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"DOES LIQUIDITY CONSTRAINT MATTER FOR RICARDIAN EQUIVALENCE?"

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Do Liquidity Constraints Matter for Ricardian Equivalence?*

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Abstract

The paper analysis a general equilibrium model with two periods, several households and a government that has to finance some expenditures in the first period. Households may have some private information either about their type (adverse selection) or about some action level chosen in the first period that affects the probability of certain states of nature in the second period (moral hazard). Trade of financial assets are intermediated by a finite collection of banks. Banks objective functions are determined in equilibrium by shareholders. Due to private information it may be optimal for the banks to introduce constraints in the set of available portfolios for each household as well as household specific asset prices. In particular, households may face distinct interest rates for holding the risk-free asset.

The government finances its expenditures either by taxing households in the first period or by issuing bonds in the first period and taxing households in the second period. Taxes may be state-dependent. Suppose government policies are *neutral*: i) government policies do not affect the distribution of wealth across households; and ii) if the government decides to tax a household in the second period there is a portfolio available for the banks that generates the same payoff in each state of nature as the household taxes. Then, Ricardian equivalence holds if and only if an appropriate boundary condition is satisfied. Moreover, at every free-entry equilibrium the boundary condition is satisfied and thus Ricardian equivalence holds. These results do not require any particular assumption on the banks' objective function. In particular, we do not assume banks to be risk neutral.

*Comments welcome. Do not quote.

1. Introduction

Ricardian equivalence states that the government decision on how to finance its expenditures, lump-sum taxes or debt, is irrelevant in the determination of households' consumption decisions. This equivalence holds in the standard Walrasian model with many periods and it was extended by Barro (1974) to overlapping generation models with bequest motives. In this last model, a sequence of generations behave as an infinitely lived household due precisely to the bequest motive.

The basic principle behind Ricardian equivalence in the Walrasian model with long lived households is remarkably simple. In this model a household faces complete markets and thus the budget constraint is completely determined by commodity prices and the household discounted wealth. The government decision between taxes in the first period or bonds in the first period and taxes in some future periods only affects the timing of payment of taxes but not the household's total (discounted) wealth. Therefore, the government policy does not affect the household budget constraint. Ricardian equivalence then follows immediately.

Since Barro's contribution there has been a long controversy on the validity of Ricardian equivalence. This controversy is only natural given the strong policy implications of this equivalence which are orthogonal to the traditional macroeconomic assumption that if the government decides to finance its expenditures with bonds instead of taxes households would increase current consumption.

There are at least three theoretical reasons why Ricardian equivalence might fail in a standard general equilibrium model: lack of bequest motives, existence of liquidity constraints and non-lump-sum taxes. The extent that these reasons are indeed relevant for the Ricardian equivalence to fail is an empirical problem. Is the lifetime of a standard household long enough so that Ricardian equivalence is a good approximation for an economy with finite lived households? How much do liquidity constraints matter? How sensitive are households consumption decisions to marginal changes in the income taxes? It is the role of the theory; however, to access under which conditions these reasons may lead the Ricardian equivalence to fail.[1]

It has long been recognized that the Ricardian equivalence requires complete

[1]The theory may also be used to investigate further implications of the assumptions. The empirical verification of these implications can then be used to validated, or not, those assumptions. Benheim (1987), for example, notices that the standard bequest motive leads to several counter-intuitive results, besides the Ricardian equivalence. This result suggests, according to Benheim, that an alternative formulation of the bequest motive may be necessary.

markets. Indeed, if households face constraints in the amount of income they may transfer across periods and states of nature then their optimal decision depends not only on their total discounted wealth but also upon the distribution of wealth across periods and states of nature. A change in the government policy may lead to a transfer of income over time that is not feasible for the household to implement and thus Ricardian equivalence may not hold.

Under which conditions does Ricardian equivalence hold if financial markets are not complete? The investigation of this question requires an explicit formulation of available financial markets as well as of the constraints each household faces in trading these assets. Moreover, these constraints should not be taken as a primitive of the model since they are usually imposed by traders upon the households due to some market imperfection or even the possibility of bankruptcy. Therefore, *the investigation of the relation between Ricardian equivalence and liquidity constraints requires a model in which the constraints faced by households in trading financial assets are determined in equilibrium.*

The existence of constraints on financial assets appears naturally in models with asymmetric information, and in particular in both adverse selection and moral hazard models. In these models the constraints faced by households are a function of both the equilibrium prices and the government policy. The validity of the Ricardian equivalence, therefore, depends on how these constraints adjust to changes in the government policy.

This paper investigates the robustness of Ricardian equivalence to the existence of constraints in trading financial assets and, in particular, to the existence of liquidity constraints. We consider a general equilibrium model with two periods and uncertainty about the second period. There is a finite collection of households which either have private information about the likelihood of certain states of nature in the second period (adverse selection) or choose an action in the first period which determines these probabilities but this action is not observable by any other agent (moral hazard).

Households may transfer income across periods and states of nature by trading financial assets, which are intermediated by a finite collection of banks. Due to private information it may be optimal for the banks to introduce constraints in the set of available portfolios for each household. In equilibrium households face endogenous constraints on trading financial assets and asset prices. In particular, households may face distinct risk-free interest rates.

Each bank is endowed with an objective function determined by its shareholders. We assume that when determining the banks policies shareholders cannot take into account that they may also be clients of the bank.[2] Each bank chooses

[2]We return to this point later.

a contract to be offered to each type of household which specifies asset prices and the constraints on the set of portfolios the household may buy. Banks take into account the private information held by household and as a result both asset prices and constraints are usually household specific.

There is a government which has to finance some expenditures in the first period either by taxing the households in that period or by issuing bonds in the first period and taxing the households in the second period. It is assumed that the second period taxes lies on the span of the existing financial assets: for each household there is a portfolio available for the banks that in each state of nature generates the same payoff as the second period taxes. It is also assumed that the government taxes are lump-sum and each household faces the same present value of taxes under both policies, where the present value is computed by using asset prices faced by the government. We refer to this class of policies as *neutral*. Notice that due to asymmetric information the asset prices faced by a household may be distinct from the ones faced by the government. Therefore, *a household may not face the same amount of discounted taxes under both government policies when the present value is computed using the asset prices she faces in the market.*

The leading example of alternative government policies are: i) taxing the households in the first period; or ii) issuing risk-free bonds in the first period and imposing state independent taxes in the second period. Suppose that there is a risk free asset. Then, the policies are neutral provided that the discounted value of second period taxes is equal to the first period taxes for each household, when the discounted fact used is the interest rate faced by the government.

We show that Ricardian equivalence holds in this economy provided that the government policies are neutral and the following boundary condition is satisfied: given a change in the government policy no household can get strictly better off by withdrawing from trading financial assets. The necessity of the boundary condition should be intuitive. A change in the government policy can have real effect if it provides the households access to a distribution of income across periods that the household strictly prefers but which does not maximize banks' objective functions and thus it is not offered in the market under the original policy.

Ricardian equivalence follows almost immediately from the basic definition of equilibrium, provided that the boundary condition is satisfied. Suppose government instead to taxing households in the first period decides to issue bonds in the first period and tax in the second period. Consider for each bank and household the contract that, if accepted by the household, generates the same feasible set of allocations for the household as the set generated by the previous contract under the original government policy. The household accepts this contract if and only if the boundary condition is satisfied. Indeed, the only difference between the

alternatives faced by the household under the new policy/new contract and the original policy/original contract is the utility derived from rejecting all offers and not trading any financial assets since the change in government policy leads to a change in the household initial distribution of income across states of nature. If the boundary condition is satisfied then the household accepts the new contract and chooses the same allocations and actions as before.

Notice, however, that under these new contracts each bank usually faces a different distribution of payoff across states of nature, which may not be optimal. Suppose the government policy lies on the span of the existing financial assets. Then each bank can choose a portfolio that generates the same payoff as the new contracts offered to the households. Thus, by selling the appropriate portfolio each bank can offer the new contracts for the households and still obtains the same payoff in each period and state of nature as under the original government policy.

Therefore, by offering alternative contracts to the households and by trading an appropriate portfolio each bank neutralizes the effects of a change of the government policy upon households' budget constraint and still obtains the same distribution of payoffs across periods and states of nature. It follows almost immediately from this observation that this new strategy for each bank must also be a Nash equilibrium in the supply of contracts, given that the original banks strategies under the previous government policy were also an equilibrium. Moreover, the shareholders choice of each bank objective function policy only depends on their budget constraint and the banks attainable portfolios, which are the same under both government policies. Finally, simple computations show that all markets clear under these new strategies for the banks. Hence, Ricardian equivalence must hold. This argument generalizes almost immediately to economies with many periods. The notation in this case, however, may get very cumbersome.

The basic argument to show Ricardian equivalence can be carry over in simpler models than the one considered in this paper. The cost of relying on a simplified model, however, is that the appropriate definition of neutral policies required to ensure Ricardian equivalence may impose artificially few conditions due precisely to the simplified assumptions. Indeed, and that is a central result of the paper, Ricardian equivalence holds provided that the set of government policies considered does not affect the distribution of wealth and does not implement policies unavailable for the banks. The exact meaning of both conditions, however, depends crucially on a precise specification of uncertainty, assets available for banks, government and households as well as the nature of the asymmetric information. It is precisely to capture all restrictions on the available government policies required to ensure Ricardian equivalence that lead us to choose a rather

general model which, however, has the inconvenient cost of making the presentation of the basic structure rather cumbersome. Nevertheless, once the basic assumptions and definitions are presented the proofs follow almost immediately.

It should be notice that the characterization of Ricardian equivalence does not require risk neutrality of the banks and it is compatible with endogenous determination of the banks' objective function by the existing shareholders. In the standard Walrasian model with production and complete markets shareholders agree in equilibrium about the firms' objective functions, which leads to the standard assumption that firms maximize profits. Once markets are incomplete, however, this result no longer holds. In general households may disagree about the marginal value of income in future states of nature and, in particular, about the firms' objective function. There are several alternatives to determine the firms' problem in the incomplete markets case and each alternative depends on a careful specification about when shareholders decide upon the firms' objective function, which rules are used to reach an agreement, existence or not of side payments among shareholders, etc. In our model we refer to firms as *banks* since their only role is to intermediate the trade of financial assets.

We assume that for each bank there is a function that determines banks' objective function for each possible set of prices, shareholders and households' income in the several states of nature. This formulation is compatible with several specifications of the precise process of determination of the banks' policies. Moreover, we show that the validity of Ricardian equivalence does not depend on the particular process chosen.

Following the contribution of Rothschild and Stiglitz (1976), it has been traditional in the literature of asymmetric information to assume that insurance companies, or in our terminology banks, are risk neutral and to define equilibrium by a *free-entry condition*: an equilibrium in the supply of contracts is defined as a finite set of banks, for each bank a strategy that maximizes its objective function given all remaining banks' strategies, and, finally, that there is no incentive for a new bank to enter in the market.

This definition of equilibrium departs from standard Nash equilibrium in the sense that the number of players is not specified at the outset but it is rather determined in equilibrium. Of course, one can always model free-entry equilibrium as a Nash equilibrium of a game with infinitely many banks. Then, a free-entry equilibrium, as defined by Rothschild and Stiglitz for example, is a Nash equilibrium of this large game such that only finitely many firms have offers accepted by the households.[3] We say that a bank is *active* in an equilibrium if its offers

[3] One should notice that the existence of inactive banks play a crucial role in several arguments advanced by Rothschild and Stiglitz.

are accepted by some household.

Clearly, a free-entry equilibrium is always a Nash equilibrium for an appropriately chosen set of banks. On the other hand, every Nash equilibrium in the model with finitely many banks is also a free-entry equilibrium provided that some bank chooses to be inactive in equilibrium. Unfortunately, not every Nash equilibrium in a model with finitely many banks is a free-entry equilibrium. Indeed, if all existing banks are actives then there may be a profitable desviation for a new bank, even though there is no profitable desviation for the active ones.[4] We use the equivalence between Nash with at least one inactive bank and free-entry equilibrium to show that *in every free-entry equilibrium the boundary property is satisfied and thus Ricardian equivalence holds.*

This result contrasts with Yotsuzuka (1987) who provides examples in a standard adverse selection model where Ricardian equivalence may not hold with risk-neutral banks. His examples, however, rely on the fact that *a change in the government policy implies changes in distribution of wealth across households.* Our results, on the other hand, rely on the government policies being neutral and, in particular, require the distribution of discounted wealth across households to be independent of the government policy.[5]

The next section presents the basic model. For simplicity, we first present the model with complete contracts. We then show that the basic model generalizes almost immediately to the case with incomplete contracts. In the third section we define neutral policies and provide the basic characterization of Ricardian equivalence. We then define free-entry equilibrium and show that in every free-entry equilibrium Ricardian equivalence holds. Finally, we compare our results to Yotsuzuka's.

2. A Basic Model with Asymmetric Information

2.1. An Outline of the Model

We consider a general equilibrium model with two periods and uncertainty in the second period. There is a finite number of types of households and a large number of households of each type. Households may have some private information either

[4]Rosenthal and Weiss (1984) provides an example where a mixed strategy Nash equilibrium with n firms is *not* a non-entry equilibrium, i.e. even though there is no profitable desviation for any of the firms active in the market an inactive new firm is strictly better off by not being inactive.

[5]We emphasize, once more, that since in our model the asset prices faced by a household may be different from the one faced by the government, a household present value of wealth calculated using the asset prices she faces on the market may depend on the government policy.

about their type (adverse selection) or about some action level chosen in the first period that affects the probability of certain states of nature in the second period (moral hazard).

There is a government that has to finance some expenditures in the first period. The government can either tax households in the first period or sell a portfolio in the first period and finance the second period portfolio payoff by taxing the households. It is assumed that the present value of each household total taxes does not depend on the particular government policy chosen. The present value of taxes is computed using the asset prices available for the government, which may not the equal to the asset prices faced by the households. In this section we restrict the analysis to state independent taxes. Later we consider the case of state dependent taxes as well.

There is a set of banks which offer a menu of contracts to each type of household. Each contract specifies a set of alternative portfolios and asset prices. Each bank is endowed with an objective function, which depends on the banks net payoffs in each period and sate of nature. This objective function is determined by the shareholders in equilibrium. Each bank takes into account the existence of asymmetric information and the government policy and chooses to offer the set of contract that maximizes its objective function, given its conjecture about the remaining banks strategies. Typically, these offers are household specific: different types of households face different contracts. Given the government policy, banks offers and relative prices, a household choose to accept the offer the maximizes her indirect utility. Equilibrium requires conjectures to be fulfilled and markets to clear. In what follows we provide a complete description of the model.

We first present a simplified version of the model with complete contracts: there are no restrictions on the set of contracts a bank may offer to each type of household. In the end of the section we show that the model generalizes almost immediately to the case with incomplete contracts and where households trade financial assets.

2.2. Time and Uncertainty

Consider an exchange economy with 2 periods, T types of households, a continuum of identical households of each type, J banks and a government. Let α_t be the fraction of type t household in the population. To simplify the notation, but without any loss of generality, assume $\alpha_t = 1/T$ for every t.

There are S aggregate states of nature in the second period, indexed by $s = 1, \ldots, S$. In each aggregate state of nature in the second period a type t household faces S^t individual states, indexed by $s^t = 1, \ldots, S^t$. There are C

commodities in the first period and in each state of nature in the second period. Let $G^t := C(SS^t + 1)$ be the total number of goods for each type t household in this economy. We refer to the first period as state of nature $s = 0$ and we identify a given commodity by a triple (s, s^t, c), where c refer to the commodity physical characteristics, s^t the individual state of nature and s the aggregate state of nature in which the commodity is available. We refer to a state of nature for a household as the pair (s, s^t).

Suppose a given fraction $\alpha_t^{\prime}$ of type t households has probability $\Pi_t(s, s^t)$ of being in the individual state s^t when the aggregate state of nature is s. We assume that in the aggregate state of nature s in the second period a fraction $\alpha_t^{\prime}\Pi_t(s, s^t)$ of these households will be in the individual state s^t. This assumption is a generalization of Malinvaud's (1972) definition of individual risk.

In several applications, one would like to assume that the individual risk to be independent and independently distributed across households of the same type and thus, by relying in some law of large numbers, to obtain that in the second period in each aggregate state of nature s^t there is no uncertainty about the fraction of the population in each possible individual state. Unfortunately, as noticed by Feldman and Giles (1985) and Judd (1985) there is no law of large number for a continuum of random variables. There are, however, several approaches to deal with problem. Hammond and Lisboa (1997) provides a particular approach and further references.

2.3. Financial Assets

There are $I \geq 1$ financial assets available for the government and banks with payoffs contingent on the aggregate state of nature s. Each one of these i financial assets are described by a vector $y_i \in \Re^S$

$$y_i = \begin{bmatrix} y_i^1 \\ \vdots \\ y_i^S \end{bmatrix}$$

where y_i corresponds to the payoff of asset i in the aggregate state of nature s in units of accounts.[6] We assume that the first asset is the risk free asset, $y_1 = 1$ for every s^t. Let $Y \in \Re^{S \times I}$ be the corresponding payoff matrix

$$Y = \begin{bmatrix} y_1^1 & & y_I^1 \\ \vdots & \ddots & \vdots \\ y_1^S & & y_I^S \end{bmatrix}$$

[6] All results remain true if, instead, we assume all assets pay in units of a given commodity in every state of nature, say commodity 1.

Let q be the corresponding vector of asset prices.

2.4. The Household Problem

Each type t household is characterized by a tuple $(U_t, e_t, \bar{\theta}_t, A_t)$ where $e_t \in \Re_+^G$ is the household vector of initial endowments, $\bar{\theta}_t \in [0,1]^J$, $\bar{\theta}_t = \left(\bar{\theta}_t^1, \ldots, \bar{\theta}_t^J\right)$, $\bar{\theta}_t^j$ is the initial share of bank j owned by type t household,

$$\sum_t \bar{\theta}_t^j = 1 \;\; \textit{for every } j$$

$A_t \in \Re$ is the household set of unobservable actions and $U_t : \Re_+^{G^t} \times A_t \rightarrow \Re$ is the household utility function. Each household utility level depends on the consumption bundle $x_t \in \Re_+^{G^t}$ and on the choice of action $a_t \in A_t$.

If $x \in \Re_+^{G^t}$ we write

$$x = \left(x^0, x^{1,1}, \ldots, x^{S,S^t}\right), \; x^{s,s^t} \in \Re_+^C, \; s = 1, \ldots, S, \; s^t = 1, \ldots, S^t$$

where x^{s,s^t} is a vector of commodities in state (s, s^t). We assume that the household choice of action cannot be observable by any other agent. Notice that the standard household model can be obtained by taking $A_t = \{0\}$. To simplify the notation *we assume the households utility functions to be strictly quasi-concave and A_t to be convex for every t.*

Suppose type t household faces a vector of income transfer $y_t \in \Re^{SS^t+1}$

$$y_t = \begin{pmatrix} y_t^0 \\ y_t^{1,1} \\ \vdots \\ y_t^{S,S^t} \end{pmatrix}$$

where y_t^{s,s^t} specifies a net flow of income in units of account in state (s, s^t) in units of accounts. In the first period type t household takes as given commodity prices in each aggregate state of nature $p^s \in \Re_+^C$, stock prices $\mu \in \Re^J$, government policy $\mathcal{P}$ and for every j the bank j net profits ψ_j^s in each aggregate state s, which for the time being we assume to depend only on the aggregate state of nature. The government policy specifies how much taxes $\tau_t^s(\mathcal{P})$ each type t household pays in each aggregate state of nature s in units of accounts and it depends on the particular policy $\mathcal{P}$ chosen by the government. *Notice that we do not allow*

taxes to depend on the individual states of nature and we also restrict the analysis to equilibrium prices that only depend on the aggregate state of nature. Let

$$p = \left(p^0, p^1, \ldots, p^S\right) \in \Re_+^G,\ G := C(S+1)$$

be the vector of relative prices.

A type t household solves the following problem

$$\max_{x_t, a_t, \theta_t} U_t(x_t, a_t) \ \ \textit{subject to}$$

$$\begin{aligned} p^0 \left(x_t^0 - e_t^0\right) + y_t^0 + \left(\psi^0 - \mu\right)\bar{\theta}_t + \mu\theta_t + \tau_t^0(\mathcal{P}) \leq 0 \\ p^s\left(x_t^{s,s^t} - e_t^{s,s^t}\right) + y_t^{s,s^t} + \theta_t\psi^s + \tau_t^s(\mathcal{P}) \leq 0 \ \ \forall s, s^t \end{aligned}$$

Let

$$\left\{x_t(y_t, p, \mu, \psi, \mathcal{P}), a_t(y_t, p, \mu, \psi, \mathcal{P}), \theta_t(y_t, p, \mu, \psi, \mathcal{P})\right\}$$

be the household optimal choice of allocations, actions, portfolios and shares. With some abuse of notation, let

$$U_t(y_t, p, \mu, \psi, \mathcal{P}) = U_t(x_t, a_t) \ \ \textit{where} \ \ (x_t, a_t) = (x_t(p, \mu, \psi, \mathcal{P}), a_t(p, \mu, \psi, \mathcal{P}))$$

be the household indirect utility, where the optimal choice of allocation and action is uniquely determined since the utility function is assumed to be strictly quasi-concave.

Suppose instead the household faces J pairs of alternative offers of income transfer

$$\left\{\left(y_t^j\right)_{j=1}^J\right\}$$

We assume the household can either accept at most a single offer or refuse all offers and do not trade assets. In order to formalize these assumptions, let $y(n)$ be the null contract

$$y_t^0(n) = y_t^{s,s^t}(n) = 0 \ \ \textit{for every} \ \left(s, s^t\right)$$

Let

$$Y_t := \left\{\left(y_t^j\right)_j\right\} \cup \{(y(n))\}$$

The household then chooses the contract that maximizes the indirect utility

$$y_t^* \in \arg\max_{y_t \in Y_t} U_t(y_t, p, \mu, \psi, \mathcal{P})$$

2.5. The Banks Problem

There are J banks in this economy which intermediate the trade of financial assets. In the spirit of asymmetric information models, however, the probabilities of the second period may either be a household's private information or depend on the action level and consumption bundle chosen by the household in the first period. Banks behave strategically and each one chooses to offer the contracts that maximize its objective function taking into account the remaining banks offers and how households behave for a given set of offered contracts and relative prices.

Let

$$\Pi_{t',h'}\left(s, s^t; \left(x^0_{t,h}, a_{t,h}\right)_{t,h}\right) \in (0,1)$$

be the probability of state (s, s^t) of a household (h', t') in case for every h and t the h household of type t chooses the first period allocation $x^0_{t,h}$ and action $a_{t,h}$.[7] We assume that households of the same type face the same probability as a function of their own actions, given the remaining households actions. More formally, let $\chi_{(t',h',h'')}$ be the function that permutes the labels of households h' and h'' of type t'

$$\chi_{(t',h',h'')}(t,h) = \begin{cases} h \;\; if \;\; t \neq t' \;\; or \;\; if \;\; t = t' \;\; and \;\; h \neq h', h'' \\ h' \; if \; t = t' \; and \; h = h'' \\ h'' \; if \; t = t' \; and \; h = h' \end{cases}$$

Then, for every t', h' and h'' the following holds

$$\Pi_{t',h'}\left(s, s^t; \left(x^0_{t,h}, a_{t,h}\right)_{t,h}\right) = \Pi_{t',h''}\left(s, s^t; \left(x^0_{\chi_{(t',h',h'')}(t,h)}, a_{\chi_{(t',h',h'')}(t,h)}\right)_{t,h}\right)$$

Once more, we do not exclude the trivial case

$$\Pi_{t'}\left(s, s^t; \left(x^0_{t,h}, a_{t,h}\right)_{t,h}\right) = \Pi_{t'}\left(s, s^t\right) \;\; for \; all \;\; \left(\left(x^0_{t,h}, a_{t,h}\right)_{t,h}\right) \;\; and \;\; \left(s, s^t\right)$$

Following the spirit of Rothschild and Stiglitz (1976), bank j may also not know the probability of certain individual states which, however, are privately known by the households. All banks, however, have the same information which is described by a partition of the set of households

$$\mathcal{J}_i \subset \{1, \ldots, T\}, \; i = 1, \ldots, n$$

[7] The probability of a household individual state may depends only on the household own choice of action and allocation.

where

$$\mathcal{J}_i \cap \mathcal{J}_j = \emptyset \ if \ i \neq j, \ \bigcup_i \mathcal{J}_i = \{1, \dots, T\}$$

Moreover, we assume that if $t, t' \in \mathcal{J}_i$ for some i then $S^t = S^{t'}$.

In the first period banks take government policy as given and offer contracts for each household. Each contract specifies how much a household may trade of each financial asset and at which price. We assume that each bank can make T offers.[8] Banks may also trade assets with payoff contingent on the aggregate state of nature.

In order to define the banks strategic space more formally, let $\mathcal{Y}(t)$ be the space of income transfers $\Re^{SS^t+1}$. A bank strategy space is given by

$$\mathcal{S}_j := \left\{ \bigotimes_{t=1}^{T} \mathcal{Y}(t) \right\} \times \Re^I$$

A strategy $s \in \mathcal{S}_j$ is also written as $s = ((y_t)_t, l_j)$ where $y_t \in \mathcal{Y}(t)$ is bank j offer to type t household and l_j is the bank demand for assets with payoff contingent on the aggregate states of nature. Without loss of generality we assume that bank j offers satisfy the following incentive compatibility constraints

$$U_{t'}(y_t, p, q, \psi, \mu, \mathcal{P}) \leq U_{t'}(y_{t'}, p, q, \psi, \mu, \mathcal{P}) \ \ \forall t \ and \ t \ s.t. \ t, t' \in \mathcal{J}_i, \ for \ every \ i$$

In order to complete the description of the bank problem we need to specify the banks utility, or objective, function. Suppose banks make offers $(\bar{y}) := \left(y_t^j\right)_{j,t}$ and choose portfolio $(l_j)_j$. Let also be given households expected profits, relative prices and government policy $(p, q, \psi, \mu, \mathcal{P})$. Finally, let households' choice of actions and consumption bundles be

$$(x, a) = (x_t(\bar{y}, p, \mu, \psi, \mathcal{P}), a_t(\bar{y}, p, \mu, \psi, \mathcal{P}))_t$$

If type t household accept bank i offer then the bank has payoff

$$-y_t^{s,s^t} \ \ in \ state \ \left(s, s^t\right)$$

Recall we have assumed that households can accept at most a single offer. Let π_t^i be the probability that type t household accept bank i offer. If type t household strictly prefer bank i offer to all other banks' offer

$$U_t(y_t, p, q, \psi, \mu, \mathcal{P}) > U_t(y_t, p, q, \psi, \mu, \mathcal{P}) \ \ for \ all \ j \neq i$$

[8] We assume that banks can make as many offers as there are types of households. This assumption simplifies the notation but is not necessary for any of the following results. We return to this point later.

then the household will accept bank i offer for sure

$$\pi_t^i(\bar{y}, p, \mu, \psi, \mathcal{P}) = 1$$

Since all households of the same type are identical then the bank's offer is accepted by all households of the type t. If type t household strictly prefers bank $j \neq i$ offer then bank i offer is refused for sure

$$\pi_t^i(\bar{y}, p, \mu, \psi, \mathcal{P}) = 0$$

Finally, if the household is indifferent among offers from I banks, including bank i, which strictly dominates all remaining offers then

$$\pi_t^i(\bar{y}, p, \mu, \psi, \mathcal{P}) = \frac{1}{I}$$

In this case we assume that with probability 1 the bank's offer is accepted by $1/I$ of the households of this type. Given these assumptions, π_t^i *is also the fraction of households of type t who accept firm i offer.*

By construction, bank i offer to type t household is accepted with probability one by a certain fraction of those households, π_t^i. In the aggregate state of nature s there is a fraction $\pi_t^i \Pi_t(s, s^t)$ in the individual state s^t with probability 1, for every s^t. Therefore, *the only uncertainty faced by the banks is the aggregate uncertainty s.*

Bank i payoff in the first period is given by

$$\nu_i^0 = -\sum_t \pi_t^i(\cdot)\, y_t^0 - ql_i$$

and in the aggregate state s the payoff is given by

$$\nu_i^s = y^s l_i - \sum_t \sum_{s^t>0} \Pi_t\left(s, s^t; \left(x_{t',h}^0, a_{t',h}\right)_{t',h}\right) \pi_t^i(\cdot)\, y_t^{s,s^t}$$

Let $\nu_i = \left(\nu_i^0, \nu_i^1, \ldots, \nu_i^S\right)$. Bank i utility is then given by

$$\Xi_i(\nu_i) = \sum_{s=1}^{S} \Pi\left(s; \left(x_{t,h}^0, a_{t,h}\right)_{t,h}\right) \xi_i^s\left(\nu_i^s, \nu_i^0\right)$$

where

$$\Pi\left(s; \left(x_{t,h}^0, a_{t,h}\right)_{t,h}\right)$$

is the probability of the aggregate state of nature s and $\xi_i^s : \Re^2 \rightarrow \Re$ is bank i state s utility function. Notice that we have assumed the bank total utility, or objective, function to be state separable but not necessarily time separable or state independent. In particular, we do not assume this function to be linear. We discuss in the next section how the banks' objective functions is determined and why in this framework it is natural to have a bank not necessarily maximizing expected profits.

2.6. Determination of Banks' Objective Function

In the standard Walrasian model with production and complete markets one can show that shareholders unanimously prefer the firms to maximize profits and then bank j objective function Ξ_j is fully determined in equilibrium.[9] When markets are incomplete and information may be asymmetric in general households disagree about the marginal value of income in distinct states of nature.[10] In particular, shareholders may disagree about the optimal objective function for a firm.

There several approaches to solve the firm problem in the incomplete markets case. A particular approach is to allow the original owners of the firm to negotiate the firm policy with side payments among themselves up to a unanimous agrement has been reached. Stock market then opens and households trade shares. Alternatively, the firm policy may be specified by the new owners based in some majority rule.

A careful specification of all these steps in the process of determination of banks objective function is necessary in order to analyze a general equilibrium model with production and incomplete markets. In any of these formulations, however, the outcome is an objective function for the firm which depends on the economy primitives and prices. The particular formulation chosen leads to a particular relation between these variables and the firm objective function, which completes the description of equilibrium.[11]

In the model considered in this paper, firms only intermediate the trade of financial assets, which naturally leads to us referring to them as *banks*. We will show later that the *Ricardian equivalence holds for a large class of approaches to determine the banks' objective function*. In order to show this result, and to prevent having to consider several particular cases, we assume that the process of determining the banks objective function can be described in abstract terms by a general function that for each set of economic fundamentals, prices and

[9] See Magill and Quinzii (1996, chap.6).

[10] See Geanakoplos and Polemarcharkis (1986).

[11] Magill and Quinzii (1996, chap. 6) present a very complete exposition of the theory of the firm with incomplete markets and also provides further references.

shareholders expected stream of income gives an objective function for each bank. We also allowed the households to expect the price of a bank share to be a function of the objective function the shareholders choose for the bank.

Recall that households indirect utility only depends on the commodity prices and their income in each state of nature. Let

$$\omega_t^{s,s^t} := p^{s^t} e_t^{s,s^t} + y_t^{s,s^t} + \tau_t^s (\mathcal{P})$$

The effects of both the government policy as well as the constraints faced by the households on financial markets are summarized on households stream of income ω_t. Consider the set of banks possible objective functions

$$\mathcal{A}' := \left\{ \Xi : \Re_{++}^{S+1} \to \Re / \ \Xi\left(\nu^0, \nu^1, \ldots, \nu^S\right) = \sum_{s=1}^{S} \Pi(s; \cdot)\, \xi_i^s\left(\nu^s, \nu^0\right) \right\}$$

Without any loss of generality, we restrict the set of possible objective functions to the set of *monotonic* functions[12]

$$v > v' \Rightarrow \Xi(v) > \Xi(v')$$

Given the economic fundamentals we assume that for each bank j there is a function

$$\begin{array}{c} \Xi_j : \left(\bigotimes_{t=1}^{T}\left([0,1]^J \times \Re^{SS^t+1}\right)\right) \times \Re_+^G \times \Re^I \to \mathcal{A} \\ \left(\left(\bar{\theta}_t, \omega_t\right)_t, p, q\right) \to \Xi_j\left(\cdot; \left(\bar{\theta}_t, \omega_t\right)_t, p, q\right) \end{array}$$

that for each initial distribution of shares, households conjectures of stream of income, commodity, stock and asset prices gives the choice of bank j objective function. Finally the following function $\mu_j(\cdot)$ gives the households' conjecture of asset prices, which may depend upon equilibrium prices, households' distribution of income across periods and households' choice of banks' objective function.

$$\begin{array}{c} \mu_j : \left(\bigotimes_{t=1}^{T}\left([0,1]^J \times \Re^{SS^t+1}\right)\right) \times \Re_+^G \times \Re^I \times \mathcal{A} \to \Re \\ \left(\left(\bar{\theta}_t, \omega_t\right)_t, p, q, \Xi_j\right) \to \mu_j\left(\Xi_j, \left(\bar{\theta}_t, \omega_t\right)_t, p, q\right) \end{array}$$

One should notice that we have excluded the possibility of a household using its control of a bank in order to select a particular restriction in trading financial assets. Given the banks' objective function, the set of offered contracts is determined taking into account both the possible net profits in each state of nature and the effects of these contracts on the probability of the states of nature but not their effects on the shareholders indirect utility which would affect the

[12] If $v = \left(v^0, v^1, \ldots, v^S\right) \in \Re^{S+1}$ then we say that $v > v'$ if $v^s \geq v^{s\prime}$ and $v \neq v'$.

banks' objective function themselves. *We do not allow the set of offered contracts to depend on the fact that a particular household with whom the bank trades is also a shareholder.* This assumption seems to be empirically justified. In several countries it is forbidden for banks to lend money to its shareholders and in general there are several regulations that tend to prevent a shareholder to obtain a privileged trading with his/her bank.

2.7. The Government

There is a government who has to finance some net trades $g \in \Re^C$ in the first period. In order to finance these expenditures the government has to choose between two alternative policies. The first policy is to tax households in the first period. The second policy is to finance the first period expenditures by selling financial assets $b_g \in \Re^I$ with payoff contingent upon the second period aggregate states of nature and finance this portfolio in the second period by taxing the households. We refer to the first policy as policy $\mathcal{T}$, and to the second policy as police $\mathcal{B}(b_g)$. *Notice that if the government decides to finance its expenditures by policy $\mathcal{B}(b_g)$ then it must trade assets b_g with payoff contingent upon the aggregate state of nature Y, which are precisely the assets available for the banks as well.* Therefore, the government does not have access to financial assets that are not available for the banks.

In policy $\mathcal{T}$ type t household is taxed in the amount τ_t^0 in the first period, while in policy $\mathcal{B}(b_g)$ the household is taxed in the amount τ_t^s in each state s in the second period. *Government policies are assumed to be balanced*

$$\begin{aligned} p^0 g &= \sum_t \tau_t^0 = q b_g \\ y^s b_g &= \sum_t \Pi_t(s)\, \tau_t^s \textit{ for every } s = 1, \ldots, S \end{aligned}$$

2.8. Incomplete Contracts

In the previous sections of the model we have assumed that banks can offer any contract to each type of household. In this section we how the previous model generalizes to the case with incomplete contracts. Suppose there are $I^t \geq 1$ financial assets available for each type t of household in this economy with payoff contingent upon the household states of nature (s, s^t). Each financial asset i is

described by a vector $y_i(t) \in \Re^{SS^t}$

$$y_i(t) = \begin{pmatrix} y_i^{1,1}(t) \\ \vdots \\ y_i^{S,S^t}(t) \end{pmatrix}$$

where $y_i^{s,s^t}(t)$ is asset i promise of payment in the state (s, s^t). All assets pay in units of account.[13] We assume that the first financial asset is the risk-free asset $y_1^{s,s^t}(t) = 1$ *for every* (s, s^t). Let $Y(t)$ be the corresponding payoff matrix

$$Y(t) = \begin{bmatrix} y_1^{1,1}(t) & & y_I^{1,1}(t) \\ \vdots & \ddots & \vdots \\ y_1^{S,S^t}(t) & & y_I^{S,S^t}(t) \end{bmatrix}$$

Let $q_t \in \Re^{I^t}$ be the corresponding vector of asset prices faced by type t households.

Finally, we assume that every aggregate financial asset is also available for the households: if y_i is an asset with payoff contingent upon the aggregate states of nature then the vector $y_i(t) \in \Re^{S \times S^t}$ given by $y_i^{s,s^t}(t) = y_i^s$ for every s^t and s is an available financial asset for type t, for every t.

Suppose a bank offer to a type of household is a contract that specifies asset prices q_t and a set of portfolios a household may buy from the bank. The available set of portfolios can be described by I^t constraints on trading financial assets represented by the function $\phi_t : \Re^{I^t} \to \Re^{I^t}$. A portfolio $b_t \in \Re^{I^t}$ is said to be *feasible* for a type t household if

$$\phi_t(b_t) \leq 0$$

Notice that we have impose no restrictions on the constraints functions. In particular, it may be the case that

$$\phi_t^i(b) = 0 \ \textit{for every}\ i = 1, \ldots, I^t$$

in which case type t household faces no restriction on trading financial assets. On the other hand, a constraint may take the form of

$$\phi_t^1(b) = \|b - b_t^*\|,\ \phi_t^i(b) = 0\ \textit{for every}\ b\ \textit{and}\ i > 1$$

for a given b_t^*. In this case the unique feasible household's choice of portfolio is $b = b_t^*$.

[13] All results remain true if, instead, we assume all assets pay in units of a given commodity in every state of nature, say commodity 1.

A household then solves the following problem

$$\max_{x_t,a_t,\theta_t,b_t} U_t(x_t,a_t) \quad \textit{subject to}$$

$$\phi_t(b_t) \le 0$$

$$p^0\left(x_t^0 - e_t^0\right) + q_t b_t + \left(\psi^0 - \mu\right)\bar{\theta}_t + \mu\theta_t + \tau_t^0(\mathcal{P}) \le 0$$

$$p^s\left(x_t^{s,s^t} - e_t^{s,s^t}\right) - y^{s,s^t}(t)b_t + \theta_t\psi^s + \tau_t^s(\mathcal{P}) \le 0 \quad s = 1,\ldots,S, \; s^t = 1,\ldots,S^t$$

As before, given each bank offers a type t household chooses the contract that maximizes its indirect utility function.

Notice that given prices and government policy both the household and the bank problems only depend on the vector income payments derived from the firm choice of asset prices and the portfolio chosen by the household among the feasible set proposed by the firm. Suppose instead of offering contract $\left(\phi_t^j, q_t^j\right)$ bank j offers

$$y_t^j := \left(y_t^{0,j},\ldots,y_t^{SS^t,j}\right) \in \Re^{SS^t+1}$$

where

$$\begin{bmatrix} y_t^{1,1} \\ \vdots \\ y_t^{S,S^t} \end{bmatrix} = Y(t)b_t$$

$$y_t^0 = q_t^j b_t$$

and

$$b_t \in b_t\left(p,\mu,\psi,\left(\bar{\phi},\bar{q}\right),\mathcal{P}\right)$$

Then, both type t household and bank j obtain the same payoff as if the bank has offered $\left(\phi_t^j, q_t^j\right)$. Thus, we can restrict the bank strategy space without any loss of generality to

$$\mathcal{S}_j := \mathcal{Y} \times \Re^I$$

where

$$\mathcal{Y}(t) \; : = \left\{ y = \left(y^0, y^{1,1},\ldots,y^{S,S^t}\right) \in \Re^{SS^t+1} / \begin{bmatrix} y^{1,1} \\ \vdots \\ y^{S,S^t} \end{bmatrix} \in spanY(t) \right\}$$

$$\mathcal{Y} \; : = \bigotimes_{t=1}^{T} \mathcal{Y}(t)$$

A strategy for bank j is $s = ((y_t)_t, l_j)$. The vector $y_t = \left(y_t^{s,s^t}\right)_{s,s^t}$ is the bank offer to type t household, where y_t^{s,s^t} is the income contingent to state (s, s^t) in units of account. In this alternative formulation, given the government policy $\mathcal{P}$, the remaining banks strategies

$$y^{-j} := \left(y^1, \ldots, y^{j-1}, y^{j+1}, \ldots, y^J\right),\ y^j := \left(y_1^j, \ldots, y_{t_j}^j\right),\ y_t^j \in \mathcal{Y}\ \forall\ j \textit{ and } t$$

prices and households expectation about profits (p, q, μ, ψ) each bank j is solving

$$\max_{(y \in \mathcal{Y}, l^j)} \Xi_j \left(\left(x^0 \left(y^{-j}, y^j\right), a\left(y^{-j}, y^j\right)\right) q, y^j, l^j\right)$$

which is precisely the same objective function we obtained before, except that the domain of the banks offers must be appropriately re-defined. Therefore, by a suitable definition of $\mathcal{Y}(t)$ we can immediately generalize the previous model with complete contracts to the case with incomplete contracts.

2.9. The Timing of the Model and Equilibrium

The timing of the model is as follows. Government announces its policy. Households conjecture constraints on financial markets, asset, stock and commodity prices and choose banks objective functions. Given its objective function, each bank j conjectures the remaining banks strategies, commodity, stock and asset prices and announce a set of offers. Households choose at most a single contract, consumption bundles and banks shares satisfying the budget constraint. Equilibrium requires all conjectures to be fulfilled and markets to clear.

Definition 2.1. *A **Rational Expectation Equilibrium** is a set*

$$\left\{p, q, \mu, (x_t, a_t, \theta_t)_t, \left(\Xi_j, y^j, l_j\right)_j, \mathcal{P}\right\}$$

such that:
i) the government chooses a policy $\mathcal{P} \in \{\mathcal{B}(b_g), \mathcal{T}\}$;
ii) for each t the vector (x_t, a_t, θ_t) solves type t household problem given prices p, banks strategies $\left(y_t^j\right)_j$, the profits ψ associated with these strategies and government policy $\mathcal{P}$;
iii) the strategy (y^j, l_j) is bank j best response given the objective function Ξ_j, prices, households expected profits and policy $(p, q, \mu, \psi, \mathcal{P})$ and that it expects the remaining banks strategies to be (y^{-j});

iv) shareholders conjecture of share prices are correct and they agree on bank j policy function for every j

$$\Xi_j(\cdot) = \Xi_j\left(\cdot; \left(\bar{\theta}_t, \omega_t\right)_t, p, q\right)$$
$$\mu_j = \mu_j\left(\Xi_j, \left(\bar{\theta}_t, \omega_t\right)_t, p, q\right)$$
$$\omega_t^{s,s^t} = p^s e_t^{s,s^t} + y_t^{s,s^t} + \tau_t^s(\mathcal{P})$$

v) government budget balances

$$p^0 g = \sum_t \tau_t^0 = q b_g$$
$$y^s b_g = \sum_t \Pi_t(s)\, \tau_t^s$$

vi) markets clear

$$\sum_t \theta_t^j = 1$$
$$\sum_t \left(x_t^0 - e_t^0\right) + g = 0$$
$$\sum_t \sum_{s^t > 0} \Pi_t\left(s, s^t\right)\left(x_t^{s,s^t} - e_t^{s,s^t}\right) = 0 \text{ for every } s$$
$$\sum_j l_j = 0 \text{ if policy } \mathcal{T}$$
$$\sum_j l_j = b_g \text{ if policy } \mathcal{B}(b_g)$$

Definition 2.2. *We say that **Ricardian equivalence** holds if the set of equilibrium commodity allocations and prices does not depend on the government policy.*

3. Ricardian Equivalence

3.1. Neutral Policies

In this section we define *neutral government policies.* Essentially, two government policies are said to be neutral if they both are associated with the same distribution of wealth across households. It is immediate to provide examples where if government policies are not neutral then Ricardian equivalence does not hold. Suppose for example there are two types of households and under one policy the government taxes just one type of household where under the alternative policy

only the other type pay taxes. If taxes are high enough then, for an appropriate distribution of initial endowments, under each policy only one type of household consumes. This trivial argument does not require any asymmetry information or even the existence of banks. Therefore, the neutrality of government policy should be a natural requirement for Ricardian equivalence to hold.[14]

In order to provide a formal definition of neutrality, suppose the policies available for the government are $\{\mathcal{T}, \mathcal{B}(b_g)\}$. Following the notation use in the previous section, if the government chooses policy $\mathcal{T}$ it finances its expenditures in the first period by taxing the households in that period then each type t household is taxed τ_t^0 in the first period and nothing otherwise. If the government chooses policy $\mathcal{B}(b_g)$ then it finances its first period expenditures by selling the portfolio b_g and the second period payment of this portfolio by taxing type t household in the amount τ_t^s in each state s in second period and nothing in the first period.

Definition 3.1. *We say that the policies $\{\mathcal{T}, \mathcal{B}(b_g)\}$ are* **neutral** *if for every t there are b_g^t such that*

$$\begin{array}{l} i)\ y^s b_g^t = \tau_t^s \text{ for each } s \\ ii)\ \tau_t^0 = q b_g^t \end{array}$$

Notice that this formal definition can be easily extended to any pair of government policies $\left\{\mathcal{B}\left(b_g'\right), \mathcal{B}(b_g)\right\}$, with $b_g \neq b_g'$.

Suppose the government is considering between the neutral policies $\{\mathcal{T}, \mathcal{B}(b_g)\}$. For every t the portfolio b_g^t generates for the government the same payoff in the second period as taxing type t households in the amount τ_t^s in each state of nature (s, s^t). Then, the present value of this portfolio for the government is qb_g^t. If both policies are associated with the same distribution of wealth across households then the present value of taxes a type pays must also be the same. In order to evaluate the present value of future taxes we use the asset prices faced by the government. By assumption, under policy $\mathcal{B}(b_g)$ for each type of household there is a portfolio that generates the same payoff as the second period taxes and, from the government point of view, the present value of the taxes is equal to value of the corresponding portfolio. Notice, however, that households may face different asset prices then the government's. *Therefore, the household present value of wealth computed using the asset prices she faces in the market may change under alternative government policies, even if these policies are neutral!*

Notice that the very definition of neutral policy requires the existence of a portfolio b_g^t and therefore a contract $y_t \in \mathcal{Y}(t)$ since markets may be incomplete.

[14]We will later discuss Yotsuzuka model where government policies are *not neutral* and thus Ricardian equivalence may fail.

If such a portfolio or contract do not exist then a change in government policy may lead to a distribution of income across second period states of nature for a household that it is not feasible for the banks to implement. But then, a change in government policy may lead to a distribution of income for a household that is not possible for a bank to offer and, thus, may trivially have real effects. Hence, Ricardian equivalence may fail if government policies are not neutral.

3.2. Basic Characterization of Ricardian Equivalence

In this section we show that Ricardian equivalence holds if and only if a boundary condition is satisfied. This boundary condition states that at a proposal equilibrium a change of the government policy cannot strictly improve some household that decides to withdraw from financial markets. This result should be intuitive. In our general framework, competition among banks may not lead to strategies that maximize each household's welfare, given the set of prices. In particular, there may exists an alternative strategy that improves households' welfare and provides non-negative expected profits given the remaining banks strategies. Notice that this possibility may occur even if banks are assumed to be risk neutral, as in the original Rothschild and Stiglitz model. In this model, if each bank can only offer a single contract per household then there are equilibrium which are Pareto dominated by incentive compatible allocations.[15] Therefore, a change in government policy may implement a Pareto improving allocation and thus Ricardian equivalence fails in this case.

Definition 3.2. *Fix a rational expectation equilibrium*

$$\left\{p^*, q^*, \mu^*, (x_t^*, a_t^*, \theta_t^*)_t, \left(\Xi_j^*, y^{j*}, l_j^*\right)_j, \mathcal{P}^*\right\}$$

Let $(x_t(0,\mathcal{P}), a_t(0,\mathcal{P}))$ be the type t household optimal demand under policy $\mathcal{P}$, prices (p^*, q^*, μ^*) and non-trade in financial assets, $b_t = 0$. We say that this rational expectations equilibrium satisfies the **boundary property** *if for every t*

$$U_t(a_t(0,\mathcal{P}), x_t(0,\mathcal{P})) \leq U_t(x_t^*, a_t^*) \quad \mathcal{P} \neq \mathcal{P}^*$$

Proposition 3.3. *Fix a rational expectation equilibrium*

$$\left\{p^*, q^*, \mu^*, (x_t^*, a_t^*, \theta_t^*)_t, \left(\Xi_j^*, y^{j*}, l_j^*, \hat{y}^j\right)_j, \mathcal{P}^*\right\}$$

[15] See Chassagnon and Chiappori (1996).

Then, the allocations $(x_t^*, a_t^*, \theta_t^*)_t$ and prices (p^*, q^*, μ^*) are also an equilibrium allocation and prices under a neutral policy $\mathcal{P} \neq \mathcal{P}^*$ for some appropriately chosen banks' strategies if and only if the boundary property is satisfied.

Proof. Suppose the government has chosen policy $\mathcal{T}$ and let

$$\left\{p(\mathcal{T}), q(\mathcal{T}), \mu(\mathcal{T}), (x_t(\mathcal{T}), a_t(\mathcal{T}), \theta_t(\mathcal{T}))_t, \left(\Xi_j(\mathcal{T}), y^j(\mathcal{T}), l_j(\mathcal{T})\right)_j, \mathcal{T}\right\}$$

be an equilibrium. The existence of a solution to the banks problem requires the non-arbitrage condition[16] to be satisfied

$$\exists \lambda \in \Re_{++}^S \text{ such that } q = \lambda Y$$

Suppose instead the government decides for the neutral policy $\mathcal{B}(b_g)$. By definition of neutral policies, for every t there is a portfolio b_g^t that generate the same payoff in the aggregate state of nature as second period taxes for type t household under policy $\mathcal{B}(b_g)$

$$y^s b_g^t = \tau_t^s$$

Consider the set of strategies for the banks

$$\left(y^j(\mathcal{B})\right)_j$$

where

$$\begin{aligned} y_t^{0,j}(\mathcal{B}) &: = y_t^{0,j}(\mathcal{T}) - \tau_t^0 \\ y_t^{s,s^t,j}(\mathcal{B}) &: = y_t^{s,s^t,j}(\mathcal{T}) + \tau_t^s \end{aligned}$$

By the neutrality assumption

$$\begin{bmatrix} y_t^{1,1,j}(\mathcal{T}) \\ \vdots \\ y_t^{S,S^t,j}(\mathcal{T}) \end{bmatrix}, \begin{bmatrix} \tau_t^1 \\ \vdots \\ \tau_t^S \end{bmatrix} \in spanY(t)$$

and thus we have

$$\begin{bmatrix} y_t^{1,1,j}(\mathcal{B}) \\ \vdots \\ y_t^{S,S^t,j}(\mathcal{B}) \end{bmatrix} = \begin{bmatrix} y_t^{1,1,j}(\mathcal{T}) \\ \vdots \\ y_t^{S,S^t,j}(\mathcal{T}) \end{bmatrix} + \begin{bmatrix} \tau_t^1 \\ \vdots \\ \tau_t^S \end{bmatrix} \in \mathcal{Y}(t) = spanY(t)$$

[16] See, for example, Magill and Quinzii (1996, chap. 2).

Hence y^j is a feasible strategy for firm j. Let $l_j(\mathcal{B})$ be defined as follows

$$l_j(\mathcal{B}) = l_j(\mathcal{T}) + \sum_t \pi_t^j b_g^t$$

Finally, let

$$(p(\mathcal{B}), q(\mathcal{B}), \mu(\mathcal{B})) := (p(\mathcal{T}), q(\mathcal{T}), \mu(\mathcal{T}))$$

Then the households feasible set is identical for both government policies and thus they must choose the same allocation provided boundary property is satisfied. Indeed, notice that

$$\begin{aligned} p^0(\mathcal{T})\left(x_t^0 - e_t^0\right) + \left(\psi^0 - \mu\right)\bar{\theta}_t + \mu\theta_t &\leq y_t^0(\mathcal{T}) - \tau_t^0 \Leftrightarrow \\ p^0(\mathcal{B})\left(x_t^0 - e_t^0\right) + \left(\psi^0 - \mu\right)\bar{\theta}_t + \mu\theta_t &\leq y_t^0(\mathcal{B}) \end{aligned}$$

and for every $s > 0$

$$\begin{aligned} p^s(\mathcal{T})\left(x_t^{s,s^t} - e_t^{s,s^t}\right) + \theta_t\psi^s &\leq y_t^{s,s^t}(\mathcal{T}) \Leftrightarrow \\ p^s(\mathcal{B})\left(x_t^{s,s^t} - e_t^{s,s^t}\right) + \tau_t^s + \theta_t\psi^s &\leq y_t^{s,s^t}(\mathcal{B}) \end{aligned}$$

Moreover, under the boundary condition if the government chooses policy $\mathcal{B}(b_g)$ every household is better off by accepting the banks offers and consuming $x_t(\mathcal{T})$ then refusing the offers and consuming $x_t(0, \mathcal{B})$. Therefore,

$$x_t(\mathcal{B}) = x_t(\mathcal{T})$$

is a solution to the household problem under policy $\mathcal{B}$, prices $(p(\mathcal{B}), \psi(\mathcal{T}), q(\mathcal{B}), \mu(\mathcal{B}))$ and offers $y(\mathcal{B})$.

Recall that π_t^j is the fraction of households of type t that accept firm j offer and in particular $\sum_j \pi_t^j = 1$ for every t. Then, under $l(\mathcal{B})$ financial assets market clear

$$\begin{aligned} \sum_j l_j(\mathcal{B}) &= \sum_j \left(l_j^1(\mathcal{T}) + \sum_t \pi_t^j b_g^t\right) \\ &= \left(\sum_j l_j^1(\mathcal{T}) + \sum_t b_g^t \sum_j \pi_t^j\right) \\ &= \left(0 + \sum_t b_g^t\right) \\ &= b_g \end{aligned}$$

Notice that since each household faces the same distribution of income and prices the banks policy function is the same under both government policies

$$\begin{aligned}
\Xi_j(\mathcal{T}) &= \Xi_j\left(\left(\bar{\theta}_{tt}, \omega_t(\mathcal{T})\right)_t, p(\mathcal{T}), q(\mathcal{T})\right) \\
&= \Xi_j\left(\left(\bar{\theta}_{tt}, \omega_t(\mathcal{B})\right)_t, p(\mathcal{B}), q(\mathcal{B})\right) \\
&= \Xi_j(\mathcal{B}) \\
\mu_j(\mathcal{T}) &= \mu_j\left(\left(\bar{\theta}_{tt}, \omega_t(\mathcal{T})\right)_t, p(\mathcal{T}), q(\mathcal{T})\right) \\
&= \mu_j\left(\left(\bar{\theta}_{tt}, \omega_t(\mathcal{B})\right)_t, p(\mathcal{B}), q(\mathcal{B})\right) \\
&= \mu_j(\mathcal{B})
\end{aligned}$$

It remains to show that the set of strategies is a Nash equilibrium in the supply of assets. Suppose for some bank there is a profitable desviation. Then by a reversed construction of strategies there was also a profitable desviation under the policy $\mathcal{T}$. But this would contradict the definition of an equilibrium. Thus, each bank obtains the same payoff in each state of nature under policy $\mathcal{T}$ as under policy $\mathcal{B}$, $\psi(\mathcal{T}) = \psi(\mathcal{B})$. Therefore, every equilibrium allocation under policy $\mathcal{T}$ which satisfied the boundary property can also be supported as an equilibrium allocation under policy $\mathcal{B}(b_g)$. By a symmetric argument, every equilibrium allocation under policy $\mathcal{B}(b_g)$ can also be supported as an equilibrium allocation under policy $\mathcal{T}$.

Finally, suppose the boundary property is not satisfied then under prices $(p(\mathcal{T}), q(\mathcal{T}), \mu(\mathcal{T}))$ then at least one type t household can choose an allocation $x_t(0, \mathcal{B})$ which strictly dominates $x_t(\mathcal{T})$. Therefore, either prices won't change at the new equilibrium but the type t household would choose a distinct allocation or prices will change. In any case, Ricardian equivalence does not hold. A symmetric argument holds for policy $\mathcal{B}(b_g)$. The proof is then complete. □

3.3. Free-Entry Equilibrium

It is standard in the asymmetric information literature with Bertrand competition to assume that banks are risk neutral and to use a free-entry condition as the equilibrium concept.[17] A free-entry equilibrium requires every bank to be maximizing its objective function given the remaining banks strategies and, furthermore, that there is no incentive for a new bank to enter in the market and offer contracts that are accepted by some household. As we discussed in the introduction, Nash and free-entry equilibrium are not equivalent concepts. There is, however, an immediate equivalence between Nash equilibrium with at least one inactive bank and a free-entry equilibrium: given an economy with J banks every Nash equilibrium with at least one inactive bank is a free-entry equilibrium

[17]See for example Rothschild and Stiglitz (1976) and Hellwig (1988).

and every free-entry equilibrium is a Nash equilibrium for an appropriate choice of the number of banks.

Given the equivalence between Nash equilibrium with inactive banks and free-entry equilibrium *we refer to a rational expectation equilibrium as a* **free-entry equilibrium** *if there is at least one bank has all its offers accepted with probability zero by every household.* In this section we show that in every free-entry equilibrium Ricardian equivalence holds provided that banks prefer more income than less.

The next result requires households demand function to be a continuous function of the set of offered contracts, which in the general equilibrium literature usually follows from some variation of the survival assumption.[18] Due to the possibility of incomplete markets a stronger version of the survival assumption is actually necessary in order to guarantee positive consumption in every state of nature. The following assumption is a weak version of the *Inada condition* and it states that the utility of not consuming any bundle at any given state of nature is arbitrarily small. Let

$$X(t) := \left\{x \in \Re_+^{G^t} / \, x^s \neq 0 \textit{ for all } s\right\}, \ A_t = [0,1] \textit{ or } A_t = \{0\} \textit{ for every } t$$

We assume that $U_t : X(t) \times A_t \to \Re \in C^0$ *is strictly monotonic*

$$x > x' \Rightarrow U_t(x,a) > U_t(x',a)$$

and

$$\{x(n)\} \subset X(t), x(n) \to x, \ x^{s,s^t} \to 0 \textit{ implies } U_t(x) \to -\infty$$

One can alternatively assume that the indifference curves do not intersect the axis, as in the standard smooth general equilibrium model.[19]

Proposition 3.4. *Ricardian equivalence holds at every free-entry equilibrium provided that government policies are neutral.*

Proof. Let $\mathcal{T}$ be the government policy at a given equilibrium, $\mathcal{B}(b_g)$ be the alternative policy, $U_t(y, \mathcal{P})$ the household indirect utility given contract y and policy $\mathcal{P}$ and that all prices are as in the given equilibrium, and, finally, let $(x_t(y,\mathcal{P})\, a_t(y,\mathcal{P}))$ be the corresponding household's allocation and action.

[18] McKenzie (1981) discusses the role of the survival assumption in general equilibrium models and provides further references.

[19] See, for example, Balasko (1988).

Suppose the boundary condition does not hold and let t be a household which gets strictly better with the alternative government policy

$$U_t\left(0, \mathcal{B}\left(b_g\right)\right) > U_t\left(y_t^*, \mathcal{T}\right) \Rightarrow x_t^{s,s^t}\left(0, \mathcal{B}\left(b_g\right)\right) > 0 \; for\ every\ \left(s, s^t\right)$$

By definition of a free-entry equilibrium, there is a bank, say bank 1, whose offers are not accepted by any household. Let y_t be the contract that mimics the effect of policy $\mathcal{B}\left(b_g\right)$ for type t household, where

$$U_t\left(y_t, \mathcal{T}\right) = U_t\left(0, \mathcal{B}\left(b_g\right)\right)$$

By assumption of neutral policies there is a portfolio b_g^t such that

$$y^s b_g^t = \tau_t^s \;\; for\ every\ s \;\; and \;\; q b_g^t = \tau_t^0 = y_t^0$$

Notice that preferences are strict monotonic and thus in equilibrium $p \gg 0$. Moreover

$$x_t^{s,s^t}\left(0, \mathcal{B}\left(b_g\right)\right) > 0 \Rightarrow x_t^{s,s^t}\left(y_t, \mathcal{T}\right) > 0$$

which gives

$$p^s x_t^{s,s^t}\left(y_t, \mathcal{T}\right) > 0$$

Then, by standard demand function arguments $\left(x_t\left(y_t, \mathcal{T}\right), a_t\left(y_t, \mathcal{T}\right)\right)$ must be a continuous function of the offered contract in some neighborhood of $\hat{y}_t$. In particular, we can construct an alternative offer $\hat{y}_t$ such that it provides smaller income for the household in the first period

$$\hat{y}_t^0 < y_t^0$$

and same payoff in the second period $\hat{y}_t^{s,s^t} = y_t^{s,s^t}$ and it satisfies

$$U_t\left(x_t\left(y_t, \mathcal{T}\right), a_t\left(y_t, \mathcal{T}\right)\right) > U_t\left(x_t\left(\hat{y}_t, \mathcal{T}\right), a_t\left(\hat{y}_t, \mathcal{T}\right)\right) > U_t\left(x_t\left(y_t^*, \mathcal{T}\right), a_t\left(y_t^*, \mathcal{T}\right)\right)$$

Notice that the payoff generated by the offer $\hat{y}_t$ in each aggregate state of nature does not depend on the particular household who accepts the bank's offer. In particular, this payoff is generated by the portfolio b_g^t. Therefore, independently of the household who accepts bank $1's$ offer

$$\hat{y}_t^0 < q b_g^t = \tau_t^0$$

Suppose bank 1 offers contract $\hat{y}_t$ and sells portfolio b_g^t in addition to its portfolio decision under the original candidate equilibrium, l_1. It then obtains a payoff in every state s in the second period

$$y^s l_1 + \tau_t^s - y^s b_g^t = y^s l_1$$

and in the first period

$$-ql_1 - \hat{y}_t^0 + qb_t^g > -ql_1 - qb_t^g + qb_t^g = -ql_1$$

Therefore, the strategy $\left(\hat{y}_t, l_1 - b_g^t\right)$ strictly improves the bank payoff in the first period and does not change the firm payoff in any other state of nature in the second period. Therefore, the bank is strictly better of choosing this strategy then by following its original strategy. But this contradicts the definition of equilibrium. Therefore, it is not possible for the boundary condition to be violated and thus Ricardian equivalence must hold. The symmetric argument works if $\mathcal{T}$ is the government policy at a given equilibrium and $\mathcal{B}(b_g)$ is the alternative policy. The proof is then complete. □

4. State Dependent Taxes

4.1. Neutral Policies

In the previous sections we assumed that taxes do not depend on the individual state of nature. The previous results can be extended for a certain class of taxes that do depend on the individual states of nature. The definition of neutral policies in this case, however, gets quite cumbersome.

Definition 4.1. *We say that the policies $\{\mathcal{T}, \mathcal{B}(b_g)\}$ are* **neutral** *if for every t there are b_g^t and $y_t \in \mathcal{Y}(t)$ such that*

i) $y_t^{s,s^t} = \tau_t^{s,s^t}$ *for every* (s, s^t)
ii) $y^s b_g^t = \sum_{s^t} \Pi_t(s, s^t)\, \tau_t^{s,s^t}$ *for each* s
iii) $\tau_t^0 = qb_g^t$

Moreover, if $t, t' \in \mathcal{J}_i$ then

$$iv)\ \sum_{s^t} \Pi_{t'}\left(s, s^t\right) \tau_t^{s,s^t} = \sum_{s^t} \Pi_t\left(s, s^t\right) \tau_t^{s,s^t} \text{ for every } s$$

Notice that the very definition of neutral policy requires the existence of a portfolio b_g^t and a contract $y_t \in \mathcal{Y}(t)$ since markets may be incomplete. If such a portfolio or contract do not exist then a change in government policy may lead to a distribution of income across second period states of nature for a household that it is not feasible for the banks to implement. But then, a change in government policy may lead to a distribution of income for a household that is not possible

for a bank to offer and, thus, may trivially have real effects. Hence, Ricardian equivalence may fail if government policies are not neutral.

Finally, the role of assumption *(iv)* is to guarantee that the government does not have more information then the banks and thus it is not able to implement changes in the households income distribution that banks cannot reproduce. Moreover, it also guarantees that changes in the government policies do not have an effect on the distribution of wealth due to the inability of the government to differentiate among different types of households. Indeed, suppose this assumption does not hold. Under policy $\mathcal{B}(b_g)$ if the government cannot differentiate between types t and t' it must impose the same taxes in each state of nature. But then, if assumption *(iv)* does not hold, in some aggregate state of nature households total expected taxes are different. In particular, types t and t' pay different (discounted) taxes. On the other hand, under policy $\mathcal{T}$ both households pay the same taxes in the first period since, once more, the government cannot differentiate between these types. But then a change in the government policy does lead to a change in the distribution of wealth across households. We will show later that Yotsuzuka model violates precisely this condition and that indeed in his model a change in the government policy does lead to a change in the distribution of wealth.

It is easy to verify that every pair of policies which are neutral under the previous definition are also neutral under this alternative definition. Moreover, all previous results extend almost immediately for this case as well.

4.2. Yotsuzuka Model and Non-Neutral Policies

In the previous section we showed that Ricardian equivalence holds at every free-entry equilibrium provided that government policies are neutral. This result contrasts with Yotsuzuka (1987) who provides examples in a standard adverse selection model where Ricardian equivalence may not hold with risk-neutral banks and free-entry equilibrium. His examples, however, rely on the fact that a change in the government policy implies changes in distribution of wealth across households. Therefore, *the government policies considered by Yotsuzuka are not neutral.*

In Yotsuzuka model households have uncertainty about their second period income, which may be high or low. If a household faces low income then she defaults and, in particular, does not pay taxes. Therefore, if the government decides to finance its expenditures by issuing bonds in the first period and taxing households in the second period then taxes must be state dependent. Since the government does not distinguish between high and low risk households, it

must tax both types of households in the same amount in each state of nature in the second period. But this implies that under this government policy the low risk household must pay higher (discounted) taxes then if the government taxes households in the first period, while the reverse happens to the high risk households. But then, a change in policy by the government has implications over the distribution of wealth across households. It is not surprising that Ricardian equivalence fails in Yotsuzuka model.

The surprising aspect of Yotsuzuka paper, quite the contrary, is that Ricardian equivalence still holds under a very specific formulation of the adverse selection model. This formulation corresponds to Jaynes (1978) version of the original Rothschild and Stiglitz model where by enlarging banks' strategic space and by assuming that households cannot commit to accept a single bank offer its is argued that: i) an equilibrium always exits; and ii) some pooling of contracts may be observed in equilibrium. A central feature of Jaynes model is that contracts are not observable, contrarily to the original Rothschild and Stiglitz model. *Each bank's offer specifies not only the amount of insurance and its price but also if the bank is going to make public the fact that the household has accepted the bank offer.*

Ricardian equivalence, under the set of policies considered by Yotsuzuka, holds for any equilibrium in the Jaynes model. Yotsuzuka also provides examples where Ricardian equivalence does not hold for the standard Rothschild and Stiglitz separating equilibrium. Since in the former model the information about contracts is an endogenous variable Yotsuzuka refers to Jaynes model as a generalization of the Rothschild and Stiglitz model and concludes that Ricardian equivalence is re-established once the information flow about contracts is appropriately treated as an endogenous variable.

This conclusion does not seem accurate. A precise formulation of Jaynes model is provided by Hellwig (1988), who shows that the equilibrium solution obtained by Jaynes relies strongly on a very specific timing of the model. In particular, *natural variations of the sequence of movements in this dynamic game do not produce Jaynes equilibrium but rather the standard Rothschild and Stiglitz separating equilibrium outcome, even when contracts are not observable and each bank offer specifies if the bank is going to make public the set of households who have accepted it.* Hellw!! But in this separating equilibrium outcome, as shown by Yotsuzuka himself, Ricardian equivalence may not hold for the non-neutral polices he considered. Therefore, Yotsuzuka's claim that with a more general class of model Ricardian equivalence with non-neutral policies is re-established does not seem entirely correct.

The peculiarity of Yotsuzuka result, however, should not obscure an important

remark. Due to asymmetric information, the set of policy available to the government may not neutral and the existing alternative policies may have distribution effects. Therefore, Ricardian equivalence may not hold. It is common, however, in the macroeconomic literature on taxation to assume the existence of a representative household. But in this case government policies are always neutral and, in particular, the income distribution effects of alternative government policies, and hence the eventual failure of Ricardian equivalence, is not investigated.

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