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"International Integration and Long-Run Persistence of the GNP Distribution"

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International integration and long-run persistence of the GNP distribution (Preliminary version)

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Abstract

We use the Ramsey model of growth elaborated by Bliss [1995] and Ventura [1997] to show how international integration results in long-run persistence of GNPs distribution, while allowing, under certain conditions on parameters, for convergence during the transition. First, we provide relationships which explicitly relate, in the neighborhood of the steady-state, the magnitude of conditional convergence or divergence to the fundamentals of the economies. Second, we present an analysis of the Cobb Douglas case with a broad class of utility functions and show that there is always transitional convergence with this technology. Third, directions for testing the model against the traditional closed-economy setting are proposed. These lead to adding specific and world-wide regressors to traditional growth regressions.

1. Introduction

Absolute convergence, when it exists, appears to be gradually slowing down. Martin [1997] reports that the speed of convergence among European regions fell from 2% to 1.3% in the 80's. Similarly, Barro and Sala-I-Martin [1995] (chap. 11) have evidenced a significant fall in the speed of convergence among Japanese prefectures after 1955.

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Interestingly, this empirical phenomenon is particularly clear in economies which interact strongly with one another. However, the importance of openness and integration has been downplayed or overlooked by the traditional convergence literature (see for example Barro and Sala-I-Martin [1995]). Regions (e.g. US States, Japanese prefectures, Europeans regions, etc.) have all been singled out in this literature because a similar steady state level was a plausible assumption, not because they were integrated. Nevertheless, they *are* integrated, in the sense that we can expect that factor prices are equalized across regions.

The objective of this article is to assess how important economic integration – globalization – is for the convergence of real wealth and real income among regions or countries. How does factor price equalization shape the dynamics of income distribution across regions or countries ? The approach developed in this article is a contribution to a better understanding of the relationship between interdependence, convergence and growth.

We use the neoclassical model that was constructed by Chatterjee [1994] and Caselli and Ventura [1996] to study distributive dynamics among infinitely lived individuals in a particular economy. We follow Bliss [1995] and Ventura [1997], who have applied the model to study inequality among countries. Our basic framework is a general equilibrium growth model, as this is a simple and natural way to deal with interactions among economies. Factor price equalization is achieved without any restrictions and the optimal behavior of Ramsey savers determines the dynamics of wealth and income.

Bliss [1995] has shown that in this setting, globalization promotes long-run income inequality. More precisely, factor price equalization deters convergence so that initial differences in income persist forever. However, Caselli and Ventura [1996] and Ventura [1997] have shown that convergence *may* exist during the transition of the aggregate economy toward its steady-state. In contrast with the closed model, conditional convergence is due to the homothetic properties of intertemporal demand – not diminishing returns. These authors focus on the case where the technology is CES and the instantaneous utility function is logarithmic. Unfortunately, these articles do not indicate exactly how these theoretical results can be confronted with empirical evidence on the distribution of income across regions or countries. There remains an important gap to bridge between these theoretical criticisms and their empirical counterparts. An important preoccupation of the paper is therefore to provide an empirical strategy for testing the effect of integration on convergence.

The findings of this paper are as follows. First, we provide one possible explanation for the deceleration of convergence reported above, as we simultaneously have a model that can display convergence during the transition and long-run persistence. The relationship between the dynamics of cross-section inequality and

aggregate growth is made explicit by a linearization around the steady-state. This local characterization makes it possible to relate the magnitude of the convergence or divergence effect to the parameters describing the fundamentals of economies. Second, we complete some results obtained by Caselli and Ventura [1996] by using a broad class of utility function. More precisely, we show that conditional convergence occurs as soon as technology is Cobb-Douglas and utility is CRRA. Third, we develop testing strategies that are based on the linearized version of the model. A panel data study, which consists to estimate an "extended" system of growth rate regressions with fixed effect, can potentially discriminate between the model of the integrated economy presented below and the traditional model of autarchic economies.

The paper is organized as follows. In the second section, we present the basic setup adapted from Caselli and Ventura [1996]. The third section contains the core results of the article. We detail how the distribution dynamics displays both persistence and conditional convergence or divergence. Section 4 provides a testing strategy. We conclude in the last section.

2. Basic framework

2.1. Structure of the world economy and technology

2.1.1. Static structure

The world economy consists of a collection of countries indexed by their relative labor productivity $\theta \in]0, \theta_{\max}]$. In *per capita* terms, the aggregate national technology is given by:

$$Y(\theta) = f\left[K(\theta), A\theta\right] + A\phi(\theta), \tag{2.1}$$

where $Y(\theta)$ and $K(\theta)$ are respectively the domestic product and the domestic capital stock of country θ, both in *per capita* terms. A is the worldwide level of technological efficiency growing at a constant exogenous rate x and $f(\cdot)$ is a neoclassical production function. $\phi(\theta)$ is a constant parameter, either positive or negative. $\phi(\theta) < 0$ means that a subsistence consumption level absorbs part of the output. $\phi(\theta) > 0$ can be viewed as a fixed rent increasing output, for example some production using only specific factors that country θ alone possesses.

$(\theta, \phi(\theta))$ is therefore time-invariant and characterizes the technology used by country θ. To do away with certain technical difficulties, these quantities are specified in intensive terms[1]. We will use lower-case letters to denote intensive variables: for all *per capita* variable Z, $z = Z/A$.

[1] Ben-David [1998] studies the case in which the subsistence level is not indexed on technological change. Interestingly, endogenous formation of clubs may arise.

We therefore have three sources of country heterogeneity: capital stock $K(\theta)$, labor productivity θ and the parameter $\phi(\theta)$. We note $q(B)$ the number of countries with productivity index $\theta \in B$. We normalize θ and the size of the world economy so that $\int_0^{\theta_{\max}} \theta q(d\theta) = \int_0^{\theta_{\max}} q(d\theta) = 1$. It is assumed that the world population grows at the exogenous rate n and that the population is identical in any country[2]. In this context, A is the average labor productivity in the world economy. We note the average value of the $\phi(\theta)'s$ as $\phi \equiv \int_0^{\theta_{\max}} \phi(\theta) q(d\theta)$. Under the condition $\phi \neq 0$, we then define the relative value of $\varphi(\theta)$ as $\phi_R(\theta) \equiv \phi(\theta)/\phi$.

Before we proceed to integrate the world economy, let us first recall that in autarchy the quantity $f\left[K(\theta), \theta A\right] + \phi(\theta)A$ is both the GNP and the GDP of country θ. From this point on, we assume that the world economy is integrated through perfect capital mobility. It is now necessary to distinguish between the capital owned by a given country, noted $k(\theta)$, and the capital used by that country, noted $\hat{k}(\theta)$. The quantity $k(\theta) - \hat{k}(\theta)$ is accordingly the portion of country $\theta's$ capital installed abroad. The gross rental rate of capital is noted $r + \delta$, with $\delta > 0$ the capital depreciation rate. With an integrated world market for capital, $r + \delta$ must be identical across countries. In each country, competitive firms equate the gross marginal productivity of capital to ~~the world~~ gross rental rate:

$$f_1\left[\hat{k}(\theta), \theta\right] = r + \delta. \tag{2.2}$$

With $f(\cdot)$ homogeneous of degree one, equation (2.2) implies $\hat{k}(\theta) = \theta \hat{k}(1)$. With our normalization, world and average quantities are equal:

$$\hat{k} \equiv \int \hat{k}(\theta) q(d\theta) = \int k(\theta) q(d\theta) \equiv k. \tag{2.3}$$

k is the world capital stock, the average installed capital stock as well as the capital stock installed in the average country. The capital stock installed in θ therefore satisfies:

$$\hat{k}(\theta) = \theta \hat{k} = \theta k. \tag{2.4}$$

We can now write the GNP of country θ as

$$y(\theta) = \theta f(k) + \phi(\theta) + (r + \delta)(k(\theta) - \theta k), \tag{2.5}$$

where $f(k) \equiv f(k, 1)$.

The real wage in country θ satisfies:

$$w(\theta) = \theta f_2(\hat{k}, \theta) = \theta f_2(k, 1) = \theta w. \tag{2.6}$$

[2] This does not imply any loss of generality, while it simplifies the notation. Our results obtain with any distribution of the world population.

Hence, in the integrated economy, factor prices ($r(t)$ and $w(t)$) are identical in any country, for all t.

The world output is:

$$y = \int \left(f\left[\hat{k}(\theta), \theta\right] + \phi(\theta) \right) q(d\theta) = f(k) + \phi.$$

As is well known, the complete integration of the world economy implies instantaneous *conditional* convergence (at the precise moment when the world economy is integrated) of the gross domestic product (adjusted from $\phi(\theta)$):

$$\hat{y}(\theta) - \phi(\theta) = \theta(y - \phi), \text{ for all } \theta. \tag{2.7}$$

In contrast, there is no instantaneous convergence effect affecting the GNP $y(\theta)$. This quantity depends on country $\theta's$ wealth, as is shown in equation (2.5).

2.1.2. Capital mobility and trade

One could argue that this representation of the world economy is irrelevant for analyzing real world dynamics simply because it relies on the unrealistic assumption of perfect capital mobility. One way to address this difficulty is to realize that this is equivalent to assuming international trade in goods. So long as countries do not specialize, trade would still result in complete integration, that is, in a unique set of factor prices in any country. This result is long known to trade theorists. In essence what is traded through goods are the factors embodied in them. It is therefore easy to formally show that the model above is observationally equivalent to a two-sector model (one good by sector) in which goods are traded in the world economy – provided that there is no specialization in any country. Integration explicitly based on trade is presented in Ventura [1997]. In this article, a critical assumption concerning the factor intensity in each sector ensures that complete specialization is impossible.

As a result of this equivalence, most of our findings will be valid in the two economic structures - capital mobility and international trade. We have to point out, however, that the GDP $\hat{y}(\theta)$ has no meaning in an economy without capital mobility. In this case, both GNP and GDP are equal to $y(\theta)$.

2.1.3. Dynamic structure

Time is continuous. The national capital of country θ is driven by:

$$\dot{k}(\theta) = \theta w + rk(\theta) + \phi(\theta) - c(\theta) - (n + x)k(\theta), \quad k(\theta, 0) \text{ given}, \tag{2.8}$$

with $c(\theta)$ the consumption level of country θ.

We rule out Ponzi games by assuming:

$$\lim_{t \to \infty} e^{-R(0,t)} e^{(x+n)t} k(\theta, t) \geq 0, \tag{2.9}$$

where $R(0,t) \equiv \int_0^t r(s)ds$.

Aggregating these equations over countries yields the law of motion of the world capital stock:

$$\dot{k} = f(k) + \phi - (\delta + x + n)k - c, \; k(0) \text{ given.} \tag{2.10}$$

2.2. Households

The representative household of country θ maximizes:

$$U(\theta) = \int_0^\infty \frac{C(\theta, t)^{1-\sigma} - 1}{1 - \sigma} e^{nt} e^{-\rho t} dt, \tag{2.11}$$

subject to constraints (2.8-2.9) and taking the time paths of world prices as given. $C(\theta, t)$ is *per capita* consumption in country θ at time t, $\sigma > 0$ the inverse of the constant intertemporal elasticity of substitution, and $\rho > 0$ the utility discount rate. In contrast with autarchy, the assumption that households maximize intertemporal utility – as opposed to the choice of some exogenous saving rates – is central to the convergence results of the integrated model[3]. This point will become clearer later.

With these preferences, it is possible to interpret $\phi(\theta)$ as a measure of intertemporal flexibility: the larger $\phi(\theta)$, the more flexible is country θ in its intertemporal allocation of consumption. With a negative $\phi(\theta)$, country θ will not be able to substitute consumption through time above a certain level. Conversely, a positive $\phi(\theta)$ means that there is at any time a constant source of output which is by definition completely independent of country θ's time allocation problem and therefore raises its ability to substitute consumption intertemporally. A significant analytical advantage of dealing with an heterogeneity in $\phi(\theta)$ over an heterogeneity in σ – another measure of intertemporal flexibility – is to facilitate aggregation[4]. Note that by simply defining $C_{sg}(\theta) = C(\theta) - \phi(\theta)e^{xt}$ together with technology $y(\theta) = f[k(\theta), \theta]$, it is possible to rewrite the maximization problem above with a standard Stone-Geary intertemporal utility function. This setting is fully equivalent to the one used here, but makes the economic interpretation of a positive $\phi(\theta)$ more difficult.

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[3] It is a well-known fact that the key convergence properties of the Solow model are preserved in a Cass-Koopmans setting: the economy converges to a unique steady-state; the transitional growth rate is decreasing in the stock of capital at any time.

[4] The case of heterogeneity in σ is analyzed in Béraud [1998].

As pointed out by Caselli and Ventura [1996], these preferences thus make it possible to describe the aggregate world economy as a hypothetical representative country endowed with exactly average world characteristics. Aggregate paths are found by solving the usual autarchy problem, which leads to:

$$\dot{c} = c\left[\sigma^{-1}\left(f'(k) - \delta - \rho\right) - x\right], \tag{2.12}$$
$$\dot{k} = f(k) + \phi - c - (n + \delta + x)k, \quad k(0) > 0 \text{ given.} \tag{2.13}$$

This system is appended with the transversality condition, that is, inequality (2.9) taken as an equality. As is well known, we need the following condition on parameters so that this condition always holds in equilibrium:

$$\rho > n + (1 - \sigma)x. \tag{2.14}$$

In the rest of the paper we will find convenient to note the discounted flow of any variable x as: $\tilde{x}(t) \equiv \int_t^{\infty} x(\tau)e^{-R(t,\tau)}e^{(\tau-t)(x+n)}d\tau$.

Because of the homothetic properties of preferences, the consumption of any country θ can be written as a linear function of its total wealth, for all t[5]:

$$c(\theta, t) = \nu(t)a(\theta, t), \tag{2.15}$$

where

$$a(\theta, t) = k(\theta, t) + \theta\tilde{w}(t) + \phi(\theta)\tilde{1}(t), \tag{2.16}$$

and

$$\nu(t) = \left[\int_t^{\infty} e^{R(t,\tau)(1-\sigma)/\sigma - (\tau-t)(\rho/\sigma - n)}d\tau\right]^{-1}. \tag{2.17}$$

The key point in (2.15) is that $\nu(t)$, the propensity to consume out of total wealth, is the same for any country θ, depending only on the aggregate behavior of the world economy. This fact implies that the total wealth of any country grows at a rate given by the world economy alone. A simple expression for that growth rate can be found by taking the time derivative of (2.16)[6] and substituting from (2.8):

$$\frac{\dot{a}(\theta)}{a(\theta)} = r - \nu - n - x. \tag{2.18}$$

This is a crucial feature of the model. In a world where factor prices are equated across countries by factor mobility and/or international trade, and where

[5] Combine the intertemporal budget constraint $\tilde{c}(t) = a(t)$ with the integral version of (2.12).
[6] Notice that $\dot{\tilde{x}} = (r - n - x)\tilde{x} - x$.

preferences are homothetic (in total wealth), all countries accumulate total wealth at precisely the same rate – regardless of relative levels at any time. Remember that $a(\theta)$ is national wealth, which is the sort of wealth that matters in a world where countries may export capital – if we want to be able to interpret the integration as capital mobility. This obviously is an important departure from the vision of the world economy as a collection of neoclassical closed economies. In that sort of economy, poor countries always grow faster that rich ones, both in terms of total wealth and in terms of capital. We will now turn to the growth rates of national stocks of capital in the integrated economy.

3. Transitional convergence and long-run persistence

3.1. Transitional convergence

An interesting form of the law of motion of country θ's capital obtains by substituting (2.15) into (2.8):

$$\dot{k}(\theta) = (r - \nu - x - n)k(\theta) + (w - \nu\tilde{w})\theta + (1 - \nu\tilde{1})\phi(\theta). \tag{3.1}$$

Let $h(\theta) \equiv k(\theta)/k$ denote the relative capital of country θ. Equation (3.1) and its aggregate version then imply:

$$\dot{h}(\theta) = -(\psi_1 + \psi_2)h(\theta) + \psi_1\theta + \psi_2\phi_R(\theta), \tag{3.2}$$

where ψ_1 and ψ_2 are two measures of convergence defined as

$$\psi_1 \equiv \frac{w - \nu\tilde{w}}{k} \text{ and } \psi_2 \equiv \phi\frac{1 - \nu\tilde{1}}{k}.$$

Note that both ψ_1 and ψ_2 are invariant across countries. Relationship (3.2) shows how the distribution of financial wealth changes over time. This expression is interesting because the three sources of heterogeneity are clearly isolated. What do we learn? First, note that the value of $\psi \equiv \psi_1 + \psi_2$ determines the extent of the *conditional convergence effect*. Imagine that countries do not differ in their fundamentals θ and $\phi(\theta)$. Then, from (3.2), it can be seen that the distribution of financial wealth will shrink if ψ is positive and will expand if ψ is negative. ψ is thus the instantaneous speed of conditional convergence. This speed applies to any country of the world economy.

Second, *absolute convergence* can also be analyzed through equation (3.2). These dynamics can be visualized with figure 3.1. Let the position $(h(\theta), \theta, \phi_R(\theta))$ of a country θ be represented as indicated in this diagram. Assume that the fundamentals θ and $\phi_R(\theta)$ are located on a vertical straight line. Assume further

Figure 3.1:

that there is some heterogeneity in the fundamentals of the world economy, so that $\theta, \phi_R(\theta) \neq 1$. It can be seen from figure 3.1 that there is absolute convergence if and only if $h(\theta)$ approaches the value 1 on the vertical straight line $(\theta, \phi_R(\theta))$. Equation (3.2) informs us that $h(\theta)$ approaches the segment $[\theta, \phi_R(\theta)]$ if and only if ψ_1 and ψ_2 are positive.

Others situations can be described by noting that the two straight lines $(h(\theta), \theta)$ and $(h(\theta), \phi_R(\theta))$ divide the plan in four areas. One can see that the distributions of θ and $\phi_R(\theta)$ are repulsive or attractive depending on the sign of ψ_1 and ψ_2.

For a particular country, the occurrence of absolute catching-up depends on *both* the sign of ψ_1 and ψ_2 and the position of its fundamentals on the vertical line.

Knowing the time paths of the world-wide values ψ_1 and ψ_2, one will know how countries move toward their long-run position. In the subsection 3.3, we study how these coefficients change in the neighborhood of the world steady-state. The occurrence of local conditional convergence or divergence depends on the parameters describing the fundamentals of the economies. In the subsection 3.4, we study the Cobb-Douglas case and show for a broad class of utility functions that conditional convergence holds globally during the transition. But this is by no means a general result of the model. Caselli and Ventura [1996] have shown that the model with a CES production function can display transitional divergence or even successive periods of convergence and divergence, a phenomenon reminiscent of a Kuznets curve.

In fact, in this economy, a key determinant of transitional convergence is the elasticity of substitution between factors. Consider (2.16) and (2.18): regardless of

the share of financial wealth in its total wealth, any country will find it optimal to accumulate the latter at the same rate. Think of a high elasticity of substitution. As the world economy accumulates capital, the demand for labor will tend to be relatively low because the economy will increasingly substitute capital for labor. So will be the discounted flow of wages, a component of total wealth ("human" wealth). Finally, one sees that to keep total wealth on an optimal path, countries poorly endowed with financial wealth will need to accumulate it at a quicker rate than the financially rich. An alternative expression for ψ illustrates this point. From (2.18):

$$\psi = \frac{\dot{k}}{k} - \frac{\dot{a}}{a}. \tag{3.3}$$

This relationship shows that if, *in aggregate terms*, the growth rate of capital is higher than that of total wealth, there will be transitional conditional convergence.

Another point is worth emphasizing here. The important underlying mechanism for transitional convergence is the homothetic property of household preferences, not diminishing returns to capital like in the autarchic model. In this economy, transitional divergence in capital stocks is plausible even though technology is one with diminishing returns.

3.2. Long-run persistence of the cross-section distribution

Integrating (3.2) yields:

$$h(\theta,t) = \lambda_1(0,t)h(\theta,0) + \lambda_2(0,t)\theta + \lambda_3(0,t)\phi_R(\theta). \tag{3.4}$$

where

$$\lambda_1(0,t) = \exp[-\int_0^t \psi(s)ds], \tag{3.5}$$

$$\lambda_2(0,t) = \int_0^t \psi_1(s)\lambda_1(s,t)ds, \tag{3.6}$$

$$\lambda_3(0,t) = \int_0^t \psi_2(s)\lambda_1(s,t)ds. \tag{3.7}$$

What is interesting in this relationship is that, again, $\lambda_1(0,t)$, $\lambda_2(0,t)$ and $\lambda_3(0,t)$ are independent of θ. This equation therefore provides a very transparent decomposition of the distribution of financial wealth at any point in time, making explicit the respective contributions of the initial financial wealth distribution, the distribution of labor productivity and the distribution of the $\phi(\theta)'s$. Not surprisingly, given the construction of (3.4), the $\lambda_i's$ sum up to unity, for all (t,t')[7]:

$$\lambda_1(t,t') + \lambda_2(t,t') + \lambda_3(t,t') = 1. \tag{3.8}$$

[7] One way to do this is to sum (3.4) over the $\theta's$.

The variable $\lambda_1(t,t')$ is a measure of cumulated conditional convergence between t and t', as opposed to $\psi(t')$, which may be viewed as the instantaneous conditional convergence at instant t'. It can be seen by direct examination of (3.4) taken between t and t' that if $\lambda_1(t,t')$ is less that unity, then there will be cumulated conditional convergence over that period of time.

What does (3.4) tell us on the asymptotic distribution of financial wealth? In the long run the distribution reads:

$$h(\theta,\infty) = \lambda_1(0,\infty)h(\theta,0) + \lambda_2(0,\infty)\theta + \lambda_3(0,\infty)\phi_R(\theta). \qquad (3.9)$$

There are no particular reasons why $\lambda_1(0,\infty)$ should be equal to zero. Evidently, the distribution of long-run financial wealth will be also influenced by the other two sources of heterogeneity, respectively in θ and $\phi(\theta)$. But the important fact is that we have a situation of long-run persistence of the financial wealth distribution. In other words, there may well be transitional convergence – or divergence –, as was shown in section 3.1, but in all cases these dynamics will come to an halt as the world economy proceeds towards a steady-state. This result is in line with the one obtained by Bliss [1995].

Note that long-run persistence of initial distributions applies in exactly the same way to consumption: just consider (2.15) and (2.16). Income distribution dynamics are slightly more complex because the shares of respectively capital and labor incomes in total income may vary over time. It is nonetheless not difficult to show that long-run persistence also applies. Defining the share of capital income in total income $\alpha(t) \equiv r(t)k(t)/y(t)$ and the share of labor income as $\beta(t) \equiv w(t)/y(t)$, an expression for the relative income of country θ, $y_R(\theta,t) \equiv y(\theta,t)/y(t)$, can be found by substitution into (3.4):

$$y_R(\theta,t') = \gamma_1(t,t')y_R(\theta,t) + \gamma_2(t,t')\theta + \gamma_3(t,t')\phi_R(\theta), \qquad (3.10)$$

where

$$\gamma_1(t,t') = \frac{\alpha(t')}{\alpha(t)}\lambda_1(t,t'), \qquad (3.11)$$

$$\gamma_2(t,t') = \beta(t') + \alpha(t')\lambda_2(t,t') - \frac{\alpha(t')}{\alpha(t)}\beta(t)\lambda_1(t,t'). \qquad (3.12)$$

$$\gamma_3(t,t') = 1 - \gamma_1(t,t') - \gamma_2(t,t'). \qquad (3.13)$$

A different way of writing equation (3.10) is to make explicit the income "target" of country θ:

$$y_R(\theta,t') = \gamma_1(t,t')y_R(\theta,t) + (1-\gamma_1(t,t'))y^{\star}_R(\theta,[t,t']), \qquad (3.14)$$

where the income target $y_R^\star(\theta,[t,t'])$ is defined as:

$$y_R^\star(\theta,[t,t']) = \frac{\gamma_2(t,t')}{\gamma_2(t,t')+\gamma_3(t,t')}\theta + \frac{\gamma_3(t,t')}{\gamma_2(t,t')+\gamma_3(t,t')}\phi_R(\theta). \tag{3.15}$$

On the one hand, these results are reminiscent of the autarchic model, in the sense that the dynamics is still guided by a gap between the current value of the variables (e.g. relative income $y_R(t)$) and some long run "target". On the other hand, observe that in the integrated economy, this target is time-varying and, more importantly, can never be attained. The model contrasts sharply with the long-run behavior of economies in the autarchic model. One key result of that setting is that any economy converges to a unique steady-state level of wealth and income – conditional on structural parameters such the labor productivity and preferences. This process will eventually iron out initial income differences. Here, on the contrary, the effects of initial wealth will be felt forever. Each country will reach its own particular steady-state level of financial wealth and income – even if conditioned on structural parameters. Put differently, there are no stationary wealth and income distributions.

It is important, however, to remember that these results obtain in a utility-maximizing environment. In the absence of any externality, any country in this model always achieves a higher intertemporal utility level in an integrated world economy, even if its steady-state wealth and consumption levels would have been higher in autarchy.

3.3. Characterization around the world steady-state

One way to understand how the distribution of financial wealth changes as the world economy grows towards its steady-state is to linearize equation (3.4) around the aggregate steady-state. To do this, we need to study the coefficients $\lambda_i(t,t')$, $i=1,2,3$, or alternatively $\psi(t),\psi_1(t)$ and $\psi_2(t)$, around the steady-state.

Note that for this purpose it is easiest "to eliminate time" and to work with aggregate quantities as functions of the capital stock k, not t. A well-known example of this are the policy rules $c(k), a(k), w(k)$,etc. Similarly, we define $\lambda_1(k,k')$ as the value taken by the coefficient λ_1 when the world economy starting from k moves to k'. In the same fashion, we can define $\lambda_i(k,k')$, $i=2,3$, $\psi_i(k)$, $i=1,2$ and $\psi(k)$.

Let $k^\star$ denote the steady state aggregate level of capital. We want to show how the coefficients λ_i, $i=1,2,3$, ψ_i, $i=1,2$ and ψ behave when the aggregate economy converges toward its steady-state. This requires a local study of the functions $\lambda_i(k,k')$, $\psi_i(k)$ and $\psi(k)$ around $k^\star$.

First, these functions satisfy: $\lambda_1(k^\star, k^\star) = 1$, $\lambda_2(k^\star, k^\star) = \lambda_3(k^\star, k^\star) = 0$, and $\psi_1(k^\star) = \psi_2(k^\star) = \psi(k^\star) = 0$. Second,we define $\eta_i^\star$, $i = 1, 2, 3$ as the semi-elasticities of the function $\lambda_i(k, k^\star), i = 1, 2, 3$ with respect to k, evaluated at $k = k^\star$:

$$\eta_i^\star \equiv k^\star \left(\frac{\partial \lambda_i(k, k^\star)}{\partial k} \right)_{k=k^\star}, \quad i = 1, 2, 3. \tag{3.16}$$

Taking the first order Taylor expansion of $\lambda_i(k, k')$ near the steady-state yields:

$$\lambda_i(k, k^\star) \approx \lambda_i(k^\star, k^\star) + \eta_i^\star \left[\frac{k - k^\star}{k^\star} \right], \quad i = 1, 2, 3. \tag{3.17}$$

Moreover, since the coefficients $\lambda_i, i = 1, 2, 3$ sum up to unity, we have:

$$\eta_1^\star + \eta_2^\star + \eta_3^\star = 0. \tag{3.18}$$

Assuming that the initial world capital stock $k(0)$ is not too far off $k^\star$, and neglecting the second order terms, the long-run distribution of relative financial wealth is given by:

$$h(\theta, \infty) = \left[1 - \eta_1^\star \left(\frac{k^\star}{k(0)} - 1 \right) \right] h(\theta, 0) - \eta_2^\star \left(\frac{k^\star}{k(0)} - 1 \right) \theta - \eta_3^\star \left(\frac{k^\star}{k(0)} - 1 \right) \phi_R(\theta). \tag{3.19}$$

This equation can be interpreted as follows. Imagine that the world economy is below its steady-state ($k_0 < k^\star$), so that the world economy experiences an episode of growth at a cumulated rate $k^\star/k(0) - 1$. Then, the initial distribution of the $h(\theta, 0)'s$ shrinks or expands – depending on the sign of $\eta_1^\star$. If $\eta_1^\star > 0$, aggregate growth implies a phenomenon of conditional convergence (in relative terms). Relative productivity level θ has a positive influence on long-run relative wealth as long as $\eta_2^\star < 0$. The influence of $\phi(\theta)$ depends on the sign of $\eta_3^\star$.

In Appendices B and C, we show that the coefficients $\eta_i^\star, i = 1, 2, 3$ can be expressed as functions of the parameters of the model:

$$\eta_1^\star = 1 - \frac{k^\star}{c^\star} \left[\rho + \sigma(x + \mu) - (n + x) \right], \tag{3.20}$$

$$\eta_2^\star = \frac{k^\star}{c^\star} \sigma \mu - \frac{w^\star}{c^\star}, \tag{3.21}$$

$$\eta_3^\star = -\frac{\phi}{c^\star}, \tag{3.22}$$

where $\mu > 0$ is the speed of convergence of the world economy toward its steady-state (see Appendix A).

Moreover, we show in the appendices that:

$$\psi'(k^\star) = -\mu\frac{\eta_1^\star}{k^\star}, \quad \psi_1'(k^\star) = \mu\frac{\eta_2^\star}{k^\star}, \quad \psi_2'(k^\star) = \mu\frac{\eta_3^\star}{k^\star}. \tag{3.23}$$

These equations provide a full characterization of the cross-section dynamics around the steady-state. In particular, we see how the parameters describing world preferences and technology influence the distribution of wealth across countries.

One of the most interesting results is the negative relationship between μ, the world-wide speed of convergence, and $\eta_1^\star$, a measure of the conditional convergence effect. We see that a slow adjustment of the world economy translates into a low long-run persistence of the distribution of financial wealth. Ventura [1997] identified this property of the model, but without quantifying it. Knowing that ϵ, the elasticity of substitution of the production function, and σ, the inverse of the intertemporal elasticity of substitution, both impact negatively on μ, one can conclude that the size of the long-run persistence effect decreases with these parameters. By contrast, this size increases with ϕ, the average level of the $\phi(\theta)$'s.

From (3.20), one can see that the most realistic configuration of the parameters leads to a conditional convergence effect[8], *i.e.*, that the world steady-state is located on the decreasing part of an hypothetical Kuznets curve. A conditional divergence effect would take place only for high values of μ and σ. Also, the influence of $\phi(\theta)$ on its long-run wealth is always positive. The influence of the relative productivity level θ depends on the sign of $\eta_2^\star$. Some kind of substitution effect between financial and human wealths cannot be ruled out. Indeed, $\eta_2^\star > 0$ would mean that countries poorly endowed in terms of θ make up for their low labor productivity by accumulating financial wealth at a faster rate. It could be the case for high values of σ and μ[9].

Starting from (3.10), we can use the same analysis to study the behavior of the distribution of relative gross income $y_R(\theta)$. Appendix D focuses on the determination of the semi-elasticities $\omega_i, i = 1,2,3$ of the coefficients γ_i. The results obtained for the distribution of relative wealth $h(\theta)$ must be amended so as to take into account the fact that respective factor shares in total income may be variable around the aggregate steady state. This exercise clarifies the influence of a high elasticity of substitution on the income distribution. First, a high value of ϵ tends to depress the growth of wages and hence favors conditional convergence of relative wealth, as was noted above. But we have a second effect here. A high value of ϵ also implies a strong decrease of the share of wage in total income

[8] [At this point, it is important to remind that $\rho + \sigma x$ is the net rate of interest.]

[9] Remember that the $\eta_i^\star$ relate to one another by (3.18). Consequently, conditional divergence implies some substitution effect.

when the world converges toward its steady-state. This second effect weakens conditional convergence.

3.4. The Cobb-Douglas case

We now turn to some global properties of the model. Caselli and Ventura [1996] have established the convergence property of the Cobb-Douglas economy with logarithmic instantaneous utility. Using a different type of proof, we will show that this property generalizes to the CRRA specification. Remember, also, that our preference are equivalent to standard Stone-Geary preferences.

We now specialize the production function as

$$f(k) = k^{\alpha} \text{ with } 0 < \alpha < 1. \tag{3.24}$$

We further restrict the $\phi(\theta)'s$ to be interpreted as country-specific production only, that is, for all θ: $\phi(\theta) \geq 0$. It is then possible to show that in this case, under some mild condition on parameters, there is conditional convergence at any time during the transition. To see this, expand $\psi(t)$ as

$$\psi(t) = \frac{\int_t^{\infty} \left[\left((w(t) + \phi) \right) e^{R(t,\tau)(1-\sigma)/\sigma - (\tau - t)(\rho/\sigma - n)} - (w(\tau) + \phi)\, e^{-R(t,\tau)} \right] d\tau}{\nu^{-1}(t) k(t)}.$$

We show by *reductio ad absurdum* that $\psi(t) \geq 0$. Suppose $\psi(t) < 0$. This implies that there exists at least one time interval $[\tau_a, \tau_b[\in [t, \infty[$ such that for all $\tau \in [\tau_a, \tau_b[$, $w(\tau) + \phi > (w(t) + \phi) e^{R(t,\tau)/\sigma - (\tau - t)(\rho/\sigma - n)}$. This in turn implies that there exist at least another time interval $[\tau_c, \tau_d[\in [t, \tau_a[$ such that for all $\tau \in [\tau_c, \tau_d[$, $\frac{\dot{w}(\tau)}{w(\tau)+\phi} > \frac{1}{\sigma}\left[r(\tau) - \rho \right] + x = \frac{\dot{c}(\tau)}{c(\tau)}$. This contradicts the fact that, under the condition

$$\alpha(x + n + \delta)/(\rho + \sigma x + \delta) \leq 1/\sigma, \tag{3.25}$$

a Cobb-Douglas technology implies $\frac{\dot{w}(\tau)}{w(\tau)} \leq \frac{\dot{c}(\tau)}{c(\tau)}$ for all τ,[10] which is turns implies $\frac{\dot{w}(\tau)}{w(\tau)+\phi} \leq \frac{\dot{c}(\tau)}{c(\tau)}$ for all $\phi \geq 0$. We conclude that under (3.25) we have $\psi(t) \geq 0$ for all t, that is, there is conditional convergence for all t.

Notice that condition (3.25) is not particularly strong. It is satisfied in most calibrations of the model in autarchy, for example those of Barro and Sala-I-Martin [1995].

This result is interesting because of the empirical relevance of the Cobb-Douglas production function. Also, the Cobb-Douglas case illustrates how the

[10] This result has been established by Barro and Sala-I-Martin [1995], page 89. Under condition (3.25), they show that the consumption/output ratio always grows, which leads to $\frac{\dot{c}(\tau)}{c(\tau)} \geq \frac{\dot{w}(\tau)}{w(\tau)}$ for all τ.

elasticity of substitution between factors impacts on conditional convergence. The elasticity is here exactly equal to unity, which is an upper borderline case for a neoclassical production function. A higher setting would violate the Inada conditions and result in perpetual growth, as was established by Jones and Manuelli [1990].

4. Strategies for empirical testing

In this section, we give directions along which the above results could be tested, leaving actual estimations to further research. A key preoccupation is to focus on the discriminating predictions of the model – particularly, on what distinguishes the integrated world economy from a collection of autarchic countries. A prediction of the model which clearly possesses this discriminating property is the long-run persistence of initial conditions, in other words, that convergence, when it exists, will never be complete. To simplify our exposition in this section, we assume that θ is the only source of heterogeneity in the fundamentals. More precisely: $\phi(\theta) = 0$ for all θ.

4.1. Integrated economy vs autarchic economies

A natural route to explore the difference between autarchic and integrated economies is to express the distribution of relative incomes as a function of past relative incomes and relative fundamentals. In the integrated economy, relationship (3.14) is available. By contrast, such expression does not exist in the autarchic case as the dynamics of world wide income depends on the distribution of capital across countries.

A way to solve this difficulty is to linearize the autarchic dynamics around the steady state, as the distribution of capital among countries is uniquely determined near the steady state. For any country θ, we have:

$$y(\theta, t') - y^{\star}(\theta) = e^{-\mu(t'-t)} \left(y(\theta, t) - y^{\star}(\theta) \right), \tag{4.1}$$

with μ the speed of convergence of a country toward its steady-state and $y^{\star}(\theta) = \theta y^{\star}$ the steady-state income of country θ. Note that we used the same symbol μ for the speed of convergence because such speed is exactly equal to the μ characterizing the aggregate dynamics of the integrated economy.

From this, we obtain:

$$y_R(\theta, t') = e^{-\mu(t'-t)} y_R(\theta, t) + \left(1 - e^{-\mu(t'-t)}\right) \theta. \tag{4.2}$$

The integrated-economy analogue to this expression, with $\phi(\theta) = 0$, comes directly from (3.14):

$$y_R(\theta, t') = \gamma_1(t, t') y_R(\theta, t) + (1 - \gamma_1(t, t'))\theta. \tag{4.3}$$

The comparison (4.2) and (4.3) is straightforward. The term $e^{-\mu(t'-t)}$ will tend to zero as t grows, whereas its integrated-economy counterpart, the quantity $\gamma_1(t, t')$, will not. Initial conditions have a persistent effect in the integrated economy. This persistence phenomenon does not exist in the autarchic economy and convergence will always translate into complete catching-up.

Another way of comparing the respective predictions of the two models is to look at growth rates. To do this, we now have to use the linearized expression of $\gamma_1(t, t')$ (see Appendix D). We therefore assume that the world economy is not too far from a steady state. From (4.2), an approximate expression for the distance between national growth rates and aggregate growth rates in the autarchic model is:

$$g(\theta, t, t') - g(t, t') = \left(1 - e^{-\mu(t'-t)}\right)\left[\frac{\theta}{y_R(\theta, t)} - 1\right], \tag{4.4}$$

where[11]

$$g(\theta, t, t') \equiv \frac{y(\theta, t') - y(\theta, t)}{y(\theta, t)} \text{ and } g(t, t') \equiv \frac{y(t') - y(t)}{y(t)}.$$

A linearized version of (4.3) provides an equivalent expression in the integrated economy[12]:

$$g(\theta, t, t') - g(t, t') = \left(1 - e^{-\mu(t'-t)}\right)\omega_1^\star\left(\frac{y^\star}{y(t)} - 1\right)\left[\frac{\theta}{y_R(\theta, t)} - 1\right]. \tag{4.5}$$

The comparison between (4.4) and (4.5) is again quite transparent. The behavior of an integrated economy differs from its autarchic counterpart in one key aspect: the term $\omega_1^\star\left(y^\star/y(t) - 1\right)$, which is absent from equation (4.4) can be viewed as an additional "convergence deceleration" term: as the *aggregate* economy proceeds toward its steady state, the deviation of the growth rate of country θ from the mean is clearly annihilated faster in the integrated economy than it is in the autarchic model.

[11] In the above expression and in the rest of the section, we use the approximation

$$g(\theta, t, t') - g(t, t') = \frac{y_R(\theta, t') - y_R(\theta, t)}{y_R(\theta, t)}.$$

[12] This expression has been obtained using the fact that $1 - \gamma_1(t, t') \approx \omega_1^\star\left(\frac{y(t')}{y(t)} - 1\right) \approx (1 - e^{-\mu(t'-t)})\omega_1^\star\left(\frac{y^\star}{y(t)} - 1\right)$.

4.2. A modified growth rate regression

It is possible to construct a test of the model by including both equations (4.4) and (4.5) in the following regression:

$$g(\theta,t,t') - g(t,t') = a \times \left\{\left(1 - e^{-\mu(t'-t)}\right)\left[\frac{\theta}{y_R(\theta,t)} - 1\right]\right\}$$
$$+ \quad (1-a) \times \left\{\omega_1^\star\left(1 - e^{-\mu(t'-t)}\right)\left(\frac{y^\star}{y(t)} - 1\right)\left[\frac{\theta}{y_R(\theta,t)} - 1\right]\right\}. \qquad (4.6)$$

The value $a = 1$ would corroborate the autarchic model while the integrated economy would be excepted to yield the value $a = 0$. Developing this formula gives:

$$\begin{aligned} g(\theta,t,t') - g(t,t') &= \left(1 - e^{-\mu(t'-t)}\right) \times \theta\left(a - (1-a)\omega_1^\star\right)\frac{1}{y_R(\theta,t)} \\ &+ \left(1 - e^{-\mu(t'-t)}\right) \times \theta\left((1-a)\omega_1^\star y^\star\right)\frac{1}{y(\theta,t)} \\ &- \left(1 - e^{-\mu(t'-t)}\right) \times \left((1-a)\omega_1^\star y^\star\right)\frac{1}{y(t)} \\ &+ \left(1 - e^{-\mu(t'-t)}\right) \times \left((1-a)\omega_1^\star - a\right). \end{aligned}$$

This implies the following reduced form:

$$g(\theta,t,t') - g(t,t') = \theta\alpha_1\frac{1}{y_R(\theta,t)} + \theta\alpha_2\frac{1}{y(\theta,t)} + \alpha_3\frac{1}{y(t)} + \alpha_4. \qquad (4.7)$$

In addition, we know that, in both the autarchic and the integrated models, the world-wide cumulated rate of growth satisfies the following reduced form:

$$g(t,t') = \beta_1\frac{1}{y(t)} + \beta_2 \qquad (4.8)$$

where $\beta_1 = (1 - e^{-\mu(t'-t)})y^\star$ and $\beta_2 = -(1 - e^{-\mu(t'-t)})$.

Equations (4.7-4.8) form a system of "augmented" growth rate regressions. Namely, in equation (4.7), only two additional regressors, $(y(\theta,t))^{-1}$and $(y(t))^{-1}$, account for the relationship between growth in the aggregate and dynamics of inequalities across nations. These equations have to be estimated using panel data.

A first approach to the empirical testing of the model is to verify that coefficients α_2 and α_3 are significantly different from zero. A second and more structural approach would provide estimates of a and $\omega_1^\star$. The estimation of a will refine the test on α_2 and α_3, while the estimation of $\omega_1^\star$ would provide information on whether there is convergence or divergence of GNPs.

5. Conclusion

This model provides a theoretical explanation for the deceleration of convergence among integrated economies. When interactions are taken into account in a Ramsey model of economic growth, initial conditions durably influence long-run GNPs. Since it is possible to simultaneously have convergence during the transition, convergence will necessarily slow down as the world economy proceeds towards its steady state. Using panel data analysis, our characterization makes it possible to confront the model with empirical evidence.

This analysis highlights the importance of interactions among economies for understanding patterns of convergence. We have shown that, in the long run, the influence of integration may run contrary to the traditional notion of "equalizing exchange". Again, it is important to remember that in this intertemporal utility setting and without any sort of externality, the counterpart of the long-run persistence result is an improvement of dynamic utility for all countries.

In addition to the actual testing of the model, two directions can be outlined for further research on integration and growth. First, less than perfect integration should be analyzed, for example through adjustment costs or investment irreversibility, as in Vellutini [1997]. Note that the persistence result may be affected by the presence of liquidity constraints as introduced by Huggett [1997]. These extensions could obviously bring additional realism to the model, especially when applied to the world economy. Second, the result of long-run GNP persistence could be offset by mechanisms of technology diffusion. This is clearly a very important extension of the model, as shown by the recent literature on the subject, for example Basu and Weil [1998] who explore the idea of gradual technology diffusion through a mechanisms of "appropriate technology", or Eeckhout and Jovanovic [1998] who show how non-trivial external effects give rise to inequality in productivity.

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Technical appendices

A. The world economy around its steady-state

Around the steady-state $(c^\star, k^\star)$, the world economy dynamics is characterized by the Jacobian matrix:

$$J = \begin{bmatrix} 0 & c^\star\sigma^{-1}f''(k^\star) \\ -1 & f'(k^\star) - (\delta + n + x) \end{bmatrix}. \tag{A.1}$$

The characteristic polynomial associated to J is:

$$P(\mu) = \mu^2 - (\rho + \sigma x - (n + x))\mu + c^\star\sigma^{-1}f''(k^\star). \tag{A.2}$$

It is well known that $P(\mu)$ has two roots of opposite signs $\mu_1 > 0$ and $\mu_2 < 0$. Moreover, we know that $\mu_1 + \mu_2 = \mathrm{Tr}(J) = \rho + \sigma x - (n + x)$. $\mu = |\mu_2| > 0$ is the *speed of convergence* of the world economy toward its steady-state. As $\mu_1 \times \mu_2 = \mathrm{Det}(J) = c^\star\sigma^{-1}f''(k^\star)$, we obtain:

$$\mu(\mu + \rho + \sigma x - (n + x)) = -c^\star\sigma^{-1}f''(k^\star). \tag{A.3}$$

μ can be expressed as a function of the parameters of worldwide preferences and technology. Define ϵ and α as representing respectively the elasticity of substitution between factors of the world technology and the share of capital income in total income[13]:

$$\epsilon \equiv -\frac{f'(k)(f(k) + \phi - kf'(k))}{k(f(k) + \phi)f''(k)}, \quad \alpha \equiv \frac{kf'(k)}{f(k) + \phi}. \tag{A.4}$$

Let $r^\star = \rho + \sigma x$ be the (net) interest rate in he steady-state. The world speed of adjustment is then given by:

$$\mu = \frac{1}{2}\left[-(r^\star - (n + x)) + \sqrt{(r^\star - (n + x))^2 + \frac{4}{\epsilon\sigma}(r^\star + \delta)(1 - \alpha)\frac{c^\star}{k^\star}}\right]. \tag{A.5}$$

Moreover, with our definitions, the ratio $c^\star/k^\star$ satisfies:

$$\frac{c^\star}{k^\star} = \frac{r^\star + \delta}{\alpha} - (\delta + n + x).$$

It is useful to study the local properties of the policy rule $c(k)$ associated to the competitive world dynamics. We eliminate time in the differential system (2.12,2.13) so that:

$$c'(k) = \frac{\sigma^{-1}(f'(k) - \delta - \rho)c(k) - xc(k)}{f(k) + \phi - c(k) - (\delta + x + n)k}. \tag{A.6}$$

[13] Including ϕ in these expressions facilitates forthcoming algebrical manipulations.

Since 0/0 is indeterminate, (A.6) is uninformative on the value of $c'(k^\star)$. However, by applying L'Hôpital's rule, it is easy to show that $c'(k^\star)$ solves $P(\mu) = 0$ with $P(\mu)$ the characteristic polynomial associated to J. Consequently, there are two candidates for $c'(k^\star)$: the two eigenvalues μ_1 and μ_2, corresponding respectively to the stable and unstable arms of the phase diagram. It is easy to see that the stable arm is increasing in the (k, c) plan so that:

$$c'(k^\star) = \mu_1 = \mu + \rho + \sigma x - (n + x), \tag{A.7}$$

is the relevant solution.

B. Properties of $\lambda_1(k, k^\star)$ and $\psi(k)$ around the steady-state

$\lambda_1(t, t')$ can alternatively be expressed in terms of the aggregate capital stocks $k(t)$ and $k(t')$. Substituting (3.3) into (3.5) provides a convenient expression:

$$\lambda_1(k, k') = \frac{a(k')/a(k)}{k'/k} \tag{B.1}$$

From this, we have

$$\eta_1^\star = 1 - (k^\star/a^\star)a'(k^\star). \tag{B.2}$$

We need to characterize the policy rule $a(k)$ near the steady-state $k^\star$. Knowing that $\dot{a} = (r - n - x)a - c$ and using the expression of the aggregate dynamic system, we have:

$$a'(k) = \frac{(f'(k) - \delta - n - x)a(k) - c(k)}{f(k) + \phi - c(k) - (\delta + n + x)k}.$$

Again, by applying L'Hôpital's rule to this ratio of functions, we see that $a'(k^\star)$ satisfies:

$$-c'(k^\star)a'(k^\star) - a^\star f''(k^\star) + c'(k^\star) = 0,$$

so that:

$$a'(k^\star) = 1 - a^\star \frac{f''(k^\star)}{c'(k^\star)}.$$

Further, knowing that $c^\star/a^\star = \rho + \sigma x - (n + x) = \nu^\star$, we get:

$$a'(k^\star) = 1 + \frac{\sigma\mu}{\rho + \sigma x - (n + x)}. \tag{B.3}$$

From this last expression, we conclude that:

$$\eta_1^\star = 1 - \frac{k^\star}{c^\star}\left[\rho + \sigma(x + \mu) - (n + x)\right]. \tag{B.4}$$

The instantaneous speed of conditional convergence $\psi(k)$ relates to the function $\lambda_1(k,k')$ through the expression:

$$\frac{\partial \log \lambda_1(k,k')}{\partial k} = \frac{\psi(k)}{f(k)+\phi-c(k)-(n+\delta+x)k}.$$

In $k=k^\star$, we apply the L'Hôpital's rule once more, using (A.7) to show:

$$\left(\frac{\partial \log \lambda_1(k,k^\star)}{\partial k}\right)_{k=k^\star} = \frac{\eta_1(k^\star)}{k^\star} = \frac{\psi'(k^\star)}{-\mu}.$$

Finally:

$$\psi'(k^\star) = -\mu\frac{\eta_1(k^\star)}{k^\star}. \tag{B.5}$$

C. Properties of $\lambda_3(k,k^\star)$ and $\psi_2(k)$ around the steady-state

Remember that:

$$\lambda_3(t,t') = \int_t^{t'} \psi_2(s)\lambda_1(s,t')ds.$$

Making the change of variable $t \rightarrow k$, we obtain:

$$\lambda_3(k,k') = \int_k^{k'} \frac{\psi_2(u)\lambda_1(u,k')}{f(u)+\phi-c(u)-(\delta+n+x)u}du.$$

By differentiating this equality with respect to k:

$$\lambda_3'(k) \equiv \frac{\partial \lambda_3(k,k^\star)}{\partial k} = \frac{-\psi_2(k)\lambda_1(k,k^\star)}{f(k)+\phi-c(k)-(\delta+n+x)k}.$$

Knowing that $\lambda_1(k^\star,k^\star)=1$ and $\psi_2(k^\star)=0$, we apply the L'Hôpital rule to show that:

$$\lambda_3'(k^\star) = \frac{\psi_2'(k^\star)}{\mu}. \tag{C.1}$$

We now turn to the evaluation of the derivative $\psi_2'(k^\star)$. The function $\psi_2(k)$ satisfies $\psi_2(k) = \phi[1-\nu(k)\tilde{1}(k)]/k$. Differentiating this expression with respect to k yields:

$$\psi_2'(k) = -\phi\frac{k\left[\nu'(k)\tilde{1}(k)+\nu(k)\tilde{1}'(k)\right]+(1-\nu(k)\tilde{1}(k))}{k^2}.$$

This expression has to be evaluated at $k=k^\star$. First, we need $\nu'(k^\star)$. By definition, $\nu(k)=c(k)/a(k)$ so that:

$$\frac{\nu'(k)}{\nu(k)} = \frac{c'(k)}{c(k)} - \frac{a'(k)}{a(k)}.$$

It is easy to check that $\nu^\star = \nu(k^\star) = \rho + \sigma x - (x+n)$. Knowing (A.7) and (B.3), one gets:

$$\nu'(k^\star) = -\frac{1}{a^\star}\mu(1-\sigma), \tag{C.2}$$

with μ the speed of adjustment of the world economy.

Second, we need $\tilde{\mathrm{I}}'(k^\star)$. Using the expression:

$$\tilde{\mathrm{I}}(t) = \int_t^\infty e^{-\int_t^\tau (r(s)-n-x)ds} d\tau,$$

By making the change of variable $t \to k$:

$$\tilde{\mathrm{I}}'(k) = \frac{(f'(k)-n-x-\delta)\tilde{\mathrm{I}}(k) - 1}{f(k)+\phi-c(k)-(\delta+x+n)k}.$$

For $k = k^\star$, we apply the L'Hôpital rule to this expression:

$$\tilde{\mathrm{I}}'(k^\star) = \frac{f''(k^\star)\tilde{\mathrm{I}}(k^\star) + \tilde{\mathrm{I}}'(k^\star)(f'(k^\star)-\delta-x-n)}{f'(k^\star)-c'(k^\star)-(\delta+x+n)k}.$$

Knowing that $\tilde{\mathrm{I}}(k^\star) = (\rho+\sigma x-(x+n))^{-1}$ and that the denominator is equal to μ, one can see that:

$$\tilde{\mathrm{I}}'(k^\star) = \frac{\sigma\mu}{c^\star(\rho+\sigma x-(n+x))}. \tag{C.3}$$

Using equations (C.1) (C.2) and (C.3), we see that:

$$\psi_2'(k^\star) = -\frac{\mu\phi}{k^\star c^\star}. \tag{C.4}$$

Knowing that $\lambda_3'(k^\star) = \psi_2'(k^\star)/\mu$ and $\eta_3^\star = k^\star\lambda_3'(k^\star)$, we finally obtain:

$$\eta_3^\star = -\phi/c^\star. \tag{C.5}$$

D. Properties of $\gamma_i(k,k')$

Let $\gamma_1(k,k')$, $\gamma_2(k,k')$ and $\gamma_3(k,k')$ be functions of capital stock, corresponding to the coefficients $\gamma_i, i = 1,2,3$ of equation (3.10). The objective of this section is to find closed-form expressions for the semi-elasticities $\omega_i^\star, i = 1,2,3$ of these functions around the world steady-state.

From (3.11) we have:

$$\omega_1^\star = \eta_1^\star - \frac{k^\star\alpha'(k^\star)}{\alpha^\star}; \tag{D.1}$$

with $\alpha(k) = kf'(k)/(f(k)+\phi)$. The elasticity of α around the steady-state can be computed using definitions (A.4). We obtain:

$$\frac{k^\star \alpha'(k^\star)}{\alpha^\star} = (1-\alpha^\star)(1-1/\epsilon^\star).$$

This equation, together with (B.4), implies:

$$\omega_1^\star = 1 - \frac{k^\star}{c^\star}\left[\rho + \sigma(x+\mu) - (n+x)\right] - (1-\alpha^\star)(1-(1/\epsilon^\star)). \tag{D.2}$$

Note that when $\epsilon^\star = 1$, which corresponds to the Cobb-Douglas case with $\phi = 0$, $\omega_1^\star$ is equal to $\eta_1^\star$.

Despite its simplicity, an inconvenient of equation (D.2) is that it is not homogeneous in the parameters describing the world fundamentals. For instance, μ depends on $\epsilon^\star$. One way to circumvent this difficulty is to use (A.3) to show that:

$$\frac{k^\star \alpha'(k^\star)}{\alpha^\star} = 1 - \frac{k^\star(\rho+\sigma x+\delta)}{c^\star + (n+x+\delta)k^\star} - \frac{k^\star}{c^\star}\frac{\sigma\mu(\mu+\rho+\sigma x-(n+x))}{\rho+\sigma x+\delta}.$$

By combining this expression with (B.4), we obtain a homogenous expression for $\omega_1^\star$. However, this expression is complex. It simplifies in the case $\delta = n = x = 0$:

$$\omega_1^\star = \frac{k^\star}{c^\star}\frac{\sigma\mu^2}{\rho}, \tag{D.3}$$

which is always positive. This means that in the absence of depreciation, demographic growth and technological progress there is always conditional convergence in gross income around the steady state.

The expression of $\omega_3^\star$ can be obtained from (3.13). It is easy to show that:

$$\omega_3^\star = \alpha^\star\frac{\phi}{c^\star} - (1-\alpha^\star-\beta^\star)\left[\omega_1^\star - \alpha^\star\right], \tag{D.4}$$

where $\beta^\star = w^\star/y^\star$ is the share of wages in total income.

Knowing that the $\gamma_i's$ sum up to unity, we have $\omega_2^\star = -\omega_1^\star - \omega_3^\star$. Hence:

$$\omega_2^\star = -\alpha^\star\frac{\phi}{c^\star} + (\alpha^\star+\beta^\star)\omega_1^\star - \alpha^\star(1-\alpha^\star-\beta^\star). \tag{D.5}$$

For $\phi = 0$, this reduces to $\omega_2^\star = -\omega_1^\star$.