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The Effects of Uncertainty about the Timing of Deals on Consumer Behavior

Priscilla Yung Medeiros
(Northwestern University)

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Coordenação:
Prof. Luis Henrique. B. Braido
e-mail: lbraido@fgv.br

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Priscilla Yung Medeiros[2]

Kellogg School of Management

Northwestern University

November 4, 2007

Abstract

I examine the effects of uncertainty about the timing of deals (i.e. temporary price cuts or sales) on consumer behavior in a dynamic inventory model of consumer choice. I derive implications for purchase behavior and test them empirically, using two years of scanner data for soft drinks. I find that loyal consumers' decisions, both about the allocation of their purchases over time and the quantity to be purchased in a particular deal, are affected by the uncertainty about the timing of the deal for the product. Loyal consumers buy a higher fraction of their overall purchases during deals as the uncertainty decreases. This effect increases with an increase in the product's share of a given consumer's purchase in the same category or if the consumer stockpiles (i.e., is a shopper). During a particular deal, loyal shoppers increase the quantity they purchase the more time that has passed since the previous deal, and the higher the uncertainty about the deals' timing. For the non-loyal consumers these effects are not significant. These results hold for products that are frequently purchased, like soft-drinks and yogurt, but do not hold for less frequently purchased products, such as laundry detergents. The findings suggest that manufacturers and retailers should incorporate the effects of deals' timing on consumers' purchase' decisions when deriving optimal pricing strategies.

[1] I wish to thank David Bell for the data, Igal Hendel and Scott Stern for their guidance and Eric Anderson, David Besanko, Alexei Alexandrov, Alberto Salvo, Minjae Song and participants of the Management and Strategy seminar at Kellogg for useful comments. Comments and suggestions are greatly appreciated.

[2] Please address correspondence to Priscilla Yung Medeiros, Management and Strategy Department, Kellogg School of Management, 2001 Sheridan Road, Evanston, IL, 60208. Email: p-medeiros@kellogg.northwestern.edu.

1 Introduction

It is well documented in both the marketing and economics literatures, that *deals*, defined as temporary price-reductions or discount sales, are a key component of firms' pricing strategies for both non-durable and durable goods. As a result, the nature of the consumer response to deals is of substantial importance to both academics and managers. Recent studies demonstrate that demand anticipation (at low prices consumers store for future consumption) is present in many frequently purchased non-durable goods[3]. Therefore, we should expect consumers to strategically time their purchases to coincide with deals. Then uncertainty about the timing of a deal for a brand might affect consumers' purchases decisions.

The objective of this paper is to test and understand whether and how uncertainty about deals' timing affects consumers' decisions, about both the allocation of their purchases over time and the quantity purchased at the time of a particular deal. I also investigate how these effects vary across different types of consumers and product categories. I develop a dynamic model of consumer choice where consumers' just form beliefs about future prices when they are uncertain about deals' timing. Otherwise they know the entire price distribution. I use the model to derive implications for purchase behavior and test them empirically using two years of scanner data for soft drinks, laundry detergents, and yogurts. The bottom line from the empirical results is that loyal shopper consumers' decisions, both on the allocation of their purchases over time, and on the quantity purchased at a particular deal, are affected by the timing of the brand's deals. And, unlike previous studies, I am able to support this claim with scanner data. This result holds for products that are frequently purchased, like soft-drinks and yogurts, but not for products that are less frequently purchased, like laundry detergents. This result also has distinct managerial implications. It suggests that it is crucial for both manufacturers and retailers to incorporate the effects of deals' timing on consumers' purchasing decisions when deriving optimal pricing strategies.

Similarly to other analysts of this phenomenon[4], I investigate how uncertainty about deals' timing affects consumers' choices using a dynamic context and let consumers endogenously choose the amount

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[4] Examples of papers in this literature are Assunção and Meyer (1993) and Krishna (1994).

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The objective of this paper is to test and understand whether and how uncertainty about deals' timing affects consumers' decisions, about both the allocation of their purchases over time and the quantity purchased at the time of a particular deal. I also investigate how these effects vary across different types of consumers and product categories. I develop a dynamic model of consumer choice where consumers' just form beliefs about future prices when they are uncertain about deals' timing. Otherwise they know the entire price distribution. I use the model to derive implications for purchase behavior and test them empirically using two years of scanner data for soft drinks, laundry detergents, and yogurts. The bottom line from the empirical results is that loyal shopper consumers' decisions, both on the allocation of their purchases over time, and on the quantity purchased at a particular deal, are affected by the timing of the brand's deals. And, unlike previous studies, I am able to support this claim with scanner data. This result holds for products that are frequently purchased, like soft-drinks and yogurts, but not for products that are less frequently purchased, like laundry detergents. This result also has distinct managerial implications. It suggests that it is crucial for both manufacturers and retailers to incorporate the effects of deals' timing on consumers' purchasing decisions when deriving optimal pricing strategies.

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to consume. The unique aspect of the model presented here is the crucial assumption about the way consumers form their beliefs about future prices. Consumers form these beliefs only when they are uncertain about deals' timing. In the case of predictable deal patterns, consumers know the entire price distribution. This assumption differs from these previous works and also from other related works in the economics literature that studies demand anticipation using a dynamic inventory model of consumer choice. They all assume either that consumers form expectations about future prices according to a Markov process or that the way expectations are formed is the same independent of the level of uncertainty about deals' timing. I then describe the optimal consumer behavior in both cases. I take the case of predictable deal patterns as the benchmark and derive implications for purchase behavior. I then consider the case of an unpredictable deal pattern and compare the implied purchasing behavior to the behavior under the benchmark case.

In the model, consumers purchase for two reasons: for current consumption (endogenously determined) and to build inventory. How much consumers buy in each period depends on their current inventory level, the current shock to utility from consumption and current prices. In the unpredictable case, how much consumers buy also depends on their beliefs about future prices while in the predictable case it depends on the number of weeks between two consecutive deals, which consumers know in advance. Prices can take on two values: on deal, p_L, and off deal, p_H such that $p_L<p_H$. Consumers know both prices, they do not know before coming to the store which price will be offered.

I focus on four types of consumers: loyal shopper, loyal non-shopper, non-loyal shopper, and non-loyal non-shopper. Both kinds of loyal consumers are in the market for their particular brand often; in some sense they need the brand. They choose the brand most of the time regardless of price. When their favorite brand is not offered at a deal they generally do not substitute for another brand. Instead they either buy their favorite brand at the regular price or consume it from inventory. In the theoretical model of consumer behavior, I define *loyal* as the consumer whose marginal utility from consumption of the brand is high enough such that he is willing to purchase this brand at the regular price if necessary, i.e. his stock is zero. Non-loyal consumers have no compelling need to buy a brand. They buy a brand only if its

price is low enough. Empirically, I identify a *loyal* as the consumer whose share of purchases on a particular brand is at least 70% of his total purchases in the category.

A *shopper* consumer buys not only for immediate consumption, but also to stockpile. In the model, *shopper* is defined as the consumer who has a storage cost low enough such that he is able to stockpile for the average number of weeks between two consecutive deals. Empirically, I identify a *shopper* by the characteristics of his purchase pattern that arise from his stockpiling behavior. For instance, a consumer for whom the difference of the time to the next purchase is larger for purchases on sale than for purchases not on sale is defined as a shopper. The intuition is that these consumers buy a larger quantity when purchasing on sale to be able to last longer without purchasing.

There are two implications derived from the model. The first is that, at the time of a particular deal, loyal shopper consumers increase their quantity purchased the more time has passed since the previous deal and the higher the uncertainty about deals' timing. The second implication is that, as compared to other consumers, loyal shoppers buy a higher fraction of their overall purchases during deals, and, as a result, save more, as uncertainty about deals' timing decreases. Non-shopper consumers don't stock up so they buy only to the extent of their consumption. Non-loyal consumers may be affected in the same way by the uncertainty about the deal timing of other competitive brands.

In terms of empirical analysis, this paper also contributes to the literature on the effects of deal patterns on purchase decisions. Previous works rely on experiments to test their model implications[5]. To the best of my knowledge, I am the first to use scanner data to study how uncertainty about the timing of deals affects consumers' purchase decisions. Unlike previous work I also investigate how these effects vary across different types of consumers and product categories.

The theoretical implications are tested using two years of scanner data for soft drinks, laundry detergents, and yogurts. These data were collected using scanning devices in nine supermarkets, belonging to five different chains, in two sub-markets of Chicago. The store level data includes weekly prices, quantities and promotional activities. The household level data follows the purchases of 1,042

[5] Examples of previous works are Krishna (1994) and Meyer and Assunção (1990).

households over a period of 104 weeks. I know when each household visited a store, how much was spent in each visit, which products were bought, and where they were bought.

I define the unpredictability of the deal pattern of a product as the coefficient of variation of the number of weeks between two consecutive deals[6]. The higher the standard deviation of the number of weeks between two consecutive deals, for a given average number of weeks between two consecutive deals, the less predictable the deal pattern is. I define consumers' gains from buying a higher fraction of their overall purchases during deals as savings. The savings are defined as the difference between how much consumers actually spent and how much they would have spent if they had bought the same amount randomly and therefore, paid the average of the prices they observed at their trips to the store.

I first estimate the effects of uncertainty about deals' timing on savings separately for loyal and non-loyal consumers after controlling for other relevant characteristics of the deal pattern, such as frequency of deals (percentage of the total time the product was offered during a deal), and average discount (average of all the percentage discounts for which the product was offered). Then I include all consumers together in a single regression and add two new independent variables: the actual shares each consumer buys of each brand in the category and an interaction term between uncertainty about deals' timing and shares. I also interact loyalty with the shopper classification. The main finding is that uncertainty about deals' timing significantly affects loyal consumers' savings. Loyal consumers buy a higher fraction of their overall purchases during deals (save more), as uncertainty about deals' timing decreases. This effect increases with an increase in the product's share of a given consumer's purchase in the category or if the consumer stockpiles (is a shopper). For non-loyal consumers these effects are not significant.

I also regress the quantity purchased per visit to the store on price, promotional activities, number of weeks since the previous deal, uncertainty about deals' timing and an interaction term between the last two. I find that loyal and shopper consumers increase the quantity they purchase on a particular deal both as more time passes since the previous deal and as there is greater uncertainty about deals' timing. These

[6] The Coefficient of variation is the standard deviation of the number of weeks between two consecutive deals divided by the average number of weeks between two consecutive deals. This measure is defined per UPC, per store, per consumer.

results for consumers' decisions, both for the allocation of their purchases over time, and for the quantity purchased during a particular deal, hold for products that are frequently purchased, like soft-drinks and yogurts, but do not hold for products that are less frequently purchased, like laundry detergents.

I assume that prices and deals' timing are exogenously given at the present work. An interesting extension is to determine manufacturers' and retailers' optimal pricing strategies incorporating the effects of deals' timing on consumer behavior and compare the results to their actual decisions. I show here how to find the optimal pricing strategy for the case of a monopolist manufacturer. I find situations where the monopolist manufacturer can achieve a higher profit from offering predictable deals instead of a constant price. I also discuss situations where firms can profit from unpredictability such as using it as a source of increasing overall consumption of the brand, as it is documented that for some products, the presence of additional inventory on hand induces additional consumption per period.

The remainder of the paper is organized as follows. In section two I review the relevant literature. In section three I present the dynamic inventory model of consumer choice and derive testable implications. In section four I describe the data and identification strategies. In section five I present the empirical results for the effects of uncertainty about deals' timing on consumers' allocation of their purchases over time. In section six I show the empirical results for the effects of uncertainty about deals' timing on the quantity purchased during a particular deal. In section seven I present a cross-category analysis. In section eight I discuss the managerial implications. In section nine I conclude.

2 Related Research

This paper contributes to two main streams of research. In the following sections, I review the relevant research in these two areas and discuss how they relate to my work.

2.1 Literature on Demand Anticipation

One branch of the literature on demand anticipation focuses on exploring the more fundamental question of whether data supports the argument that demand anticipation is an important effect of promotion. In the marketing literature, Gupta (1988) distinguishes three components of household response: category purchase timing, brand choice, and purchase quantity. In the coffee category, Gupta finds that the percentage of own-brand sales elasticity with respect to a particular promotion that is due to brand-switching elasticity is 84%, that is due to purchase acceleration elasticity is 14%, and that is due to quantity elasticity is 2%. Other follow-up papers extend Gupta's (1988) approach, and generalize for many categories and brands (e.g. Chintagunta (1993) and Bell, Chiang and Padmanabhan (1999)). Across these decomposition studies the authors find that, on average, brand switching accounts for the vast majority of total elasticity. In contrast, Van Heerde, Gupta & Wittink (2003) offer a complementary decomposition measure based, instead of elasticity, on unit sales. The authors apply their new method to previously reported elasticity decomposition results and find that the cross-brand component, instead of accounting for 75% of the total elasticity effect, is actually 33%. This gives new evidence to the importance of demand anticipation effect of sales promotion.

In the economics literature, the question of whether data supports demand anticipation to be an important effect of promotion, is studied testing implications derived from a dynamic inventory model of consumer choice. Boizot, Robin and Visser (2001) present a dynamic inventory model that they test using consumer dairy data. They show that duration from previous purchase increases in current price and declines in past price, and quantity purchased increases in past prices. Hendel and Nevo (2006b) also present a dynamic inventory model and use it to derive implications about observable variables that stem from storing, but would not be expected under static framework. Using scanner data on three different product categories (laundry detergents, yogurt and soft-drinks) they find the results to be consistent with an inventory model. They find that aggregate demand increases as a function of duration from previous sale, and this effect differs between sale and non-sale periods. They also find that when buying on sale households tend to buy more quantity, buy earlier and postpone their next purchase.

This paper presents a dynamic inventory model of consumer choice based on Hendel and Nevo (2006b). A drawback from Hendel and Nevo (2006b) and some other papers in this literature is the assumption that either consumers form expectations about future prices according to a Markov process or that the way the expectations are formed is the same independent of the level of uncertainty about deals' timing. While this is a fine assumption for many applications, it is too restrictive if one's final goal is to study the effects of deals' timing on consumers' purchase decisions. Instead I assume that consumers' beliefs about future prices just follow a Markov process in the case that consumer is uncertain about deals' timing (unpredictable deal pattern). In the case in which deals are predictable (there is no uncertainty), consumer's know the entire price distribution.

Another branch of the literature on demand anticipation focuses on structurally estimating dynamic inventory models of consumer choice. Erdem, Imai and Keane (2003) construct a structural model of demand in which consumers can store different varieties of the product. They focus on the role of price expectations and differences between short run and long run price responses. They show that temporary price cuts primarily generate purchase acceleration and category expansion, rather than brand switching. Sun, Neslin and Srinivasan (2003) also show that brand-switching elasticities are overestimated by stand-alone logit models. Hendel and Nevo (2006a) structurally estimate a dynamic inventory model of consumer choice using scanner data on laundry detergents and show that static demand models create biased price elasticity estimates.

This paper makes a first step in identifying the importance of letting consumers' form beliefs about future prices only when they are uncertain about deals' timing. An interesting future extension is to apply the estimation methods developed in previous papers (Erdem, Imai and Keane (2003) and Hendel and Nevo (2006a)) to structurally estimate the model presented here and generate normative pricing implications.

2.2 Literature on the Effects of Deal Patterns on Consumer Behavior

There is a stream of literature which builds rational models of purchasing under price uncertainty and investigates how deal patterns influence consumers' purchase behavior. The starting point for much of this research is Golabi's (1985) zero-order model (prices in each period are independent of prices in prior periods) for the case of a single good and constant consumption rate. Golabi's model is extended by Krishna (1992) to the multiple brand case and by Assunção and Meyer (1993) to accommodate variable consumption rate and first-order dealing patterns. The model presented here is close to Assunção and Meyer (1993) in the sense that consumers endogenously chose the amount to consume. However, most of implications derived in Assunção and Meyer (1993) are based on the special case of markets characterized by bimodal prices where consumers' expectations about future prices are represented by a first order Markov process. Again, the way expectations are formed is the same independent of the level of uncertainty on deals' timing.

Of these studies, the paper closest to mine is Krishna (1994). She explores the effect of deal patterns on consumer behavior by developing a normative purchase quantity model that incorporates all deal patterns. One of the implications of her model is that the average quantity purchased during deals should be larger when there is greater certainty about deals' timing. I also investigate how certainty about deals' timing affects consumers' choices using a dynamic context. However, in contrast to her work, I let consumers endogenously choose the amount to consume (instead of assuming a constant consumption rate). I also explicitly account for consumer heterogeneity and differences across categories.

There is a significant lack of empirical research on the effects of deal patterns on purchase decisions. Krishna (1994) tests some of her model implications in a laboratory experiment. Meyer and Assunção (1990) also use an experiment to report how consumers make rational sequential purchase decisions with imperfect knowledge about future prices using different shapes of the distribution of prices and its trend over time. To the best of my knowledge, I am the first to use scanner data to study how uncertainty about deals' timing affects consumers' purchase decisions, and how the effects vary across different types of consumers and product categories.

3 Model of Consumer Behavior

In this section, I present the dynamic inventory model of consumer choice. First, I describe the basic setup of the model. Next, I describe how consumers form their expectations about future prices. Then, I describe the optimal consumer behavior under both the predictable and unpredictable cases. I take the case of predictable deal pattern as the benchmark and derive implications for purchase behavior. I then consider the case of unpredictable deal pattern and compare the implied purchasing behavior to the behavior under the benchmark case. From this comparison I derive two theoretical implications that are tested with scanner data.

3.1 The Basic Setup

Household h purchases for two reasons: current consumption and to build inventories. At each period t, household h decides the amount it wants to consume, c_{ht}, and the quantity it wants to purchase, q_{ht}, of each single product. The household derives a utility from consumption that is described by the following equation:

$$u(c_{ht}) = \beta_h log(c_{ht} + v_{ht}) \tag{1}$$

where β_h is the marginal utility from consumption and v_{ht} is a shock to utility. The shock to utility, v_{ht}, introduces randomness in the household's needs, unobserved by the researcher. Low realizations of v_{ht} increase the household's need, making it more inelastic. Households know the current realization of the shock when they reach the store. But they don't know the future realizations of the shock. I assume that v_{ht} can take on three values, $v_{ht} \in \{0, 1, 2\}$, with equal probabilities. I also assume that the shocks are i.i.d. across each type h of households.

Household h also buys to take advantage of deals and to store for future consumption. The cost of storing inventory is given by:

$$C_{ht}(i_{ht}) = \theta_h i_{ht} \tag{2}$$

where i_{ht} is the inventory level of household h at period t, and θ_h is the marginal disutility of storing inventory.

Prices can take on two values: on deal, p_L, and off deal, p_H such that $p_L<p_H$. Consumers are aware of both prices, they do not know before coming to the store which price will be offered.

Define d_{ht} as follows, with each consumer at each date having a potentially different d_{ht}:

$$d_{ht} = \begin{cases} 0 & stay\ home \\ 1 & visit\ store \end{cases} \tag{3}$$

Each consumer is given an exogenously determined vector of d_{ht}'s. Consumers do not decide on when to go to the store, they just know when are the next times they are going to be in the store. At each period consumers visit a store, they must decide on the quantity to purchase. They observe the price of each good even if they decide not to purchase the good. At all periods consumers decide on the quantity to consume. When consumers do not visit the store, they do not observe the price of the good. This assumption introduces heterogeneity in consumers' beliefs about future prices. Since different consumers might visit stores at different periods, they possibly experience different price distributions for the same good, at the same store.

The consumer's problem can be represented as:

$$\max_{c_{ht}, q_{ht}} \sum_{t=0}^{\infty} \delta^t E\left[\beta_h \log(c_{ht} + v_{ht}) - C_{ht}(i_{ht}) - d_{ht}\gamma_h p_t q_{ht} \middle| \Psi_{ht}\right] s.t.$$

$$C_h(i_{ht}) = \theta_h i_{ht}$$

$$i_{ht} = i_{h,t-1} + d_{ht} q_{ht} - c_{ht} \tag{4}$$

$$i_{ht} \geq 0 \quad c_{ht} \geq 0 \quad q_{ht} \geq 0$$

where Ψ_{ht} is the information set at time t, and δ the discount factor. At each time t, household h derives non-negative utility from current consumption of the good. At time t, household h also incurs the cost of storing, whenever it ends period t with a positive inventory, and the cost of purchase, whenever it visits a store and decides to purchase a positive amount. Quantity not consumed is stored as inventory.

3.2 Information Set

The contents of the information set, Ψ_{ht}, depend on the type of deal pattern being offered. Deal patterns can either be *predictable* (no uncertainty about deals' timing) or *unpredictable* (some level of uncertainty about deals' timing). More precisely define by D the number of periods the good is offered on deal. Also define by N_z the number of weeks between two consecutive deals for $z=1,...,D-1$ and $N=[N_1,...,N_{D-1}]$. $\mu(N)$ stands for the average number of weeks between two consecutive deals and $\sigma(N)$ the respective standard deviation. A product has a predictable deal pattern when $\sigma(N)=0$. Any deal pattern such that $\sigma(N)>0$ is classified as unpredictable. For instance, a product that is promoted on alternate weeks or every 3 weeks has a predictable deal pattern.

In the case in which deals are unpredictable, the information set at time t consists of the beginning of the period inventory, i_{ht-1}, current prices, p_t, the shock to utility from consumption, v_{ht}, and the vector of d_{ht}'s: $\Psi_{ht}=\{i_{ht-1}, p_t, v_{ht}, d_{ht}, d_{ht-1}, d_{ht-2},...\}$. Consumers' expectations about future prices are represented by a first-order Markov process with two prices, a deal price (p_L) which is thought to occur with probability $\pi_{H,L}$ if p_H was the price in the previous period, and $\pi_{L,L}$ if p_L was the price in the previous period, and a regular price (p_H) which is thought to occur with probability $\pi_{L,H}$ and $\pi_{H,H}$ after the occurrence of price p_L and p_H. Formally the probability function can be described by the Markov chain:

$$\begin{array}{cccc} & & \multicolumn{2}{c}{\textit{price at } t+1} \\ & & p_L & p_H \\ \textit{price at } t & p_L & \pi_{L,L} & \pi_{L,H} \\ & p_H & \pi_{H,L} & \pi_{H,H} \end{array} \tag{5}$$

The transition probabilities describe how predictable, consumers believe, the deal pattern is. Consumers form their expectations (give value to the transition probabilities) during an initial learning period. Those beliefs are defined per product, per consumer, per store. Consumers that visited the same store at the same periods have identical beliefs. Consumers use these transition probabilities defined at this initial learning period to make decisions on the quantities to buy and consume. The utility maximization problem described in (4) happens after this initial learning period is over. There is no further learning in my model.

The closer the transition probabilities, $\pi_{L,H}$ and $\pi_{H,L}$, are to 0 and 1, the more predictable the deal pattern is. The closer $\pi_{L,H}$ and $\pi_{H,L}$ are to 0.5, the more unpredictable the deal pattern is. Due to the Markov assumption, the only predictable deal pattern that can be represented by the Markov chain is deals happening every odd period, i.e., $\pi_{L,H}=\pi_{H,L}=1$ or the cases of constant price, i.e., either $\pi_{L,H}$ or $\pi_{H,L}$ equal to 0. For any other deals' timing consumers face some source of uncertainty.

We expect consumers to have more information about deals' timing when deals are predictable than when there is some uncertainty about timing. Due to the Markov assumption, I am not able to distinguish between a predictable deals pattern, like deals happening every 3 weeks, from an unpredictable one. To solve this issue I consider that consumers know the entire price distribution when deals are predictable. They know exactly when it was the last deal. Therefore, for the case of a predictable deal pattern, consumers information set at time t consists not only of the beginning of the period inventory, $i_{h,t-1}$, current prices, p_t, shock to utility from consumption, v_{ht}, and the vector of d_{ht}'s, but also of the number of weeks between two consecutive deals, N: $\Psi_{ht}=\{i_{ht-1}, p_t, v_{ht}, d_{ht}, N, d_{ht-1}, d_{ht-2},\ldots\}$.

3.3 Consumer Behavior

Independently of the type of deal pattern being offered, in each period consumers compare the costs of holding inventory and the benefits from buying at a current price instead of future expected prices. Their decisions also depend on the exogenously determined vector of d_{ht}'s. Consumers might want to purchase more at a given visit to store for consumption at the later periods they are going to stay home.

To simplify, in the following analysis I consider the case in which all consumers visit the store every period, i.e, $d_{ht}=1$ for $\forall t$ and h.

At the regular price consumers might purchase for immediate consumption, depending on their inventory, price sensitivity, γ, marginal utility from consumption, β, and the realization of the random shock to utility. Consumers do not buy for storage at the regular price unless I relax the assumption that $d_{ht}=1$ for $\forall t$. Ceteris paribus[7], at the regular price consumers' decisions on how much to purchase are the same, independent of the type of deal pattern being offered.

At the deal price consumers might purchase for immediate consumption depending on their inventory, price sensitivity, γ, marginal utility from consumption, β, and the realization of the random shock to utility. However, consumers might also purchase for storage. This last decision depends on their storage cost, θ, discount factor, δ, the regular price, p_H, deal price, p_L, and, in the case consumers are uncertain about deals' timing, also on their expectations about future prices. In the case of predictable deal pattern, the decision to purchase for storage also depends on the number of weeks between two consecutive deals, N.

In the case of an unpredictable deal pattern, ceteris paribus, the smaller the probability that after a promotion has been offered another promotion occurs, $\pi_{L,L}$, and the higher the probability that after a regular price has been offered another regular price occurs, $\pi_{H,H}$, the bigger the quantity purchased at a particular deal. Of course this conclusion depends, among other things, on the storage cost and discount factor. If the storage cost is too high or discount factor too small this conclusion might not hold. And for a given range of average number of weeks between two consecutive deals, $\mu(N)$, an increase in the variance of the price distribution leads to a decrease in $\pi_{L,L}$ and to an increase in $\pi_{H,H}$.

In the case of a predictable deal pattern, ceteris paribus, the bigger the number of periods the product is offered at the regular price, N, the bigger the quantity purchased at a particular deal. Again this conclusion depends, among other things, on the storage cost and discount factor. If the storage cost is too

[7] Given the same inventory level, price sensitivity, γ, marginal utility from consumption, β, and the realization of the random shock to utility.

high or discount factor too small this conclusion might not hold. The optimal consumer's purchase decisions are described in proposition 1.The generalization of proposition 1 including shocks to utility, proposition 2, is described in the appendix. Proofs are provided in the appendix. In all the following analysis I drop the subscript *h*, to simplify notation.

Proposition 1: *(Benchmark case) Consider the case of a predictable deal pattern. Prices are cyclic with a cycle defined by one period of deal price followed by N consecutive periods of regular price. Assume no shocks to utility ($v_t = 1$). The optimal quantities to purchase and consume for the N+1 periods of the cycle can be described as:*

$$c^*_j = \frac{\beta}{\gamma \vartheta_j} - 1 \qquad for \qquad j = 1,\dots,n \tag{6}$$

$$q^*_1 = \sum_{j=1}^{n} c^*_j, q^*_2 = \dots = q^*_n = 0 \tag{7}$$

$$q^*_H = q^*_{n+1} = \dots = q^*_{N+1} = c^*_{n+1} = \dots = c^*_{N+1} = \frac{\beta}{\gamma p_H} - 1 \tag{8}$$

$$n = \max k \middle| \vartheta_k < p_H, k \in IN \tag{9}$$

$$\begin{cases} \vartheta_1 = p_L \\ \vartheta_k = \dfrac{p_L + \theta \sum_{j=0}^{k-2} \delta^j}{\delta^{k-1}} \qquad for \qquad k = 2,\dots,n \end{cases} \tag{10}$$

The intuition for proposition 1 is the following. First define υ_k as the virtual price. It is the unit cost of purchasing at the first period low price for a later consumption at period $j=k$, where $k=2,\dots,n$. It is composed by the cost of purchasing at the deal price, p_L, plus the cost of carrying the inventory up to period k of consumption. Consumers compare the virtual price with the cost of purchasing the same unit at period k regular price, when deciding on the amount to be purchased for storage at the first period deal price. For all periods such that the virtual price is smaller than the regular price it is optimal to purchase in advance at the first period promotional price. n is the last period from the N periods of regular price where

the virtual price is smaller than the regular price. During these $n>1$ periods of the cycle there is no need for purchase. Consumption comes from inventory. For the remaining periods of the cycle, from $t=n+1$ to $t=N+1$, purchases are only for immediate consumption.

A main difference of consumers' purchase behavior under predictable deal pattern and unpredictable deal pattern is the probability of overstocking and understocking. Consumers *overstock* when they have a positive inventory on hand when a deal occurs. Consumers *understock* when they have zero or insufficient inventory during a regular price when it would be ex-post optimal to have a positive inventory. If we consider no shocks to utility, the probability of overstocking and understocking is zero for the benchmark case, and positive for the unpredictable deal pattern.

In order to derive implications for consumers' decisions on the allocation of their purchases over time, I first describe (Proposition 3) how consumers' gains from buying a higher fraction of their overall purchases during deals vary with the parameters of the model in the benchmark case.

Proposition 3: *In the case of predictable deal pattern, consumers' gains from buying a higher fraction of their overall purchases during deals increases as p_H, v_t and δ increases or as θ, p_L and N decreases.*

Given that the fraction of overall purchases during deals is given by the ratio n/N, and n is the number of periods where the virtual price is smaller than the regular price, the smaller the virtual price and the higher the regular price the larger is n and consequently, the higher the ratio. The virtual price increases as p_L, θ and N increases or as δ decreases. Also low realizations of the shocks to utility increase consumers' needs, increasing the probability of purchase at the regular price.

3.4 Testable Implications

I now focus on those predictions of the model that help us understand how consumers' purchase decisions vary with uncertainty about deals' timing, for a given average number of weeks between two consecutive deals. I am also interested in how this behavior varies across different types of consumers. In particular, I

am interested on four main types of consumers: loyal shopper, loyal non-shopper, non-loyal shopper, and non-loyal non-shopper.

Loyal is the consumer whose marginal utility from consumption of the good is high enough such that he is willing to purchase at the regular price if necessary, i.e. stock is zero. More formally, loyal is the consumer for whom $\beta_h > \gamma_h p_H$. *Non-loyal* is the consumer whose marginal utility from consumption of the good is smaller than the same threshold, i.e., $\beta_h \leq \gamma_h p_H$. These consumers are not willing to perform an inter-temporal substitution of this product.

Shopper is the consumer who has a storage cost low enough such that he is able to stockpile for the average number of weeks between two consecutive deals. More formally, shopper is the consumer for whom $\theta_h < \dfrac{\left(\delta^{\mu(N)_h - 1}\right) p_H - p_L}{\sum_{j=0}^{\mu(N)_h - 2} \delta^j}$. Note that for the benchmark case a shopper has $n=N$, i.e., he is able to have 100% of his purchases during deals. *Non-shopper* is the consumer who has a storage cost higher than the same threshold.

The following two implications are derived using the case of unpredictable deal pattern and comparing to the benchmark case. Proofs are provided in the Appendix.

Implication 1: *Loyal shopper consumers increase their quantity purchased on a particular deal both as more time passes since the previous deal and the higher the uncertainty about deals' timing*[8].

The first part of this implication, namely that the quantity purchased on a particular deal increases as more time passes since the previous deal, comes from the fact that purchases in non-deal periods are only for consumption. As the number of weeks from the last deal increases, inventory declines (both because consumers do not buy for storage at non-deal periods and because consumption might be positive in most of these periods). And since the quantity purchased increases as inventory decreases we get the

[8] For a given average number of weeks between two consecutive deals, all else constant, and also for the long term stationary probability of the low price state big enough, $\Pi_L \in [1/2, 1]$.

result. For non-loyal consumers their quantity purchased on a particular deal does not necessarily increases as inventory on this product decreases.

The rest of this implication is a consequence of the fact that as $\pi_{L,L}$ decreases and $\pi_{H,H}$ increases, the quantity purchased at a particular deal increases. And for a given range of average number of weeks between two consecutive deals, an increase in the variance of the price distribution leads to a decrease in $\pi_{L,L}$ and an increase in $\pi_{H,H}$. More precisely, this result holds in the case where the average number of weeks between two consecutive deals is not too large, so that the long-term stationary probability of the low price state is big enough, $\Pi_L \in [1/2, 1]$.

Implication 2: *Loyal shopper consumers buy a higher fraction of their overall purchases during deals as uncertainty about deals' timing decreases.*

For the case of unpredictable deal pattern, the probability that the next deal happens earlier/later than expected is positive. This probability increases as uncertainty about deals' timing increases. I showed in Implication 1 that the quantity purchased at a particular deal increases as uncertainty about deals' timing increases. If a deal happens earlier than expected, consumers have overstocked and if it happens later, consumers have understocked. In the case consumers have understocked, and if they are loyal, they purchase at the regular price. If consumers are non-loyal, they might find it optimal not to purchase the product at the regular price. Therefore, the higher the uncertainty about deals' timing the bigger is the number of loyal consumer's purchases at the regular price. Consequently the smaller is the fraction of overall purchases during deals.

Comparing to the benchmark case with no shock to utility, loyal shopper consumers have 100% of their overall purchases during deals since $n=N$. So loyal shopper consumers can only do worst when they are uncertain about deals' timing[9].

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[9] These implications are similar to those showed in implication 1 of Krishna (1994) coming from a different model. Concerning deals' timing the implications of her model are that the probability of overstocking (having a positive inventory when the next deal occurs) is smaller the greater the certainty of when the next deal will occur. She also finds that the proportion of

4 Data and Identification Strategies

In this section, I first present a description of the dataset. Then I discuss how the main variables of the model are identified from consumers' purchase decisions.

4.1 Data Description

I use the Stanford Market Basket Dataset consisting of scanner data for 1,042 households in the Chicago Metropolitan area, collected between June 1991 and June 1993 in two submarkets (494 urban panelists and 548 suburban) for seven different stores. This dataset has two components, store and household-level data. From the household level data I know when a household visited a store, how much was spent in each visit, which products were bought, and where it was bought. The store level data includes weekly prices, aggregate quantity sold and promotional activities. The data is available for twenty-four product categories.

I focus on the soft-drinks category. This is a category of particular interest for the questions analyzed here. First, because it is a category frequently purchased, non-perishable, and easy to store. We expect most consumers to purchase not only for immediate consumption, but also to stockpile. Second, because it is also a frequently promoted category. Moreover, there is a significant difference in the way the same products are promoted across stores. I also replicate the results for two other categories: yogurts and laundry detergents. Both are less frequently promoted than soft-drinks but they are still non-perishable[10] and easy to store.

I define a product as a brand. For each brand I include the 4 highest market shares UPCs as long as they can be argued to be perceived as the same product. When aggregating UPCs, I am implicitly

quantity purchased on deal is larger and the buyer's average cost is smaller when deals' timing is more certain. These implications are similar to my implication 2. Krishna (1994) also finds that the average quantity purchased on deal is larger when deals' timing is more certain. Compare that to implication 1 where I find that higher uncertainty leads to higher quantity purchased at a particular deal. One result seems to contradict the other. Not necessarily. The quantity purchased on a particular deal increases as uncertainty about deals' timing increases for a given level of inventory. If this level of inventory is higher because uncertainty about deals' timing lead consumers to overstock, then consumers might purchase less in a particular deal than they would have purchased in the benchmark case. Whether the average quantity purchased on deal is smaller when deals' timing is more uncertain depends on how much consumers overstocked and understocked over time.

[10] Unlike detergents and soft-drinks, yogurt can be stored for a limited time only. Nevertheless, relative to the frequency of visits to the store, yogurt is still a storable product, especially because I just consider here the smallest size of 6oz.

assuming that consumers have stronger preferences for the brand, but are indifferent among the different UPCs included in a brand. I also require the prices of these UPCs that constitute a brand to be at least 90% correlated for most of the stores at the dataset. The reason for this requirement is not to introduce measurement errors in my definition of deal.

The soft drinks category embraces several sub-categories such as cola, flavored soda and club soda/mixer all of which can be divided into regular, low calorie and caffeine free. The two main brands are Coke and Pepsi that dominate most of the cola and low-calorie cola sub categories. The flavored soda sub-category is less concentrated. The products in the soft drinks category are sold in either cans (that can be sold as singles or bundled into 6, 12 or 24-unit packs) or bottles (that can be sold in different sizes such as 16 oz. 1, 2 and 3 liter). I focus on 2 liter bottle colas. I also focus on the two main brands, Pepsi and Coke. Table 1 shows the percent market share for each selected product (UPC) at this selected 2 liter bottle cola sub-category. I included only four out of the seven stores I have data for soft-drinks, as for only these four stores the prices of the UPCs that constitutes a brand are at least 90% correlated.

The yogurt category is very concentrated at the brand level with two main brands: Dannon and Yoplait. These brands are offered in many different flavors (like vanilla, strawberry, cherry, peach, raspberry, among others) that also differs by fat contents. Yogurts are also sold in different sizes such as 6 oz., 8 oz., 16 oz., and 32 oz. I focus on regular 6 oz. yogurts. I also focus on the two main brands: Dannon and Yoplait. Table 1 shows the percent market share for each selected product (UPC) at this selected 6 oz. regular yogurt sub-category. Unlike detergents and soft-drinks, yogurts can be stored for a limited time only, especially after the unit is opened. This is why I focus on the smallest size. This size is more storable than the bigger sizes. The smallest size is also more frequently promoted than the other sizes.

Laundry detergents come in two main forms: liquid and powder. Liquid detergents account for 70% of the quantity sold. The leading firms are Procter and Gamble, which produces Tide and Cheer, and Unilever, which produces All, Wisk and Surf. I focus on liquid detergents. Liquid detergents are sold in different sizes such as 32 oz., 64 oz., 96 oz., 128 oz., and 256 oz. I focus on 128 oz. liquid laundry detergents. I also focus on two brands: Wisk and All, both produced by Unilever. Purex is the leading

brand for the 128 oz. liquid detergent market but there is missing data for prices. Tide is also among the top brands for this selected market but again there is missing data for prices. Table 1 shows the percent market share for each selected product (UPC) at this selected 128 oz. liquid detergent sub-category. I focus on the 128 oz. because this size is more frequently promoted than the other sizes and also preferable for storage[11]. I included only five out of the seven stores I have data for detergents, as for only these five stores the prices of the UPCs that constitutes a brand are at least 90% correlated.

I do not account for some possible substitution effects at the three selected categories. For instance, at the soft-drinks category I assume cans and bottles are different products and I do not account for possible substitution effects between them. The reason is that I expect cans to be more useful for individual consumption while large bottles are more used for parties and big families. I also do not account for substitution effects between liquid and powder detergents or between different sizes of yogurts again because I expect consumers to perceive them as different products.

In the model of consumer behavior, I assumed that consumers form their expectations (give value to the transition probabilities) about future prices during an initial learning period. For consumers to learn about the prices in a specific store they need to visit this store frequently enough. This is why in my empirical application I only include those households that visited a particular store at least 20 times. I also only include households that purchased at least 6 (4, 2) units of a particular brand, either Coke or Pepsi (Dannon or Yoplait, Wisk or All)[12], at a particular store during the 104 weeks. These restrictions considerably reduce the size of my sample.

4.2 Identification Strategies and Preliminary Analysis

In the model section I showed that consumers' purchase decisions of a product over time and at a particular deal are affected by the product's deals' timing if consumers are loyal to the product. One main result is that loyal shopper consumers buy a higher fraction of their overall purchases during deals as

[11] Given that there are gains in buying a larger size given by quantity discounts (non-linear pricing) and differently from the other categories, the product does not suffer any alteration after opened.

[12] I used different cutoffs for different categories because the average total quantity purchased per consumer, per store, is significantly different for each of the three categories.

uncertainty about deals' timing decreases. In order to test this implication empirically I first need to define a deal. Next, I need to identify what are the relevant characteristics about deals that affect consumers' purchase decisions and how to measure their allocation of purchases over time. Finally, I need to identify the four types of consumers: loyal shopper, loyal non-shopper, non-loyal shopper, and non-loyal non-shopper.

4.2.1 Definition of Deal

Consistent with the previous literature, I define regular price as the modal price for each product (UPC), at each store along the 104 weeks of data. Deal is any deviation at least 5% below the modal price.

4.2.2 Relevant Characteristics about Deals

There are four main characteristics about deals that may affect consumers' purchase decisions:

Average Discount. Products are offered on deals with different percent discounts off the regular price. Average discount is the average of all the observed percent discounts the product was offered on, over the 104 weeks.

Table 2 shows summary statistics on the *average discount*. The first four columns show statistics on the percent discounts off the regular price. The statistics are calculated per UPC, per store. The last column shows the standard deviation, across all stores, of the average percent discount off the regular price. The figures suggest that each store offers different percent discounts for the same product over time. Stores also differ from each other on the average discount the same product is offered on. Soft-drinks have the highest average discounts, followed by yogurts and detergents. The yogurt category has the higher variation on the average discount the same product is offered on, across stores.

Frequency of Deals. This is the percentage of weeks each product was offered on deal, independently of the particular percent discount offered on a particular deal.

Table 3 shows summary statistics on *frequency of deals*. The first column shows the average, across all stores, of the percent of weeks a deal was offered for each product at each store. The second column

shows the standard deviation, across all stores, of the percent of weeks a deal was offered for each product at each store. On average, soft-drinks are offered on deal half of the time followed by yogurts, 21% of the time, and by detergents, 15% of the time. There is also a significant variation on the frequency each product is offered on deal across stores.

Average Duration: Average number of weeks between two consecutive deals. This measure is related to frequency of deals. The higher the average duration is the smaller the frequency.

Variation of the Duration: Standard deviation of the number of weeks between two consecutive deals. It is related to the predictability of the deal pattern. The higher the variation of the duration the less predictable (more uncertain) the deal pattern is.

Table 4 shows summary statistics on *average duration*[13]. On average products are offered on deal with one week interval for soft-drinks, five weeks interval for yogurts and six weeks interval for detergents. Again there is variation of the average duration across stores (part B of the table). The maximum number of weeks between two consecutive deals observed is seventeen for soft-drinks but it can reach 50 weeks for detergents. Table 5 shows summary statistics on *variation of the duration.* The first column shows that the *average duration* varies over time. The second column shows that how much the *average duration* varies over time also varies across stores. Some stores have more certain deals' timing, with a smaller *variation of the duration,* while others have more uncertain deals' timing, with a bigger *variation of the duration.*

The main characteristic I am interested on is the unpredictability (uncertainty) of deals' timing. In the model of consumer behavior predictability of the deal pattern was described by the transition probabilities of the Markov chain for prices. In the empirical application I define *unpredictability* of the deal pattern of a product as the coefficient of variation of the number of weeks between two consecutive deals, i.e:

$$\begin{aligned} \textit{Unpredictability} &= \textit{Coefficient of Variation of Duration} \\ &= (\textit{Variation of the Duration}) \,/\, (\textit{Average Duration}) \end{aligned}$$

[13] Note that the minimum number of weeks between two consecutive deals is one which means that the product was offered on deal in consecutive weeks.

4.2.3 Measures of Allocation of Total Purchase Over Time

A direct way of measuring how much consumers concentrate their purchases on deals over time is to calculate the fraction that was bought on sale, from the total amount purchased. Another way of measuring it, is by using the difference between how much consumers actually spent and how much they would have spent if they had bought the same amount randomly and therefore, paid the average of the prices they observed at their trips to the store. These are my two measures of allocation of purchases over time:

Fraction: Fraction that was bought on sale, from the total amount purchased.

Savings: Difference between how much consumers actually spent and how much they would have spent if they had bought the same amount randomly and paid the average observed price. Note that the average observed price is different for each consumer as consumers visit the stores at different periods and consequently, observe different prices.

To find this last measure I first calculate how much each consumer spent in each UPC, at each store, over the 104 weeks. Consistent with the model, I assume that the decision of visiting a store is exogenous. This decision does not depend on the deals' characteristics of a particular UPC or brand. I record the exact weeks each consumer visited each store, to buy any type of product, and not just the products considered here. Average observed price is the average of all the prices offered per UPC, per store, at the weeks the consumer visited the store. I also record the amount each consumer purchased of each UPC in each store. Savings is the difference between the actually amount spent and the total amount they would have spent if they bought the same quantity for the average observed price.

4.2.4 Types of Consumers

There are four types of consumers: loyal shopper, loyal non-shopper, non-loyal shopper, and non-loyal non-shopper. In the model, I defined loyal as the consumer whose marginal utility from consumption of the good is high enough such that he is willing to purchase the product at the regular price if necessary.

However I do not have information on the marginal utility of consumption. Instead, I identify as a loyal the consumer whose share of purchases on a particular brand is at least 70% of his total purchase in the category[14]. A Non-loyal is the consumer whose share of purchases on a particular brand is less than 70% of his total purchase in the category. Consumers who are loyal to one brand are automatically non-loyal to the other brand. Some consumers are non-loyal to both brands considered at each category.

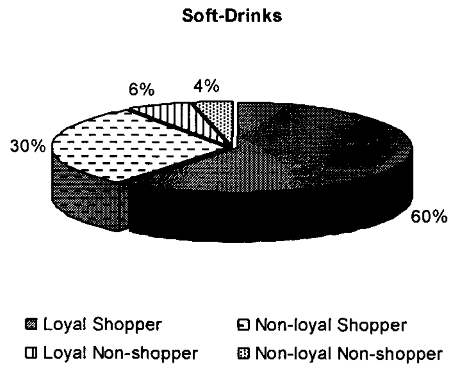
A shopper consumer buys not only for immediate consumption, but also to stockpile. They can be loyal to a brand or not. In the model, shopper is defined as the consumer who has a storage cost low enough such that he is able to stockpile for the average number of weeks between two consecutive deals. However I don't have information on the storage cost of each household. Instead, I identify a shopper by characteristics of his purchase pattern that arises from his stockpiling behavior. Some characteristics, like households buy more on deals, are consistent not only with small storage cost and stockpiling, but also with an alternative theory: when prices go down households consume more. This is why I look at other characteristics and use different robust checks on the definition of shoppers. For now I define a shopper as any consumer who presents the first characteristic and at least one or more of the other three characteristics:

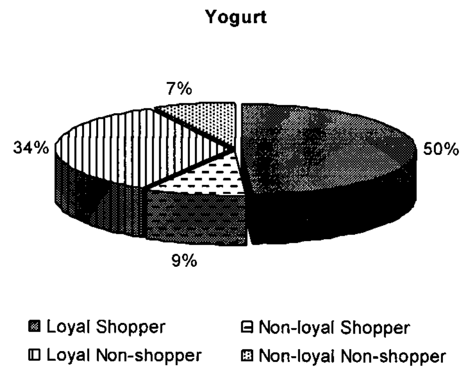
- *The difference of the time to the next purchase is larger for purchases on sale than for purchases not on sale.* That's because consumers buy a larger quantity when purchasing on sale to stay longer without purchasing.
- *The difference between the average quantity purchased on sale and out of sale is positive.* The intuition is that, if stockpiling, consumers buy a larger quantity when purchasing on sale.
- *The average time from the previous purchase is shorter for purchases on sale than for purchases not on sale.* In other words, even if the consumer does not have a current consumption need for the product, he still buys on sale to stockpile.
- *The probability the previous purchase was not on sale given that the current purchase was not on sale is higher.* The intuition is that since non-sale purchases have a lower inventory threshold, a

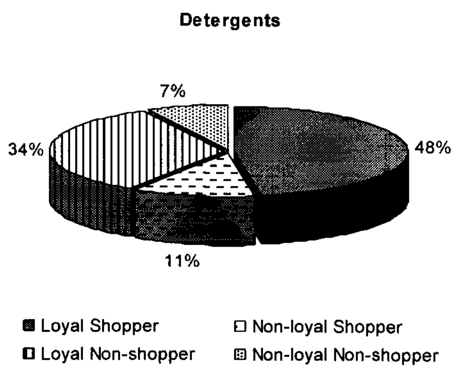
[14] Later I include a robust check on this definition of loyalty.

non-sale purchase informs us that inventories are low which in turn means, other things equal, that the last purchase was not on sale.

To check the robustness of this definition I also use, alternatively, the first characteristic together with each of the other three characteristics as the definition of a shopper. The results I present in the empirical sections are robust for these different definitions of a shopper. Note that the first and the last two characteristics could only arise from a stockpiling theory and not from an increased consumption theory. Finally non-shopper is a consumer who does not present any of the four characteristics stated above. Table 6 presents summary statistics of household's savings, proportion bought on sales and characteristics of deal patterns calculated per household, per brand, per store for both the loyal and non-loyal groups for soft-drinks, yogurt, and detergents. The distribution of households in my sample across the four types is summarized at the figures below for each category.







5 Results on Allocation of Purchases Over Time

I now turn to the implications of the model presented in section three. Those implications generate several testable hypotheses. I focus here on the soft-drinks category. The results for the two other categories, detergents and yogurts, are discussed separately in section seven.

Hypothesis 1. *(from Implication 2) Loyal consumers buy a higher fraction of their overall purchases during deals (save more in monetary terms) as uncertainty about deals' timing decreases. This effect is not significant for non-loyal consumers.*

Hypothesis 1 is derived from implication 2 of the model. The intuition is the following. When facing a deal period, consumers buy more in order to stockpile for future consumption. When facing a non-deal period, they may consume from inventory. The ability of stocking the right amount at the deal period to prevent purchase at the regular price depends, among other things, on how precise is the information on when the next deal is. For a given average duration, as the variation of the duration decreases, the information consumers have on when the next deal is becomes more precise. So the smaller the coefficient of variation of duration (the smaller the unpredictability), the more consumers are able to buy on deal periods. We should also expect this effect to be significant just for loyal consumers. Those are the consumers willing to perform inter-temporal substitution. Non-loyal consumers do not follow the deals of the brand they are not loyal to and as so, should not be affected by its timing.

I first estimate the effect of uncertainty about deals' timing on savings separately for loyal consumers and non-loyal consumers after controlling for other relevant characteristics of the deal pattern, such as frequency of deals (percentage of the total time the product was offered on deal), and average discount (average of all the percentage discounts the product was offered on). Under this specification I am implicitly assuming that all loyal (non-loyal) consumers respond to unpredictability in the same way. Then I include all consumers together in a single regression and add two new independent variables: the actual shares each consumer buys of each brand in the category and an interaction term between

unpredictability and shares. This second specification checks whether different degrees of loyalties imply different responses to unpredictability. In both specifications the identification comes from the variation of deals' timing for the same brand across stores and the variation of the observed deal pattern for the same brand at the same store across consumers.

Table 7 presents the results, for the soft-drinks category, of regressing savings on unpredictability, average discount, frequency of deals, total number of trips for each store and for each consumer and total number of units purchased for each brand, per consumer, per store. A unit of observation is the value of the respective variable per household, per brand, per store averaged across time. I also include brand and store dummies. The first column shows the results for the regressions just for loyal consumers. The main finding is that uncertainty about deals' timing significantly affects loyal consumers' savings. The coefficient of unpredictability is significant and negative. For a given average duration, as the variation of the duration increases the savings decrease. Loyal consumers buy a higher fraction of their overall purchases during deals (save more in monetary terms), as uncertainty about deals' timing decreases. This result is also robust to another measure of gains from concentrating purchases on deals, fraction. The second column shows the results for the regressions just for non-loyal consumers. The effect of unpredictability on savings/fraction is not significant. The third column shows the results for the regressions that include all the consumers together and add two new independent variables: shares and the interaction term between unpredictability and shares[15]. They support the claim that the results are robust to the definition of loyalty. Not only unpredictability affects the allocation of purchases over time, but this effect increases with an increase in the product's share of a given consumer's purchase in the category.

Implication 2 also states that the effect of uncertainty about deals' timing on loyal consumers' allocation of purchases over time increases if the consumer stockpiles (is a shopper).

Hypothesis 2. *The effects of deals' timing on allocation of purchases over time increases if the loyal consumer stockpiles.*

[15] The interaction term is defined as deviations from the averages namely:
[(coef. of variation of duration) – (average coef. of variation of duration)]*[share – (average share for loyal consumers)].

Table 8 presents the results, for the soft-drinks category, of regressing savings on unpredictability, average discount, frequency of deals, total number of trips and total number of units purchased per brand, per consumer, per store. I estimate the effect of unpredictability on savings separately for shopper consumers (first column), loyal shopper consumers (second column), and non-loyal shopper consumers (third column). The coefficient of unpredictability is significant and negative for the regressions including just loyal shopper consumers. The results are not significant for the regressions including just shoppers.

The data also includes two types of promotional activities: feature and display. The feature measures if the product was advertised by the retailer, in other words, if a flyer was sent to consumers that week. The display measures if the product was displayed differently than usual within the store that week. So, instead of looking directly at the price distribution, consumers could have used feature and/or display as an alternative source of information to learn about deals. This creates my third testable hypothesis:

Hypothesis 3. *Consumers use display and/or feature as alternative sources of information about deals. If so, uncertainty about feature's timing and/or display's timing affects the fraction of the overall purchases loyal consumers do during deals.*

Table 9 presents summary statistics, for the soft drinks category, on the characteristics of deal patterns using display and feature as the main source of information about deals, instead of prices.

I first estimate the effect of unpredictability on savings separately for loyal consumers and non-loyal consumers using display as the main source of information about deals, after controlling for other relevant characteristics of the deal pattern. The results were not significant[16]. Uncertainty about display's timing does not affect loyal consumers' fraction of their overall purchases during deals.

Next I estimate the effect of unpredictability on savings separately for loyal consumers and non-loyal consumers using feature as the main source of information about deals, after controlling for other

[16] Since the results were not significant I omitted the outcome of the regressions in the present work.

relevant characteristics of the deal pattern. Table 10 presents the results for these regressions. The results are very similar to the ones found using prices (table 7). Again, the coefficient of unpredictability is significant and negative. Uncertainty about features' timing affects loyal consumers' fraction of their overall purchases during deals. A possible explanation for these results is that consumers associate more feature with deal than display. So they use feature as a source of information about deals but not displays.

Finally I include both unpredictability of deals' timing and features' timing in the same specification. The results turn out to be individually insignificant due to multicollinearity, since feature and deals are very correlated. However, both coefficients are jointly significant.

6 Results on the Quantity Purchased at a Particular Deal

I now look at consumers decisions at each particular deal. Hypothesis 4 is derived from implication 1 of the model.

Hypothesis 4. *(Implication 1) Loyal shopper consumers increase their quantity purchased at a particular deal both as more time passes since the previous deal, and the higher the uncertainty about deals' timing.*

To test this hypothesis I use the first 26 weeks of purchases for each consumer as the learning period[17]. For this period I calculate the variation of the duration (standard deviation of the number of weeks between two consecutive deals) for each household. With the remaining weeks I regress log of quantity purchased per consumer, per visit, per brand, per store on log of price, the variation of the duration, number of weeks since the previous deal, promotional activities and an interaction term between number of weeks since the previous deal and variation of the duration. The identification comes from both the variation of the observed deal pattern for the same brand at the same store across consumers and from the variation of the quantity purchased at a deal period for the same consumer over time.

[17] I checked the robustness of the results to the definition of initial learning period by looking at different initial learning period's intervals. Qualitatively the results are robust to the different definitions examined.

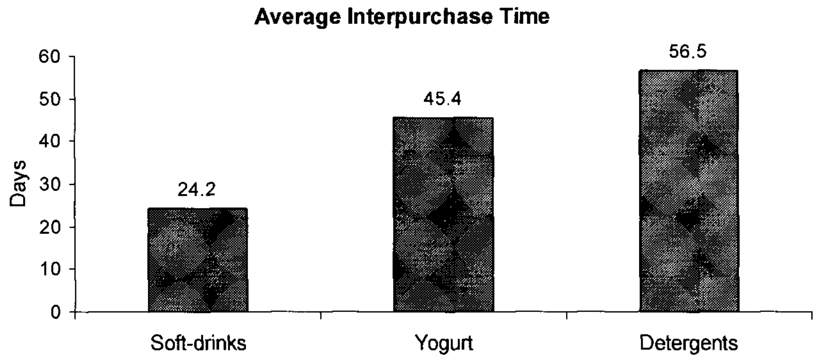
Table 11 presents the results for the soft-drinks category. The first column of table 11 presents the results for the loyal shopper group for purchases only at deal periods. I find that, as expected, the coefficient on number of weeks from previous deal is positive and significant and the coefficient on the interaction term is positive and significant. These results support my claim, not only the quantity purchased at a particular deal increases as more time passes since the previous deal, but also this effect is bigger as uncertainty about deals' timing increases. This result does not hold for non-loyal consumers. The effect of uncertainty about deals' timing on quantity purchased is also not significant for purchases at the off deal periods.

7 Cross-Category Comparison

I replicate the same regressions described in sections 5 and 6 for the two other categories: detergents and yogurts. Table 12 presents the results, for the yogurt category, on the effects of uncertainty about deals' timing on the allocation of purchases over time. The main finding is that uncertainty about deals' timing significantly affects loyal consumers' savings. This effect increases with an increase in the product's share of a given consumer's purchase in the category or if the consumer stockpiles (is a shopper). The results were not significant for detergents. I also replicated the regression using uncertainty about features' timing and displays' timing as alternative sources of information about deals. The results were not significant for both yogurts and detergents.

Finally I replicated the regressions for the effects of uncertainty about deals' timing on the quantity purchased at a particular deal. For detergents, I find that the quantity purchased at a particular deal increases as more time passes since the previous deal, but the effect of uncertainty about deals' timing on the quantity purchased was not significant. Table 13 presents the results for yogurts. The main finding is that loyal shopper consumers increase their quantity purchased on a particular deal both as more time passes since the previous deal and the higher the uncertainty about deals' timing.

An important difference between the three categories is the frequency of purchase. The figure below shows the average annual interpurchase time (in days) for households that make at least two purchases in the category during the year, between 1983 and 1997, using aggregate data from IRI's 1986 Marketing Factbook. The data are collected for more than 20,000 participating scanner panel households living in 12 different domestic U.S. markets.



Detergents are the least frequently purchased of the three, followed by yogurts and soft-drinks. Note that the figure above shows the average annual interpurchase time for the entire category. Given that I use data just for 6oz. yogurts and 128 oz. detergents, I expect the difference between the specific UPCs I use for detergents and yogurts to be even bigger than the one described in the figure. Therefore, uncertainty about deals' timing just affects consumers purchase' decisions if the product is frequently purchased. One possible explanation is that when consumers don't buy the product frequently enough, they are not able to learn about the deals' timing.

8 Managerial Implications

The previous empirical findings have important pricing implications. For instance, suppose retailers adopt a constant mark-up strategy and pass-through all the deals offered by a manufacturer of a particular brand to the final consumer. The manufacturer/retailer might have a hard time selling its products at the regular price if a large fraction of consumers behaves strategically, trying to match their purchases with deals, and deals are too predictable. One implication is that unpredictability can be used as mechanism to avoid the

most attractive type of consumer, the one that would be willing to purchase otherwise at the regular price, to concentrate all his purchases on deals.

Therefore, manufacturers and retailers should incorporate the effects of deals' timing on consumers' purchase' decisions when determining optimal pricing strategies. I am not trying to argue here whether they already incorporate these effects or not. This is actually an interesting research question. My point is that to generate normative pricing strategies one need to account for the effect of uncertainty about deals' timing on consumers purchase decisions. I show here how to find the optimal pricing strategy for the case of a monopolist manufacturer. Next, I discuss situations where managers can use unpredictability of deals in their favor.

To keep things as easy as possible, suppose a monopolist manufacturer is able either to sell directly to the final consumer or to dictate prices to the retailer so that I can ignore the vertical relationship between the manufacturer and the retailer. In reality we know that retailers adopt more complex mark-up strategies and there is not a monopolist brand but an oligopoly of brands in a single product category. Ideally a complete pricing model should include the vertical relationship between manufacturer and retailers and a game-theoretic framework to account for the competitive strategies among the different brands. However this is beyond the scope of the present work.

8.1 Monopolist Model and Algorithm

Suppose a monopolist manufacturer faces two different types of final consumers, $h=\{1,2\}$. A fraction Θ of the manufacturer's potential consumers are type 1. Define by K the marginal cost of production that I assume to be constant over time and δ the discount rate. Given the optimal regular price, the seller must determine the optimal discount size and the optimal deal pattern where p_H is the regular price and $p_L=\lambda p_H$ is the deal price with $\lambda\in[0,1]$. To find the optimal price distribution the monopolist first decides if it is optimal to have a predictable or unpredictable deal pattern. If a predictable deal pattern is optimal, the monopolist should also choose N, the optimal number of weeks between two consecutive deals. If an

unpredictable deal pattern is optimal, the monopolist decides on the probabilities of transition between regular price and deal price[18]. In any case the monopolist profit maximization problem can be described as:

$$\max_{[(\pi_{L,H},\pi_{H,L};N],p_L,p_H} \sum_{t=0}^{\infty} \delta^t \{[\Theta q^*_{1t}(p_t) + (1-\Theta) q^*_{2t}(p_t)](p_t - K)\} \tag{11}$$

s.t. q^*_{1t} and q^*_{2t} are the arguments that maximizes consumer problem.

In order to show under what conditions offering deals is a better strategy than charging a unique constant price I need to derive the optimal constant price. In a situation of constant price, $p_t=p_c$, the dynamic consumer maximization problem described in (4) simplifies to a static one with $i^*_{ht}=i^*_{h,t-1}=0$ and $c_{ht}=q_{ht}$. Assuming the shocks to utility are i.i.d. across each type of consumers, the optimal constant price can be described as[19]:

$$p_c = p^* = \sqrt{AK} \tag{12}$$

where

$$A = \Theta \frac{\beta_1}{\gamma_1} + (1-\Theta)\frac{\beta_2}{\gamma_2} \tag{13}$$

I therefore propose the following five-step procedure to examine whether offering deals is a better strategy than charging a unique price. The first step consists on choosing/estimating parameter specifications for the consumer and firm behavior as described in equations (4) and (11). In the second step, given these parameter specifications, I find the optimal constant price, as described by equations

18 This is a simplification of the monopolist real decision. The monopolist actually decides on the average and standard deviation of the number of weeks between two consecutive deals. These two measures can be mapped, not one to one, into a transition probability.

19 The derivation is provided in the Appendix.

(12) and (13), and the correspondent profit. In the third step I suppose the monopolist finds it optimal to offer an unpredictable deal pattern and I use the algorithm described in the appendix to solve for the optimal pricing strategies, given the parameter specification. This algorithm involves finding the fixed point to the consumer maximization problem and searching for the combination of transition probabilities, $(\pi_{L,L}, \pi_{H,H})$ and prices, (p_L, p_H) that achieves the highest profit. In the fourth step I suppose the monopolist finds it optimal to offer a predictable deal pattern. Given the description in Proposition 1 of the optimal consumer behavior for the predictable (benchmark) case, I derive the optimal prices and length of the cycle, (p_L, p_H, N), as described at the appendix, for the respective parameter specification. Finally I compare the profits achieved with 1) optimal constant price, 2) the optimal combination of $(\pi_{L,L}, \pi_{H,H})$ and (p_L, p_H) in the case of an unpredictable deal pattern and 3) the optimal combination (p_L, p_H, N) in the case of a predictable deal pattern and conclude on the optimal strategy.

8.2 Discussion

In the case all consumers do not stockpile or have a very high storage cost the monopolist's optimal strategy is to charge a constant price. In the alternative case the demand is composed of two types of consumers – loyal non-shopper with small price sensitivity and non-loyal shopper with very high price sensitivity – and the second group is the largest one, the monopolist find it optimal to offer predictable deals. The intuition is first that offering deals is better strategy than offering a constant price because the less attractive group is large enough so that it is interesting for the seller to target them too instead of selling just to the loyal group. Second, predictable deal pattern achieves a higher profit than unpredictable deal pattern because the non-loyal group is too price sensitive and consequently unpredictability does not increase the amount they purchase at a particular deal in comparison to the predictable case.

An interesting venue of future research is to investigate, using the algorithm described above, situations where unpredictable deal pattern is the optimal strategy. I could not find any situation where unpredictability achieves a higher profit, beyond gains from discounting, in comparison with the predictable case. Ways to extend the model in order to find unpredictability as the optimal outcome are

the following. Unpredictable deals can be used as a source of increasing overall consumption of the brand. For some product categories, like bacon, salted snacks and soft drinks, there is an additional consumption per period induced by the presence of additional inventory on hand, as reported by recent studies (Ailawadi and Neslin (1998), Bell et al (1999) and Sun (2005)). And since I found that the less predictable the deal pattern is, the higher the quantity purchased at a particular deal, this extra inventory might trigger higher consumption rates. I should also include competition and the possibility of losing a big number of sales at the deal periods due to the fact that the competitor also knows when the deal is coming (if predictable) and offers it before, so that consumers store from the competitive brand.

9 Conclusions

I showed that loyal shopper consumers' decisions, both about the allocation of their purchases over time, and about the quantity purchased during a particular deal, are affected by the product's deal pattern. And, unlike previous studies, I support this claim with scanner data.

I develop a dynamic model of consumer choice where consumers are forward looking and buy in advance, at lower prices, to stock for future consumption. In the model, consumers form beliefs about future prices only when they are uncertain about deals' timing. In the case of a predictable deal pattern, consumers know the entire price distribution. This model generates implications for purchase behavior that I test empirically.

I use scanner data on soft drinks, laundry detergents and yogurts. For soft-drinks and yogurts I find several pieces of evidence consistent with the model. (1) The more predictable the deal pattern, the higher the fraction of the overall purchase that loyal consumers buy during deals. (2) This effect increases with an increase in the product's share of the consumer's purchase in the category. (3) This effect also increases if the loyal consumer stockpiles. (4) The effect is not significant for non-loyal consumers. (5) The more time that has passed since the previous deal and the less predictable the deal pattern, the more loyal shopper consumers increase the quantity they purchase during a particular deal.

The results are not significant for laundry detergents. An important difference between the three categories is the frequency of purchase. Detergents are the least frequently purchased of the three, followed by yogurt and soft-drinks. Therefore, uncertainty about deals' timing just affects consumers purchase decisions if the product is frequently purchased. One possible explanation is that when consumers don't buy the product frequently enough, they are not able to learn about the deals' timing. An interesting extension would be to replicate the results for more product categories and understand the product characteristics for which uncertainty about deals' timing affects consumer behavior.

The findings suggest that it is crucial for both manufacturers and retailers to incorporate the effects of deal patterns on consumer purchases' decisions when deriving optimal pricing strategies. In the present work, I assume that prices and deals' timing are exogenously given. An interesting extension would be to determine manufacturers and retailers optimal pricing strategies incorporating the effects of deals' timing on consumer behavior. I show here how to find the optimal pricing strategy for the case of a monopolist manufacturer. I find situations where the monopolist manufacturer can achieve a higher profit from offering predictable deals instead of constant price but cannot find any situation where unpredictability achieves a higher profit, beyond gains from discounting, in comparison with the predictable case. One needs to extend the model presented here to find unpredictability as the best outcome. There are many different ways the model can be extended: including competition, looking at the vertical relationship between manufacturer and retailer, allowing for more than two prices, and exploring the possibility of asymmetric information between consumers and sellers. One could also adopt a different utility function.

I also assumed that, for the case of unpredictable deal patterns, consumers form their expectations during an initial learning period and that there is no further learning after that. An interesting extension would be to allow consumers to Bayesian update their beliefs about future prices.

An important next step to be pursued is to structurally estimate the parameters of the dynamic inventory model of consumer choice with scanner data and generate normative pricing implications. This way I can

compare my results with the real strategies adopted by the firms. I could also use this model to investigate the welfare implications of some countries' restrictions on the use of promotions in retail markets.

Appendix

Proof of Proposition 1: Assume no shock to utility, $v_t=1$. If consumers purchase just for immediate consumption (no stockpiling) the dynamic problem described in (4) simplifies to a static one. From the F.O.C. of the static maximization problem, the optimal quantity to purchase and consume is[20]:

$$c^* = q^* = \frac{\beta}{\gamma p_j} - 1 \tag{A1}$$

Assume the first deal price takes place at $t=1$. At this period consumer purchases for immediate consumption and to stockpile. The amount he purchases for immediate consumption is given by equation (A1) with $p_j=p_L$. Since consumer knows the number of weeks between two consecutive deals, N, the amount he purchases to stockpile is given by the following general decision rule: *"At the deal period, purchase in advance for consumption at the subsequent n periods, where n is an integer, and* $n \in [1,N+1]$. *If* $n=1$, *it is not worth purchasing in advance for future consumption. So* $q^*_1=c^*_1$. *If* $n>1$, *purchase at the first period the sum of the consumptions of the subsequent n periods, i.e.,* $q_1^* = \sum_{j=1}^{n} c_j^*$. *In this case, there is no need for purchase at the later n periods, i.e,* $q^*_2=\ldots=q^*_n=0$."

To determine the optimal consumption at those n periods, and the optimal n, I define by υ_k the virtual price (equation 10). It is the unit cost of purchasing at the first period low price, p_L, plus the cost of carrying (storage cost + discount factor) the inventory up to period k of consumption, where $k=2,\ldots,n$. Consumers compare the virtual price, υ_k, with the cost of purchasing the same unit at period k regular price, p_H, when deciding on the amount to be purchased for storage at the first period deal price. For all periods such that $\upsilon_k<p_H$ it is optimal to purchase in advance at the first period promotional price.

[20] In the following derivations I omitted the index for consumer's type, h, to simplify the notation.

Therefore n is the last period from the N periods of regular price for which $\upsilon_k < p_H$ (equation 9). If $\upsilon_k > p_H$ for all k then $n=1$. For these n periods the optimal consumption is given by equation (A1) with $p_j = \upsilon_j$.

For the periods $j=n+1,\dots,N+1$, inventory is zero, since n is the last period of zero purchase and consumption from inventory. Purchases are only for immediate consumption and the optimal amount to consume and purchase is given by equation (A1) with $p_j = p_H$.□

Proposition 2: Generalization of Proposition 1 including shocks to utility. The optimal quantities to purchase and consume for the $N+1$ periods of the cycle can be described as:

i) For $t=1$ where $p_t = p_L$:

$$q^*_1 = \sum_{j=1}^{n} c^e_j \tag{A2}$$

$$c^*_1 = c^e_1 = \frac{\beta}{\gamma p_L} - v_1 \tag{A3}$$

$$c^e_j = \frac{\beta}{\gamma \vartheta_j} - 1 \quad for \quad j = 2,\dots,n \tag{A4}$$

$$n = max\, k \big| \vartheta_k < p_H, k \in IN \tag{A5}$$

$$\vartheta_k = \frac{p_L + \theta \sum_{j=0}^{k-2} \delta^j}{\delta^{k-1}} \quad for \quad k = 2,\dots,n \tag{A6}$$

ii) For $t=2,\dots,N+1$ where $p_t = p_H$:

$$c^v_j = \frac{\beta}{\gamma \vartheta_j} - v_j \tag{A7}$$

$$c^H_j = \frac{\beta}{\gamma p_H} - v_j \tag{A8}$$

For j=2,...,n:

$$\begin{cases} if \quad i_{j-1} \geq c_j^v \Rightarrow q_j^* = 0, c_j^* = c_j^v \\ if \quad i_{j-1} < c_j^v \Rightarrow q_j^* = c_j^H - i_{j-1}, c_j^* = c_j^H \end{cases} \tag{A9}$$

For $j=n+1,...,N+1$:

$$\begin{cases} if \quad i_{j-1} \geq c_j^H \Rightarrow q_j^* = 0, c_j^* = c_j^H \\ if \quad i_{j-1} < c_j^H \Rightarrow q_j^* = c_j^H - i_{j-1}, c_j^* = c_j^H \end{cases} \tag{A10}$$

Proof of Proposition 2: Consider the case of purchase for immediate consumption. From the F.O.C. of the static maximization problem, the optimal quantity to purchase and consume is:

$$c^* = q^* = \frac{\beta}{\gamma p_j} - v_j \tag{A11}$$

where $v_j \in \{0,1,2\}$.

Assume the first deal price takes place at $t=1$. At this period consumer purchases for immediate consumption and to stockpile. The amount he purchases for immediate consumption is given by equation (A11) with $p_j=p_L$. Since consumer knows the number of weeks between two consecutive deals, N, but does not know a priori the realization of the future shocks to utility, the amount he purchases to stockpile for future consumption is given by the new decision rule: "*At the deal period, purchase in advance for consumption at the subsequent n periods, where n is an integer and n* $\in [1,N+1]$. *If* $n=1$, *it is not worth purchasing in advance for future consumption. So* $q^*_1=c^*_1$. *If* $n>1$, *purchase at the first period the sum of the expected consumptions of the subsequent n periods, i.e.,* $q_1^* = \sum_{j=1}^{n} c_j^e$. *Since this purchase is based on the expected consumption, consumers might need to purchase during these n periods. Whether consumers need to purchase or not depends on the realizations of the shocks to utility.*"

To determine the optimal consumption at those n periods, the expected consumption, and the optimal n, I define by υ_k the virtual price. This is the same definition as before. For all periods such that

$\upsilon_k < p_H$ it is optimal to purchase in advance at the first period promotional price. Therefore n is the last period from the N periods of regular price for which $\upsilon_k < p_H$. At $t=1$, the expected future consumption is described by equation (A4). It follows from the assumption that $v_t \in \{0,1,2\}$ with equal probabilities and, consequently, $E_1(v_t)=1$ for $\forall t$. For these n periods the optimal consumption is given by equation (A11) with $p_j = \upsilon_k$, or $p_j = p_H$. Since for these n periods $\upsilon_k < p_H$, it follows that $c^v > c^H$. Consumers consider the virtual price as the price available whenever they are able to fulfill their higher consumption needs, c^v, with inventory. In this case there is no need for purchase at the period and consumption is according to c^v. Consumers consider the regular price as the price available whenever they are not able to fulfill their higher consumption needs, c^v, with inventory. In this case, consumption is according to c^H and the total amount to be purchased, if necessary, is given by the difference between c^H and the inventory left from the previous period, i_{j-1}.

For the periods $j=n+1,\dots,N+1$, $\upsilon_k > p_H$. Consumers consider p_H as the available price and consumption is according to c^H. The total amount to be purchased, if necessary, is the difference between c^H and the inventory left from the previous period, i_{j-1}. □

Proof of Implication 1: Consider a deal period, t, and a consumer whose last purchase on deal was in period t-j. Consumers only purchase positive amounts in off deal periods for immediate consumption. As j grows, inventory declines, since at non-deal periods consumers may also consume from inventory. Therefore, the expected amount to be purchased on the deal period, t, conditional on having purchased at t-j, increases in j. In other words, the higher the number of weeks from the previous deal, the higher the amount purchased at the deal period t.[21]

The quantity purchased on a deal period, t, also increases in σ^2, the variance of the price distribution. This claim holds for the case where the average number of weeks between two consecutive deals is not too large, such that the long-term stationary probability of the low price state is big enough,

[21] That result is also showed in Hendel and Nevo (2006b).

$\Pi_L \in [1/2, 1]$. The stationary probability vector, Π, is defined as the vector whose elements can be computed by taking the limit:

$$\lim_{k \to \infty} \pi_{i,j}^{k} = \Pi_j \tag{A12}$$

This vector is the eigenvector of the probability matrix, associated with eingenvalue 1. Using the probability matrix described in (5) and this fact that $\pi\Pi = \pi$:

$$(\Pi_L \quad \Pi_H)\begin{pmatrix} \pi_{L,L} - 1 & \pi_{L,H} \\ \pi_{H,L} & \pi_{H,H} - 1 \end{pmatrix} = 0 \tag{A13}$$

And using the fact that $\Pi_L + \Pi_H = 1$ I find that:

$$\Pi_L = \frac{\pi_{H,L}}{\pi_{H,L} + \pi_{L,H}} \tag{A14}$$

$$\Pi_H = \frac{\pi_{L,H}}{\pi_{H,L} + \pi_{L,H}} \tag{A15}$$

From (A14) and (A15) I calculate the average and variance of the price distribution:

$$\mu = p_L \Pi_L + p_H \Pi_H \tag{A16}$$

$$\sigma^2 = \Pi_L p_L^2 + \Pi_H p_H^2 - \Pi_L^2 p_L^2 - \Pi_H^2 p_H^2 - 2 p_L p_H \Pi_L \Pi_H \tag{A17}$$

As the variance increases, and for $\Pi_L \in [1/2, 1]$,both the probability that there is a deal tomorrow given that there was a deal today decreases, $\partial\pi_{L,L}/\partial\sigma^2<0$, and the probability that there is a regular price tomorrow given that there was a regular price today increases, $\partial\pi_{H,H}/\partial\sigma^2>0$ for. To see this I use the chain rule and I substitute at (A17) the fact that $\Pi_H=1- \Pi_L$:

$$\frac{\partial \sigma^2}{\partial \pi_{j,j}} = \frac{\partial \sigma^2}{\partial \Pi_j} \frac{\partial \Pi_j}{\partial \pi_{j,j}} \tag{A18}$$

for $j=L,H$.

$$\frac{\partial \Pi_j}{\partial \pi_{j,k}} = -\frac{\pi_{k,j}}{(\pi_{k,j}+\pi_{j,k})^2} < 0 \Rightarrow \frac{\partial \Pi_j}{\partial \pi_{j,j}} > 0 \tag{A19}$$

for $k=H$ if $j=L$ and $k=L$ if $j=H$.

$$\frac{\partial \sigma^2}{\partial \Pi_j} = (1-2\pi_j)(p_H - p_L)^2 \tag{A20}$$

For $\Pi_L \in [0,1/2)$ and from equation (A20) I conclude that $\partial\sigma^2/\partial\Pi_L>0$ and $\partial\sigma^2/\partial\Pi_H<0$. For $\Pi_L \in [1/2,1]$ I conclude that $\partial\sigma^2/\partial\Pi_L \leq 0$ and $\partial\sigma^2/\partial\Pi_H \geq 0$. Therefore, $\partial\pi_{L,L}/\partial\sigma^2<0$ and $\partial\pi_{H,H}/\partial\sigma^2>0$ for $\Pi_L \in [1/2,1]$.

The rest of the implication is a consequence of that fact that as $\pi_{L,L}$ decreases and $\pi_{H,H}$ increases, the quantity purchased at a particular deal increases for loyal shopper consumers. The other types of consumers may not necessarily increase the quantity purchased at a particular deal period given that the total amount they are willing to purchase to store for future consumption is restricted by either a high storage cost and/or by a low marginal utility of consumption the good (not willing to perform inter-temporal substitution).□

Derivation of the Optimal Constant Price: In a situation of constant price, $p_t=p_c$ for $\forall t$, the dynamic consumer maximization problem described in (4) simplifies to a static one. The solution to this problem is described in equation (A11). Since the shocks to utility are i.i.d. across each type h of consumers, the monopolist maximization problem can be described as:

$\Pi_L \in [1/2,\ 1]$. The stationary probability vector, Π, is defined as the vector whose elements can be computed by taking the limit:

$$\lim_{k\to\infty} \pi_{i,j}^{k} = \Pi_j \tag{A12}$$

This vector is the eigenvector of the probability matrix, associated with eingenvalue 1. Using the probability matrix described in (5) and this fact that $\pi\Pi = \pi$:

$$\begin{pmatrix}\Pi_L & \Pi_H\end{pmatrix}\begin{pmatrix}\pi_{L,L}-1 & \pi_{L,H}\\ \pi_{H,L} & \pi_{H,H}-1\end{pmatrix} = 0 \tag{A13}$$

And using the fact that $\Pi_L + \Pi_H = 1$ I find that:

$$\Pi_L = \frac{\pi_{H,L}}{\pi_{H,L}+\pi_{L,H}} \tag{A14}$$

$$\Pi_H = \frac{\pi_{L,H}}{\pi_{H,L}+\pi_{L,H}} \tag{A15}$$

From (A14) and (A15) I calculate the average and variance of the price distribution:

$$\mu = p_L\Pi_L + p_H\Pi_H \tag{A16}$$

$$\sigma^2 = \Pi_L p_L^2 + \Pi_H p_H^2 - \Pi_L^2 p_L^2 - \Pi_H^2 p_H^2 - 2p_L p_H \Pi_L \Pi_H \tag{A17}$$

As the variance increases, and for $\Pi_L \in [1/2,\ 1]$,both the probability that there is a deal tomorrow given that there was a deal today decreases, $\partial\pi_{L,L}/\partial\sigma^2<0$, and the probability that there is a regular price tomorrow given that there was a regular price today increases, $\partial\pi_{H,H}/\partial\sigma^2>0$ for. To see this I use the chain rule and I substitute at (A17) the fact that $\Pi_H=1-\Pi_L$:

$$\frac{\partial \sigma^2}{\partial \pi_{j,j}} = \frac{\partial \sigma^2}{\partial \Pi_j} \frac{\partial \Pi_j}{\partial \pi_{j,j}} \qquad \text{(A18)}$$

for $j=L,H$.

$$\frac{\partial \Pi_j}{\partial \pi_{j,k}} = -\frac{\pi_{k,j}}{(\pi_{k,j} + \pi_{j,k})^2} < 0 \Rightarrow \frac{\partial \Pi_j}{\partial \pi_{j,j}} > 0 \qquad \text{(A19)}$$

for $k=H$ if $j=L$ and $k=L$ if $j=H$.

$$\frac{\partial \sigma^2}{\partial \Pi_j} = (1 - 2\pi_j)(p_H - p_L)^2 \qquad \text{(A20)}$$

For $\Pi_L \in [0,1/2)$ and from equation (A20) I conclude that $\partial\sigma^2/\partial\Pi_L > 0$ and $\partial\sigma^2/\partial\Pi_H < 0$. For $\Pi_L \in [1/2,1]$ I conclude that $\partial\sigma^2/\partial\Pi_L \leq 0$ and $\partial\sigma^2/\partial\Pi_H \geq 0$. Therefore, $\partial\pi_{L,L}/\partial\sigma^2 < 0$ and $\partial\pi_{H,H}/\partial\sigma^2 > 0$ for $\Pi_L \in [1/2,1]$.

The rest of the implication is a consequence of that fact that as $\pi_{L,L}$ decreases and $\pi_{H,H}$ increases, the quantity purchased at a particular deal increases for loyal shopper consumers. The other types of consumers may not necessarily increase the quantity purchased at a particular deal period given that the total amount they are willing to purchase to store for future consumption is restricted by either a high storage cost and/or by a low marginal utility of consumption the good (not willing to perform inter-temporal substitution).□

Derivation of the Optimal Constant Price: In a situation of constant price, $p_t = p_c$ for $\forall t$, the dynamic consumer maximization problem described in (4) simplifies to a static one. The solution to this problem is described in equation (A11). Since the shocks to utility are i.i.d. across each type h of consumers, the monopolist maximization problem can be described as:

$$\max_{p_c}\left[\Theta\left(\frac{\beta_1}{\gamma_1 p_c}-1\right)+(1-\Theta)\left(\frac{\beta_2}{\gamma_2 p_c}-1\right)\right](p_c-K) \tag{A21}$$

From the F.O.C. of the above maximization problem I find the optimal constant price as described in equations (12) and (13). □

Algorithm to Determine the Optimal Combination ($\pi_{L,L}$, $\pi_{H,H}$) and (p_H, p_L) in the Case of Unpredictable Deal Pattern: Suppose the monopolist finds it optimal to offer an unpredictable deal pattern. The algorithm used to solve for the optimal pricing strategies is the following. First, I find the policy functions for consumption and quantity to be purchased for any given prices and transition probabilities[22]. From the contraction mapping theorem it is easy to prove that the consumer problem described in (4) has a unique fixed point. To solve for the policy functions I perform a grid search over a finite set of possible values of consumption and quantity to be purchased. I consider that consumers decide on integer quantities to purchase and consume. I also discretize the state space.

Define by q^*_f, the floor of the optimal unconstrained quantity, q^*, and by q^*_c, the ceiling of the optimal unconstrained quantity. By construction q^*_c-q^*_f=1. Therefore, for a given constant price, the optimal integer quantity, $q^*int(v)$, consumers choose to purchase and consume at period t is:

$$q_t^*\ int(v)=\max\left[U_t(q_{ct}^*),U_t(q_{ft}^*)\middle|v_t=\{-1;0;1\}\right] \quad \forall t \tag{A22}$$

where $U_t(.)$ stands for the static utility for period t.

The monopolist compares, for each type h of consumers, the prices that make consumers indifferent between the ceiling and the floor quantities. Using the fact that $q^*_c - q^*_f = 1$, after some trivial calculation I find that:

[22] I assume no shock to utility, v_t=1, in my derivations.

$$p\,int(v) = \frac{\beta}{\gamma} log\left(1 + \frac{1}{q_c^* + v} \right) \tag{A23}$$

Those are all the possible candidates for optimal constant price accounting for integer restrictions. The optimal constant price is an ε smaller than the price that among all these candidates maximizes profit. These prices are also the candidates for deal price and regular price in the case of unpredictable deal pattern. Given that prices must be above the marginal cost[23] or above the price that set demand equal to zero[24], denoted by p_{max}, all the possible combinations of regular price and deal price are in the intervals $p_H \in [p_c, p_{max}]$ and $p_L \in [K, p_c]$ where p_c stands for optimal constant price. Given this finite set of prices' candidates and also considering a finite range of possible values for the transition probabilities, I find the optimal combination of (s_1,s_2) and (p_L,p_H) that maximizes profit.

Derivation of the Optimal Pricing Strategy in the Case of Predictable Deal Pattern: Suppose the monopolist find it optimal to offer a predictable deal pattern. Assume no shock to utility, $v_t=1$. Define by n_h, the last period consumer type h consumes from inventory and suppose $n_1>n_2$[25]. The monopolist problem can be described as:

$$\max_{p_L, p_H, N} \left[\Theta q_{1,1} + (1-\Theta) q_{2,1} + \sum_{t=n_2+1}^{n_1} \delta^t (1-\Theta) q_{2,t} + \sum_{t=n_1+1}^{N} \delta^t \left(\Theta q_{1,t} + (1-\Theta) q_{2,t} \right) \right] (p_H - K)\ s.t. \tag{A24}$$

$q_{1,1}$, $q_{2,1}$, $q_{1,t}$, $q_{2,t}$, n_1, n_2 are the arguments that maximizes the consumer problem.

Consumers decide on integer quantities to purchase and consume. Define by c^*_f, the floor of the optimal unconstrained consumption, c^*, and by c^*_c, the ceiling of the optimal unconstrained consumption. The optimal integer consumption, c^*int, can be described as:

23 Or the price that makes the more price sensitive consumer purchases the maximum amount he can purchase per visit, given that this price is higher than the marginal cost.
24 That is the price that makes the less price sensitive type of consumer indifferent between purchasing or not the good.
25 If the inverse is true, $n_2>n_1$, the same logic applies with the proper modifications in equation (A27).

$$c_t^* \; int = max[U_t(c_{ct}^*), U_t(c_{ft}^*)]$$

(A25)

The possible candidates for p_L are all the prices that at each period make consumers indifferent between consuming one extra unit or not. Since I don't know ex ante n_h, and given that the choice of p_L affects n_h, I start with the assumption that $n=N$.

After the period $t=min\{n_1, n_2\}$, some consumers might start to purchase at the regular price. The possible candidates for p_H are all the prices that at each period make consumers indifferent between consuming one extra unit or not. I then form a finite set of possible candidates for (p_L,p_H) and calculate the corresponding ns. With all the (p_L,p_H,n) possible combinations I finally calculate the optimal quantities to be purchased and consumed and obtain the corresponding profits. The optimal (p_L,p_H,n) is the one that achieves the highest profit.

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Table 1

Percent Market Share for each UPC at the Selected Markets

Selected 2 Liter Bottle Cola Market

Pepsi	28.5
Coke Classic	15.8
Diet Coke	15.8
Diet Pepsi	14.7
Caffeine Free Diet Pepsi	9.2
Caffeine Free Diet Coke	6.8
Caffeine Free Pepsi	6.0
Caffeine Free Coke	3.2

Selected 6 oz. Yogurt Market

Yoplait Custard LMN	17.0
Dannon Blended STB	13.5
Yoplait STB	13.5
Yoplait VAN	12.3
Dannon Blended PCH	12.3
Dannon Blended RSB	11.7
Dannon Blended BUB	10.8
Yoplait Custard BNA	8.9

Selected 128 oz. Liquid Laundry Detergent Market

All Regular	37.4
Wisk Regular	29.1
All Free N Clear USC	17.9
Wisk Power Plus Regular	10.2
Wisk USC	3.1
Wisk Power PlusUSC	2.3

Table 2

A) Summary Statistics on Percent Discounts Off the Regular Price

Calculated per UPC, per Store

	Average for All Stores	Std per Store, Averaged Across Stores	Minimum for All Stores	Maximum for All Stores
Soft-drinks				
Coke Classic	26.0	11.6	5.0	61.5
Caffeine Free Coke	25.6	11.5	5.0	61.5
Diet Coke	26.4	11.4	5.0	61.5
Caffeine Free Diet Coke	26.3	11.4	5.0	61.5
Pepsi	23.7	11.3	5.0	61.5
Caffeine Free Pepsi	23.6	11.3	5.8	61.5
Diet Pepsi	23.7	11.2	5.0	61.5
Caffeine Free Diet Pepsi	23.8	11.1	5.0	61.5
Yogurt				
Yoplait Custard LMN	20.2	9.0	6.6	55.4
Yoplait STB	27.4	11.1	6.3	55.4
Yoplait VAN	27.4	11.1	6.3	55.4
Yoplait Custard BNA	27.0	10.0	6.3	55.4
Dannon Blended STB	28.0	13.0	5.9	57.6
Dannon Blended PCH	28.6	13.4	5.5	57.6
Dannon Blended RSB	26.7	10.7	5.5	57.6
Dannon Blended BUB	27.9	12.9	5.9	59.8
Detergents				
All Regular	18.8	10.4	5.0	51.7
All Free N Clear USC	19.8	10.9	5.0	51.7
Wisk Regular	21.1	9.1	5.1	45.0
Wisk USC	24.6	6.8	5.1	45.0

Table 2

B) Standard Deviation, Across Stores, of the Average Percent Discount Off the Regular Price

Averages Calculated per UPC, per Store, over 104 weeks

Soft-drinks		Yogurt		Detergents	
Coke Classic	5.0	Yoplait Custard LMN	5.0	All Regular	6.0
Caffeine Free Coke	5.1	Yoplait STB	11.2	All Free N Clear USC	6.1
Diet Coke	4.9	Yoplait VAN	11.2	Wisk Regular	4.0
Caffeine Free Diet Coke	4.9	Yoplait Custard BNA	12.6	Wisk USC	7.0
Pepsi	6.6	Dannon Blended STB	12.9		
Caffeine Free Pepsi	6.4	Dannon Blended PCH	11.0		
Diet Pepsi	6.7	Dannon Blended RSB	13.8		
Caffeine Free Diet Pepsi	6.6	Dannon Blended BUB	12.6		

Table 3

Summary Statistics on the Percent of Weeks each UPC was Offered on Deal

Calculated Per UPC, Per Store

Soft-drinks			Yogurt			Detergents		
	Average for All Stores	Std Across Stores		Average for All Stores	Std Across Stores		Average for All Stores	Std Across Stores
Coke Classic	52.4	14.6	Yoplait Custard LMN	20.5	4.0	All Regular	22.1	15.2
Caffeine Free Coke	52.1	14.0	Yoplait STB	15.6	8.8	All Free N Clear USC	16.2	12.7
Diet Coke	51.7	15.6	Yoplait VAN	15.6	8.8	Wisk Regular	20.1	16.2
Caffeine Free Diet Coke	52.2	15.3	Yoplait Custard BNA	14.1	8.8	Wisk USC	19.0	15.5
Pepsi	54.1	13.5	Dannon Blended STB	24.1	16.4			
Caffeine Free Pepsi	54.1	13.5	Dannon Blended PCH	29.1	22.6			
Diet Pepsi	54.8	14.2	Dannon Blended RSB	23.6	15.9			
Caffeine Free Diet Pepsi	54.1	15.0	Dannon Blended BUB	23.2	13.4			

Table 4

A) Summary Statistics on the Number of Weeks between Two Consecutive Deals

Calculated per UPC, per Store

Soft-drinks				Yogurt				Detergents			
	Average	Min	Max		Average	Min	Max		Average	Min	Max
Coke Classic	2.0	1	17	Yoplait Custard LMN	4.6	1	48	All Regular	9.3	1	52
Caffeine Free Coke	2.0	1	17	Yoplait STB	9.9	1	42	All Free N Clear USC	6.3	1	19
Diet Coke	2.1	1	17	Yoplait VAN	6.7	1	42	Wisk Regular	8.0	1	20
Caffeine Free Diet Coke	2.0	1	17	Yoplait Custard BNA	4.3	1	32	Wisk USC	8.1	1	20
Pepsi	1.9	1	13	Dannon Blended STB	5.3	1	27				
Caffeine Free Pepsi	1.9	1	13	Dannon Blended PCH	6.6	1	40				
Diet Pepsi	1.9	1	13	Dannon Blended RSB	6.1	1	28				
Caffeine Free Diet Pepsi	1.9	1	13	Dannon Blended BUB	5.6	1	27				

Table 4

B) Standard Deviation, Across Stores, of the Average Number of Weeks between Two Consecutive Deals

Averages Calculated per UPC, per Store, over 104 weeks

Soft-drinks		Yogurt		Detergents	
Coke Classic	0.5	Yoplait Custard LMN	1.3	All Regular	8.7
Caffeine Free Coke	0.5	Yoplait STB	7.6	All Free N Clear USC	3.4
Diet Coke	0.6	Yoplait VAN	3.6	Wisk Regular	6.4
Caffeine Free Diet Coke	0.6	Yoplait Custard BNA	0.8	Wisk USC	6.6
Pepsi	0.4	Dannon Blended STB	3.3		
Caffeine Free Pepsi	0.4	Dannon Blended PCH	4.1		
Diet Pepsi	0.4	Dannon Blended RSB	5.5		
Caffeine Free Diet Pepsi	0.5	Dannon Blended BUB	3.3		

Table 5

Summary Statistics on the Standard Deviation of the Number of Weeks between Two Consecutive Deals

Standard Deviation Calculated per UPC, per Store, over 104 weeks

Soft-drinks			Yogurt			Detergents		
	Average for All Stores	Std Across Stores		Average for All Stores	Std Across Stores		Average for All Stores	Std Across Stores
Coke Classic	1.3	1.2	Yoplait Custard LMN	8.2	3.8	All Regular	10.1	8.0
Caffeine Free Coke	1.3	1.1	Yoplait STB	8.8	4.3	All Free N Clear USC	3.0	2.9
Diet Coke	1.3	1.2	Yoplait VAN	9.8	4.6	Wisk Regular	3.7	1.6
Caffeine Free Diet Coke	1.3	1.2	Yoplait Custard BNA	6.9	1.1	Wisk USC	3.7	1.4
Pepsi	1.2	1.0	Dannon Blended STB	5.6	2.0			
Caffeine Free Pepsi	1.2	1.0	Dannon Blended PCH	7.2	4.5			
Diet Pepsi	1.2	1.0	Dannon Blended RSB	5.3	2.9			
Caffeine Free Diet Pepsi	1.2	1.0	Dannon Blended BUB	5.8	2.4			

Table 6

Summary Statistics on Characteristics of Deal Patterns and Allocation of Purchases Over Time

Calculated per Household, per Brand, per Store

Soft-drinks	mean	std	min	max
Loyal Group				
Savings (%)	21.3	16.3	-14.4	72.6
Fraction (%)	77.7	21.6	0.0	100.0
Average Duration (weeks)	3.3	1.5	1.4	9.6
Variation of the Duration (weeks)	2.9	1.6	0.4	9.6
Unpredictability (variation of the duration / average duration)	0.9	0.2	0.2	1.4
Discount (%)	25.1	4.5	12.9	32.8
Frequency of Deals (%)	48.3	10.9	31.0	76.7
Non-Loyal Group				
Savings (%)	22.4	14.6	-16.4	57.7
Fraction (%)	79.1	22.1	0.0	100.0
Average Duration (weeks)	3.0	1.2	1.4	9.8
Variation of the Duration (weeks)	2.7	1.6	0.9	10.3
Unpredictability (variation of the duration / average duration)	0.9	0.2	0.4	1.6
Discount (%)	26.8	3.9	15.0	33.7
Frequency of Deals (%)	51.0	11.9	30.0	75.9
All Consumers				
Savings (%)	21.7	15.7	-16.4	72.6
Fraction (%)	78.2	21.7	0.0	100.0
Average Duration (weeks)	3.2	1.4	1.4	9.8
Variation of the Duration (weeks)	2.9	1.6	0.4	10.3
Unpredictability (variation of the duration / average duration)	0.9	0.2	0.2	1.6
Discount (%)	25.6	4.4	12.9	33.7
Frequency of Deals (%)	49.2	11.3	30.0	76.7

Yogurt	mean	std	min	max
Loyal Group				
Savings (%)	18.6	25.8	-19.2	73.2
Fraction (%)	77.4	29.8	0.0	100.0
Average Duration (weeks)	6.5	4.4	1.0	28.0
Variation of the Duration (weeks)	8.1	4.3	0.3	22.8
Unpredictability (variation of the duration / average duration)	1.6	1.6	0.6	8.3
Discount (%)	35.3	9.6	7.3	45.8
Frequency of Deals (%)	19.8	7.2	1.7	42.9
Non-Loyal Group				
Savings (%)	9.9	22.6	-16.6	63.7
Fraction (%)	73.4	23.4	28.6	100.0
Average Duration (weeks)	5.7	4.0	1.2	20.3
Variation of the Duration (weeks)	7.5	3.5	3.6	17.3
Unpredictability (variation of the duration / average duration)	2.3	3.1	0.5	12.8
Discount (%)	31.7	12.5	8.5	41.9
Frequency of Deals (%)	20.7	8.7	5.3	48.6
All Consumers				
Savings (%)	17.1	25.4	-19.2	73.2
Fraction (%)	76.7	28.8	0.0	100.0
Average Duration (weeks)	6.3	4.3	1.0	28.0
Variation of the Duration (weeks)	8.0	4.2	0.3	22.8
Unpredictability (variation of the duration / average duration)	1.9	2.3	0.5	12.8
Discount (%)	34.7	10.2	7.3	45.8
Frequency of Deals (%)	20.0	7.5	1.7	48.6

Detergents	mean	std	min	max
Loyal Group				
Savings (%)	12.9	17.8	-12.9	71.2
Fraction (%)	63.4	41.3	0.0	100.0
Average Duration (weeks)	8.7	5.7	1.3	39.0
Variation of the Duration (weeks)	22.5	8.4	3.4	62.3
Unpredictability (variation of the duration / average duration)	6.2	3.1	0.2	23.9
Discount (%)	15.2	6.1	5.8	29.2
Frequency of Deals (%)	31.0	8.5	4.3	45.8
Non-Loyal Group				
Savings (%)	15.2	17.5	-4.9	60.2
Fraction (%)	75.0	37.7	0.0	100.0
Average Duration (weeks)	8.2	5.2	2.1	38.0
Variation of the Duration (weeks)	20.5	4.6	8.2	29.9
Unpredictability (variation of the duration / average duration)	6.2	1.3	2.8	8.0
Discount (%)	14.0	5.1	6.6	25.1
Frequency of Deals (%)	31.1	5.4	21.1	39.7
All Consumers				
Savings (%)	13.3	17.7	-12.9	71.2
Fraction (%)	65.6	40.7	0.0	100.0
Average Duration (weeks)	8.5	5.4	1.3	39.0
Variation of the Duration (weeks)	22.1	7.9	3.4	62.3
Unpredictability (variation of the duration / average duration)	6.2	2.8	0.2	23.9
Discount (%)	14.9	5.9	5.8	29.2
Frequency of Deals (%)	31.0	8.0	4.3	45.8

Table 7

Does Uncertainty About the Timing of Deals Affect Allocation of Total Purchase Over Time?

Results for Soft-Drinks Category

Dependent Variable: Percentage Savings / Fraction Bought on Sale

Explanatory Variable	Loyal Group (Standard Error)		Non-Loyal Group (Standard Error)		All Consumers (Standard Error)	
	Savings	Fraction	Savings	Fraction	Savings	Fraction
Unpredictability	-0.133	-0.275	0.081	-0.124	-0.119	-0.289
	(0.089)	(0.118)	(0.096)	(0.149)	(0.080)	(0.112)
Average Discount	2.589	3.307	0.332	-1.303	1.587	1.615
	(0.967)	(1.281)	(1.298)	(2.026)	(0.759)	(1.060)
Frequency of Deals	0.222	0.234	0.318	0.434	0.357	0.469
	(0.351)	(0.464)	(0.509)	(0.795)	(0.285)	(0.398)
Shares	No	No	No	No	0.018	-0.007
					(0.042)	(0.058)
Interaction term	No	No	No	No	-0.390	-0.417
(shares x unpredictability)					(0.167)	(0.233)
Number of Observations	150	150	75	75	225	225

Notes: Numbers in parentheses are standard errors. Results considered households that visited at least 20 times each store and purchased at least 6 units. Loyal is a consumer whose share of purchases on a particular brand (Coke or Pepsi) is at least 70% of his total purchase in the category. All regressions include brand and store dummy variables and some other controls not presented here such as total number of trips for each store and total number of units purchased for each brand, per consumer, per store.

Table 8

The Effect of Timing of Deals on Allocation of Total Purchase Over Time for Loyal Shopper Consumers

Results for Soft-Drinks Category

Dependent Variable: Percentage Savings / Fraction Bought on Sale

Explanatory Variable	Shopper Group (Standard Error)		Shopper Loyal Group (Standard Error)		Shopper Non-loyal Group (Standard Error)	
	Savings	Fraction	Savings	Fraction	Savings	Fraction
Unpredictability	-0.031	-0.205	-0.176	-0.330	0.075	-0.150
	(0.065)	(0.090)	(0.086)	(0.114)	(0.100)	(0.157)
Average Discount	1.443	1.764	2.921	3.870	-0.160	-1.224
	(0.797)	(1.111)	(0.980)	(1.307)	(1.397)	(2.190)
Frequency of Deals	0.414	0.535	0.376	0.248	0.265	0.710
	(0.295)	(0.410)	(0.342)	(0.456)	(0.576)	(0.903)
Number of observations	203	203	136	136	67	67

Notes: Numbers in parentheses are standard errors. Results considered households that visited at least 20 times each store and purchased at least 6 units. Loyal is a consumer whose share of purchases on a particular brand (Coke or Pepsi) is at least 70% of his total purchase in the category. All regressions include brand and store dummy variables and some other controls not presented here such as total number of trips for each store and total number of units purchased for each brand, per consumer, per store.

Table 9

Summary Statistics on the Characteristics of Deal Patterns Using Feature and Display

Results for Soft-Drinks Category

Calculated per Household, per Brand, per Store

		mean	std	min	max
	Loyal Group				
Feature as the main source of information on deals	Unpredictability (variation of the duration / average duration)	0.9	0.2	0.2	1.4
	Average Duration (weeks)	3.7	1.4	2.1	10.1
	Variation of the Duration (weeks)	2.7	1.4	0.9	8.0
	Frequency of Deals (%)	42.6	4.8	27.8	55.4
Display as the main source of information on deals	Unpredictability (variation of the duration / average duration)	0.8	0.3	0.2	1.6
	Average Duration (weeks)	11.4	6.7	4.1	38.1
	Variation of the Duration (weeks)	8.8	4.9	2.8	32.1
	Frequency of Deals (%)	14.7	4.6	5.3	28.4
	Non-Loyal Group				
Feature as the main source of information on deals	Unpredictability (variation of the duration / average duration)	0.8	0.2	0.4	1.6
	Average Duration (weeks)	3.4	1.1	2.3	8.0
	Variation of the Duration (weeks)	2.8	1.4	1.0	7.7
	Frequency of Deals (%)	42.9	4.8	31.6	61.4
Display as the main source of information on deals	Unpredictability (variation of the duration / average duration)	0.8	0.2	0.0	1.3
	Average Duration (weeks)	10.9	5.7	4.1	30.2
	Variation of the Duration (weeks)	7.9	4.4	0.0	24.4
	Frequency of Deals (%)	15.9	5.0	4.0	26.7

Table 10

Does Uncertainty About the Timing of Features Affect Allocation of Total Purchases Over Time?

Results for Soft-Drinks Category

Dependent Variable: Percentage Savings / Fraction Bought on Sale

Explanatory Variable	Loyal Group (Standard Error)		Non-Loyal Group (Standard Error)	
	Savings	Fraction	Savings	Fraction
Unpredictability	-0.156	-0.267	0.113	-0.020
	(0.097)	(0.130)	(0.100)	(0.159)
Average Discount	2.4	2.996	0.633	-1.034
	(0.961)	(1.283)	(1.319)	(2.088)
Frequency of Deals	0.572	0.476	0.524	0.523
	(0.395)	(0.528)	(0.486)	(0.769)
Number of observations	150	150	75	75

Notes: Numbers in parentheses are standard errors. Results considered households that visited at least 20 times each store and purchased at least 6 units. Loyal is a consumer whose share of purchases on a particular brand (Coke or Pepsi) is at least 70% of his total purchase in the category. All regressions include brand and store dummy variables and some other controls not presented here such as total number of trips for each store and total number of units purchased for each brand, per consumer, per store.

Table 11

Does Uncertainty About the Timing of Deals Affect the Quantity Purchased at a Particular Deal?

Results for Soft-Drinks Category

Dependent Variable: Log of Quantity Purchased at a Deal Period

Explanatory Variable	Loyal Shopper Group (Standard Error)	Non-Loyal Group (Standard Error)
log (price)	-0.545	-0.324
	(0.103)	(0.145)
Variation of the Duration	0.009	0.027
	(0.010)	(0.014)
Average Discount	0.952	-0.865
	(0.456)	(0.557)
Number of Weeks from Previous Deal	5.096	2.852
	(3.109)	(5.552)
(Number of Weeks from Previous Deal)^2	-14.809	-78.463
	(30.036)	(65.411)
Interaction term between Number of Weeks and Variation of the Duration	1.664	0.844
	(0.862)	(1.212)
Number of observations	1118	441

Notes: Numbers in parentheses are standard errors. Results considered households that visited at least 20 times each store and purchased at least 6 units. Loyal is a consumer whose share of purchases on a particular brand (Coke or Pepsi) is at least 70% of his total purchase in the category. All regressions include brand and store dummy variables. Number of weeks from previous deal is divided by 100.

Table 12

Does Uncertainty About the Timing of Deals Affect Allocation of Total Purchase Over Time?

Results for Yogurt Category

Dependent Variable: Percentage Savings / Fraction Bought on Sale

Explanatory Variable	Loyal (Standard Error)		Shopper (Standard Error)		Loyal Shopper (Standard Error)		All Consumers (Standard Error)	
	Savings	Fraction	Savings	Fraction	Savings	Fraction	Savings	Fraction
Unpredictability	-0.078	-0.134	-0.132	-0.191	-0.152	-0.219	-0.050	-0.082
	(0.049)	(0.065)	(0.082)	(0.109)	(0.088)	(0.132)	(0.020)	(0.054)
Average Discount	0.388	0.137	2.516	2.669	5.119	5.624	0.417	0.110
	(1.083)	(1.451)	(1.236)	(1.639)	(1.709)	(2.569)	(1.001)	(0.935)
Frequency of Deals	0.743	1.528	0.836	1.870	1.281	2.549	0.689	1.535
	(0.721)	(0.966)	(0.725)	(0.961)	(0.822)	(0.045)	(0.647)	(0.865)
Shares	No	No	No	No	No	No	0.007	-0.116
							0.093	(0.355)
Interaction term	No	No	No	No	No	No	-0.026	-0.082
							(0.015)	(0.048)
Number of observations	124	124	87	87	73	73	149	149

Notes: Numbers in parentheses are standard errors. Results considered households that visited at least 20 times each store and purchased at least 4 units. Loyal is a consumer whose share of purchases on a particular brand (Wisk or All) is at least 70% of his total purchase in the category. All regressions include brand and store dummy variables and some other controls not presented here such as total number of trips for each store and total number of units purchased for each brand, per consumer, per store.

Table 13

Does Uncertainty About the Timing of Deals Affect the Quantity Purchased at a Particular Deal?

Results for Yogurt Category

Dependent Variable: Log of Quantity Purchased at a Deal Period

Explanatory Variable	Loyal Shopper Group (Standard Error)	Non-Loyal Group (Standard Error)
log (price)	-0.620	-0.560
	(0.220)	(0.294)
Variation of the Duration	0.021	0.014
	(0.012)	(0.045)
Average Discount	0.357	0.280
	(0.215)	(1.056)
Number of Weeks from Previous Deal	4.705	2.805
	(2.732)	(5.642)
(Number of Weeks from Previous Deal)^2	12.756	-84.805
	(20.057)	(78.440)
Interaction term between Number of Weeks and Variation of the Duration	2.284	1.125
	(1.102)	(1.942)
Number of observations	685	203

Notes: Numbers in parentheses are standard errors. Results considered households that visited at least 20 times each store and purchased at least 4 units. Loyal is a consumer whose share of purchases on a particular brand (Wisk or All) is at least 70% of his total purchase in the category. All regressions include brand and store dummy variables. Number of weeks from previous deal is divided by 100.