative could be potentially misleading.

In a Bayesian framework, however, those problems do not arise: nuisance parameters are integrated out of the joint conditional probability distribution and testing for changes in regime becomes an issue of model selection using Bayes factors. Throughout, β is assumed to be known and r is kept fixed and equal to 1.
With this in mind, this work employs a Gibbs-Sampling strategy as follows:

1. Set i=1 and give starting values $\tilde{S}_T^{(0)} = \{S_1^{(0)}, S_2^{(0)}, ..., S_T^{(0)}\}'$, $\Sigma_m^{(0)}$;

2. Generate $B^{(i)}$ from $p(B|\beta, \Sigma^{(i-1)}, \tilde{S}_T^{(i-1)}, Y)$ and $\Sigma_m^{(i)}$ from $p(\Sigma_m|B^{(i)}, \tilde{S}_T^{(i-1)}, Y)$;

3. Generate $(p_{00}, p_{11})^{(i)}$ from $p(p_{00}, p_{11}|\tilde{S}_T^{(i-1)})$;

4. Generate $\tilde{S}_T^{(i)}$ from $p(\tilde{S}_T^{(i-1)}|\Theta^{(i)}, Y)$, where $\Theta = \{B, \Sigma_m, p_{00}, p_{11}\}$, using the multi-move Gibbbs Sampling (forward filtering, backward sampling);

5. Set i=i+1 and go back to 2.

where $p(\Xi|\eta)$ denotes the posterior distribution of Ξ conditional on η, for any two random variables (Ξ, η).

3.2.1 Identification restrictions

The Gibbs sampling algorithm is not able to identify neither the state variables nor the transition probabilities if the true data generating process in not regime dependent. For this reason I follow the methodology in Koop e Potter (1999) and restrict, *a priori*, that each state occurs at least 15% and at most 85% of the time at the states vector $\tilde{S}_T$. That is, for state m,

$$15 \leq \sum_{t=1}^{\tau} s_{m,t}/\tau \leq 85\%$$

If this restriction is not satisfied at any run i of the Gibbs sampler make $\tilde{S}_T^{(i)} = \tilde{S}_T^{(i-1)}$.

3.2.2 Priors, posteriors and sampling scheme

Inference on equation 3.3 requires no specific structures for priors, i.e., they are allowed to be diffuse (except for the cointegrating vector). However, the computation of Bayes factors require them to be proper. Following Sugita (2006), priors are chosen as:

$$\begin{aligned}
\Sigma_m &\sim IW(S_m, h_m) \\
vec(B) &\sim MN(P, \Omega_B) \\
p_{00} &\sim \text{beta}(u_{00}, u_{01}) \\
p_{11} &\sim \text{beta}(u_{11}, u_{10})
\end{aligned} \tag{3.4}$$

where IW and MN stand for the inverse wishart and matrix normal distributions respectively, S has the same dimensions of Σ_m, h_m represents degrees of freedom in IW,P the mean of vec(B) and Ω_B is assumed block diagonal, implying prior independence between Γ and α.

This formulation also implies prior independence between Σ_m and B, i.e.:

$$p(B|\Sigma_1, \Sigma_0) = p(B)p(\Sigma_1, \Sigma_0)$$

The posterior distributions are all derived in Sugita (2006) [1] and result in:

$$\begin{aligned} \Sigma_m|B, \tilde{S}_T, Y &\sim IW([Y_m - W_m B]'[Y_m - W_m B] + S_m, h_m + \tau_m) \\ p_{00}|\tilde{S}_T, Y &\sim \text{beta}(u_{00} + m_{00}, u_{01} + m_{01}) \\ p_{11}\tilde{S}_T, Y &\sim \text{beta}(u_{11} + m_{11}, u_{10} + m_{10}) \\ vec(B)|\Sigma_m, \tilde{S}_T, Y &\sim MN(vec(B_*), M_*) \end{aligned} \tag{3.5}$$

with m_{ij} representing the number of transitions from state i to state j and $\tau_m = \sum_{t=1}^{\tau} s_{m,t}$. Also:

$$M_* = \left\{\Omega_B^{-1} + \sum_{m=0}^{1} [\Sigma_m^{-1} \otimes (W_m' W_m)]\right\}^{-1}$$

and

$$vec(B_*) = M_* \left\{\Omega_B^{-1} P + \sum_{m=0}^{1} [(\Sigma_m^{-1} \otimes I) vec(W_m' Y_m)]\right\}$$

To sample from $\tilde{S}_T$ the choice is made for the multi-move Gibbs sampler. A forward filtering is performed based on Hamilton (1989) and backward sampling from the relation:

$$\Pr(S_t = 1|S_{t+1}, Y, \Theta) = \frac{\Pr(S_{t+1}|S_t = 1)\Pr(S_t = 1|\Theta, Y)}{\sum_{j=0}^{1} \text{pr}(S_{t+1}|S_t = j)\Pr(S_t = j|\Theta, Y)} \tag{3.6}$$

for t=T-1,T-2...1.
Once equation 3.6 is computed, let u be a realization of $U \sim U_{[0,1]}$, the uniform distribution. Set $S_t = 1$ if $\Pr(S_t = 1|S_{t+1}, Y, \Theta) \geq u$ and 0 if contrary.

[1] In case of homocedasticity the model is simplified and can be also be seen in Sugita (2008). Strachan e Inder (2004) offer a deeper discussion on the posterior distribution of the cointegrating vector.

4 Montecarlo experiment and application

The upcoming sections are devoted to testing and implementing the model. For this purpose, the hyperparameters used in the prior distributions were chosen considering complete unawareness about the true parameter values. Table 4.0.1 briefly discusses each of the arguments[1]:

Table 4.0.1 – Prior Hyperparameters

Parameter of interest	Prior Distribution	Hyper-parameter	Value	Comment
Σ_m	IW(S_m, h)	S_m	$\mathbf{I}/1000$	Diminish precision of prior parameters
Σ	IW(S,h)	h	2	Given the posterior set sufficiently small to be irrelevant compared to τ
vec(B)	$MN(P, \Omega_B)$	P	$\mathbf{0}$	Prior vectorial mean of the coefficient matrix. Irrelevant for large Ω_B
B	$MN(P, \Omega_B)$	Ω_B	$\mathbf{I}$*1000	Ensure large variance of prior distribution.
p_{ii}	beta(u_{ii}, u_{ij})	(u_{ii}, u_{ij})	(9,1)	Being set small enough are dominated by the number of transitions in each states vector.

Note: $i, j = 0, 1$ and $i \neq j$ in the last line of the table.

Except for the prior hyperparameters of the transition probabilities' distributions[2] the following exercises were performed with different prior parameters to insure robustness. As desired (and also pointed by Sugita (2008)), these choices have no impact on model inference.

[1] Dimensions omitted.

[2] This observation is relevant for small sample sizes. For reference, see properties of the beta distribution in Kim e Nelson (1999)

4.1 Artificial data

To asses the validity of the model with the restriction $\alpha_{(S_t=0)} = (0,0)'$, bidimensional systems of size T=(250,500,1000) with transition probabilities $(p_{11}, p_{00}) \in \{(0.98, 0.95), (0.95, 0.95), (1, 0)\}$, $\alpha_{(S_t=1)} = \alpha_1 = -[0.2, 0.1]'$, $\alpha_{(S_t=0)} = \alpha_0 = [0, 0]'$, $\beta = [1, -0.95]'$, $\mu_{(S_t=1)} = \mu_1 = [0.01, 0.01]'$, $\mu_{(S_t=0)} = \mu_0 = -\mu_1$, $\Sigma_{(S_t=1)} = \Sigma_1 = \frac{1}{1000}\begin{bmatrix} 1 & 0.9 \\ 0.9 & 1 \end{bmatrix}$ and $\Sigma_{(S_t=0)} = \Sigma_0 = \frac{1}{1000}\begin{bmatrix} 1.3 & 0.3 \\ 0.3 & 1.3 \end{bmatrix}$ were simulated.

In this setup I'm able to test:

$$\begin{aligned} M_0 : \quad & \Delta X_t = \mu_1 + \alpha_1 \beta X_{t-1} + \varepsilon_t \\ M_1 : \quad & \Delta X_t = \mu_{(S_t)} + \alpha_{(S_t)} \beta X_{t-1} + \varepsilon_t \\ M_2 : \quad & \Delta X_t = \mu_{(S_t)} + \alpha_{(S_t)} \beta X_{t-1} + \tilde{\varepsilon}_t \end{aligned}$$

with $\varepsilon_t \sim N(0, \Sigma_t)$ and $\tilde{\varepsilon}_t \sim N(0, \Sigma_{S_t})$.

Table 4.1.1 shows the means, standard deviations and high frequency intervals of the sampled posterior distributions with size T=1000 and $(p_{11}, p_{00}) = (0.98, 0.95)$.
For the rest of the set of tested probabilities we see that all of the parameters lie inside a 95% frequency interval. Appendix B shows the results for generated models with T=500 and T=250. In general lines, smaller sample sizes tend to generate wider probability density intervals for the estimates, which also contain the true parameter values.
To save space, results for $(p_{11}, p_{00}) = (0.95, 0.95)$ are not reported and are available upon request [3].

[3] Notice that $(p_{11}, p_{00}) = (1, 0)$ is not identified under models M1 and M2, although a qualitative result can be drawn: the sampling indicates that the true model is not switching states on the cointegration relation, making p_{11} tend to 1 and p_{00} to 0. This would also be the case for $(p_{11}, p_{00}) = (0, 1)$.

Table 4.1.1 – Parameter estimates using Bayesian framework with 30,000 iterations and burn-in period of 10,000. Abusing notation, subscript i in μ_{mi} represents the i-th entry of the vector $\mu(S_t = m)$. Same is true for α_i, which will be reported only for $m = 1$ since it is fixed otherwise. True model: M_2

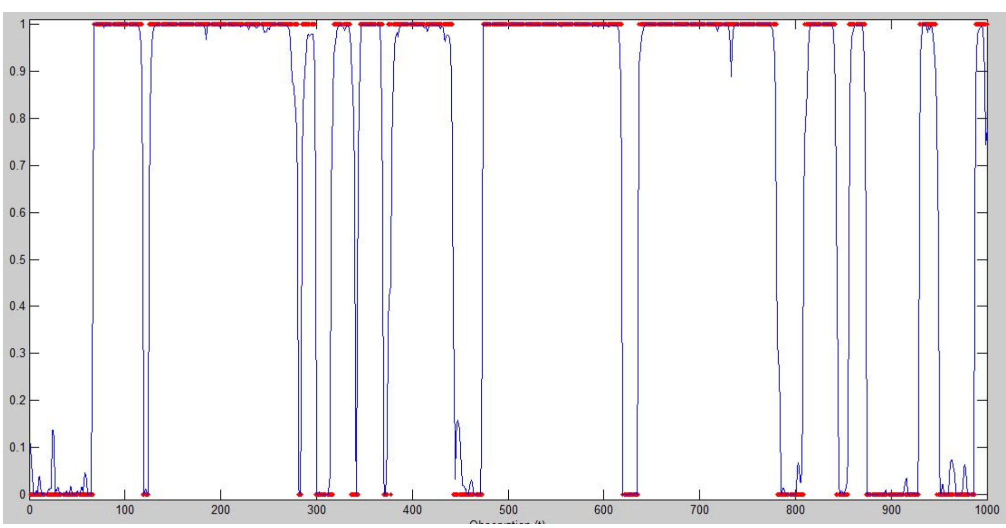
Parameter	True value	Mean	Median	Std. Dev.	95% HFI
α_1	-0.2	-0.2044	-0.2041	0.0322	[-0.2674, -0.1421]
α_2	-0.1	-0.1097	-0.1095	0.0300	[-0.1697, -0.0514]
μ_{01}	-0.01	-0.0069	-0.0069	0.0024	[-0.0118, -0.0020]
μ_{02}	-0.01	-0.0085	-0.0085	0.0026	[-0.0138, -0.0031]
μ_{11}	0.01	0.01102	0.01101	0.0012	[0.00859, 0.01351]
μ_{12}	0.01	0.01055	0.01055	0.0012	[0.00325, 0.01292]
p_{00}	0.95	0.93556	0.9375	0.0193	[0.8925, 0.9697]
p_{11}	0.98	0.97747	0.9784	0.0066	[0.9630, 0.9888]

$$|\Sigma_1| = \det \begin{bmatrix} 0.0010 & 8.6122e-04 \\ 8.6122e-04 & 9.2464e-04 \end{bmatrix} = 1.8909e-07,$$

$$|\Sigma_2| = \det \begin{bmatrix} 0.0011 & 2.079e-04 \\ 2.079e-04 & 0.0013 \end{bmatrix} = 1.4733e-07.$$

Figure 8 shows smoothed probabilities $P(S_t = 1|Y)$ plotted against the generated state vector $\tilde{S}_T$, indicating good performance in identifying Markovian states.

Figure 8 – Smoothed probabilities (blue line) capturing the true generated state vector (red dots)



Posterior probability for M_2 when the true model is M_2 generated with T=1000 (when compared to models M_0 and M_1) is 1. As shown in Sugita (2008) and Sugita (2006), model

selection via Bayes factors (approximated by BIC Shwarz Criteria) is sensitive to sample size, being more able to capture non-linearities in the data as it increases.

4.1.1 MCMC diagnostics

Tables 4.1.2, 4.1.3, 4.1.4 and 4.1.5 report effective sample size and convergence diagnostics by Geweke (1992), Heidelberger e Welch (1983), Raftery e Lewis (1992) and Raftery e Lewis (1995) respectively.

Table 4.1.2 shows the number of effectively independent draws generated by the MCMC simulation for each parameter, after burn-in. It is a measure of sample size corrected by the autocorrelation in the chain.

Table 4.1.2 – Effective chain size after correcting for autocorrelation. 20,000 iterations after burn-in

Parameter	Effective Size
α_1	16,713.306
α_2	16,284.565
μ_{01}	14,621.949
μ_{02}	14,057.290
μ_{11}	16,763.774
μ_{12}	16,513.425
p_{00}	8,241.985
p_{11}	7,811.379

it indicates fairly good mixing of the chains, even for the transition probabilities.

Table 4.1.3 presents a test based on comparing the difference of the means of two non-overlapping parts of the chain which are, for instance, the first 10% and last 50% quantiles. This measure has asymptotic normal distribution. Its results would not reject the null of equality of the means even at a 1% level, which indicates stationarity of the distribution:

Table 4.1.3 – Geweke diagnostic: comparing means of the first 10% and last 50% of the chain. The proposed statistic is constructed on the difference of the means and has asymptotic standard normal distribution. Tested at a 5% significance level.

Parameter	Statistic	Diagnostic
α_1	-1.5186	Equal
α_2	-1.4579	Equal
μ_{01}	-1.4541	Equal
μ_{02}	-0.4264	Equal
μ_{11}	0.4480	Equal
μ_{12}	0.4899	Equal
p_{00}	1.7713	Equal
p_{11}	0.4274	Equal

Table 4.1.4 reinforces the results found in Geweke's test. Its diagnostics are based on a two step procedure:

- In step 1 a Cramer-von-Mises statistics is employed to test the null of stationarity of the sample. It is done firstly on the whole sample and then successively discarding the first 10% until the null is accepted or half of the sample is discarded. If the test is passed (null accepted), we move to the next step.
- Half of the width of a confidence interval is calculated using the part of the sample not discarded in the previous step. If the ratio of the mean over the "halfwidth" measure is sufficiently small, the test is passed, indicating stationarity.

Table 4.1.4 – Heidelberg and Welch diagnosis: a two step stationarity test

Parameter	p-Value	Stationarity result	Halfwidth result
α_1	0.0929	Passed	Passed
α_2	0.0640	Passed	Passed
μ_{01}	0.2504	Passed	Passed
μ_{02}	0.6237	Passed	Passed
μ_{11}	0.3426	Passed	Passed
μ_{12}	0.3589	Passed	Passed
p_{00}	0.1679	Passed	Passed
p_{11}	0.8847	Passed	Passed

On the same direction is table 4.1.5 which, reports Raftery-Lewis diagnostics. It is a convergence test based on the precision of the estimation of a prespecified quantile of the

distribution given, an acceptance tolerance. Then it runs a pilot sampler and suggests the number of iterations necessary to achieve convergence, a burn-in period and a dependence factor, which can be interpreted as a proportional increase in the number of iterations necessary due to chain autocorrelation. Base on these criteria the whole set of parameters seem to have been well generated by the employed algorithm.

Table 4.1.5 – Raftery-Lewis diagnostics with quantile = 0.025, accuracy = $\pm 5 \times 10^{-3}$ and 95% probability

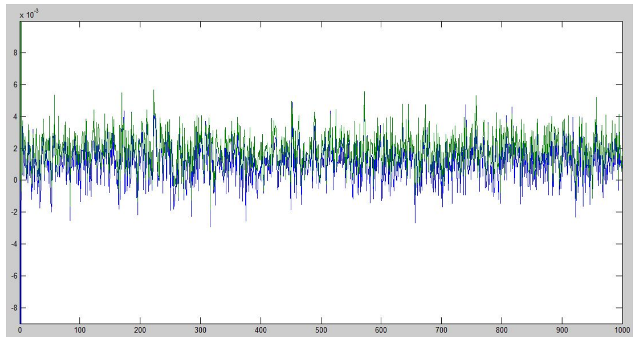
Parameter	Burn-in(M)	Total iterations (N)	Integrating/Dependence Factor
α_1	2	3,746	1.03
α_2	2	3,746	1.02
μ_{01}	2	3,746	1.01
μ_{02}	3	3,746	1.07
μ_{11}	2	3,746	1.02
μ_{12}	2	3,746	1.03
p_{00}	4	3,746	1.33
p_{11}	4	3,746	1.31

Altogether with a visual inspection of the autocorrelation plots in figure 11 (see appendix A) and the results in this section one can conclude that this model is able to capture the Markov switching behavior of a two states error correction model and sample the parameters reasonably well, delivering good convergence results.

4.2 Tick-by-tick aggregated transaction prices

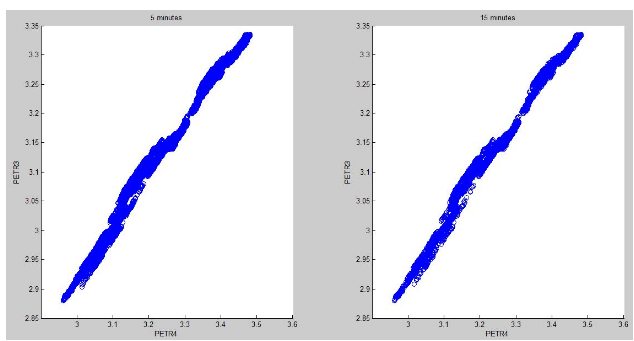
As can be noticed, the distribution of the returns of the stocks present a very elevated kurtosis. Also, there appears to be some persistence of squared the stocks in study.
This is troublesome in this framework: assuming identification restrictions as in Koop e Potter (1999), the states vector is repeated more than 99% of the times of the Gibbs sampling iterations, resulting in lack of identification of the parameters. For sake of illustration, figure 9 shows the first 1000 iterations of the algorithm for Petrobras in 15 minutes aggregation intervals when sampling the vector μ_1. It turns out that this is the same output delivered when sampling μ_1 and α_1, indicating a useless result.

Figure 9 – Problematic sampling for Petrobras



Looking at figure 10, it suggests that there exists a possible regime dependent α that is different from (0,0) in both states. To give this extra flexibility for the model I let α_0 be jointly estimated in B. The same behavior as above is reached and the algorithm is stopped. A very similar characteristic is presented when running the model for Vale, either restricting the loading parameter or not.

Figure 10 – Dispersion plots for Petrobras



A visual inspection of figure 7, however, clearly suggests a long run relation between common and preferred stocks.
The results of this section show that strong non-normality characteristics of the data are not well addressed by a Markov error correction model. Inspection of data characteristics suggest an approach able to capture greater persistence in volatility and leptokurtic characteristics of high frequency financial data.

5 Conclusion

The theoretical contributions of Fernandes e Scherrer (2012) came as a turning point on the evaluation of information shares, in the sense of simplicity and robustness. Also their findings about its non constancy during periods of market distress opened a front of research for modeling the high frequency data using a Markov switching framework.
Building on those results this paper looks at the model proposed by Sugita (2006), which circumvents issues of non identifiability and inference that arise in standard likelihood-based methods and applies to the Brazilian financial market context, using Petrobras' and Vale's common and preferred stocks traded at BM&FBOVESPA.
Great care with data handling, however, is crucial for working within this framework since stationarity is assumed at each given state: presence of too much noise might well invalidate the model.

As seen in the simulated exercise, the Bayesian strategy seems to well identify the Markov Switching behavior of the series. By marginally modifying the model in Sugita (2006) great convergence results are obtained using Montecarlo simulations.

Dealing with high frequency data, however, exacerbates stylized facts of financial time series[1]. In particular, the presence of kurtosis and persistent heteroskedastic behavior in the distribution of the returns may well undermine Johansen's test due to a lack of power. It turns out that these characteristics also bring difficulties on the identification of the nuisance parameters under the presented Bayesian framework. Indeed, the chosen sample present characteristics that undermine identification and convergence of the proposed model.

Future work might want to devote special attention to such effects by imposing a more flexible structure for the variance/covariance matrix. Additionally, Koop, Leon-Gonzalez e Strachan (2011) propose a model where the whole cointegration space [2] evolves through time, which could be more adequate to the context.

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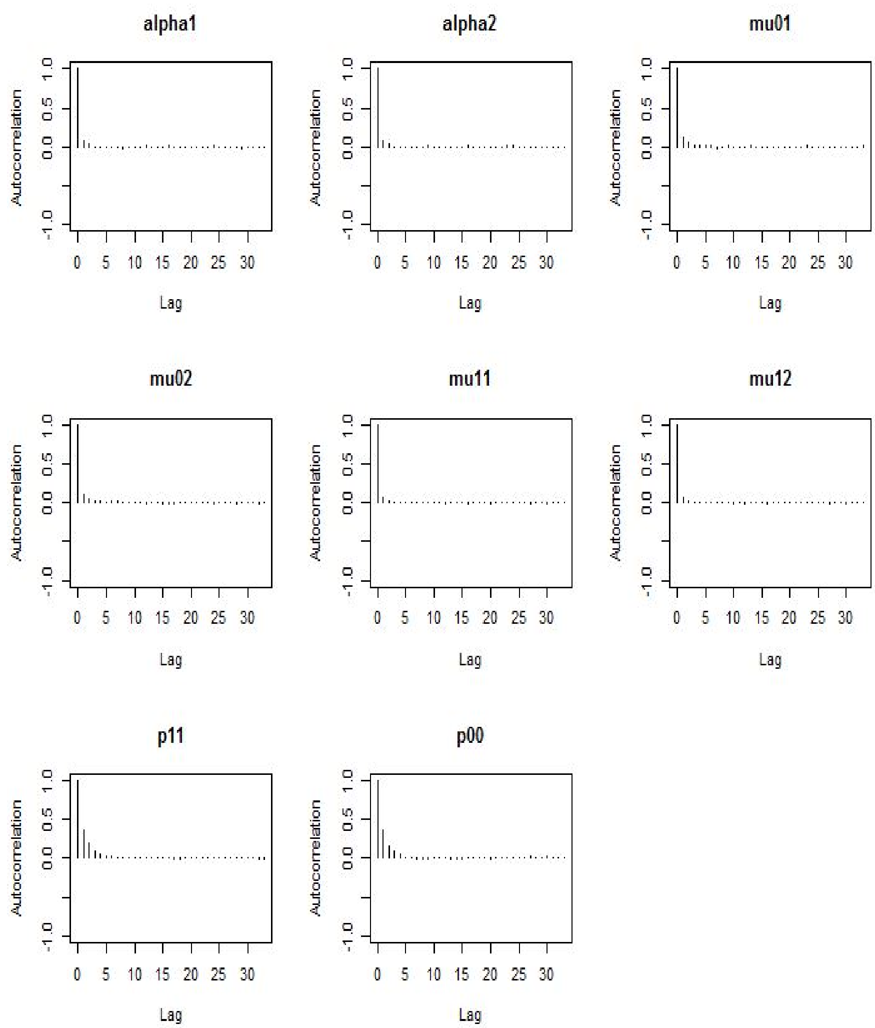
1 See Tsay (2010) for reference.
2 as well as its loadings

A MCMC autocorrelation plots

This section is devoted for a visual inspection on the autocorrelation generated by the sampling scheme employed in section 4, with 20 thousand iterations left after discarding the first 10 thousand as burn-in period. Interpretation of the results is given in chapter 4, together with other standard MCMC diagnostics.

A.1 Montecarlo experiment

Figure 11 – Autocorrelation plots for each individual parameter



B Bayesian estimates with different sample sizes

Table B.0.1 show posterior distribution characteristics of simulated data under model M_2 for sample size T=500 and T=250 respectively. Interpretation is given in chapter 4.

Table B.0.1 – Parameter estimates with T=500

Parameter	True value	Mean	Median	Std. Dev.	95% HFI
α_1	-0.2	-0.2157	-0.2041	0.0360	[-0.2866, -0.1463]
α_2	-0.1	-0.1315	-0.1095	0.0348	[-0.1995, -0.0633]
μ_{01}	-0.01	-0.0080	-0.0079	0.0033	[-0.0145, -0.00149]
μ_{02}	-0.01	-0.0088	-0.0088	0.0034	[-0.0138, -0.0031]
μ_{11}	0.01	0.00946	0.00944	0.0017	[0.0061, 0.01286]
μ_{12}	0.01	0.00995	0.00994	0.0016	[0.0067, 0.01323]
p_{00}	0.95	0.95017	0.95281	0.0020	[0.9042, 0.9814]
p_{11}	0.98	0.98157	0.98258	0.0075	[0.9644, 0.9933]

$$|\Sigma_1| = \det \begin{bmatrix} 9.9211e-04 & 8.6018e-04 \\ 8.6018e-04 & 9.4305e-04 \end{bmatrix} = 1.9569e-07,$$

$$|\Sigma_2| = \det \begin{bmatrix} 0.0013 & 4.0161e-04 \\ 4.0161e-04 & 0.0012 \end{bmatrix} = 1.3222e-06.$$

Table B.0.2 – Parameter estimates with T=500

Parameter	True value	Mean	Median	Std. Dev.	95% HFI
α_1	-0.2	-0.1806	-0.1802	0.0342	[-0.2485, -0.1142]
α_2	-0.1	-0.1141	-0.1139	0.0315	[-0.1766, -0.0524]
μ_{01}	-0.01	-0.0100	-0.0100	0.0037	[-0.0174, -0.0026]
μ_{02}	-0.01	-0.0121	-0.0121	0.0039	[-0.0199, -0.0045]
μ_{11}	0.01	0.00680	0.00682	0.0029	[0.00010, 0.01257]
μ_{12}	0.01	0.00548	0.00549	0.0027	[3.0e-05, 0.01088]
p_{00}	0.95	0.96337	0.96734	0.0211	[0.9042, 0.9814]
p_{11}	0.98	0.97940	0.98169	0.0118	[0.9644, 0.9933]

$$|\Sigma_1| = \det \begin{bmatrix} 0.00126 & 0.00111 \\ 0.00111 & 0.00114 \end{bmatrix} = 2.1443e-07,$$

$$|\Sigma_2| = \det \begin{bmatrix} 0.00115 & 1.8956e-04 \\ 1.8956e-04 & 0.00130 \end{bmatrix} = 1.4735e-06.$$

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