

RATIONAL EXPECTATIONS AND THE SERVICE OF PUBLIC DEBT: A RECURSIVE APPROACH

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Abstract

This paper uses dynamic programming to study the time consistency of optimal macroeconomic policy in economies with recurring public deficits. To this end, a general equilibrium recursive model introduced in Chang (1998) is extended to include government bonds and production. The original model presents a Sidrauski economy with money and transfers only, implying that the need for government financing through the inflation tax is minimal. The extended model introduces government expenditures and a deficit-financing scheme, analyzing the Sargent-Wallace (1981) problem: recurring deficits may lead the government to default on part of its public debt through inflation. The methodology allows for the computation of the set of all sustainable stabilization plans even when the government cannot pre-commit to an optimal inflation path. This is done through value function iterations, which can be done on a computer. The parameters of the extended model are calibrated with Brazilian data, using as case study three Brazilian stabilization attempts: the Cruzado (1986), Collor (1990) and the Real (1994) plans. The calibration of the parameters of the extended model is straightforward, but its numerical solution proves unfeasible due to a dimensionality problem in the algorithm arising from limitations of available computer technology. However, a numerical solution using the original algorithm and some calibrated parameters is obtained. Results indicate that in the absence of government bonds or production only the Real Plan is sustainable in the long run. The numerical solution of the extended algorithm is left for future research.

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1. Introduction

Economists have continually studied the optimality of macroeconomic policy. One important facet of this study involves the time consistency of optimal policies associated with the financing of public deficits. Macroeconomic theory predicts that permanent fiscal deficits may induce an economy to default implicitly on its debt through inflation. Rational agents, who have expectations about the probability of default, take a gamble when purchasing government bonds and an equilibrium is attained in the economy. The purpose of this paper is two fold: first, to extend a model introduced in Chang (1998) to include government bonds and production to study this problem in a recursive dynamic analytical framework; second, to use the Brazilian experience to test its feasibility. The new model uses as a point of departure (1) the recursive model developed in Chang (1998) based on Calvo (1978); (2) a two-period model introduced in Calvo (1988). The latter studies the interaction between rational agents and government authorities in a public deficit financing scheme in which there is a likelihood of default on public debt. It allows for the possibility of two equilibria: a good, Pareto-optimal one with low interest rates and inflation, and a bad Pareto-inefficient equilibrium, with high interest rates and high inflation, an implicit default. Calvo concludes that in the absence of government pre-commitment, characterized through the issuance of indexed-bonds or an upper bound on interest rates, the economy settles in the Pareto-inefficient one, a dynamically inconsistent result. One problem with this model is that it does not allow for the confrontation of his hypothesis with time series data.

To confront Calvo's hypothesis with time-series data, Chang (1998) is extended to include non-indexed government bonds and production and is calibrated with Brazilian data. Chang (1998) is based on Calvo (1978), which assumes a simple Sidrauski economy with money and transfers only. The absence of government expenditures in the model eliminates the need for an onerous deficit-financing scheme through the implementation of heavy inflation taxes. The extended model attempts to address the Sargent-Wallace (1981) problem posed in Calvo (1988) from a recursive dynamic perspective. The extended model is calibrated with Brazilian data using as case studies three Brazilian stabilization attempts: the Cruzado (1986), the Collor (1990), and the Real plans (1994). The advantage of the new formulation is that it allows for the computation of the set of all dynamically consistent government plans, even when the government cannot pre-commit to an optimal inflation path. This is obtained through value function iteration, done on a computer. The numerical solution of the original model was straightforward. Results obtained indicate that in the absence of government bonds or production, only the Real Plan is sustainable in the long run. The Brazilian experience supports well the predictions of the model: that macroeconomic policies that deviate from optimal path may not be sustainable in the long run. The numerical solution of the extended model proves unfeasible due to a dimensionality problem in the algorithm arising from limitations in the available computer technology and is left for future research.

The paper is organized as follows: Part 1 is the introduction; Part 2 presents the model; Part 3 conducts the calibration; Part 4 computes the numerical solution and Part 5 concludes.

2. The model

Time is discrete and there are infinite periods: $t = 0, 1, 2, \dots, \infty$. There is a welfare maximizing government in a closed economy who sets marginal tax rates and seignorage to smooth out the tax burden over time. There is one representative consumer and one consumption good that is consumed or invested. There is production given by $F(k_t)$, the only source of income. There is depreciation, δ , and no population growth. There are two assets: money and nominal government bonds. Households take initial capital stock, k_0 , initial money holdings, M_0 , initial stock of government bonds, $b_0 = 0$, and marginal tax rate, τ_t as given.

Preferences:

The representative household's preferences over consumption and money holding are given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)]$$

Functions u and v satisfy:

- (1) $u : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ is C^2 , strictly concave, and strictly increasing.
- (2) $v : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ is C^2 , and strictly concave.
- (3) $\lim_{c \rightarrow 0} u'(c) = \lim_{m \rightarrow 0} v'(m) = \infty$
- (4) There is a finite satiation level of money, $m^f > 0$ such that $v'(m^f) = 0$

Furthermore, consumption, the price level and the real value of money holding at the end of period t are respectively given by c_t, p_t, m_t where

$$m_t = \frac{M_t}{P_t}$$

$$q_t = \frac{1}{P_t}, \text{ the inverse of the price level}$$

Money supply:

Money grows at a constant rate μ_t .

Real money growth, inflation and the satiation level of money holdings are respectively given by

$$\frac{P_{t+1}}{P_t} m_{t+1} = (1 + \mu_t) m_t$$

$$\pi_t = \frac{(P_{t+1} - P_t)}{P_t}$$

$$m = m^f > 0$$

Government Bonds:

Bonds are one-period to maturity, zero coupon government bonds, where

$Q_t = \$1$ is the normalized price of a bond

B_t is the number of bonds

$b_t = \frac{B_t}{P_t}$ is the real value of a bond at period t

$R_{nt} = \frac{(1 - Q_t)}{Q_t}$ is nominal return on bonds at period t

$(1 + r_{gt}) = \frac{(1 + R_{nt})P_t}{P_{t+1}}$ is the real rate of return on bonds, or $(1 + r_{gt}) = \frac{(1 + R_{nt})}{(1 + \pi_t)}$

$\frac{b_{t+1}}{(1 + r_{gt})}$ is the real present value of a bond that matures at period $t+1$

$b_0 = 0$ is the initial stock of bonds

$b = b^f > 0$ is an upper bound on bond holdings

Taxes

τ_t is a proportional tax levied at each period t . Consumers take τ_t as given.

$\tau_t F(k_t)$ is the total tax revenue, a fraction of total production.

Deficit financing

The level of g_t at each period t is a government decision exogenous to the model. At each period, the government chooses how much money to create or to withdraw from circulation and how many bonds to issue or to retire, and must satisfy:

$$g_t - \tau_t F(k_t) = \left[\frac{b_{t+1}}{(1 + r_{gt})} - b_t \right] + \left[(1 + \pi_t)m_{t+1} - m_t \right]$$

Default

$\frac{p_t}{p_{t+1}}$ is the share of bonds not defaulted at period t

$$\frac{p_t}{p_{t+1}} = (1 - \theta_t) = \frac{1}{(1 + \pi_t)}$$

$\theta_t = \frac{\pi_t}{(1 + \pi_t)}$ is the share of bonds defaulted at period t

The consumer problem

The household will maximize

∞

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)] \quad \text{s.t.}$$

$$c_t + k_{t+1} + (1 + \pi_t)m_{t+1} + \frac{b_{t+1}}{(1+r_g)} = (1 - \tau_t)F(k_t) + k_t + m + b_t$$

$$k_t = (1 - \delta)k_t + i$$

The government problem

Government will maximize its representative consumers' utility

∞

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)] \quad \text{s.t.}$$

$$g_t - \tau_t F(k_t) = \left[\frac{b_{t+1}}{(1+r_g)} - b_t \right] + \left[(1 + \pi_t)m_{t+1} - m_t \right] \quad \text{where}$$

$g_t - \tau_t F(k_t)$ is the government primary deficit at period t

The consumer value function:

States: k, m, b

Controls: c', k', m', b'

Prices: p, p'

$$V(k, m, b) = \max_{k', m', b'} \left\{ u[(1 - \tau)F(k) + m + b + k - [k' + (1 + \pi)m' + \frac{b'}{(1+r_g)}]] + v(m) \right. \\ \left. + V(k', m', b') \right\}$$

FOC's:

$$k' : -(1)u_c[(1 - \tau)F(k) + m + b + k - [k' + (1 + \pi)m' + \frac{b'}{(1+r_g)}]] + \beta V_k(k', m', b') = 0$$

$$m' : -(1 + \pi)u_c[(1 - \tau)F(k) + m + b + k - [k' + (1 + \pi)m' + \frac{b'}{(1+r_g)}]] + \beta V_m(k', m', b') = 0$$

$$b' : \frac{-(1)}{(1+r_g)}u_c[(1 - \tau)F(k) + m + b + k - [k' + (1 + \pi)m' + \frac{b'}{(1+r_g)}]] + \beta V_b(k', m', b') = 0$$

The government value function:

$$W(m, b) = \max_{b', \tau} \left\{ u[(1 - \tau)F(k + m + b + k - [k' + (1 + \pi)m' + \frac{b'}{(1+r_g)}])] + v(m) \right. \\ \left. + W(m', b') \right\} \text{s.t.}$$

$$g - \tau F(k) = \left[\frac{b'}{(1+r_g)} - b \right] + [(1 + \pi)m' - m]$$

Competitive Equilibrium

A *policy* is a sequence of nominal money growth rates, $\mu = (\mu_0, \mu_1, \dots)$, a sequence of tax rates, $\tau = (\tau_0, \tau_1, \dots)$ and a sequence of bonds $b = (b_0, b_1, \dots)$ s.t. $\mu \in [\underline{\mu}, \bar{\mu}] \equiv [\underline{\Pi}, \bar{\Pi}] \equiv \Pi$ and all $t \geq 0$. An *allocation* is a set of nonnegative sequences of consumption $c = (c_0, c_1, \dots)$, capital, $k = (k_0, k_1, \dots)$, real money demands, $m = (m_0, m_1, \dots)$, and bonds, $b = (b_0, b_1, \dots)$.

Given an initial capital stock, k_0 , an initial nominal money holding M_0 , an initial stock of bonds, $b_0 = 0$, and a government policy (μ, τ, b) , a CE is formed by an allocation (c, k, m, b) , a value function V and a vector of prices (π, r_g) if:

(i) The FOC's of the consumer problem are satisfied:

$$(1) \quad k' : (-1)u_c(c) + \beta V_k(k', m', b') = 0$$

$$(2) \quad m' : (-1)(1 + \pi)u_c(c) + \beta V_m(k', m', b') = 0$$

$$(3) \quad b' : \frac{(-1)}{(1 + r_g)}u_c(c) + \beta V_b(k', m', b') = 0$$

(ii) Markets clear every period:

(4) Money demand equals money supply:

$$m_{t+1} = \frac{(1 + \mu_t)}{(1 + \pi_t)} m_t$$

(5) Bond demand equals to bond supply:

$$b_{t+1} = (1 + r_{gt})\{g_t - \tau_t F(k_t) + b_t - [(1 + \pi_t)m_{t+1} - m_t]\}$$

(6) Total expenditures equals total resources:

$$c_t + k_{t+1} = F(k_t) - \tau_t F(k_t)$$

The Continuation Value of Equilibrium

The *continuation* of a CE is a CE. In other words, if $(m, \tau, \mu, b) \in \text{CE}$, then $(m_t, \tau_t, \mu_t, b_t) \in \text{CE}$. Following Chang (1998), the continuation value of equilibrium, will be used as an artificial state variable in the Ramsey problem with bonds. The three envelope conditions will yield a vector of promises made at period t : the promised marginal value of capital, the promised marginal utility of money, and the promised marginal value of bonds, respectively given by

$$V_k = [(1 - \tau)F_k(k)]u_c(c) = \phi'_k$$

$$V_m = u_c(c) + v_m(m) = \phi'_m$$

$$V_b = u_c(c) = \phi'_b$$

Therefore, all the promises made at period t are summarized by $\phi' = (\phi'_k, \phi'_m, \phi'_b)$

Steady State

In steady state $c_{t+1} = c_t = \bar{c}$, $k_{t+1} = k_t = \bar{k}$, $m_{t+1} = m_t = \bar{m}$, and $b_{t+1} = b_t = \bar{b}$. These are obtained from the solution of the system of equations and unknowns formed by the definition of CE. The equilibrium conditions for the money and bonds market given for the maximization problem above are respectively

$$m_{t+1} = \frac{(1 + \mu_t)}{(1 + \pi_t)} m_t$$

$$b_{t+1} = (1 + r_{gt}) \{ g_t - \tau_t F(k_t) + b_t - [(1 + \pi_t) m_{t+1} - m_t] \}$$

The simplified Steady State consumer Euler equations are:

$$\bar{k} : \frac{1}{\beta} = (1 - \tau) F_k(\bar{k})$$

$$\bar{m} : u_c(\bar{c}) = \frac{\beta}{(1 + \pi - \beta)} v_m(\bar{m})$$

$$\bar{b} : \frac{1}{\beta} = (1 + r_g)$$

The Ramsey Problem with government bonds and perfect commitment:

Under perfect commitment the government's problem is to choose a policy μ and an associated CE such that there is no other CE that results in higher consumer welfare.

The problem is to choose (m, τ, μ, b) in CE to maximize

∞

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)] \text{ s.t.}$$

$$c_t + k_{t+1} = F(k_t) - \tau_t F(k_t)$$

The Ramsey problem must be solved recursively. From the FOC's and the continuation value of equilibrium defined above we have that all the promises made at t are given by:

$$(1 - \tau_t) F_k(k_t) = \phi'_k, \text{ the promised marginal value of capital}$$

$$u_c(c) + v_m(m) = \phi'_m, \text{ the promised marginal utility of money}$$

$$u_c(c) = \phi'_b, \text{ the promised marginal value of bonds}$$

$$\phi' = (\phi'_k, \phi'_m, \phi'_b), \text{ the vector of promises made at period } t$$

Let the set of initial promises consistent with CE be defined by Ω , where $\exists(m, \tau, \mu, b) \in \Omega$:

$$\Omega = \{ (m, \tau, \mu, b) \in CE \mid \phi \in \mathbb{R}^3 : (([(1 - \tau_0) F_k(k_0)] u_c(c_0)), (u_c(c_0) + v_m(m_0)), (u_c(c_0))) \}$$

The government problem becomes a $W^*(\phi)$:

Maximize

∞

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)] \text{ s.t.}$$

$$(m_t, \tau_t, \mu_t, b_t) \in \Gamma(\phi) \text{ where}$$

$\phi = (\phi_k, \phi_m, \phi_b)$, a vector of initial promises, and

$$\Gamma(\phi) = \{(m_t, \tau_t, \mu_t, b_t) \in CE \mid \phi = (\phi_k, \phi_m, \phi_b)\}$$

Therefore, $\Gamma(\phi) = \{(m_t, \tau_t, \mu_t, b_t) \in CE \mid \phi = ((1 - \tau_0)F_k(k_0), (u_c(c_0) + v_m(m_0)), (u_c(c_0)))\}$

Given a vector of initial promises $\phi = (\phi_k, \phi_m, \phi_b)$ in Ω , $\Gamma(\phi)$ is a non-empty, compact subset of CE.

Since the objective function is continuous, $W^*(\phi)$ is well defined on Ω . If $W^*(\cdot)$ were known, then the problem

would be to obtain the max of $W^*(\phi)$ on Ω . Thus, a dynamic programming formulation can be obtained.

$W^*(\phi)$ satisfies the functional equation

$$W(\phi) = \text{Max} \{[u(c) + v(m)] + \beta W(\phi')\} \text{ s.t.}$$

$$1) (m, \tau, \mu, b, \phi') \in E \times \Omega$$

$$2) \phi = (\phi_k, \phi_m, \phi_b) \text{ where}$$

$$\phi_k = (1 - \tau)F_k(k)u_c(c)$$

$$\phi_m = u_c(c) + v_m(m)$$

$$\phi_b = u_c(c)$$

$$3) g - \tau F(k) = \left[\frac{b'}{(1 + r_g)} - b \right] + [(1 + \pi)m' + m]$$

$$4) (1 - \tau)F_k(k)u_c(c) = \beta \phi'_k$$

$$u_c(c) + v_m(m) = \beta \phi'_m$$

$$u_c(c) = \beta \phi'_b$$

If a bounded function $W : \Omega \rightarrow \mathbb{R}^3$ satisfies the above functional equation, then $W = W^*$. If the set of all possible artificial states Ω can be computed, the Ramsey problem can be solved recursively. The set Ω can be computed by using the fact that it is the fixed point of a particular operator.¹ Let W be a subset of \mathbb{R}^3 . A new subset $B(W)$ will be defined as:

¹ Chang (1998)

$$B(W) = \{ \phi \in \mathbb{R}^3 : \exists (m, \tau, \mu, b, \phi') \in E \times W \mid$$

$$\phi = (\phi_k, \phi_m, \phi_b), \text{ where}$$

$$\phi_k = (1 - \tau)F_k(k)u_c(c)$$

$$\phi_m = u_c(c) + v_m(m)$$

$$\phi_b = u_c(c)$$

$$g - \tau F(k) = \left[\frac{b'}{(1 + r_g)} - b \right] + \left[(1 + \pi)m' + m \right] \text{ and}$$

$$(1 - \tau)F_k(k)u_c(c) = \beta \phi'_k$$

$$u_c(c) + v_m(m) = \beta \phi'_m$$

$$u_c(c) = \beta \phi'_b \text{ hold } \}$$

$$\text{Then, } W \subseteq B(W) \Rightarrow B(W) \subseteq \text{ and } = B(\text{ })$$

Thus, Ω can be computed by iterating on B . Once Ω is known, the functional equation can be solved to obtain W^* . The introduction of the vector of artificial state variables $\phi = (\phi_k, \phi_m, \phi_b)$ takes care of the requirement that the new Ramsey Problem must be consistent with a perfect-foresight CE.

The Ramsey Problem with no Pre-commitment

Next we assume that the government cannot pre-commit to an infinite policy sequence. The CE is defined as a set of Sustainable Plans as described below.² The goal is to compute the set of all SPs recursively. The incentive constraint is handled through the introduction of the continuation value of the equilibrium, which is the vector of promises $\phi = (\phi_k, \phi_m, \phi_b)$ and is used as artificial state variables. After any history, the continuation of a SP must be consistent with a CE for the infinite future. To ensure CE for the infinite future, the set of SPs must include two vectors of artificial state variables. The first is the one corresponds to the three FOC's; the second is the vector promises made at period t . The SPs and sustainable outcomes will be defined next.

Sustainable Plans

Sustainable Plans will now be defined as in Chang (1998). Let a money growth history and a government strategy be defined as:

$$\text{History: } \mu' = (\mu_0, \mu_1, \dots, \mu_t), \text{ restricted to } \mu \in [\underline{\mu}, \bar{\mu}] \equiv [\underline{\pi}, \bar{\pi}] = \Pi$$

$$\text{Strategy: } \sigma' = (\sigma_0, \sigma_1, \dots, \sigma_t) \text{ such that } \sigma_0 \in \Pi \text{ and } \sigma_t : \Pi^{t-1} \rightarrow \Pi$$

The strategy space available to the government will be restricted to those consistent with CE.³

² Procedure is the same as in Chang (1998)

³ As in Chang (1998)

Let CE_{Π} be the set of infinite horizon sequences of money growth consistent with CE . Thus, let

$$\mu \in \Pi^{\infty}$$

$$CE_{\Pi} = \{ \exists (\mu, m, \tau, b) | (m, \tau, b) \in CE \}$$

A strategy σ' is admissible if after any history μ^{t-1} , the continuation of $\mu' \in CE_{\Pi}$.

An allocation rule α is a sequence $\alpha = (\alpha_0, \alpha_1, \dots, \alpha_t)$ s.t. for each t , $\alpha_t : \Pi^t \rightarrow (0, m^f) \times \tau \times (0, b^f)$, where $\alpha_t(\mu') = \{m_t(\mu'), \tau_t(\mu'), b_t(\mu')\}$ denotes the real value of money, taxes and bonds respectively. Next, the government will be restricted to choose an admissible policy. It will announce for the future a policy that is consistent with the existence of CE. A government strategy σ and an allocation rule α constitute a **Sustainable Plan** if:

- (i) σ is admissible
- (ii) α is competitive given the strategy σ

After any history μ^{t-1} , the continuation of σ is optimal for the government, that is, the sequence μ_t induced by the strategy σ after a money growth history μ^{t-1} maximizes

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(m_t)] \text{ over } CE_{\Pi}, \text{ given } \alpha.$$

The continuation of a SP is a SP. Thus, any SP induces a CE sequence (m, τ, μ, b) and any CE sequence is a **sustainable outcome** if induced by some SP.⁴ Next, the SP is characterized recursively.

Consider first a “best” deviation. Let $\Theta = \{(m, \tau, \mu, b) \in CE \mid \exists \text{ a SP whose outcome is } (m, \tau, \mu, b)\}$ be the set of all sustainable outcomes. Define S as

$$S = \{(W, \phi) \mid \exists \text{ a SO } (m, \tau, \mu, b) \in \Theta \text{ with } W \text{ s.t. } \phi = (\phi_k, \phi_m, \phi_b), \text{ the vector of promises made at } t\}$$

The set S is the set of all pairs of continuation values and promises that may emerge in the first period of a SP. S will be characterized in a recursive way. Any SO implies an initial value W and an initial promise ϕ . The pair (W, ϕ) must belong to S .

In the first period, a SP must describe an initial optimal action μ_i and for each possible deviation from μ_i , i.e., each $\mu \in CE^0$, the SP must specify the real quantity of money $m(\mu)$, taxes $\sigma(\mu)$ and bonds $b(\mu)$. The SP will specify a continuation value, $W_{t+1}(\mu)$ and a continuation “promise” $\phi_{t+1}(\mu)$, which must belong to S . Let Z denote a subset of $W \times \Omega$. Define a new set D as

$$D(Z) \subseteq W \times \Omega$$

⁴ This follows from the Abreu, Pearce & Stachetti operator described in Chang (1998). The existence of SPs has already been proven in Chang (1998). The only difference is that the dimension has been increased with the inclusion of k, b .

$D(Z) = \{ (W, \phi) \mid \exists \text{ a } \mu \in CE^0, \text{ and for each } \mu \in CE^0, \exists \text{ a vector } (m(\mu), \sigma(\mu), b(\mu), W_{t+1}(\mu), \phi_{t+1}(\mu)) \text{ in } (0, m^f) \times T \times (0, b^f) \times Z \text{ s.t.}:$

- 1) $W = u(c(\mu) + v(m(\mu))) + \beta W'(\mu)$
- 2) $\phi = (\phi_k, \phi_m, \phi_b)$, where

$$\phi_k = [(1 - \tau(\mu))F_k(k)]u_c(c(\mu))$$

$$\phi_m = u_c(c(\mu)) + v_m(m(\mu))$$

$$\phi_b = u_c(c(\mu)) \} \text{ and for all } \mu \in CE^0,$$
- 3) $W \geq u[c(\mu)] + v[m(\mu)] + \beta W'(\mu)$
- 4) $g - \tau(\mu)F(k) = \left[\frac{b'(\mu)}{(1+r_g)} - b(\mu) \right] + \left[(1+\pi(\mu))m' + m(\mu) \right]$
- 5) $(1 - \tau(\mu))F_k(k)u_c(c(\mu)) = \beta\phi'_k$

$$u_c(c(\mu)) + v_m(m(\mu)) = \beta\phi'_m$$

$$u_c(c(\mu)) = \beta\phi'_b$$

Constraints 4 & 5 are necessary to ensure that the continuation of a SP after any deviation is consistent with a CE. If $Z \subseteq D(Z)$, then $D(Z) \subseteq S$ and $S = D(S)$, S is the largest fixed point of D .

The algorithm to compute S is given by $S_0 = W \times \Omega$, $S_n = D(S_{n-1})$, $n = 1, 2, \dots$. The sequence converges to S .

Therefore, it is possible to compute S and a SP corresponding to any (W, ϕ) in S .

Next, consider a deviation that minimizes welfare losses to society.⁵ Let μ be a deviation from equilibrium.

Let Z be a compact set s.t. $S \subseteq Z \subseteq W \times \Omega$. Define P as

$P(\mu; Z) = \text{Min } u[c] + v(m) + \beta W'$ s.t

- 1) $g - \tau F(k) = \left[\frac{b'}{(1+r_g)} - b \right] + \left[(1+\pi)m' + m \right]$
- 2) $(m, \tau, b, W_{t+1}, \phi_{t+1}) \in (0, m^f) \times T \times (0, b^f) \times Z$
- 3) $(1 - \tau)F_k(k)u_c(c) = \beta\phi'_k$
- 4) $u_c(c) + v_m(m) = \beta\phi'_m$
- 5) $u_c(c) = \beta\phi'_b$

If Z were equal to S , then $P(\mu; Z)$ would be the worst possible continuation after the deviation h in CE^0_π .

This is extended to allow for punishments supported by (W, ϕ) in sets $Z > S$. Let

$BR(Z) = \text{Max } P(\mu; Z)$ subject to $\mu \in CE^0_\pi$

If $Z = S$, then $BR(Z)$ would be the government's *best deviation*. Finally, define $E(Z)$ as

⁵ The Cronshaw and Luenberger Operator in Chang (1998)

$E(Z) = \{ (W, \phi) \in W \times \Omega \mid \exists (m, \tau, \mu, b, W', \theta') \in E \times Z \text{ s.t. all the following conditions hold:}$

$$1) \quad g - \tau F(k) = \left[\frac{b'}{(1+r_g)} - b \right] + \left[(1+\pi)m' + m \right]$$

$$2) \quad (1-\tau)F_k(k)u_c(c) = \beta\phi'_k$$

$$3) \quad u_c(c) + v_m(m) = \beta\phi'_m$$

$$4) \quad u_c(c) = \beta\phi'_b$$

$$5) \quad W = u(c) + v(m) + \beta W'$$

$$6) \quad \phi = (\phi_k, \phi_m, \phi_b)$$

$$\phi_k = (1-\tau)F_k(k)u_c(c)$$

$$\phi_m = u_c(c) + v_m(m)$$

$$\phi_b = u_c(c)$$

$$7) \quad \text{and } W \geq \text{BR}(Z) \}$$

If $Z = S$, $E(Z)$ would include all pairs (W, ϕ) that could be enforced by a threat of reverting to the least favorable continuation for the government. To obtain the algorithm, we use the fact that

$$S \subseteq W \times \Omega \text{ and } Z_0 = W \times \Omega. \text{ For all } n = 1, 2, Z_n = E(Z_{n-1})$$

$\{Z_n\}$ is a sequence of compact sets that include S . Therefore, the set of sustainable outcomes and an algorithm to compute it are obtained.

3. Calibration

The calibration procedures conducted herein follow standard guidelines present in the literature,⁶ and involve mapping the model into observed features of the Brazilian data. Going from the theoretical framework to the quantitative analysis involves three steps: first, choosing a parametric class; second, constructing data measurements that are consistent with the parametric class chosen, establishing a correspondence between the parametric class chosen and the observed data. This requires re-organizing the data in such way that is consistent with the model; finally, assigning values to the above-mentioned parameters of the model, which involves setting parameter values so that the behavior of the model matches the features of the measured data. For this model economy, this implies estimating values to the parameters of technology, productivity A , capital share, θ , depreciation δ , preferences, α , ρ , β , and tax policy, τ .⁷

3.1 Functional Forms

The first step in the calibration procedure is to choose the functional forms used. The analytical framework underlying the model developed herein involves utility maximization with money in the utility function in a monetary economy with production and two assets: money and bonds. Functional forms were chosen to ensure that assumptions (1)-(4) of the theoretical model, as described in Chapter 3, are met.

Utility:

Utility is represented by

$$U(c_t, m_t) = u(c_t) + v(m_t)$$

where

$$u(c_t) = \log(c)$$

$$v(m) = m_f m + \frac{m^2}{2},$$

m_f is the satiation level of money

Production:

The model assumes a *Cobb-Douglas* production function with constant returns to scale and depreciation (δ).

$$F(K) = AK^\theta L^{1-\theta}$$

K is the total capital stock

L is the economically active population.

A is a measure of average overall technological change or *average productivity*

⁶ Kydland & Prescott (1982), Cooley & Hansen (1992)

⁷ The main sources of time series data used for calibration of the model were: Getulio Vargas Foundation (FGV), the Brazilian Institute of Applied Economics (IPEA), the Brazilian Central Bank, the Brazilian Treasury Department, the Brazilian Institute of Geography and Statistics (IBGE) and the IMF.

- θ is share of capital in measured GDP
 δ is the depreciation rate

In per worker terms, the production function becomes

$$F(k) = Ak^\theta, \text{ where } k = \frac{K}{L}$$

3.2 Calibration of the Parameters

The next step is to calibrate the parameters:

Preferences: ρ, β, α

Technology: θ, A, δ

Policy: μ, τ

Preferences:

Utility is given by

$$\sum_{t=0}^{\infty} u(c_t) + v(m_t) \text{ where}$$

$$u(c_t) = \ln(c_t)$$

$$v(m_t) = am + \frac{m^2}{2}$$

Thus, the discounted lifetime utility becomes

$$\sum_{t=0}^{\infty} \beta^t \left(\ln(c_t) + \left[m_f m + \frac{m^2}{2} \right] \right)$$

The simplified steady state consumer Euler equations obtained above are:

$$\bar{k} : \frac{1}{\beta} = (1 - \tau) F_k(\bar{k})$$

$$\bar{m} : u_c(\bar{c}) = \frac{\beta}{(1 + \pi - \beta)} v_m(\bar{m})$$

$$\bar{b} : \frac{1}{\beta} = (1 + \rho)$$

The subjective rate of time preference, ρ - in steady state, return to government bonds is equal to return to capital.

The discount factor, β – was estimated as per the definition

$$\beta = \frac{1}{1 + \rho}$$

The satiation level of money, m_f – was calibrated using the simplified steady state euler equation :

$$u_c(\bar{c}) = \frac{\beta}{(1 + \bar{\pi} - \beta)} v_m(\bar{m})$$

$$u_c(\bar{c}) = \frac{1}{\bar{c}}$$

$$v_m(\bar{m}) = m_f - \bar{m}$$

$$\frac{1}{\bar{c}} = \frac{\beta}{1 + \bar{\pi} - \beta} \left\{ m_f - \bar{m} \right\}$$

$$\xi = \frac{\beta}{(1 + \bar{\pi} - \beta)}$$

$$m_f = \frac{1}{\bar{c}\xi} + \bar{m}$$

$$\bar{c} = f(\bar{k}) - \delta\bar{k} - (\bar{g} + \bar{X}) \quad \text{or}$$

$$\bar{c} = f(\bar{k}) - \bar{i} - (\bar{g} + \bar{X})$$

where i is the net investment per quarter, or gross investment less depreciation.

Technology: the share of capital θ and productivity parameter A – were initially calibrated by regressing real GDP on net fixed capital stock and economically active population. Originally, quarterly data was used. Since these parameters do not enter the numerical solution using the original algorithm, this was not re-calculated with yearly data.

$$1 + \rho = (1 - \tau)F_k(\bar{k})$$

$$F(k) = Ak^\theta$$

$$Lny = LnA + \theta Lnk$$

Depreciation (δ) – A quarterly series based on average yearly levels was constructed using as point of departure the gross capital stock and net capital stock series constructed by IPEA.

3.3 Calibration Results

Initially, all parameters were calibrated with quarterly data for the entire period studied: 1980-1997. Next, the exercise was repeated with yearly data. The calibration was repeated for the period range encompassed by each of the three major stabilization plans, Cruzado, Collor and Real, as well as for the pre-Cruzado period. It is worth mentioning that beginning in the 60s, Brazil experienced numerous shocks that included a military takeover, resulting in institutional changes, price controls, indexations, hyperinflation, etc. Therefore, the economy was not

near a steady state during this period. Consequently, the results obtained in the first calibration attempt (1980-1997) presented distortions reflecting these facts. For this reason, the fundamental parameters β and m_f were calibrated again using data from the 50s and 60s, when the economy resembled closer steady state equilibrium. Calibration results are reported on Table 1. Sample averages are reported on Table 2.

The estimation of the preference parameters was straightforward. The parameters ρ and β followed directly from the assumption that in steady state all interest rates in the economy are equal. Real interest rates were estimates from the Fisher equation, by obtaining the difference between the nominal interest rates on government bonds and inflation. The Cobb-Douglas production function parameters were obtained by running a standard restricted form of the growth accounting regression, by regressing per worker output growth, $\log(Y/L)$, on capital per worker growth, $\log(K/L)$. The full-range sample was used as reference model. The Durbin-Watson for the initial specification of the reference model indicated the presence of serial correlation, indicating that different factors other than capital and labor affected output, These could include the institutional framework underlying the different stabilization attempts.

The most difficult parameter to calibrate was the average tax rate, τ . This was due to the fact that several taxes were created and eliminated during the period studied, making their consolidation extremely difficult. Historical data on taxes that are no longer in effect (for example, the export tax) is no longer available from the usual sources prior to 1992. Historical series are available only for total federal fiscal income prior to 1992. Historical data for state and municipal taxes are not available, with the exception of sales tax. Obtaining and consolidating this information would require an amount effort and time beyond what is feasible for the conclusion of this paper, and will be left for future research. Since taxes do not affect the steady state equilibrium of the model, a narrower period range for the study was allowed. This made possible estimating the averages with the exiting data sets, since the narrower data range does not affect the steady state equilibrium or the numerical solution of the model and results. The calibrated tax parameter was 14.5%. Due to these problems, the tax rate calibrated herein is underestimated, but can be used as a point of departure for this research. The breakdown of the tax structure is reported on Table 3. Figures 1 and 2 summarize data on inflation, interest rates, M1 and government bonds. A more elaborate calibration will be left for future work.

Figure 1
Quarterly Inflation and Nominal Interest Rates, 1980-1998

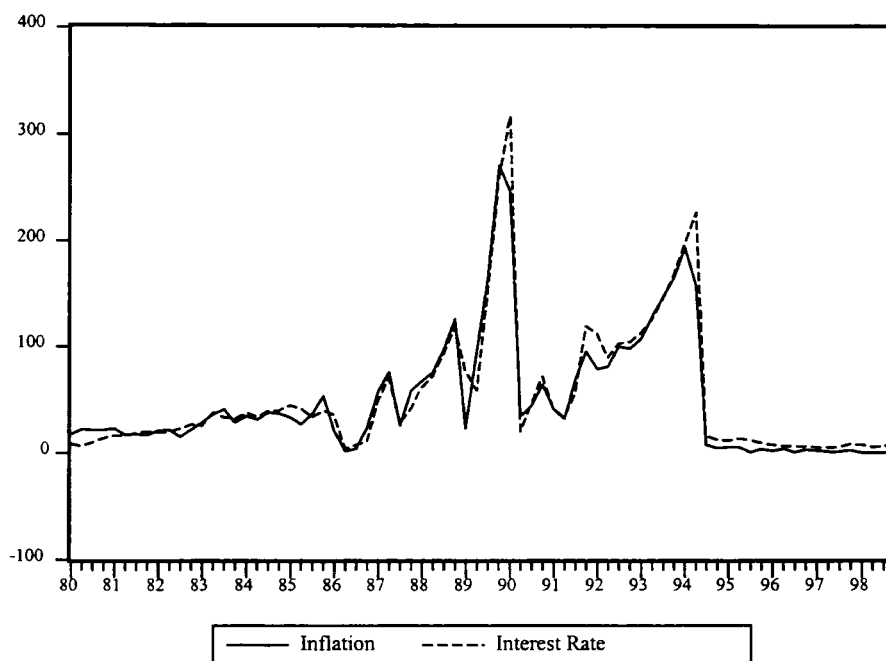


Figure 2
Real M1 and G-Bonds per worker as GDP ratio, 1998 Real.

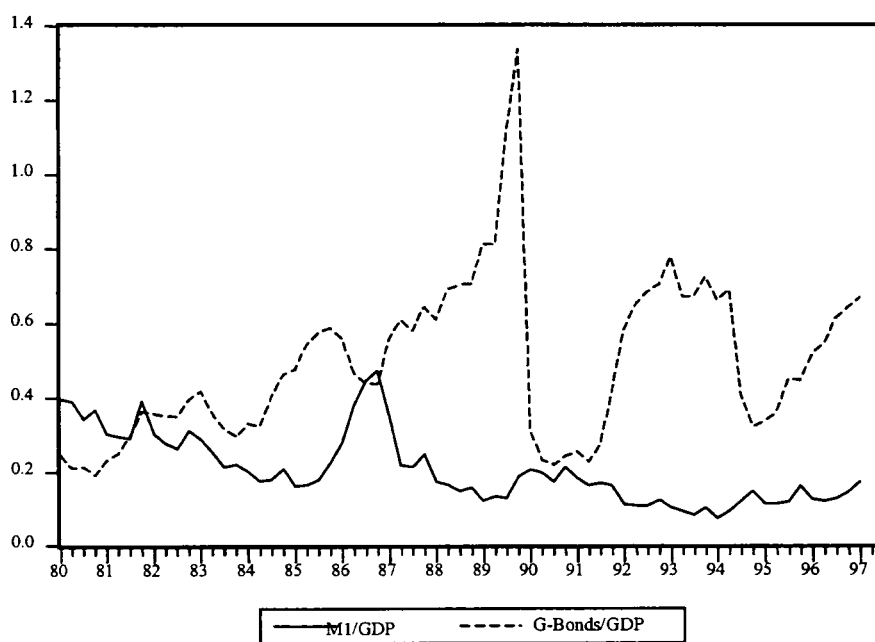


Table 1.a: Calibration Results

Period	Preferences		
	ρ	β	$\% m_f^*$
1980-1997 Full Range Period	0.4357600	0.697556	5.36
1980-1986 Pre Cruzado	0.239808	0.806575	7.29
1987-1990 Cruzado Plan	1.311219	0.432672	5.49
1990-1994 Collor Plan	1.187509	0.457140	3.15
1994-1997 Real Plan	0.240482	0.806138	3.43
1987-1994 High Inflation Period	1.247832	0.444873	3.31
Steady State	0.05	0.95	25

Values estimated using yearly data

Averages for the year

*** Estimated using average money balances as GDP ratio*

Table 1.b: Calibration Results (Quarterly Data)

Period	Preference		Technology			Policy
	ρ	β	m_f	θ	δ^*	τ^{**}
1980-1997	0.046185	0.95585	667	0.489549	0.30000	0.144
1980-1986 Pre Cruzado	-0.005287	1.00531	1,026	0.422361	0.25888	N/A
1987-1990 Cruzado Plan	0.000193	0.999806	574	0.592313	0.30608	N/A
1990-1994 Collor Plan	0.077443	0.928124	307	0.422091	0.33472	N/A
1994-1997 Real Plan	0.056869	0.94619	433	0.497986	0.35122	0.144
1987-1994 High Inflation	0.038818	0.963965	440			N/A

Values estimated using quarterly data

*Average for the year

**Data available from 1992 on

Table 2: Sample Averages

Period	$\bar{\pi}\%$	$\bar{r}\%$	\bar{c}^*	\bar{y}^*	\bar{k}^{**}	\bar{g}^*	\bar{m}^{**}	\bar{b}^{**}
1980-1997	218	43	9,582	12,433	18,057	714	667	1,520
1980-1986 Pre Cruzado	167	24	10,834	14,068	20,736	718	1026	1,304
1987-1989 Cruzado Plan	1052	131	10,445	13,801	19,167	783	574	1,927
1990-1994 Collor Plan	1080	119	9,729	12,773	13,279	711	307	1,386
1995-1997 Real Plan	7	24	9,451	12,592	16,518	826	433	1,662
1987-1994 High Inflation	1066	125	10,087	13,287	16,223	747	440	1,657

* Values in real terms per worker, yearly data, 1998 Real

** Values in real terms per worker, stock, 1998 Real

Table 3. Tax Structure

TOTAL TAXES 1998 Prices				Total Withheld				Total Income		ICMS Sales Tax		IPI - Tax on Industrialized Products		PLAN FOR GOV WORKERS Social Security Contribution	
Unit: R\$ Million (Yearly)				Total Withheld	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio
Year	Nominal	Price index(July)	GDP 1998 Prices												
1992	619,493	7 12E-04	714,207,276	8076.82	0.011	5,647.0	0.008	41.3	0.067	8,457.5	0.012	286.7	0.0004		
1993	14,116,170	1 33E-02	738,548,713	10120.06	0.014	6,420.0	0.009	856.8	0.061	9,780.7	0.013	253.0	0.0003		
1994	360,919,362	6 61E-01	740,295,203	11492.82	0.016	7,533.9	0.010	25,539.7	0.071	10,423.3	0.014	1,241.8	0.0017		
1995	658,141,237	8 25E-01	820,152,984	17377.93	0.021	11,507.0	0.014	47,198.9	0.072	13,635.1	0.017	2,100.8	0.0026		
1996	751,506,870	9 06E-01	842,323,504	18293.61	0.022	15,371.3	0.018	55,682.0	0.074	15,511.6	0.018	2,477.1	0.0029		
1997	864,111,026	9 65E-01	879,958,826	20872.75	0.024	15,651.1	0.018	59,564.0	0.069	16,833.5	0.019	2,643.9	0.0030		
Average					1.786%			Average	1.281%	Average	6.881%	Average	1.556%	Average	0.182%

TOTAL TAXES 1998 Prices				IOF Financial Transactions Tax		ITR Real Estate Taxes		NET PROFIT TAX		COFINS Social Security Tax on Firm Revenue	
Unit: R\$ Million (Yearly)				Total Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio
Year	GDP Nominal	Price index(July)	GDP 1998 Prices								
1992	619,493	7 12E-04	714,207,276	2,362.5	0.003	17.8	0.008	2,297.0	0.003	3,590.0	0.005
1993	14,116,170	1 33E-02	738,548,713	3,024.3	0.004	29.4	0.009	3,526.4	0.005	5,452.4	0.007
1994	360,919,362	6 61E-01	740,295,203	3,668.9	0.005	17.5	0.010	4,499.1	0.006	10,717.9	0.014
1995	658,141,237	8 25E-01	820,152,984	3,223.5	0.004	104.7	0.014	5,852.0	0.007	15,225.8	0.019
1996	751,506,870	9 06E-01	842,323,504	2,854.7	0.003	262.0	0.018	6,597.9	0.008	17,892.1	0.021
1997	864,111,026	9 65E-01	879,958,826	3,784.9	0.004	209.3	0.018	7,698.5	0.009	19,033.9	0.022
Average					0.400%	Average	1.281%	Average	0.630%	Average	1.472%

TOTAL TAXES GDP Nominal Values				EXPORT TAX		PIS/PASEP Social Security Contribution		FUNDAF Social Security Contribution		IMPORT TARIFFS	
Unit: R\$ Million (Yearly)				Total Expt Tax Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Revenue	GDP Ratio	Total Imp T Revenue	GDP Ratio
Year	GDP Nominal	Price index(July)	GDP 1998 Prices								
1992	619,493	7 12E-04	714,207,275,995	1.7	0.0000024	4,086.0	0.006	423.0	0.0006	1,596.6	0.002
1993	14,116,170	1 33E-02	738,548,713,073	1.3	0.0000017	4,615.8	0.006	213.0	0.0003	1,734.4	0.002
1994	360,919,362	6 61E-01	740,295,202,610	1.6	0.0000021	5,284.4	0.007	225.7	0.0003	2,358.0	0.003
1995	658,141,237	8 25E-01	820,152,983,684	15.9	0.0000194	6,122.5	0.007	335.9	0.0004	4,910.9	0.006
1996	751,506,870	9 06E-01	842,323,504,305	2.1	0.0000025	7,308.3	0.009	402.1	0.0005	4,227.2	0.005
1997	864,111,026	9 65E-01	879,958,825,868	3.2	0.0000037	7,597.7	0.009	398.7	0.0005	5,138.7	0.006
Average					0.0005312%	Average	0.731%	Average	0.0421%	Average	0.410%

4 Numerical Solution

This section computes the set of sustainable plans S numerically. The objective is two fold: first, to compute S using Chang's original algorithm, but with more realistic parameters. His original parameterization of the model was chosen simply to ensure that the mathematical assumptions [A1]-[A7]⁸ were met, without commitment to a real economy. The original parameterization is:

$$u(c) = 10,000 \log(c)$$

$$f(x) = 64 - (0.2x)^2$$

$$x = m(h - 1)$$

$$v(m) = 40m - \frac{m^2}{2}$$

$$\beta = 0.9$$

$$\Pi = [\underline{\Pi}, \overline{\Pi}] = [0.25, 1.25]$$

The new parameters were obtained from the calibration of the model using Brazilian data. A more ambitious goal was to increase the dimension of the algorithm to include government bonds and production, as in the extended theoretical model. This was proven to be extremely difficult due to limitations of the available computer resources, causing the computation time to increase exponentially with the increase of the size of the grid. However, even though the program was not run with the inclusion of bonds or production as in the expanded model, it is explained below how their introduction to the algorithm would be straightforward. The original algorithm was used using some of the calibrated parameters. The exercise allowed for the discussion of dynamic inconsistency of macroeconomic policies in light of the theory proposed by this model, using as case study the Brazilian economic scenario of the last twenty years. The Brazilian experience was tested in order to verify if according to this model the recent stabilization plans would be sustainable in the long run. Results indicated that in the absence of government bonds or production, only the Real Plan is sustainable in the long run. In spite of the computational difficulties found, the exercise was fruitful because it lay the foundation for the numerical solution of the extended model with the proposed increase in the dimension of the algorithm, which will be pursued in future research

4.1 The algorithm

The structure of the original algorithm is summarized in Figure 4.1. The general idea is that solution of the model is obtained using a grid approach in which the space for the simultaneous solution of the value function and constraints equations is broken into intervals and equilibrium points are searched in each interval. The algorithm is divided into distinct parts. The first is a subroutine that performs some preliminary calculations, which are used later in the program. It begins by computing the grids for money balances, *mgrid*, and nominal money growth, *hgrid*.

⁸ [A1] $u : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ is C^2 , strictly concave, and strictly increasing.

[A2] $v : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ is C^2 , and strictly concave.

[A3] $\lim_{c \rightarrow 0} u'(c) = \lim_{m \rightarrow 0} v'(m) = \infty$

[A4] There is a finite satiation level of money, $m^f > 0$ such that $v'(m^f) = 0$

Next, it computes the set of possible values of the inflation tax, X , and consumption as a function of the deadweight loss due to the inflation tax. It then computes: $u(c)$, $v(m)$, $u'(c)$ and $v'(m)$; the vector of promised marginal utility of money, $tgrid$, obtained from the estimation of an upper bound for $theta$, the highest possible promised marginal utility of money from the FOC, (with no inflation tax and highest money holdings, mf); the discounted lifetime utility, $wgrid$, the non-monetary equilibrium, $w0$, and the equilibrium under the Friedman rule, wf . These are saved for later.

The next block of the algorithm contains a series of subroutines that will compute the set of sustainable plans under no government pre-commitment, the set S . It begins introducing a matrix Z whose dimension is the length of $tgrid \times$ length of $wgrid$, and whose elements correspond to a pair (θ, w) , elements in $tgrid$ and $wgrid$. The program initializes Z as a matrix of “ones” in a parent program that will repeat each set of subroutines several times until Z converges to the set S , which is a subset of Z . Within this program there are two distinct blocks. The first computes the *best* government response function. The key idea is that when considering following an equilibrium recommendation, the government considers only the payoffs associated with the “best” deviation: one that minimizes the welfare losses to society. This is obtained through a series of tests. For each element in $wgrid$ and every possible deviation h , the subroutine tests for the “highest” worst possible deviations, as given by equations (18)-(21) in Chang (1998). Given each deviation h , promised marginal utility of money $theta$, and a desired condition for the “worst” SP, this is computed numerically by testing if the difference between each element in $wgrid$ and in the matrices generated by the set of CE equations computed for the worst possible SP lies within the 10% error tolerance, as in the perfect commitment case. The three matrices generated by this set of equations are: the utility function matrix given by $u(c) + v(m)$, the FOC and $theta$, the artificial constraint matrix. For each deviation h , the corresponding column of each one of these matrices is chosen. Every element in each column is tested for the desired condition: that the maximum value allowed for w (initialized by $wmax$) is the minimum element of the utility matrix. Additionally, the constraint imposed by the artificial state variable $theta$ is tested against $tgrid$: the difference between each element in the column of the artificial constraint $theta$ matrix and $\beta \cdot ith$ -element in $tgrid$ is checked to verify if it lies within the 10% error margin. If yes, a “one” is attributed to an index column vector, and a “zero” if otherwise, indicating as before a CE. Next, the column of minimums is compared with this index vector, and only the elements corresponding to the “ones”, i.e. the CE $thetas$, are chosen. The minimum of these is then chosen as the new maximum w . This value is then compared to a “best” value for w , which is initialized as $wmin$, the lower bound on $wgrid$. If this best value is smaller than the new maximum w computed above, then the new best becomes this value. This procedure is repeated for the next deviation h . Thus, the output of this block is a scalar, which is the best value for w and is used in the next block.

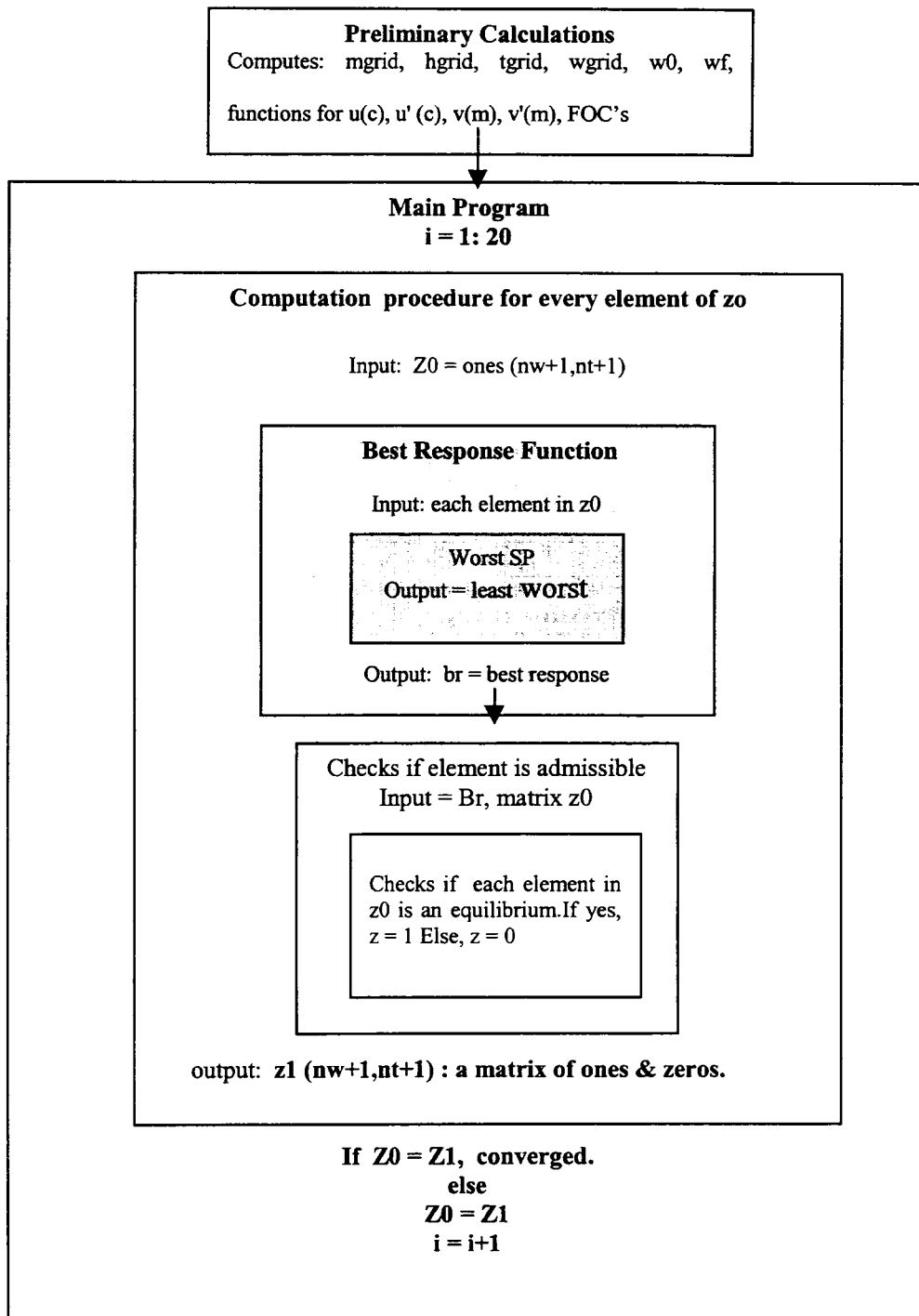
The next block in the algorithm begins with a subroutine to verify if the element in the Z matrix is admissible. It begins by comparing each element in $wgrid$ with the “best response” scalar obtained above. If the element in $wgrid$ is smaller than “best”, then a zero is given, and a one is given if otherwise. Then, it tests every element in the Z matrix with the allowed w s from the step above. Given the CE $theta$ ’s computed above and the admissible elements in $wgrid$, it tests again if the difference between (1) elements in the discounted utility matrix and each $wgrid$ and (2) the difference between the elements in the constraint matrix and $\beta \cdot ith$ - $tgrid$ lie within the

error tolerance. If yes, a “one” is given, and a “zero” otherwise. The result is a matrix of ones and zeros, where as before, ones correspond to equilibrium points. These procedures are repeated until Z converges to the set S .

The main problem with this algorithm is the number of times these subroutines must be repeated, generating endless loops, which become difficult to compute with the available computer technology. The original model, based on Calvo (1978), presents a Sidrauski economy with money and government transfers only, implying only one FOC and implying 2 artificial constraints only. Increasing the dimensionality in the model with the introduction of bonds and/or production is theoretically straightforward but would make the numerical calculations even more difficult, since it would involve more implicit loops. The introduction of government bonds in the budget constraint would involve the introduction of another FOC equation, and in turn 2 more artificial state variables. Likewise, the introduction of production would involve another FOC and 2 more artificial state variables. For each of these artificial state variables there would be a corresponding matrix whose every element would need to be tested as described above. This would also involve an increase the dimension of the original E-grid matrix ($m \times h$) to ($m \times h \times b \dots$) The numerical computations involved in increasing the dimensionality of the model to this extent have proven unfeasible with the available technology. Due to these difficulties, the actual computation of the algorithm was conducted only with the new estimated parameters, leaving the expansion in the dimensionality of the algorithm for future research.

One additional issue needs to be mentioned. Chang’s parameterization of the model reflected the fact that the need for government financing through the inflation tax is minimal, given that the original model has transfers only. The parameter used to reflect such tax burden in original government budget constraint equation is 0.02, suggesting that the deadweight loss due to inflation represents only 2% of transfers. This value seems unrealistic for economies with large government expenditures. This problem was addressed theoretically with the inclusion of government expenditures and a corresponding taxation scheme. For the numerical solution of the extended algorithm the parameter representing the tax burden will have to be adjusted accordingly. This will be addressed in future research.

Figure 4.1 Structure of the Algorithm



4.2 Numerical Results

This section summarizes the numerical results from the computed algorithm above. The numerical computations use the parameters estimated above. These parameters are normalized to match Chang's maximum feasible output in the original parameterization of the model. He chooses the satiation money to be 62% of the maximum feasible output. The m_f obtained from the FOC using Brazilian data from the last 20 was about 2% to 5% of GDP, which seemed too low. This stemmed from the fact that average money holdings in Brazil were low during and after the hyperinflation years. The next step was to calibrate the parameters using Brazilian data for years prior to the hyperinflation history. The M1/GDP ratio found for the 50s and early 60s was around 25% of GDP, which would yield an estimated m_f in the 25% to 35% range using the FOCs of the extended model. Next, US data (for sure, no hyperinflation history there), was checked, and m_f was found to be approximately in the same range, approximately 25% – 30% of GDP. Nowhere the M1/GDP ratio in the 62% range used by Chang in the original parameterization of the model was found. Next, historical data for real interest rates in the same periods were checked for years with no hyperinflation history. The conclusion was that the real interest rates for these years would yield β for the US around 0.97, and for Brazil around 0.95, much higher than the values obtained using the data from the last 20 years. The explanation to these findings is that the economic instability of the last 20 years were giving rise to distortions in the calibration of fundamental parameters such as m_f and β . The appropriate calibration was done using data from years with no hyperinflation history, whose economic performance resembled closer a "steady state" economy.

Once all of this was done, a series of exercises was conducted with the algorithm. The program was run again several times with 1/2 of the original grid: $mgrid$ with 60 intervals, $hgrid$ with 30, $tgrid$ with 30, and $wgrid$ with 29 intervals. The resulting Z matrices, the computed sets sustainable plans, S , were 30×31 matrices of ones and zeros, again, ones meaning equilibrium points corresponding to a pair (θ, w) . Two groups of separate tests, each with a different lower bound on the inverse of nominal money growth, h , were conducted. For the first, $hmin$ was set at the original $hmin$, 0.25, and for the second for the second, $hmin$ was set at 0.03. This was done to avoid ruling out the inverse of money growths observed during the hyperinflation years in Brazil, when nominal money growth rates some periods surpassed 2000% per year, yielding lower bounds for $hmin$ around 0.03. Computations for each of these two scenarios were placed on two separate panels. For each $hmin$, the tests were repeated twelve times: for m_f set at the original 40 (62%Y) level, and at 26(40%Y), 22 (35%Y) and 16 (25%Y); for each different m_f the tests were repeated for β set at the original 0.9, then at 0.95 and 0.97. For each scenario, the upper bound on the promised marginal utility of money, $thetamax$, the minimum possible equilibrium, $wmin$, the nonmonetary equilibrium, $w0$, the equilibrium with the Friedman Rule, wf and the highest possible equilibrium, $wmax$ for all tests were computed. Results from these computations are summarized in Table 6.3.1. Each test yielded a Z matrix corresponding to the computed set S . All sets were graphed and placed on the corresponding h panel.

Some facts were observed from these tests. First, the upper bound on θ , the promised marginal utility of money decreased with a decrease in m_f and increased with the decrease in $hmin$. For each $hmin$ and m_f , $thetamax$ did not change with β . Second, the lower bound on w , $wmin$, decreased with $hmin$, but $w0$, wf remained unchanged. Finally, the decrease in the lower bound on h did not affect the upper bound on w , $wmax$. In summary, lower mfs

yielded lower promises of marginal utility of money and lower $hmins$, i.e., higher money growth, yielded potentially higher promised marginal utility of money. Higher β s yielded higher $wmin$, $w0$, wf and $wmax$.

Given these facts, it was easy to compare if plans sustainable with $hmin$ set at 0.25 remained sustainable with lower $hmins$, higher β s and lower m_s , as well as Chang's conclusions, that the computed sustainable plans lied within the range of $w0$ and wf as in his original findings. These results did not seem to necessarily hold for all cases. This is observed in Panel 3, for which the program was allowed to run with the grid the same size as with the original parameters, but now using the parameters which seemed to encompass the Brazilian experience: $mf = 16$, $\beta = 0.95$, $hmin = 0.03$. Clearly, the computed sustainable plans were spread over a range much broader than this interval, with approximately 75% of the w s lying below the non-monetary equilibrium.

To check for the robustness of Chang's original results, the program was run with the original parameterization, but with a 100% increase in the size of each grid: $mgrid$ with 240 points, $hgrid$ with 100, $tgrid$ with 100 and $wgrid$ with 99 intervals. The first loop of the program took 13 days to run in a Unix server with 1GIG of RAM. The program was allowed to run for 4 weeks, however, the system crashed before it ended.

The algorithm was used to test for some observed facts of the Brazilian economy in the last 20 years, to check if according to this model, the different stabilization attempts would represent sustainable plans. To do this, the promised marginal utility of money and w s under different stabilization plans were computed. The w s were computed assuming that money balances changed in the years encompassed by each stabilization plan, and remained constant at the last value of the last period for the infinite future. Thus, the discounted lifetime utility, w , and the promised marginal utility of money were computed as follows:

Cruzado Plan:

$$w = u(f(x_{87})) + v(m_{87}) + \beta[u(f(x_{88}) + v(m_{88}))] + \beta^2[u(f(x_{89}) + v(m_{89}))] + \frac{\beta^3}{(1-\beta)}[u(f(x_{90}) + v(m_{90}))]$$

$$\theta_{Cruzado} = u'(f(x_{87})) * m_{87}$$

Collor Plan:

$$w = u(f(x_{91})) + v(m_{91}) + \beta[u(f(x_{92}) + v(m_{92}))] + \beta^2[u(f(x_{93}) + v(m_{93}))] + \frac{\beta^3}{(1-\beta)}[u(f(x_{94}) + v(m_{94}))]$$

$$\theta_{Collor} = u'(f(x_{91})) * m_{91}$$

Real Plan:

$$w = u(f(x_{95})) + v(m_{95}) + \beta[u(f(x_{96}) + v(m_{96}))] + \frac{\beta^2}{(1-\beta)}[u(f(x_{97}) + v(m_{97}))]$$

$$\theta_{Real} = u'(f(x_{95})) * m_{95}$$

The resulting pairs (θ, w) for each case was then compared to the set of sustainable plans for Brazil, graphed in Panel 3, in order to verify if each stabilization plan corresponded to a sustainable plan, a “one” on the computed set S . Results are summarized on table 6.3.2. The computed pairs (θ, w) for the Cruzado Plan and the Collor Plan were not a sustainable plans. The Real Plan was sustainable, yielding the highest sustainable promised marginal utility of money of and highest discounted utility of the three. The worst SP of the three was the Collor Plan, yielding both lowest sustainable promised marginal utility of money and lowest discounted utility. The Cruzado Plan yielded a slightly lower sustainable marginal discounted utility of money discounted utility than the Real Plan. All three plans yielded low sustainable promised marginal utility of money, with θ at about 10% of θ_{max} . The computed discounted utilities were roughly in the lower half of w_{grid} . These results are consistent with the rational expectation hypothesis, suggesting a fear of a return to hyperinflation.

Table 4.1			
	θ	w	SP^*
Period			
1987-1990 Cruzado Plan	365.62	361,340	No
1991-1994 Collor Plan	256.09	361,129	No
1995-1997 Real Plan	377.84	361,910	Yes

* Pair is a Sustainable Plan

Table 4.2: Computation Summary

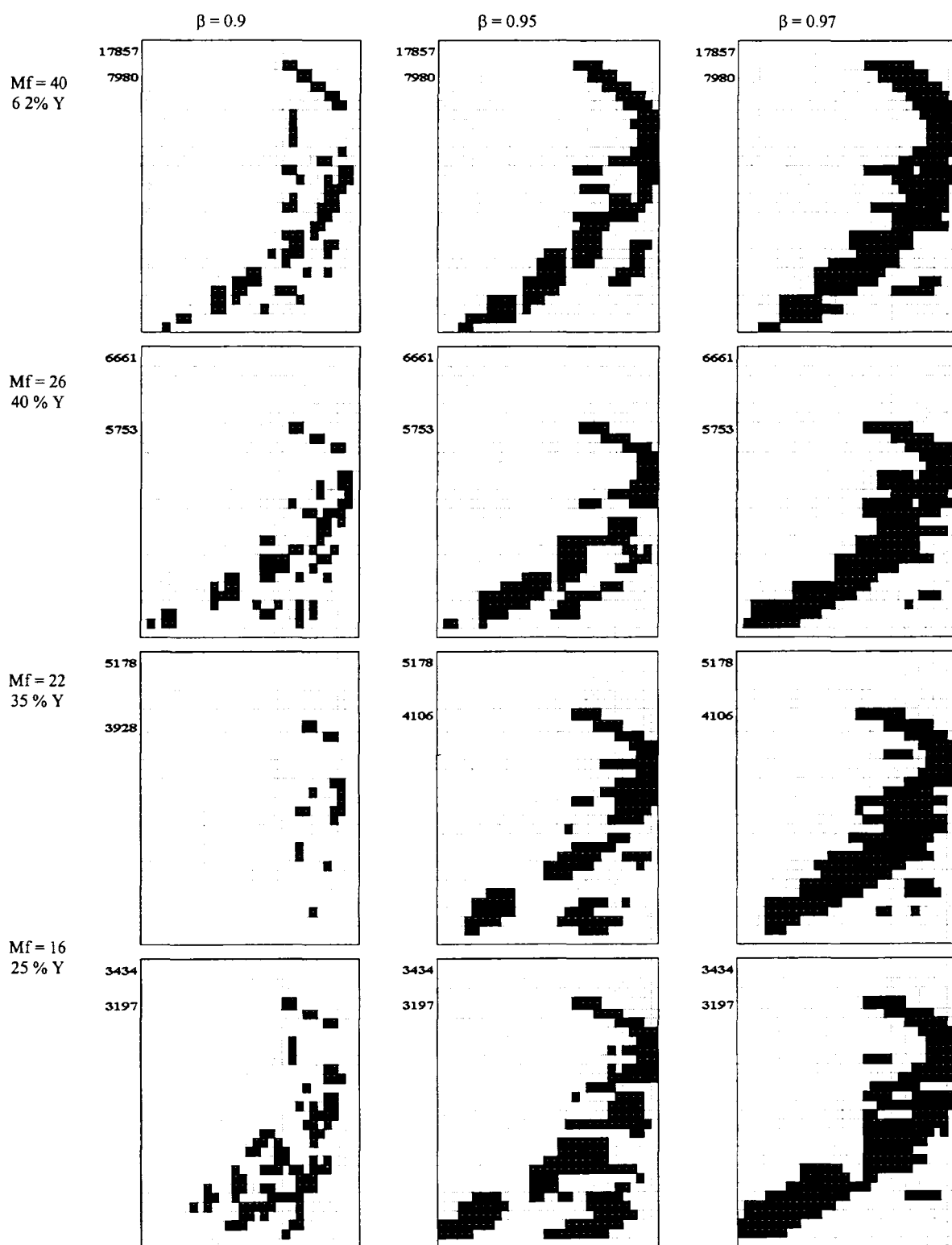
<i>Test</i>	<i>m_f</i>	<i>hmin</i>	β	<i>m0</i>	<i>xmin</i>	<i>Xmax</i>	<i>thetamax*</i>	<i>wmin</i>	<i>w0***</i>	<i>wf****</i>	<i>wmax</i>
1	16	0.25	0.9	0.38	-12	4	3434	180,823	180,677	181,812	181,898
2			0.95	8.19	-12	4	3434	361,646	363,186	363,757	363,796
3			0.97	11.31	-12	4	3434	602,743	605,960	606,305	606,327
4	16	0.03	0.9	0.38	-15	4	3679	180,225	180,677	181,812	181,898
5			0.95	8.19	-15	4	3679	360,450	363,190	363,757	363,796
6			0.97	11.31	-15	4	3679	600,749	605,960	606,305	606,327
7	22	0.25	0.9	6.38	-16.5	5.5	5178	180,934	181,817	182,876	183,038
8			0.95	14.19	-16.5	5.5	5178	361,868	365,466	366,003	366,076
9			0.97	17.31	-16.5	5.5	5178	603,113	609,760	610,085	610,127
10	22	0.03	0.9	6.38	-21.3	5.5	6006	179,645	181,817	182,876	183,038
11			0.95	14.19	-21.3	5.5	6006	359,289	365,466	366,003	366,076
12			0.97	17.31	-21.3	5.5	6006	598,816	609,760	610,085	610,127
13	26	0.25	0.9	10.38	-19.5	6.5	6661	180,965	182,780	183,771	183,998
14			0.95	18.19	-19.5	6.5	6661	361,930	367,386	367,894	367,996
15			0.97	21.31	-19.5	6.5	6661	603,217	612,960	613,268	613,327
16	26	0.03	0.9	10.38	-25.2	6.5	8429	178,921	182,777	183,771	183,998
17			0.95	18.19	-25.2	6.5	8429	357,841	367,386	367,894	367,996
18			0.97	21.31	-25.2	6.5	8429	596,402	612,960	613,268	613,327
19	40	0.25	0.9	24.38	-30	10	17857	179,838	187,400	188,078	188,618
20			0.95	32.19	-30	10	17857	359,675	376,626	376,995	377,236
21			0.97	35.31	-30	10	17857	599,459	628,360	628,588	628,727
22	40	0.03	0.9	24.38	-38.8	10	132191	162,450	187,397	188,078	188,618
23			0.95	32.19	-38.8	10	132191	324,899	376,626	376,995	377,236
24			0.97	35.31	-38.8	10	132191	541,499	628,360	628,588	628,727

* Promised Marginal Utility of Money

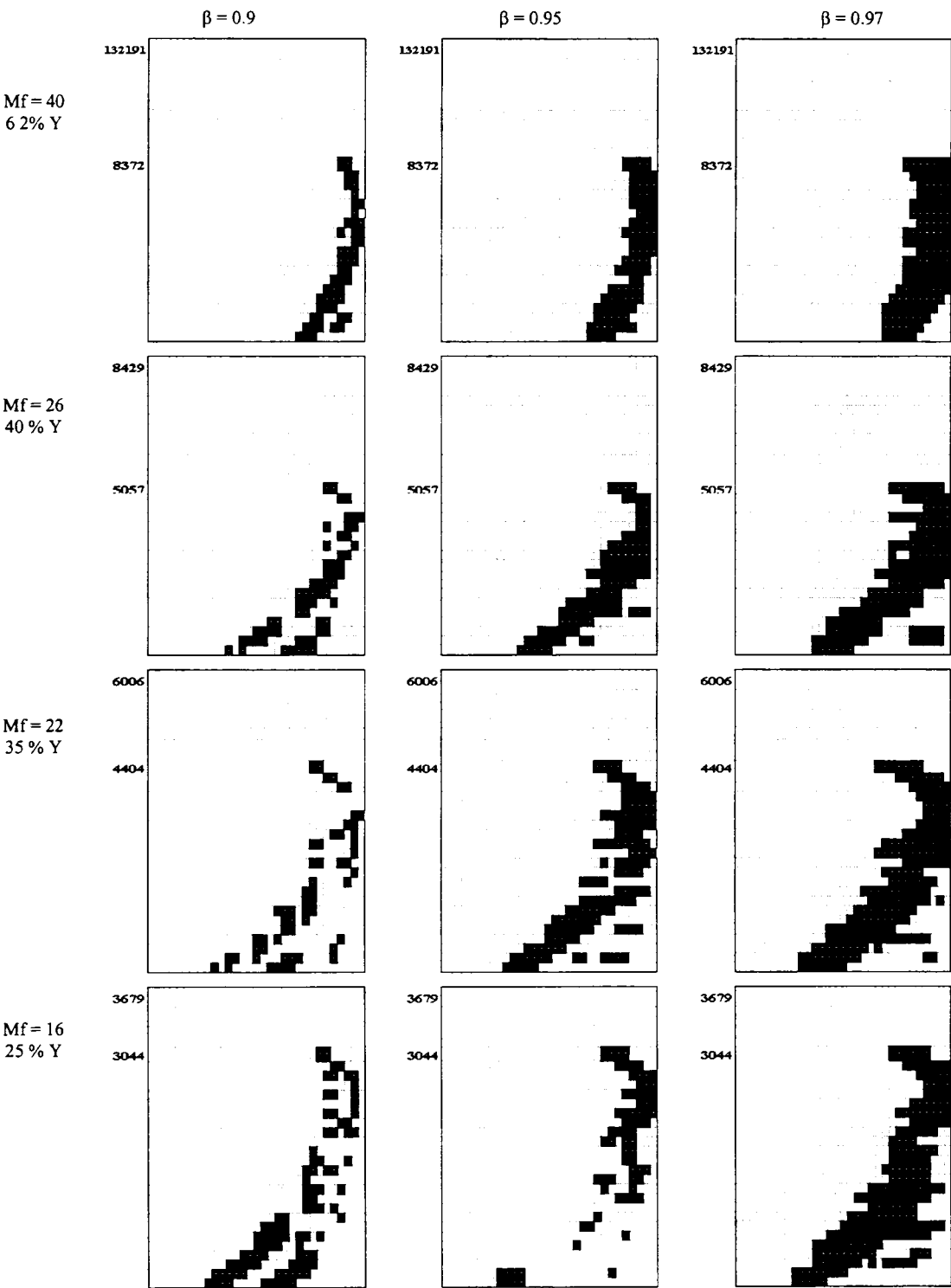
* Non-monetary Equilibrium

* Friedman Rule Equilibrium

Panel 1
 $h_{\min} = 0.25$

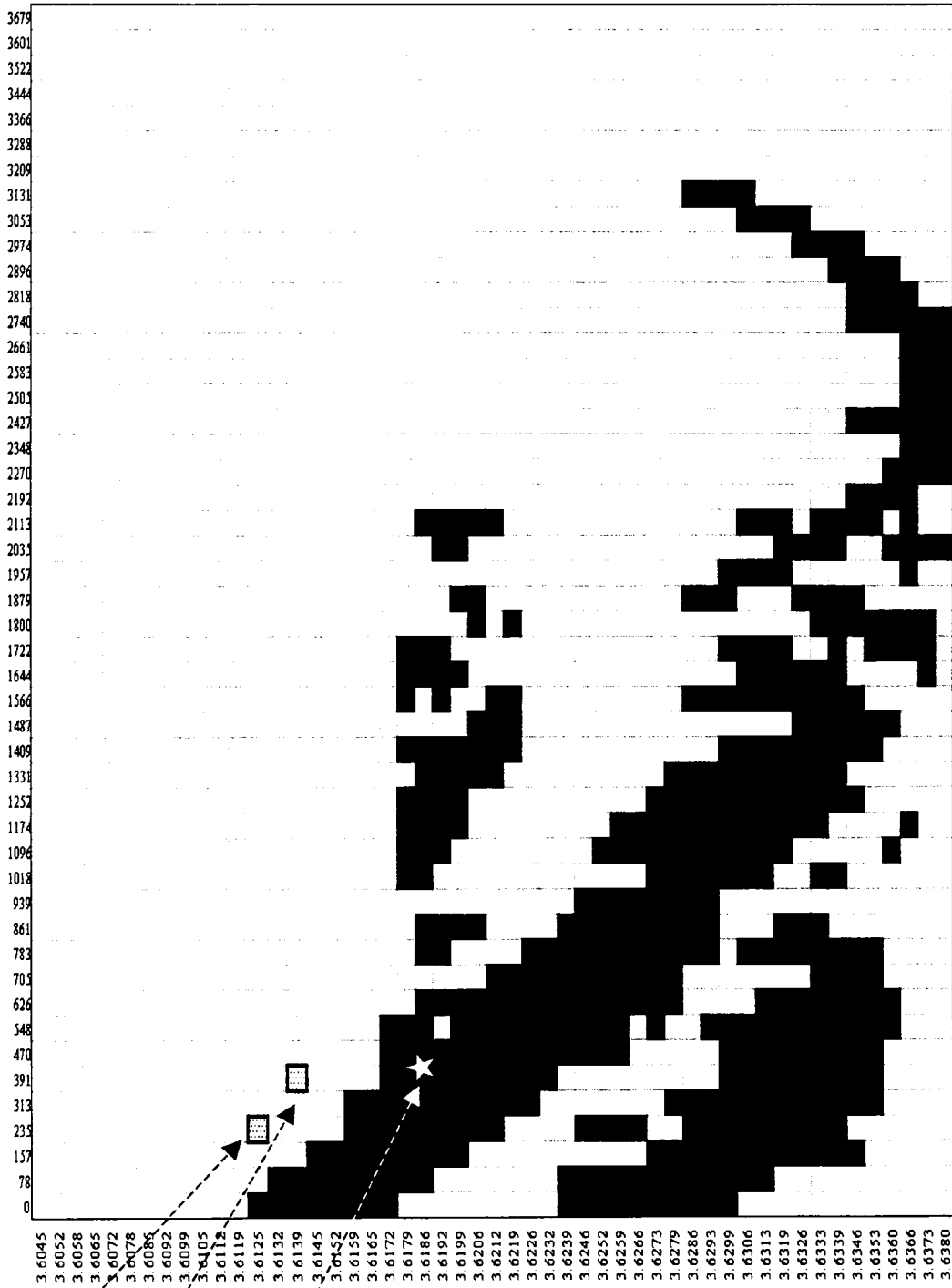


Panel 2
h min = 0.03



Panel 3: Computed S
 $\beta = 0.95$
 $hmin = 0.03$

Computed Θ



A. Computed W^*
 $\times 10^5$
B.

W_0

W_f

Collor Plan Cruzado Plan Real Plan

5 Conclusion

The recursive theoretical model used herein assumes that economies are in a steady state. Brazil for the last 25 years has not been close to a steady state. It has undergone a military take over, a number of heterodox stabilization attempts with price freezes, indexation, hyperinflation, risk of default, which is captured by the real interest rates etc. Thus, appropriate calibration of fundamental parameters of the model could not use the original data from turbulent periods (1980-1997), for which clearly the economy was not in steady state equilibrium. The accurate calibration of parameters that do not depend on policies (β , mf) were calibrated with data from years that would seem closer to a steady state economy, the 50's and 60's, thus eliminating the rational expectations effect, which suggest a fear of a return to hyperinflation. The economic scenario of the turbulent years were studied allowing the lower bound on $hmin$ to include the inverse money growth experienced during the hyperinflation years. The original model proved to work well to explain the Brazilian experience. Macroeconomic policies implementing deviations from the original promises for the Cruzado and the Collor plan were not sustainable in the long run, as predicted by the model. However, the numerical computations failed to obtain the result that the SPs were always comprised between the interval $[w0, wf]$.

The original model was expanded to study dynamic inconsistency of macroeconomic policy when there is a likelihood of implicit default on public debt through inflation. The theoretical expansion of the model was straightforward. However, the numerical computation of the extended model was difficult due to the dimensionality curse in the algorithm, resulting from the limitations of existing computer resources. The original algorithm was calibrated with Brazilian data and the Brazilian experience supported the main predictions of the model. The main contribution of the exercise was to lay the foundation for the calibration of the model and the computation of sustainable plans when increasing the dimension of the algorithm to include government bonds, production and a foreign sector. This goal will be pursued in future research.

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