FUNDAÇÃO GETULIO VARGAS

EPGE
Escola de Pós-Graduação em Economia

SEMINÁRIOS DE PESQUISA ECONÔMICA

" A Competitive Growth Model with Endogenous Fertility"

Fernando A . Veloso

(Department of Economics - University of Chicago)

LOCAL
Fundação Getulio Vargas
Praia de Botafogo, 190 - 10° andar - Auditório

DATA
15/01/98 (5ª feira)

HORÁRIO
16:00h

Coordenação: Prof. Pedro Cavalcanti Gomes Ferreira
Email: ferreira@fgv.br - ☎ (021) 536-9250

A Competitive Growth Model with Endogenous Fertility*

Fernando A. Veloso**
Department of Economics
University of Chicago
feve@cicero.spc.uchicago.edu

August 1997

Abstract

This paper investigates the interaction between endogenous fertility behavior and the distribution of income and wealth among families in a competitive market economy. We construct a growth model in which altruistic dynasties are heterogeneous in their initial stocks of physical capital. Dynasties make choices of family size along with decisions about consumption and intergenerational transfers. We show that, if the rate of time preference is increasing in the number of children and preferences over number of children satisfy a normality assumption, all steady states are characterized by equality of capital stocks and consumption among families. We also provide sufficient conditions for uniqueness of the steady state. In order to illustrate these results, we present an example in which preferences over number of children are logarithmic and the technology is Cobb-Douglas. For this combination of preferences and technology, there exists a unique egalitarian steady state. Moreover, the economy converges to this steady state in only one generation.

* I thank Avner Ahituv, Fernando Alvarez, Gary Becker, Nancy Stokey, seminar participants of the Growth and Development Workshop at the University of Chicago, and especially Robert Lucas, for comments and suggestions. Financial support from CAPES is gratefully acknowledged. All errors are mine.
** Graduate student, Department of Economics, University of Chicago.

1.Introduction

This paper investigates the interaction between endogenous fertility behavior and the distribution of income and wealth in a competitive market economy. More specifically, we examine under what conditions endogenous fertility leads to equality of income and wealth among families in the long run. The paper is related to two strands of literature, namely the literature on the distribution of income and wealth among individuals in deterministic market economies and the literature on the economic determinants of fertility behavior.

Stiglitz (1969) develops a general equilibrium model of the distribution of income and wealth among individuals, in the context of a neoclassical growth framework. Stiglitz analyses an economy in which agents are divided into a finite number of long-lived families with mortal members in each generation. Different generations of a given family are linked through intergenerational transfers. In his model, the economy-wide capital stock is determined simultaneously with the distribution of capital among individuals. Stiglitz shows that, under the assumption that bequests are a linear function of family income, the steady state level of the economy-wide capital stock per capita is globally stable and wealth and income are asymptotically evenly distributed among families[1].

It has been shown, however, that the results in Stiglitz (1969) depend crucially on the assumption that families have linear bequest rules. If one assumes that bequests are determined endogenously by utility-maximizing agents, Stiglitz's results on the long run equality of income and wealth among agents do not hold[2], unless additional restrictions are imposed on preferences[3].

This paper follows Stiglitz (1969) in integrating a model of family behavior into a model of the distribution of income and wealth[4]. It departs from his analysis by assuming that intergenerational transfers are the result of the maximization of a dynastic utility function. On the other hand, the model presented in this paper differs from infinitely-lived agent models of the distribution of income and wealth by interpreting the decision makers as families and giving content to this interpretation by explicitly considering their fertility decisions.

[1]In Stiglitz's model, different families receive the same wage rate and differ only in their per capita wealth holdings. The assumption that bequests are linear in income implies that an increase in per capita wealth by a given percentage raises bequests by a smaller percentage. As a result, the wealth per capita of the poorer families grows faster than that of the richer families.

[2]Becker (1980) analyses a model that is similar to Stiglitz (1969) in all aspects, except for the fact that he assumes that infinitely-lived agents maximize a time-additive utility function with a constant rate of time preference. He shows that the household with the lowest discount rate owns all the capital in the long run. If discount rates are equal between households, then the steady state distribution of income is indeterminate.

[3]Lucas and Stokey (1984) study a similar model in which preferences are recursive, but not necessarily additively separable over time. They show that, if preferences exhibit a property labeled increasing marginal impatience, which means that the consumer's discount factor is a decreasing function of steady state consumption, then there will exist a unique interior stationary distribution of consumption and wealth.

[4]The role of the family in generating persistence of income and wealth inequality across generations has been analysed in the context of stochastic models by Becker and Tomes (1979), Loury (1981), Becker and Tomes (1986) and Laitner (1992), among others. In these models, limited insurance opportunities are crucial for generating inequality among families in the long run.

Becker and Barro (1988) construct a partial equilibrium dynamic model in which altruistic families make fertility and bequest decisions. They show that, for a specific functional form of preferences, there exists a unique steady state and the economy converges to the steady state in one generation, because bequests per child are independent of wealth[5]. In their model, the driving force behind the stability of the steady state is the fact that wealthier agents dilute their additional resources by having more children.

This paper combines ideas from the papers above and analyses an economy in which altruistic dynasties make fertility and investment decisions along the lines developed in Becker and Barro (1988) within a competitive market framework modeled as in Stiglitz (1969). More specifically, we consider a society divided into a finite number of dynasties, in which individuals from different generations are altruistically linked. All the members of a given dynasty have the same physical capital holdings, but dynasties differ in their per capita holdings and their size (number of members). Different dynasties interact in competitive markets for goods and factor services in each period. Parents are assumed to derive utility from their consumption, number of children and the well-being of each child.

In a steady state of this economy the capital stock of each dynasty is constant over time, but not necessarily equal among dynasties. Also, all dynasties have the same fertility rate, so the shares of each dynasty in the total population are constant. We provide sufficient conditions on dynastic preferences and the costs of child rearing such that, in any steady state, all dynasties have the same capital stock. This egalitarian result follows from a normality assumption on fertility. We also provide sufficient conditions for uniqueness of the steady state.

The paper is organized as follows. Section 2 defines the setup of the model and defines a recursive competitive equilibrium for the economy. Section 3 defines a steady state and an egalitarian steady state and provides sufficient conditions for uniqueness of an egalitarian steady state. Section 4 shows that, if preferences over number of children are logarithmic, as in Lucas (1996), and if the technology is Cobb-Douglas, then there exists a unique egalitarian steady state, which is globally stable. Section 5 concludes and presents some directions for future research.

2.The Model

The setup of the model is the following. Society is divided into a finite number of dynasties. We define a dynasty or family line as a collection of agents composed of a parent and all his descendants. We assume that the economy starts out with a finite number of parents, who in turn define a finite number of dynasties, indexed by $j=1,...,M$.

[5] Alvarez (1994) reinterprets the household problem in Becker and and Barro (1988) as a dynamic portfolio problem, in which the assets are the total members of the next generation and total intergenerational transfers. In this problem, per child transfers play the role of portfolio weights. Alvarez shows that, given the preferences postulated in Becker and Barro (1988), this reformulation of the household problem has homogeneous utility functions, so the portfolio weights are independent of wealth.

Agents live two periods, the first as children, in which they do not make any economic decisions, and the second as parents. Each period is taken to be a generation. Parents are assumed to value their consumption, the number of children they have, and the lifetime utility of each child, according to preferences given by[6]:

$$u_t = W(c_t, n_t, u_{t+1})$$

where c_t denotes parental consumption, n_t denotes number of children , u_t is the utility of the parent and u_{t+1} is the utility of each child. We assume that W is strictly increasing and strictly concave in all its arguments, twice continuously differentiable and satisfies the following discounting condition:

$$0 < W_u(c_t, n_t, u_{t+1}) < 1 \qquad \forall(c_t, n_t, u_{t+1})$$

We also assume that $\lim_{c_t \to 0} W(c_t, n_t, u_{t+1}) = \infty \quad \forall(n_t, u_{t+1})$

Parents have identical preferences and supply inelastically one unit of labor. All the currently alive members of a given dynasty have the same stock of physical capital but dynasties differ in their per capita physical capital holdings. Moreover, dynasties may differ in size (number of members).

There is a large number of firms endowed with the same constant returns to scale technology, so we can assume, without loss of generality, that there is only one firm, which produces the only consumption good according to an aggregate constant returns to scale production function, described by Y=F(K,N),where K and N denote aggregate capital and labor, respectively.[7]

Let y denote output per capita and $\bar{k}$ denote the capital-labor ratio. We assume that

$$y = f(\bar{k}) \equiv F(\bar{k},1) \qquad f'(\bar{k}) > 0,\ f''(\bar{k}) < 0,\ \lim_{k \to 0} f'(\bar{k}) = \infty,\ \lim_{k \to \infty} f'(\bar{k}) = 0$$

In each period, firms sell goods to the household sector and agents supply their labor at a wage w and rent their capital to firms at a rental rate r. The economy is assumed to be competitive, so both agents and firms take prices as given.

In this economy, agents are indexed by the dynasty they belong to. Let k_i denote the capital stock of a member of dynasty i and N_i the number of members of dynasty i. A typical member of dynasty i derives his income from the wage rate w and from capital k_i, which earns rent at the rate r. Capital depreciates at the rate δ . We assume that each child has a fixed cost ϕ in units of the consumption good, so ϕn_t is the total cost of child

[6]Recursive (but not necessarily time-additive) preferences over consumption have been used by Lucas and Stokey (1984) and Dolmas (1996) in the context of optimal growth models. The formulation presented in the text has been postulated by Alvarez (1994) and Lucas (1996) in the context of endogenous fertility models.

[7]Since each agent supplies one unit of labor, the number of hours supplied is equal to the size of the population.

rearing. Parents choose a bequest k_i' for each child, so total bequests equal $n_i k_i'$. Parents also spend their resources on their own consumption c_i, so their budget constraint can be written as[8]:

$$c_i + n_i(\phi + k_i') = (1 - \delta + r)k_i + w$$

Let $a_i \equiv \dfrac{N_i}{\sum_{j=1}^{M} N_j}$ denote the share of dynasty i in the population. The state of the economy can be described by the vector $s = (k, a)$, where $k = (k_1, \ldots, k_M)$ and $a = (a_1, \ldots, a_M)$.

Let $V(k_i, s)$ denote the lifetime utility of a parent with capital k_i, who behaves optimally, when the state of the economy is s. The problem of the head of dynasty i is the following:

$$\begin{aligned} V(k_i, s) = &\max_{c_i \geq 0, n_i \geq 0, k_i' \geq 0} W\left(c_i, n_i, V\left(k_i', h(s)\right)\right) \\ &\text{s.t.} \\ &c_i + n_i\left(\phi + k_i'\right) = \left(1 - \delta + r(s)\right)k_i + w(s) \end{aligned} \tag{1}$$

where r(s) is the real rental rate of capital, expressed as a function of the state s and w(s) is the real wage rate, also expressed as a function of the state s. The function h describes the law of motion of the state, assumed to be known by the agents. Each dynasty takes wages and rental rates as given. The solution of this problem yields optimal decision rules described by the functions

$$k_i' = b(k_i, s), \quad n_i = n(k_i, s), \quad c_i = c(k_i, s) \tag{2}$$

Competition and profit maximization by firms together imply that factors are paid their marginal products and firms earn zero profits. These conditions define the functions

$$r(s) = r(k, a) = f'\left(\sum_{i=1}^{M} a_i k_i\right) \tag{3}$$

$$w(s) = w(k, a) = f\left(\sum_{i=1}^{M} a_i k_i\right) - \left(\sum_{i=1}^{M} a_i k_i\right) f'\left(\sum_{i=1}^{M} a_i k_i\right) \tag{4}$$

[8]This formulation of the budget constraint incorporates the interaction between quantity and quality of children emphasized in Becker and Lewis (1973) and Becker and Barro (1988).

Definition 1. A recursive competitive equilibrium is a value function $V: R_+^{2M+1} \to R$, policy functions $b: R_+^{2M+1} \to R_+$, $n: R_+^{2M+1} \to R_+$, $c: R_+^{2M+1} \to R_+$, and an economy-wide law of motion $h: R_+^{2M} \to R_+^{2M}$ for the state s, such that:

1)V satisfies (1), given h.

2)b, n and c are optimal decision rules, that is, they solve (1), for given h.

3)The law of motion h satisfies the following rational expectations condition:

$$\forall s, \ h(s) = \left(b(k_1, s), \ldots, b(k_M, s), \frac{n(k_1, s)}{\sum_{i=1}^{M} a_i n(k_i, s)} a_1, \ldots, \frac{n(k_M, s)}{\sum_{i=1}^{M} a_i n(k_i, s)} a_M \right) \qquad (5)$$

Notice that (5) is a collection of functional equations, since each coordinate on the right-hand side depends on h(s).

3.Steady State

Definition 2. A steady state is a state $s^* = (k^*, a^*)$ such that:

$$h(s^*) = s^*$$

where h is the equilibrium law of motion of the state. This definition states that, in a steady state, the capital stocks and population shares of all dynasties are constant. Notice that, in a steady state, the following must hold:

$$\text{i)} k_i^* = b(k_i^*, s^*), \qquad \forall i = 1, \ldots, M$$

$$\text{ii)} \text{for some } \lambda > 0, \lambda = n(k_i^*, s^*) \quad \forall i = 1, \ldots, M \ .$$

where (b, n) are the policy functions for a recursive competitive equilibrium. Condition (i) states that, for each dynasty, it is optimal to maintain the capital stock at its steady state level. Condition (ii) states that, in a steady state, all dynasties grow at a common rate λ, which is the optimal fertility rate given the distribution of capital and population shares among dynasties.

Definition 3. An egalitarian steady state is a steady state with $k_i^* = k_j^* \ \forall i, j = 1, \ldots, M$

In a steady state, the rental rate of capital and the wage rate will be constant at r and w, respectively, since the economy-wide per capita capital stock, $\bar{k}^* \equiv \sum_{i=1}^{M} a_i^* k^*_i$, is constant.

Remark 1. Let $V(k_i, s^*)$ denote the lifetime utility attained by an adult member of dynasty i in a steady state. Since the distribution of capital and population shares is constant in a steady state, the optimum value function of a parent depends upon s^* only through $\bar{k}^*$, the economy-wide per capita capital stock.[9] Hence, we can define a function $\tilde{V}$ satisfying $V(k_i, s^*) = \tilde{V}(k_i, \bar{k}^*)$ for all $s^* = (k_1^*, \ldots, k_M^*, a_1^*, \ldots, a_M^*)$ such that $\bar{k}^* = \sum_{j=1}^{M} a_i^* k_i^*$.

Consider the decision problem of the head of dynasty i when the economy is at a steady state. Let $v(k_i) \equiv \tilde{V}(k_i, \bar{k})$. Then $v(k_i)$ satisfies the following functional equation:

$$\begin{aligned} v(k_i) &= \max \; W(c_i, n_i, v(k_i')) \\ &\text{s.t.} \\ &c_i + n_i(\phi + k_i') = Rk_i + w \end{aligned} \tag{6}$$

where $R \equiv 1 - \delta + r = 1 - \delta + f'(\bar{k})$ and $w = f(\bar{k}) - \bar{k} f'(\bar{k})$[10]. The first-order conditions corresponding to this problem are:

$$n_i W_c(c_i, n_i, v(k_i')) = W_u(c_i, n_i, v(k_i')) v'(k_i') \tag{7}$$

$$W_n(c_i, n_i, v(k_i')) = W_c(c_i, n_i, v(k_i'))(\phi + k_i') \tag{8}$$

Equation (7) equates the marginal cost of an additional unit of bequest to its marginal benefit. Note that the marginal utility of consumption in the left-hand side of (7) is multiplied by the fertility rate, because of the interaction between bequests and the number of children in the budget constraint.

Equation (8) equates the marginal utility of children to its marginal cost. Note that bequest per child k_i' increases the marginal cost of children, again because of the interaction between quantity and quality of children in the budget constraint.

The envelope condition is:

---

[9]This follows from the assumption that the production function exhibits constant returns to scale and the fact that the population share a_i does not enter the decision problem of a parent.

[10]Henceforth, I will drop the superscript * that has been used to denote steady state values, in order to simplify the notation.

$$v'(k_i) = W_c(c_i, n_i, v(k_i'))R \tag{9}$$

Let $u_i = v(k_i)$. From (6)-(9) and the definition of a steady state, we can characterize the steady state with the following system of equations:

$$u_i = W(c_i, \lambda, u_i) \qquad i = 1, \dots, M \tag{10}$$

$$RW_u(c_i, \lambda, u_i) = \lambda \qquad i = 1, \dots, M \tag{11}$$

$$\frac{W_n(c_i, \lambda, u_i)}{W_c(c_i, \lambda, u_i)} = \phi + k_i \qquad i = 1, \dots, M \tag{12}$$

$$c_i + \lambda(\phi + k_i) = Rk_i + w \qquad i = 1, \dots, M \tag{13}$$

$$f'(\bar{k}) + 1 - \delta = R \tag{14}$$

$$w = f(\bar{k}) - \bar{k}f'(\bar{k}) \tag{15}$$

$$\bar{k} = \sum_{i=1}^{M} a_i k_i \tag{16}$$

$$\sum_{i=1}^{M} a_i = 1 \tag{17}$$

There are 4M+4 equations and 4M+4 unknowns. The unknowns are:

$$((c_i, k_i, u_i, a_i), i = 1, \dots, M), \lambda, \bar{k}, R, w$$

Equations (13)-(17) imply:

$$f(\bar{k}) + (1 - \delta)\bar{k} = \sum_{i=1}^{M} a_i c_i + \phi\lambda + \lambda\bar{k} \tag{18}$$

Assumption 1. Assume that $W(c, n, u) = \hat{W}(c, n) + \varphi(n)u$[11], where φ satisfies the following properties[12]:

[11]The term $\varphi(n)$ converts the utility of children into that of parents, and can be interpreted as measuring the degree of altruism of parents toward children.

i) $0 < \varphi(n) < 1 \quad \forall n$; ii) φ is strictly increasing; iii) φ is strictly concave; iv) $\varphi \in C^2$
We also assume $\hat{W}$ that satisfies the following properties:
v) $\hat{W}$ is strictly increasing; vi) $\hat{W}$ is strictly concave; vii) $\hat{W} \in C^2$

Condition (iii) of assumption 1 implies that $\varphi(n)/n$ is decreasing in n, that is, $\varphi - \varphi' n > 0$. This corresponds to the assumption that altruism per child is decreasing in the number of children[13], which is captured by the altruism function postulated in Becker and Barro (1988). This condition amounts to assuming that the effective rate of time preference is increasing in the number of children.

Conditions (ii) and (v) imply that, for given utility per child u, parental utility is strictly increasing in the number of children n. Conditions (iii) and (vi) imply that, for given u, parental utility is strictly concave in n.

Assumption 2. Assume that the production function satisfies the following properties:

a) $f'(\bar{k}) > 0$; b) $f''(\bar{k}) < 0$; c) $\lim_{\bar{k} \to 0} f'(\bar{k}) = \infty$; d) $\lim_{\bar{k} \to \infty} f'(\bar{k}) = 0$

In the subsequent analysis, it will be assumed that a steady state exists (that is, there exists a solution to (10)-(17))[14]. We will focus on the issue of uniqueness of the steady state.

In order to solve for the steady state, we will use the following strategy. First, we will postulate an economy-wide capital stock per capita $\bar{k}$. From (14) and (15), we can express the interest rate R and the wage rate w as funtions of $\bar{k}$:

$$R = R(\bar{k}) \equiv f'(\bar{k}) + 1 - \delta \tag{19}$$

$$w = w(\bar{k}) \equiv f(\bar{k}) - \bar{k} f'(\bar{k}) \tag{20}$$

Using (10)-(13), we will solve for c_i, k_i, u_i and λ for a given pair (R,w). Using (19) and (20), we will define a mapping from $\bar{k}$ to the individual capital stock $k_i, k_i = k_i(\bar{k})$. Then we will use (16) and (17) to solve for the equilibrium $\bar{k}$.

Consider a member of a dynasty i. Given the preference structure assumed above, (10) becomes

---

[12]Benhabib and Nishimura (1993) postulate preferences of the form $W(c,n,u) = c^{\sigma} + \varphi(n)u$, $0 < \sigma \le 1$ in their version of the neoclassical growth model with endogenous fertility and exogenous technological progress.

[13]It is important to notice that if $\varphi(n) = \beta$, $\beta \in (0,1)$, which corresponds to the standard assumption that the discount factor is constant, this assumption is automatically satisfied. This suggests that the assumption of decreasing altruism per child may be less restrictive than it seems.

[14]In section 4, we will provide an example of a combination of preferences and technology for which a steady state exists.

$$u_i = \hat{W}(c_i,\lambda) + \varphi(\lambda)u_i \tag{21}$$

From (21) and using the discounting assumption $0 < \varphi(n) < 1$, we can solve for u_i as a function of (c,λ):

$$u_i = \frac{\hat{W}(c_i,\lambda)}{1-\varphi(\lambda)} \equiv g(c_i,\lambda) \qquad g_c > 0 \ , \ g_\lambda > 0 \tag{22}$$

The derivatives of g have the signs above because of the assumptions that $\hat{W}$ is increasing in c and λ and $\varphi'(\lambda) > 0$.

After substituting (22) into the left-hand side of (12) and using the functional form for W from assumption 1, we can write the steady state marginal rate of substitution between children and consumption as a function of steady state consumption and fertility as follows:

$$m(c_i,\lambda) \equiv \frac{\hat{W}_n(c_i,\lambda) + \varphi'(\lambda)g(c_i,\lambda)}{\hat{W}_c(c_i,\lambda)} \tag{23}$$

From (13), we can write k_i as a function of c_i and λ (for given R and w):

$$k_i = \frac{c_i + \phi\lambda - w}{R-\lambda} \tag{24}$$

If we substitute (23) and (24) into (12), we obtain

$$m(c_i,\lambda) = \frac{\phi R + c_i - w}{R-\lambda} \tag{25}$$

Given the assumed preference structure, (11) can be rewritten as:

$$R = \frac{\lambda}{\varphi(\lambda)} \tag{26}$$

Since $\dfrac{\varphi(\lambda)}{\lambda}$ is decreasing in λ, we can solve (26) for λ as a function of R, $\lambda = \xi(R)$, where $\xi(R)$ satisfies:

$$R \equiv \frac{\xi(R)}{\varphi(\xi(R))} \tag{27}$$

Differentiating (27) implicitly with respect to R, we obtain

$$\xi'(R) = \frac{\lambda^2}{R^2(\varphi - \varphi'\lambda)} > 0$$

since $\varphi - \varphi'\lambda > 0$. If we use (27) to substitute for λ in (25), we obtain:

$$m(c_i, \xi(R)) = \frac{\phi R + c_i - w}{R(1 - \varphi(\xi(R)))} \tag{28}$$

We want to find restrictions on W such that (28) defines c_i as a function of (R, w), where W is defined in assumption 1. Throughout this analysis, we will keep (R, w) constant and view both sides of (28) as functions of c_i.

Remark 2. From assumption 2, there exists a $\tilde{k} > 0$ such that $\bar{k} \leq f(\bar{k}) \leq \tilde{k}$, for all $0 \leq \bar{k} \leq \tilde{k}$ and $f(\bar{k}) < \bar{k}$, for all $\bar{k} > \tilde{k}$. Hence, $X = [0, \tilde{k}]$ is the set of maintainable capital stocks.

Assumption 3. Assume that $\phi R(\tilde{k}) - w(\tilde{k}) > 0$, where $\tilde{k}$ is the maximum sustainable capital stock per capita.

Remark 3. Since $w(\bar{k}) / R(\bar{k})$ is strictly increasing in $\bar{k}$, assumption 2 implies that $\phi R(\bar{k}) - w(\bar{k}) > 0 \ \forall \bar{k} \in X$.

Assumption 3 imposes a lower bound on the cost of child rearing ϕ, given by $\frac{w(\tilde{k})}{R(\tilde{k})}$. One way to interpret this assumption is that it requires the net cost of producing a descendant to be positive. An additional child costs ϕ in the current period, which is worth $R\phi$ next period. Since an additional descendant will earn w next period, when he becomes an adult, the lifetime cost of an additional adult is $\phi R - w$, which is positive from assumption 2.

Assumption 4. $\varepsilon(c_i, \lambda) \equiv \frac{m_c(c_i, \lambda) c_i}{m(c_i, \lambda)} \geq 1$ for any (c_i, λ) satisfying (10)-(17).

Lemma 1 states that, in any steady state, $c_i = c_j \ \forall i, j = 1, \ldots, M$.

Lemma 1. Let assumptions 1, 3 and 4 hold. Then there is at most one c_i that solves (28) for given (R,w).

Proof: Let $\Omega(c_i, \lambda) \equiv \frac{m(c_i, \lambda)}{c_i}$

If we divide both sides of (28) by c_i and rearrange the terms, we obtain:

$$\Omega(c_i,\xi(R)) = \frac{1}{R(1-\varphi(\xi(R)))} + \frac{\phi R - w}{R(1-\varphi(\xi(R)))c_i} \tag{29}$$

From assumption 1(i) and assumption 3, the right-hand side of (29) is strictly decreasing in c_i.

Differentiating $\Omega(c_i,\xi(R))$ with respect to c_i, we obtain:

$$\Omega_c(c_i,\xi(R)) = (\varepsilon(c_i,\xi(R)) - 1)\frac{m(c_i,\xi(R))}{c_i^2} \tag{30}$$

From assumption 4 and (30), $\Omega(c_i,\xi(R))$ is weakly increasing in c_i at the steady state solution, so the left-hand side of (29) is weakly increasing in c_i at any such solution. Since the right-hand side of (29) is strictly decreasing in c_i , there exists at most one c_i satisfying (29). QED

The derivative of $\Omega(c_i,\lambda)$ with respect to c_i can be related to the aggregator W as follows:

$$\Omega_c(c_i,\lambda) = \frac{\left[\hat{W}_c\hat{W}_{cn} - (\hat{W}_n + \varphi' g)\hat{W}_{cc} + \hat{W}_c\varphi' g_c\right]c_i - \dfrac{\hat{W}_n + \varphi' g}{\hat{W}_c}}{c_i^2} \tag{31}$$

where all derivatives are evaluated at steady state values.

To gain some intuition on the restriction imposed on $\varepsilon(c_i,\lambda)$, consider the following problem[15]:

$$\begin{aligned} &\max_{c,n}\ \hat{W}(c,n) + \varphi(n)u \\ &\quad \text{s.t.} \\ &\quad c + n(\phi + k') = I \end{aligned} \tag{32}$$

In this problem, k' is held constant, so $u = v(k')$ is a parameter[16].

Definition 4. n is normal if the maximizing value of n in (32) is increasing in I for all values of ϕ and k'. This is equivalent to the condition $\hat{W}_c\hat{W}_{cn} - (\hat{W}_n + \varphi' g)\hat{W}_{cc} > 0$.[17]

---

[15]The following argument is motivated by a similar reasoning in Lucas (1996).

[16]This problem is a version of the problem stated in (6), with bequests per child k' taken as given.

[17]This condition can be obtained by differentiating implicitly the first-order conditions associated to (32) with respect to I and by requiring the derivative of the maximizing value of n with respect to I to be positive.

Definition 5. n and u are complements if the maximizing value of n in (32) is increasing in u for all values of ϕ and k'. This is equivalent to the condition $\hat{W}_c \varphi' > 0$.

If we assume that n is normal and n and u are complements in the sense defined above, the first term in the numerator of (31) will be positive at the steady state solution, since $g_c > 0$. Yet, these conditions are not enough to guarantee the existence of a unique c_i satisfying (29), since the second term in the numerator of (31) is also positive. Hence, we need the stronger condition $\varepsilon(c_i, \lambda) \geq 1$.

The intuition for this result is the following. Richer dynasties desire to have more children, since children are a normal good. Yet, they also face a higher price of children, because they invest more in each child. Hence, it might be possible to have a steady state in which two dynasties with different capital stocks and consumption find it optimal to have the same number of children. Assumption 3 requires ϕ to be large enough in order to reduce the effect of a higher bequest per child on the cost of children. The condition $\varepsilon(c_i, \lambda) \geq 1$ requires the income effect on fertility and the complementarity between fertility and utility per child to be strong enough, in the sense that the numerator of (31) has to be positive.

In light of lemma 1, we can define c_i as a function of (R, w), $c_i = \pi(R, w)$. The following proposition states that if a steady state exists, it must be egalitarian.

Proposition 1. Let assumptions 1, 3 and 4 hold. Then, if a steady state exists, it is egalitarian, that is, it satisfies $k_i = k_j \quad \forall i, j = 1, \ldots, M$.

Proof. Since, in any steady state, $c_i = \pi(R, w)$ and $\lambda = \xi(R)$, we can use (24) to obtain:

$$k_i = \frac{\pi(R, w) + \phi \xi(R) - w}{R - \xi(R)} \tag{33}$$

From (33), it is clear that $k_i = k_j \quad \forall i, j = 1, \ldots, M$. QED

From (19) and (26), we can define λ as a function of $\bar{k}$, $\lambda = \xi(R(\bar{k})) \equiv \lambda(\bar{k})$, where $\lambda(\bar{k})$ satisfies:

$$\frac{\lambda(\bar{k})}{\varphi(\lambda(\bar{k}))} \equiv R(\bar{k}) \tag{34}$$

Differentiating (34) implicitly with respect to $\bar{k}$, we obtain:

$$\dot{\lambda}(\bar{k}) = \frac{\lambda^2}{R^2(\varphi - \varphi' \lambda)} R'(\bar{k}) \quad < 0 \tag{35}$$

since $\varphi - \varphi' \lambda > 0$ and $R'(\bar{k}) = f''(\bar{k}) < 0$. Equation (35) states that when the economy-wide capital stock per capita is higher, the fertility rate is lower. The reason is that a higher $\bar{k}$ reduces the interest rate, so fertility has to be lower in order to reduce the effective rate of time preference.

From (19), (20) and (29), we can define c_i as a function of $\bar{k}$, $c_i = C(\bar{k})$, where $C(\bar{k})$ satisfies:

$$\Omega\left(C(\bar{k}),\xi(R(\bar{k}))\right) = \frac{1}{R(\bar{k})\left(1-\varphi(\xi(R(\bar{k})))\right)} + \frac{\phi - \Gamma(\bar{k})}{\left(1-\varphi(\xi(R(\bar{k})))\right)C(\bar{k})} \tag{36}$$

where $\Gamma(\bar{k}) \equiv \dfrac{w(\bar{k})}{R(\bar{k})}$.

Definition 6. c is normal if the maximizing value of c in (32) is increasing in I for all values of ϕ and k'. This is equivalent to the condition $(\hat{W}_n + \varphi' g)\hat{W}_{cn} - \hat{W}_c(\hat{W}_{nn} + \varphi'' g) > 0$.

Assumption 5. $(\hat{W}_n + \varphi' g)\hat{W}_{cn} - \hat{W}_c(\hat{W}_{nn} + \varphi'' g) > 0$.

Lemma 2. Let assumptions 1, 2, 3, 4 and 5 hold. Then $C(\bar{k})$ is strictly decreasing in $\bar{k}$.

Proof. The appendix shows that, if we differentiate (36) implicitly with respect to $\bar{k}$, we obtain:

$$C'(\bar{k}) = \frac{\left[\dfrac{(1-\varphi)\left[f(\bar{k})+(1-\delta)\bar{k}-c_i\right]+R\varphi'\xi'\left[c_i+(\phi-\Gamma)R\right]}{(1-\varphi)^2 R^2 c_i} - \Omega_\lambda \xi'\right]}{\left[\Omega_c + \dfrac{\phi-\Gamma}{(1-\varphi)c_i^2}\right]} R'(\bar{k}) \quad < 0 \tag{37}$$

The denominator of (37) is positive, since $\Omega_c \geq 0$ (which follows from assumption 4), $\phi - \Gamma > 0$ (which is equivalent to assumption 3) and $\varphi \in (0,1)$, which follows from assumption 1(i). The term Ω_λ in the numerator is the derivative of Ω with respect to λ, which is equal to:

$$\Omega_\lambda = \frac{\hat{W}_c(\hat{W}_{nn} + \varphi'' g) - (\hat{W}_n + \varphi' g)\hat{W}_{cn}}{c_i} \quad < 0$$

which is negative from assumption 5. From the feasibility condition (18), we have $f(\bar{k})+(1-\delta)\bar{k} > c_i$. Since $\xi' > 0$ and $\varphi' > 0$, the term inside brackets in the numerator of (37) is positive. Since $R'(\bar{k}) = f''(\bar{k}) < 0$, we have established that $C'(\bar{k}) < 0$. QED

The intuition for this result is the following. A higher $\bar{k}$ is associated with lower fertility, which raises the marginal rate of substitution between number of children and consumption, from the assumption that c is normal. The normality condition on fertility requires consumption to decline. A change in $\bar{k}$ also affects the cost of fertility through changes in w and R. By using the fact that, in equilibrium, R and w are related to marginal productivities, one can observe that the net (negative) effect of an increase in the wage rate on the cost of child rearing dominates, which reduces consumption further (this is captured by the first term in the numerator of (37)).

From (19), (20) and (24) and substituting $c_i = C(\bar{k})$ and $\lambda = \lambda(\bar{k})$, we can write k_i as a function of $\bar{k}$, $k_i = \Psi(\bar{k})$, which satisfies:

$$\Psi(\bar{k}) \equiv \frac{C(\bar{k}) + \phi\lambda(\bar{k}) - w(\bar{k})}{R(\bar{k}) - \lambda(\bar{k})} \tag{38}$$

Since $k_i = k_j \;\; \forall i, j = 1, \ldots, M$ and $\sum_{j=1}^{M} a_j = 1$, (16) and (38) imply that the steady state economy-wide per capita capital stock $\bar{k}$ must satisfy the following condition:

$$\Psi(\bar{k}) = \bar{k} \tag{39}$$

Equation (39) equates the desired individual capital stock to the economy-wide capital stock per capita. The next proposition provides conditions under which (39) has a unique solution for $\bar{k}$.

Proposition 2. Let assumptions 1, 2, 3, 4 and 5 hold. Then, if a steady state exists, the economy-wide capital stock per capita $\bar{k}$ is unique.

Proof. If we view both sides of (39) as functions of $\bar{k}$, the right-hand side is just the 45 degree line. If we differentiate (38) implicitly with respect to $\bar{k}$ and rearrange the terms, we obtain:

$$\Psi'(\bar{k}) = \frac{C'(\bar{k}) + (\phi + k_i)\lambda'(\bar{k}) + f''(\bar{k})(\bar{k} - k_i)}{1 - \delta + f'(\bar{k}) - \lambda} \tag{40}$$

An increase in $\bar{k}$ reduces desired fertility and consumption, which tends to reduce the desired individual capital stock. Yet, if $k_i \neq \bar{k}$, there are income effects associated to an increase in $\bar{k}$. Wages increase by $-f''\bar{k}$ and interest income falls by $f''k_i$. If k_i is large

relative to $\bar{k}$, the fall in interest income may outweigh the increase in w, so k_i may increase even though desired c and λ are smaller. When $k_i = \bar{k}$, (40) is reduced to

$$\Psi'(\bar{k}) = \frac{C'(\bar{k}) + (\phi + k_i)\lambda'(\bar{k})}{1 - \delta + f'(\bar{k}) - \lambda} \qquad < 0 \tag{41}$$

since $C'(\bar{k}) < 0$ from (37), $\lambda'(\bar{k}) < 0$ from (35) and $1 - \delta + f'(\bar{k}) = R(1-\varphi) > 0$, from assumption 1. Hence, the function $\Psi(\bar{k})$ has a negative slope when it crosses the 45 degree line, which implies that it can cross it only once, establishing the desired result.

4. Log Preferences[18]

Let $W(c,n,u) = (1-\beta)\log(c) + \eta \log(n) + \beta u \qquad 0 < \beta < 1 \qquad \eta > 0$
Let $f(\bar{k}) = A\bar{k}^{\alpha} \qquad 0 < \alpha < 1 \qquad \delta = 1$

4.1. Steady State

For this specification of preferences, $\varphi(n) = \beta$, so $\frac{\varphi(n)}{n} = \frac{\beta}{n}$ is strictly decreasing in n, as required by assumption 1. It is clear that assumption 2 is satisfied.

In this example, the function g is given by

$$u_i = g(c_i, \lambda) = \frac{(1-\beta)\log(c_i) + \eta \log(\lambda)}{1-\beta} \tag{42}$$

The steady state marginal rate of substitution between number of children and consumption is

$$\frac{W_n(c_i, \lambda, u_i)}{W_c(c_i, \lambda, u_i)} = \frac{\eta c_i}{(1-\beta)\lambda} = m(c_i, \lambda) \tag{43}$$

The function $\Omega(c_i, \lambda) \equiv \frac{m(c_i, \lambda)}{c_i}$ is given by

$$\Omega(c_i, \lambda) = \frac{\eta}{(1-\beta)\lambda} \tag{44}$$

[18] The preferences used in this example have been studied in Lucas (1996).

which does not depend on c_i. From (43), $\varepsilon(c_i, \lambda) = 1$, so assumption 4 is satisfied. It is straightforward to verify that assumption 3 requires $\phi > \frac{(1-\alpha)}{\alpha} A^{\frac{1}{1-\alpha}}$. Assumption 5 amounts to $W_{nn} = -\frac{\eta}{\lambda^2} < 0$, which holds in this example.

Since assumptions 1-5 are satisfied for this example, propositions 1 and 2 imply that there exist at most one steady state for this combination of preferences and technology, and it must be egalitarian.

For this example, we can solve the steady state equations (10)-(17) and obtain a closed-form solution for the steady state values of consumption, capital and fertility, which are given by

$$c_i = \frac{(1-\beta)A}{1-\beta+\eta}\left[\frac{\phi\alpha\beta(1-\beta+\eta)}{\eta-\alpha\beta(1-\beta+\eta)}\right]^{\alpha}$$

$$k_i = \bar{k} = \frac{\phi\alpha\beta(1-\beta+\eta)}{\eta-\alpha\beta(1-\beta+\eta)}$$

$$\lambda = \alpha\beta A\left[\frac{\eta-\alpha\beta(1-\beta+\eta)}{\phi\alpha\beta(1-\beta+\eta)}\right]^{1-\alpha}$$

We need the additional restriction $\eta - \alpha\beta(1-\beta+\eta) > 0$ to guarantee existence of a steady state with positive consumption, capital and fertility. It can be easily verified that this restriction is satisfied if $\eta > \beta$, since $\alpha \in (0,1)$.

4.2. Stability

In this subsection, we analyse the stability of the steady state computed above. We assume that the economy starts at time t=0. Let superscrips index dynasties and subscripts denote the time period (assumed to be a generation). We assume that there are M dynasties, with initial capital stock $k^i{}_0$, $i = 1,\ldots, M$. The dynamic system associated to the recursive competitive equilibrium defined in section 2 is described by the following equations:

$$u^i{}_t = (1-\beta)\log(c^i{}_t) + \eta\log(n^i{}_t) + \beta u^i{}_{t+1} \tag{45}$$

$$\frac{c^i{}_{t+1}}{c^i{}_t} = \frac{\beta R_{t+1}}{n^i{}_t} \tag{46}$$

$$\frac{c^i{}_t}{n^i{}_t} = \frac{(1-\beta)(\phi + k^i{}_{t+1})}{\eta} \tag{47}$$

$$c^i_t + n^i_t\left(\phi + k^i_{t+1}\right) = R_t k^i_t + w_t \tag{48}$$

$$R_t = 1 - \delta + f'\left(\bar{k}_t\right) = \alpha A \bar{k}_t^{\alpha-1} \tag{49}$$

$$w_t = f\left(\bar{k}_t\right) - \bar{k}_t f'\left(\bar{k}_t\right) = (1-\alpha) A \bar{k}_t^{\alpha} \tag{50}$$

$$\bar{k}_t = \sum_{j=1}^{M} a_t^j k^j_t \tag{51}$$

$$a^i_{t+1} = \frac{n^i_t}{\sum_{j=1}^{M} a^j_t n^j_t} a^i_t \tag{52}$$

$$\sum_{i=1}^{M} a^i_t = 1 \tag{53}$$

where $u^i_t = v\left(k^i_t, s_t\right)$ and $s_t = \left(k^1_t, \ldots, k^M_t, a^1_t, \ldots, a^M_t\right)$. Equation (45) is derived from the Bellman equation in (1), (46) is the first-order condition for bequests, (47) is the first-order condition for fertility, (48) is the budget constraint, (49) and (50) relate factor prices to the marginal productivities, (51) defines the economy-wide capital stock per capita, (52) describes the evolution of the population shares in each dynasty and (53) states that the sum of the shares must sum up to one.

From (46) and (47), we find that at t=1, the following holds:

$$c^i_1 = \frac{\beta(1-\beta)R_1}{\eta}\left(\phi + k^i_1\right) \tag{54}$$

From (47) and (48), we can obtain another expression for consumption at t=1:

$$c^i_1 = \frac{(1-\beta)\left(w_1 + R_1 k^i_1\right)}{1-\beta+\eta} \tag{55}$$

From (54) and (55), we obtain

$$k^i_1 = \frac{\phi\beta(1-\beta+\eta)}{\eta-\beta(1-\beta+\eta)} - \frac{\eta}{\eta-\beta(1-\beta+\eta)}\left(\frac{w_1}{R_1}\right) \tag{56}$$

Equation (56) implies that $k^i_1 = k^j_1 \ \forall i,j$. Hence, at t=1, capital stocks are equal among dynasties, independently of their initial capital stocks. From (45)-(53), we obtain that this common value of the capital stock is equal to the steady state $\bar{k}$:

$$k^i{}_1 = k^j{}_1 = \bar{k} = \frac{\phi\alpha\beta(1-\beta+\eta)}{\eta-\alpha\beta(1-\beta+\eta)} \qquad \forall i,j = 1,\ldots,M$$

To summarize, the economy converges to the unique egalitarian steady state in only one generation. The intuition behind the stability of the steady state is the following. From (46), we have

$$\frac{c^i{}_1}{c^i{}_0} = \frac{\beta R_1}{n^i{}_0} \tag{57}$$

From (47) and (54), we have

$$n^i{}_0 = \frac{\eta(1-\beta)(w_0 + R_0 k^i{}_0)}{(1-\beta)(1-\beta+\eta)(\phi+\bar{k})} \tag{58}$$

Equation (57) shows that high fertility dynasties discount more the utility of each child, so their consumption grow less than the consumption of low fertility dynasties, and so does their capital stock. From (58), it follows that richer parents (higher $k^i{}_0$) dilute their wealth by having more children than poorer parents. Hence, the capital stock of poorer parents grows faster than that of richer parents, leading to convergence of the per capita capital stock among dynasties. For log preferences and Cobb-Douglas technology, this convergence takes only one generation.

5.Conclusion

In this paper, we construct a growth model in which altruistic dynasties are heterogeneous in their initial stocks of physical capital. Parents make choices of family size along with decisions about consumption and intergenerational transfers. We show that, if the rate of time preference is increasing in the number of children and preferences satisfy a normality assumption, then families will have the same stock of physical capital per capita in the long run. Moreover, this common level of the capital stock is unique. If preferences are logarithmic and the technology is Cobb-Douglas, the economy converges to the unique egalitarian steady state in one generation.

In the model presented in this paper, the equality of wealth and income among families in the long run is related to the fact that fertility is positively related to income. A variety of empirical studies have documented a positive correlation between family income and fertility[19]. However, there are also several studies which find a negative correlation between fertility and family income[20]. The studies that find a positive relation

[19]See, for example, Becker (1960), Mincer (1963), Simon (1974) and Wahl (1985).

[20]See Willis (1973), Ben-Porath (1973) and Mulligan (1993). Simon (1974) documents a negative relation between income and fertility at lower income levels and a positive relation at upper levels.

between fertility and income usually control for variables that are related to the productivity of the household in the marketplace, especially wages and level of education. When these variables are not controlled for, a negative relation between fertility and family income is usually observed.

Becker, Murphy and Tamura (1990) construct a growth model in which agents with higher stocks of human capital have lower fertility rates and invest more in each child than agents with low human capital. This negative relation between fertility and human capital arises because human capital increases the time cost of raising children. In their model, endogenous fertility behavior leads to long run inequality among families endowed with different initial human capital stocks.[21]

This paper is intended to be the first step within a broader research project, which will combine ideas from Stiglitz (1969), Becker and Barro (1988) and Becker, Murphy and Tamura (1990) and incorporate heterogeneity in physical and human capital among families. The goal of this research will be to investigate how the composition of wealth between physical and human capital affects fertility decisions and how these in turn affect inequality among families in the long run.[22]

The idea is that heterogeneity in physical capital will tend to be eliminated as agents with more physical capital will have more children. On the other hand, agents with more human capital will have fewer children and will invest more in each child, which tends to magnify initial differences in human capital across dynasties. Hence, since income is derived from these two stocks, there will be two forces affecting income inequality in opposing directions. The final outcome will depend, among other factors, on how the relative returns to physical and human capital change as the economy grows.

Appendix

Proof of Lemma 2. If we differentiate (36) implicitly with respect to $\bar{k}$, we obtain

$$\left[\Omega_c + \frac{(\phi - \Gamma)}{(1-\varphi)c_i^{\,2}}\right] C'(\bar{k}) = -\frac{R'(\bar{k})}{(1-\varphi)R^2} - \frac{\Gamma'(\bar{k})}{(1-\varphi)c_i} + \frac{R\varphi'\xi'\left[c_i + (\phi - \Gamma)R\right]R'(\bar{k})}{(1-\varphi)^2 R^2 c_i} - \Omega_\lambda \xi' R' \qquad (59)$$

[21]Using data from the PSID, Mulligan (1993) finds that the correlation between fertility choice and various costs of rearing children, including parents' level of education, can account for between one third and one half of the correlation between parents' and children's economic status, for several measures of the latter, including consumption, family income and education.

[22]Veloso (1997) shows that the introduction of human capital into the model presented in section 2 can lead to long run heterogeneity in income among families, if human capital is assumed to increase the cost of child rearing.

Since $\Gamma(\bar{k}) \equiv \frac{w(\bar{k})}{R(\bar{k})}$, we have $\Gamma'(\bar{k}) = \frac{w'(\bar{k})R - wR'(\bar{k})}{R^2}$. From (19), we have $R'(\bar{k}) = f''(\bar{k})$. From (20), we have $w'(\bar{k}) = -\bar{k}R'(\bar{k})$

This implies that $\Gamma'(\bar{k}) = \frac{-R'(\bar{k})[w(\bar{k}) + \bar{k}R(\bar{k})]}{R^2}$

From (19) and (20), we obtain

$$\Gamma'(\bar{k}) = \frac{-R'(\bar{k})[f(\bar{k}) + (1-\delta)\bar{k}]}{R^2} \tag{60}$$

If we substitute (60) into the right-hand side of (59) and rearrange the terms, we obtain

$$\left[\Omega_c + \frac{(\phi - \Gamma)}{(1-\varphi)c_i^2}\right]C'(\bar{k}) = \left[\frac{(1-\varphi)[f(\bar{k}) + (1-\delta)\bar{k} - c_i] + R\varphi'\xi'[c_i + (\phi - \Gamma)]R}{(1-\varphi)^2 R^2 c_i}\right]R'(\bar{k})$$

$$-\Omega_\lambda \xi' R'(\bar{k})$$

which gives the expression in the text.

References

Alvarez, Fernando (1994). "A Baby Step in the Theory of Fertility." Ph.D. Dissertation, Department of Economics, University of Minnesota.

Barro, Robert, and Gary S. Becker (1989). "Fertility Choice in a Model of Economic Growth." *Econometrica* 57(2):481-501.

Becker, Gary S. (1960). "An Economic Analysis of Fertility." In: *Demographic and Economic Change in Developed Countries*. Princeton: Princeton University Press, for the National Bureau of Economic Research.

Becker, Gary S., and Robert Barro (1988). "A Reformulation of the Economic Theory of Fertility." *Quarterly Journal of Economics* 103:1-25.

Becker, Gary S., and Gregg H. Lewis (1973). "On the Interaction Between the Quantity and Quality of Children." *Journal of Political Economy* 2(2):279-88.

Becker, Gary S., Kevin M. Murphy, and Robert F. Tamura (1990). "Economic Growth, Human Capital, and Population Growth." *Journal of Political Economy* 98:S12-S37.

Becker, Gary S., and Nigel Tomes (1979). "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility." *Journal of Political Economy* 87(6):1153-89.

Becker, Gary S., and Nigel Tomes (1986). "Human Capital and the Rise and Fall of Families." *Journal of Labor Economics* 4(3)(2):S1-S39.

Becker, Robert A. (1980). "On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households." *Quarterly Journal of Economics* 375-82.

Benhabib, Jess, and Kazuo Nishimura (1993). "Endogenous Fertility and Growth." In: *General Equilibrium, Growth, and Trade II, The Legacy of Lionel McKenzie*. Edited by R. Becker, M. Boldrin, R. Jones, and W. Thomson. Academic Press Inc.

Ben-Porath, Yoram (1973). "Economic Analysis of Fertility in Israel: Point and Counterpoint." *Journal of Political Economy* 81(2)(2):S202-S233.

Dolmas, Jim (1996). "Balanced-Growth-Consistent Recursive Utility." *Journal of Economic Dynamics and Control* 20:657-80.

Laitner, John (1992). "Random Earnings Differences, Lifetime Liquidity Constraints, and Altruistic Intergenerational Transfers." *Journal of Economic Theory* 58(2):135-70.

Loury, Glenn C. (1981). "Intergenerational Transfers and the Distribution of Earnings." *Econometrica* 49(4):843-67.

Lucas, Robert E., Jr. (1988). "On the Mechanics of Economic Development." *Journal of Monetary Economics* 22:3-42.

Lucas, Robert E., Jr. (1996). "Ricardian Equilibrium: A Neoclassical Exposition." mimeo, Department of Economics, University of Chicago.

Lucas, Robert E., Jr., and Nancy L. Stokey (1984). "Optimal Growth with Many Consumers." *Journal of Economic Theory* 32:139-71.

Mincer, Jacob (1963). "Market Prices, Opportunity Costs, and Income Effects." In: *Measurement in Economics*. Edited by C. F. Christ et al. Stanford, Stanford University Press.

Mulligan, Casey B. (1993). "Intergenerational Altruism, Fertility, and the Persistence of Economic Status." Ph.D. Dissertation, Department of Economics, University of Chicago.

Simon, Julian L. (1974). *The Effects of Income on Fertility*. Monograph no. 19. Chapel Hill: Carolina Population Center, University of North Carolina.

Stiglitz, Joseph E. (1969). "Distribution of Income and Wealth among Individuals." *Econometrica* 37(3):382-97.

Veloso, Fernando A. (1997). "A Competitive Growth Model with Endogenous Fertility", Thesis Proposal, Department of Economics, University of Chicago.

Wahl, Jenny B. (1985). "Fertility in America: Historical Patterns and Wealth Effects on the Quantity and Quality of Children." Ph.D. Dissertation, Department of Economics, University of Chicago.

Willis, Robert J. (1973). "A New Approach to the Economic Theory of Fertility Behavior." *Journal of Political Economy* 81(2)(2):S14-S64.