FUNDAÇÃO GETÚLIO VARGAS
ESCOLA de PÓS-GRADUAÇÃO em
ECONOMIA

Lucas Alves Estevam de Lima

Taxation of Couples: a Mirrleesian Approach to Collective Households

Rio de Janeiro
2015

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Dissertação para obtenção do grau de mestre apresentada à Escola de Pós-Graduação em Economia

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Ficha catalográfica elaborada pela Biblioteca Mario Henrique Simonsen/FGV

Lima, Lucas Alves Estevam de
Taxation of couples: a mirrleesian approach to collective households / Lucas Alves Estevam de Lima. – 2015.
23 f.

Dissertação (mestrado) - Fundação Getulio Vargas, Escola de Pós-Graduação em Economia.
Orientador: Carlos da Costa.
Inclui bibliografia.

1. Impostos. 2. Família – Impostos. 3. Negociação. I. Costa, Carlos Eugênio da. II. Fundação Getulio Vargas. Escola de Pós-Graduação em Economia. III. Título.

CDD – 336.2

LUCAS ALVES ESTEVAM DE LIMA

TAXATION OF COUPLES: A MIRRLEESIAN APPROACH TO COLLECTIVE HOUSEHOLDS.

Dissertação apresentada ao Curso de Mestrado em Economia da Escola de Pós-Graduação em Economia para obtenção do grau de Mestre em Economia.

Data da defesa: 29/04/2015

ASSINATURA DOS MEMBROS DA BANCA EXAMINADORA

Carlos Eugênio Ellery Lustosa da Costa
Orientador (a)

Leandro Gorno

Érica Diniz Oliveira

Érica Diniz Oliveira

Resumo

Esta dissertação apresenta uma abordagem alternativa para o problema de taxação de famílias. Mais especificamente modelamos a decisão familiar com um modelo de barganha de Nash em que os o governo determina de forma ótima as utilidades de discórdia. Demonstramos um Princípio da Revelação para esse modelo de forma a reduzir a classe de mecanismos possíveis, além disso calculamos os ganhos do mecanismo ótimo em relação a outros mecanismos razoáveis por meio de exemplos. Discutimos algumas implicações associadas ao mecanismo ótimo.

Palavras-chave: Taxação Ótima, Abordagem Coletiva de Famílias, Barganha de Nash, Princípio da Revelação

Abstract

This dissertation presents a alternative approach to deal with family taxation problem. More specifically we model the family decision as a Nash Bargaining where the planner can choose optimally the disagreement utilities. We prove a Revelation Principle for this model so we can consider a smaller set of mechanisms, besides that we compute optimal mechanism gains compared to other reasonable mechanism through examples. We discuss some implications of the optimal mechanism.

Keywords: Optimal Taxation, Household Collective Approach, Nash Bargaining, Revelation Principle

Contents

1 Introduction

Even though the majority of individuals live as couples, optimal tax theory is almost entirely based on the assumption that agents do not interact as spouses.

This is not due to tax theorists not recognizing the importance of taking these extra-market interactions. Rather, some of the auxiliary assumptions that were commonly used to model household behavior and the taxation of couples, in particular, made it unnecessary to treat the case of married agents separately.

In fact, if the family as a whole acts like it was an unique agent, in the sense that the family ordering of bundles is consistent with the usual preferences axioms, and if the planner agrees with this division inside family, then the problem can be reframed as a more tradicional taxation problem. This could be called an unitary approach.

Most of the literature on taxation of couples has made those assumptions and looked at the problem from a multidimensional screening perspective. Kleven et al. [2009] This is a natural direction of expansion as most of the taxation literature answers consider only a single non-observable ability and in a family we need at least two, except if there is a bijective relation between one spouse and the other.

Here we look at a different angle, we relax the unitary assumption taking into account the recent developments in family economics and consider a collective approach to household behavior—Chiappori [1988], Browning and Chiappori [1998]. More precisely, we assume that families choices are the outcome of a Nash bargain, which threat points may be directly affected by the mechanism designed by the planner.

The key elements in our approach are the following. First although we assume that types are not known by the planner or other agents in general, we allow spouses to know each other's types. Agents are able to cooperate to attain utilities in the frontier of their utility opportunity sets, yet they diverge as to which point in the frontier they prefer. That is, we are in the realm of the collective model of Browning and Chiappori [1998]. In particular, the household decision process is characterized by a Nash bargain which disagreement utility profiles depend on the tax policy.

After a literature review about the family, taxation and family taxation literatures we first introduce our model, discuss some of the assumptions and briefly talk about alternatives. Then we justify our approach through an application of the revelation principle.

We then discuss some of the insights that can be gain from the model and look at some different examples in which the channels governing those insights are made explicit. Also, given the revelation principle all other mechanism are dominated by the one suggested, but it still interesting to see why other seemingly reasonable mechanisms may

be strictly inferior.

2 Literature Review

2.1 Family Taxation

The family taxation literature has focused mainly in cases where the family decision is equivalent to the decision of a single agent[1] (unitary models) or in cases where the household members are treated separately, ignoring that each member has information about other members in the family. Seminal works in those area were made by Kleven et al. [2009] and, Boskin and Sheshinski [1983].

Kleven et al. [2009] consider a multidimensional screening problem in which the family is unitary, the secondary earner is restricted to have only two possible types, the marriage decision isn't modeled, the types of each member of a couple are uncorrelated. Also they assume no income effect on labor supply and separability in the disutility of working between each member of the couple, *i.e.* changes of labor supply of one spouse doesn't affect the other marginal disutility of working.

They characterize some qualitative properties of the optimal taxation, focusing especially on the question of jointness of taxation. But some of their results are lost when we drop the assumption of uncorrelated type. Frankel [2014]

Boskin and Sheshinski [1983] look to a separable linear taxation system and shows that to tax equally husband and wife is ineficient, in the sense that it produces a dead weight loss substantially larger than when the planner uses two tax systems. The rationale behind the result is that as wifes have a much more elastic labor supply than the husband the earning taxes should differ. The analysis is extended to nonlinear taxation by Schroyen [2003], where he consider that couples fill the tax individually but the labor supply decision is made by the household as a whole, this restrict the planner as he need to avoid tax arbitrages inside the families, *i.e.* redistribution of leisure and income inside the family to avoid taxes.

In essence the focus has been on extending the framework to allow some multidimensionality, as the family naturally has a multidimensional set of abilities. At the same time in full multidimensional model the optimum characterization problem is usually untractable, especially because local incentive compatibility constraints cease to be enough for global incentive compatibility. And qualitatively we can't say much about the opti-

[1] And the planner agrees with the decision inside the family, in the sense that given the allocations the planner wouldn't want to redistribute inside the household.

mum. Rochet and Choné [1998]

Another possibility less explored is depart from the unitary approach and consider a collective approach.Chiappori [1988], Browning and Chiappori [1998] The optimal taxation problem becomes really hard for the planner in an arbitrary collective model, especially because we can't apply the revelation principle. So we need to add some restrictions on how the planner can affect intrahousehold decisions.

2.2 Family and Collective Decision

The family literature has some branches focusing either on the gains of marriage, decision inside the family and formation of families, we ignore the family formation issue. In this project we consider a household production function following the work of Becker [1965] and we give more structure to the family decision assuming a Nash Bargaining. Nash [1950]

The collective decision literature, started with the Nash Bargaining solution, has focused especially in axioms that define uniquely a solution to the bargaining problem. Usually the allocation is partially dependent on a disagreement utility point. How this point is decided is dependent on the environment and what assumptions are reasonable to that environment. The most common disagreement points are the rational threat point, where each agent chooses his disagreement action trying to maximize his payoff in the agreement equilibrium, or an equilibrium of the noncooperative game that arises in disagreement. The decision between the two options is related to the ability to commit of each member in the bargain. Thomson [1994], Myerson [1997]

Usually we have the frontier of utility attainable in agreement and disagreement, but in our context is exactly the ability of the planner to give different utility frontier that allow him to influence the intrahousehold allocation. It is natural as the usual interpretation of bargaining consider that the agents are playing a fixed game and if they decide to don't cooperate, then they still in the same game. Here the game is not fixed. Besides that if we don't consider that the planner can change the agreement and disagreement indepently we lose the revelation principle.

Another relevant contribution is Lundberg and Pollak. They propose an bargaining family model where the threat point is a noncooperative equilibrium within the marriage(the separate spheres bargaining) instead of being the threat of divorce, this can be used to explaing how changes in policy that only affect married couples can still affect the equilibrium allocation. Our model, although different, builds in the same intuition by allowing the planner to set a threat point inside the marriage as a way to change the equilibrium allocation without changing the singles' utility.

2.3 Taxation

The relevant taxation literature in our context was initiated by Mirrlees [1971] and consider the problem of implementing a tax system that maximizes a social welfare function subject to government budget constraint and individuals' incentive compatibility. This literature imposes independency between the private type of each agent or, equivalently, that the tax system can't condition in the whole set of announcements to decide the tax faced by a single individual, it can only uses this individual announcement and the known distribution of types. Guesnerie [1995]

In our context this independency is unreasonable as it would imply that each spouse has no information about the other spouse ability.

Also the usual approach in the literature is to find the optimal allocation and then show that this allocation can be implemented with a tax system. Following this we look to the optimal allocation in our model and we plan to show in future versions that this can be implemented with three tax systems, one for couples filling jointly, one for couples filling separately and one for singles.

3 Model

The economy is inhabited by a large number of agents. Each agent is identified by a set of characteristics $\theta \in \Theta$, with preferences,$\succsim_\theta$, defined over consumptions bundles $\mathbf{c} \in \mathbf{X} \subseteq R^n$, which are representable by the utility function $U(\cdot, \theta)$.

Agents may be single or paired with another agent to form a family characterized by a pair $\eta = (\theta_1, \theta_2) \in \Theta^2$. The utility attained by an agent if married is allowed to differ from that attained if single. We shall however maintain the assumption that the ordering $\succsim_\theta$ is invariant to whether the agent is single or married

Consumption bundles are produced from 'market' transactions $\mathbf{x}$ according to technologies that depend, among other things, on an agents' marital status. For singles we define a function F^s from $\mathbf{X} \times \Theta \to 2^{\mathbf{X}}$. As for households, let $\mathbf{P} = \mathbf{X} \times \mathbf{X}$. We define a function $F^a : \mathbf{P} \times \Theta^2 \to 2^{\mathbf{X}}$ which describes the bundles $\mathbf{c}_1 + \mathbf{c}_2$ that a couple η can attain using input $\mathbf{x}$, i.e., $\mathbf{c}_1 + \mathbf{c}_2 \in F^a(\mathbf{x}, \eta)$.

For our purposes it will also be important to define the potential consumption attained by the family when spouses are in disagreement. We define it as a function $F^d : \mathbf{P} \times \Theta^2 \to 2^{\mathbf{X}}$, and assume that for every $\mathbf{x} \in \mathbf{P}$ and every η we have

$$F^a(\mathbf{x}, \eta) \supseteq F^d(\mathbf{x}, \eta).$$

The idea here is that if couples are in distress many feasible actions that would expand their consuption possibilities would not be undertaken. That is if spouses are unable to reach an agreement, thus the superscript a, then they need not realize the consumption potential from marriage.

These functions aim at capturing gains that arise from sharing non-rival goods, the devision of labor associated with both comparative advantage and increasing returns in household production, and other things that make are important to define the potential gains from marriage. Two things are worth mentioning. First, the gains from marriage need not be realized, which is apparent from our definition of F^d. Moreover, a bundle $(\mathbf{c}_1, \mathbf{c}_2)$ such that $\mathbf{c}_1 + \mathbf{c}_2$ is in the frontier of $F^{\mathrm{a}}(\mathbf{x}, \eta)$ is necessary but, by no means, sufficient for efficiency.

To fully characterize the optimal allocation we restrict the family's decision process by assuming that, when they cooperate, spouses solve a Nash bargaining problem. Formally, $\eta = (\theta_1, \theta_2)$, given the disagreement utility (u_1, u_2), the household solves

$$\begin{aligned} \max_{\mathbf{c}_1, \mathbf{c}_2, \mathbf{x}} \quad & (U(\mathbf{c}_1, \theta_1) - u_1)\,(U(\mathbf{c}_2, \theta_2) - u_2) \\ \text{s.t.} \quad & \mathbf{c}_1 + \mathbf{c}_2 \in F^{\mathrm{a}}(\mathbf{x}, \eta) \\ & U(\mathbf{c}_i, \theta_i) \geq u_i \\ & \mathbf{x} \in \mathbb{B}, \end{aligned}$$

where $\mathbb{B}$ denotes the couple's market budget set.

The potential sources of inefficiency in an allocation are, therefore, i) being restricted to a set smaller than $F^{\mathrm{a}}(\mathbf{x}, \eta)$ for any given $\mathbf{x}$, ii) for a given $\mathbf{x}$ choosing a bundle $\mathbf{c}_1, \mathbf{c}_2$ such that $\mathbf{c}_1 + \mathbf{c}_2 \in F^{\mathrm{a}}(\mathbf{x}, \eta)$ which does not maximize the objective (), and; iii) choosing an inefficient market bundle $\mathbf{x}'$ in the sense that there is an alternative market bundle $\mathbf{x}''$ for which the value of the program is higher under $F^{\mathrm{a}}(\mathbf{x}'', \eta)$ than under $F^{\mathrm{a}}(\mathbf{x}', \eta)$.

Finally the crucial assumption regarding the informational structure of the problem is that an agents' type is known only by the agent himself (herself) and his (her) spouse, if he or she is married.

3.1 A Direct Truthful Mechanism

To characterize the incentive feasible allocations in this economy we introduce an appropriate definition of a mechanism for our setting.

We assume that the government can distinguish between single and married agents. That is, we assume that one agent cannot claim to be single when he or she is married.

Definition 1. A mechanism $\Psi = (M^s, M^m, g^s(\cdot), g^c(\cdot,\cdot))$ is a collection of message sets for singles M^s, message sets for married agents M^m and outcome functions $g^s : M^s \to \mathbf{X}$ for singles and $g^m : M^s \times M^m \to \mathbf{P}$ for couples.

In principle the outcome function could depend on messages from all agents. In our setting, however, restricting the outcome functions only to depend on what each agent (or couple) announces is without loss.

We first describe what a direct truthful mechanim is in our setting.

For singles, the government asks each agent his or her type and assings a bundle $\mathbf{x}$ and recommends a consumption bundle $\mathbf{c} \in F^s(\mathbf{x}, \theta)$. If the agent has a spouse, however, the planner asks each agent his (her) type and his (her) spouse's type. I.e., an admissible massage by married agents is $\eta' \in \Theta^2$. Provided that agents are truthful, i.e., messages $(\tilde{\eta}_1, \tilde{\eta}_2)$ are such that $\tilde{\eta}_1 = \tilde{\eta}_2 = \eta$. The planner then assigns a bundle $(\mathbf{x}_1, \mathbf{x}_2)$ in $\mathbf{P}$ and recommends a consumption pair $(\mathbf{c}_1, \mathbf{c}_2) \in F^a((\mathbf{x}_1, \mathbf{x}_2), \eta)$.

What makes the problem special when compared to the case in which only singles inhabit the economy is the possibility of conflicting reports by spouses. Conflicing reports will not arise at the equilibrium of a truthful direct mechanism, yet, they may be relevant for determining the disagreement utilities which will ultimately determinte how spouses split the gains from marriage.

To handle this possibility we re-write $g^c(\cdot,\cdot)$ as follows. $g^c(\eta, \eta') = \Gamma^{\mathrm{a}}_{\mathrm{c}}(\eta)$, i.e., $\Gamma^{\mathrm{a}}_{\mathrm{c}} \colon \Theta^2 \to \mathbf{P}$, if $\eta = \eta'$ and $g^c(\eta, \eta') = \Gamma^{\mathrm{d}}_{\mathrm{c}}(\eta, \eta')$, i.e., $\Gamma^{\mathrm{d}}_{\mathrm{c}} \colon \Theta^2 \times \Theta^2 \to \mathbf{P}$ if $\eta \neq \eta'$.

Finally, for any x and any θ let $\varphi(x, \theta)$ be the solution to the type θ single agent utility maximization problem when he or she chose transactions x. Then we define $\Gamma^s(\theta)$ as $\varphi(g^s(\theta))$ and focus on Γ^s in all that follows.

When deciding what announcement to make, a type θ agent solves

$$\max_{\theta'} \quad U\left(\Gamma_{\mathrm{s}}(\theta'), \theta\right)$$

The single agent's program is standard. We shall now describe the programme solved by married agents.

Assume that the family is acting cooperatively. In particular, assume that spouses solve a Nash bargaining problem, which threat points are given by the disagreement

utility profile (u_1, u_2). Formally, the family $\eta = (\theta_1, \theta_2)$ solves

$$\begin{aligned} \max_{\mathbf{c}_1, \mathbf{c}_2, \eta'} \quad & (U(\mathbf{c}_1, \theta_1) - u_1)(U(\mathbf{c}_2, \theta_2) - u_2) \\ \text{s.t.} \quad & \mathbf{c}_1 + \mathbf{c}_2 \in F^{\mathrm{a}}(\Gamma_{\mathrm{c}}^{\mathrm{a}}(\eta'), \eta) \\ & U(\mathbf{c}_i, \theta_i) \geq u_i. \end{aligned}$$

We denote the solution to this problem $\mathbf{c}_i(\eta, u_1, u_2)$ and $\tau(\eta, u_1, u_2)$. The maximum is denoted by $V(\eta, u_1, u_2)$.

The crucial assumption in the collective setting is that the family does reach an efficient allocation, i.e., they solve a Pareto problem of which the Nash Bargaining problem is a special case. Cooperation in our setting thus implies not only allocating efficiently the resources attainable with transactions (x_1, x_2), but also agreeing on the announcements that will maximize the utility attainable.

To close the problem we need to explain how the threat points are defined. We assume that they are the utilities that are attained by spouses when they are not able to agree, the disagreement utilies.

To model the decision of a type θ_1 in a $\eta = (\theta_1, \theta_2)$ type family when the spouses are in disagreement we consider a pair of functions $\Delta_i \colon \Theta^2 \times \Theta^2 \times \Theta^2 \to \mathbf{P}$, $i = 1$ or 2, such that

$$\Delta_1(\eta, \eta', \eta'') + \Delta_2(\eta, \eta', \eta'') \in F^{\mathrm{d}}(\Gamma_{\mathrm{c}}^{\mathrm{d}}(\eta', \eta''), \eta).$$

These functions define how resources available at home would be split between spouses were they not able to reach an agreement and made announcements η' and η''. If a couple were to disagree, a type θ_1 agent married to a type θ_2 agent who makes an announcement η'' would choose his or her announcement by solving,

$$\max_{\eta'} \quad U\left(\Delta_1(\eta, \eta', \eta''), \theta_1\right).$$

These Δ functions are assumed to be known by all.

The other spouse's problem is similar. The utility pair (u_1, u_2) attained at the Nash equilibrium of this game by the family members is called the disagreement utility profile.

Commitment In all that follows we assume that agents cannot commit to specific actions conditional on there not being an agreement. Informally, spouses attempt an agreement, which if they cannot reach leads to another stage in which agents will choose non-cooperatively a course of action. We also assume that spliting and making an announce-

ment as single is a feasible strategy in the disagreement stage. If one of the spouses decides for this course of action in the disagreement stage the other is forced to do the same. Hence, attaining the single type θ utility is always possible for a type θ married agent.

Were we to assume that agents could commit to some actions in the disagreement game then the only possible threatpoints would be the utilities attained as singles, which we formalize with the following proposition.

Proposition 1. *If agents can commit to receive its associated single's bundle and force his or her spouse to receive his or her associated single's bundle, then threat points are given by the agents utilities as singles.*

Proof. As the Nash Bargain is efficient we know that by changing the disagreement utility one agent can only be strictly better off in equilibrium if its spouse is strictly worse off. Since an agent can always choose the pair of singles utilities in the disagreement game, no utility lower than what she or he can attain with this pair of threat points would be acceptable in equilibrium. □

3.2 Lack of Commitment

In all that follows we maintain the assumption that agents are not able to commit to an out of equilibrium action.

The first thing to note in this case is that we can avoid unmatched reports by setting, for example, $\Gamma_c^d(\eta, \eta') = 0$ if $\eta \neq \eta'$.

Under this choice of off-equilibrium punishment we define an equilibrium as follows. An equilibrium for the game induced by the direct mecanism with off-equilibrium punishment $\Gamma_c^d(\eta, \eta') = 0$ if $\eta \neq \eta'$ is such that

1. For each θ and θ' we have

$$U(\Gamma_s(\theta), \theta) \geq U(\Gamma_s(\theta'), \theta)$$

2. The functions Γ_c^a and Γ_c^d must be such that for every $\eta = (\theta_1, \theta_2)$ we have

$$V(\eta, U(\Delta_1(\eta, \eta, \eta), \theta_1), U(\Delta_2(\eta, \eta, \eta), \theta_2))$$

greater than or equal to

$$\begin{aligned}\max_{\mathbf{c}_1,\mathbf{c}_2,\eta'} \quad & \left(U(\mathbf{c}_1,\theta_1) - U\left(\Delta_1(\eta,\eta,\eta),\theta_1\right)\right)\left(U(\mathbf{c}_2,\theta_2) - U\left(\Delta_2(\eta,\eta,\eta),\theta_2\right)\right) \\ \text{s.t.} \quad & \mathbf{c}_1 + \mathbf{c}_2 \in F^{\mathrm{a}}(\Gamma^{\mathrm{d}}_{\mathrm{c}}(\eta',\eta'),\eta) \\ & U(\mathbf{c}_i,\theta_i) \geq U\left(\Delta_i(\eta,\eta,\eta),\theta_i\right)\end{aligned}$$

3. For each $\eta = (\theta_1,\theta_2)$ and η' we have

$$U\left(\Delta_1(\eta,\eta,\eta),\theta_1\right) \geq U\left(\Delta_1(\eta,\eta',\eta),\theta_1\right)$$

   and

$$U\left(\Delta_2(\eta,\eta,\eta),\theta_2\right) \geq U\left(\Delta_2(\eta,\eta,\eta'),\theta_2\right)$$

4. For each $\eta = (\theta_1,\theta_2)$ and i we have

$$U\left(\Delta_i(\eta,\eta,\eta),\theta_i\right) \geq U\left(\Gamma_{\mathrm{s}}(\theta_i),\theta_i\right)$$

5. For each $\eta = (\theta_1,\theta_2)$ we have

$$\eta \in \tau\left(\eta, U\left(\Delta_1(\eta,\eta,\eta),\theta_1\right), U\left(\Delta_2(\eta,\eta,\eta),\theta_2\right)\right)$$

6. And budget balance: If the distribution of singles is given by a measure μ_{s} and the distribution of couples is given by a measure μ_{c} then the bundle

$$\int \Gamma_{\mathrm{s}}(\theta)\,\mathrm{d}\mu_{\mathrm{s}}(\theta) + \int \Gamma^{\mathrm{a}}_{\mathrm{c}}(\eta,\eta)\,\mathrm{d}\mu_{\mathrm{c}}(\eta)$$

   is feasible.

Notice that we do not impose out-of-equilibrium budget balance.

An easy way to get 2 is to set to each η

$$\Gamma^{\mathrm{a}}_{\mathrm{c}}(\eta) \geq \Gamma^{\mathrm{d}}_{\mathrm{c}}(\eta,\eta)$$

4 Revelation Principle

The Direct Truthful Mechanism is without loss is this context. In fact, starting from the disagreement game we consider an arbitrary mechanism $\tilde{\Gamma}\colon S_1 \times S_2 \to P$. Where S_1 and S_2 are the message space for agent 1 and 2, respectively. Suppose also that the division

rule $\tilde{\Delta}_i$ is a function of type and allocation, such that

$$\tilde{\Delta}_1(\eta, \mathbf{c}) + \tilde{\Delta}_2(\eta, \mathbf{c}) \leq F_{\mathbf{c}}^{\mathrm{d}}(\mathbf{c}, \eta)$$

Let $s_i : \theta^2 \to S_i$ be an equilibrium of this game, then the direct mechanism $\Gamma : \Theta^2 \times \Theta^2 \to P$ given by

$$\Gamma(\eta', \eta'') = \tilde{\Gamma}(s_1(\eta'), s_2(\eta''))$$

has a truthful equilibrium.

In fact, consider agent 1 from family $\eta = (\theta_1, \theta_2)$, then

$$\begin{aligned} U(\tilde{\Delta}_1(\eta, \Gamma(\eta, \eta), \theta_1) &= U(\tilde{\Delta}_1(\eta, \tilde{\Gamma}(s_1(\eta), s_2(\eta)), \theta_1) \geq \\ &\geq U(\tilde{\Delta}_1(\eta, \tilde{\Gamma}(s_1(\eta'), s_2(\eta)), \theta_1) = U(\tilde{\Delta}_1(\eta, \Gamma(\eta', \eta), \theta_1) \end{aligned}$$

The same applies to agent 2.

Now consider the agreement stage. Given a mechanism $\tilde{\Gamma} : S \to P$, a message $s : \Theta^2 \to S$ and a division rule $\mathbf{c}_i : \Theta^2 \times P \to P$ are optimal if for every family $\eta = (\theta_1, \theta_2)$ with disagreement utility given by $(u_1(\eta), u_2(\eta))$ we have

$$\left(U_1(\mathbf{c}_1(\eta, \tilde{\Gamma}(s(\eta))), \theta_1) - u_1(\eta)\right) \left(U_2(\mathbf{c}_2(\eta, \tilde{\Gamma}(s(\eta))), \theta_2) - u_2(\eta)\right)$$

greater than or equal to

$$\begin{aligned} \max_{s, \mathbf{c}_1, \mathbf{c}_2} \quad & (U_1(\mathbf{c}_1, \theta_1) - u_1(\eta)) \, (U_2(\mathbf{c}_2, \theta_2) - u_2(\eta)) \\ \text{s.t.} \quad & \mathbf{c}_1 + \mathbf{c}_2 \in \tilde{\Gamma}(s) \end{aligned}$$

Now if we consider the direct mechanism $\Gamma : \Theta^2 \to P$ given by

$$\Gamma(\eta') = \tilde{\Gamma}(s(\eta'))$$

then to tell the truth in mechanism Γ is optimal.

This assume that the planner can identify when the couple are in agreement or disagreement. Another possible approach is to consider an extended revelation principle, where each agent reveals his/her type and also if they are on agreement or not. If the planner observes that both are announcing agreement then he delivers the agreement bundle, if he sees that both are announcing disagrement then he delivers the disagreement bundle. The last case is if he sees that one is announcing disagreement and the other

isn't, in that case he delivers punish as a mismatch and both attain the single's utility.

With this mechanism we can apply the reasoning above to conclude that it can be used to implement the equilibrium of any other mechanism.

5 Some comments

The fact that by changing the message space we don't affect the division rule in the disagreement stage is essential.

Also the Independence of Irrelevant Alternatives is necessary to have a clean result, for instance, if we consider a *Kalai-Smorodinsky* solution concept, then the division rule in the agreement stage will be a function of the disagreement utility and the highest utility attainable in the mechanism. So, by changing the message space we may affect the division.

We can still apply a enlarged revelation principle by keeping the optimal bundle and also bundles that maximize the attainable utility, but the result start to get clumsy and to verify if an allocation may come from a *Kalai-Smorodinsky* solution isn't completely trivial.

Without commitment we avoid the necessity of compatibility of incentives in disagreement stage by punishing infinitely unmatched announcements. One problem with this assumption is that for every (θ, θ') every matched announcement is an equilibrium[2], of course we pick the truthful equilibrium as a focal point. So one possibility is to weaken this assumption and keep some compatibility of incentives restrictions, at the same time it is the extreme case of the intuition that more information would be good by reducing the burden of compatibility of incentives in disagreement and at least qualitatively the results should be robust to this.

The existence of assignable goods can be useful through the tradicional channel as it serves as an instrument to reduce problem of compatibility of incentive in the equilibrium agreement allocation. But if the division inside of the couple is not perfectly observable, as we plan to consider in future versions, it can be used to reduce the set of possible pairs of utility attainable in disagrement.

The revelation principle (or more specific the taxation principle) isn't trivial in that context Guesnerie [1995]. When we go to arbitrary non-unitary approach we usually lose this and then need to deal with specific games. More formally, consider that the government can only choose the goods the family will receive or supply as a whole, *i.e.*

[2]If it gives at least the utility of being single

he can't decide how this goods will be allocated inside the family. Then, even if we have only one type, and so no informational problem, we can't replicate arbitrary game allocation through a truthful direct game. If we focus on a Nash bargain, for instance, it becomes clear that the problem lies in the effect of never choosen, but still possible, actions on the disagreement point. And without the revelation principle we are left with a too big universe of possible games.

One way to get the revelation principle back is to consider that the government can choose the allocation of each individual inside a family. It isn't satisfactory either, it is natural to suppose that there is a correlation between the types of the two agents in a family, in that case we usually find games that are much better than the optimal tax Guesnerie [1995]. Of course the implicit assumption is that the members of the family won't choose cooperatively their announcements.

In one sentence, when we introduce non-unitary family the interactions between the members of family tends to make the usual general mirrleesian taxation approach less effective as we lose control over the agents — when we only interact with then as a family, while they decide individually and we care about then individually — or we get to much control — through correlation between their types and the assumption that the family can't coordinate.

6 Other Mechanism

Two other reasonable mechanisms would be to ask to each individual about his private type and not about his type as a couple, *i.e* formally to ask only about the first coordinate of couple's type to the husband and the second to the wife, and to ask only one of the spouses.

By the revelation principle those mechanisms are dominated by the direct truthful mechanism.

6.1 Private direct truthful mechanism

The planner asks each agent his or her type. For singles, the mechanism is identical as the direct truthful mechanism. If the agent has a spouse, however, the planner merges the couple's announcement. Provided that agents are truthful, i.e., for a couple $\eta \in \Theta^2$ the messages $(\tilde{\theta}_1, \tilde{\theta}_2)$ are such that $(\tilde{\theta}_1, \tilde{\theta}_2) = \eta$. The planner then assigns a bundle $(\mathrm{x}_1, \mathrm{x}_2)$ in P and recommends a consumption pair $(\mathrm{c}_1, \mathrm{c}_2) \in F^a((\mathrm{x}_1, \mathrm{x}_2), \eta)$.

The biggest difference is that the planner now has to worry about lies in the disagree-

ment game, as every agent can fake his or her own type without the planner being aware.

6.2 "Patriarchal" direct truthful mechanism

The planner ask every single about his or her type, but asks only the husband of each couple about the couple's type. Provided that agents are truthful the planner then assigns a bundle (x_1, x_2) in P and recommends a consumption pair $(c_1, c_2) \in F^a((x_1, x_2), \eta)$.

Intuitively this should be generally good to men and harmful to women, but we show in an example that this can be misleading. Both mechanisms would lead to the same outcome if the disagreement utility was fixed, so the channel by which this changes implementation is through the disagreement game.

We could list other possible mechanism, for instance a "matriarchal" mechanism or a mechanism where we ask each member of a couple about his or her spouse's type, but the main idea about how the direct truthful mechanism dominated the private direct truthful mechanism can be applied to every other mechanism.

7 Examples

7.1 Example 1

Following the last section we consider some examples to understand how the direct truthful mechanism strictly dominated other reasonable options.

Assume an economy with men and women, half of the men have type θ_H and half have type θ_L, $\theta_H > \theta_L$. All women have the same type, θ_L.

The utility of a type θ agent is given by

$$v(C, L, \theta) = \log(C) - \frac{L}{\theta}$$

Half of the population is married. We suppose that the government can observe L (labor) but the division of C (consumption) between spouses is unobservable.

In this example we also restrict the planner to treat equally agents known to have same ability level. We then relax this assumption and show that the planner has incentives to treat unequally men and women even though he cares equally about them, through the incentives mechanism.

We also restrict the disagreement threat point to be the divorce threat, *i.e* the disagree-

ment bundle is exactly the single's bundle.

When a couple has type $(\theta_{\mathrm{H}}, \theta_{\mathrm{L}})$ we assume that type θ_H has all the power in disagreement. That it is he who determines unilateraly how consumption is split between himself and his spouse. In this case, it is optimal for the government to set the disagreement point to the single's utility.

When the type is $(\theta_{\mathrm{L}}, \theta_{\mathrm{L}})$ we consider that the couples share equally the consumption bundle given by the government.

If the planner has a utilitarian social welfare function given by

$$U_{\mathrm{S}}^{\mathrm{HM}} + U_{\mathrm{S}}^{\mathrm{LM}} + 2U_{\mathrm{S}}^{\mathrm{LW}} + U_{\mathrm{HL}}^{\mathrm{HM}} + U_{\mathrm{HL}}^{\mathrm{LM}} + U_{\mathrm{LL}}^{\mathrm{LM}} + U_{\mathrm{LL}}^{\mathrm{LW}}$$

where the superscrits represent the private type and the sex of the agent and the subscript state if the agent is single or married, and, if married, in which kind of couple.

The relevant IC constraints for this problem are, in the case of singles, type θ_{H}'s,

$$v(C_{\mathrm{S}}^{\mathrm{H}}, L_{\mathrm{S}}^{\mathrm{H}}, \theta_{\mathrm{H}}) \geq v(C_{\mathrm{S}}^{\mathrm{L}}, L_{\mathrm{S}}^{\mathrm{L}}, \theta_{\mathrm{H}})$$

As for couples, the type $(\theta_{\mathrm{H}}, \theta_{\mathrm{L}})$ couple's IC constraint which preclude them from announcing to be a type $(\theta_{\mathrm{L}}, \theta_{\mathrm{L}})$ couple is

$$\left(\log\left(\frac{C_{\mathrm{HL}}^{\mathrm{H}}}{C_{\mathrm{S}}^{\mathrm{H}}}\right) + \frac{L_{\mathrm{S}}^{\mathrm{H}} - L_{\mathrm{HL}}^{\mathrm{H}}}{\theta_{\mathrm{H}}}\right)\left(\log\left(\frac{C_{\mathrm{HL}}^{\mathrm{L}}}{C_{\mathrm{S}}^{\mathrm{L}}}\right) + \frac{L_{\mathrm{S}}^{\mathrm{L}} - L_{\mathrm{HL}}^{\mathrm{L}}}{\theta_{\mathrm{L}}}\right) \geq$$
$$\geq \max_{C_1}\left(\log\left(\frac{C_1}{C_{\mathrm{S}}^{\mathrm{H}}}\right) + \frac{L_{\mathrm{S}}^{\mathrm{H}} - L_{\mathrm{LL}}}{\theta_{\mathrm{H}}}\right)\left(\log\left(\frac{2C_{\mathrm{LL}} - C_1}{C_{\mathrm{S}}^{\mathrm{L}}}\right) + \frac{L_{\mathrm{S}}^{\mathrm{L}} - L_{\mathrm{LL}}}{\theta_{\mathrm{L}}}\right)$$

The first order condition to type $(\theta_{\mathrm{H}}, \theta_{\mathrm{L}})$ division of consumption

$$\left(\log\left(\frac{C_{\mathrm{HL}}^{\mathrm{L}}}{C_{\mathrm{S}}^{\mathrm{L}}}\right) + \frac{L_{\mathrm{S}}^{\mathrm{L}} - L_{\mathrm{HL}}^{\mathrm{L}}}{\theta_{\mathrm{L}}}\right) C_{\mathrm{HL}}^{\mathrm{L}} = \left(\log\left(\frac{C_{\mathrm{HL}}^{\mathrm{H}}}{C_{\mathrm{S}}^{\mathrm{H}}}\right) + \frac{L_{\mathrm{S}}^{\mathrm{H}} - L_{\mathrm{HL}}^{\mathrm{H}}}{\theta_{\mathrm{H}}}\right) C_{\mathrm{HL}}^{\mathrm{H}}$$

and the planner production function constraint, which we assume to exchange one unit of labor for one unit of consumption.

7.2 Example 2 and 3

Now suppose that the planner is no longer restricted to offer the divorce threat as the disagreement threat point (Example 2) or he can treat men and women differently (Example 3).

In example 3, even though the man in the couple (θ_H, θ_L) has all the bargaining power, the women's utility as single maybe larger than that of men. This maybe the case because she isn't affected by compatibility of incentives constraints as a single, so the planner may still distort the intrahousehold allocation to help the husband.

Notice also that as the labor is assignable the planner may have incentives to distort the economy from the efficient allocation and ask type θ_L to work.

Also the planner wants to discriminate as this reduce the cost associated with the type (θ_H, θ_L) IC constraint.

7.3 Example 4 and 5

Now we let the planner implement the direct truthful mechanism introduce above (Example 4)

In the disagreement game we ask individually to each spouse about the couple's type, as the punishment for a mismatch is to be treated as single's and not as a couple and the utility attained by each spouse with the disagreement bundle when the reports match is at least as good as the single's utility we have that to tell the truth is an equilibrium.

At the same time any other matched announcement, say (θ'_H, θ'_L) can be an equilibrium if the disagreement bundle of type (θ_H, θ_L) can be at least as good to both spouses as the single's bundle.

This multiplicity of equilibria is natural, but we can look to impose some compatibility of incentives also in the disagreement game to make the truthful revelation more credible. One way to do that is to consider that the planner asks only one of the spouses about the couples type, another posibility is to ask to each spouse only about his/her type.

In the current example to ask to each one about his/her type or to ask only to the man is the same, as the woman can only be of type θ_L.

Another way to motivate this exercise is to consider other possibles mechanisms that could be reasonable. In fact all three options would let to exactly the same outcome if the planner had no power over the disagreement utility. In other words, in agreement if the signal space is rich enough to identify the couple's type after the planner uses the announcement of both spouses them he can implement anything that he can implement asking to each one about the couples type. This is not true when we consider the disagreement game, as the couple don't act cooperatively and the planner loses power by not checking the information of one spouse with the other.

That logic should apply generally, but when we ask only the women the extra IC re-

strictions are already satisfied by the original allocation. Instead we look to the case where the planner asks only the husband (Example 5). By the revelation principle we know that he can only do worse, but it is interesting to see how this mechanism can hurt most of men in equilibrium.[3]

We have two new IC constraints

$$v(C_{\mathrm{LL}}^{\mathrm{d}}, L_{\mathrm{LL}}^{\mathrm{dM}}, \theta_{\mathrm{L}}) \geq v\left(\frac{1}{2}(C_{\mathrm{HL}}^{\mathrm{dH}} + C_{\mathrm{HL}}^{\mathrm{dL}}), L_{\mathrm{HL}}^{\mathrm{dH}}, \theta_{\mathrm{L}}\right)$$

and

$$v(C_{\mathrm{HL}}^{\mathrm{dH}}, L_{\mathrm{HL}}^{\mathrm{dH}}, \theta_{\mathrm{H}}) \geq v(2C_{\mathrm{LL}}^{\mathrm{d}} - \tilde{C}, L_{\mathrm{LL}}^{\mathrm{dM}}, \theta_{\mathrm{H}})$$

where $\tilde{C}$ is defined by

$$v(\tilde{C}, L_{\mathrm{LL}}^{\mathrm{dW}}, \theta_{\mathrm{L}}) = v(C_{\mathrm{S}}^{\mathrm{LW}}, L_{\mathrm{S}}^{\mathrm{LW}}, \theta_{\mathrm{L}})$$

In principle both constraint could be binding in equilibrium. In fact, if the planner asks only to one spouse of the couple (θ_1, θ_2), say the first, about the couple's type then he must give the same utility to every other couple (θ_1, θ') with the same sharing rule as (θ_1, θ_2).

In the following tables (1,2,3) we compare the allocation and utilities[4]

	HM/S	LM/S	LW/S	HM/HL	LW/HL	LM/LL	LW/LL	dLM/LL	dLW/LL	dHM/HL	dLW/HL
C_1	2.0000	0.4218	0.4218	2.0608	0.9240	0.9234	0.9234	0.4218	0.4218	2.0000	0.4218
C_2	2.0000	0.7461	0.7461	2.0239	0.9725	0.9523	0.9523	1.0058	1.0058	0.6981	0.4204
C_3	2.0000	0.1978	1.2395	2.0231	1.5631	0.7513	1.2395	0.1978	1.2395	2.0000	1.2395
C_4	2.0000	0.5716	1.3966	1.9656	2.1458	0.8762	1.3966	1.7482	1.7482	0.1975	1.3966
C_5	2.0000	0.6491	1.1795	2.0000	1.1795	0.8422	1.7775	1.1795	1.1795	1.6364	1.1795

Table 1: Consumption

	HM/S	LM/S	LW/S	HM/HL	LW/HL	LM/LL	LW/LL	dLM/LL	dLW/LL	dHM/HL	dLW/HL
L_1	3.1129	0	0	2.9175	0.4995	0.7835	0.7835	0	0	3.1129	0
L_2	2.5457	0.5737	0.5737	2.4858	0.7516	0.8176	0.8176	0.8723	0.8723	0.4405	0
L_3	4.6274	0	0	4.2918	0	1.3346	0	0	0	4.6274	0
L_4	5.0754	2.5705	0	4.1030	0	0	0	0.4785	0.0123	4.8440	0
L_5	5.3995	3.1489	0	2.2587	0	0	0	1.2025	0	1.6292	0

Table 2: Labor supply

The only man who benefits from the mechanism that asks only the type θ_{H} in the couple $(\theta_{\mathrm{H}}, \theta_{\mathrm{L}})$. Also notice that the woman in couple $(\theta_{\mathrm{L}}, \theta_{\mathrm{L}})$ benefits also from the change.

[3]In fact here we can use an even more elementary argument, as the planner can always ask both spouses and discart the wife's announcement.

[4]We set $\theta_{\mathrm{H}} = 2$ and $\theta_{\mathrm{L}} = 1$.

	HM/S	LM/S	LW/S	HM/HL	LW/HL	LM/LL	LW/LL
$U_4 - U_1$	-0.9813	-2.2665	1.1973	-0.6401	1.3420	0.7311	1.1973
$U_4 - U_2$	-1.2649	-2.2633	1.2006	-0.8379	1.5430	0.7343	1.2006
$U_4 - U_3$	-0.2240	-1.5093	0.1194	0.0655	0.3168	1.4883	0.1194
$U_4 - U_5$	0.1620	0.4512	0.1690	-0.9396	0.5984	0.0396	-0.2411

Table 3: Change in utility

v_1	0%
v_2	15%
v_3	72%
v_4	100%
v_5	77%

Table 4: Objective value

In table 4 we compare how much of the value of the best case (Example 4) we can attain in the other examples. We set the zero as the worst case (Example 1) and the are distributed linearly.

8 Conclusion

In this project we look to the optimal taxation of couples. Our model main features is to respect to allow the planner to have a social welfare function that depends on individual utilities instead of the aggregate family utility and to at the same time allow we model the family as solving a cooperative bargaining problem.

The main point is how the planner has incentives to create a tax system in-between the tax to couples in agreement and the tax to singles. This is due to incentives constraints that can be weakened by this in-between tax system, although it is never used in equilibrium. Through the revelation principle we show that it is the best that can be done and by an example we illustrate the channels at work and how the planner can stricly prefer the proposed mechanism to other reasonable disagreement mechanism or to no disagreement mechanism.

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